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Design of Machine Elements

Third Edition

About the Author

V B Bhandari retired as Professor and Head, Department of Mechanical Engineering at Vishwakarma Institute of Technology, Pune. He holds First-Class BE and ME degrees in Mechanical Engineering from Pune University, and his teaching experience spans over 38 years in Government Colleges of Engineering at Pune, Karad and Aurangabad. He was also a postgraduate teacher of Pune University, Shivaji University and Marathwada University. Besides being a National Scholar, he has received five prizes from Pune University during his academic career.

Professor Bhandari was a member of 'Board of Studies in Mechanical Engineering' and a member of 'Faculty of Engineering' of Pune University.

He is a Fellow of Institution of Engineers (India), a Fellow of Institution of Mechanical Engineers (India) and a Senior Member of Computer Society of India. He was a Fellow of Institution of Production Engineers (India) and a Member of American Society of Mechanical Engineers (USA).

He has presented and published twenty technical papers in national and international conferences and journals, and is also the author of *Introduction to Machine Design* published by Tata McGraw Hill Education Private Limited.

Contents

Preface

It was really a pleasure to receive an overwhelming response to the textbook *Design of Machine Elements* since it was published first in 1994. In fact, whenever I visit an engineering college in any part of the country, students and staff members of the Mechanical Engineering Department know me as the 'Machine Design author' and the book has become my identity.

Machine design occupies a prominent position in the curriculum of Mechanical Engineering. It consists of applications of scientific principles, technical information and innovative ideas for the development of a new or improved machine. The task of a machine designer has never been easy, since he has to consider a number of factors, which are not always compatible with the present-day technology. In the context of today's technical and social climate, the designer's task has become increasingly difficult. Today's designer is required to account for many factors and considerations that are almost impossible for one individual to be thoroughly conversant with. At the same time, he cannot afford to play a role of something like that of a music director. He must have a special competence of his own and a reasonable knowledge of other 'instruments.'

New to this Edition

After the publication of the second edition in 2007, it was observed that there was a need to incorporate a broader coverage of topics in the textbook to suit the content of 'Machine Design' syllabi of various universities in our country. One complete chapter on **'Design of Engine Components'** (Chapter 25) and half a chapter on **'Design of Riveted Joints'** (Chapter 8) are added to fulfill this requirement. *Design of Engine Components* includes cylinders, pistons, connecting rods, crankshafts and valve-gear mechanism. *Design of Riveted Joints* includes strength equations, eccentrically loaded joints and riveted joints in boiler shells.

Another important feature of the current edition is changing the style of solutions to numerical examples. A 'step-by-step' approach is incorporated in all solved examples of the book. This will further simplify and clarify the understanding of the examples.

Target Audience

This book is intended to serve as a textbook for all the courses in Machine Design. It covers the syllabi of all universities, technical boards and professional examining bodies such as Institute of Engineers in the country. It is also useful for the preparation of competitive examinations like UPSC and GATE.

This textbook is particularly written for the students of the Indian subcontinent, who find it difficult to adopt the textbooks written by foreign authors.

Salient Features

The main features of the book are the following:

(i) SI system of units used throughout the book

(ii) Indian standards used throughout the book for materials, tolerances, screw threads, springs, gears, wire ropes and pressure vessels

(iii) The basic procedure for selection of a machine component from the manufacturer's catalogue discussed with a particular reference to Indian products
(iv) Step by step approach of problem solving

Organization

The book is divided into 25 chapters. **Chapter 1** is an introductory chapter on machine design and discusses the various procedures, requirements, design methods and ergonomic considerations for design. **Chapter 2** is on engineering materials and describes the different kinds of irons, steels and alloys used in engineering design. **Chapter 3** explains in detail the manufacturing considerations in design. **Chapters 4 and 5** discuss the various procedures for design against static load and fluctuating load correspondingly.

Chapter 6 describes power screws in detail while **chapters 7 and 8** specify the features and varieties of threaded joints, and welded and riveted joints in that order. Similarly, **chapters 9 to 22** are each devoted to a particular design element, that is, shafts, keys and couplings; springs; friction clutches; brakes; belt drives; chain drives; rolling contact bearings; sliding contact bearings; spur gears; helical gears; bevel gears; worm gears; flywheel; cylinders and pressure vessels respectively.

Chapter 23 describes miscellaneous machine elements like oil seals, wire ropes, rope sheaves and drums. **Chapter 24** details the various statistical considerations in design. Finally, **Chapter 25** explains the design of IC engine components.

Web Resources

The readers should note that there is a website of this textbook which can be accessed at http://www.mhhe.com/bhandari/dme3e that contains the following.

For Instructors:
(i) Solution Manual
(ii) Power Point Lecture Slides

For Students:
(i) Interactive 643 Objective Type Questions
(ii) 803 Short Answer Questions
(iii) Glossary
(iv) Bibliography

The above additional information will be useful for students in preparing for competitive examinations.

Acknowledgements

For this textbook, information has been collected from various sources such as textbooks, handbooks, catalogues and journals. I would like to express my gratitude to the authors, publishers and firms who have permitted me to use their valuable material in this textbook. The following firms and individuals need special mention:

1. **A A Raimondi** of Westinghouse Electric Corporation, USA for the data on 'Dimensionless Performance Parameters of Hydrodynamic Bearings'
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6. **The Dunlop Rubber Co. (India) Ltd.**, Kolkata, for their permission to include data for the 'Selection of Dunlop belts'
7. **The Bureau of Indian Standards**, New Delhi, for its permission to include extracts from standards-IS-4218, IS-7008, IS-919, IS-1570, IS-2644, IS- 733, IS-2403, IS-3681, IS-2266, IS-3973, IS-5129, IS-4454, IS-4694, IS-210, IS-1030, IS-617, IS-813, IS-25, IS-2825, IS-2365, IS-2494 and IS-7443

I acknowledge with a deep sense of gratitude, the encouragement and inspiration received from my students, readers and teachers. I would also like to thank the following reviewers of this edition whose names are given below.

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A special thanks to the Editorial and Production teams of Tata McGraw-Hill headed by Vibha Mahajan and her enthusiastic team members—Shalini Jha, Suman Sen, Devshree Lohchab, Sohini Mukherjee and P L Pandita.

Feedback

Suggestions and comments for improvement of the book will be appreciated. They can be sent either to the publisher or to me at bhandariprof@gmail.com .

V B BHANDARI

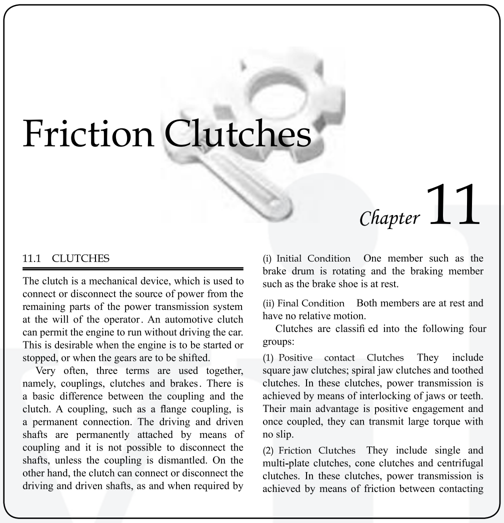
Publisher's Note

Do you have a feature request? A suggestion? We are always open to new ideas (the best ideas come from you!). You may send your comments to *tmh.mechfeedback@gmail.com* (kindly mention the title and author name in the subject line). Piracy-related issues may also be reported.

Visual Walkthrough

Introduction

Each chapter begins with an Introduction of the Machine Element designed in the chapter and its functions. This helps the reader in gaining an overview of the machine element.



Friction Clutches

Chapter 11

11.1 CLUTCHES

The clutch is a mechanical device, which is used to connect or disconnect the source of power from the remaining parts of the power transmission system at the will of the operator. An automotive clutch can permit the engine to run without driving the car. This is desirable when the engine is to be started or stopped, or when the gears are to be shifted.

Very often, three terms are used together, namely, couplings, clutches and brakes. There is a basic difference between the coupling and the clutch. A coupling, such as a flange coupling, is a permanent connection. The driving and driven shafts are permanently attached by means of coupling and it is not possible to disconnect the shafts, unless the coupling is dismantled. On the other hand, the clutch can connect or disconnect the driving and driven shafts, as and when required by

(i) Initial Condition One member such as the brake drum is rotating and the braking member such as the brake shoe is at rest.

(ii) Final Condition Both members are at rest and have no relative motion.

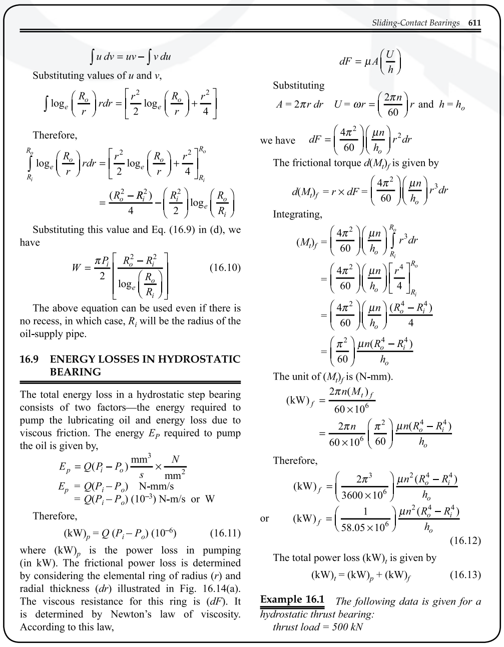
Clutches are classifi ed into the following four groups:

(1) Positive contact Clutches They include square jaw clutches; spiral jaw clutches and toothed clutches. In these clutches, power transmission is achieved by means of interlocking of jaws or teeth. Their main advantage is positive engagement and once coupled, they can transmit large torque with no slip.

(2) Friction Clutches They include single and multi-plate clutches, cone clutches and centrifugal clutches. In these clutches, power transmission is achieved by means of friction between contacting

Theoretical Considerations

Basic equations for design are derived from first principle, with a step-by-step approach.



Sliding-Contact Bearings 611

$$\int u\,dv = uv - \int v\,du$$

Substituting values of u and v,

$$\int \log_e\left(\frac{R_o}{r}\right) r\,dr = \left[\frac{r^2}{2}\log_e\left(\frac{R_o}{r}\right) + \frac{r^2}{4}\right]$$

Therefore,

$$\int_{R_i}^{R_o} \log_e\left(\frac{R_o}{r}\right) r\,dr = \left[\frac{r^2}{2}\log_e\left(\frac{R_o}{r}\right) + \frac{r^2}{4}\right]_{R_i}^{R_o} = \frac{(R_o^2 - R_i^2)}{4} - \left(\frac{R_i^2}{2}\right)\log_e\left(\frac{R_o}{R_i}\right)$$

Substituting this value and Eq. (16.9) in (d), we have

$$W = \frac{\pi P_i}{2}\left[\frac{R_o^2 - R_i^2}{\log_e\left(\frac{R_o}{R_i}\right)}\right] \qquad (16.10)$$

The above equation can be used even if there is no recess, in which case, R_i will be the radius of the oil-supply pipe.

16.9 ENERGY LOSSES IN HYDROSTATIC BEARING

The total energy loss in a hydrostatic step bearing consists of two factors—the energy required to pump the lubricating oil and energy loss due to viscous friction. The energy E_P required to pump the oil is given by,

$$E_p = Q(P_i - P_o)\frac{\text{mm}^3}{s} \times \frac{N}{\text{mm}^2}$$

$$E_p = Q(P_i - P_o) \text{ N-mm/s} = Q(P_i - P_o)(10^{-3}) \text{ N-m/s or W}$$

Therefore,

$$(\text{kW})_p = Q(P_i - P_o)(10^{-6}) \qquad (16.11)$$

where $(\text{kW})_p$ is the power loss in pumping (in kW). The frictional power loss is determined by considering the elemental ring of radius (r) and radial thickness (dr) illustrated in Fig. 16.14(a). The viscous resistance for this ring is (dF). It is determined by Newton's law of viscosity. According to this law,

$$dF = \mu A\left(\frac{U}{h}\right)$$

Substituting

$$A = 2\pi r\,dr \quad U = \omega r = \left(\frac{2\pi n}{60}\right)r \text{ and } h = h_o$$

we have $dF = \left(\frac{4\pi^2}{60}\right)\left(\frac{\mu n}{h_o}\right)r^2\,dr$

The frictional torque $d(M_t)_f$ is given by

$$d(M_t)_f = r \times dF = \left(\frac{4\pi^2}{60}\right)\left(\frac{\mu n}{h_o}\right)r^3\,dr$$

Integrating,

$$(M_t)_f = \left(\frac{4\pi^2}{60}\right)\left(\frac{\mu n}{h_o}\right)\int_{R_i}^{R_o} r^3\,dr = \left(\frac{4\pi^2}{60}\right)\left(\frac{\mu n}{h_o}\right)\left[\frac{r^4}{4}\right]_{R_i}^{R_o} = \left(\frac{4\pi^2}{60}\right)\left(\frac{\mu n}{h_o}\right)\frac{(R_o^4 - R_i^4)}{4} = \left(\frac{\pi^2}{60}\right)\frac{\mu n(R_o^4 - R_i^4)}{h_o}$$

The unit of $(M_t)_f$ is (N-mm).

$$(\text{kW})_f = \frac{2\pi n(M_t)_f}{60 \times 10^6} = \frac{2\pi n}{60 \times 10^6}\left(\frac{\pi^2}{60}\right)\frac{\mu n(R_o^4 - R_i^4)}{h_o}$$

Therefore,

$$(\text{kW})_f = \left(\frac{2\pi^3}{3600 \times 10^6}\right)\frac{\mu n^2(R_o^4 - R_i^4)}{h_o}$$

or

$$(\text{kW})_f = \left(\frac{1}{58.05 \times 10^6}\right)\frac{\mu n^2(R_o^4 - R_i^4)}{h_o} \qquad (16.12)$$

The total power loss $(\text{kW})_t$ is given by

$$(\text{kW})_t = (\text{kW})_p + (\text{kW})_f \qquad (16.13)$$

Example 16.1 *The following data is given for a hydrostatic thrust bearing:*

thrust load = 500 kN

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Properties of Materials

Exhaustive tables are provided from Indian Standards for Mechanical Properties of Engineering Materials.

of excellent corrosion resistance, it is used for pipelines in chemical and petroleum industries.

Mechanical properties of grey[2], malleable[3] and ductile[4] cast iron are given in Table 2.1.

Table 2.1 *Mechanical properties of cast iron*

Grade	*Tensile strength (Min.) (N/mm²)*	*Elongation (Min.) (%)*	*Hardness (HB)*
(A) Grey cast iron			
FG 150	150	–	130–180
FG 200	200	–	160–220
FG 220	220	–	180–220
FG 260	260	–	180–230
FG 300	300	–	180–230
FG 350	350	–	207–241
FG 400	400	–	207–270
(B) Whiteheart malleable cast iron			
WM 400	400	5	220 (Max.)
WM 350	350	3	230 (Max.)
(C) Blackheart malleable cast iron			
BM 350	350	10	150 (Max.)
BM 320	320	12	150 (Max.)
BM 300	300	6	150 (Max.)
(D) Pearlitic malleable cast iron			
PM 700	700	2	240–290
PM 600	600	3	200–250
PM 550	550	4	180–230
PM 500	500	5	160–200
PM 450	450	6	150–200
(E) Spheroidal graphite cast iron			
SG 900/2	900	2	280–360
SG 800/2	800	2	245–335
SG 700/2	700	2	225–305
SG 600/3	600	3	190–270
SG 500/7	500	7	160–240
SG 450/10	450	10	160–210
SG 400/15	400	15	130–180
SG 350/22	350	22	150 (Max.)

[2] IS 210–1993: Grey iron castings—specification.
[3] IS 14329–1995: Malleable iron castings—specification.
[4] IS 1865–1991: Iron castings with spheroidal or nodular graphite—specification.

Table 13.12 *Dimensions of standard cross-sections*

Belt section	*Pitch width W_p (mm)*	*Nominal top width W(mm)*	*Nominal Height T (mm)*	*Recommended Minimum pitch diameter of pulley (mm)*	*Permissible Minimum pitch diameter of pulley (mm)*
Z	8.5	10	6	85	50
A	11	13	8	125	75
B	14	17	11	200	125
C	19	22	14	315	200
D	27	32	19	500	355
E	32	38	23	630	500

The selection of the cross-section depends upon two factors, namely, the power to be transmitted and speed of the faster shaft. Figure 13.24 shows the range of speed and power for various cross-sections of the belt. Depending upon the power and speed of the faster pulley, a point can be plotted on this diagram and the corresponding cross-section selected. In borderline cases, alternative design calculations are made to determine the best solution.

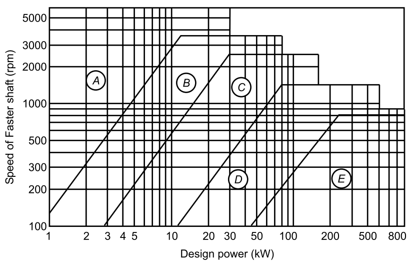

Fig. 13.24 *Selection of Cross-section of V belt*

The calculations of V-belts are based on preferred pitch diameters of pulleys and pitch lengths. The series of preferred values for pitch diameters and pitch lengths (in mm) are given in Tables 13.13 and 13.14 respectively.

The number of belts required for a given application is calculated by the following relationship:

Number of belts

$$= \frac{(\text{transmitted power in kW}) \times (F_a)}{(\text{kW rating of single V-belt}) \times (F_c) \times (F_d)}$$

$$= \frac{P \times F_a}{P_r \times F_c \times F_d} \qquad (13.18)$$

where,

P = drive power to be transmitted (kW)
F_a = correction factor for industrial service (Table 13.15)
P_r = power rating of single V-belt (from Table 13.16 to Table 13.20)
F_c = correction factor for belt length (Table 13.21)
F_d = correction factor for arc of contact (Table 13.22)

An extensive data regarding the correction factors, kW ratings of belts and other details is given in the standard as well as in manufacturer's catalogues. However, the discussion in this article is restricted to the selection procedure of general purpose belts.

Indian Standards

Indian Standards are used for Machine Elements like screw threads, belts, springs, gears, wire ropes and pressure vessels.

operation (24 h per day) such as pumps, compressors and conveyors

The values given in the above tables are only for general guidance. For a particular application, the designer should consider the past experience, the difficulties faced by the customer in replacing the bearing and the economics of breakdown costs.

15.11 LOAD FACTOR

The forces acting on the bearing are calculated by considering the equilibrium of forces in vertical and horizontal planes. These elementary equations do not take into consideration the effect of dynamic load. The forces determined by these equations are multiplied by a *load factor* to determine the dynamic load carrying capacity of the bearing. Load factors are used in applications involving gear, chain and belt drives. In gear drives, there is an additional dynamic load due to inaccuracies of the tooth profile and the elastic deformation of teeth. In chain and

15.12 SELECTION OF BEARING FROM MANUFACTURER'S CATALOGUE

The basic procedure for the selection of a bearing from the manufacturer's catalogue consists of the following steps:

(i) Calculate the radial and axial forces acting on the bearing and determine the diameter of the shaft where the bearing is to be fitted.
(ii) Select the type of bearing for the given application.
(iii) Determine the values of X and Y, the radial and thrust factors, from the catalogue. The values of X and Y factors for single-row deep groove ball bearings are given in Table 15.4. The values depend upon two ratios, $\left(\frac{F_a}{F_r}\right)$ and $\left(\frac{F_a}{C_0}\right)$, where C_0 is the static load capacity. The selection of the bearing is, therefore, done by trial and error. The static and dynamic load capacities of

Selection Procedure

When a machine component is to be selected from manufacturer's catalogue, the selection processes are discussed with a particular reference to Indian products.

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The rate of heat generated during the braking period is equal to the rate of work done by the frictional force.

Rate of heat generated = frictional force × average velocity

$= \mu N(1.047)$
$= 0.35\ (3571.43)\ (1.047)$
$= 1308.75$ N-m/s or W (iii)

Step IV *Dimensions of the block*

From Eq. (12.7),

$N = plw$
$3571.43 = (1)(2w)(w)$
$\therefore\ w = 42.26$ mm or 45 mm
$l = 2w = 90$ mm (iv)

Step V *Self-locking property*

Referring to Fig. 12.5 (a),

$a = 200$ mm $c = 50$ mm $\mu = 0.35$

$\left(\frac{a}{c}\right) = 4 \quad \therefore \left(\frac{a}{c}\right) > \mu$

The brake is not self-locking.

Example 12.5 *A double block brake is shown in Fig. 12.6. The brake drum rotates in a clockwise direction and the actuating force is 500 N. The coefficient of friction between the blocks and the drum is 0.35. Calculate the torque absorbing capacity of the brake.*

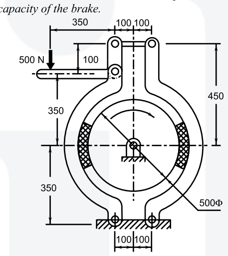

Fig. 12.6 *Double Block Brake*

Solution

Given $P = 500$ N $\mu = 0.35$ $R = 250$ mm

Step I *Force analysis*

The free-body diagram of forces acting on various parts is shown in Fig. 12.7. Considering the forces

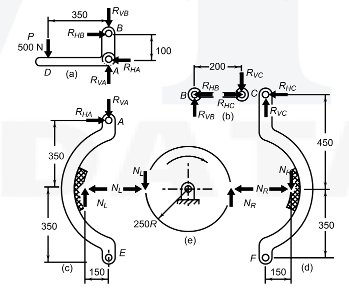

Fig. 12.7 *Free-body Diagram of Forces*

Free-Body Diagram of Forces

Whenever required, free-body diagrams are constructed to help the reader understand the forces acting on individual components.

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$\sigma_{min.} = 0$

$\sigma_a = \sigma_m = \frac{1}{2}\sigma_{max.} = \frac{1}{2}\left(\frac{750}{t}\right) = \left(\frac{375}{t}\right)$ N/mm^2

$\tan\theta = \frac{\sigma_a}{\sigma_m} = 1$ or $\theta = 45°$

The modified Goodman diagram for this example is shown in Fig. 5.45.

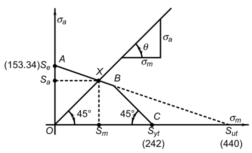

Fig. 5.45

Step III *Permissible stress amplitude*

Refer to Fig. 5.45. The coordinates of the point X are determined by solving the following two equations simultaneously.

(i) Equation of line AB

no stress concentration and the expected reliability is 50%. Calculate the number of stress cycles likely to cause fatigue failure.

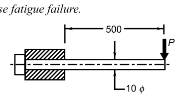

Fig. 5.46

Solution

Given For cantilever spring, $d = 10$ mm $l = 500$ mm $P = 75$ to 150 N $S_{ut} = 860$ N/mm^2 $S_{yt} = 690$ N/mm^2 $R = 50\%$

Step I *Endurance limit stress for cantilever beam*

$S_e' = 0.5S_{ut} = 0.5(860) = 430$ N/mm^2

From Fig. 5.24 (machined surface and $S_{ut} = 860$ N/mm^2),

$K_a = 0.72$

For 10 mm diameter wire, $K_b = 0.85$

For 50% reliability,

$K_c = 1.0$

$S_e = K_a K_b K_c S_e'$
$= 0.72(0.85)(1.0)(430) = 263.16$ N/mm^2

Fatigue Diagrams

Fatigue diagrams are constructed for design of machine components subjected to fluctuating loads.

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$$P_p = \left(\frac{\pi D^2}{4}\right) p' \qquad (25.69a)$$

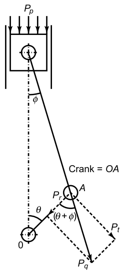


Fig. 25.23 *Force Acting on Crank*

The relationship between φ and θ is given by,

sin θ

Fig. 25.25. Due to the tangential component P_t, there are reactions $(R_1)_h$ and $(R_2)_h$ at bearings 1 and 2 respectively. Similarly, due to the radial component P_r, there are reactions $(R_1)_v$ and $(R_2)_v$ at bearings 1 and 2 respectively.

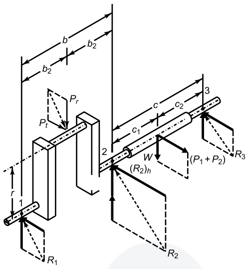


Fig. 25.24 *Centre Crankshaft at Angle of Maximum Torque*

Isometric Views

When it is difficult to understand the forces in three dimensions, isometric views are given for clear understanding.

Statistical Considerations in Design

Chapter **24**

24.1 FREQUENCY DISTRIBUTION

Statistics deals with drawing conclusions from a given or observed data. Statistical techniques are used for collection, processing, analysis and interpretation of numerical data. On the basis of statistical analysis, valid conclusions are drawn and reasonable decisions are taken. Statistics enables the engineers to understand the phenomena of variations and how to effectively predict and control them. Statistics has made valuable contributions in the areas of product design and manufacture and effective use of material and labour.

The basic data consists of observations, such as the diameters of shafts manufactured in one shift. In this case, the group of all shafts is called '*population*'. When the group is large, it is not possible to take observations of the entire population. In such cases, only a small portion of the population is examined and this portion is called a '*sample*'. *Population is defined as a collection of all elements we are studying and about which we are trying to draw conclusions. On the other hand, a sample is defined as a collection of some, but not all, of the elements of the population.* A sample is a part of the population but the converse is not true. It is easier to study a sample than studying the whole population. It costs less and takes less time. A representative sample has the characteristics of the population in the same proportions, as they are included in the entire population. Therefore, many times a sample is analysed instead of the entire population.

Let us consider an example of 100 shafts of hydrodynamic bearing, with recommended tolerance of 40e7. The shafts are manufactured on lathe and finished on grinding machine. Their diameters are measured by micrometer and the readings are tabulated in Table 24.1. The readings in Table 24.1 are called '*raw data*'. *A data is defined as the collection of numbers belonging to observations of one or more variables.* In this case, the diameter of shaft is a variable and one hundred numerical readings taken by micrometer are a data. Raw data is a data before it is arranged or analysed by any statistical method. Raw data does not show any pattern or trend of population and does not lead to any conclusions. In this chapter, the objective of statistical techniques is to rearrange and present the data in a useful way so that the decisions can be taken.

Let us rearrange the data given in Table 24.1 on the basis of diameter of shaft against the number of shafts. As shown in Table 24.2, a particular diameter such as 39.926 mm is written in the first column and each shaft with this diameter is shown by a mark X against it. Finally, the number of marks are counted and written in the last column. For example, the total number of shafts with a diameter of 39.940 mm is 8. The data in Table 24.2 is further

Statistical Considerations in Design

A separate chapter on Statistical Considerations in Design is included and examples are solved on the basis of reliability.

Example 10.5 *A helical tension spring is used in the spring balance to measure the weights. One end of the spring is attached to the rigid support while the other end, which is free, carries the weights to be measured. The maximum weight attached to the spring balance is 1500 N and the length of the scale should be approximately 100 mm. The spring index can be taken as 6. The spring is made of oil-hardened and tempered steel wire with ultimate tensile strength of 1360 N/mm² and modulus of rigidity of 81 370 N/mm². The permissible shear stress in the spring wire should be taken as 50% of the ultimate tensile strength. Design the spring and calculate*

(i) wire diameter;
(ii) mean coil diameter;
(iii) number of active coils;
(iv) required spring rate; and
(v) actual spring rate.

Solution

Given $P = 1500$ N $\quad C = 6 \quad S_{ut} = 1360$ N/mm² $\quad G = 81\,370$ N/mm² $\quad \tau = 0.5\, S_{ut}$

Step I *Wire Diameter*

The working principle of the spring balance is illustrated in Fig. 10.18. As the load acting on the spring varies from 0 to 1500 N, the pointer attached to the free end of the spring moves over a scale between highest and lowest positions. The length of the scale between these two positions of the pointer is 100 mm. In other words, the spring deflection is 100 mm when the force is 1500 N.

The permissible shear stress for spring wire is given by,

$$\tau = 0.5\, S_{ut} = 0.5\,(1360) = 680 \text{ N/mm}^2$$

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4\times 6-1}{4\times 6-4} + \frac{0.615}{6} = 1.2525$$

From Eq. (10.13),

$$\tau = K\left(\frac{8PC}{\pi d^2}\right) \quad \text{or} \quad 680 = 1.2525\left\{\frac{8\,(1500)(6)}{\pi d^2}\right\}$$

$$\therefore \qquad d = 6.5 \text{ or } 7 \text{ mm} \qquad \text{(i)}$$

Step II *Mean coil diameter*

$$D = Cd = 6\,(7) = 42 \text{ mm} \qquad \text{(ii)}$$

Step III *Number of active coils*

From Eq. (10.8),

$$\delta = \frac{8\,PD^3\,N}{Gd^4} \quad \text{or} \quad 100 = \frac{8(1500)(42)^3\,N}{(81\,370)(7)^4}$$

$$\therefore \qquad N = 21.97 \text{ or } 22 \text{ coils} \qquad \text{(iii)}$$

Step IV *Required spring rate*

$$k = \frac{P}{\delta} = \frac{1500}{100} = 15 \text{ N/mm} \qquad \text{(iv)}$$

Step V *Actual spring rate*

$$k = \frac{Gd^4}{8D^3N} = \frac{(81\,370)(7)^4}{8(42)^3(22)} = 14.98 \text{ N/mm} \qquad \text{(v)}$$

Numerical Examples

Numerical Examples solved by step by step approach are provided in sufficient number in each chapter to help the reader understand the design procedures.

Short Answer Questions

25.1 What are the functions of engine cylinder?
25.2 What are the cooling systems for engine cylinders? Where do you use them?
25.3 What are the advantages of cylinder liner?
25.4 What are dry and wet cylinder liners?
25.5 What are the desirable properties of cylinder materials?
25.6 Name the materials used for engine cylinder.
25.7 What do you understand by 'bore' of cylinder?
25.8 What are the functions of piston?
25.9 What are the design requirements of piston?
25.10 Name the materials used for engine piston.
25.11 What are the advantages and disadvantages of aluminium piston over cast iron piston?
25.12 Why is piston made lightweight?
25.13 Name two criteria for calculating the thickness of piston head.
25.14 Why is piston clearance necessary? What is its usual value?
25.15 What are the functions of piston ribs?
25.16 What is the function of the cup on piston head?
25.17 What are the functions of compression piston rings?
25.18 What are the functions of oil scraper rings?
25.19 Name the materials used for piston rings.
25.20 Why are more number of thin piston rings preferred over small number of thick rings?
25.30 What is the force on bolts of of big end connecting rod?
25.31 What is the difference between centre and overhung crankshafts?
25.32 Where do you use overhung crankshafts?
25.33 Where do you use centre crankshafts?
25.34 What is the main advantage of overhung crankshafts?
25.35 Name the materials for crankshafts.
25.36 What is the manufacturing method for crankshaft?
25.37 When do you use push rod?
25.38 Why is the design of exhaust valve more critical than that of an inlet valve?
25.39 Why is the area of inlet valve port more than that of an exhaust valve?
25.40 Why do inlet and exhaust valves have conical heads and seats?
25.41 What is the function of rocker arm?
25.42 Why is rocker arm made of I section?
25.43 Name the materials for rocker arm.
25.44 What is tappet? What is the stress in tappet?
25.45 What is the purpose of valve spring?
25.46 What is the criterion for design of push rod?

Problems for Practice

25.1 The cylinder of a four-stroke diesel engine has the following specifications:
Brake power = 3 kW
Speed = 800 rpm

Short-Answer Questions

At the end of each chapter, Short-Answer Questions are provided for the students for preparation of oral and theory examinations.

Problems for Practice

At the end of each chapter, a set of examples with answers is given as exercise to students. It is also helpful to teachers in setting classwork and homework assignments.

Problems for Practice

11.1 A single plate clutch consists of one pair of contacting surfaces. The inner and outer diameters of the friction disk are 125 and 250 mm respectively. The coefficient of friction is 0.25 and the total axial force is 15 kN. Calculate the power transmitting capacity of the clutch at 500 rpm using :
(i) uniform wear theory; and
(ii) uniform pressure theory.
[(i) 18.41 kW (ii) 19.09 kW]

11.2 An automotive single plate clutch consists of two pairs of contacting surfaces. The outer diameter of the friction disk is 270 mm. The coefficient of friction is 0.3 and the maximum intensity of pressure is 0.3 N/mm^2. The clutch is transmitting a torque of 531 N-m. Assuming uniform wear theory, calculate:

(z = 1.1258)]

11.5 A leather faced cone clutch transmits power at 500 rpm. The semi-cone angle α is 12.5°. The mean diameter of the clutch is 300 mm, while the face width of the contacting surface of the friction lining is 100 mm. The coefficient of friction is 0.2 and the maximum intensity of pressure is limited to 0.07 N/mm^2. Calculate the force to engage the clutch and the power transmitting capacity.
[1324.68 N and 9.61 kW]

11.6 A centrifugal clutch, transmitting 18.5 kW at 720 rpm, consists of four shoes. The clutch is to be engaged at 75% of the running speed. The inner radius of the drum is 165 mm, while the radius of the centre of gravity of each shoe, during engaged position, is 140 mm. The coefficient of friction is 0.25. Calculate the mass of each shoe.
[4.27 kg]

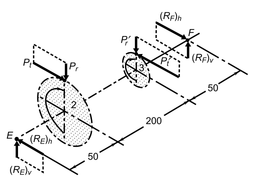


Fig. 17.30

Step II *Reactions at bearings E and F*
The forces acting in vertical and horizontal planes are shown in Fig. 17.31.

Considering vertical forces and taking moments about the bearing E,

$$P_r \times 50 + P'_r \times 250 = (R_F)_v \times 300$$
$$482.73 \times 50 + 1206.83 \times 250 = (R_F)_v \times 300$$
$$\therefore \quad (R_F)_V = 1086.15 \text{ N} \qquad \text{(i)}$$
$$P_r + P'_r = (R_F)_v + (R_E)_v$$
$$482.73 + 1206.83 = 1086.15 + (R_E)_v$$
$$\therefore \quad (R_E)_v = 603.41 \text{ N} \qquad \text{(ii)}$$

(b) Horizontal plane

Fig. 17.31

17.12 GEAR TOOTH FAILURES

There are two basic modes of gear tooth failure—breakage of the tooth due to static and dynamic loads and the surface destruction[4, 5]. The complete breakage of the tooth can be avoided by adjusting the parameters in the gear design, such as the module and the face width, so that the beam strength of the gear tooth is more than the sum of static and dynamic loads. The surface destruction or tooth wear is classified according to the basis of their primary causes. The principal types of gear tooth wear are as follows:

(i) Abrasive Wear Foreign particles in the lubricant, such as dirt, rust, weld spatter or metallic debris can scratch or brinell the tooth surface. Remedies against this type of wear are provision of oil filters, increasing surface hardness and use of high viscosity oils. A thick lubricating film developed by these oils allows fine particles to pass without scratching.

(ii) Corrosive Wear The corrosion of the tooth surface is caused by corrosive elements, such as

[4] RH Pearson–'Gear overdesign and how to avoid it'–Machine Design—May 9, 1968, vol. 40, no. 11, p. 153.
[5] Eugene E Shipley – 'Gear failures—how to recognize them, what causes them and how to avoid them?'—Machine Design—Dec 7, 1967, vol. 39, no. 28, p. 152.

References

The list of textbooks, journals and company catalogues is provided at the end of respective pages for quick reference.

Introduction

Chapter 1

If the point of contact between the product and people becomes a point of friction, then the industrial designer has failed. On the other hand, if people are made safer, more efficient, more comfortable—or just plain happier—by contact with the product, then the designer has succeeded.

Henry Dreyfuss[1]

1.1 MACHINE DESIGN

Machine design is defined as the use of scientific principles, technical information and imagination in the description of a machine or a mechanical system to perform specific functions with maximum economy and efficiency. This definition of machine design contains the following important features:

(i) A designer uses principles of basic and engineering sciences such as physics, mathematics, statics and dynamics, thermodynamics and heat transfer, vibrations and fluid mechanics. Some of the examples of these principles are
  (a) Newton's laws of motion,
  (b) D' Alembert's principle,
  (c) Boyle's and Charles' laws of gases,
  (d) Carnot cycle, and
  (e) Bernoulli's principle.

(ii) The designer has technical information of the basic elements of a machine. These elements include fastening devices, chain, belt and gear drives, bearings, oil seals and gaskets, springs, shafts, keys, couplings, and so on. A machine is a combination of these basic elements. The designer knows the relative advantages and disadvantages of these basic elements and their suitability in different applications.

(iii) The designer uses his skill and imagination to produce a configuration, which is a combination of these basic elements. However, this combination is unique and different in different situations. The intellectual part of constructing a proper configuration is creative in nature.

(iv) The final outcome of the design process consists of the description of the machine. The description is in the form of drawings of assembly and individual components.

(v) A design is created to satisfy a recognised need of customer. The need may be to perform a specific function with maximum economy and efficiency.

[1] Henry Dreyfuss–*The Profile of Industrial Designer*—Machine Design, July 22, 1967.

Machine design is the creation of plans for a machine to perform the desired functions. The machine may be entirely new in concept, performing a new type of work, or it may more economically perform the work that can be done by an existing machine. It may be an improvement or enlargement of an existing machine for better economy and capability.

1.2 BASIC PROCEDURE OF MACHINE DESIGN

The basic procedure of machine design consists of a step-by-step approach from given specifications about the functional requirements of a product to the complete description in the form of drawings of the final product. A logical sequence of steps, usually common to all design projects, is illustrated in Fig. 1.1. These steps are interrelated and interdependent, each reflecting and affecting all other steps. The following steps are involved in the process of machine design.

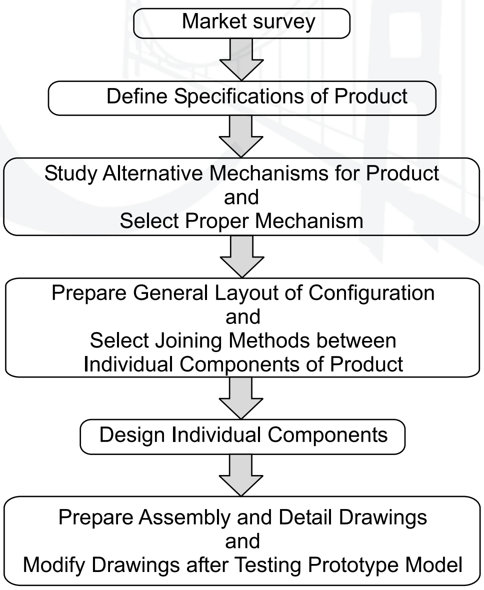


Fig. 1.1 *The Design Process*

Step 1: Product Specifications

The first step consists of preparing a complete list of the requirements of the product. The requirements include the output capacity of the machine, and its service life, cost and reliability. In some cases, the overall dimensions and weight of the product are specified. For example, while designing a scooter, the list of specifications will be as follows:

(i) Fuel consumption = 40 km/l
(ii) Maximum speed = 85 km/hr
(iii) Carrying capacity = two persons with 10 kg luggage
(iv) Overall dimensions
 Width = 700 mm
 Length = 1750 mm
 Height = 1000 mm
(v) Weight = 95 kg
(vi) Cost = Rs 40000 to Rs 45000

In consumer products, external appearance, noiseless performance and simplicity in operation of controls are important requirements. Depending upon the type of product, various requirements are given weightages and a priority list of specifications is prepared.

Step 2: Selection of Mechanism

After careful study of the requirements, the designer prepares rough sketches of different possible mechanisms for the product. For example, while designing a blanking or piercing press, the following mechanisms are possible:

(i) a mechanism involving the crank and connecting rod, converting the rotary motion of the electric motor into the reciprocating motion of the punch;
(ii) a mechanism involving nut and screw, which is a simple and cheap configuration but having poor efficiency; and
(iii) a mechanism consisting of a hydraulic cylinder, piston and valves which is a costly configuration but highly efficient.

The alternative mechanisms are compared with each other and also with the mechanism of the products that are available in the market. An approximate estimation of the cost of each alternative configuration is made and compared with the cost of existing products. This will reveal the competitiveness of the product. While selecting the final configuration, the designer should consider whether the raw materials and standard parts required for making the product are available in the market. He should also consider whether the manufacturing processes required to fabricate the non-standard components are available in the factory. Depending upon the cost-competitiveness, availability of raw materials and manufacturing facility, the best possible mechanism is selected for the product.

Step 3: Layout of Configuration

The next step in a design procedure is to prepare a block diagram showing the general layout of the selected configuration. For example, the layout of an Electrically-operated Overhead Travelling (EOT) crane will consist of the following components:

(i) electric motor for power supply;
(ii) flexible coupling to connect the motor shaft to the clutch shaft;
(iii) clutch to connect or disconnect the electric motor at the will of the operator;
(iv) gear box to reduce the speed from 1440 rpm to about 15 rpm;
(v) rope drum to convert the rotary motion of the shaft to the linear motion of the wire rope;
(vi) wire rope and pulley with the crane hook to attach the load; and
(vii) brake to stop the motion.

In this step, the designer specifies the joining methods, such as riveting, bolting or welding to connect the individual components. Rough sketches of shapes of the individual parts are prepared.

Step 4: Design of Individual Components

The design of individual components or machine elements is an important step in a design process. It consists of the following stages:

(i) Determine the forces acting on the component.
(ii) Select proper material for the component depending upon the functional requirements such as strength, rigidity, hardness and wear resistance.
(iii) Determine the likely mode of failure for the component and depending upon it, select the criterion of failure, such as yield strength, ultimate tensile strength, endurance limit or permissible deflection.
(iv) Determine the geometric dimensions of the component using a suitable factor of safety and modify the dimensions from assembly and manufacturing considerations.

This stage involves detailed stress and deflection analysis. The subjects 'Machine Design' or 'Elements of Machine Design' cover mainly the design of machine elements or individual components of the machine. Section 1.4 on Design of Machine Elements, elaborates the details of this important step in design procedure.

Step 5: Preparation of Drawings

The last stage in a design process is to prepare drawings of the assembly and the individual components. On these drawings, the material of the component, its dimensions, tolerances, surface finish grades and machining symbols are specified. The designer prepares two separate lists of components—standard components to be purchased directly from the market and special components to be machined in the factory. In many cases, a prototype model is prepared for the product and thoroughly tested before finalising the assembly drawings.

It is seen that the process of machine design involves systematic approach from known specifications to unknown solutions. Quite often, problems arise on the shop floor during the production stage and design may require modifications. In such circumstances, the designer has to consult the manufacturing engineer and find out the suitable modification.

1.3 BASIC REQUIREMENTS OF MACHINE ELEMENTS

A machine consists of machine elements. Each part of a machine, which has motion with respect to some other part, is called a machine element. It is important to note that each machine element may consist of several parts, which are manufactured separately. For example, a rolling contact bearing is a machine element and it consists of an inner race, outer race, cage and rolling elements like balls. Machine elements can be classified into two groups—*general-purpose* and *special-purpose* machine elements. General-purpose machine elements include shafts, couplings, clutches, bearings, springs, gears and machine frames Special-purpose machine elements include pistons, valves or spindles. Special-purpose machine elements are used only in certain types of applications. On the contrary, general-purpose machine elements are used in a large number of machines.

The broad objective of designing a machine element is to ensure that it preserves its operating capacity during the stipulated service life with minimum manufacturing and operating costs. In order to achieve this objective, the machine element should satisfy the following basic requirements:

(i) Strength: A machine part should not fail under the effect of the forces that act on it. It should have sufficient strength to avoid failure either due to fracture or due to general yielding.

(ii) Rigidity: A machine component should be rigid, that is, it should not deflect or bend too much due to forces or moments that act on it. A transmission shaft in many times designed on the basis of lateral and torsional rigidities. In these cases, maximum permissible deflection and permissible angle of twist are the criteria for design.

(iii) Wear Resistance: Wear is the main reason for putting the machine part out of order. It reduces useful life of the component. Wear also leads to the loss of accuracy of machine tools. There are different types of wear such as abrasive wear, corrosive wear and pitting. Surface hardening can increase the wear resistance of the machine components, such as gears and cams.

(iv) Minimum Dimensions and Weight: A machine part should be sufficiently strong, rigid and wear-resistant and at the same time, with minimum possible dimensions and weight. This will result in minimum material cost.

(v) Manufacturability: Manufacturability is the ease of fabrication and assembly. The shape and material of the machine part should be selected in such a way that it can be produced with minimum labour cost.

(vi) Safety: The shape and dimensions of the machine parts should ensure safety to the operator of the machine. The designer should assume the worst possible conditions and apply 'fail-safe' or 'redundancy' principles in such cases.

(vii) Conformance to Standards: A machine part should conform to the national or international standard covering its profile, dimensions, grade and material.

(viii) Reliability: Reliability is the probability that a machine part will perform its intended functions under desired operating conditions over a specified period of time. A machine part should be reliable, that is, it should perform its function satisfactorily over its lifetime.

(ix) Maintainability: A machine part should be maintainable. Maintainability is the ease with which a machine part can be serviced or repaired.

(x) Minimum: Life-cycle Cost: Life-cycle cost of the machine part is the total cost to be paid by the purchaser for purchasing the part and operating and maintaining it over its life span.

It will be observed that the above mentioned requirements serve as the basis for design projects in many cases.

1.4 DESIGN OF MACHINE ELEMENTS

Design of machine elements is the most important step in the complete procedure of machine design. In order to ensure the basic requirements of machine elements, calculations are carried out to find out the dimensions of the machine elements. These calculations form an integral part of the design of machine elements. The basic procedure of the design of machine elements is illustrated in Fig. 1.2. It consists of the following steps:

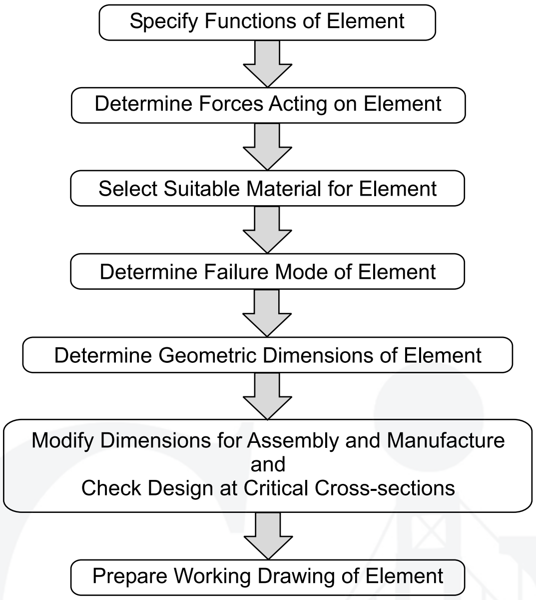


Fig. 1.2 *Basic Procedure of Design of Machine Element*

Step 1: Specification of Function

The design of machine elements begins with the specification of the functions of the element. The functions of some machine elements are as follows:

(i) ***Bearing*** To support the rotating shaft and confine its motion

(ii) ***Key*** To transmit the torque between the shaft and the adjoining machine part like gear, pulley or sprocket

(iii) ***Spring in Clock*** To store and release the energy

(iv) ***Spring in Spring Balance*** To measure the force

(v) ***Screw Fastening*** To hold two or more machine parts together

(vi) ***Power Screw*** To produce uniform and slow motion and to transmit the force

Step 2: Determination of Forces

In many cases, a free-body diagram of forces is constructed to determine the forces acting on different parts of the machine. The external and internal forces that act on a machine element are as follows:

(i) The external force due to energy, power or torque transmitted by the machine part, often called 'useful' load

(ii) Static force due to deadweight of the machine part

(iii) Force due to frictional resistance

(iv) Inertia force due to change in linear or angular velocity

(v) Centrifugal force due to change in direction of velocity

(vi) Force due to thermal gradient or variation in temperature

(vii) Force set up during manufacturing the part resulting in residual stresses

(viii) Force due to particular shape of the part such as stress concentration due to abrupt change in cross-section

For every machine element, all forces in this list may not be applicable. They vary depending on the application. There is one more important consideration. The force acting on the machine part is either assumed to be concentrated at some point in the machine part or distributed over a particular area. Experience is essential to make such assumptions in the analysis of forces.

Step 3: Selection of Material

Four basic factors, which are considered in selecting the material, are availability, cost, mechanical properties and manufacturing considerations.

For example, flywheel, housing of gearbox or engine block have complex shapes. These components are made of cast iron because the casting process produces complicated shapes without involving machining operations. Transmission shafts are made of plain carbon steels, because they are available in the form of rods, besides their higher strength. The automobile body and hood are made of low carbon steels because their cold formability is essential to press the parts. Free cutting steels have excellent machinability due to addition of sulphur. They are ideally suitable for bolts and studs because of the ease with which the thread profiles can be machined. The crankshaft and connecting rod are subjected to fluctuating forces and nickel–chromium steel is used for these components due to its higher fatigue strength.

Step 4: Failure Criterion

Before finding out the dimensions of the component, it is necessary to know the type of failure that the component may fail when put into service. The machine component is said to have 'failed' when it is unable to perform its functions satisfactorily. The three basic types of failure are as follows:

(i) failure by elastic deflection;
(ii) failure by general yielding; and
(iii) failure by fracture.

In applications like transmission shaft, which is used to support gears, the maximum force acting on the shaft is limited by the permissible deflection. When this deflection exceeds a particular value (usually, 0.001 to 0.003 times of span length between two bearings), the meshing between teeth of gears is affected and the shaft cannot perform its function properly. In this case, the shaft is said to have 'failed' due to elastic deflection. Components made of ductile materials like steel lose their engineering usefulness due to large amount of plastic deformation. This type of failure is called failure by yielding. Components made of brittle materials like cast iron fail because of sudden fracture without any plastic deformation. There are two basic modes of gear-tooth failure—breakage of tooth due to static and dynamic load and surface pitting. The surface of the gear tooth is covered with small 'pits' resulting in rapid wear. Pitting is a surface fatigue failure. The components of ball bearings such as rolling elements, inner and outer races fail due to fatigue cracks after certain number of revolutions. Sliding contact bearings fail due to corrosion and abrasive wear by foreign particles.

Step 5: Determination of Dimensions

The shape of the machine element depends on two factors, viz., the operating conditions and the shape of the adjoining machine element. For example, involute profile is used for gear teeth because it satisfies the fundamental law of gearing. A V-belt has a trapezoidal cross-section because it results in wedge action and increases the force of friction between the surfaces of the belt and the pulley. On the other hand, the pulley of a V-belt should have a shape which will match with the adjoining belt. The profile of the teeth of sprocket wheel should match the roller, bushing, inner and outer link plates of the roller chain. Depending on the operating conditions and shape of the adjoining element, the shape of the machine element is decided and a rough sketch is prepared.

The geometric dimensions of the component are determined on the basis of failure criterion. In simple cases, the dimensions are determined on the basis of allowable stress or deflection. For example, a tension rod, illustrated in Fig. 1.3, is subjected to a force of 5 kN. The rod is made of plain carbon

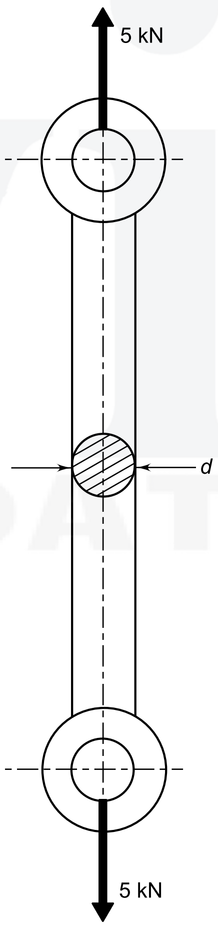


Fig. 1.3 *Tension Rod*

steel and the permissible tensile stress is 80 N/mm^2. The diameter of the rod is determined on the basis of allowable stress using the following expression:

$$\text{stress} = \frac{\text{force}}{\text{area}} \quad \text{or} \quad 80 = \frac{(5 \times 10^3)}{\left(\frac{\pi d^2}{4}\right)}$$

Therefore,

$$d = 8.92 \text{ or } 10 \text{ mm}$$

As a second example, consider a transmission shaft, shown in Fig. 1.4, which is used to support a gear. The shaft is made of steel and the modulus

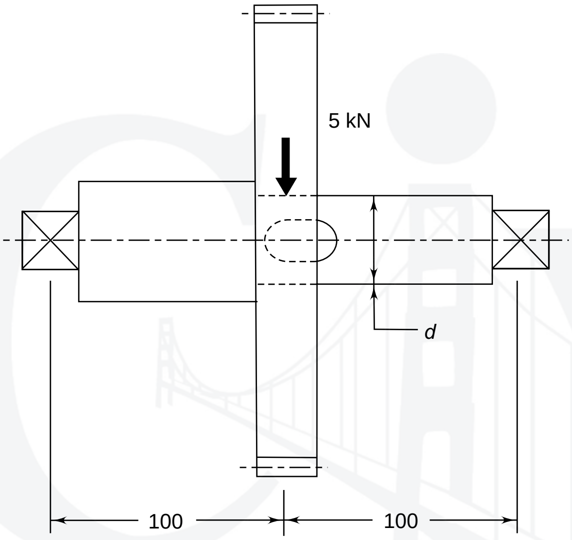


Fig. 1.4 *Transmission Shaft*

of elasticity is 207 000 N/mm^2. For proper meshing between gear teeth, the permissible deflection at the gear is limited to 0.05 mm. The deflection of the shaft at the centre is given by,

$$\delta = \frac{Pl^3}{48EI} \quad \text{or} \quad 0.05 = \frac{(5 \times 10^3)(200)^3}{48(207\,000)\left(\frac{\pi d^4}{64}\right)}$$

Therefore,

$$d = 35.79 \text{ or } 40 \text{ mm}$$

The following observations are made from the above two examples:

(i) Failure mode for the tension rod is general yielding while elastic deflection is the failure criterion for the transmission shaft.

(ii) The permissible tensile stress for tension rod is obtained by dividing the yield strength by the factor of safety. Therefore, yield strength is the criterion of design. In case of a transmission shaft, lateral deflection or rigidity is the criterion of design. Therefore, modulus of elasticity is an important property for finding out the dimensions of the shaft.

Determination of geometric dimensions is an important step while designing machine elements. Various criteria such as yield strength, ultimate tensile strength, torsional or lateral deflection and permissible bearing pressure are used to find out these dimensions.

Step 6: Design Modifications

The geometric dimensions of the machine element are modified from assembly and manufacturing considerations. For example, the transmission shaft illustrated in Fig. 1.4 is provided with steps and shoulders for proper mounting of gear and bearings. Revised calculations are carried out for operating capacity, margin of safety at critical cross-sections and resultant stresses taking into consideration the effect of stress concentration. When these values differ from desired values, the dimensions of the component are modified. The process is continued till the desired values of operating capacity, factor of safety and stresses at critical cross-sections are obtained.

Step 7: Working Drawing

The last step in the design of machine elements is to prepare a working drawing of the machine element showing dimensions, tolerances, surface finish grades, geometric tolerances and special production requirements like heat treatment. The working drawing must be clear, concise and complete. It must have enough views and cross-sections to show all details. The main view of the machine element should show it in a position, it is required to occupy in service. Every dimension must be given. There should not be scope for guesswork and a necessity for scaling the drawing. All dimensions that are important for proper assembly and interchangeability must be provided with tolerances.

1.5 TRADITIONAL DESIGN METHODS

There are two traditional methods of design—*design by craft evolution* and *design by drawing*. Bullock cart, rowing boat, plow and musical instruments are some of the products, which are produced by the craft-evolution process. The salient features of this age-old technique are as follows:

(i) The craftsmen do not prepare dimensioned drawings of their products. They cannot offer adequate justification for the designs they make.

(ii) These products are developed by trial and error over many centuries. Any modification in the product is costly, because the craftsman has to experiment with the product itself. Moreover, only one change at a time can be attempted and complete reorganization of the product is difficult.

(iii) The essential information of the product such as materials, dimensions of parts, manufacturing methods and assembly techniques is transmitted from place to place and time to time by two ways. First, the product, which basically remains unchanged, is the main source of information. The exact memory of the sequence of operations required to make the product is second source of information. There is no symbolic medium to record the design information of the product.

With all these weaknesses, the craft-evolution process has successfully developed some of the complex structures. The craft-evolution method has become obsolete due to two reasons. This method cannot adapt to sudden changes in requirement. Secondly, the product cannot be manufactured on a mass scale.

The essential features of design by drawing method are as follows:

(i) The dimensions of the product are specified in advance of its manufacture.

(ii) The complete manufacturing of the product can be subdivided into separate pieces, which can be made by different people. This division of work is not possible with craft-evolution.

(iii) When the product is to be developed by trial and error, the process is carried out on a drawing board instead of shop floor. The drawings of the product are modified and developed prior to manufacture.

In this method, much of the intellectual activity is taken away from the shop floor and assigned to design engineers.

1.6 DESIGN SYNTHESIS

Design synthesis is defined as the process of creating or selecting configurations, materials, shapes and dimensions for a product. It is a decision making process with the main objective of optimisation. There is a basic difference between design analysis and design synthesis. In design analysis, the designer assumes a particular mechanism, a particular material and mode of failure for the component. With the help of this information, he determines the dimensions of the product. However, design synthesis does not permit such assumptions. Here, the designer selects the optimum configuration from a number of alternative solutions. He decides the material for the component from a number of alternative materials. He determines the optimum shape and dimensions of the component on the basis of mathematical analysis.

In design synthesis, the designer has to fix the objective. The objective can be minimum cost, minimum weight or volume, maximum reliability or maximum life. The second step is mathematical formulation of these objectives and requirements. The final step is mathematical analysis for optimisation and interpretation of the results. In order to illustrate the process of design synthesis, let us consider a problem of designing cylindrical cans. The requirements are as follows:

(i) The cylindrical can is completely enclosed and the cost of its material should be minimum.

(ii) The cans are to be stored on a shelf and the dimensions of the shelf are such that the radius of the can should not exceed R_{max}.

The following notations are used in the analysis:

r = radius of can
h = height of can
A = surface area of can
V = volume of can

Therefore,

$$A = 2\pi r^2 + 2\pi rh \qquad \text{(a)}$$
$$V = \pi r^2 h \qquad \text{(b)}$$

Substituting Eq. (b) in Eq. (a),

$$A = 2\pi r^2 + \frac{2V}{r} \qquad \text{(c)}$$

For minimum cost of material of the can,

$$\frac{dA}{dr} = 0 \quad \text{or} \quad 4\pi r - \frac{2V}{r^2} = 0$$

or $$r = \left(\frac{V}{2\pi}\right)^{1/3}.$$

Let us call this radius as r_1 giving the condition of minimum material. Therefore,

$$r_1 = \left(\frac{V}{2\pi}\right)^{1/3} \qquad \text{(d)}$$

In order to satisfy the second requirement,

$$0 < r < R_{max.} \qquad \text{(e)}$$

In Eqs (d) and (e), r_1 and $R_{max.}$ are two independent variables and there will be two separate cases as shown in Fig. 1.5.

Case (a)

$$r_1 > R_{max.}$$

The optimum radius will be,

$$r = R_{max.} \qquad \text{(i)}$$

Case (b)

$$r_1 < R_{max.}$$

The optimum radius will be

$$r = r_1 \qquad \text{(ii)}$$

It is seen from the above example, that design synthesis begins with the statement of requirements, which are then converted into mathematical expressions and finally, equations are solved for optimisation.

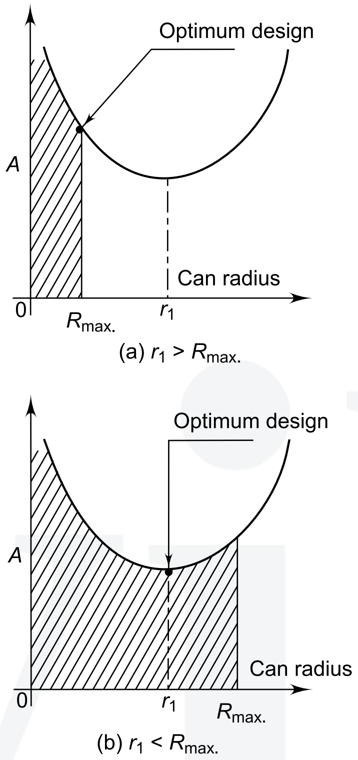


Fig. 1.5 *Optimum Solution to Can Radius*

1.7 USE OF STANDARDS IN DESIGN

Standardization is defined as obligatory norms, to which various characteristics of a product should conform. The characteristics include materials, dimensions and shape of the component, method of testing and method of marking, packing and storing of the product. The following standards are used in mechanical engineering design:

(i) Standards for Materials, their Chemical Compositions, Mechanical Properties and Heat Treatment For example, Indian standard IS 210 specifies seven grades of grey cast iron designated as FG 150, FG 200, FG 220, FG 260, FG 300, FG 350 and FG 400. The number indicates ultimate tensile strength in N/mm^2. IS 1570 (Part 4) specifies chemical composition of various grades of alloy steel. For example, alloy steel designated by 55Cr3 has 0.5–0.6% carbon, 0.10–0.35% silicon, 0.6–0.8% manganese and 0.6–0.8% chromium.

(ii) Standards for Shapes and Dimensions of Commonly used Machine Elements The machine elements include bolts, screws and nuts, rivets, belts and chains, ball and roller bearings, wire ropes, keys and splines, etc. For example, IS 2494 (Part 1) specifies dimensions and shape of the cross-section of endless V-belts for power transmission. The dimensions of the trapezoidal cross-section of the belt, viz. width, height and included angle are specified in this standard. The dimensions of rotary shaft oil seal units are given in IS 5129 (Part 1). These dimensions include inner and outer diameters and width of oil seal units.

(iii) Standards for Fits, Tolerances and Surface Finish of Component For example, selection of the type of fit for different applications is illustrated in IS 2709 on 'Guide for selection of fits'. The tolerances or upper and lower limits for various sizes of holes and shafts are specified in IS 919 on 'Recommendations for limits and fits for engineering'. IS 10719 explains method for indicating surface texture on technical drawings. The method of showing geometrical tolerances is explained in IS 8000 on 'Geometrical tolerancing on technical drawings'.

(iv) Standards for Testing of Products These standards, sometimes called 'codes', give procedures to test the products such as pressure vessel, boiler, crane and wire rope, where safety of the operator is an important consideration. For example, IS 807 is a code of practice for design, manufacture, erection and testing of cranes and hoists. The method of testing of pressure vessels is explained in IS 2825 on 'Code for unfired pressure vessels'.

(v) Standards for Engineering Drawing of Components For example, there is a special publication SP46 prepared by Bureau of Indian Standards on 'Engineering Drawing Practice for Schools and Colleges' which covers all standards related to engineering drawing.

There are two words—standard and code—which are often used in standards. *A standard is defined as a set of specifications for parts, materials or processes*. The objective of a standard is to reduce the variety and limit the number of items to a reasonable level. On the other hand, *a code is defined as a set of specifications for the analysis, design, manufacture, testing and erection of the product*. The purpose of a code is to achieve a specified level of safety.

There are three types of standards used in design office. They are as follows:

(i) Company standards They are used in a particular company or a group of sister concerns.

(ii) National standards These are the IS (Bureau of Indian Standards), DIN (German), AISI or SAE (USA) or BS (UK) standards.

(iii) International standards These are prepared by the International Standards Organization (ISO).

Standardization offers the following advantages:

(a) The reduction in types and dimensions of identical components to a rational number makes it possible to manufacture the standard component on a mass scale in a centralised process. For example, a specialised factory like Associated Bearing Company (SKF) manufactures ball and roller bearings, which are required by all engineering industries. Manufacture of a standard component on mass production basis reduces the cost.

(b) Since the standard component is manufactured by a specialised factory, it relieves the machine-building plant of the laborious work of manufacturing that part. Availability of standard components like bearings, seals, knobs, wheels, roller chains, belts, hydraulic cylinders and valves has considerably reduced the manufacturing facilities required by the individual organisation.

(c) Standard parts are easy to replace when worn out due to interchangeability. This facilitates servicing and maintenance of machines. Availability of standard spare parts is always assured. The work of servicing and maintenance can be carried out even at an ordinary service station. These factors reduce the maintenance cost of machines.

(d) The application of standard machine elements and especially the standard units (e.g. couplings, cocks, pumps, pressure reducing valves and electric motors) reduce the time and effort needed to design a new machine. It is no longer necessary to design, manufacture and test these elements and units, and all that the designer has to do is to select them from the manufacturer's catalogues. On the other hand, enormous amount of work would be required to design a machine if all the screws, bolts, nuts, bearings, etc., had to be designed anew each time. Standardization results in substantial saving in the designer's effort.

(e) The standards of specifications and testing procedures of machine elements improve their quality and reliability. Standard components like SKF bearings, Dunlop belts or Diamond chains have a long-standing reputation for their reliability in engineering industries. Use of standard components improves the quality and reliability of the machine to be designed.

In design, the aim is to use as many standard components as possible for a given machine. The selection of standard parts in no way restricts the creative initiative of the designer and does not prevent him from finding better and more rational solutions.

1.8 SELECTION OF PREFERRED SIZES

In engineering design, many a times, the designer has to specify the size of the product. The '*size*' of the product is a general term, which includes different parameters like power transmitting capacity, load carrying capacity, speed, dimensions of the component such as height, length and width, and volume or weight of the product. These parameters are expressed numerically, e.g., 5 kW, 10 kN or 1000 rpm. Often, the product is manufactured in different sizes or models; for instance, a company may be manufacturing seven different models of electric motors ranging from 0.5 to 50 kW to cater to the need of different customers. Preferred numbers are used to specify the 'sizes' of the product in these cases.

French balloonist and engineer *Charles Renard* first introduced preferred numbers in the 19th century. The system is based on the use of geometric progression to develop a set of numbers. There are five basic series[2], denoted as R5, R10, R20, R40 and R80 series, which increase in steps of 58%, 26%, 12%, 6%, and 3%, respectively. Each series has its own *series factor*. The series factors are given in Table 1.1.

Table 1.1 *Series factors*

R5 Series	$\sqrt[5]{10} = 1.58$
R10 Series	$\sqrt[10]{10} = 1.26$
R20 Series	$\sqrt[20]{10} = 1.12$
R40 Series	$\sqrt[40]{10} = 1.06$
R80 Series	$\sqrt[80]{10} = 1.03$

The series is established by taking the first number and multiplying it by a series factor to get the second number. The second number is again multiplied by a series factor to get the third number. This procedure is continued until the complete series is built up. The resultant numbers are rounded and shown in Table 1.2. As an example, consider a manufacturer of lifting tackles who wants to introduce nine different models of capacities ranging from about 15 to 100 kN. Referring to the R10 series, the capacities of different models of the lifting tackle will be 16, 20, 25, 31.5, 40, 50, 63, 80 and 100 kN.

Table 1.2 *Preferred numbers*

R5	*R10*	*R20*	*R40*
1.00	1.00	1.00	1.00
			1.06
		1.12	1.12
			1.18
	1.25	1.25	1.25
			1.32
		1.40	1.40
			1.50

(*Contd*)

[2] IS 1076–1985: Preferred Numbers (in three parts).

Table 1.2 *Contd*

R5	*R10*	*R20*	*R40*
1.60	1.60	1.60	1.60
			1.70
		1.80	1.80
			1.90
	2.00	2.00	2.00
			2.12
		2.24	2.24
			2.36
2.50	2.50	2.50	2.50
			2.65
		2.80	2.80
			3.00
	3.15	3.15	3.15
			3.35
		3.55	3.55
			3.75
4.00	4.00	4.00	4.00
			4.25
		4.50	4.50
			4.75
	5.00	5.00	5.00
			5.30
		5.60	5.60
			6.00
6.30	6.30	6.30	6.30
			6.70
		7.10	7.10
			7.50
	8.00	8.00	8.00
			8.50
		9.00	9.00
			9.50
10.00	10.00	10.00	10.00

It is observed from Table 1.2 that small sizes differ from each other by small amounts, while large sizes by large amounts. In the initial stages, the product is manufactured in a limited quantity and use is made of the R5 series. As the scale of production is increased, a change over is made from R5 to R10 series, introducing new sizes of intermediate values of R10 series. Preferred numbers minimise unnecessary variation in sizes. They assist the designer in avoiding selection of sizes in an arbitrary manner. The complete range is covered by minimum number of sizes, which is advantageous to the producer and consumer.

There are two terms, namely, '*basic series*' and '*derived series*', which are frequently used in relation to preferred numbers. R5, R10, R20, R40 and R80 are called *basic series*. Any series that is formed on the basis of these five basic series is called *derived series*. In other words, derived series are derived from basic series. There are two methods of forming derived series, namely, reducing the numbers of a particular basic series or increasing the numbers.

In the first method, a derived series is obtained by taking every second, third, fourth or *p*th term of a given basic series. Such a derived series is designated by the symbol of the basic series followed by the number 2, 3, 4 or *p* and separated by '/' sign. If the series is limited, the designation also includes the limits inside the bracket. If the series is unlimited, at least one of the numbers of that series is mentioned inside the bracket. Let us consider the meaning of these designations.

(i) Series R 10/3 (1, … ,1000) indicates a derived series comprising of every third term of the R10 series and having the lower limit as 1 and higher limit as 1000.

(ii) Series R 20/4 (…, 8, …) indicates a derived series comprising of every fourth term of the R20 series, unlimited in both sides and having the number 8 inside the series.

(iii) Series R 20/3 (200, …) indicates a derived series comprising of every third term of the R20 series and having the lower limit as 200 and without any higher limit.

(iv) Series R 20/3 (…200) indicates a derived series comprising of every third term of the R20 series and having the higher limit as 200 and without any lower limit.

In the second method, the derived series is obtained by increasing the numbers of a particular basic series. Let us consider an example of a derived series of numbers ranging from 1 to 1000 based on the R5 series. From Table 1.2, the numbers belonging to the R5 series from 1 to 10 are as follows:

1, 1.6, 2.5, 4, 6.3, 10

The next numbers are obtained by multiplying the above numbers by 10. They are as follows:

16, 25, 40, 63, 100

The same procedure is repeated and the next numbers are obtained by multiplying the above numbers by 10.

160, 250, 400, 630, 1000

Therefore, the complete derived series on the basis of R5 series is as follows:

1, 1.6, 2.5, 4, 6.3, 10, 16, 25, 40, 63, 100, 160, 250, 400, 630, 1000

The advantage of derived series is that one can obtain geometric series for any range of numbers, that is, with any value of the first and the last numbers. Also, one can have any intermediate numbers between these two limits.

Example 1.1 *Find out the numbers of the R5 basic series from 1 to 10.*

Solution

Step I *Calculation of series factor*

The series factor for the R5 series is given by

$$\sqrt[5]{10} = 1.5849$$

Step II *Calculation of numbers*

The series R5 is established by taking the first number and multiplying it by a series factor to get the second number. The second number is again multiplied by a series factor to get the third number. This procedure is continued until the complete series is built up. The numbers thus obtained are rounded.

First number = 1

Second number = 1 (1.5849) = 1.5849 = (1.6)

Third number = $(1.5849)(1.5849) = (1.5849)^2$
$= 2.51 = (2.5)$

Fourth number = $(1.5849)^2(1.5849) = (1.5849)^3$
$= 3.98 = (4)$

Fifth number = $(1.5849)^3(1.5849) = (1.5849)^4$
$= (6.3)$

Sixth number = $(1.5849)^4(1.5849) = (1.5849)^5$
$= (10)$

In above calculations, the rounded numbers are shown in brackets.

Example 1.2 *Find out the numbers of R20/4(100, ..., 1000) derived series.*

Solution

Step I *Calculation of series factor*

The series factor for the R20 series is given by

$$\sqrt[20]{10} = 1.122$$

Step II *Calculation of ratio factor*

Since every fourth term of the R20 series is selected, the ratio factor (ϕ) is given by,

$$\phi = (1.122)^4 = 1.5848$$

Step III *Calculation of numbers*

First number = 100

Second number = 100(1.5848) = 158.48 = (160)

Third number = $100(1.5848)(1.5848) = 100(1.5848)^2$
$= 251.16 = (250)$

Fourth number = $100(1.5848)^2(1.5848)$
$= 100(1.5848)^3 = 398.04 = (400)$

Fifth number = $100(1.5848)^3(1.5848)$
$= 100(1.5848)^4 = 630.81 = (630)$

Sixth number = $100(1.5848)^4(1.5848)$
$= 100(1.5848)^5 = 999.71 = (1000)$

In the above calculations, the rounded numbers are shown in brackets. The complete series is given by

100, 160, 250, 400, 630 and 1000

Example 1.3 *A manufacturer is interested in starting a business with five different models of tractors ranging from 7.5 to 75 kW capacities. Specify power capacities of the models. There is an expansion plan to further increase the number of models from five to nine to fulfill the requirement of farmers. Specify the power capacities of the additional models.*

Solution

Part I *Starting Plan*

Step I *Calculation of ratio factor*

Let us denote the ratio factor as (ϕ). The derived series is based on geometric progression. The power rating of five models will as follows,

$7.5(\phi)^0$, $7.5(\phi)^1$, $7.5(\phi)^2$, $7.5(\phi)^3$ and $7.5(\phi)^4$

The maximum power rating is 75 kW. Therefore,

$$7.5(\phi)^4 = 75 \quad \text{or} \quad \phi = \left(\frac{75}{7.5}\right)^{1/4}$$

$$= (10)^{1/4} = \sqrt[4]{10} = 1.7783$$

Step II Power rating of models

Rating of first model = (7.5) kW

Rating of second model = 7.5(1.7783) = 13.34
= (13) kW

Rating of third model = $7.5(1.7783)^2$ = 23.72
= (24) kW

Rating of fourth model = $7.5(1.7783)^3$ = 42.18
= (42) kW

Rating of fifth model = $7.5(1.7783)^4$ = 75.0
= (75) kW

Part II Expansion Plan

Step III Calculation of ratio factor

When the number of models is increased to nine, the power rating of nine models will be as follows:

$7.5(\phi)^0$, $7.5(\phi)^1$, $7.5(\phi)^2$, $7.5(\phi)^3$, $7.5(\phi)^4$, …, $7.5(\phi)^8$

The maximum power rating is 75 kW. Therefore,

$$7.5(\phi)^8 = 75 \quad \text{or} \quad \phi = \left(\frac{75}{7.5}\right)^{1/8}$$

$$= (10)^{1/8} = 1.3335$$

Step IV Power rating of models

The power rating of the nine models will be as follows:

First model = 7.5 $(1.3335)^0$ = (7.5) kW
Second model = 7.5 $(1.3335)^1$ = 10.00 = (10) kW
Third model = 7.5 $(1.3335)^2$ = 13.34 = (13) kW
Fourth model = 7.5 $(1.3335)^3$ = 17.78 = (18) kW
Fifth model = 7.5 $(1.3335)^4$ = 23.72 = (24) kW
Sixth model = 7.5 $(1.3335)^5$ = 31.62 = (32) kW
Seventh model = 7.5 $(1.3335)^6$ = 42.17 = (42) kW
Eighth model = 7.5 $(1.3335)^7$ = 56.24 = (56) kW
Ninth model = 7.5 $(1.3335)^8$ = 74.99 = (75) kW

Part III Power capacities of additional models

It is observed that there are four additional models having power ratings as 10, 18, 32 and 56 kW.

Example 1.4 *It is required to standardize eleven shafts from 100 to 1000 mm diameter. Specify their diameters.*

Solution

Step I Calculation of ratio factor

The diameters of shafts will be as follows:

$100(\phi)^0$, $100(\phi)^1$, $100(\phi)^2$, $100(\phi)^3$, …, $100(\phi)^{10}$

The maximum diameter is 1000 mm. Therefore,

$$100(\phi)^{10} = 1000 \quad \text{or} \quad \phi = \left(\frac{1000}{100}\right)^{1/10}$$

$$= (10)^{1/10} = \sqrt[10]{10}$$

Therefore the diameters belong to the R10 series.

Step II Calculation of shaft diameters

Since the minimum diameter is 100 mm, the values of the R10 series given in Table 1.2 are multiplied by 100. The diameter series is written as follows:

100, 125, 160, 200, 250, 315, 400, 500, 630, 800 and 1000 mm

1.9 AESTHETIC CONSIDERATIONS IN DESIGN

Each product has a definite purpose. It has to perform specific functions to the satisfaction of customer. The contact between the product and the people arises due to the sheer necessity of this functional requirement. The functional requirement of an automobile car is to carry four passengers at a speed of 60 km/hr. There are people in cities who want to go to their office at a distance of 15 km in 15 minutes. So they purchase a car. The specific function of a domestic refrigerator is to preserve vegetables and fruits for a week. There is a housewife in the city who cannot go to the market daily and purchase fresh vegetables. Therefore, she purchases the refrigerator. It is seen that such functional requirements bring products and people together.

However, when there are a number of products in the market having the same qualities of efficiency, durability and cost, the customer is attracted towards the most appealing product.

External appearance is an important feature, which not only gives grace and lustre to the product but also dominates sale in the market. This is particularly true for consumer durables like automobiles, household appliances and audiovisual equipment.

The growing realisation of the need of aesthetic considerations in product design has given rise to a separate discipline known as '*industrial design*'. The job of an industrial designer is to create new forms and shapes, which are aesthetically pleasing. The industrial designer has, therefore, become the fashion maker in hardware.

Like in fashion, the outward appearance of a product undergoes many changes over the years. There are five basic forms—step, stream, taper, shear and sculpture. The *step form* is similar to the shape of a 'skyscraper' or multistorey building. This involves shapes with a vertical accent rather than a horizontal. The *stream* or *streamline form* is seen in automobiles and aeroplane structures. The *taper form* consists of tapered blocks interlocked with tapered plinths or cylinders. The *shear form* has a square outlook, which is suitable for free-standing engineering products. The *sculpture form* consists of ellipsoids, paraboloids and hyperboloids. The sculpture and stream forms are suitable for mobile products like vehicles, while step and shear forms are suitable for stationary products.

There is a relationship between functional requirement and appearance of the product. In many cases, functional requirements result in shapes which are aesthetically pleasing. The evolution of the streamlined shape of the Boeing is the result of studies in aerodynamics for effortless speed. The robust outlook and sound proportions of a high-capacity hydraulic press are the results of requirements like rigidity and strength. The objective of chromium plating of the parts of household appliances is corrosion resistance rather than pleasing appearance.

Selection of proper colour is an important consideration in product aesthetics. The choice of colour should be compatible with the conventional ideas of the operator. Many colours are associated with different moods and conditions. Morgan has suggested the meaning of colours that are given in Table 1.3.

Table 1.3 *Meaning of colour*

Colour	*Meaning*
Red	Danger-Hazard-Hot
Orange	Possible danger
Yellow	Caution
Green	Safety
Blue	Caution-Cold
Grey	Dull

The external appearance of the product does not depend upon only the two factors of form and colour. It is a cumulative effect of a number of factors such as rigidity and resilience, tolerances and surface finish, motion of individual components, materials, manufacturing methods and noise. The industrial designer should select a form which is in harmony with the functional requirements of the product. The economics and availability of surface-treating processes like anodizing, plating, blackening and painting should be taken into account before finalising the external appearance of the product.

1.10 ERGONOMIC CONSIDERATIONS IN DESIGN

Ergonomics is defined as the relationship between man and machine and the application of anatomical, physiological and psychological principles to solve the problems arising from man–machine relationship. The word 'ergonomics' is coined from two Greek words—'*ergon*', which means 'work' and '*nomos*', which means 'natural laws'. Ergonomics means the natural laws of work. From design considerations, the topics of ergonomic studies are as follows:

(i) Anatomical factors in the design of a driver's seat
(ii) Layout of instrument dials and display panels for accurate perception by the operators
(iii) Design of hand levers and hand wheels
(iv) Energy expenditure in hand and foot operations

(v) Lighting, noise and climatic conditions in machine environment

Ergonomists have carried out experiments to determine the best dimensions of a driver's seat, the most convenient hand or foot pressure or dimensions of levers and hand wheels.

The machine is considered as an entity in itself in machine design. However, ergonomists consider a man–machine joint system, forming a closed loop as shown in Fig. 1.6. From display instruments, the operator gets the information about the operations of the machine. If he feels that a correction is necessary, he will operate the levers or controls. This, in turn, will alter the performance of the machine, which will be indicated on display panels. The contact between man and machine in this closed-loop system arises at two places—display instruments, which give information to the operator, and controls with which the operator adjusts the machine.

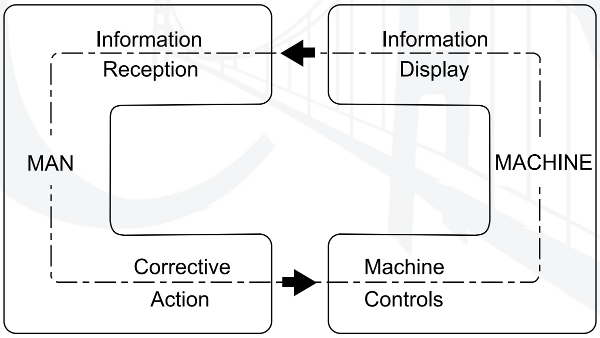


Fig. 1.6 *Man–Machine Closed-Loop System*

The visual display instruments are classified into three groups:

(i) Displays giving quantitative measurements, such as speedometer, voltmeter or energy meter

(ii) Displays giving the state of affairs, such as the red lamp indicator

(iii) Displays indicating predetermined settings, e.g., a lever which can be set at 1440 rpm, 720 rpm or 'off' position for a two-speed electric motor.

Moving scale or dial-type instruments are used for quantitative measurements, while lever-type indicators are used for setting purposes. The basic objective behind the design of displays is to minimise fatigue to the operator, who has to observe them continuously. The ergonomic considerations in the design of displays are as follows:

(i) The scale on the dial indicator should be divided in suitable numerical progression like 0–10–20–30 and not 0–5–30–55.

(ii) The number of subdivisions between numbered divisions should be minimum.

(iii) The size of letters or numbers on the indicator should be as follows:

$$\text{Height of letter or number} \geq \frac{\text{Reading distance}}{200}$$

(iv) Vertical figures should be used for stationary dials, while radially oriented figures are suitable for rotating dials.

(v) The pointer should have a knife-edge with a mirror in the dial to minimise parallax error.

The controls used to operate the machines consist of levers, cranks, hand wheels, knobs, switches, push buttons and pedals. Most of them are hand operated. When a large force is required to operate the controls, levers and hand wheels are used. When the operating forces are light, push buttons or knobs are preferred. The ergonomic considerations in the design of controls are as follows:

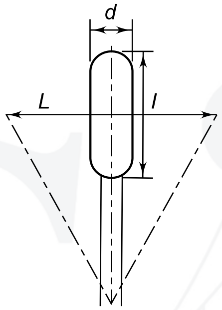
(i) The controls should be easily accessible and logically positioned. The control operation should involve minimum motions and avoid awkward movements.

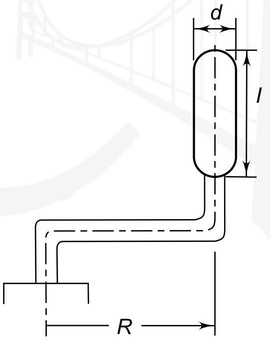
(ii) The shape of the control component, which comes in contact with hands, should be in conformity with the anatomy of human hands.

(iii) Proper colour produces beneficial psychological effects. The controls should be painted in red colour in the grey background of machine tools to call for attention.

The aim of ergonomics is to reduce the operational difficulties present in a man–machine joint system, and thereby reduce the resulting physical and mental stresses.

The shape and dimensions of certain machine elements like levers, cranks and hand wheels are decided on the basis of ergonomic studies. The resisting force, i.e., the force exerted by the operator without undue fatigue is also obtained by ergonomic considerations. Ergonomic textbooks[3, 4] give exhaustive details of the dimensions and resisting forces of different control elements. In this article, we will restrict the consideration to levers, cranks and hand wheels, which are frequently required in machine design. The nomenclature for the dimensions of lever, crank and hand wheel are shown in Fig. 1.7. Their dimensions and magnitude of resisting force are given in Tables 1.4 to 1.6.





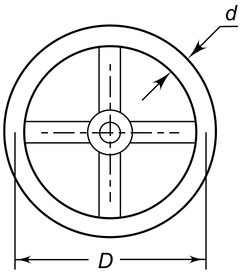


Fig. 1.7 *Control Elements: (a) Lever (b) Crank (c) Hand Wheel*

Table 1.4 *Dimensions and resisting force for lever*

	d	*l*	*L*	*P*
Minimum	40	75	–	–
Maximum	70	–	950	90

d = handle diameter (mm)
l = grasp length (mm)
L = displacement of lever (mm)
P = resisting force (N)

Table 1.5 *Dimensions and resisting force for crank with heavy load (more than 25 N)*

	d	*l*	*R*	*P*
Minimum	25	75	125	–
Maximum	75	–	500	40

d = handle diameter (mm)
l = grasp length (mm)
R = crank radius (mm)
P = resisting force (N)

Table 1.6 *Dimensions and resisting force for hand wheel*

	D	*d*	*P*
Minimum	175	20	–
Maximum	500	50	240

D = mean diameter of hand wheel (mm)
d = rim diameter (mm)
P = resisting force with two hands (N)

1.11 CONCURRENT ENGINEERING

Conventional design process is sequential, where the main activities are executed in a sequence as shown in Fig. 1.8. It begins with market survey, with the objective of finding out the requirements of the customer. This information is then handed over to the design department in the form of a '*product brief*'. The design department prepares the design and makes a few prototype samples for testing. The assembly and detail drawings are then prepared and passed on to the production department for their approval. Usually, the

[3] B.M. Pulat—*Fundamentals of Industrial Ergonomics*—Prentice Hall Inc.—1992.
[4] Hywel Murell—*Ergonomics—Man in his Working Environment*'—Chapman and Hall—1986.

production department suggests changes in design from manufacturing considerations and the drawings are sent back to the design department. This process of sending the drawings by the design department to the production department and from the production department back to the design department continues till the design is finalised and in between valuable time is lost. Engineers who design new products and manufacturing personnel who have to figure out how to make the product are often at odds because of their different backgrounds and points of view. Many times, designers are creative artists who overlook the capabilities of the plant's machinery. Manufacturing engineers on the other hand, are realistic. They prefer standard materials, simple manufacturing methods, standard components and as few of them as possible. Due to this difference in background, many a times the designer comes up with a new product and throws it 'over the wall' to the manufacturing personnel, who says "We can't make this". Then there is a lot of finger pointing. In the *sequential design process*, the production department suggests a number of design modifications, after the design is finalised. In past, this has resulted in time consuming re-designs and missed time schedules in a number of projects. Personality conflicts and departmental barriers often create more problems. Due to these reasons, sequential design is often called 'over the wall' design.

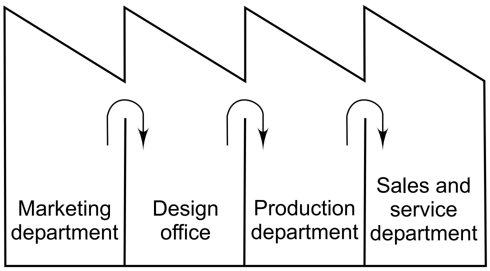


Fig. 1.8 *Sequential Design Process*

In recent years, there is a fundamental shift in the way the designs are prepared. The sequential design process is being replaced by *simultaneous* or *concurrent engineering*, where various activities are carried out in parallel, instead of in series. The trend is to bring the design and manufacturing activities together as a single engineering discipline.

Concurrent engineering is defined as the design process that brings both design and manufacturing engineers together during the early phases of design process. In this process, a team of specialists examines the design from different angles as shown in Fig. 1.9. The specialists include a manufacturing engineer, tool engineer, field personnel, reliability engineer and safety engineer. They consider various aspects of the product such as feasibility, manufacturability, assembly, testability, performance, reliability, maintainability, safety and cost. All these aspects are simultaneously considered early in the design stage. For example, manufacturing and assembly is simultaneously considered with stress analysis. This results in smaller number of modifications in the design at a later stage and reduces the 'time interval' from the conceptual stage to the marketing stage.

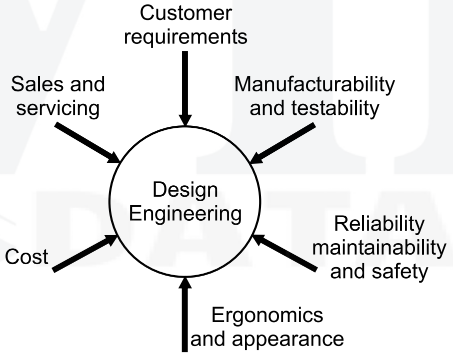


Fig. 1.9 *Simultaneous Design Process*

An example of a company making measuring instruments is interesting[5]. Keithley Instruments, Solon, USA applied the concept of concurrent engineering in development of a digital multi-meter. The number of parts in the new instrument were reduced from 131 to 76, the number of assembly screws were reduced from 30 to 8 and assembly time was reduced by 35% requiring only one screwdriver instead of multiple assembly tools.

[5] 'Design for Manufacture: key to success'—Machine Design—Vol. 64, No. 10—May, 1992.

Short-Answer Questions

1.1 Define machine design.
1.2 What is the final outcome of a machine design process?
1.3 Name the various requirements of a product giving suitable example.
1.4 What are the basic requirements of a machine element?
1.5 What are the steps involved in design of a machine element?
1.6 Define design synthesis.
1.7 Distinguish between design synthesis and design analysis.
1.8 What is standardization?
1.9 What are the three basic types of standards used in a design office?
1.10 What do you understand by size of a product? Give examples.
1.11 What are preferred numbers?
1.12 How many basic series are used? How will you denote them?
1.13 What is a derived series?
1.14 How will you form a derived series?
1.15 What is industrial design?
1.16 Define ergonomics.
1.17 Explain man–machine joint system.
1.18 What is concurrent engineering?
1.19 Distinguish between sequential design and concurrent engineering.

Problems for Practice

1.1 Find out the numbers of R10 basic series from 1 to 10.

1.2 Find out the numbers of R20/3 (200,…) derived series.
[*200, 280(282.5), 400(399.03), 560(563.63), 800(796.13), 1120(1124.53), …*] (*ϕ = 1.4125*)

1.3 It is required to standardise load-carrying capacities of dumpers in a manufacturing unit. The maximum and minimum capacities of such dumpers are 40 and 630 kN, respectively. The company is interested in developing seven models in this range. Specify their load carrying capacities.
[*40, 63(63.33), 100(100.26), 160(158.73), 250(251.31), 400(397.87), 630(629.90)*]
(*ϕ = 1.5832*)

1.4 It is required to standardise 11 speeds from 72 to 720 rpm for a machine tool. Specify the speeds.
[*72, 90.64, 114.11, 143.65, 180.84, 227.66, 286.60, 360.80, 454.22, 571.81, 719.85 rpm*]

Engineering Materials

Chapter 2

2.1 STRESS–STRAIN DIAGRAMS

A very useful information concerning the behaviour of material and its usefulness for engineering applications can be obtained by making a tension test and plotting a curve showing the variation of stress with respect to strain. A *tension test* is one of the simplest and basic tests and determines values of number of parameters concerned with mechanical properties of materials like strength, ductility and toughness. The following information can be obtained from a tension test:

(i) Proportional limit
(ii) Elastic limit
(iii) Modulus of elasticity
(iv) Yield strength
(v) Ultimate tension strength
(vi) Modulus of resilience
(vii) Modulus of toughness
(viii) Percentage elongation
(ix) Percentage reduction in area

The specimen used in a tension test is illustrated in Fig. 2.1. The shape and dimensions of this specimen are standardised. They should conform to IS 1608 : 1972[1]. The cross-section of the specimen can be circular, square or rectangular. The standard gauge length l_0 is given by,

$$l_0 = 5.65\sqrt{A_0},$$

where A_0 is the cross-sectional area of the specimen.

For circular cross-section,

$$l_0 \approx 5d_0$$

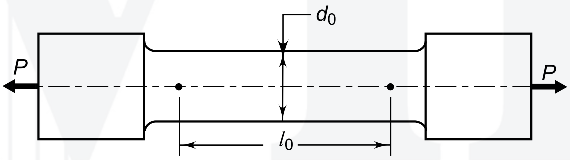


Fig. 2.1 *Specimen of Tension-test*

In a tension test, the specimen is subjected to axial tension force, which is gradually increased and the corresponding deformation is measured. Initially, the gauge length is marked on the specimen and initial dimensions d_0 and l_0 are measured before starting the test. The specimen is then mounted on the machine and gripped in the jaws. It is then subjected to an axial tension force, which is increased by suitable increments. After each increment, the amount by which the gauge length l_0 increases, i.e., deformation of gauge length, is measured by an extensometer. The procedure of measuring the tension force and corresponding deformation is continued till fracture

[1] IS 1608–1972: Method for tensile testing of steel products.

occurs and the specimen is broken into two pieces. The tensile force divided by the original cross-sectional area of the specimen gives stress, while the deformation divided by gauge length gives the strain in the specimen.

Therefore, the results of a tension test are expressed by means of stress–strain relationship and plotted in the form of a graph. A typical stress–strain diagram for ductile materials like mild steel is shown in Fig. 2.2. The following properties of a material can be obtained from this diagram:

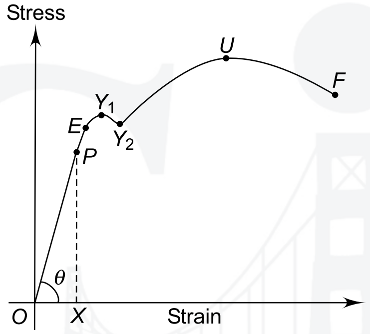


Fig. 2.2 *Stress–Strain Diagram of Ductile Materials*

(i) Proportional Limit It is observed from the diagram that stress–strain relationship is linear from the point O to P. OP is a straight line and after the point P, the curve begins to deviate from the straight line. *Hooke's law* states that stress is directly proportional to strain. Therefore, it is applicable only up to the point P. *The term proportional limit is defined as the stress at which the stress–strain curve begins to deviate from the straight line*. Point P indicates the proportional limit.

(ii) Modulus of Elasticity *The modulus of elasticity or Young's modulus (E) is the ratio of stress to strain up to the point P*. It is given by the slope of the line OP. Therefore,

$$E = \tan\theta = \frac{PX}{OX} = \frac{\text{stress}}{\text{strain}}$$

(iii) Elastic Limit Even if the specimen is stressed beyond the point P and up to the point E, it will regain its initial size and shape when the load is removed. This indicates that the material is in elastic stage up to the point E. Therefore, E is called the elastic limit. *The elastic limit of the material is defined as the maximum stress without any permanent deformation*.

The proportional limit and elastic limit are very close to each other, and it is difficult to distinguish between points P and E on the stress–strain diagram. In practice, many times, these two limits are taken to be equal.

(iv) Yield Strength When the specimen is stressed beyond the point E, plastic deformation occurs and the material starts yielding. During this stage, it is not possible to recover the initial size and shape of the specimen on the removal of the load. It is seen from the diagram that beyond the point E, the strain increases at a faster rate up to the point Y_1. In other words, there is an appreciable increase in strain without much increase in stress. In case of mild steel, it is observed that there is a small reduction in load and the curve drops down to the point Y_2 immediately after yielding starts. The points Y_1 and Y_2 are called the upper and lower yield points, respectively. For many materials, the points Y_1 and Y_2 are very close to each other and in such cases, the two points are considered as same and denoted by Y. The stress corresponding to the yield point Y is called the yield strength. *The yield strength is defined as the maximum stress at which a marked increase in elongation occurs without increase in the load.*

Many varieties of steel, especially heat-treated steels and cold-drawn steels, do not have a well-defined yield point on the stress–strain diagram. As shown in Fig. 2.3, the material yields gradually after passing through the elastic limit E. If the loading is stopped at the point Y, at a stress level slightly higher than the elastic limit E, and the specimen is unloaded and readings taken, the curve would follow the dotted line and a permanent set or plastic deformation will exist. The strain corresponding to this permanent deformation is indicated by OA. *For such materials, which do not exhibit a well-defined yield point, the yield strength is defined as the stress corresponding to* *a permanent set of 0.2% of gauge length*. In such cases, the yield strength is determined by the *offset method*. A distance *OA* equal to 0.002 mm/mm strain (corresponding to 0.2% of gauge length) is marked on the *X*-axis. A line is constructed from the point *A* parallel to the straight line portion *OP* of the stress–strain curve. The point of intersection of this line and the stress–strain curve is called *Y* or the yield point and the corresponding stress is called 0.2% yield strength.

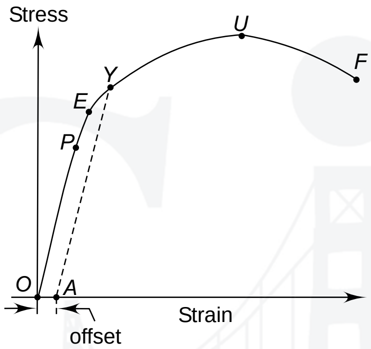


Fig. 2.3 *Yield Stress by Offset Method*

The terms *proof load* or *proof strength* are frequently used in the design of fasteners. Proof strength is similar to yield strength. It is determined by the offset method; however the offset in this case is 0.001 mm/mm corresponding to a permanent set of 0.1% of gauge length. *0.1% Proof strength, denoted by symbol Rp0.1, is defined as the stress which will produce a permanent extension of 0.1% in the gauge length of the test specimen*. The proof load is the force corresponding to proof stress.

(v) Ultimate Tensile Strength We will refer back to the stress–strain diagram of ductile materials illustrated in Fig. 2.2. After the yield point Y_2, plastic deformation of the specimen increases. The material becomes stronger due to strain hardening, and higher and higher load is required to deform the material. Finally, the load and corresponding stress reach a maximum value, as given by the point *U*. *The stress corresponding to the point U is called the ultimate strength. The ultimate tensile strength is the maximum stress that can be reached in the tension test.*

For ductile materials, the diameter of the specimen begins to decrease rapidly beyond the maximum load point *U*. There is a localised reduction in the cross-sectional area, called *necking*. As the test progresses, the cross-sectional area at the neck decreases rapidly and fracture takes place at the narrowest cross-section of the neck. This fracture is shown by the point *F* on the diagram. The stress at the time of fracture is called *breaking strength*. It is observed from the stress–strain diagram that there is a downward trend after the maximum stress has been reached. The breaking strength is slightly lower than the ultimate tensile strength.

The stress–strain diagram for brittle materials like cast iron is shown in Fig. 2.4. It is observed that such materials do not exhibit the yield point. The deviation of the stress–strain curve from straight line begins very early and fracture occurs suddenly at the point *U* with very small plastic deformation and without necking. Therefore, ultimate tensile strength is considered as failure criterion in brittle materials.

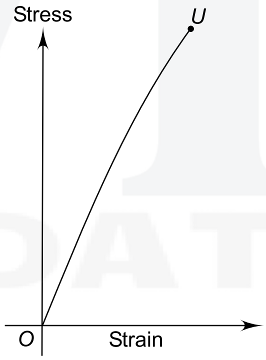


Fig. 2.4 *Stress-Strain Diagram of Brittle Materials*

(vi) Percentage Elongation After the fracture, the two halves of the broken test specimen are fitted together as shown in Fig. 2.5(b) and the extended gauge length *l* is measured. *The percentage elongation is defined as the ratio of the increase in the length of the gauge section of the specimen to original gauge length, expressed in per cent*. Therefore,

$$\text{percentage elongation} = \left(\frac{l - l_0}{l_0}\right) \times 100$$

Ductility is measured by percentage elongation.

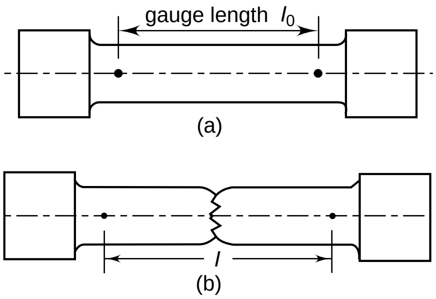


Fig. 2.5 *Determination of Percentage Elongation: (a) Original Test Piece (b) Broken Test Piece*

(vii) Percentage Reduction in Area *Percentage reduction in area is defined as the ratio of decrease in cross-sectional area of the specimen after fracture to the original cross-sectional area, expressed in per cent.* Therefore,

$$\text{percentage reduction in area} = \left(\frac{A_0 - A}{A_0}\right) \times 100$$

where,

A_0 = original cross-sectional area of the test specimen

A = final cross-sectional area after fracture

Percentage reduction in area, like percentage elongation, is a measure of the ductility of the material. If porosity or inclusions are present in the material or if damage due to overheating of the material has occurred, the percentage elongation as well as percentage reduction in area are drastically decreased. Therefore, percentage elongation or percentage reduction in area is considered as an *index of quality* for the material.

2.2 MECHANICAL PROPERTIES OF ENGINEERING MATERIALS

Materials are characterised by their properties. They may be hard, ductile or heavy. Conversely, they may be soft, brittle or light. The mechanical properties of materials are the properties that describe the behaviour of the material under the action of external forces. They usually relate to elastic and plastic behaviour of the material. Mechanical properties are of significant importance in the selection of material for structural machine components. In this article, we will consider the following mechanical properties:

(1) strength
(2) elasticity
(3) plasticity
(4) stiffness
(5) resilience
(6) toughness
(7) malleability
(8) ductility
(9) brittleness
(10) hardness

Strength is defined as the ability of the material to resist, without rupture, external forces causing various types of stresses. Strength is measured by different quantities. Depending upon the type of stresses induced by external loads, strength is expressed as tensile strength, compressive strength or shear strength. Tensile strength is the ability of the material to resist external load causing tensile stress, without fracture. Compressive strength is the ability to resist external load that causes compressive stress, without failure. The terms yield strength and ultimate tensile strength are explained in the previous article.

Elasticity is defined as the ability of the material to regain its original shape and size after the deformation, when the external forces are removed. All engineering metals are elastic but the degree of elasticity varies. Steel is perfectly elastic within a certain elastic limit. The amount of elastic deformation which a metal can undergo is very small. During the elastic deformation, the atoms of the metal are displaced from their original positions but not to the extent that they take up new positions. Therefore, when the external force is removed, the atoms of the metal return to their original positions and the metal takes back its original shape.

Plasticity is defined as the ability of the material to retain the deformation produced under the load on a permanent basis. In this case, the external forces deform the metal to such an extent that it cannot fully recover its original dimensions. During plastic deformation, atoms of the metal are permanently displaced from their original positions and take up new positions. The ability of some metals to be extensively deformed in the plastic range without fracture is one of the useful engineering properties of materials. For example, the extensive plastic deformability of low carbon steels enables automobile parts such as the body, hood and doors to be stamped out without fracture. The difference between elasticity and plasticity is as follows:

(i) Elasticity is the ability of a metal to regain its original shape after temporary deformation under an external force. Plasticity is the ability to retain the deformation permanently even after the load is removed.

(ii) The amount of elastic deformation is very small while plastic deformation is relatively more.

(iii) During elastic deformation, atoms of metal are temporarily displaced from their original positions but return back when the load is removed. During plastic deformation, atoms of metal are permanently displaced from their original positions and take up new positions.

(iv) For majority of materials, the stress–strain relationship is linear in the elastic range and non-linear in the plastic range.

(v) Elasticity is an important consideration in machine-tool components while plasticity is desirable for components made by press working operations.

Stiffness or rigidity is defined as the ability of the material to resist deformation under the action of an external load. All materials deform when stressed, to a more or less extent. For a given stress within elastic limit, the material that deforms least is the stiffest. *Modulus of elasticity* is the measure of stiffness. The values of the modulus of elasticity for aluminium alloy and carbon steel are 71 000 and 207 000 N/mm^2 respectively. Therefore, carbon steel is stiffer than aluminium alloy. Stiffness is an important consideration in the design of transmission shafting.

Resilience is defined as the ability of the material to absorb energy when deformed elastically and to release this energy when unloaded. A resilient material absorbs energy within elastic range without any permanent deformation. This property is essential for spring materials. Resilience is measured by a quantity, called *modulus of resilience*, which is the strain energy per unit volume that is required to stress the specimen in a tension test to the elastic limit point. It is represented by the area under the stress–strain curve from the origin to the elastic limit point.

Toughness is defined as the ability of the material to absorb energy before fracture takes place. In other words, toughness is the energy for failure by fracture. This property is essential for machine components which are required to withstand impact loads. Tough materials have the ability to bend, twist or stretch before failure takes place. All structural steels are tough materials. Toughness is measured by a quantity called *modulus of toughness*. Modulus of toughness is the total area under stress–strain curve in a tension test, which also represents the work done to fracture the specimen. In practice, toughness is measured by the *Izod and Charpy impact testing machines*. Toughness decreases as the temperature increases. The difference between resilience and toughness is as follows:

(i) Resilience is the ability of the material to absorb energy within elastic range. Toughness is the ability to absorb energy within elastic and plastic range.

(ii) Modulus of resilience is the area below the stress–strain curve in a tension test up to the yield point. Modulus of toughness is the total area below the stress–strain curve.

(iii) Resilience is essential in spring applications while toughness is required for components subjected to bending, twisting, stretching or to impact loads. Spring steels are resilient while structural steels are tough.

Figures 2.6(a) and (b) show the difference between modulii of resilience and toughness.

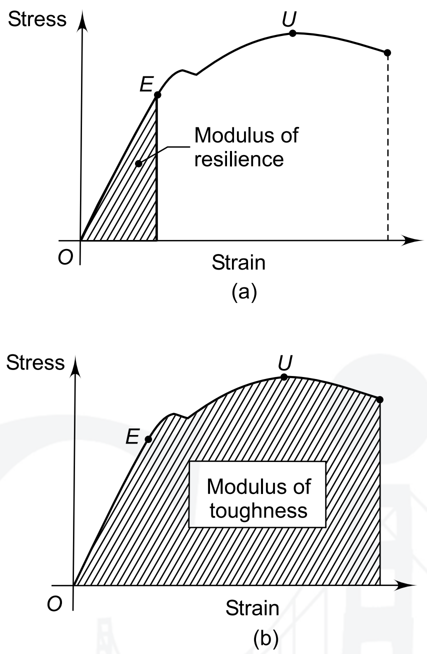


Fig. 2.6 *Modulii of Resilience and Toughness*

Malleability is defined as the ability of a material to deform to a greater extent before the sign of crack, when it is subjected to compressive force. The term 'malleability' comes from a word meaning 'hammer', and in a narrow sense, it means the ability to be hammered out into thin sections. Malleable metals can be rolled, forged or extruded because these processes involve shaping under compressive force. Low carbon steels, copper and aluminium are malleable metals. In general, malleability increases with temperature. Therefore, processes like forging or rolling are hot working processes where hot ingots or slabs are given a shape.

Ductility is defined as the ability of a material to deform to a greater extent before the sign of crack, when it is subjected to tensile force. In other words, ductility is the permanent strain that accompanies fracture in a tension test. Ductile materials are those materials which deform plastically to a greater extent prior to fracture in a tension test. Mild steel, copper and aluminium are ductile materials. Ductile metals can be formed, drawn or bent because these processes involve shaping under tension. Ductility is a desirable property in machine components which are subjected to unanticipated overloads or impact loads. Ductility is measured in units of percentage elongation or percentage reduction in area in a tension test. The ductility of metal decreases as the temperature increases because metals become weak at increasing temperature. All ductile materials are also malleable; however, the converse is not always true. Some metals are soft but weak in tension and, therefore, tend to tear apart under tension. Both malleability as well as ductility are reduced by the presence of impurities in the metal. The difference between malleability and ductility is as follows:

(i) Malleability is the ability of a material to deform under compressive force. Ductility is the ability to deform under tensile force.

(ii) Malleability increases with temperature, while ductility decreases with increasing temperature.

(iii) All ductile materials are also malleable, but the converse is not true.

(iv) Malleability is an important property when the component is forged, rolled or extruded. Ductility is desirable when the component is formed or drawn. It is also desirable when the machine component is subjected to shock loads.

Brittleness is the property of a material which shows negligible plastic deformation before fracture takes place. Brittleness is the opposite to ductility. A brittle material is that which undergoes little plastic deformation prior to fracture in a tension test. Cast iron is an example of brittle material. In ductile materials, failure takes place by yielding. Brittle components fail by sudden fracture. *A tensile strain of 5% at fracture in a tension test is considered as the dividing line between ductile and brittle materials*. The difference between ductility and brittleness is as follows:

(i) Ductile materials deform to a greater extent before fracture in a tension test. Brittle materials show negligible plastic deformation prior to fracture.

(ii) Steels, copper and aluminium are ductile materials, while cast iron is brittle.

(iii) The energy absorbed by a ductile specimen before fracture in a tension test is more, while brittle fracture is accompanied by negligible energy absorption.

(iv) In ductile materials, failure takes place by yielding which is gradual. Brittle materials fail by sudden fracture.

Hardness is defined as the resistance of the material to penetration or permanent deformation. It usually indicates resistance to abrasion, scratching, cutting or shaping. Hardness is an important property in the selection of material for parts which rub on one another such as pinion and gear, cam and follower, rail and wheel and parts of ball bearing. Wear resistance of these parts is improved by increasing surface hardness by case hardening. There are four primary methods of measuring hardness—Brinell hardness test, Rockwell hardness test, Vicker hardness test and Shore scleroscope. In the first three methods, an indenter is pressed onto the surface under a specific force. The shape of the indenter is either a ball, pyramid or cone. The indenters are made of diamond, carbide or hardened steel, which are much more harder than the surface being tested. Depending upon the cross-sectional area and depth of indentation, hardness is expressed in the form of an empirical number like *Brinell hardness number*. In a Shore scleroscope, the height of rebound from the surface being tested indicates the hardness. Hardness test is simpler than tension test. It is non-destructive because a small indentation may not be detrimental to the performance of the product.

Hardness of the material depends upon the resistance to plastic deformation. Therefore, as the hardness increases, the strength also increases. For certain metals like steels, empirical relationships between strength and hardness are established. For steels,

$$S_{ut} = 3.45\ (\text{BHN})$$

where S_{ut} is ultimate tensile strength in N/mm^2.

2.3 CAST IRON

Cast iron is a generic term, which refers to a family of materials that differ widely in their mechanical properties. *By definition, cast iron is an alloy of iron and carbon, containing more than 2% of carbon*. In addition to carbon, cast iron contains other elements like silicon, manganese, sulphur and phosphorus. There is a basic difference between steels and cast iron. Steels usually contain less than 1% carbon while cast iron normally contains 2 to 4% carbon. Typical composition of ordinary cast iron is as follows:

carbon = 3.0 – 4.0%
silicon = 1.0 – 3.0%
manganese = 0.5 – 1.0%
sulphur = up to 0.1%
phosphorus = up to 0.1%
iron = remainder

The mechanical properties of cast iron components are inferior to the parts, which are machined from rolled steels. However, even with this drawback, cast iron offers the only choice under certain conditions. From design considerations, cast iron offers the following advantages:

(i) It is available in large quantities and is produced on a mass scale. The tooling required for the casting process is relatively simple and inexpensive. This reduces the cost of cast iron products.

(ii) Cast iron components can be given any complex shape without involving costly machining operations.

(iii) Cast iron has a higher compressive strength. The compressive strength of cast iron is three to five times that of steel. This is an advantage in certain applications.

(iv) Cast iron has an excellent ability to damp vibrations, which makes it an ideal choice for machine tool guides and frames.

(v) Cast iron has more resistance to wear even under the conditions of boundary lubrication.

(vi) The mechanical properties of cast iron parts do not change between room temperature and 350°C.

(vii) Cast iron parts have low notch sensitivity.

Cast iron has certain drawbacks. It has a poor tensile strength compared to steel. Cast iron parts are section-sensitive. Even with the same chemical composition, the tensile strength of a cast iron part decreases as the thickness of the section increases. This is due to the low cooling rate of thick sections. For thin sections, the cooling rate is high, resulting in increased hardness and strength. Cast iron does not offer any plastic deformation before failure and exhibits no yield point. The failure of cast iron parts is sudden and total. Cast iron parts are, therefore, not suitable for applications where permanent deformation is preferred over fracture. Cast iron is brittle and has poor impact resistance. The machinability of cast iron parts is poor compared to the parts made of steel.

Cast irons are classified on the basis of distribution of carbon content in their microstructure. There are three popular types of cast iron—grey, malleable and ductile. *Grey cast iron* is formed when the carbon content in the alloy exceeds the amount that can be dissolved. Therefore, some part of carbon precipitates and remains present as 'graphite flakes' distributed in a matrix of ferrite or pearlite or their combination. When a component of grey cast iron is broken, the fractured surface has a grey appearance due to the graphite flakes. Grey cast iron is specified by the symbol FG followed by the tensile strength in N/mm^2 for a 30-mm section. For example, FG 200, in general, means a grey cast iron with an ultimate tensile strength of 200 N/mm^2. Grey cast iron is used for automotive components such as cylinder block, brake drum, clutch plate, cylinder and cylinder head, gears and housing of gear box, flywheel and machine frame, bed and guide.

White cast iron is formed when most of the carbon content in the alloy forms iron carbide and there are no graphite flakes. *Malleable cast iron* is first cast as white cast iron and then converted into malleable cast iron by heat treatment. In malleable cast iron, the carbon is present in the form of irregularly shaped nodules of graphite called 'temper' carbon. There are three basic types of malleable cast iron—blackheart, pearlitic and whiteheart—which are designated by symbols BM, PM and WM, respectively and followed by minimum tensile strength in N/mm^2. For example,

(i) BM 350 is blackheart malleable cast iron with a minimum tensile strength of 350 N/mm^2;

(ii) PM 600 is pearlitic malleable cast iron with a minimum tensile strength of 600 N/mm^2; and

(iii) WM 400 is whiteheart malleable cast iron with a minimum tensile strength of 400 N/mm^2.

Blackheart malleable cast iron has excellent castability and machinability. It is used for brake shoes, pedal, levers, wheel hub, axle housing and door hinges. Whiteheart malleable cast iron is particularly suitable for the manufacture of thin castings which require ductility. It is used for pipe fittings, switch gear equipment, fittings for bicycles and motorcycle frames. Pearlitic malleable iron castings can be hardened by heat treatment. It is used for general engineering components with specific dimensional tolerances.

Ductile cast iron is also called *nodular cast iron* or *spheroidal graphite cast iron*. In ductile cast iron, carbon is present in the form of spherical nodules called 'spherulites' or 'globules' in a relatively ductile matrix. When a component of ductile cast iron is broken, the fractured surface has a bright appearance like steel. Ductile cast iron is designated by the symbol SG (spheroidal graphite) followed by the minimum tensile strength in N/mm^2 and minimum elongation in per cent. For example, SG 800/2 is spheroidal graphite cast iron with a minimum tensile strength of 800 N/mm^2 and a minimum elongation of 2%. Ductile cast iron is used for crankshaft, heavy duty gears and automobile door hinges. Ductile cast iron combines the processing advantages of grey cast iron with the engineering properties of steel. Spheroidal graphite cast iron is dimensionally stable at high temperatures and, therefore, used for furnace doors, furnace components and steam plants. Because of excellent corrosion resistance, it is used for pipelines in chemical and petroleum industries.

Mechanical properties of grey[2], malleable[3] and ductile[4] cast iron are given in Table 2.1.

Table 2.1 *Mechanical properties of cast iron*

Grade	*Tensile strength (Min.) (N/mm²)*	*Elongation (Min.) (%)*	*Hardness (HB)*
(A) Grey cast iron			
FG 150	150	–	130–180
FG 200	200	–	160–220
FG 220	220	–	180–220
FG 260	260	–	180–230
FG 300	300	–	180–230
FG 350	350	–	207–241
FG 400	400	–	207–270
(B) Whiteheart malleable cast iron			
WM 400	400	5	220 (Max.)
WM 350	350	3	230 (Max.)
(C) Blackheart malleable cast iron			
BM 350	350	10	150 (Max.)
BM 320	320	12	150 (Max.)
BM 300	300	6	150 (Max.)
(D) Pearlitic malleable cast iron			
PM 700	700	2	240–290
PM 600	600	3	200–250
PM 550	550	4	180–230
PM 500	500	5	160–200
PM 450	450	6	150–200
(E) Spheroidal graphite cast iron			
SG 900/2	900	2	280–360
SG 800/2	800	2	245–335
SG 700/2	700	2	225–305
SG 600/3	600	3	190–270
SG 500/7	500	7	160–240
SG 450/10	450	10	160–210
SG 400/15	400	15	130–180
SG 350/22	350	22	150 (Max.)

[2] IS 210–1993: Grey iron castings—specification.
[3] IS 14329–1995: Malleable iron castings—specification.
[4] IS 1865–1991: Iron castings with spheroidal or nodular graphite—specification.

2.4 BIS SYSTEM OF DESIGNATION OF STEELS

A large number of varieties of steel are used for machine components. Steels are designated by a group of letters or numbers indicating any one of the following three properties:[5, 6]

(i) tensile strength;
(ii) carbon content; and
(iii) composition of alloying elements.

Steels, which are standardised on the basis of their tensile strength without detailed chemical composition, are specified by two ways—a symbol Fe followed by the minimum tensile strength in N/mm^2 or a symbol FeE followed by the yield strength in N/mm^2. For example, Fe 360 indicates a steel with a minimum tensile strength of 360 N/mm^2. Similarly, FeE 250 indicates a steel with a minimum yield strength of 250 N/mm^2.

The designation of plain carbon steel consists of the following three quantities:

(i) a figure indicating 100 times the average percentage of carbon;
(ii) a letter C; and
(iii) a figure indicating 10 times the average percentage of manganese.

As an example, 55C4 indicates a plain carbon steel with 0.55% carbon and 0.4% manganese. A steel with 0.35–0.45% carbon and 0.7–0.9% manganese is designated as 40C8.

The designation of unalloyed free cutting steels consists of the following quantities:

(i) a figure indicating 100 times the average percentage of carbon;
(ii) a letter C;
(iii) a figure indicating 10 times the average percentage of manganese;
(iv) a symbol 'S', 'Se', 'Te' or 'Pb' depending upon the element that is present and which makes the steel free cutting; and
(v) a figure indicating 100 times the average percentage of the above element that makes the steel free cutting.

As an example, 25C12S14 indicates a free cutting steel with 0.25% carbon, 1.2% manganese and 0.14% sulphur. Similarly, a free cutting steel with an average of 0.20% carbon, 1.2% manganese and 0.15% lead is designated as 20C12Pb15.

The term 'alloy' steel is used for low and medium alloy steels containing total alloying elements not exceeding 10%. The designation of alloy steels consists of the following quantities:

(i) a figure indicating 100 times the average percentage of carbon; and
(ii) chemical symbols for alloying elements each followed by the figure for its average percentage content multiplied by a factor. The multiplying factor depends upon the alloying element. The values of this factor are as follows:

Elements	Multiplying factor
Cr, Co, Ni, Mn, Si and W	4
Al, Be, V, Pb, Cu, Nb, Ti, Ta, Zr and Mo	10
P, S, N	100

In alloy steels, the symbol 'Mn' for manganese is included only if the content of manganese is equal to or greater than 1%. The chemical symbols and their figures are arranged in descending order of their percentage content.

As an example, 25Cr4Mo2 is an alloy steel having average 0.25% of carbon, 1% chromium and 0.2% molybdenum. Similarly, 40Ni8Cr8V2 is an alloy steel containing average 0.4% of carbon, 2% nickel, 2% chromium and 0.2% vanadium. Consider an alloy steel with the following composition:

carbon = 0.12–0.18%
silicon = 0.15–0.35%
manganese = 0.40–0.60%
chromium = 0.50–0.80%

[5] IS 1762–1974: Code for designation of steels.
[6] IS 1570–1978: Schedule for wrought steels.

The average percentage of carbon is 0.15%, which is denoted by the number (0.15 × 100) or 15. The percentage content of silicon and manganese is negligible and, as such, they are deleted from the designation. The significant element is chromium and its average percentage is 0.65. The multiplying factor for chromium is 4 and (0.65 × 4) is 2.6, which is rounded to 3. Therefore, the complete designation of steel is 15Cr3. As a second example, consider a steel with the following chemical composition:

carbon = 0.12–0.20%
silicon = 0.15–0.35%
manganese = 0.60–1.00%
nickel = 0.60–1.00%
chromium = 0.40–0.80%

The average percentage of carbon is 0.16% and multiplying this value by 100, the first figure in the designation of steel is 16. The average percentage of silicon and manganese is very small and, as such, the symbols Si and Mn are deleted. Average percentages of nickel and chromium are 0.8 and 0.6, respectively, and the multiplying factor for both elements is 4. Therefore,

nickel: 0.8 × 4 = 3.2 rounded to 3 or Ni3
chromium: 0.6 × 4 = 2.4 rounded to 2 or Cr2.

The complete designation of steel is 16Ni3Cr2.

The term '*high alloy steels*' is used for alloy steels containing more than 10% of alloying elements. The designation of high alloy steels consists of the following quantities:

(i) a letter 'X';
(ii) a figure indicating 100 times the average percentage of carbon;
(iii) chemical symbol for alloying elements each followed by the figure for its average percentage content rounded off to the nearest integer; and
(iv) chemical symbol to indicate a specially added element to attain desired properties, if any.

As an example, X15Cr25Ni12 is a high alloy steel with 0.15% carbon, 25% chromium and 12% nickel. As a second example, consider a steel with the following chemical composition:

carbon = 0.15–0.25%
silicon = 0.10–0.50%
manganese = 0.30–0.50%
nickel = 1.5–2.5%
chromium = 16–20%

The average content of carbon is 0.20%, which is denoted by a number (0.20 × 100) or 20. The major alloying elements are chromium (average 18%) and nickel (average 2%). Hence, the designation of steel is X20Cr18Ni2.

2.5 PLAIN CARBON STEELS

Depending upon the percentage of carbon, plain carbon steels are classified into the following three groups:

(i) Low Carbon Steel Low carbon steel contains less than 0.3% carbon. It is popular as '*mild steel*'. Low carbon steels are soft and very ductile. They can be easily machined and easily welded. However, due to low carbon content, they are unresponsive to heat treatment.

(ii) Medium Carbon Steel Medium carbon steel has a carbon content in the range of 0.3% to 0.5%. It is popular as *machinery steel*. Medium carbon steel is easily hardened by heat treatment. Medium carbon steels are stronger and tougher as compared with low carbon steels. They can be machined well and they respond readily to heat treatment.

(iii) High Carbon Steel High carbon steel contains more than 0.5% carbon. They are called *hard steels* or *tool steels*. High carbon steels respond readily to heat treatments. When heat treated, high carbon steels have very high strength combined with hardness. They do not have much ductility as compared with low and medium carbon steels. High carbon steels are difficult to weld. Excessive hardness is often accompanied by excessive brittleness.

Plain carbon steels are available in the form of bar, tube, plate, sheet and wire. The mechanical properties of plain carbon steels[7] are given in Table 2.2.

[7] IS 1570 (Part 2/Sec1)–1979: Schedules for wrought steels—carbon steels (unalloyed steels).

Table 2.2 *Mechanical properties of plain carbon steels*

Grade	*Tensile strength (Min.) (N/mm²)*	*Yield strength (Min.) (N/mm²)*	*Hardness (HB)*	*Elongation %*
7C4	320	–	–	27
10C4	340	–	–	26
30C8	500	400	179	21
40C8	580	380	217	18
45C8	630	380	229	15
50C4	660	460	241	13
55C8	720	460	265	13
60C4	750	–	255	11
65C6	750	–	255	10

(**Note:** Minimum yield strength = 55% of minimum tensile strength)

Many times, a designer is faced with the aspect of choosing a correct carbon content for a particular application. This is an important decision because by merely changing the carbon content, one can get totally different properties of steels.

The guidelines for deciding carbon content in plain carbon steels are as follows:

(i) In applications like automobile bodies and hoods, the ability of the material to deform to a grater extent or 'ductility' is the most important consideration. Such a material should have high ductility. Ductility is measured in terms of percentage elongation. It is observed from Table 2.2, that lower the percentage of carbon, higher is the percentage of elongation or ductility. Therefore, a plain carbon steel, like 7C4, which has a lower percentage of carbon and a higher percentage of elongation, is selected for these parts.

(ii) In applications like gears, machine tool spindles and transmission shafts, strength, toughness and response to heat treatment are important considerations. In these components, the surface is heavily stressed, while the stresses in the core are of comparatively small magnitude. These components require a soft core and a hard surface. This is achieved by case hardening of gears, shafts and spindles. Medium and high carbon steels, such as 40C8, 45C8, 50C4, 55C8, and 60C4 which are stronger, tougher and respond readily to heat treatment are, therefore, selected for these components. They can also be machined well to the required accuracy.

(iii) Spring wires are subjected to severe stress and strength is the most important consideration in selection of their material. High carbon steel, such as 65C6, having maximum tensile strength, is selected for helical and leaf springs.

(iv) Low and medium carbon steels can be satisfactorily welded. However, low carbon steels are the most easily welded. Higher the percentage of carbon in steel, more difficult it is to weld. Therefore, welded assemblies are made of low and medium carbon steels.

(v) Low and medium carbon steels can be satisfactorily forged. However, low carbon steels that are very soft and ductile, are the most easily forged. Higher the percentage of carbon in steel, the more difficult it is to forge the part. Therefore, forged components such as levers, rocker arm, yoke or tie rod are made of low carbon steel 30C8. However, there are some forged components like connecting rod and crankshaft, which also require heat treatment after forging. They are made of medium carbon steel 40C8, which responds readily to heat treatment.

(vi) All steels have essentially the same modulus of elasticity. Thus, if rigidity is the requirement of the component, all steels perform equally well. In this case, the least costly steel should be selected.

Some of the important applications of plain carbon steels are as follows:

7C4 Components made by severe drawing operation such as automobile bodies and hoods

10C4 Case hardened components such as cams and cam shaft, worm, gudgeon pin, sprocket and spindle

30C8 Cold formed and case hardened parts such as socket, tie rod, yoke, lever and rocker arm

40C8 Transmission shaft, crank shaft, spindle, connecting rod, stud and bolts

45C8 Transmission shaft, machine tool spindle, bolts and gears of large dimensions

50C4 Transmission shaft, worm, gears and cylinder

55C8 Components with moderate wear resistance such as gears, cam, sprocket, cylinder and key

60C4 Machine tool spindle, hardened bolt and pinion

65C6 Coil and leaf springs

2.6 FREE CUTTING STEELS

Steels of this group include carbon steel and carbon–manganese steel with a small percentage of sulphur. Due to addition of sulphur, the machinability of these steels is improved. *Machinability is defined as the ease with which a component can be machined.* It involves three factors—the ease of chip formation, the ability to achieve a good surface finish and ability to achieve an economical tool life. Machinability is an important consideration for parts made by automatic machine tools. Typical applications of free cutting steels are studs, bolts and nuts. Mechanical properties of free cutting steels[8] are given in Table 2.3.

Table 2.3 *Mechanical properties of free cutting steels (cold drawn bars) (20–40 mm diameter)*

Grade	*Tensile strength (Min.) (N/mm²)*	*Elongation (Min.) (%)*
10C8S10	460	10
14C14S14	520	11
25C12S14	560	10
40C10S18	600	10
40C15S12	640	8

2.7 ALLOY STEELS

Alloy steel is defined as carbon steel to which one or more alloying elements are added to obtain certain beneficial effects. The commonly added elements include silicon, manganese, nickel, chromium, molybdenum and tungsten. The term '*alloy steels*' usually refers to '*low*' alloy steels containing from about 1 to 4 per cent of alloying elements. On the other hand, stainless and heat-resisting steels are called '*high*' alloy steels. Plain carbon steels are successfully used for components subjected to low or medium stresses. These steels are cheaper than alloy steels. However, plain carbon steels have the following limitations:

[8] IS 1570 (Part 3)–1979: Schedules for wrought steel—carbon and carbon-manganese free cutting steels.

(i) The tensile strength of plain carbon steels cannot be increased beyond 700 N/mm^2 without substantial loss in ductility and impact resistance.
(ii) Components with large section thickness cannot be produced with martensitic structure. In other words, plain carbon steels are not deep hardenable.
(iii) Plain carbon steels have low corrosion resistance.
(iv) Medium carbon steels must be quenched rapidly to obtain a fully martensitic structure. Rapid quenching results in distortion and cracking in heat-treated components.
(v) Plain carbon steels have poor impact resistance at low temperatures.

To overcome these deficiencies of plain carbon steels, alloy steels have been developed. Alloy steels cost more than plain carbon steels. However, in many applications, alloy steel is the only choice to meet the requirements. Alloy steels have the following advantages:

(i) Alloy steels have higher strength, hardness and toughness.
(ii) High values of hardness and strength can be achieved for components with large section thickness.
(iii) Alloy steels possess higher hardenability, which has great significance in heat treatment of components.
(iv) Alloy steels retain their strength and hardness at elevated temperatures.
(v) Alloy steels have higher resistance to corrosion and oxidation compared with plain carbon steels.

Alloying elements can affect constitution, characteristics and behaviour of these steels. The effects of major alloying elements are as follows:

(i) Silicon Silicon is present in almost all steels. It increases strength and hardness without lowering the ductility. Silicon is purposely added in spring steel to increase its toughness.

(ii) Manganese Most steels contain some manganese remaining from the deoxidisation and desulphurisation processes. However, when it exceeds 1 per cent, it is regarded as an alloying element. Manganese is one of the least expensive alloying elements. It increases hardness and strength. It also increases the depth of hardening. Manganese is an important alloying element in free cutting steels.

(iii) Nickel Nickel increases strength, hardness and toughness without sacrificing ductility. It increases hardenability of steel and impact resistance at low temperature. The main effect of nickel is to increase toughness by limiting grain growth during the heat treatment process.

(iv) Chromium Chromium increases hardness and wear resistance. Chromium steel components can be readily hardened in heavy sections. They retain strength and hardness at elevated temperatures. Chromium steels containing more than 4 per cent chromium have excellent corrosion resistance.

(v) Molybdenum Molybdenum increases hardness and wear resistance. It resists softening of steel during tempering and heating.

(vi) Tungsten Tungsten and molybdenum have similar effects. It is an expensive alloying element and about 2 to 3 per cent tungsten is required to replace 1 per cent of molybdenum. It is an important alloying element in tool steels.

Many times, a designer is faced with the aspect of choosing the correct alloying element of steels for a particular application. This is an important decision because by merely changing an alloying element, one can get totally different combinations of mechanical properties for steels.

The guidelines for selecting alloy steels are as follows:

(i) Spring wires are subjected to severe stresses, and strength is the most important consideration in selection of their material. Silicon increases strength. Therefore, silicon steel, such as 55Si7, is selected for helical and leaf springs.
(ii) In case of highly stressed screws, bolts and axles, high strength and toughness are important considerations. Nickel increases strength and toughness without loss of

ductility. Therefore, nickel steel such as 40Ni14 is used for these severely stressed components.

(iii) In applications like gears, surface hardness, wear resistance and response to heat treatment are important considerations. In these components, the surface is heavily stressed, while the stresses in the core are of comparatively small magnitude. These components require a soft core and a hard surface. Chromium increases hardness and wear resistance. Also, chromium steels are readily hardened in heavy sections. Therefore, chromium steels, such as 40Cr4 is selected for all types of gears.

(iv) In a number of components like gears, cams, camshafts, and transmission shafts, combined properties such as hardness and toughness, strength and ductility are required. This is achieved by using nickel and chromium as alloying elements and selecting proper heat treatment. Nickel–chromium steels, like 16Ni3Cr2 or 30Ni16Cr5, which combines hardness and toughness, are selected for these parts.

Some of the important applications of alloy steels are as follows:

55Si7 Leaf and coil springs

37C15 Axle, shaft and crankshaft

35Mn6Mo3 Bolt, stud, axle, lever and general engineering components

16Mn5Cr4 Gears and shaft

40Cr4 Gears, axle and steering arm

50Cr4 Coil, laminated and volute springs

40Cr4Mo2 Shaft, axle, high tensile bolt, stud and popeller shaft

40Cr13Mo10V2 Components subjected to high tensile stresses

40Ni14 Severely stressed screw, nut and bolt

16Ni3Cr2 Gears, transmission components, cam and camshaft

30Ni16Cr5 Heavy duty gears

35Ni5Cr2 Gear shaft, crankshaft, chain parts, camshaft and planetary gears

40Ni6Cr4Mo2 General machine parts, nuts and bolts, gears, axles, shafts and connecting rod

40Ni10Cr3Mo6 High strength machine components, bolts and studs, axles and shafts, gears and crankshafts

Mechanical properties of alloy steels[9] are given in Table 2.4 on next page.

2.8 OVERSEAS STANDARDS

Cast iron and steel are the essential ingredients in any product. A large variety of steel and cast iron is developed for a number of applications. In our country, collaborations with foreign industries have resulted in use of different overseas standards and designations. Some important designations for ferrous materials are as follows,[10, 11, 12]

(i) The American Society for Testing Materials (ASTM) has classified grey cast iron by means of a number. This class number gives minimum tensile strength in kpsi. For example, ASTM Class No. 20 has minimum ultimate tensile strength of 20000 psi. Similarly, a cast iron with minimum ultimate tensile strength of 50000 psi is designated as ASTM Class No. 50. Commonly used ASTM classes of cast iron are 20, 25, 30, 35, 40, 50 and 60.

[9] IS 1570 (Part 4)–1988: Schedules for wrought steels—Alloy steels with specified chemical composition and mechanical properties.

[10] SAE J402: 1984–SAE Numbering system for wrought or rolled steel (SAE standard).

[11] 'SAE Handbook'—Vol.1—'Materials'—Society of Automotive Engineers Inc., 1987.

[12] 'Metals Handbook'—Vol.1—'Properties and selection: Iron, steels and high performance alloys'—American Society of Metals Inc., 1990.

Table 2.4 *Mechanical properties of alloy steels*

Grade	*Tensile strength (N/mm^2)*	*0.2 percent Proof stress (Min.) (N/mm^2)*	*Elongation (Min.) (%)*	*Hardness (HB)*	*Condition*
37C15	590–740	390	18	170–217	Oil–hardened and tempered
35Mn6Mo3	690–840	490	14	201–248	Oil-hardened and tempered
16Mn5Cr4	790 (Min.)	–	10	–	Case-hardened steel (core properties)
40Cr4	690–840	490	14	201–248	Oil-hardened and tempered
40Cr4Mo2	700–850	490	13	201–248	Oil-hardened and tempered
40Cr13Mo10V2	1340 (Min.)	1050	8	363 (Min.)	Oil-hardened and tempered
40Ni14	790–940	550	16	229–277	Oil-hardened and tempered
16Ni3Cr2	690 (Min.)	–	15	–	Case-hardened steel (core properties)
30Ni16Cr5	1540 (Min.)	1240	8	444 (Min.)	Oil-hardened and tempered
35Ni5Cr2	690–840	490	14	201–248	Oil-hardened and tempered
40Ni6Cr4Mo2	790–940	550	16	229–277	Oil-hardened and tempered
40Ni10Cr3Mo6	990–1140	750	12	285–341	Oil-hardened and tempered

(ii) In Germany, Deutches Institut Fuer Normung (DIN) has specified grey cast iron by minimum ultimate tensile strength in kgf/mm^2. For example, GG-12 indicates grey cast iron with minimum ultimate tensile strength of 12 kgf/mm^2. Similarly, grey cast iron with minimum ultimate tensile strength of 26 kgf/mm^2 is designated as GG-26. The common varieties of grey cast iron according to DIN standard are GG-12, GG-14, GG-18, GG-22,GG-26 and GG-30.

(iii) A numbering system for carbon and alloy steels is prescribed by the Society of Automotive Engineers (SAE) of USA and American Iron and Steel Institute (AISI). It is based on chemical composition of the steel. The number is composed of four or five digits. The first two digits indicate the type or alloy classification. The last two or three digits give the carbon content. Since carbon is the most important element in steel, affecting the strength and hardness, it is given proper weightage in this numbering system. The basic numbers for various types of steel are given in Table 2.5. For example, plain carbon steel has 1 and 0 as its first two digits. Thus, a steel designated as 1045 indicates plain carbon steel with 0.45% carbon. Similarly, a nickel–chromium steel with 1.25% Ni, 0.60% Cr and 0.40% carbon is specified as SAE 3140.

The AISI number for steel is the same as the SAE number. In addition, there is a capital letter A, B, C, D or E that is prefixed to the number. These capital letters indicate the manufacturing process of steel. The meaning of these letters is as follows:

A—Basic open-hearth alloy steel
B—Acid Bessemer carbon steel
C—Basic open-hearth carbon steel
D—Acid open-hearth carbon steel
E—Electric furnace alloy steel

The British system designates steel is a series of numbers known as 'En' series. The En number of a steel has no correlation either with the chemical composition such as carbon content and types of alloying element or mechanical properties such as ultimate tensile strength. For example, the number 3 in En3 steel has no relationship with the carbon content, alloying element or strength of steel. Table 2.6 shows the overseas equivalent designations of some popular varieties of steel.

Table 2.5 *Basic numbering system of SAE and AISI steels*

Material	*SAE or AISI Number*
Carbon steels	1xxx
plain carbon	10xx
free-cutting, screw stock	11xx
Chromium steels	5xxx
low chromium	51xx
medium chromium	52xxx
corrosion and heat resisting	51xxx
Chromium–nickel–molybdenum steels	86xx
Chromium–nickel–molybdenum steels	87xx
Chromium–vanadium steels	6xxx
1.00% Cr	61xx
Manganese steels	13xx
Molybdenum steels	4xxx
carbon–molybdenum	40xx
chromium–molybdenum	41xx
chromium–nickel–molybdenum	43xx
nickel–molybdenum; 1.75% Ni	46xx
nickel-molybdenum; 3.50% Ni	48xx
Nickel–chromium steels	3xxx
1.25% Ni, 0.60% Cr	31xx
1.75% Ni, 1.00% Cr	32xx
3.50% Ni, 1.50% Cr	33xx
Silicon–manganese steels	9xxx
2.00% Si	92xx
Nickel steels	2xxx
3.5% Ni	23xx
5.0% Ni	25xx

2.9 HEAT TREATMENT OF STEELS

The heat treatment process consists of controlled heating and cooling of components made of either plain carbon steel or alloy steel, for the purpose of changing their structure in order to obtain certain desirable properties like hardness, strength or ductility. The major heat treatment processes are as follows:

(i) **Annealing** consists of heating the component to a temperature slightly above the critical temperature, followed by slow cooling. It reduces hardness and increases ductility.

(ii) **Normalising** is similar to annealing, except that the component is slowly cooled in air. It is used to remove the effects of the previous heat treatment processes.

Table 2.6 *Overseas equivalent designations of steel*

BIS designation	*En Number*	*SAE*	*AISI*	*DIN*
Plain-carbon steels				
7C4	2A	1010	C 1010	17210
10C4	32A	1012	C 1012	17155
30C8	5	1030	C 1030	–
45C8	43B	1045	C 1045	17200
50C4	43A	1049, 1050	C 1049, C 1050	–
55C8	43J, 9K	1055	C 1055	–
60C4	43D	1060	C 1060	17200
65C6	42B	1064	C 1064	17222
Free cutting steels				
10C8S10	–	1109	C 1109	–
14C14S14	7A, 202	1117, 1118	C 1117, C 1118	–
25C12S14	7	1126	C 1126	–
40C10S18	8M	1140	C 1140	–
40C15S12	15AM	1137	C 1137	–
Alloy steels				
40Cr4	18	5135	5135	–
40Ni14	22	2340	2340	–
35Ni5Cr2	111	3140	3140	1662
30Ni16Cr5	30A	–	-	–
40Ni6Cr4Mo2	110	4340	4340	17200
27C15	14B	1036	C 1036	17200
37C15	15, 15A	1041, 1036	C 1041, C 1036	17200
50Cr4V2	47	6150	6150	17221

(iii) **Quenching** consists of heating the component to the critical temperature and cooling it rapidly in water or air. It increases hardness and wear resistance. However, during the process, the component becomes brittle and ductility is reduced.

(iv) **Tempering** consists of reheating the quenched component to a temperature below the transformation range, followed by cooling at a desired rate. It restores the ductility and reduces the brittleness due to quenching.

The recommended hardening and tempering treatments and temperature ranges can be obtained from the standards. The selection of a proper heat treatment process depends upon the desirable properties of the component.

2.10 CASE HARDENING OF STEELS

In number of situations, the stress distribution across the cross-section of a component is not uniform. In some cases, the surface is heavily stressed, while the stress in the core is of comparatively small magnitude. The examples of this type of stress distribution are gear, cam and rolling contact bearing. The surface failure of these components can be avoided by case hardening.

Case hardening can be achieved by the following two ways:

(i) by altering the structure at the surface by local hardening, e.g., flame or induction hardening.

(ii) by altering the structure as well as the composition at the surface, e.g., case carburising, nitriding, cyaniding and carbo-nitriding.

Flame hardening consists of heating the surface above the transformation range by means of a flame, followed by quenching. The distortion of the component is low because the bulk of the work piece is not heated. Flame hardening can be done in stages, such as hardening of tooth by tooth of a gear blank. The minimum case depth obtained by this process is 1 mm, although case depths up to 6 mm are quite common. Flame hardening is recommended under the following situations:

(i) where the component is large;

(ii) where a small area of the work piece is to be hardened; and

(iii) where dimensional accuracy is desirable.

Carbon steels containing more than 0.4% carbon are generally employed for flame hardening.

The **induction-hardening process** consists of heating the surface by induction in the field of an alternating current. The amount of heat generated depends upon the resistivity of the material. Induction hardening produces case depths as small as 0.1 mm. There is not much difference between flame and induction hardening, except for the mode of heating and minimum case depth.

Case carburising consists of introducing carbon at the surface layer. Such a component has a high-carbon surface layer and a low-carbon core with a gradual transformation from one zone to the other. Different methods are used to introduce carbon, but all involve heating from 880 to 980°C. The carburising medium can be solid, liquid or gas. Case carburising is recommended for case depths up to 2 mm.

Carbo-nitriding consists of introducing carbon and nitrogen simultaneously at the surface layer. The component is heated from 650 to 920°C in the atmosphere of anhydrous ammonia and then quenched in a suitable medium. Nitrogen is concentrated at the surface and backed up by a carburised case. Medium carbon steels are carbo-nitrided with case depths up to 0.6 mm. The process gives a higher wear resistance compared to the case-carburising process. *Cyaniding* is similar to carbo nitriding except that the medium is liquid.

Nitriding consists of exposing the component to the action of nascent nitrogen in a gaseous or liquid medium from 490 to 590°C. This process does not involve any subsequent quenching. The gaseous medium consists of dry ammonia. The liquid medium can be of cyanides and cyanates. The nitrided case consists of two zones—a brittle white zone next to the surface, consisting of nitrides, followed by a tougher diffusion zone, where nitrides are precipitated in the matrix. Case depths up to 0.1 mm are obtained by this process. Nitrided components are used for applications requiring high resistance to abrasion, high endurance limit and freedom from distortion. The disadvantages of this process are as follows:

(i) The components cannot be used for concentrated loads and shocks;

(ii) The case depth is limited to 0.5 mm; and

(iii) Considerable time is required for the process due to long cycle time.

The applications of the nitrided component are indexing worms, high-speed spindles and crankshafts.

2.11 CAST STEEL

Pouring molten steel of desired chemical composition into a mould and allowing the steel to solidify produces cast steel components. Steel castings can be made from any type of carbon and low alloy steels. Cast steel components and wrought steel components of equivalent chemical composition respond similarly to heat treatment, have the same weldability and have similar physical and mechanical properties. However, cast steel components do not exhibit the effects of directionality on mechanical properties that are typical of wrought steels. This non-directional characteristic of cast steel components is advantageous where application involves multi-directional forces. Compared with cast iron components, the cast steel parts are lighter for the same strength. However, cast steels are costly and their superior mechanical properties are justified only in certain applications. Cast steel in its liquid form has poor fluidity compared with cast iron. Therefore, the wall thickness or section thickness cannot be made less than 6.5 mm. During solidification, cast steel shrinks to a greater extent. This results in residual stresses which cannot be completely relieved by the normalising process. Poor fluidity and excessive contraction should be taken into account while designing a component made of cast steel. It is always advisable to divide complicated one piece castings into simple parts and join them at the stage of assembly.

There are two varieties of steel castings: carbon steel castings and high tensile steel castings.[13, 14] *Carbon steel castings* are used for heavy duty machinery, highly stressed parts and gears. There are five grades of carbon steel castings. They are classified on the basis of minimum yield stress and tensile strength values respectively, expressed in N/mm^2. For example, a carbon steel casting of grade 200–400 has a yield stress of 200 N/mm^2 and an ultimate tensile strength of 400 N/mm^2. A carbon steel casting of yield stress and ultimate tensile strength of 280 and 520 N/mm^2 respectively, will be designated as a carbon steel casting of grade 280–520. The mechanical properties of carbon steel castings are given in Table 2.7.

Table 2.7 *Mechanical properties of carbon steel castings*

Grade	*Yield stress (Min.) (N/mm^2)*	*Tensile strength (Min.) (N/mm^2)*	*Elongation (Min.) (%)*
200–400	200	400	25
230–450	230	450	22
260–520	260	520	18
280–520	280	520	18
340–570	340	570	15

High tensile steel castings have higher strength, good toughness and high resistance to wear. These castings are used in transportation equipment and agricultural machinery. There are five grades of high tensile steel castings which are classified according to the ultimate tensile strength, for example CS 640 is a steel casting with minimum ultimate tensile strength of 640 N/mm^2. A high tensile steel casting with minimum ultimate tensile strength of 1030 N/mm^2 is designated as CS 1030. The mechanical properties of high tensile steel castings are given in Table 2.8.

Table 2.8 *Mechanical properties of high tensile steel castings*

Grade	*Tensile strength (Min.) (N/mm^2)*	*Yield stress[1] (Min.) (N/mm^2)*	*Elongation (Min.) (%)*	*Hardness (HB)*
CS 640	640	390	15	190
CS 700	700	580	14	207
CS 840	840	700	12	248
CS 1030	1030	850	8	305
CS 1230	1230	1000	5	355

Note: 1. Yield stress is 0.5 per cent proof stress.

2.12 ALUMINIUM ALLOYS

Aluminium alloys are recent in origin compared with copper or steel. However, due to a unique combination of certain mechanical properties, they have become the most widely used nonferrous metal. Aluminium alloys offer the following design advantages:

[13] IS 1030—1998: Carbon steel castings for general engineering purposes—specification.
[14] IS 2644–1994: High strength steel castings for general engineering and structural purposes—specification.

(i) Low Specific Gravity The relative density of aluminium alloys is 2.7 compared with 7.9 of steel, i.e., roughly one third of steel. This results in light weight construction and reduces inertia forces in applications like connecting rod and piston, which are subjected to reciprocating motion.

(ii) Corrosion Resistance Aluminium has high affinity for oxygen and it might be expected that aluminium components will oxidise or rust very easily. However, in practice it has an excellent resistance to corrosion. This is due to the thin but very dense film of oxide (alumina skin) which forms on the surface of metal and protects it from further atmospheric attack. It is due to alumina skin that there is comparatively dull appearance on the surface of a polished aluminium component.

(iii) Ease of Fabrication Aluminium alloys have a face-centered cubic crystal structure with many slip planes. This makes the material ductile and easily shaped. They can be cast, rolled, forged or extruded. Aluminium alloys can be formed, hot or cold, with considerable ease. They can be machined easily if suitable practice and proper tools are used. They can be joined by fusion welding, resistance welding, soldering and brazing. Due to excellent malleability, it is possible to produce very thin aluminium foil suitable for food packaging.

(iv) High Thermal Conductivity As a material for constructional components, aluminium has poor strength compared with steel. In '*soft*' condition, the tensile strength of pure aluminium is only 90 N/mm^2, while even in work-hardened state, it is no more than 135 N/mm^2. Hence, for engineering applications, aluminium is alloyed in order to obtain high strength to weight ratio. Some of the high strength aluminium alloys with suitable heat treatment have tensile strength in excess of 600 N/mm^2. There are two varieties of aluminium alloys–wrought and cast—which are used for machine components. Wrought aluminium alloys are available in the form of plates, sheets, strips, wires, rods and tubes. There are three methods of casting aluminium alloys, viz., sand casting, gravity die casting and pressure die casting.

Aluminium alloys are designated by a particular numbering system. The numbers given to alloying elements are given in Table 2.9.

Table 2.9

Aluminium	—	1	Magnesium	—	5
Copper	—	2	Magnesium silicide	—	6
Manganese	—	3	Zinc	—	7
Silicon	—	4	Other elements	—	8

Cast aluminium alloys are specified by a '*four-digit*' system while wrought alloys by a '*five-digit*' system[15, 16, 17, 18]. The meaning of these digits is as follows:

First digit It identifies the major alloying element.

Second digit It identifies the average percentage of the major alloying element, halved and rounded off.

Third, fourth and fifth digits They identify the minor alloying elements in order of their decreasing percentage.

As an example, consider an aluminium alloy casting with 9.8% Cu, 1.0% Fe and 0.25% Mg.

First digit	identification of copper	:	2
Second digit	(9.8/2 = 4.9 or 5)	:	5
Third digit	identification of iron	:	8

[15] IS 617–1994: Cast aluminium and its alloys—ingots and casting for general engineering purposes—specification.
[16] IS 733–1983: Specifications for wrought aluminium and aluminium alloy bars, rods and sections for general engineering purposes.
[17] IS 5052–1993: Aluminium and its alloys—Temper designations.
[18] SP: 1–1967: Comparison of Indian and overseas standards on aluminium alloy castings—Bureau of Indian Standards.

Fourth digit identification of magnesium : 5
Complete designation = 2585

Some of the important applications of cast aluminium alloy are as follows:

Alloy 4450 Engine cylinder blocks, castings for valve body and large fan blades

Alloy 4600 Intricate and thin-walled castings, motor housings, water cooled manifolds and pump casings

Alloy 2280 Connecting rods and flywheel housings

Alloy 2285 (Y-alloy) Pistons and cylinder heads

Alloy 2250 Castings for hydraulic equipment

Alloy 4652 Pistons for internal combustion engines.

The alloy 4600 is used for pressure die casting parts. It has excellent fluidity, which facilitates the production of complex castings of large surface area and thin walls. The mechanical properties of aluminium alloy castings are given in Table 2.10.

The important applications of wrought aluminium alloy are as follows:

Alloy 24345 Heavy duty forging and structures

Alloy 24534 Stressed components of aircraft

Alloy 54300 Welded structures and tank

Alloy 64430 Roof truss and deep-drawn container

Alloy 74530 Welded pressure vessels.

Table 2.10 *Mechanical properties of cast aluminium alloys*

Alloy	*Condition*	*Tensile Strength (Min.) (N/mm^2)*		*Elongation (Min.) (%)*	
		Sand-cast	*Chill-cast*	*Sand-cast*	*Chill-cast*
4450	M	135	160	2	3
	T5	160	190	1	2
	T7	160	225	2.5	5
	T6	225	275	–	2
4600	M	165	190	5	7
2280	T4	215	265	7	13
	T6	275	310	4	9
2285	T6	215	280	–	–
2550	M	–	170	–	–
4652	T5	–	210	–	–
	T6	140	200	–	–
	T7	175	280	–	–

(M = as cast; T5 = precipitation treated; T4 = solution treated; T7 = solution treated and stabilised; T6 = solution and precipitation treated)

The mechanical properties of wrought aluminium alloy are given in Table 2.11 on next page.

2.13 COPPER ALLOYS

Copper possesses excellent thermal and electrical conductivity. It can be easily cast, machined and brazed. It has good corrosion resistance. However, even with these advantages, pure copper is not used in any structural application due to its poor strength. The tensile strength of copper is about 220 N/mm^2. Pure copper is mainly used for electrical and thermal applications. For structural applications, copper alloys are used instead of pure copper. Some of the popular copper alloys are brass, bronze, gunmetal and monel metal. Their properties and applications are briefly discussed in this article.

Table 2.11 *Mechanical properties of wrought aluminium and aluminium alloys*

Alloy	Condition	Diameter(mm) over	Diameter(mm) upto	0.2 per cent proof stress (N/mm^2) Min.	0.2 per cent proof stress (N/mm^2) Max.	Tensile strength (N/mm^2) Min.	Tensile strength (N/mm^2) Max.	Elongation (Min.) (%)
24345	M	–	–	90	–	150	–	12
	O	–	–	–	175	–	240	12
	W	–	10	225	–	375	–	10
		10	75	235	–	385	–	10
		75	150	235	–	385	–	8
		150	200	225	–	375	–	8
	WP	–	10	375	–	430	–	6
		10	25	400	–	460	–	6
		25	75	420	–	480	–	6
		75	150	405	–	460	–	6
		150	200	380	–	430	–	6
24534	M	–	–	90	–	150	–	12
	O	–	–	–	175	–	240	12
	W	–	10	220	–	375	–	10
		10	75	235	–	385	–	10
		75	150	235	–	385	–	8
		150	200	225	–	375	–	8
54300	M	–	150	130	–	265	–	11
	O	–	150	125	–	–	350	13
64430	M	–	–	80	–	110	–	12
	O	–	–	–	–	–	150	16
	W	–	150	120	–	185	–	14
		150	200	100	–	170	–	12
	WP	–	5	225	–	295	–	7
		5	75	270	–	310	–	7
		75	150	270	–	295	–	7
		150	200	240	–	280	–	6
74530	W[1]	–	6	220	–	255	–	9
		6	75	230	–	275	–	9
		75	150	220	–	265	–	9
	WP	–	6	245	–	285	–	7
		6	75	260	–	310	–	7
		75	150	245	–	290	–	7

Note: [1] Naturally aged for 30 days (M = as manufactured; O = annealed; W = solution treated and naturally aged; WP = solution and precipitation treated).

(i) Brass The most commonly used copper alloy is brass. It is an alloy of copper and zinc. Sometimes, it may contain small amounts of tin, lead, aluminium and manganese. Brass has the following advantages:

(i) The tensile strength of brass is higher than that of copper.
(ii) Brass is cheaper than copper.
(iii) Brass has excellent corrosion resistance.
(iv) Brass has better machinability.
(v) Brass has good thermal conductivity.

The strength and ductility of brass depend upon the zinc content. As the amount of zinc increases, the strength of brass increases and ductility decreases. The best combination of strength and ductility is obtained when the amount of zinc is 30 per cent. Brass can be used either in rolled condition or as cast. Some of the commonly used varieties of brass are yellow brass, naval brass, cartridge brass and muntz metal. Typical applications of brass in the field of mechanical engineering are tubes for condensers and heat exchangers, automotive radiator cores, rivets, valve stems and bellow springs.

(ii) Bronze Bronze is an alloy of copper and elements other than zinc. In some cases, bronze may contain a small amount of zinc. There are three important varieties of bronze—aluminium bronze, phosphor bronze and tin bronze. *Aluminium bronze* contains 5 to 10 per cent aluminium. It has excellent corrosion resistance, high strength and toughness, low coefficient of friction and good damping properties. It is used for hydraulic valves, bearings, cams and worm gears. The colour of aluminium bronze is similar to that of 22 carat gold and it is frequently called '*imitation*' gold. Aluminium bronze is difficult to cast because its casting temperature is around 1000°C. At this temperature, aluminium oxidises and creates difficulties in casting.

Phosphor bronze contains about 0.2 per cent phosphorus. The effect of phosphorus is to increase the tensile strength and corrosion resistance and reduce the coefficient of friction. In cast form, phosphor bronze is widely used for worm wheels and bearings. In wrought form, it is used for springs, bellows, pumps, valves and chemical equipment. *Tin bronze* contains up to 18 per cent tin and sometimes small amounts of phosphorus, zinc or lead. It has excellent machinability, wear resistance and low coefficient of friction. It is used for pump castings, valve fittings and bearings. High prices of both copper and tin, put limitations on the use of tin bronze.

In general, all varieties of bronze have the following advantages:

(i) excellent corrosion resistance;
(ii) low coefficient of friction; and
(iii) higher tensile strength than copper or brass.

The main limitation of bronze is its high cost.

(iii) Gunmetal Gunmetal is an alloy of copper which contains 10% tin and 2% zinc. The presence of zinc improves fluidity of gunmetal during casting process. Zinc is considerably cheaper than tin. Therefore, the total cost of gunmetal is less than that of bronze. In cast form, gunmetal is used for bearings. It has excellent corrosion resistance, high strength and low coefficient of friction.

(iv) Monel Metal Monel metal is a copper–nickel alloy. It contains 65% nickel and 32% copper. It has excellent corrosion resistance to acids, alkalis, brine water, sea water and other chemicals. It is mainly used for handling sulphuric and hydrochloric acids. It is also used for pumps and valves for handling the chemicals in process equipment.

2.14 DIE CASTING ALLOYS

The die casting process consists of forcing the molten metal into a closed metal die. This process is used for metals with a low melting point. The advantages of the die casting process are as follows:

(i) Small parts can be made economically in large quantities.
(ii) Surface finish obtained by this method is excellent and requires no further finishing.
(iii) Very thin sections or complex shapes can be obtained easily.

The drawbacks include the cost of dies and restriction on the size of the component, since only small parts can be die cast.

Die casting alloys are made from zinc, aluminium and magnesium. Brass can be die cast but its casting temperature is high. Zinc die castings are more popular due to their high strength, long die life and moderate casting temperature. Aluminium and magnesium die castings are light in weight but their casting temperature is higher than that of zinc die castings.

2.15 CERAMICS

Ceramics can be defined as a compound of metallic and non-metallic elements with predominantly 'ionic' interatomic bonding. The word '*ceramic*' is derived from the Greek word keramos which means '*potter's clay*'. Traditional ceramics include refractories, glass, abrasives, enamels and insulating materials. However, many substances, which are now classed as ceramics in fact, contain no clay. Modern ceramics include metal oxides, carbides, borides, nitrides and silicates. Some of their examples are Magnesia (M_gO), Alumina (Al_2O_3), Zirconia (Z_rO_2), Beryllia (B_eO), Silicon carbide (S_iC) and Tungsten carbide (T_iC).

The advantages of modern engineering ceramics are as follows:

(i) Most of the ceramics possess high hardness. This increased hardness is due to the operation of strong covalent bonds between atoms in their crystal structure. Materials like silicon carbide and boron nitride are the examples of ceramics with high hardness. It is this property which makes them useful as abrasive powder and cutting tools.

(ii) Ceramics have high melting points. Materials such as magnesia and alumina have melting points of 2800°C and 2040°C respectively. This property makes them excellent refractory materials for the lining of the furnaces.

(iii) Ceramics are good thermal insulators. In most of the ceramics, there are no free conducting electrons and heat is conducted only by transfer of vibration energy from one atom to another. This is unlike free electrons in metals. Hence, ceramics possess excellent insulating property. This is another reason to use them as refractory material.

(iv) Ceramics have extremely high electrical resistivity. Hence, they are used for electrical insulators. Porcelain is a popular insulating material. Alumina is used for spark-plug insulation.

(v) The densities of ceramics are low compared with those of engineering metals. This results in lightweight components.

(vi) Ceramics are chemically resistant to most of the acids, alkalis and organic substances. They are also unaffected by oxygen. This increases the durability of ceramic components.

Ceramics have certain drawbacks. Their main disadvantages are as follows:

(i) Ceramics are brittle in nature. They are highly susceptible to stress concentration. Presence of even a micro-crack may lead to failure because it acts as a stress raiser. In ceramics, there is no plastic deformation like metals and no redistribution of stresses. This results in brittle fracture like cast iron components.

(ii) In ceramics, ductility is almost zero. This is mainly due to the presence of small voids in the structure of ceramic parts.

(iii) Ceramics have poor tensile strength.

(iv) There is a wide variation in strength values of ceramics. Even in identical specimens, the properties vary due to variation of internal pores. Hence, in design of ceramic components, a statistical approach is essential for calculating the values of strength.

(v) Ceramics are difficult to shape and machine.

In this article, we will consider only recent applications of advanced ceramics in the automobile industry. A number of parts of automotive engines are nowadays made of ceramics. They include cylinder liners, pistons, valves and engine blocks. The principal advantages of ceramic engine components over conventional metal parts are as follows:

(i) ability to withstand higher operating temperature;
(ii) excellent wear and corrosion resistance;
(iii) lower frictional loss;
(iv) ability to operate without cooling system; and
(v) light weight construction with low inertia force.

Research is being conducted on gas turbine engines that employ ceramic rotors, stators, regenerators and combustion housings. Other applications include turbine blades for aircraft engines and surface coatings for the engine parts.

2.16 PLASTICS

Plastics are synthetic materials processed by heat and pressure. They are perhaps the most widely used group of polymers. There are two terms, which are used in understanding the construction of plastics, viz., monomer and polymer. *A monomer is a group of atoms that constitutes one unit of a polymer chain*. When monomers are subjected to heat and pressure, they join together to form a chain called *polymer*. *A polymer is a non-metallic organic compound of high molecular weight consisting of a very long chain of monomers*. The process of combining monomers into polymers is called *polymerization*. Figure 2.7 shows the construction of typical monomers and their corresponding polymers. In this figure, atoms of carbon, hydrogen and other elements are represented by their chemical symbols and their bonds by radial lines.

When a short polymer chain is lengthened by adding more and more monomer units, the material becomes more dense and passes from gaseous state to liquid state, from liquid state to semi-solid state and finally becomes a tough solid material. Let us consider the example of addition of (CH_2) unit to a polymer chain as shown in Fig. 2.8.

(i) Initial composition is (CH_4) which is methane gas.
(ii) Addition of one unit of (CH_2) to a methane molecule results in heavier ethane gas with (C_2H_6) composition.
(iii) Further addition of (CH_2) units to the ethane chain results in pentane, which is in liquid form with (C_5H_{12}) composition.

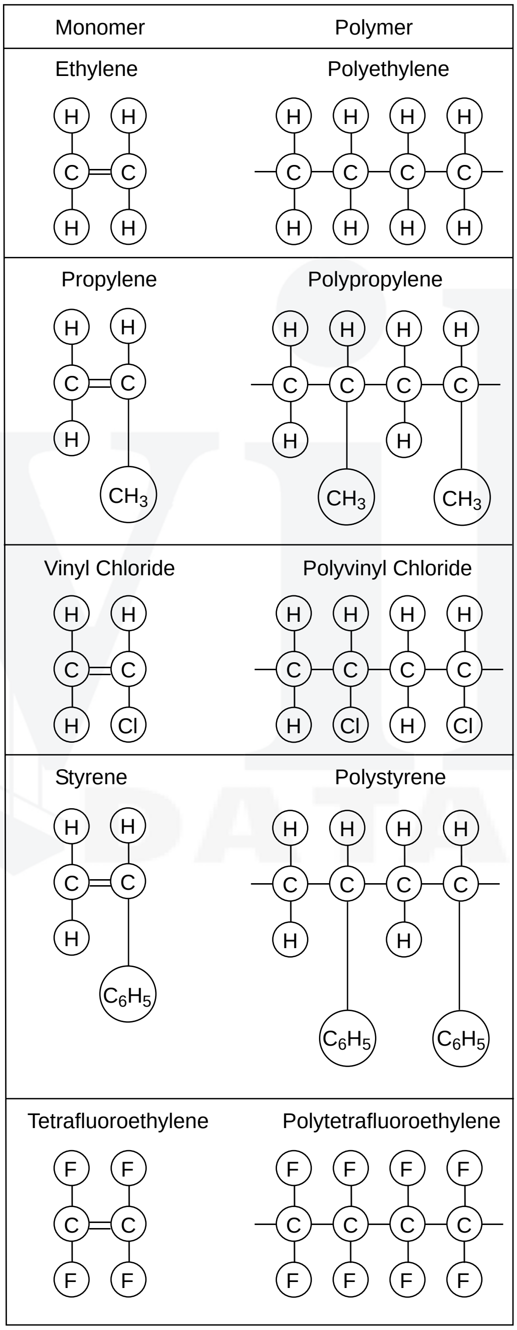


Fig. 2.7 *Construction of Monomer and Polymer*

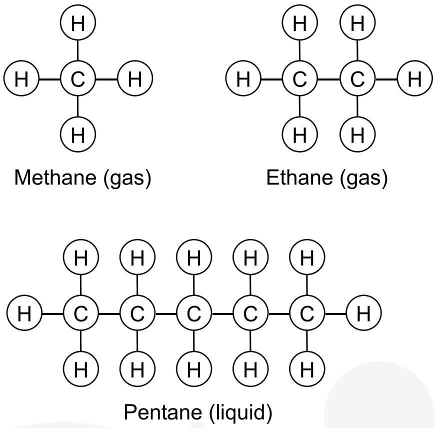


Fig. 2.8 *Monomer Chains*

(iv) If the process of adding (CH_2) units to the pentane chain is continued, paraffin wax is obtained. It is in a semi-solid stage with (C_{18} H_{38}) composition.

(v) If the process is further continued, a solid plastic called low-density polyethylene is obtained at approximately (C_{100} H_{102}) composition.

(vi) In the next stage, high-density polyethylene is obtained. It contains about half-million (CH_2) units in a single chain. It is a very tough solid plastic.

Adding a terminal link called *terminator*, which satisfies the bonds at each end of the chain, stops the linking of monomer units.

Knowing the construction of a monomer and polymer, we can define the term *plastic* at this stage. *A plastic can be defined as a solid material consisting of an organic polymer of a long molecular chain and high molecular weight*. It may also contain some additives like fillers, plasticisers, flame retardants and pigments. A *filler* is an inert foreign substance added to a polymer to improve certain properties such as tensile and compressive strengths, abrasion resistance, toughness and dimensional and thermal stability. Filler materials include finely powdered sawdust, silica floor and sand, clay, limestone and talc. A *plasticiser* is low-molecular weight polymer additive that improves flexibility, ductility and toughness and reduces brittleness and stiffness. They include polyvinyl chloride and acetate copolymers. A *flame retardant* is an additive which increases flammability resistance. Most polymers are flammable in their pure form. A flame retardant interferes with the combustion process and prevents burning. A *pigment* or *colourant* imparts a specific colour to the plastic material.

Plastics are divided into two basic groups depending on their behaviour at elevated temperatures, viz., thermoplastics and thermosetting plastics. *A thermoplastic is a polymeric material which softens when heated and hardens upon cooling. A thermosetting plastic is a polymeric material, which once having cured or hardened by a chemical reaction does not soften or melt upon subsequent heating*. In short, a thermoplastic softens with heat while a thermosetting plastic does not. A thermoplastic material can be moulded and remoulded repeatedly. This difference in properties of thermoplastic and thermosetting plastic materials is due to molecular structures of their polymer chains. A thermoplastic material has a linear polymer chain while a thermosetting plastic material consists of a cross-linked polymer chain as

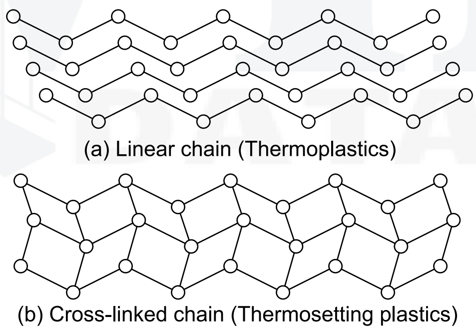


Fig. 2.9 *Linear and Cross-linked Polymer Chains*

shown in Fig. 2.9. The difference between the two categories of plastic is as follows:

(i) A thermoplastic material has a linear polymer chain. A thermosetting plastic material has cross-linked polymer chain.

(ii) A thermoplastic material can be softened, hardened or resoftened repeatedly by the application of heat. Alternate heating and cooling can reshape them. On the other hand, thermosetting plastic materials, once set and hardened, cannot be remelted or reshaped.

(iii) Thermoplastic materials can be recycled. Therefore, thermoplastic components are environmentally friendly. It is not possible to recycle a thermosetting plastic material. Disposal of components made of thermosetting plastic material, after their useful life, creates a problem.

(iv) Molecules in a linear chain can slide over each other. Therefore, thermoplastic materials are flexible. On the other hand, cross-linked thermosetting materials are more rigid. Their rigidity increases with the number of cross-links.

(v) Common examples of thermoplastic materials are polyethylene, polypropylene, polyvinylchloride (PVC), polystyrene, polytetrafluoethylene (PTFE) and nylon. Common examples of thermosetting plastic materials are phenolics, aminos, polyesters, epoxies and phenal-formaldehyde.

As a material for machine component, plastics offer the following advantages:

(i) Plastic materials have low specific gravity resulting in lightweight construction. The specific gravity of the heaviest plastic is 2.3 compared with 7.8 of cast iron.

(ii) Plastics have high corrosion resistance in any atmospheric condition. This is the most important advantage of plastic materials over metals. Many varieties of plastic materials are acid-resistant and can endure chemicals for a long period of time. PVC has excellent resistance to acids and alkalis.

(iii) Some plastic materials have low coefficient of friction and self-lubricating property. The coefficient of friction of polytetrafluoroethylene, commonly called *Teflon*, is as low as 0.04. Such materials are ideally suitable for bearings.

(iv) Fabrication of plastic components is easy. Raw material is available in the form of powders, granules or compressed pills. The raw material is converted into plastic parts by compression moulding, injection moulding, transfer moulding or extrusion process. Compression moulding is commonly used for components made of thermosetting plastic materials. Injection moulding is widely used for parts made of thermoplastics. Complicated parts performing several functions can be moulded from plastic material in a single operation.

Plastic materials have the following disadvantages:

(i) Plastic materials have poor tensile strength compared with other construction materials. The tensile strength of plastic materials varies from 10 N/mm^2 to 80 N/mm^2.

(ii) Mechanical properties of engineering metals do not vary much within the range of ambient temperatures encountered in practice. For many polymers, particularly thermoplastic materials, the mechanical properties vary considerably with temperature in the ambient region. For example, a thermoplastic material may have a tensile strength of 70 N/mm^2 at 0°C, falling to 40 N/mm^2 at 25°C and further to 10 N/mm^2 at 80°C.

(iii) A number of polymeric materials display viscoelastic mechanical behaviour. These materials behave like a glass at low temperatures, a rubbery solid at intermediate temperatures and a viscous liquid as the temperature is further increased. Therefore, such materials are elastic at low temperature and obey Hooke's law and at high temperatures, a liquid-like behaviour prevails. At intermediate temperatures, the rubberlike solid state exhibits combined mechanical characteristics of these two extremes. This condition is called *viscoelasticity*.

(iv) Many plastic materials are susceptible to time-dependent deformation when the stress level is maintained constant. Such deformation is called *creep*. This type of deformation is significant even at room temperature and under moderate stresses, which are even below the yield strength of the material. Due to creep, a machine component of plastic material under load may acquire a permanent set even at room temperature.

Although a large number of plastics are developed, we will consider a few materials in this article. These materials are mainly used for machine components. The names in brackets indicate popular trade names of the material.

(i) Polyamide (Nylon, Capron Nylon, Zytel, Fosta) Polyamide is a thermoplastic material. It has excellent toughness and wear resistance. The coefficient of friction is low. It is used for gears, bearings, conveyor rollers and automotive cooling fans.

(ii) Low-density Polyethylene (Polythene) It is a thermoplastic material. It is flexible and tough, light in weight, easy to process and a low-cost material. It is used for gaskets, washers and pipes.

(iii) Acetal (Delrin) It is a thermoplastic and a strong engineering material with exceptional dimensional stability. It has low coefficient of friction and high wear resistance. It is used for self-lubricating bearings, cams and running gears.

(iv) Polyurethane (Duthane, Texin) It is a thermoplastic and a tough, abrasion-resistant and impact-resistant material. It has good dimensional properties and self-lubricating characteristics and is used for bearings, gears, gaskets and seals.

(v) Polytetrafluoroethylene (Teflon) It is a thermoplastic material. It has low coefficient of friction and self-lubricating characteristics. It can withstand a wide range of temperatures from –260 to + 250°C. It is ideally suitable for self-lubricating bearing.

(vi) Phenolic It is a thermosetting plastic material. It has low cost with a good balance of mechanical and thermal properties. It is used in clutch and brake linings as filler material. Glass reinforced phenolic is used for pulleys and sheaves.

The mechanical properties of plastic materials are given in Table 2.12.

Table 2.12 *Mechanical properties of plastics*

Material	*Specific gravity*	*Tensile strength (N/mm^2)*	*Compressive strength (N/mm^2)*
Polyamide	1.04–1.14	70	50–90
Low-density Polythene	0.92–0.94	7–20	–
Acetal	1.41–1.42	55–70	–
Polyurethane	1.21–1.26	35–60	25–80
Teflon	2.14–2.20	10–25	10–12
Phenolic	1.30–1.90	30–70	–

2.17 FIBRE REINFORCED PLASTICS

Fibre reinforced plastic (FRP) is a composite material in which the low strength of the polymeric material is increased by means of high strength fibres. There are two main constituents of fibre reinforced plastic, viz., matrix and fibres. The function of the *matrix* is to provide a rigid base for holding the fibres in correct position. The function of the *fibres* is to transmit the load acting on the component. The bond between the surface of the fibres and surrounding matrix is usually chemical. The matrix protects the fibres from surface damage and from the action of environment. The fibres used in composite should be long enough so that the bonding force between the surface of the fibre and the surrounding matrix is greater than the tensile strength of the fibre.

Two types of fibres are widely used, viz., glass and carbon fibres. The advantages of glass reinforced plastics (GRP) are as follows:

(i) Glass can be easily drawn into fibres from the molten state.
(ii) Glass is cheaper and readily available.
(iii) Glass fibre is relatively strong.
(iv) Glass is chemically inert with respect to plastic matrix materials.

The disadvantages of glass reinforced plastic are as follows:

(i) Glass reinforced plastic has poor rigidity and stiffness.

(ii) Its application is limited up to a temperature of 300°C.

Glass reinforced plastic is used for automotive bodies, pipes, valve bodies, pump casings and storage containers. It is more popular for vehicle bodies due to low specific gravity resulting in lightweight construction.

The advantages of carbon reinforced plastics (CRP) are as follows:

(i) Carbon fibre has maximum strength compared with all other fibre materials.
(ii) Carbon fibre retains its strength at elevated temperature.
(iii) Moisture, acids and solvents at ambient temperature do not affect carbon fibre.
(iv) Carbon reinforced plastic is relatively cheap.

There is one limitation for carbon reinforced plastic. Manufacturing techniques required to produce carbon fibre are relatively complicated. Carbon reinforced plastic is used for pressure vessels, aircraft components and casings of rocket motors.

Matrix material used in GRP and CRP should remain stable at the temperatures encountered in the application. It should not be affected by moisture and surrounding atmosphere. Both thermoplastic and thermosetting plastics are used as matrix materials. Nylon is a commonly used thermoplastic, while phenolic resins are popular thermosetting plastics for the matrices of the reinforced composites.

As an engineering material for structural components, fibre reinforced plastic offers the following advantages:

(i) It has low specific gravity resulting in lightweight construction.
(ii) It has high specific strength and modulus of elasticity.
(iii) It has good resistance to fatigue failure, particularly parallel to the direction of the fibres.
(iv) It has good resistance to corrosion.

The term *specific strength* means the ratio of the tensile strength to the specific gravity of the material.

The disadvantages of fibre reinforced plastic are as follows:

(i) A composite material containing fibres in a single direction is extremely anisotropic. The tensile strength of such a material in a direction perpendicular to that of fibres may be 5% or even less than that measured in the direction of fibres. Many times, the transverse strength is less than that of the matrix material because of the presence of discontinuities and insufficient binding between the fibres and the surrounding matrix material.
(ii) The design of components made of fibre reinforced plastics is complex. It is necessary to know the direction of principal stresses in such components. The fibres are aligned along the direction of principal stresses.
(iii) The manufacturing and testing of fibre reinforced components is highly specialised.

Fibre reinforced composite is a comparatively new material. It is being increasingly used for machine and structural parts such as motor shafts, gears and pulleys. Such materials are 'custom-made' materials, which combine the desirable characteristics of two or more materials in a given required manner.

2.18 NATURAL AND SYNTHETIC RUBBERS

Natural rubber is obtained from rubber latex, which is a milky liquid obtained from certain tropical trees. It is a low cost elastomer. Different varieties of rubber are obtained by adding carbon, silica and silicates. *Vulcanised rubber* is obtained by adding sulphur, which is followed by heating. Addition of carbon makes the rubber hard. Natural rubber, in hard and semi-hard conditions, is used for belts, bushes, flexible tubes and vibration mounts. It is also used for production of coatings, protective films and adhesives. Rubber coatings provide protection in a chemical environment.

Synthetic rubber has properties similar to those of natural rubber. It can be thermoplastic or a thermosetting plastic. It is, however, costlier than natural rubber. A few applications of synthetic rubber are as follows:

(i) Chloroprene (Neoprene) Conveyors and V belts, brake diaphragms and gaskets

(ii) Nitrile Butadiene (NBR) Bushes for flexible coupling and rubber rollers

(iii) Polysulfide (Thikol) Gaskets, washers and diaphragms

(iv) Chlorosulfonyl Polyethylene (Hypalon) Tank lining, high temperature conveyor belts, seals and gaskets

(iv) Silicone Seals, gaskets and O-rings

2.19 CREEP

When a component is under a constant load, it may undergo progressive plastic deformation over a period of time. This time-dependent strain is called creep. *Creep is defined as slow and progressive deformation of the material with time under a constant stress*. Creep deformation is a function of stress level and temperature. Therefore, creep deformation is higher at higher temperature and creep becomes important for components operating at elevated temperatures. Creep of bolts and pipes is a serious problem in thermal power stations. The material of steam or gas turbine blades should have a low creep rate, so that blades can remain in service for a long period of time before having to be replaced due to their reaching the maximum allowable strain. These blades operate with very close clearances and permissible deformation is an important consideration in their design. Design of components working at elevated temperature is based on two criteria. Deformation due to creep must remain within permissible limit and rupture must not occur during the service life. Based on these two criteria, there are two terms—creep strength and creep rupture strength. *Creep strength of the material is defined as the maximum stress that the material can withstand for a specified length of time without excessive deformation. Creep rupture strength of the material is the maximum stress that the material can withstand for a specified length of time without rupture.*

An idealised creep curve is shown in Fig. 2.10. When the load is applied at the beginning of the creep test, the instantaneous elastic deformation *OA* occurs. This elastic deformation is followed

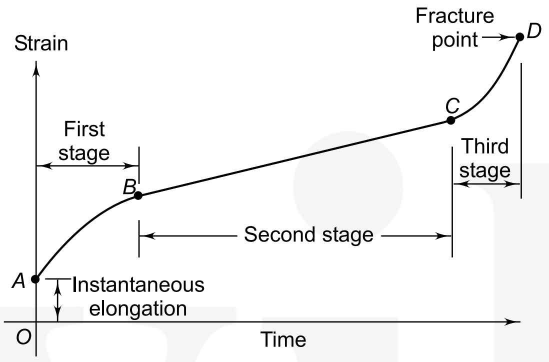


Fig. 2.10 *Creep Curve*

by the creep curve *ABCD*. Creep occurs in three stages. The first stage called *primary creep* is shown by *AB* on the curve. During this stage, the creep rate, i.e., the slope of the creep curve from *A* to *B* progressively decreases with time. The metal strain hardens to support the external load. The creep rate decreases because further strain hardening becomes more and more difficult. The second stage called *secondary creep* is shown by *BC* on the curve. During this stage, the creep rate is constant. This stage occupies a major portion of the life of the component. The designer is mainly concerned with this stage. During secondary creep, recovery processes involving highly mobile dislocations counteract the strain hardening so that the metal continues to elongate at a constant rate. The third stage called *tertiary creep* is shown by *CD* on the creep curve. During this stage, the creep rate is accelerated due to necking and also due to formation of voids along the grain boundaries. Therefore, creep rate rapidly increases and finally results in fracture at the point *D*. Creep properties are determined by experiments and these experiments involve very long periods stretching into months.

2.20 SELECTION OF MATERIAL

Selection of a proper material for the machine component is one of the most important steps in the process of machine design. The best material is one which will serve the desired purpose at minimum cost. It is not always easy to select such a material and the process may involve the trial and error method. The factors which should be considered while selecting the material for a machine component are as follows:

(i) Availability The material should be readily available in the market, in large enough quantities to meet the requirement. Cast iron and aluminium alloys are always available in abundance while shortage of lead and copper alloys is a common experience.

(ii) Cost For every application, there is a limiting cost beyond which the designer cannot go. When this limit is exceeded, the designer has to consider other alternative materials. In cost analysis, there are two factors, namely, cost of material and the cost of processing the material into finished goods. It is likely that the cost of material might be low, but the processing may involve costly manufacturing operations.

(iii) Mechanical Properties Mechanical properties are the most important technical factor governing the selection of material. They include strength under static and fluctuating loads, elasticity, plasticity, stiffness, resilience, toughness, ductility, malleability and hardness. Depending upon the service conditions and the functional requirement, different mechanical properties are considered and a suitable material is selected. For example, the material for the connecting rod of an internal combustion engine should be capable to withstand fluctuating stresses induced due to combustion of fuel. In this case, endurance limit becomes the criterion of selection. The piston rings should have a hard surface to resist wear due to rubbing action with the cylinder surface, and surface hardness is the selection criterion. In case of bearing materials, a low coefficient of friction is desirable while clutch or brake lining requires a high coefficient of friction. The material for automobile bodies and hoods should have the ability to be extensively deformed in plastic range without fracture, and plasticity is the criterion of material selection.

(iv) Manufacturing Considerations In some applications, machinability of material is an important consideration in selection. Sometimes, an expensive material is more economical than a low priced one, which is difficult to machine. Free cutting steels have excellent machinability, which is an important factor in their selection for high strength bolts, axles and shafts. Where the product is of complex shape, castability or ability of the molten metal to flow into intricate passages is the criterion of material selection. In fabricated assemblies of plates and rods, weldability becomes the governing factor. The manufacturing processes, such as casting, forging, extrusion, welding and machining govern the selection of material.

Past experience is a good guide for the selection of material. However, a designer should not overlook the possibilities of new materials.

2.21 WEIGHTED POINT METHOD

In recent years, systematic methods have been developed for selection of materials. One such method is the weighted point method. It consists of the following four steps:

(i) The first step consists of the study of the given application and preparing a list of the desirable properties of the material for the application.

(ii) The desirable properties are then assigned values. The approximate range of these properties, such as yield strength, endurance strength, hardness, etc., is specified.

(iii) The desirable properties are divided into two groups—Go-no-go *parameters* and *discriminating parameters*. The Go-no-go parameters are the constraints. As an example, if a material is not available, or if it cannot be fabricated into a given shape, it is totally rejected. This is a screening step and only those materials, which meet the essential requirement, are allowed further consideration.

(iv) The discriminating parameters are the properties of the material which can be given quantitative values. The weightage depends upon the importance of that particular property in the given application. As an example, in case of a connecting rod, the endurance strength may be given a weighting factor of 5, compared with the cost having a weighting factor of 1. In general, the weighting factor varies from 1 to 5, with 1 for the poorest and 5 for the best.

Then each property of the candidate material is assigned a rating, ranging from 1 to 5, depending upon how closely it meets the requirements. These ratings are multiplied by the weighting factors for each property. These numbers are finally added and materials are arranged in descending order of their total points.

The main drawback of this method is the skill and judgement required for assigning the weightage. The results may not be numerically correct; however, one can get a priority list of materials for a given application.

Example 2.1 *It is required to select a material by the weighted point method. There are four candidate materials, viz., low alloy steel, plain carbon steel, stainless steel and chromium steel, which have passed through screening test. For a particular application, the designer has given a 5-point weightage for ultimate tensile strength, 3-point weightage for hardenability and 2-point weightage for cost-economy. Table 2.13 gives the data for the candidate materials.*

Table 2.13

Sr. No.	Material Property	*Materials* Low alloy steel	Plain carbon steel	Stainless steel	Chromium steel
1	Ultimate tensile strength (N/mm^2)	850	850	1200	950
2	Hardenability Index	60	80	30	100
3	Cost (Rs / unit)	40	50	100	80

Select the most suitable material for the given application.

Solution

Part I *Calculation of weightage points for low alloy steel*

Step I *Points for ultimate tensile strength*

The sum of ultimate tensile strength of four materials is given by

$$850 + 850 + 1200 + 950 = 3850$$

Therefore, for low alloy steel, the per cent strength is given by

$$\frac{850}{3850} = 0.22$$

Since weightage for strength is 5, the points for low alloy steel are given by

$$0.22 \times 5 = 1.1 \qquad \text{(a)}$$

Step II *Points for hardenability index*

The sum of hardenability index of four materials is given by

$$60 + 80 + 30 + 100 = 270$$

Therefore, for low alloy steel, the per cent hardenability index is given by

$$\frac{60}{270} = 0.222$$

Since weightage for hardenability is 3, the points for low alloy steel are given by

$$0.22 \times 3 = 0.666 \qquad \text{(b)}$$

Step III *Points for cost*

The points for cost are inversely proportional because a material with lower cost or points is a better material.

The sum of cost factor is given by

$$\frac{1}{40} + \frac{1}{50} + \frac{1}{100} + \frac{1}{80} = 0.025 + 0.02$$
$$+ 0.01 + 0.0125 = 0.0675$$

Therefore, for low alloy steel, the per cent for cost factor is given by

$$\frac{0.0250}{0.0675} = 0.37$$

Since weightage for cost factor is 2, the points for low alloy steel are given by

$$0.37 \times 2 = 0.74 \qquad \text{(c)}$$

Step IV *Total points*

From (a), (b) and (c), the total points for low alloy steel are given by

$$1.1 + 0.666 + 0.74 = 2.506$$

Part II *Tabulation of weightage points*

Similarly, total points for other materials are calculated and given in Table 2.14.

Table 2.14

	Material Property	*Low alloy steel*	*Plain carbon steel*	*Stainless steel*	*Chromium steel*
(a)	Tensile strength				
	Per cent	0.22	0.22	0.312	0.247
	Points	1.10	1.10	1.56	1.235
(b)	Hardenability				
	Per cent	0.222	0.296	0.111	0.37
	Points	0.666	0.888	0.333	1.11
(c)	Cost				
	Per cent	0.37	0.296	0.148	0.185
	Points	0.74	0.592	0.296	0.37
	Total Points	2.506	2.58	2.189	2.715

Part III *Selection of material*

The list of material according to descending order of points will be

(i) Chromium steel (2.715 points)
(ii) Plain carbon steel (2.58 points)
(iii) Low alloy steel (2.506 points)
(iv) Stainless steel (2.189 points)

Therefore, for this particular application, chromium steel is selected as the best material for the component.

Short-Answer Questions

2.1 What is cast iron?
2.2 What is the percentage of carbon in cast iron and steel?
2.3 What are the advantages of cast iron form design considerations?
2.4 What are the disadvantages of cast iron form design considerations?
2.5 What is grey cast iron?
2.6 How will you designate grey cast iron?
2.7 Name the components made of grey cast iron.
2.8 What are white cast irons?
2.9 How will you designate white cast iron?
2.10 Name the components made of white cast iron.
2.11 What is spheroidal graphite cast iron?
2.12 How will you designate plain carbon steels?
2.13 What is high alloy steel?
2.14 How will you designate high alloy steels?
2.15 What is X20Cr18Ni2 designation of steel?
2.16 What is low carbon steel?
2.17 What is medium carbon steel?
2.18 What is high carbon steel?

2.19 What is mild steel?

2.20 What is the percentage of carbon in mild steel?

2.21 Define alloy steel.

2.22 Name the various alloying elements in 'alloy' steels.

2.23 What are the advantages of alloy steel?

2.24 What are the important components made of alloy steels?

2.25 Compare cast iron and cast steel components.

2.26 Compare steel and cast steel components.

2.27 Name the components made of carbon steel castings?

2.28 Name the components made of high tensile steel castings?

2.29 What are the advantages of aluminum alloy for mechanical components?

2.30 Name the components made of aluminum alloy castings?

2.31 Name the components made of wrought aluminum alloy.

2.32 What are the advantages of copper alloys from design considerations?

2.33 What are the disadvantages of copper alloys from design considerations?

2.34 What are ceramics?

2.35 What are the advantages and drawbacks of ceramics?

2.36 What are the applications of ceramics in engineering industries?

2.37 What is plastic?

2.38 What is a monomer? Give its examples.

2.39 What is a polymer? Give its examples.

2.40 What are the types of plastics?

2.41 What is a thermoplastic? Give its examples.

2.42 What is a thermosetting plastic? Give its examples.

2.43 What is Teflon? Where do you use it?

2.44 What is fibre reinforced plastic (FRP)?

2.45 What are the advantages of fibre reinforced plastics?

2.46 What are the disadvantages of fibre reinforced plastics?

2.47 What is creep?

2.48 Explain the situations where creep is a serious problem.

2.49 What are the factors to be considered for selection of material for a machine component?

2.50 Explain the principle of weighted point method for selection of material for a machine component.

Manufacturing Considerations in Design

Chapter 3

3.1 SELECTION OF MANUFACTURING METHOD

Manufacturing of the product is an important link in the chain of events that begins with the concept of a probable product and ends with a competitive product in the market place. Product design, selection of materials and processing the materials into finished components are closely related to one another. Manufacturing can be considered as processing the available material into useful components of the product, e.g., converting a mild steel sheet into car body, converting a billet of cast iron into a machine tool bed or converting a steel bar into a transmission shaft. The manufacturing processes can be broadly classified into the following three categories:

(i) Casting Processes In these processes, molten metals such as cast iron, copper, aluminium or non-metals like plastic are poured into the mould and solidified into the desired shape, e.g., housing of gear box, flywheel with rim and spokes, machine tool beds and guides.

(ii) Deformation Processes In these processes, a metal, either hot or cold, is plastically deformed into the desired shape. Forging, rolling, extrusion, press working are the examples of deformation processes. The products include connecting rods, crankshafts, I-section beams, car bodies and springs.

(iii) Material Removal or Cutting Processes In these processes, the material is removed by means of sharp cutting tools. Turning, milling, drilling, shaping, planing, grinding, shaving and lapping are the examples of material removal processes. The products include transmission shafts, keys, bolts and nuts.

In addition, there are joining processes like bolting, welding and riveting. They are essential for the assembly of the product.

Many times, a number of manufacturing methods are available to make the component. In such cases, the optimum manufacturing method is selected by considering the following factors:

(i) Material of the component
(ii) Cost of manufacture
(iii) Geometric shape of the component
(iv) Surface finish and tolerances required
(v) Volume of production

One of the easiest methods to convert the raw material into the finished component is *casting*. There are several casting processes such as sand casting, shell-mould casting, permanent mould casting, die casting, centrifugal casting or investment casting. *Sand casting* is the most popular casting process. The advantages of sand casting process as a manufacturing method are as follows:

(i) The tooling required for casting process is relatively simple and inexpensive. This reduces the cost. Sand casting is one of the cheapest methods of manufacturing.

(ii) Almost any metal such as cast iron, aluminium, brass or bronze can be cast by this method.

(iii) Any component, even with a complex shape, can be cast. There is no limit on the size of the component. Even large components can be cast.

The disadvantages of the sand casting process are as follows:

(i) It is not possible to achieve close tolerances for cast components. Therefore, cast components require additional machining and finishing, which increases cost.

(ii) Cast components have a rough surface finish.

(iii) Long and thin sections or projections are not possible for cast components.

One of the important deformation processes is *forging*. In forging, the metal in the plastic stage, rather than in the molten stage, is forced to flow into the desired shape. There are a number of forging processes such as hand forging, drop forging, press forging or upset forging. The *drop-forging method* accounts for more than 80% of the forged components. The advantages of forging as a manufacturing method are as follows:

(i) The fibrelines of a forged component can be arranged in a predetermined way to suit the direction of external forces that will act on the component when in service. Therefore, forged components have inherent strength and toughness. They are ideally suitable for applications like connecting rods and crankshafts.

(ii) In forging, there is relatively good utilisation of material compared with machining.

(iii) Forged components can be provided with thin sections, without reducing the strength. This results in lightweight construction.

(iv) The tolerances of forged parts can be held between close limits, which reduce the volume of material removal during the final finishing stages.

(v) The forging process has a rapid production rate and good reproducibility.

The disadvantages of the forging process are as follows:

(i) Forging is a costly manufacturing method. The equipment and tooling required for forging is costly.

(ii) Forging becomes economical only when the parts are manufactured on a large scale.

Material removal or cutting processes are the most versatile and most common manufacturing methods. Almost every component is subjected to some kind of machining operation in its final finishing stage. Metal removal processes are broadly divided into three categories—metal cutting processes, grinding processes and unconventional machining processes. Depending upon the shape of machined surfaces, the metal removal processes are selected in the following way:

(i) For machining flat surfaces, shaping, planing and milling processes are usually used. A flat surface can also be machined on a lathe by the facing operation. Broaching and surface grinding are finishing methods for flat surfaces.

(ii) For machining external cylindrical surface, turning on lathe is a popular method. Such surfaces are finished by the cylindrical grinding method.

(iii) For machining internal cylindrical surfaces, drilling and boring are popular processes. Reaming and cylindrical grinding are finishing processes.

The advantages of metal cutting processes as a manufacturing method are as follows:

(i) Almost any metal can be machined.

(ii) It is possible to achieve close tolerances for machined components.

(iii) Machined components have a good surface finish.

The disadvantages of machining processes are as follows:

(i) Machining processes are costly and the rate of production is low compared with casting or forging.

(ii) It is not possible to machine thin sections or projections.
(iii) There is wastage of material during material removal process.

In drilling operation, the cost of the hole increases linearly with the depth of the hole. However, when the depth is more than three times the diameter, the cost increases more rapidly.

3.2 DESIGN CONSIDERATIONS OF CASTINGS

Complex parts, which are otherwise difficult to machine, are made by the casting process using sand mould. Almost any metal can be melted and cast. Most of the sand cast parts are made of cast iron, aluminium alloys and brass. The size of the sand casting can be as small as 10 g and as large as 200×10^3 kg. Sand castings have irregular and grainy surfaces and machining is required if the part is moving with respect to some other part or structure. Cast components are stable, rigid and strong compared with machined or forged parts. Typical examples of cast components are machine tool beds and structures, cylinder blocks of internal combustion engines, pumps and gear box housings.

Poor shaping of a cast iron component can adversely affect its strength more than the composition of the material. Before designing castings, the designer should consult the foundry man and the patternmaker, whose cooperation is essential for a successful design. The general principles for the design of casting[1] are as follows:

Always Keep the Stressed Areas of the Part in Compression Cast iron has more compressive strength than its tensile strength. The balanced sections with equal areas in tension and compression are not suitable for cast iron components. The castings should be placed in such a way that they are subjected to compressive rather than tensile stresses as illustrated in Fig. 3.1. When tensile stresses are unavoidable, a clamping device such as a tie rod or a bearing cap as illustrated in Fig. 3.2 should be considered. The clamping device relieves the cast iron components from tensile stresses.

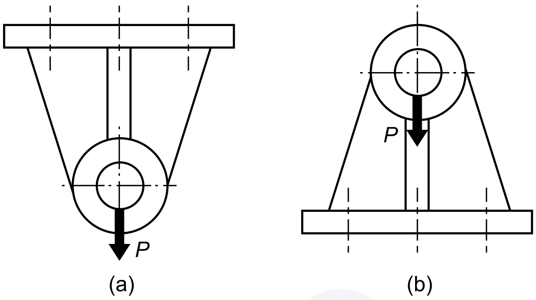


Fig. 3.1 *(a) Incorrect (Part in Tension) (b) Correct (Part in Compression)*

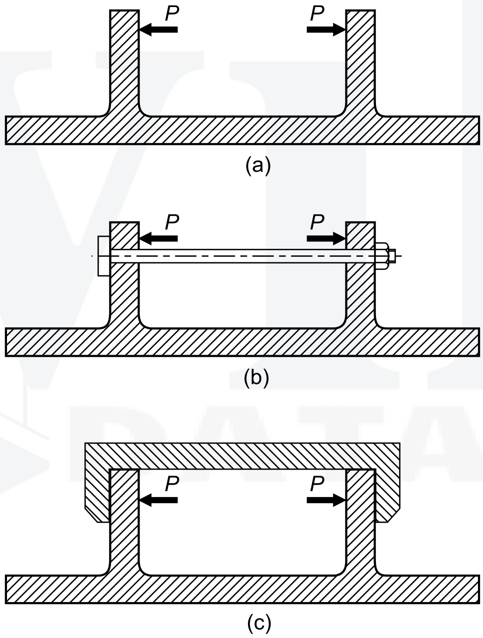


Fig. 3.2 *(a) Original Component (b) Use of Tie-rod (c) Use of Bearing-cap*

Round All External Corners It has two advantages—it increases the endurance limit of the component and reduces the formation of brittle chilled edges. When the metal in the corner cools faster than the metal adjacent to the corner, brittle chilled edges are formed due to iron carbide.

[1] John B. Caine, 'Five ways to improve grey iron-castings', Machine Design, vol. 36, no. 25, October 22, 1964.

Appropriate fillet radius, as illustrated in Fig. 3.3 reduces the stress concentration. The values of the corner radii for different section thickness are given in Table 3.1.

Table 3.1

Wall thickness (mm)	*Inside corner radius (mm) (minimum)*
0–30	10
30–50	15
50–80	20
80–120	30

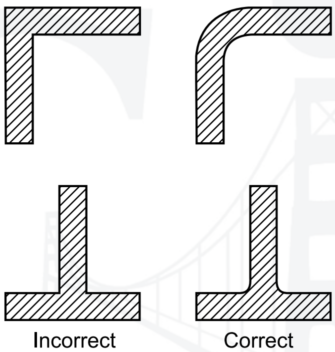


Fig. 3.3 *Provision of Fillet Radius*

Wherever Possible, the Section Thickness throughout should be Held as Uniform as Compatible with Overall Design Considerations Abrupt changes in the cross-section result in high stress concentration. If the thickness is to be varied at all, the change should be gradual as illustrated in Fig. 3.4.

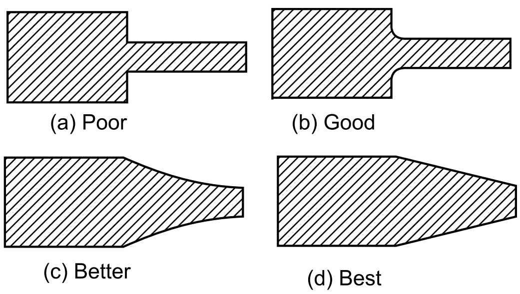


Fig. 3.4 *Change in Section-thickness*

Avoid Concentration of Metal at the Junctions At the junction as shown in Fig. 3.5, there is a concentration of metal. Even after the metal on the surface solidifies, the central portion still remains in the molten stage, with the result that a shrinkage cavity or blowhole may appear at the centre. There are two ways to avoid the concentration of metal. One is to provide a cored opening in webs and ribs, as illustrated in Fig. 3.6. Alternatively, one can stagger the ribs and webs, as shown in Fig. 3.7.

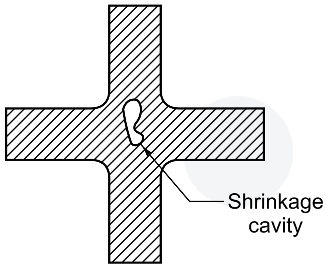


Fig. 3.5

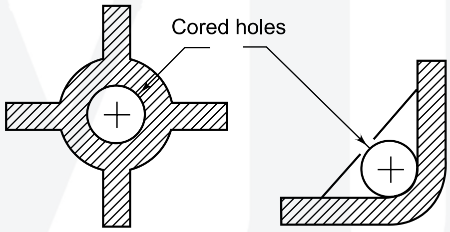


Fig. 3.6 *Cored Holes*

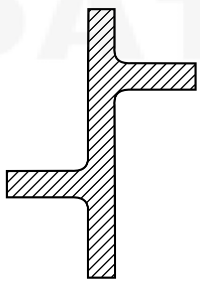


Fig. 3.7 *Staggered Ribs*

Avoid Very Thin Sections In general, if the thickness of a cast iron component is calculated from strength considerations, it is often too small. In such cases, the thickness should be increased to certain practical proportions. The minimum section thickness depends upon the process of casting, such as sand casting, permanent mould casting or die casting. The minimum thickness for a grey cast iron component is about 7 mm for parts up to 500 mm long, which gradually increases to 20 mm for large and heavy castings.

Shot Blast the Parts wherever Possible The shot blasting process improves the endurance limit of the component, particularly in case of thin sections.

Some ways to improve the strength of castings are illustrated in Figs 3.8 to 3.11. In Fig. 3.8, the inserted stud will not restore the strength of the original thickness. The wall adjacent to the drilled hole should have a thickness equivalent to the thickness of the main body. Figure 3.9 shows cored holes in webs or ribs. Oval-shaped holes are preferred with larger dimensions along the direction of forces. Patterns without a draft make a mould difficult and costly. A minimum draft of 3° should be provided, as illustrated in Fig. 3.10. Outside bosses should be omitted to facilitate a straight pattern draft as shown in Fig. 3.11.

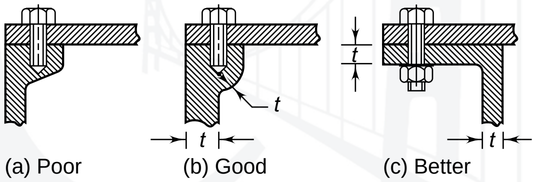


Fig. 3.8 *Uniform Wall-thickness*

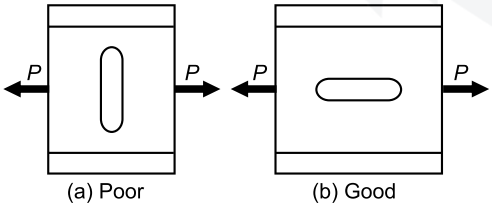


Fig. 3.9 *Cored Holes in Ribs*

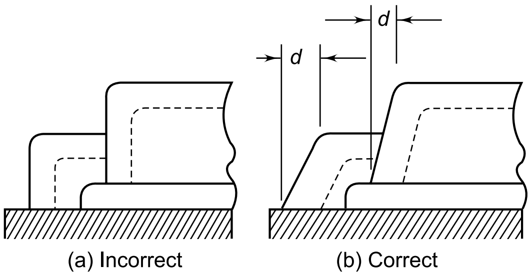


Fig. 3.10 *Provision of Draft*

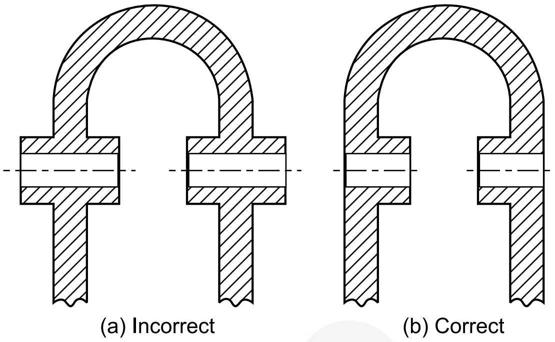


Fig. 3.11

3.3 DESIGN CONSIDERATIONS OF FORGINGS

Forged components are widely used in automotive and aircraft industries. They are usually made of steels and non-ferrous metals. They can be as small as a gudgeon pin and as large as a crankshaft. Forged components are used under the following circumstances:

(i) Moving components requiring light weight to reduce inertia forces, e.g., connecting rod of IC engines.

(ii) Components subjected to excessive stresses, e.g., aircraft structures.

(iii) Small components that must be supported by other structures or parts, e.g., hand tools and handles.

(iv) Components requiring pressure tightness where the part must be free from internal cracks, e.g., valve bodies.

(v) Components whose failure would cause injury and expensive damage are forged for safety.

In order to obtain maximum benefit from forged components, the following principles should be adopted:

(i) While designing a forging, advantage should be taken of the direction of fibre lines. The grain structure of a crankshaft manufactured by the three principal methods, viz., casting, machining and forging, is shown in Fig. 3.12. There are no fibre lines in the cast component and the grains are scattered. In case of a component prepared by machining methods, such as turning or milling, the original fibre lines of rolled stock are broken. It is only in case of forged parts that the fibre lines are arranged in a favourable way to withstand stresses due to external load. While designing a forging, the profile is selected in such a way that fibre lines are parallel to tensile forces and perpendicular to shear forces. Machining that cuts deep into the forging should be avoided, otherwise the fibre lines are broken and the part becomes weak.

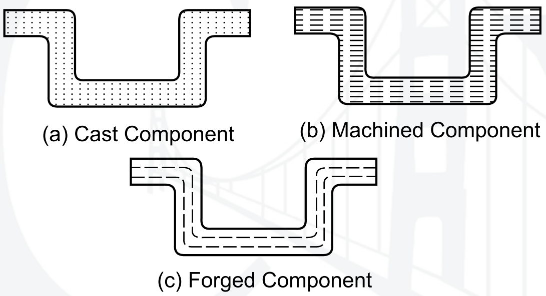


Fig. 3.12 *Grain Structure*

(ii) The forged component should be provided with an adequate draft as illustrated in Fig. 3.13. The draft angle is provided for an easy removal of the part from the die impressions. The angles α and β are drafts for outside and inside surfaces. As the material cools, it shrinks, and a gap is formed between the outer surface of the forging and the inner surfaces of the die cavity, with the result that the draft angle for the outer surface is small. On the other hand, when the material cools, its inner surfaces tend to shrink and grip the projecting surface of the die, with the result that the draft angle for the inner surface is large. For steels, the recommended values of α and β are 7° and 10° respectively.

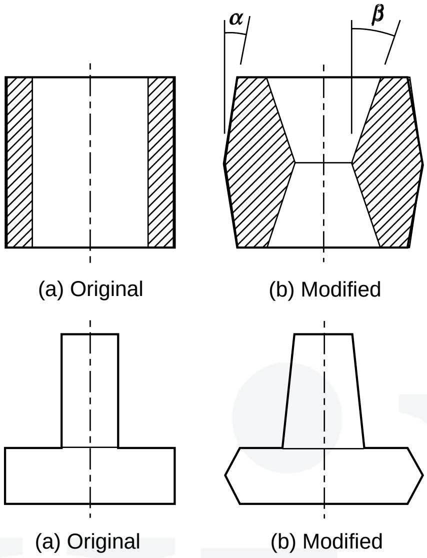


Fig. 3.13 *Draft for Forgings*

(iii) There are two important terms related to forgings, as illustrated in Fig. 3.14. The *parting line* is a plane in which the two halves of the forging dies meet and in which flash is formed. A *forging plane* is a plane, which is perpendicular to the die motion. In most of the cases, the parting line and forging plane coincide, as illustrated in Fig. 3.15. There are two basic principles for the location of the parting line—the parting line should be in one plane as far as possible and it should divide the forging into two equal parts. When the parting line is broken, as shown in Fig. 3.16, it results in unbalanced forging forces, which tend to displace the two die halves. Such forces are balanced either by a counter lock or by forging the two components simultaneously in a mirror-image position. A parting line that divides the forging into two halves ensures the minimum depth to which the steel must flow to fill the die impressions.

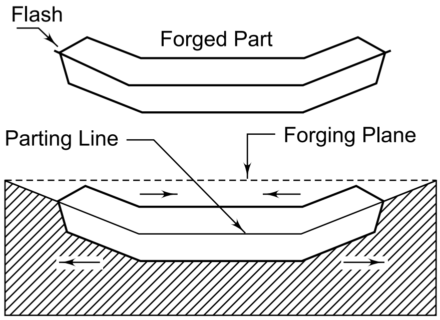


Fig. 3.14

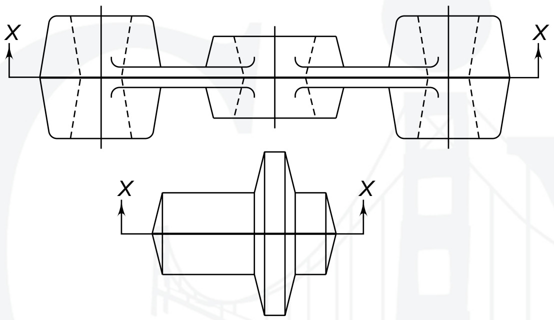


Fig. 3.15 *Location of Parting Line and Forging Plane (XX)*

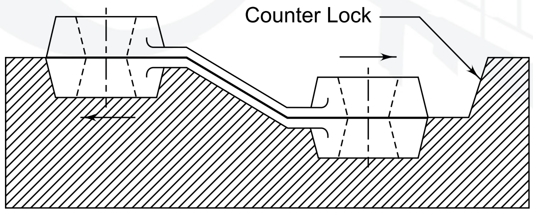


Fig. 3.16 *Unbalanced Forces*

(iv) The forging should be provided with adequate fillet and corner radii. A small radius results in folds on the inner surface and cracks on the outer surface. A large radius is undesirable, particularly if the forged component is to be machined, during which the fibre lines are broken. Sharp corners result in increasing difficulties in filling the material, excessive forging forces, and poor die life. The magnitude of fillet radius depends upon the material, the size of forging and the depth of the die cavity. For moderate size steel forgings, the minimum corner radii are 1.5, 2.5 and 3.5 mm for depths up to 10, 25 and 50 mm respectively.

(v) Thin sections and ribs should be avoided in forged components. A thin section cools at a faster rate in the die cavity and requires excessive force for plastic deformation. It reduces the die life, and the removal of the component from the die cavities becomes difficult. For steel forgings, the recommended value of the minimum section thickness is 3 mm.

A properly designed forging is not only sound with regard to strength but it also helps reduce the forging forces, improves die life and simplifies die design. If the design is poor, the best of steel and forging methods will not give a satisfactory component.

3.4 DESIGN CONSIDERATIONS OF MACHINED PARTS

Machined components are widely used in all industrial products. They are usually made from ferrous and non-ferrous metals. They are as small as a miniature gear in a wristwatch and as large as a huge turbine housing. Machined components are used under the following circumstances:

(i) Components requiring precision and high dimensional accuracy
(ii) Components requiring flatness, roundness, parallelism or circularity for their proper functioning
(iii) Components of interchangeable assembly
(iv) Components, which are in relative motion with each other or with some fixed part

The general principles for the design of machined parts are as follows:

(i) Avoid Machining Machining operations increase cost of the component. Components made by casting or forming methods are usually cheaper. Therefore, as far as possible, the designer should avoid machined surfaces.

(ii) Specify Liberal Tolerances The secondary machining operations like grinding or reaming are costly. Therefore, depending upon the functional requirement of the component, the designer should specify the most liberal dimensional and geometric tolerances. Closer the tolerance, higher is the cost.

(iii) Avoid Sharp Corners Sharp corners result in stress concentration. Therefore, the designer should avoid shapes that require sharp corners.

(iv) Use Stock Dimensions Raw material like bars are available in standard sizes. Using stock dimensions eliminates machining operations. For example, a hexagonal bar can be used for a bolt, and only the threaded portion can be machined. This will eliminate machining of hexagonal surfaces.

(v) Design Rigid Parts Any machining operation such as turning or shaping induces cutting forces on the components. The component should be rigid enough to withstand these forces. In this respect, components with thin walls or webs should be avoided.

(vi) Avoid Shoulders and Undercuts Shoulders and undercuts usually involve separate operations and separate tools, which increase the cost of machining.

(vii) Avoid Hard Materials Hard materials are difficult to machine. They should be avoided unless such properties are essential for the functional requirement of the product.

3.5 HOT AND COLD WORKING OF METALS

The temperature at which new stress-free grains are formed in the metal is called the *recrystallization temperature*. There are two types of metal deformation methods, namely, hot working and cold working. *Metal deformation processes that are carried out above the recrystallization temperature are called hot working processes*. Hot rolling, hot forging, hot spinning, hot extrusion, and hot drawing are hot working processes. *Metal deformation processes that are carried out below the recrystallization temperature are called cold working processes*. Cold rolling, cold forging, cold spinning, cold extrusion, and cold drawing are *cold working processes*. Hot working processes have the following advantages:

(i) Hot working reduces strain hardening.
(ii) Hot rolled components have higher toughness and ductility. They have better resistance to shocks and vibrations.
(iii) Hot working increases the strength of metal by refining the grain structure and aligning the grain of the metal with the final counters of the part. This is particularly true of forged parts.
(iv) Hot working reduces residual stresses in the component.

Hot working processes have the following disadvantages:

(i) Hot working results in rapid oxidation of the surface due to high temperature.
(ii) Hot rolled components have poor surface finish than cold rolled parts.
(iii) Hot working requires expensive tools.

Cold working processes have the following advantages:

(i) Cold rolled components have higher hardness and strength.
(ii) Cold worked components have better surface finish than hot rolled parts.
(iii) The dimensions of cold rolled parts are very accurate.
(iv) The tooling required for cold working is comparatively inexpensive.

Cold working processes have the following disadvantages:

(i) Cold working reduces toughness and ductility. Such components have poor resistance to shocks and vibrations.
(ii) Cold working induces residual stresses in the component. Proper heat treatment is required to relieve these stresses.

3.6 DESIGN CONSIDERATIONS OF WELDED ASSEMBLIES

Welding is the most important method of joining the parts into a complex assembly. Design of welded joints is explained in a separate chapter on welded joints. In this article, general principles in design of welded assemblies are discussed. The guidelines are as follows:

(i) Select the Material with High Weldability In general, a low carbon steel is more easily welded than a high carbon steel. Higher carbon content tends to harden the welded joint, as a result of which the weld is susceptible to cracks. For ease in welding, maximum carbon content is usually limited to 0.22 per cent[2].

(ii) Use Minimum Number of Welds Distortion is a serious problem in welded assemblies. It creates difficulties in maintaining correct shape, dimensions and tolerances of finished assemblies. A metallic plate or component does not distort, when it is heated or cooled as a total unit uniformly and it has freedom to expand or contract in all directions. In welding, however, only the adjoining area of the joint is heated up, which has no freedom to expand or contract. Uneven expansion and contraction in this adjoining area and parent metal results in distortion. When distortion is prevented by clamping fixtures, residual stresses are built up in the parts and annealing is required to relieve these stresses. Since distortion always occurs in welding, the design should involve a minimum number of welds and avoid over welding. It will not only reduce the distortion but also the cost.

(iii) Do not Shape the Parts Based on Casting or Forging In designing a welded assembly to replace a casting, it is incorrect to duplicate its appearance or shape by providing protrusions, brackets and housing. The designer should appreciate that welded assemblies are different from castings, having an appearance of their own. A correctly designed welded assembly is much lighter than the corresponding casting. It should reflect its lightweight characteristic, flexibility and economy of the material.

(iv) Use Standard Components The designer should specify standard sizes for plates, bars and rolled sections. Non-standard sections involve flame cutting of plates and additional welding. Standard tubular sections should be used for torsional resistance. As far as possible, the designer should select plates of equal thickness for a butt joint.

(v) Avoid Straps, Laps and Stiffeners The stiffness of a plate can be increased by making bends, indentations in the form of ribs or corrugations by press working. If at all a stiffener is required to provide rigidity to the plate, it should be designed properly with minimum weight. Use of a separate stiffener involves additional welding increasing distortion and cost.

(vi) Select Proper Location for the Weld There are two aspects of selecting the correct location for a welded joint. The welded joint should be located in an area where stresses and deflection are not critical. Also, it should be located at such a place that the welder and welding machine has unobstructed access to that location. It should be possible to carry out pre-weld machining, post-weld heat treatment and finally weld inspection at the location of the weld.

(vii) Prescribe Correct Sequence of Welding The designer should consider the sequence in which the parts should be welded together for minimum distortion. This is particularly important for a complex job involving a number of welds. An incorrect sequence of welding causes distortion and sometimes cracks in the weld metal due to stress concentration at some point in fabrication. A correct welding sequence distributes and balances the forces and stresses induced by weld contraction. For example, to prevent angular distortion in a double V butt joint, a sequence is recommended to lay welding runs alternatively on opposite sides of the joint.

Examples of 'incorrect' and 'correct' ways of welded design are illustrated from Fig. 3.17 to Fig. 3.19. In Fig. 3.17(a), it is necessary to prepare bevel edges for the components prior to welding operation. This preparatory work can be totally eliminated by making a slight change in the arrangement of components, which is shown in Fig. 3.17(b). Many times, fabrication is carried out by cutting steel plates followed by welding. The aim

[2] IS 1871–1965: Commentary on Indian Standard wrought steels for general engineering purposes.

of the designer is to minimise scrap in such process. This is illustrated in Fig. 3.18. The circular top plate and annular bottom plate are cut from two separate plates resulting in excess scrap as shown in Fig. 3.18(a). Making a slight change in design, the top plate and annual bottom plate can be cut from one plate reducing scrap and material cost, which is shown in Fig. 3.18(b). Accumulation of welded joints results in shrinkage stresses. A method to reduce this accumulation is illustrated in Fig. 3.19.

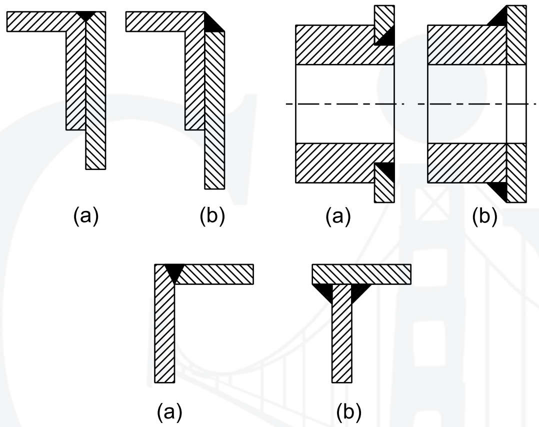


Fig. 3.17 *Saving of Preparatory Bevelling: (a) Incorrect (b) Correct*

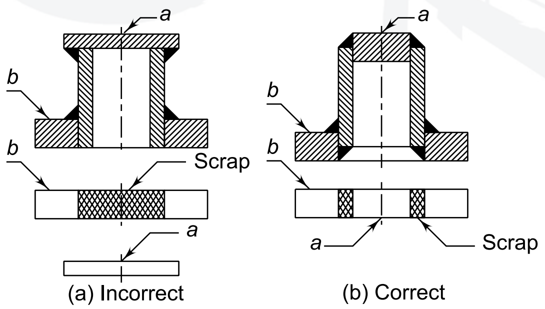


Fig. 3.18 *Reduction of Scrap*

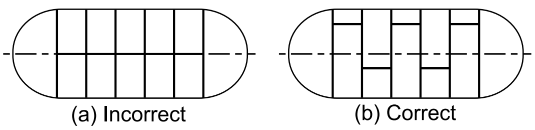


Fig. 3.19 *Avoiding Weld Accumulation*

3.7 DESIGN FOR MANUFACTURE AND ASSEMBLY (DFMA)

The design effort makes up only about 5% of the total cost of a product. However, it usually determines more than 70% of the manufacturing cost of the product. Therefore, at best only 30% of the product's cost can be changed once the design is finalised and drawings are prepared. Using statistical process control or improving manufacturing methods during production stage after the design has been finalised has marginal or little effect on the product's cost. The best strategy to lower product cost is to recognise the importance of manufacturing early in the design stage. Design for manufacture and assembly are simple guidelines formulated by Bart Huthwaite, Director of Institute for Competitive Design, Rochester, USA[3]. These guidelines, although simple, are used to simplify design, decrease assembly cost, improve product reliability and reduce operation time required to bring a new product into the market. Little technology is required to implement these guidelines. The guidelines are as follows:

(i) Reduce the Parts Count Design engineers should try for a product design that uses a minimum number of parts. Fewer parts result in lower costs. It also makes the assembly simpler and less prone to defects.

(ii) Use Modular Designs Modular design reduces the number of parts being assembled at any one time and also simplifies final assembly. Field service becomes simple, fast and cheap because dismantling is faster and requires fewer tools.

(iii) Optimize Part Handling Parts should be designed so that they do not become tangled, struck together or require special handling prior to assembly. Flexible parts such as those with wires or cables should be avoided because they complicate automated assembly. Whenever possible, parts should retain the same orientation from the point of manufacture to assembly.

[3] Bart Huthwaite—*Checklist for DFM*—Machine Design—February 8, 1990—vol. 8, no. 3.

(iv) Assemble in the Open Assembly operation should be carried out in clear view. This is important for manual assembly. It also decreases the chances of manufacturing defects slipping past the inspector.

(v) Do not Fight Gravity Design products so that they can be assembled from the bottom to top along the vertical axis. This allows simple robots and insertion tools because gravity is used to assist the assembly process. On the other hand, expensive clamping fixtures are required for assembly along horizontal axes.

(vi) Design for Part Identity For both manual and automated assembly, symmetric parts are easier to handle and orient. As assembly rate increases, symmetry becomes more important. Features should be added to enhance symmetry. Asymmetric parts should be designed so that their other surfaces make them easily identifiable. Asymmetry can be added or exaggerated to force correct alignment and orientation and make mistakes impossible.

(vii) Eliminate Fasteners Fasteners are a major obstacle to efficient assembly and should be avoided wherever possible. They are difficult to handle and can cause jamming, if defective. *In manual assembly, the cost of driving a screw can be six to ten times the cost of the screw itself.* If the use of fasteners cannot be avoided, limit the number of different types of fasteners used.

(viii) Design Parts for Simple Assembly Assembling parts can be a major challenge in automated operations. Misalignment is a serious problem when parts from different vendors are put together. Part compliance is the ability of one part to move so that it is seated properly with another. The product should be designed for part compliance. Features such as added chamfers on both parts and adequate guiding surfaces make assembly faster and more reliable.

(ix) Reduce, Simplify and Optimise Manufacturing Process The number of processes needed to assemble a product should be kept to a minimum. Processes that are difficult to control, such as welding or brazing, should be avoided.

3.8 TOLERANCES

Due to the inaccuracy of manufacturing methods, it is not possible to machine a component to a given dimension. The components are so manufactured that their dimensions lie between two limits—maximum and minimum. The basic dimension is called the *normal* or *basic size*, while the difference between the two limits is called *permissible tolerance*. *Tolerance is defined as permissible variation in the dimensions of the component*. The two limits are sometimes called the *upper* and *lower deviations*. These definitions are illustrated in Fig. 3.20, with reference to shaft and hole in clearance fit.

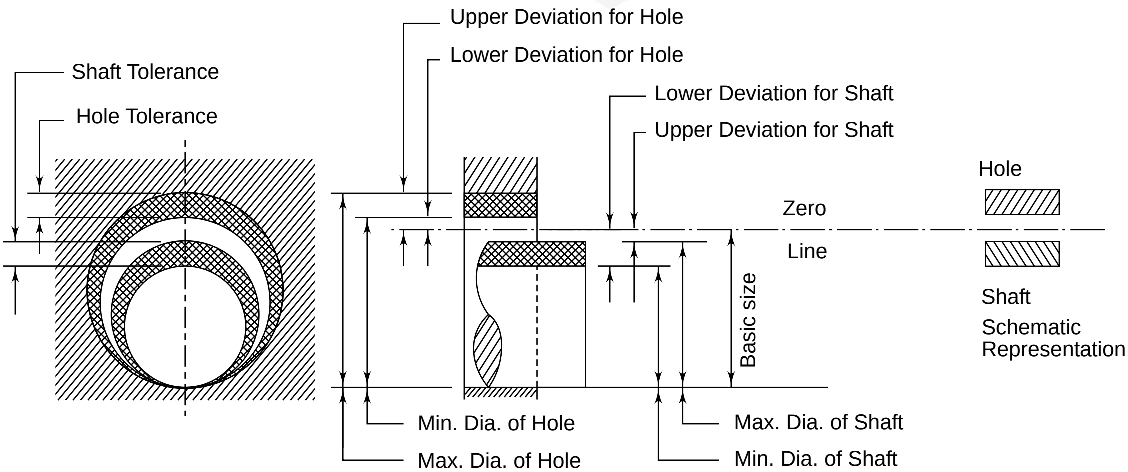


Fig. 3.20 *Tolerance Nomenclature*

There are two systems of specification for tolerances, namely, unilateral and bilateral. In the *unilateral system*, one tolerance is zero, while the other takes care of all permissible variation in basic size. For example,

$$100^{+0.04}_{+0.00} \quad \text{or} \quad 100^{+0.00}_{-0.04}$$

In case of *bilateral tolerances*, the variations are given in both directions from normal size. The upper limit in this case is the basic size plus non-zero positive tolerance, and the lower limit is the basic size plus non-zero negative tolerance. For example,

$$25^{\pm 0.4} \quad \text{or} \quad 25^{+0.4}_{-0.2}$$

In bilateral tolerances, the two tolerances are often equal, but this is not a necessary condition. Unilateral tolerances are used for shafts and holes.

3.9 TYPES OF FITS

When two parts are to be assembled, the relationship resulting from the difference between their sizes before assembly is called a fit. Depending upon the limits of the shaft and the hole, fits are broadly classified into three groups—clearance fit, transition fit and interference fit. *Clearance fit is a fit which always provides a positive clearance between the hole and the shaft over the entire range of tolerances*. In this case, the tolerance zone of the hole is entirely above that of the shaft. *Interference fit is a fit which always provides a positive interference over the whole range of tolerances*. In this case, the tolerance zone of the hole is completely below that of the shaft. *Transition fit is a fit which may provide either a clearance or interference, depending upon the actual values of the individual tolerances of the mating components*. In this case, the tolerance zones of the hole and the shaft overlap. These definitions are illustrated in Fig. 3.21.

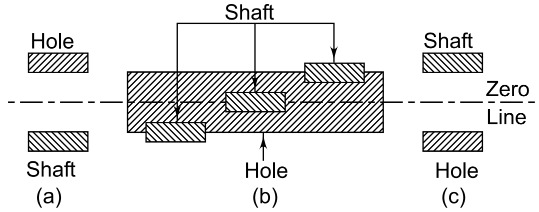


Fig. 3.21 *Types of Fits: (a) Clearance Fit (b) Transition Fit (c) Interference Fit*

There are two basic systems for giving tolerances to the shaft and the hole, namely, the hole-basis system and the shaft-basis, system. In the *hole-basis system*, the different clearances and interferences are obtained by associating various shafts with a single hole, whose lower deviation is zero. The system is illustrated in Fig. 3.22. In this case, the size of the hole is the basic size, and the clearance or interference is applied to the shaft dimension. The system is denoted by the symbol 'H'. This system has an advantage over the shaft-basis system, because holes are machined by standard drills or reamers having fixed dimensions, while the shafts can be turned or ground to any given dimension. Due to this reason, the hole-basis system is widely used.

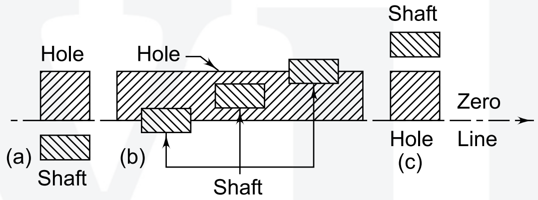


Fig. 3.22 *Hole Basis System: (a) Clearance Fit (b) Transition Fit (c) Interference Fit*

In the *shaft-basis system*, the different clearances or interferences are obtained by associating various holes with a single shaft, whose upper deviation is zero. This principle is illustrated in Fig. 3.23. In this system, the size of the shaft is the basic size, while the clearance or interference is applied to the dimensions of the hole. The system is denoted by the symbol 'h'. The shaft-basis system is popular in industries using semi-finished or finished shafting, such as bright bars, as raw material.

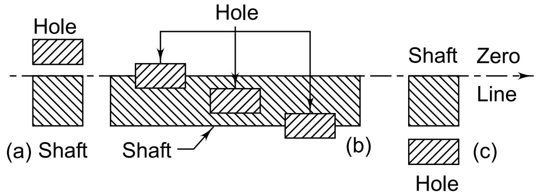


Fig. 3.23 *Shaft Basis System: (a) Clearance Fit (b) Transition Fit (c) Interference Fit*

3.10 BIS SYSTEM OF FITS AND TOLERANCES

According to a system recommended by the Bureau of Indian Standards,[4, 5, 6] tolerance is specified by an alphabet, capital or small, followed by a number, e.g., H7 or g6. The description of tolerance consists of two parts—*fundamental deviation* and *magnitude of tolerance*, as shown in Fig. 3.24. The fundamental deviation gives the location of the tolerance zone with respect to the zero line. It is indicated by an alphabet—capital letters for holes and small letters

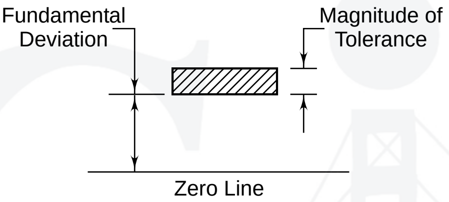


Fig. 3.24 *Description of Tolerance*

for shafts. The magnitude of tolerance is designated by a number, called the *grade*. *The grade of tolerance is defined as a group of tolerances, which are considered to have the same level of accuracy for all basic sizes*. There are eighteen grades of tolerances with designations IT1, IT2, …, IT17, and IT18. The letters of symbol IT stand for 'International Tolerance' grade. The tolerance for a shaft of 50 mm diameter as the basic size, with the fundamental deviation denoted by the letter 'g' and the tolerance of grade 7 is written as 50g7.

The fit is indicated by the basic size common to both components followed by symbols for tolerance of each component. For example,

$$50\ H8/g7 \quad \text{or} \quad 50\ H8\text{-}g7 \quad \text{or} \quad 50\,\frac{H8}{g7}$$

The formulae for calculating the fundamental deviation and magnitude of tolerance of various grades are given in the standards. However, the designer is mainly concerned with readymade tables for determining tolerances. Table 3.2 and Tables 3.3a and 3.3b give tolerances for holes and shafts up to 100-mm size. For other sizes and grades, one can refer to relevant standards.

Table 3.2 *Tolerances for holes of sizes up to 100 mm (H5 to H11)*

Diameter steps in mm		*H*							
		5	*6*	*7*	*8*	*9*	*10*	*11*	*5–11*
over	*to*				*es*				*ei*
0	3	+4	+6	+10	+14	+25	+40	+60	0
3	6	+5	+8	+12	+18	+30	+48	+75	0
6	10	+6	+9	+15	+22	+36	+58	+90	0
10	18	+8	+11	+18	+27	+43	+70	+110	0
18	30	+9	+13	+21	+33	+52	+84	+130	0
30	50	+11	+16	+25	+39	+62	+100	+160	0
50	80	+13	+19	+30	+46	+74	+120	+190	0
80	100	+15	+22	+35	+54	+87	+140	+220	0

[4] IS 919—1993: ISO System of limits and fits (in two parts).
[5] IS 2709—1964: Guide for selection of fits.
[6] IS 2101—1990: Recommendations for limits and fits for sizes above 500 mm and up to 3150 mm.

Table 3.3a *Tolerances for shafts of sizes up to 100 mm (from d to h)*

Diameter steps in mm		*d*					*e*					*f*				*g*			*h*						
		8–11	*8*	*9*	*10*	*11*	*6–9*	*6*	*7*	*8*	*9*	*6–8*	*6*	*7*	*8*	*6–7*	*6*	*7*	*5–10*	*5*	*6*	*7*	*8*	*9*	*10*
over	*to*	*es*	*ei*				*es*	*ei*				*es*	*ei*			*es*	*ei*		*es*	*ei*					
0	3	–20	–34	–45	–60	–80	–14	–20	–24	–28	–39	–6	–12	–16	–20	–2	–8	–12	0	–4	–6	–10	–14	–25	–40
3	6	–30	–48	–60	–78	–105	–20	–28	–32	–38	–50	–10	–18	–22	–28	–4	–12	–16	0	–5	–8	–12	–18	–30	–48
6	10	–40	–62	–76	–98	–130	–25	–34	–40	–47	–61	–13	–22	–28	–35	–5	–14	–20	0	–6	–9	–15	–22	–36	–58
10	18	–50	–77	–93	–120	–160	–32	–43	–50	–59	–75	–16	–27	–34	–43	–6	–17	–24	0	–8	–11	–18	–27	–43	–70
18	30	–65	–98	–117	–149	–195	–40	–53	–61	–73	–92	–20	–33	–41	–53	–7	–20	–28	0	–9	–13	–21	–33	–52	–84
30	50	–80	–119	–142	–180	–240	–50	–66	–75	–89	–112	–25	–41	–50	–64	–9	–25	–34	0	–11	–16	–25	–39	–62	–100
50	80	–100	–146	–174	–220	–290	–60	–79	–90	–106	–134	–30	–49	–60	–76	–10	–29	–40	0	–13	–19	–30	–46	–74	–120
80	100	–120	–174	–207	–260	–340	–72	–94	–107	–126	–159	–36	–58	–71	–90	–12	–34	–47	0	–15	–22	–35	–54	–87	–140

Table 3.3b *Tolerances for shafts of sizes up to 100 mm (from j to s)*

Diameter steps in mm		*j*						*k*			*m*			*n*			*p*			*r*			*s*			
		5		6		7		5	6	5–6	6	7	6–7	6	7	6–7	6	7	6–7	5	6	5–6	5	6	7	5–7
over	*to*	*es*	*ei*	*es*	*ei*	*es*	*ei*	*es*		*ei*	*es*		*ei*	*es*		*ei*	*es*		*ei*	*es*		*ei*	*es*			*ei*
0	3	+2	–2	+4	–2	+6	–4	+4	+6	0	+8	–	+2	+10	+14	+4	+12	+16	+6	+14	+16	+10	+18	+20	+24	+14
3	6	+3	–2	+6	–2	+8	–4	+6	+9	+1	+12	+16	+4	+16	+20	+8	+20	+24	+12	+20	+23	+15	+24	+27	+31	+19
6	10	+4	–2	+7	–2	+10	–5	+7	+10	+1	+15	+21	+6	+19	+25	+10	+24	+30	+15	+25	+28	+19	+29	+32	+38	+23
10	18	+5	–3	+8	–3	+12	–6	+9	+12	+1	+18	+25	+7	+23	+30	+12	+29	+36	+18	+31	+34	+23	+36	+39	+46	+28
18	30	+5	–4	+9	–4	+13	–8	+11	+15	+2	+21	+29	+8	+28	+36	+15	+35	+43	+22	+37	+41	+28	+44	+48	+56	+35
30	50	+6	–5	+11	–5	+15	–10	+13	+18	+2	+25	+34	+9	+33	+42	+17	+42	+51	+26	+45	+50	+34	+54	+59	+68	+43
50	80	+6	–7	+12	–7	+18	–12	+15	+21	+2	+30	+41	+11	+39	+50	+20	+51	+62	+32	+55	+61	+42	+69	+75	+86	+56
80	100	+6	–9	+13	–9	+20	–15	+18	+25	+3	+35	+48	+13	+45	+58	+23	+59	+72	+37	+66	+73	+54	+86	+93	+106	+71

Note: 1. In Tables 3.1 and 3.2a and b, the tolerances are given in microns (1 micron = 0.001 mm).
2. es = upper deviation and ei = lower deviation.

3.11 SELECTION OF FITS

The guidelines for the selection of clearance fits are as follows:

(i) The fits H7-d8, H8-d9 and H11-d11 are loose running fits, and are used for plumber-block bearings and loose pulleys.

(ii) The fits H6-e7, H7-e8 and H8-e8 are loose clearance fits, and are used for properly lubricated bearings, requiring appreciable clearances. The finer grades are used for heavy-duty, high-speed bearings and large electric motors.

(iii) The fits H6-f6, H7-f7 and H8-f8 are normal running fits, widely used for grease or oil lubricated bearings having low temperature rise. They are also used for shafts of gearboxes, small electric motors and pumps.

(iv) The fits H6-g5, H7-g6 and H8-g7 are expensive from manufacturing considerations. They are used in precision equipment, pistons, slide valves and bearings of accurate link mechanisms.

The typical types of transition fits are H6-j5, H7-j6 and H8-j7. They are used in applications where slight interference is permissible. Some of their applications are spigot and recess of the rigid coupling and the composite gear blank, where a steel rim is fitted on an ordinary steel hub.

The general guidelines for the selection of interference fits are as follows:

(i) The fit H7-p6 or H7-p7 results in interference, which is not excessive but sufficient to give non-ferrous parts a light press fit. Such parts can be dismantled easily as and when required, e.g., fitting a brass bush in the gear.

(ii) The fit H6-r5 or H7-r6 is a medium drive fit on ferrous parts, which can be easily dismantled.

(iii) The fits H6-s5, H7-s6 and H8-s7 are used for permanent and semi-permanent assemblies of steel and cast iron parts. The amount of interference in these fits is sufficiently large to provide a considerable gripping force. They are used in valve seats and shaft collars.

The selection of interference fit depends upon a number of factors, such as materials, diameters, surface finish and machining methods. It is necessary to calculate the maximum and minimum interference in each case. The torque transmitting capacity is calculated for minimum interference, while the force required to assemble the parts is decided by the maximum value of interference.

3.12 TOLERANCES AND MANUFACTURING METHODS

The specification of machining method for a given component depends upon the grade of tolerances specified by the designer. The approximate relationship between the grade of tolerance and the manufacturing method with desirable accuracy under normal working conditions, is as follows:

Grade 16	Sand casting–flame cutting
Grade 15	Stamping
Grade 14	Die casting–moulding
Grade 11	Drilling–rough turning–boring
Grade 10	Milling–slotting–planning–rolling–extrusion
Grade 9	Horizontal and vertical boring–turning on automatic lathes
Grade 8	Turning, boring and reaming on centre, capstan and turret lathes
Grade 7	High precision turning–broaching–honing
Grade 6	Grinding–fine honing
Grade 5	Lapping–fine grinding–diamond boring
Grade 4	Lapping

In this chapter, the hole-basis system is discussed. The manufacturing processes for obtaining different grades of tolerances for holes are as follows:

H5	Precision boring–fine internal grinding–honing
H6	Precision boring–honing–hand reaming
H7	Grinding–broaching–precision reaming
H8	Boring–machine reaming

Finer grades of tolerances result in costly manufacturing methods. They are to be specified only on grounds of functional requirement. Coarse grades of tolerances reduce the cost of production. The designer should always take into consideration the available manufacturing facilities and their cost competitiveness before specifying the grade of tolerances.

Example 3.1 *The main bearing of an engine is shown in Fig. 3.25. Calculate*

(i) the maximum and minimum diameters of the bush and crank pin; and

(ii) the maximum and minimum clearances between the crank pin and bush.

Suggest suitable machining methods for both.

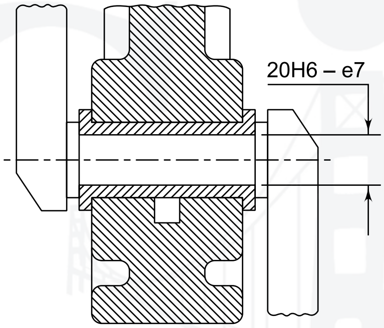


Fig. 3.25 *Engine Bearing*

Solution

Step I *Maximum and minimum diameters of the bush and crank pin*

The tolerances for the bush and crank pin from Tables 3.2 and 3.3a are as follows:

Bush:

$$20^{+0.013}_{+0.000} \quad \text{or} \quad \frac{20.013}{20.000}\ \text{mm}$$

Crankpin:

$$20^{-0.040}_{-0.061} \quad \text{or} \quad \frac{19.960}{19.939}\ \text{mm} \qquad \text{(i)}$$

Step II *Maximum and minimum clearances*

Maximum clearance = 20.013 – 19.939 = 0.074 mm

Minimum clearance = 20.000 – 19.960 = 0.040 mm (ii)

Step III *Machining methods*

The bush (H6) is usually machined by precision boring, honing or hand reaming methods. The crank pin (e7) is machined by high precision turning or broaching.

Example 3.2 *The valve seat fitted inside the housing of a pump is shown in Fig. 3.26. Calculate*

(i) the maximum and minimum diameters of the housing and valve seat; and

(ii) the magnitude of the maximum and minimum interferences between the seat and housing.

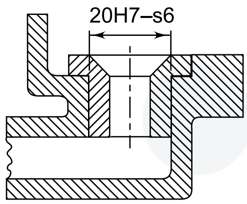


Fig. 3.26 *Valve Seat*

Solution

Step I *Maximum and minimum diameters of the housing and valve seat*

From Tables 3.2 and 3.3b, the tolerances for valve seat and housing are as follows:

Housing:

$$20^{+0.021}_{+0.000} \quad \text{or} \quad \frac{20.021}{20.000}\ \text{mm}$$

Valve seat (outer diameter):

$$20^{+0.048}_{+0.035} \quad \text{or} \quad \frac{20.048}{20.035}\ \text{mm} \qquad \text{(i)}$$

Step II *Maximum and minimum interferences*

Minimum interference = 20.035 – 20.021 = 0.014 mm

Maximum interference = 20.048 – 20.000 = 0.048 mm (ii)

3.13 SELECTIVE ASSEMBLY

The selective assembly is a process of sorting the manufactured components into different groups according to their sizes, and then assembling the components of one group to the corresponding components of a matching group. In this method, larger shafts are assembled with larger holes and smaller shafts with smaller holes. This results in closer clearance or interference without involving costly machining methods.

As an example, a hydrodynamic journal bearing with a recommended class of fit 50 H8-d8 is considered. From Tables 3.2 and 3.3a, the limiting dimensions are as follows:

Bearing:

$$50^{+0.039}_{+0.000} \quad \text{or} \quad \frac{50.039}{50.000}\ \text{mm}$$

Journal:

$$50^{-0.080}_{-0.119} \quad \text{or} \quad \frac{49.920}{49.881}\ \text{mm}$$

Maximum clearance = 50.039 – 49.881 = 0.158 mm
Minimum clearance = 50.000 – 49.920 = 0.080 mm

The range of clearance is from 0.080 to 0.158 mm. Suppose it is desired to hold the clearance in a narrow range from 0.100 to 0.140 mm. Then the bearings and the journals are divided into two groups with the following limiting dimensions:

	Bearings	*Journals*
Group A	50.039 / 50.020	49.920 / 49.900
Group B	50.020 / 50.000	49.900 / 49.881

Group A

Maximum clearance = 50.039 – 49.900 = 0.139 mm
Minimum clearance = 50.020 – 49.920 = 0.100 mm

Group B

Maximum clearance = 50.020 – 49.881 = 0.139 mm
Minimum clearance = 50.000 – 49.900 = 0.100 mm

It is seen that both groups have clearance in the range 0.100 to 0.139 mm.

In the above example, only two groups are considered. When the parts are sorted into more number of groups, the range of clearance is further reduced. Selective assembly results in finer clearance or interference, even with large manufacturing tolerances. The main drawbacks of this method are as follows:

(i) Hundred per cent inspection is required for components and there is additional operation of sorting the components.

(ii) Interchangeability is affected and servicing or replacement of worn-out components becomes difficult.

(iii) The method can be used only when a large number of components are manufactured, otherwise some groups may not contain sufficient number of components.

Selective assembly is particularly useful in case of interference fits where a tight control over the range of interference is essential to avoid loosening of mating components.

Example 3.3 *The recommended class of fit for the hub shrunk on a shaft is 50H7-s6. However, it is necessary to limit the interference from 0.030 to 0.050 mm between the hub and the shaft. Specify the groups for selective assembly.*

Solution

Step I *Maximum and minimum diameters of hub and shaft*

From Tables 3.2 and 3.3b,

Hub:

$$50^{+0.025}_{+0.000} \quad \text{or} \quad \frac{50.025}{50.000}\ \text{mm}$$

Shaft:

$$50^{+0.059}_{+0.043} \quad \text{or} \quad \frac{50.059}{50.043}\ \text{mm}$$

Step II *Maximum and minimum interferences*

Maximum interference = 50.059 – 50.000 = 0.059 mm
Minimum interference = 50.043 – 50.025 = 0.018 mm

Step III *Selection of groups*

Suppose the components are sorted into three groups (A, B and C) with the following dimensions:

Group	*A*	*B*	*C*
Hub	50.008 / 50.000	50.016 / 50.008	50.025 / 50.016
Shaft	50.048 / 50.043	50.055 / 50.048	50.059 / 50.055

Step IV *Maximum and minimum interferences for A, B and C*

The interferences I_A, I_B and I_C are as follows:

I_A max = 50.048 – 50.000 = 0.048 mm

I_B max = 50.055 – 50.008 = 0.047 mm

I_C max = 50.059 – 50.016 = 0.043 mm

The maximum interference is within the limit of 0.050 mm.

I_A min = 50.043 – 50.008 = 0.035 mm

I_B min = 50.048 – 50.016 = 0.032 mm

I_C min = 50.055 – 50.025 = 0.030 mm

The minimum interference is above the limit of 0.03 mm.

3.14 TOLERANCES FOR BOLT SPACING

When two or more components are assembled by means of bolts, it is often required to specify tolerances for the centre to centre distance between bolts. As shown in Fig. 3.27, two plates *A* and *B* are assembled by bolts with a centre distance of $(a \pm x)$. Consider the worst geometric situation, when the following conditions exist:

(i) The holes are of the smallest size, D_{min},

(ii) The bolts are of the largest size, d_{max},

(iii) The plate *A* has a minimum spacing $(a - x)$, and

(iv) The plate *B* has maximum spacing $(a + x)$.

Considering the plate *A*, the centre to centre distance between bolts is given by

$$(a - x) + 2\left(\frac{D_{min.}}{2}\right) - 2\left(\frac{d_{max.}}{2}\right)$$

or $\quad (a - x) + (D_{min.} - d_{max.}) \qquad$ (a)

Similarly, considering the plate *B*, the centre to centre distance is given by

$$(a + x) - 2\left(\frac{D_{min.}}{2}\right) + 2\left(\frac{d_{max.}}{2}\right)$$

or $\quad (a + x) - (D_{min.} - d_{max.}) \qquad$ (b)

Equating the expressions (a) and (b),

$$x = (D_{min.} - d_{max.}) \qquad (3.1)$$

The tolerance on centre distance is given by

$$(a \pm x)$$

In case there are a number of holes, one hole is considered as the master or reference hole and the location of other holes should be provided with the required tolerance.

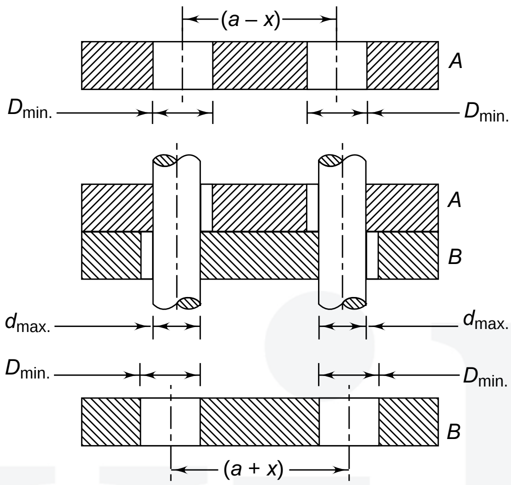


Fig. 3.27 *Assembly of Two Plates*

Example 3.4 *Two bolts, with a centre distance of 50 mm, are used to assemble two plates. The recommended class of fit between bolts and holes is 12H11-d11. Specify tolerance for the centre to centre distance between bolts.*

Solution

Step I *Maximum and minimum diameters of hole and bolt*

Hole: 12H11 (Table 3.2)

$$12^{+0.110}_{+0.000} \quad \text{or} \quad \frac{12.110}{12.000}\ \text{mm}$$

Bolt: 12d11 (Table 3.3a)

$$12^{-0.050}_{-160} \quad \text{or} \quad \frac{11.950}{11.840}\ \text{mm}$$

Step II *Calculation of x*

From Eq. (3.1),

$$x = (D_{min.} - d_{max.}) = (12.000 - 11.950) = 0.05\ \text{mm}$$

Step III *Tolerance for the centre distance*

The tolerance for the centre distance is given as

$$50 \pm 0.05\ \text{mm}$$

3.15 SURFACE ROUGHNESS

The surface roughness or surface finish plays an important role in the performance of certain machine elements. Some of the examples are as follows:

(i) Friction and wear increases with surface roughness, adversely affecting the performance of bearings.

(ii) Rough surfaces have a reduced contact area in interference fits, which reduces the holding capacity of the joints.

(iii) The endurance strength of the component is greatly reduced due to poor surface finish.

(iv) The corrosion resistance is adversely affected by poor surface finish.

It is necessary for the designer to specify an optimum surface finish from the considerations of functional requirement and the cost of manufacture. A magnified profile of the surface is shown in Fig. 3.28. There are two methods to specify surface roughness – **cla** value and **rms** value.

The **cla** value is defined as

$$\text{cla} = \frac{1}{L}\int_0^L y\,dx \tag{3.2}$$

and the **rms** (root mean square) value as

$$\text{rms} = \left[\frac{1}{L}\int_0^L y^2\,dx\right]^{1/2} \tag{3.3}$$

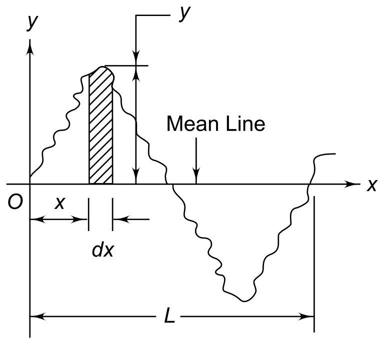


Fig. 3.28 *Surface Roughness*

A widely used symbol for surface finish is shown in Fig. 3.29. The sides of the symbol are inclined at 60° to the surface and the number indicates the surface roughness (**rms**) in microns. The surface roughness produced by various machining methods is given in Table 3.4.

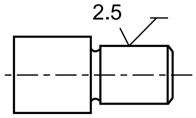


Fig. 3.29 *Symbol for Surface Roughness*

Table 3.4 *Surface roughness (**rms**) in microns*

Machining method	*Roughness (microns)*
turning–shaping–milling	12.5–1.0
boring	6.5–0.5
drilling	6.25–2.5
reaming	2.5–0.5
surface grinding	6.25–0.5
cylindrical grinding	2.5–0.25
honing–lapping	0.5–0.05
polishing–buffing	0.5–0.05

Typical values of surface roughness for machine components are as follows:

1.5 micron	—	Gear shafting and bores
0.75 micron	—	Bronze bearings
0.40 micron	—	Splined shafts, O-ring grooves, gear teeth and ball bearings
0.30 micron	—	Cylinder bores and pistons
0.20 micron	—	Crankshaft, connecting rod, cams and hydraulic cylinders

Short-Answer Questions

3.1 What is a casting process? Give examples of components made by casting.

3.2 What is deformation process? Give examples of components made by deformation process.

3.3 What is cutting process? Give examples of components made by cutting process.

3.4 What are the advantages of the sand casting process?

3.5 What are the disadvantages of the sand casting process?

3.6 What are the advantages of the forging process?
3.7 What are the disadvantages of the forging process?
3.8 What are the advantages of the cutting process as a manufacturing method?
3.9 What are the disadvantages of the cutting process as a manufacturing method?
3.10 What are the principles of designing cast iron components?
3.11 Compare grain structure of a crankshaft manufactured by casting, forging and machining processes.
3.12 How will you select direction of fibre lines in forged components?
3.13 Where do you use machined components? Give practical examples.
3.14 What are the principles for the design of machined components?
3.15 What are the advantages of the hot working process?
3.16 What are the disadvantages of the hot working process?
3.17 What are the advantages of the cold working process?
3.18 What are the disadvantages of the cold working process?
3.19 What are the principles for design of welded assemblies?
3.20 What is DFM? What is DFMA?
3.21 What are the principles of Design for Manufacture and Assemblies (DFMA)?
3.22 What is tolerance?
3.23 What are unilateral and bilateral tolerances?
3.24 What is fit?
3.25 What is a clearance fit? Give examples.
3.26 What is a transition fit? Give examples.
3.27 What is an interference fit? Give examples.
3.28 What is the shaft-basis system for giving tolerances?
3.29 What is the hole-basis system for giving tolerances?
3.30 What are the advantages of the hole-basis system over the shaft-basis system?
3.31 What is fundamental deviation?
3.32 How will you designate fundamental deviation?
3.33 How will you designate magnitude of tolerance?
3.34 What are the guidelines for selection of clearance fits? Give examples.
3.35 What are the guidelines for selection of transition fits? Give examples.
3.36 What are the guidelines for selection of interference fits? Give examples.
3.37 What is selective assembly?
3.38 Distinguish between interchangeable and selective assemblies.
3.39 What are the advantages of selective assembly?
3.40 What are the disadvantages of selective assembly?
3.41 Explain the symbol for surface roughness.

Problems for Practice

3.1 The bush of the small end of a connecting rod is shown in Fig. 3.30. Calculate

(i) the maximum and minimum diameters of the bush and connecting rod; and

(ii) the maximum and minimum interference between them.

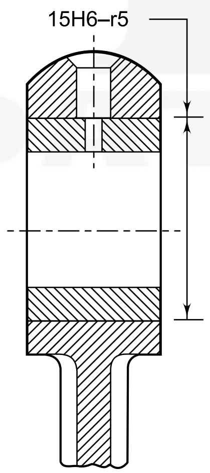


Fig. 3.30

[*(i)* $\dfrac{15.031}{15.023}$ *mm and* $\dfrac{15.011}{15.000}$ *mm*

(ii) 0.031 and 0.012 mm]

3.2 The exhaust valve of an IC engine is shown in Fig. 3.31. There is a clearance fit between the valve stem and its guide and an interference fit between the valve seat and its housing. Determine

(i) diameters of the valve stem
(ii) inner diameters of guide for valve stem
(iii) the clearances between the stem and guide
(iv) diameters of the valve seat
(v) inner diameters of housing of the valve seat
(vi) the interferences between the valve seat and its housing

$$[(i)\ \frac{4.970}{4.952}\ mm \quad (ii)\ \frac{5.012}{5.000}\ mm \quad (iii)\ 0.06 \text{ and } 0.03\ mm \quad (iv)\ \frac{20.044}{20.035}\ mm \quad (v)\ \frac{20.013}{20.000}\ mm \quad (vi)\ 0.044 \text{ and } 0.022\ mm]$$

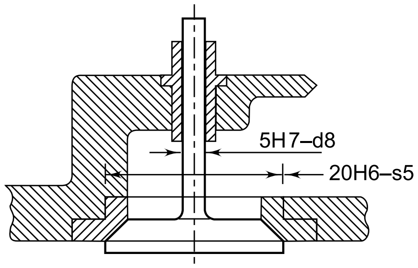


Fig. 3.31

3.3 The recommended class of fit for a hydrodynamic bearing is 40H6-e7. The maximum and minimum clearances are limited to 0.08 and 0.06 mm respectively. Specify groups for selective assembly.

The five groups are as follows:

Group	A	B	C	D	E
Hole (mm)	$\frac{40.016}{40.013}$	$\frac{40.013}{40.010}$	$\frac{40.010}{40.007}$	$\frac{40.007}{40.004}$	$\frac{40.004}{40.000}$
Shaft (mm)	$\frac{39.950}{39.945}$	$\frac{39.945}{39.940}$	$\frac{39.940}{39.935}$	$\frac{39.935}{39.930}$	$\frac{39.930}{39.925}$

Design against Static Load

Chapter 4

4.1 MODES OF FAILURE

A static load is defined as a force, which is gradually applied to a mechanical component and which does not change its magnitude or direction with respect to time. It was discussed in Chapter 2 that engineering materials are classified into two groups—ductile and brittle materials. A ductile material is one which has a relatively large tensile strain before fracture takes place. On the other hand, a brittle material has a relatively small tensile strain before fracture. A tensile strain of 5% is considered to be the dividing line between brittle and ductile materials. Structural steels and aluminium are ductile materials, while cast iron is an example of a brittle material.

A mechanical component may fail, that is, may be unable to perform its function satisfactorily, as a result of any one of the following three modes of failure[1]:

(i) failure by elastic deflection;
(ii) failure by general yielding; and
(iii) failure by fracture.

In applications like transmission shaft supporting gears, the maximum force acting on the shaft, without affecting its performance, is limited by the permissible elastic deflection. Lateral or torsional rigidity is considered as the criterion of design in such cases. Sometimes, the elastic deflection results in unstable conditions, such as buckling of columns or vibrations. The design of the mechanical component, in all these cases, is based on the permissible lateral or torsional deflection. The stresses induced in the component are not significant and the properties of the material, such as yield strength or ultimate tensile strength, are not of primary importance. The modules of elasticity and rigidity are the important properties and the dimensions of the component are determined by the load-deflection equations.

A mechanical component made of ductile material loses its engineering usefulness due to a large amount of plastic deformation after the yield point stress is reached. Considerable portion of the component is subjected to plastic deformation, called *general yielding*. There is a basic difference between general yielding and localized yielding. The *localized yielding* in the region of stress concentration is restricted to a very small portion of the component and is not considered significant. The yield strength of a material is an important property when a component is designed against failure due to general yielding.

Components made of brittle material cease to function satisfactorily because of the sudden fracture without any plastic deformation. The

[1] FB Seely and JO Smith, *Advanced Mechanics of Materials,* John Wiley and Sons. Inc.

failure in this case is sudden and total. In such cases, ultimate tensile strength of the material is an important property to determine the dimensions of these components.

The failure by elastic deflection is separately considered in the chapter on transmission shafting. The discussion in this chapter is restricted to the design of components on the strength basis.

4.2 FACTOR OF SAFETY

While designing a component, it is necessary to provide sufficient reserve strength in case of an accident. This is achieved by taking a suitable factor of safety (*fs*).

The factor of safety is defined as

$$(fs) = \frac{\text{failure stress}}{\text{allowable stress}}$$

or $$(fs) = \frac{\text{failure load}}{\text{working load}}$$

The allowable stress is the stress value, which is used in design to determine the dimensions of the component. It is considered as a stress, which the designer expects will not be exceeded under normal operating conditions. For ductile materials, the allowable stress σ is obtained by the following relationship:

$$\sigma = \frac{S_{yt}}{(fs)} \tag{4.1}$$

For brittle materials, the relationship is,

$$\sigma = \frac{S_{ut}}{(fs)} \tag{4.2}$$

where S_{yt} and S_{ut} are the yield strength and the ultimate tensile strength of the material respectively.

There are a number of factors which are difficult to evaluate accurately in design analysis. Some of the factors are as follows:

(i) Uncertainty in the magnitude of external force acting on the component

(ii) Variations in the properties of materials like yield strength or ultimate strength

(iii) Variations in the dimensions of the component due to imperfect workmanship

In addition to these factors, the number of assumptions made in design analysis, in order to simplify the calculations, may not be exactly valid in working conditions. The factor of safety ensures against these uncertainties and unknown conditions.

The magnitude of factor of safety depends upon the following factors:

(i) Effect of Failure Sometimes, the failure of a machine element involves only a little inconvenience or loss of time, e.g., failure of the ball bearing in a gearbox. On the other hand, in some cases, there is substantial financial loss or danger to the human life, e.g., failure of the valve in a pressure vessel. The factor of safety is high in applications where failure of a machine part may result in serious accidents.

(ii) Type of Load The factor of safety is low when the external force acting on the machine element is static, i.e., a load which does not vary in magnitude or direction with respect to time. On the other hand, a higher factor of safety is selected when the machine element is subjected to impact load. This is due to the fact that impact load is suddenly applied to the machine component, usually at high velocities.

(iii) Degree of Accuracy in Force Analysis When the forces acting on the machine component are precisely determined, a low factor of safety can be selected. On the contrary, a higher factor of safety is necessary when the machine component is subjected to a force whose magnitude or direction is uncertain and unpredictable.

(iv) Material of Component When the component is made of a homogeneous ductile material like steel, yield strength is the criterion of failure. The factor of safety is usually small in such cases. On the other hand, a cast iron component has non-homogeneous structure and a higher factor of safety based on ultimate tensile strength is chosen.

(v) Reliability of Component In certain applications like continuous process equipment, power stations or defense equipment, high reliability of components is expected. The factor of safety increases with increasing reliability.

(vi) Cost of Component As the factor of safety increases, dimensions of the component, material requirement and cost increase. The factor of safety is low for cheap machine parts.

(vii) Testing of Machine Element A low factor of safety can be chosen when the machine component can be tested under actual conditions of service and operation. A higher factor of safety is necessary, when it is not possible to test the machine part or where there is deviation between test conditions and actual service conditions.

(viii) Service Conditions When the machine element is likely to operate in corrosive atmosphere or high temperature environment, a higher factor of safety is necessary.

(ix) Quality of Manufacture When the quality of manufacture is high, variations in dimensions of the machine component are less and a low factor of safety can be selected. Conversely, a higher factor of safety is required to compensate for poor manufacturing quality.

The selection of magnitude of the factor of safety is one of the difficult tasks faced by the designer. The guidelines for selection of quantitative values of the factor of safety are as follows:

(i) For cast iron components, *ultimate tensile strength* is considered to be the failure criterion. Failure occurs when the maximum stress in the component due to external force exceeds the ultimate tensile strength even once. Cast iron components have a non-homogeneous structure. Many times, there are residual stresses in the component. To account for these factors, a large factor of safety, usually 3 to 5, based on ultimate tensile strength, is used in the design of cast iron components.

(ii) For components made of ductile materials like steel and which are subjected to external static forces, *yield strength* is considered to be the criterion of failure. When such components are overloaded and the stress due to external force exceeds the yield strength of the material, there is a small amount of plastic deformation, which usually does not put the component out of service. Ductile components have a homogeneous structure and the residual stresses can be relieved by proper heat treatment. The stress analysis is more precise in case of static forces. Due to these reasons, the factor of safety is usually small in such cases. The recommended factor of safety is 1.5 to 2, based on the yield strength of the material.

(iii) For components made of ductile materials and those subjected to external fluctuating forces, *endurance limit* is considered to be the criterion of failure. Such components fail on account of fatigue. Fatigue failure depends upon the amplitude of fluctuating stresses and the number of stress cycles. The nature of fatigue failure is discussed in Section 5.5. The number of factors affect endurance limit, such as stress concentration, notch sensitivity, surface finish and even the size of the component. Therefore, the endurance limit of the component is reduced to account for these factors. The recommended factor of safety based on this endurance limit of component is usually 1.3 to 1.5.

(iv) The design of certain components such as cams and followers, gears, rolling contact bearings or rail and wheel is based on the calculation of contact stresses by the Hertz' theory. Failure of such components is usually in the form of small pits on the surface of the component. *Pitting* is surface fatigue failure, which occurs when contact stress exceeds the surface endurance limit. The damage due to pitting is local and does not put the component out of operation. The surface endurance limit can be improved by increasing the surface hardness. The recommended factor of safety for such components is 1.8 to 2.5 based on surface endurance limit.

(v) Certain components, such as piston rods, power screws or studs, are designed on the basis of *buckling consideration*. Buckling is elastic instability, which results in a sudden large lateral deflection. The critical buckling load depends upon yield strength, modulus of elasticity, end conditions and radius of gyration of the column. The recommended factor of safety is 3 to 6 based on the critical buckling load of such components.

The above-mentioned values of the factor of safety are based on past experience. They are applicable under normal circumstances. However, a higher factor of safety is chosen under the following conditions:

(i) The magnitude and nature of external forces acting on the machine component cannot be precisely estimated.

(ii) It is likely that the material of the machine component has a non-homogeneous structure.

(iii) The machine component is subjected to impact force in service.

(iv) There is possibility of residual stresses in the machine component.

(v) The machine component is working in a corrosive atmosphere.

(vi) The machine part is subjected to high temperatures during operation.

(vii) The failure of the machine part may hazard the lives of people (hoist, lifting machinery and boilers) and substantial loss to property.

(viii) It is not possible to test the machine component under actual conditions of service and there is variation in actual conditions and standard test conditions.

(ix) Higher reliability is demanded in applications like components of aircrafts.

(x) There is possibility of abnormal variation in external load on some occasions.

(xi) The quality of manufacture of the machine part is poor.

(xii) The exact mode of failure of the component is unpredictable.

(xiii) There is stress concentration in a machine component.

A higher factor of safety increases the reliability of the component. However, it increases the dimensions, the volume of material and consequently the cost of the machine component. In recent years, attempts have been made to obtain precise values of factor of the safety based on statistical considerations and reliability analysis.

4.3 STRESS–STRAIN RELATIONSHIP

When a mechanical component is subjected to an external static force, a resisting force is set up within the component. *The internal resisting force per unit area of the component is called stress*. The stresses are called *tensile* when the fibres of the component tend to elongate due to the external force. On the other hand, when the fibres tend to shorten due to the external force, the stresses are called *compressive stresses*. A tension rod subjected to an external force P is shown in Fig. 4.1. The tensile stress is given by,

$$\sigma_t = \frac{P}{A} \tag{4.3}$$

where

σ_t = tensile stress (N/mm^2)

P = external force (N)

A = cross-sectional area (mm^2)

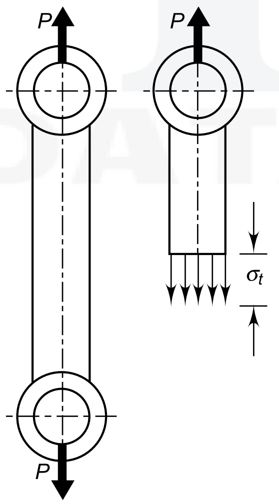


Fig. 4.1 *Tensile Stress*

Many times, the unit for stress or strength is taken as MPa.

1 MPa = 1 mega Pascal = 10^6 Pascal

$$= (10^6)\left(\frac{N}{m^2}\right) = (10^6)\frac{N}{[10^3\,mm]^2} = 1 N/mm^2$$

Therefore, two units (N/mm^2) and (MPa) are same. In this book, the unit (N/mm^2) is used. However, it can be replaced by (MPa) without any conversion factor.

The strain is deformation per unit length. It given by

$$\varepsilon = \frac{\delta}{l} \tag{4.4}$$

where,

ε = strain (mm/mm)
δ = elongation of the tension rod (mm)
l = original length of the rod (mm)

According to Hooke's law, the stress is directly proportional to the strain within elastic limit. Therefore,

$$\sigma_t \,\alpha\, \varepsilon$$

or

$$\sigma_t = E\,\varepsilon \tag{4.5}$$

where E is the constant of proportionality known as *Young's modulus* or *modulus of elasticity* (in N/mm^2 or MPa).

For carbon steels, $E = 207\,000\ N/mm^2$
For grey cast iron, $E = 100\,000\ N/mm^2$

Substituting Eqs (4.3) and (4.4) in Eq. (4.5),

$$\delta = \frac{Pl}{AE} \tag{4.6}$$

A component subjected to a compressive force is shown in Fig. 4.2. The compressive stress σ_c is given by,

$$\sigma_c = \frac{P}{A} \tag{4.7}$$

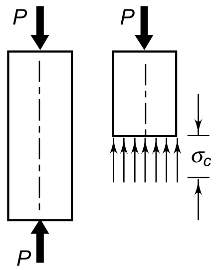


Fig. 4.2 *Compressive Stress*

The following assumptions are made in the analysis of stress and strain:

(i) The material is homogeneous.
(ii) The load is gradually applied.
(iii) The line of action of force P passes through the geometric axis of the cross-section.
(iv) The cross-section is uniform.
(v) There is no stress concentration.

4.4 SHEAR STRESS AND SHEAR STRAIN

When the external force acting on a component tends to slide the adjacent planes with respect to each other, the resulting stresses on these planes are called direct shear stresses. Two plates held together by means of a rivet are shown in Fig. 4.3 (a). The average shear stress in the rivet is given by

$$\tau = \frac{P}{A} \tag{4.8}$$

where,

τ = shear stress (N/mm^2 or MPa)
A = cross-sectional area of the rivet (mm^2)

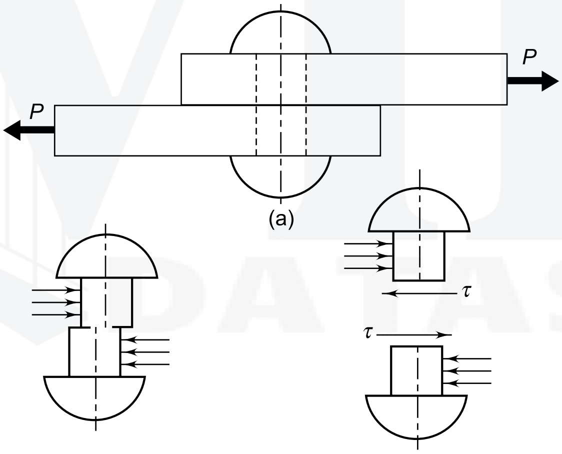


Fig. 4.3 *(a) Riveted Joint (b) Shear Deformation (c) Shear Stress*

A plane rectangular element, cut from the component and subjected to shear force, is shown in Fig. 4.4(a). Shear stresses cause a distortion in the original right angles. *The shear strain (γ) is defined as the change in the right angle of a shear element.* Within the elastic limit, the stress–strain relationship is given by

$$\tau = G\gamma \tag{4.9}$$

where,

γ = shear strain (radians)

G is the constant of proportionality known as *shear modulus* or *modulus of rigidity* (in N/mm^2 or MPa).

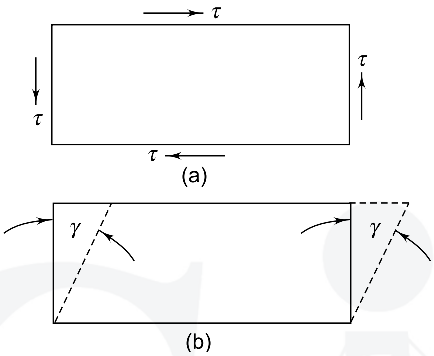


Fig. 4.4 *(a) Element Loaded in Pure Shear (b) Shear Strain*

For carbon steels, $G = 80\,000$ N/mm^2

For grey cast iron, $G = 40\,000$ N/mm^2

The relationship between the modulus of elasticity, the modulus of rigidity and the Poisson's ratio is given by,

$$E = 2G(1 + \mu) \tag{4.10}$$

where μ is Poisson's ratio. *Poisson's ratio* is the ratio of strain in the lateral direction to that in the axial direction.

For carbon steels, $\mu = 0.29$

For grey cast iron, $\mu = 0.21$

The permissible shear stress is given by,

$$\tau = \frac{S_{sy}}{(fs)} \tag{4.11}$$

where,

S_{sy} = yield strength in shear (N/mm^2 or MPa)

It will be proved at a later stage that the yield strength in shear is 50% of the yield strength in tension, according to the principal shear stress theory of failure.

4.5 STRESSES DUE TO BENDING MOMENT

A straight beam subjected to a bending moment M_b is shown in Fig. 4.5(a). The beam is subjected to a combination of tensile stress on one side of the neutral axis and compressive stress on the other. Such a stress distribution can be visualized by bending a thick leather belt. Cracks will appear on the outer surface, while folds will appear on the inside. Therefore, the outside fibres are in tension, while the inside fibres are in compression. The bending stress at any fibre is given by,

$$\sigma_b = \frac{M_b\, y}{I} \tag{4.12}$$

where,

σ_b = bending stress at a distance of y from the neutral axis (N/mm^2 or MPa)

M_b = applied bending moment (N-mm)

I = moment of inertia of the cross-section about the neutral axis (mm^4)

The bending stress is maximum in a fibre, which is farthest from the neutral axis. The distribution of stresses is linear and the stress is proportional to the distance from the neutral axis.

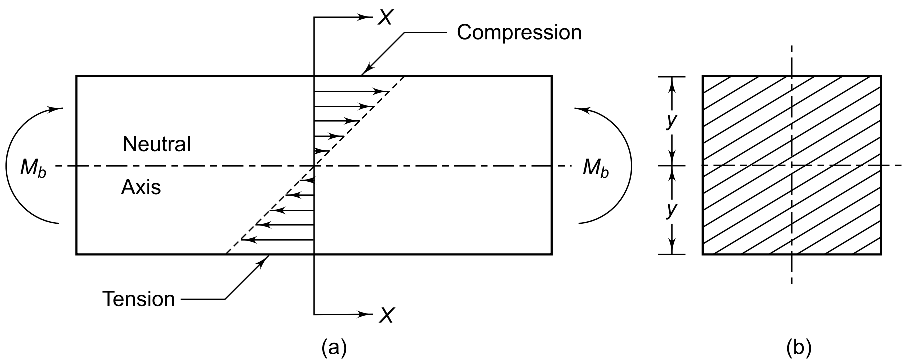


Fig. 4.5 *(a) Distribution of Bending Stresses (b) Section at XX*

Equation (4.12) is based on the following assumptions:

(i) The beam is straight with uniform cross-section.

(ii) The forces acting on the beam lie in a plane perpendicular to the axis of the beam.

(iii) The material is homogeneous, isotropic and obeys Hooke's law.

(iv) Plane cross-sections remain plane after bending.

The moment of inertia in Eq. (4.12) is the area moment of inertia.

For a rectangular cross-section,

$$I = \frac{bd^3}{12} \tag{4.13}$$

where,

b = distance parallel to the neutral axis (mm)

d = distance perpendicular to the neutral axis (mm)

For a circular cross-section,

$$I = \frac{\pi d^4}{64} \tag{4.14}$$

where d is the diameter of the cross-section.

When the cross-section is irregular, as shown in Fig. 4.6, the moment of inertia about the centroidal axis X_g is given by,

$$I_{xg} = \int y^2\, dA \tag{4.15}$$

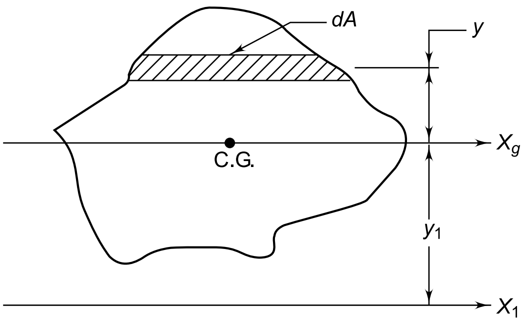


Fig. 4.6 *Parallel-axis Theorem*

The parallel-axis theorem for this area is given by the expression,

$$I_{x1} = I_{xg} + Ay_1^2 \tag{4.16}$$

where,

I_{x1} = moment of inertia of the area about X_1 axis, which is parallel to the axis X_g, and located at a distance y_1 from X_g

I_{xg} = moment of inertia of the area about its own centroidal axis

A = area of the cross-section.

In design of machine elements like transmission shaft, axle or lever, it is required to find out the maximum bending moment by constructing the bending moment diagram. There is a particular sign convention for bending moment diagram, which is illustrated in Fig. 4.7. For positive bending, the bending moment diagram is constructed on the positive side of the Y-axis. For negative bending, the diagram is on the negative side of the Y-axis. There is a simple way to remember positive bending. Imagine the crescent shaped moon—it is positive bending.

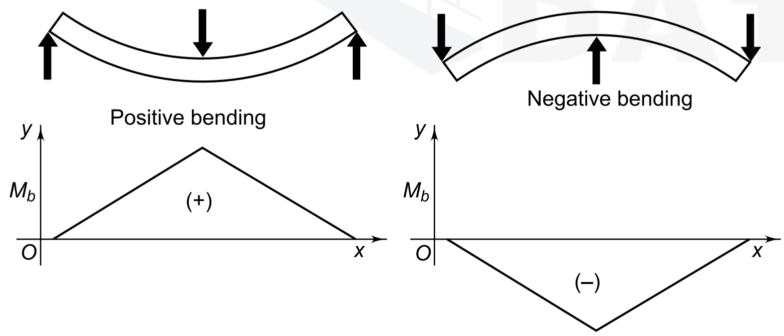


Fig. 4.7 *Sign Convention for Bending Moment*

4.6 STRESSES DUE TO TORSIONAL MOMENT

A transmission shaft, subjected to an external torque, is shown in Fig. 4.8 (a). *The internal stresses, which are induced to resist the action* *of twist, are called torsional shear stresses*. The torsional shear stress is given by

$$\tau = \frac{M_t\ r}{J} \tag{4.17}$$

where,

τ = torsional shear stress at the fibre (N/mm^2 or MPa)

M_t = applied torque (N-mm)

r = radial distance of the fibre from the axis of rotation (mm)

J = polar moment of inertia of the cross-section about the axis of rotation (mm^4)

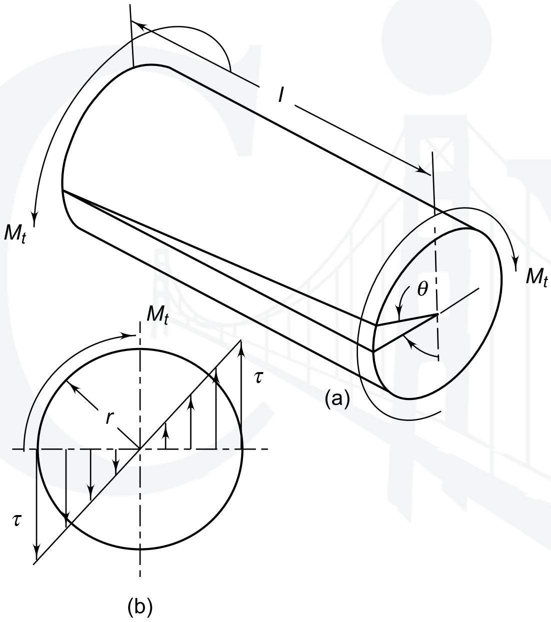


Fig. 4.8 *(a) Shaft Subjected to Torsional Moment (b) Distribution of Torsional Shear Stresses*

The distribution of torsional shear stresses is shown in Fig. 4.8 (b). The stress is maximum at the outer fibre and zero at the axis of rotation. The angle of twist is given by

$$\theta = \frac{M_t l}{JG} \tag{4.18}$$

where,

θ = angle of twist (radians)

l = length of the shaft (mm)

Equations (4.17) and (4.18) are based on the following assumptions:

(i) The shaft is straight with a circular cross-section.

(ii) A plane transverse section remains plane after twisting.

(iii) The material is homogeneous, isotropic and obeys Hooke's law.

The polar moment of inertia of a solid circular shaft of diameter d is given by

$$J = \frac{\pi d^4}{32} \tag{4.19}$$

For a hollow circular cross-section,

$$J = \frac{\pi\ (d_o^4 - d_i^4)}{32} \tag{4.20}$$

Substituting Eqs (4.19) and (4.20) in Eq. (4.18) and converting θ from radians to degrees,

$$\theta = \frac{584\ M_t l}{G d^4} \quad \text{(deg)} \tag{4.21}$$

$$\theta = \frac{584\ M_t l}{G\ (d_o^4 - d_i^4)} \quad \text{(deg)} \tag{4.22}$$

In many problems of machine design, it is required to calculate torque from the power transmitted and the speed of rotation. This relationship is given by,

$$kW = \frac{2\pi n M_t}{60 \times 10^6} \tag{4.23}$$

where,

kW = transmitted power (kW)

M_t = torque (N-mm)

n = speed of rotation (rpm)

4.7 ECCENTRIC AXIAL LOADING

In Section 4.3, it was assumed that the line of action of force passes through the centroid of the cross-section. There are certain mechanical components subjected to an external force, tensile or compressive, which does not pass through the centroid of the cross-section. A typical example of such an eccentric loading is shown in Fig. 4.9(a). According to the principle of statics, the eccentric force P can be replaced by a parallel force P passing through the centroidal axis along with a couple ($P \times e$) as shown in Figs 4.9(b) and (c) respectively. In Fig. 4.9(b), the force P causes a uniformly distributed tensile stress of magnitude (P/A). In Fig. 4.9(c), the couple causes bending stress of magnitude (Pey/I). The resultant stresses at the cross-section are obtained by the principle of superimposition of stresses. They are given by,

$$\sigma = \frac{P}{A} \pm \frac{Pey}{I} \tag{4.24}$$

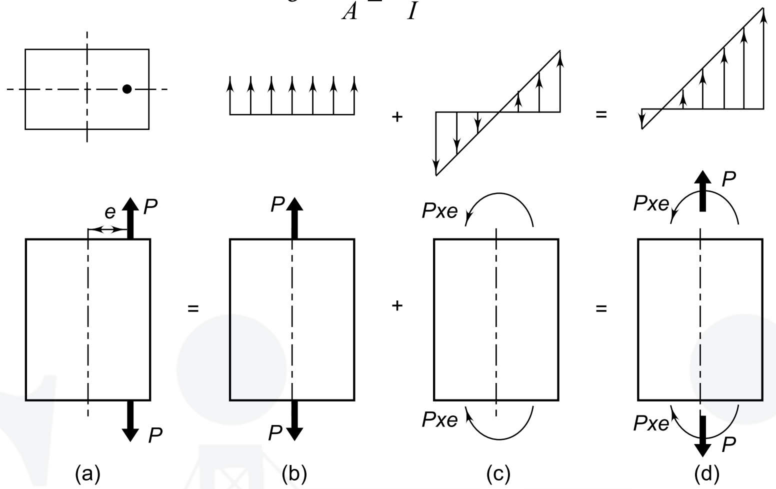


Fig. 4.9 *Eccentric Axial Load*

4.8 DESIGN OF SIMPLE MACHINE PARTS

In this chapter, design of simple machine parts is illustrated with the help of examples. The following points should be remembered in solving such problems:

(i) The dimensions of simple machine parts are determined on the basis of pure tensile stress, pure compressive stress, direct shear stress, bending stress or torsional shear stress. The analysis is simple but approximate, because a number of factors such as principal stresses, stress concentration, and reversal of stresses is neglected. Therefore, a higher factor of safety of up to 5 is taken to account for these factors.

(ii) It is incorrect to assume allowable stress as data for design. The allowable stress is to be obtained from published values of ultimate tensile strength and yield strength for a given material by Eqs (4.1) and (4.2) respectively.

(iii) It will be proved at a later stage that according to maximum shear stress theory, the yield strength in shear is 50% of the yield strength in tension. Therefore,

$$S_{sy} = 0.5\, S_{yt}$$

and the permissible shear stress (τ) is given by

$$\tau = \frac{S_{sy}}{(fs)}$$

The above value of allowable shear stress is used in determination of dimensions of the component.

The design analysis in this chapter is approximate and incomplete. The purpose of such problems is to illustrate the application of design equations to find out the geometric dimensions of the component. In practice, much more stress analysis is involved in the design of machine parts.

Example 4.1 *Two plates, subjected to a tensile force of 50 kN, are fixed together by means of three rivets as shown in Fig. 4.10 (a). The plates and rivets are made of plain carbon steel 10C4 with a tensile yield strength of 250 N/mm². The yield strength in shear is 50% of the tensile yield strength, and the factor of safety is 2.5. Neglecting stress concentration, determine*

(i) the diameter of the rivets; and
(ii) the thickness of the plates.

Solution

Given $P = 50 \times 10^3$ N $\quad S_{yt} = 250$ N/mm^2
$(fs) = 2.5$

Step I *Permissible shear stress for rivets*

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.5\ S_{yt}}{(fs)} = \frac{0.5\ (250)}{(2.5)} = 50\ \text{N/mm}^2$$

Step II *Diameter of rivets*

Since there are three rivets,

$$3\left[\frac{\pi}{4}d^2\right]\tau = P \quad \text{or} \quad 3\left[\frac{\pi}{4}d^2\right]50 = 50 \times 10^3$$

$\therefore \quad d = 20.60$ or 22 mm (i)

Step III *Permissible tensile stress for plates*

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{250}{2.5} = 100\ \text{N/mm}^2$$

Step IV *Thickness of plates*

As shown in Fig. 4.10(b),

$$\sigma_t\,(200 - 3d)\,t = P$$

or $\quad 100(200 - 3 \times 22)\,t = 50 \times 10^3$

$\therefore \quad t = 3.73$ or 4 mm (ii)

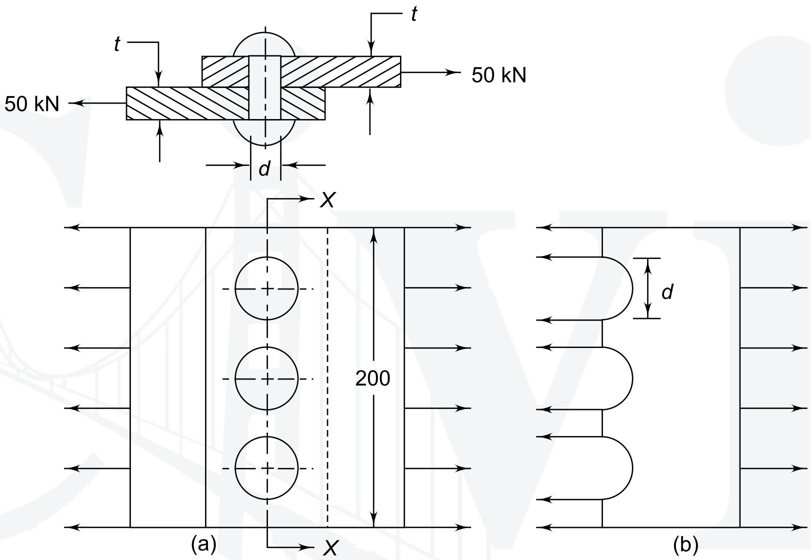


Fig. 4.10 *(a) Riveted Joint (b) Tensile Stress in Plate*

4.9 COTTER JOINT

A cotter joint is used to connect two co-axial rods, which are subjected to either axial tensile force or axial compressive force. It is also used to connect a rod on one side with some machine part like a crosshead or base plate on the other side. It is not used for connecting shafts that rotate and transmit torque. Typical applications of cotter joint are as follows:

(i) Joint between the piston rod and the crosshead of a steam engine

(ii) Joint between the slide spindle and the fork of the valve mechanism

(iii) Joint between the piston rod and the tail or pump rod

(iv) Foundation bolt

The principle of wedge action is used in a cotter joint. A cotter is a wedge-shaped piece made of a steel plate. The joint is tightened and adjusted by means of a wedge action of the cotter. The construction of a cotter joint, used to connect two rods *A* and *B* is shown in Fig. 4.11. Rod-*A* is provided with a socket end, while rod-*B* is provided with a spigot end. The socket end of rod-*A* fits over the spigot end of rod-*B*. The socket as well as the spigot is provided with a narrow rectangular slot. A cotter is tightly fitted in this slot passing through the socket and the spigot. The cotter has uniform thickness and the width dimension *b* is given a slight taper. The taper is usually 1 in 24. The taper is provided for the following two reasons:

(i) When the cotter is inserted in the slot through the socket and the spigot and pressed by means of hammer, it becomes tight due to wedge action. This ensures tightness of the joint in operating condition and prevents loosening of the parts.

(ii) Due to its taper shape, it is easy to remove the cotter and dismantle the joint.

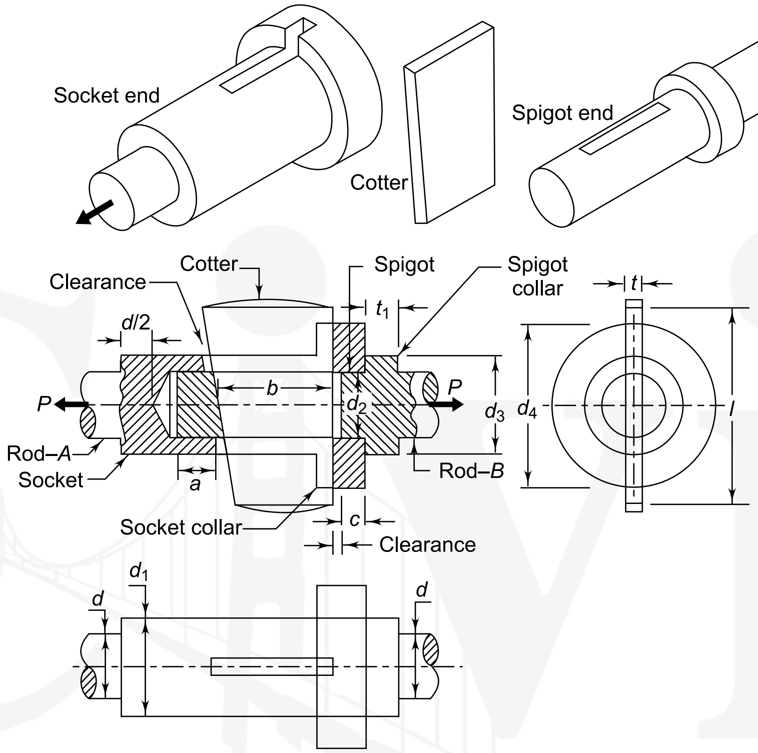


Fig. 4.11 *Cotter Joint*

The taper of the cotter as well as slots is on one side. Machining a taper on two sides of a machine part is more difficult than making a taper on one side. Also, there is no specific advantage of a taper on two sides. A clearance of 1.5 to 3 mm is provided between the slots and the cotter. When the cotter is driven in the slots, the two rods are drawn together until the spigot collar rests on the socket collar. The amount by which the two rods are drawn together is called the *draw* of the cotter. The cotter joint offers the following advantages:

(i) The assembly and dismantling of parts of the cotter joint is quick and simple. The assembly consists of inserting the spigot end into the socket end and putting the cotter into their common slot. When the cotter is hammered, the rods are drawn together and tightened. Dismantling consists of removing the cotter from the slot by means of a hammer.

(ii) The wedge action develops a very high tightening force, which prevents loosening of parts in service.

(iii) The joint is simple to design and manufacture.

Free body diagram of forces acting on three components of cotter joint, viz., socket, cotter and spigot is shown in Fig. 4.12. This diagram is constructed by using the principle that actions and reactions are equal and opposite. The forces are determined in the following way,

(i) Consider rod–A with a socket end. The rod is subjected to a horizontal force P to the left. The sum of all horizontal forces acting on the rod A with socket must be equal to zero. Therefore, there should be a force P to the right acting on the socket. This force is shown by two parts, each equal to $(P/2)$ on the socket end.

(ii) Consider rod-B with the spigot end. The rod is subjected to a force P to the right. The sum of all horizontal forces acting on rod-B must be equal to zero. Therefore, there should be a force P to the left on the spigot end.

(iii) The forces shown on the cotter are equal and opposite reactions of forces acting on the spigot end of rod-B and the socket end of rod-A.

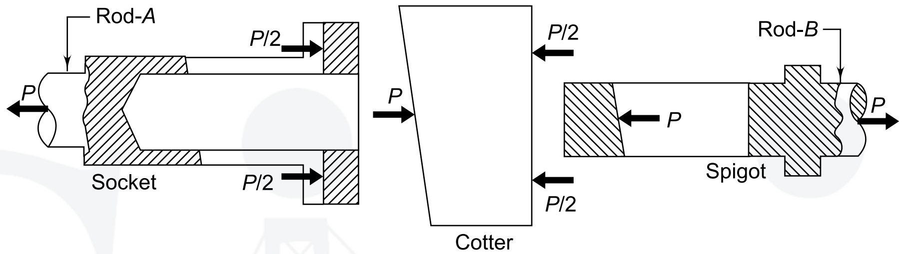


Fig. 4.12 *Free Body Diagram of Forces*

For the purpose of stress analysis, the following assumptions are made:

(i) The rods are subjected to axial tensile force.
(ii) The effect of stress concentration due to the slot is neglected.
(iii) The stresses due to initial tightening of the cotter are neglected.

In Fig. 4.11, the following notations are used

P = tensile force acting on rods (N)
d = diameter of each rod (mm)
d_1 = outside diameter of socket (mm)
d_2 = diameter of spigot or inside diameter of socket (mm)
d_3 = diameter of spigot-collar (mm)
d_4 = diameter of socket-collar (mm)
a = distance from end of slot to the end of spigot on rod-B (mm)
b = mean width of cotter (mm)
c = axial distance from slot to end of socket collar (mm)
t = thickness of cotter (mm)
t_1 = thickness of spigot-collar (mm)
l = length of cotter (mm)

In order to design the cotter joint and find out the above dimensions, failures in different parts and at different cross-sections are considered. Based on each type of failure, one strength equation is written. Finally, these strength equations are used to determine various dimensions of the cotter joint.

(i) Tensile Failure of Rods Each rod of diameter d is subjected to a tensile force P. The tensile stress in the rod is given by,

$$\sigma_t = \frac{P}{\left[\frac{\pi}{4} d^2\right]}$$

or $$d = \sqrt{\frac{4P}{\pi \sigma_t}} \tag{4.25a}$$

where σ_t is the permissible tensile stress for the rods.

(ii) Tensile Failure of Spigot Figure 4.13(a) shows the weakest cross-section at XX of the spigot end, which is subjected to tensile stress.

$$\text{Area of section at } XX = \left[\frac{\pi}{4} d_2^2 - d_2 t\right]$$

Therefore, tensile stress in the spigot is given by,

$$\sigma_t = \frac{P}{\left[\frac{\pi}{4} d_2^2 - d_2 t\right]}$$

or $$P = \left[\frac{\pi}{4} d_2^2 - d_2 t\right] \sigma_t \tag{4.25b}$$

From the above equation, the diameter of spigot or inner diameter of socket (d_2) can be determined by assuming a suitable value of t. The thickness of the cotter is usually determined by the following empirical relationship,

$$t = 0.31d \qquad (4.25c)$$

(iii) Tensile Failure of Socket Figure 4.14(a) shows the weakest section at YY of the socket end, which is subjected to tensile stress. The area of this section is given by,

$$\text{area} = \left[\frac{\pi}{4}(d_1^2 - d_2^2) - (d_1 - d_2)t\right]$$

The tensile stress at section YY is given by,

$$\sigma_t = \frac{P}{\text{area}}$$

or $$P = \left[\frac{\pi}{4}(d_1^2 - d_2^2) - (d_1 - d_2)t\right]\sigma_t \qquad (4.25d)$$

From the above equation, the outside diameter of socket (d_1) can be determined.

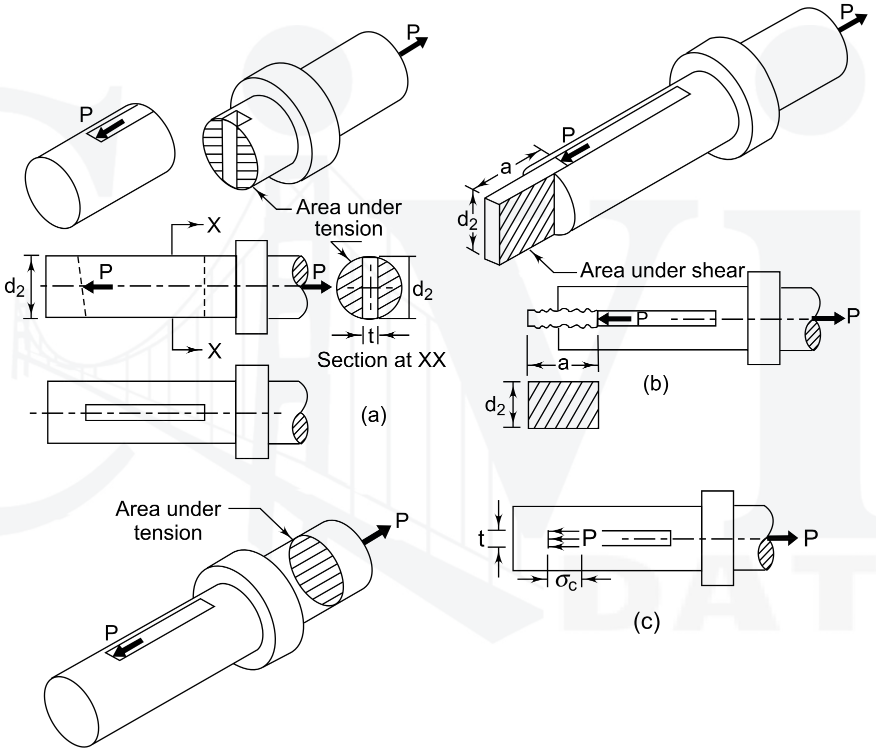


Fig. 4.13 *Stresses in Spigot End (a) Tensile Stress (b) Shear Stress (c) Compressive Stress*

(iv) Shear Failure of Cotter The cotter is subjected to double shear as illustrated in Fig. 4.15. The area of each of the two planes that resist shearing failure is (bt). Therefore, shear stress in the cotter is given by,

$$\tau = \frac{P}{2\,(bt)}$$

or $$P = 2\,bt\tau \qquad (4.25e)$$

where τ is permissible shear stress for the cotter. From Eq. (4.25e), the mean width of the cotter (b) can be determined.

(v) Shear Failure of Spigot End The spigot end is subjected to double shear as shown in Fig. 4.13(b). The area of each of the two planes that resist shear failure is (ad_2). Therefore, shear stress in the spigot end is given by,

$$\tau = \frac{P}{2\,(ad_2)}$$

or $$P = 2\,ad_2\tau \qquad (4.25f)$$

where τ is the permissible shear stress for the spigot. From Eq. (4.25f), the dimension a can be determined.

(vi) Shear Failure of Socket End The socket end is also subjected to double shear as shown in Fig. 4.14(b). The area of each of the two planes that resist shear failure is given by,

$$\text{area} = (d_4 - d_2)\, c$$

Therefore, shear stress in the socket end is given by,

$$\tau = \frac{P}{2\,(d_4 - d_2)\, c}$$

or $$P = 2\,(d_4 - d_2)\, c\, \tau \qquad (4.25g)$$

From the above equation, the dimension c can be determined.

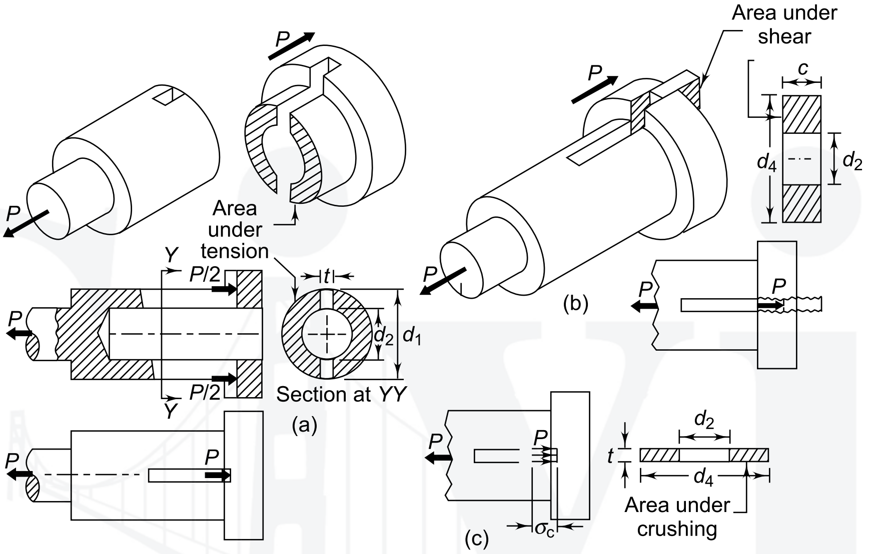


Fig. 4.14 *Stresses in Socket End (a) Tensile Stress (b) Shear stress (c) Compressive Stress*

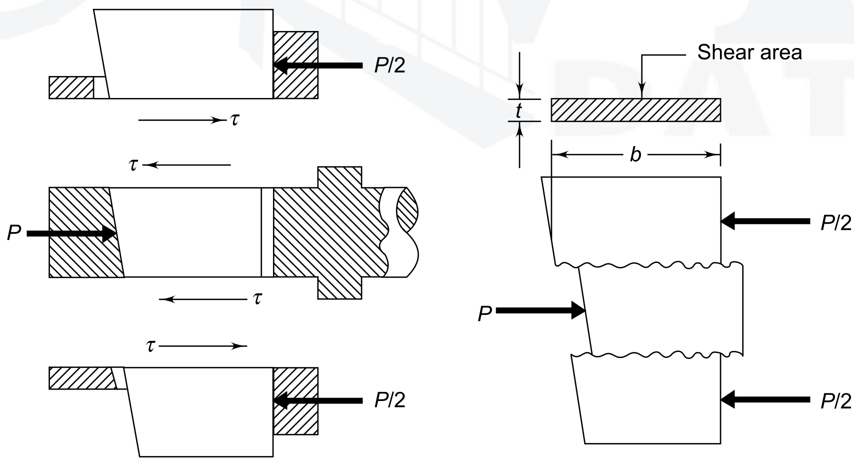


Fig. 4.15 *Shear Failure of Cotter*

(vii) Crushing Failure of Spigot End As shown in Fig. 4.13(c), the force P causes compressive stress on a narrow rectangular area of thickness t and width d_2 perpendicular to the plane of the paper. The compressive stress is given by,

$$\sigma_c = \frac{P}{t d_2} \qquad (4.25h)$$

(viii) Crushing Failure of Socket End As shown in Fig. 4.14(c), the force P causes compressive stress on a narrow rectangular area of thickness t. The other dimension of rectangle, perpendicular to the plane of paper is $(d_4 - d_2)$. Therefore, compressive stress in the socket end is given by,

$$\sigma_c = \frac{P}{(d_4 - d_2)t} \qquad (4.25i)$$

(ix) Bending Failure of Cotter When the cotter is tight in the socket and spigot, it is subjected to shear stresses. When it becomes loose, bending occurs. The forces acting on the cotter are shown in Fig. 4.16(a). The force P between the cotter and spigot end is assumed as uniformly distributed over the length d_2. The force between the socket end and cotter is assumed to be varying linearly from zero to maximum with triangular distribution. The cotter is treated as beam as shown in Fig. 4.16(b). For triangular distribution,

$$x = \frac{1}{3}y = \frac{1}{3}\left(\frac{d_4 - d_2}{2}\right) = \left(\frac{d_4 - d_2}{6}\right)$$

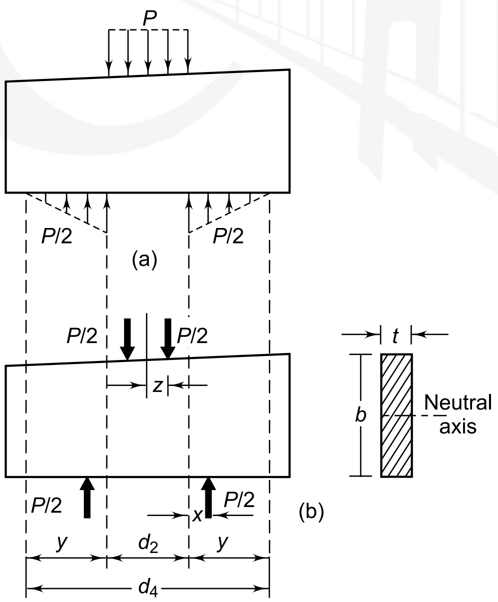


Fig. 4.16 *Cotter Treated as Beam (a) Actual Distribution of Forces (b) Simplified Diagram of Forces*

The bending moment is maximum at the centre. At the central section,

$$M_b = \frac{P}{2}\left[\frac{d_2}{2} + x\right] - \frac{P}{2}(z)$$

$$= \frac{P}{2}\left[\frac{d_2}{2} + \frac{d_4 - d_2}{6}\right] - \frac{P}{2}\left[\frac{d_2}{4}\right]$$

$$= \frac{P}{2}\left[\frac{d_2}{4} + \frac{d_4 - d_2}{6}\right]$$

Also, $I = \dfrac{tb^3}{12}$ $\qquad y = \dfrac{b}{2}$

and $\sigma_b = \dfrac{M_b y}{I}$

Therefore,

$$\sigma_b = \frac{\frac{P}{2}\left[\frac{d_2}{4} + \frac{d_4 - d_2}{6}\right]\frac{b}{2}}{\left(\frac{tb^3}{12}\right)} \qquad (4.25j)$$

The applications of strength equations from (4.25a) to (4.25j) in finding out the dimensions of the cotter joint are illustrated in the next example and the design project. In some cases, the dimensions of a cotter joint are calculated by using empirical relationships, without carrying out detail stress analysis. In such cases, following standard proportions can be used,

$d_1 = 1.75d$	$d_2 = 1.21d$
$d_3 = 1.5\,d$	$d_4 = 2.4\,d$
$a = c = 0.75\,d$	$b = 1.6\,d$
$t = 0.31\,d$	$t_1 = 0.45\,d$

Clearance = 1.5 to 3 mm
Taper for cotter = 1 in 32

4.10 DESIGN PROCEDURE FOR COTTER JOINT

The basic procedure to calculate the dimensions of the cotter joint consists of the following steps:

(i) Calculate the diameter of each rod by Eq. (4.25a),

$$d = \sqrt{\frac{4P}{\pi\sigma_t}}$$

(ii) Calculate the thickness of the cotter by the empirical relationship given in Eq. (4.25c),

$$t = 0.31\,d$$

(iii) Calculate the diameter d_2 of the spigot on the basis of tensile stress. From Eq. (4.25b),

$$P = \left[\frac{\pi}{4} d_2^2 - d_2\, t\right] \sigma_t$$

When the values of P, t and σ_t are substituted, the above expression becomes a quadratic equation.

(iv) Calculate the outside diameter d_1 of the socket on the basis of tensile stress in the socket, from Eq. (4.25d),

$$P = \left[\frac{\pi}{4}(d_1^2 - d_2^2) - (d_1 - d_2)\, t\right] \sigma_t$$

When values of P, d_2, t and σ_t are substituted, the above expression becomes a quadratic equation.

(v) The diameter of the spigot collar d_3 and the diameter of the socket collar d_4 are calculated by the following empirical relationships,

$$d_3 = 1.5\, d$$
$$d_4 = 2.4\, d$$

(vi) The dimensions a and c are calculated by the following empirical relationship,

$$a = c = 0.75\, d$$

(vii) Calculate the width b of the cotter by shear consideration using Eq. (4.25e) and bending consideration using Eq. (4.25j) and select the width, whichever is maximum between these two values.

$$b = \frac{P}{2\tau t} \qquad \text{or} \qquad b = \sqrt{\frac{3P}{t\sigma_b}\left[\frac{d_2}{4} + \frac{d_4 - d_2}{6}\right]}$$

(viii) Check the crushing and shear stresses in the spigot end by Eqs. (4.25h) and (4.25f) respectively.

$$\sigma_c = \frac{P}{td_2}$$
$$\tau = \frac{P}{2ad_2}$$

(ix) Check the crushing and shear stresses in the socket end by Eqs (4.25i) and (4.25g) respectively.

$$\sigma_c = \frac{P}{(d_4 - d_2)\, t}$$
$$\tau = \frac{P}{2\,(d_4 - d_2)\, c}$$

(x) Calculate the thickness t_1 of the spigot collar by the following empirical relationship,

$$t_1 = 0.45\, d$$

The taper of the cotter is 1 in 32.

Example 4.2 *It is required to design a cotter joint to connect two steel rods of equal diameter. Each rod is subjected to an axial tensile force of 50 kN. Design the joint and specify its main dimensions.*

Solution

Given $P = (50 \times 10^3)$ N

Part I *Selection of material*

The rods are subjected to tensile force and strength is the criterion for the selection of the rod material. The cotter is subjected to direct shear stress and bending stresses. Therefore, strength is also the criterion of material selection for the cotter. On the basis of strength, the material of the two rods and the cotter is selected as plain carbon steel of Grade 30C8 ($S_{yt} = 400$ N/mm^2).

Part II *Selection of factor of safety*

In stress analysis of the cotter joint, the following factors are neglected:

(i) initial stresses due to tightening of the cotter; and

(ii) stress concentration due to slot in the socket and the spigot ends.

To account for these factors, a higher factor of safety is used in the present design. The factor of safety for the rods, spigot end and socket end is assumed as 6, while for the cotter, it is taken as 4. There are two reasons for assuming a lower factor of safety for the cotter. They are as follows:

(i) There is no stress concentration in the cotter.

(ii) The cost of the cotter is small compared with the socket end or spigot end. If at all, a failure is going to occur, it should occur in the cotter rather than in the spigot or socket end. This is ensured by assuming a higher factor of safety for the spigot and socket ends, compared with the cotter.

It is assumed that the yield strength in compression is twice the yield strength in tension.

Part III *Calculation of permissible stresses*

The permissible stresses for rods, spigot end and socket end are as follows:

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{400}{6} = 66.67 \text{ N/mm}^2$$

$$\sigma_c = \frac{S_{yc}}{(fs)} = \frac{2S_{yt}}{(fs)} = \frac{2(400)}{6} = 133.33 \text{ N/mm}^2$$

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.5S_{yt}}{(fs)} = \frac{0.5\,(400)}{6}$$

$$= 33.33 \text{ N/mm}^2$$

Permissible stresses for the cotter are as follows:

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{400}{4} = 100 \text{ N/mm}^2$$

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.5S_{yt}}{(fs)} = \frac{0.5(400)}{4} = 50 \text{ N/mm}^2$$

Part IV *Calculation of dimensions*

The dimensions of the cotter joint are determined by the procedure outlined in Section 4.10.

Step I *Diameter of rods*

$$d = \sqrt{\frac{4P}{\pi\sigma_t}} = \sqrt{\frac{4(50\times10^3)}{\pi(66.67)}} = 30.90 \text{ or } 32 \text{ mm}$$

Step II *Thickness of cotter*

$$t = 0.31\,d = 0.31(32) = 9.92 \text{ or } 10 \text{ mm}$$

Step III *Diameter (d_2) of spigot*

$$P = \left[\frac{\pi}{4}d_2^2 - d_2 t\right]\sigma_t$$

$$50\times10^3 = \left[\frac{\pi}{4}d_2^2 - d_2(10)\right](66.67)$$

or $\quad d_2^2 - 12.73d_2 - 954.88 = 0$

Solving the above quadratic equation,

$$d_2 = \frac{12.73 \pm \sqrt{12.73^2 - 4(-954.88)}}{2}$$

$\therefore \quad d_2 = 37.91$ or 40 mm

Step IV *Outer diameter (d_1) of socket*

$$P = \left[\frac{\pi}{4}(d_1^2 - d_2^2) - (d_1 - d_2)t\right]\sigma_t$$

$$50\times10^3 = \left[\frac{\pi}{4}(d_1^2 - 40^2) - (d_1 - 40)\,(10)\right](66.67)$$

or $\quad d_1^2 - 12.73d_1 - 2045.59 = 0$

Solving the above quadratic equation,

$$d_1 = \frac{12.73 \pm \sqrt{12.73^2 - 4(-2045.59)}}{2}$$

$\therefore \quad d_1 = 52.04$ or 55 mm

Step V *Diameters of spigot collar (d_3) and socket collar (d_4)*

$$d_3 = 1.5d = 1.5(32) = 48 \text{ mm}$$

$$d_4 = 2.4d = 2.4(32) = 76.8 \text{ or } 80 \text{ mm}$$

Step VI *Dimensions a and c*

$$a = c = 0.75d = 0.75(32) = 24 \text{ mm}$$

Step VII *Width of cotter*

$$b = \frac{P}{2\tau t} = \frac{50\times10^3}{2(50)(10)} = 50 \text{ mm} \qquad \text{(a)}$$

or

$$b = \sqrt{\frac{3P}{t\sigma_b}\left[\frac{d_2}{4} + \frac{d_4 - d_2}{6}\right]}$$

$$= \sqrt{\frac{3\,(50\times10^3)}{(10)\,(100)}\left[\frac{40}{4} + \frac{80-40}{6}\right]}$$

$$= 50 \text{ mm} \qquad \text{(b)}$$

From (a) and (b),

$$b = 50 \text{ mm}$$

Step VIII *Check for crushing and shear stresses in spigot end*

$$\sigma_c = \frac{P}{td_2} = \frac{50\times10^3}{(10)(40)} = 125 \text{ N/mm}^2$$

$$\tau = \frac{P}{2\,ad_2} = \frac{50\times10^3}{2(24)(40)} = 26.04 \text{ N/mm}^2$$

$\therefore \quad \sigma_c < 133.33$ N/mm^2 and $\tau < 33.33$ N/mm^2

Step IX *Check for crushing and shear stresses in socket end*

$$\sigma_c = \frac{P}{(d_4 - d_2)\,t}$$

$$= \frac{50\times10^3}{(80-40)(10)} = 125 \text{ N/mm}^2$$

$$\tau = \frac{P}{2\,(d_4 - d_2)\,c}$$

$$= \frac{50 \times 10^3}{2(80-40)(24)} = 26.04 \text{ N/mm}^2$$

$\therefore \quad \sigma_c < 133.33 \text{ N/mm}^2 \quad \text{and} \quad \tau < 33.33 \text{ N/mm}^2$

The stresses induced in the spigot and the socket ends are within limits.

Step X *Thickness of spigot collar*

$t_1 = 0.45d = 0.45(32) = 14.4$ or 15 mm

The taper for the cotter is 1 in 32.

Part V *Dimensioned sketch of cotter joint*

The dimensions of various components of the cotter joint are shown in Fig. 4.17.

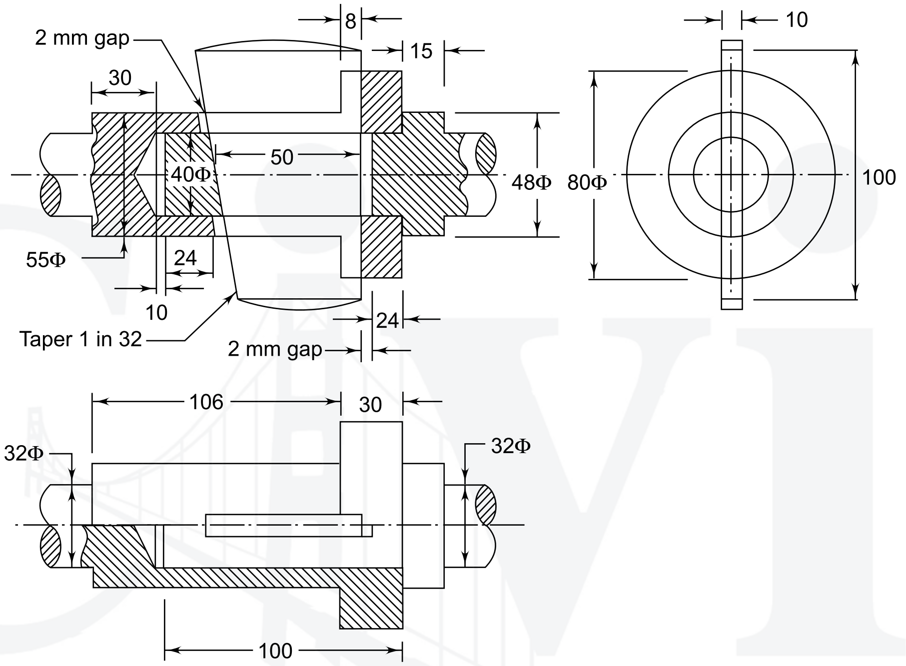


Fig. 4.17 *Dimensions of Cotter Joint*

Example 4.3 *Two rods are connected by means of a cotter joint. The inside diameter of the socket and outside diameter of the socket collar are 50 and 100 mm respectively. The rods are subjected to a tensile force of 50 kN. The cotter is made of steel 30C8 (S_{yt} = 400 N/mm^2) and the factor of safety is 4. The width of the cotter is five times of thickness. Calculate:*

(i) *width and thickness of the cotter on the basis of shear failure; and*

(ii) *width and thickness of the cotter on the basis of bending failure.*

Solution

Given $S_{yt} = 400 \text{ N/mm}^2 \quad (fs) = 4$

$P = (50 \times 10^3) \text{ N} \quad d_4 = 100 \text{ mm} \quad d_2 = 50 \text{ mm}$

Step I *Permissible stresses for cotter*

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{400}{4} = 100 \text{ N/mm}^2$$

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.5S_{yt}}{(fs)} = \frac{0.5(400)}{4} = 50 \text{ N/mm}^2$$

Step II *Width and thickness on the basis of shear failure*

$$b = 5t$$

From Eq. (4.25e),

$P = 2bt\tau \quad \text{or} \quad 50 \times 10^3 = 2(5t)\,t\,(50)$

$\therefore \quad t = 10 \text{ mm} \quad \text{and} \quad b = 5t = 50 \text{ mm} \qquad \text{(i)}$

Step III *Width and thickness on the basis of bending failure*

$d_4 = 100 \text{ mm} \qquad d_2 = 50 \text{ mm}$

From Eq. (4.25j),

$$\sigma_b = \frac{\frac{P}{2}\left[\frac{d_2}{4} + \frac{d_4 - d_2}{6}\right]\frac{b}{2}}{\left(\frac{tb^3}{12}\right)}$$

$$100 = \frac{\frac{50 \times 10^3}{2}\left[\frac{50}{4} + \frac{(100-50)}{6}\right]\frac{(5t)}{2}}{\left[\frac{t(5t)^3}{12}\right]}$$

$\therefore$ $t = 10.77$ or 12 mm and $b = 5t = 60$ mm (ii)

Example 4.4 *Two rods, made of plain carbon steel 40C8 (S_{yt} = 380 N/mm²), are to be connected by means of a cotter joint. The diameter of each rod is 50 mm and the cotter is made from a steel plate of 15 mm thickness. Calculate the dimensions of the socket end making the following assumptions:*

(i) the yield strength in compression is twice of the tensile yield strength; and

(ii) the yield strength in shear is 50% of the tensile yield strength.

The factor of safety is 6.

Solution

Given S_{yt} = 380 N/mm² $(fs) = 6$ $t = 15$ mm $d = 50$ mm

Step I *Permissible stresses*

$$\sigma_c = \frac{S_{yc}}{(fs)} = \frac{2S_{yt}}{(fs)} = \frac{2(380)}{6} = 126.67 \text{ N/mm}^2$$

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.5S_{yt}}{(fs)} = \frac{0.5(380)}{6} = 31.67 \text{ N/mm}^2$$

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{380}{6} = 63.33 \text{ N/mm}^2$$

Step II *Load acting on rods*

$$P = \frac{\pi}{4} d^2 \sigma_t \quad \text{or} \quad P = \frac{\pi}{4}(50)^2 (63.33)$$

$$= 124\,348.16 \text{ N}$$

Step III *Inside diameter of socket (d_2)*

From Eq. (4.25b),

$$P = \left[\frac{\pi}{4} d_2^2 - d_2 t\right]\sigma_t$$

$$124\,348.16 = \left[\frac{\pi}{4} d_2^2 - d_2(15)\right](63.33)$$

or $d_2^2 - 19.1 d_2 - 2500 = 0$

Solving the above quadratic equation,

$$d_2 = \frac{19.1 \pm \sqrt{19.1^2 - 4(-2500)}}{2}$$

$\therefore$ $d_2 = 60.45$ or 65 mm (i)

Step IV *Outside diameter of socket (d_1)*

From Eq. (4.25d),

$$P = \left[\frac{\pi}{4}(d_1^2 - d_2^2) - (d_1 - d_2)t\right]\sigma_t$$

$$124\,348.16 = \left[\frac{\pi}{4}(d_1^2 - 65^2) - (d_1 - 65)(15)\right](63.33)$$

or $d_1^2 - 19.1 d_1 - 5483.59 = 0$

Solving the above quadratic equation,

$$d_1 = \frac{19.1 \pm \sqrt{19.1^2 - 4(-5483.59)}}{2}$$

$\therefore$ $d_1 = 84.21$ or 85 mm (ii)

Step V *Diameter of socket collar (d_4)*

From Eq. (4.25i),

$$\sigma_c = \frac{P}{(d_4 - d_2)t}$$

or $$126.67 = \frac{124\,348.16}{(d_4 - 65)(15)}$$

$\therefore$ $d_4 = 130.44$ or 135 mm (iii)

Step VI *Dimensions a and c*

From Eq. (4.25f),

$$a = \frac{P}{2d_2\tau} = \frac{124\,348.16}{2(65)(31.67)} = 30.20 \text{ or } 35 \text{ mm} \quad \text{(iv)}$$

From Eq. (4.25g),

$$c = \frac{P}{2(d_4 - d_2)\tau} = \frac{124\,348.16}{2(135-65)(31.67)}$$

$$= 28.04 \text{ or } 30 \text{ mm} \quad \text{(v)}$$

4.11 KNUCKLE JOINT

Knuckle joint is used to connect two rods whose axes either coincide or intersect and lie in one plane. The knuckle joint is used to transmit axial tensile force. The construction of this joint permits limited angular movement between rods, about the axis of the pin. This type of joint is popular in machines and structures. Typical applications of knuckle joints are as follows:

(i) Joints between the tie bars in roof trusses.

(ii) Joints between the links of a suspension bridge.

(iii) Joints in valve mechanism of a reciprocating engine.

(iv) Fulcrum for the levers.

(v) Joints between the links of a bicycle chain.

A knuckle joint is unsuitable to connect two rotating shafts, which transmit torque. The construction of a knuckle joint, used to connect two rods *A* and *B* subjected to tensile force *P*, is shown in Fig. 4.18. An eye is formed at the end of rod-*B*, while a fork is formed at the end of rod-*A*. The eye fits inside the fork and a pin passes through both the fork and the eye. This pin is secured in its place by means of a split-pin. Due to this type of construction, a knuckle joint is sometimes called a *forked-pin joint.* In rare applications, a knuckle joint is used to connect three rods—two with forks and a third with the eye.

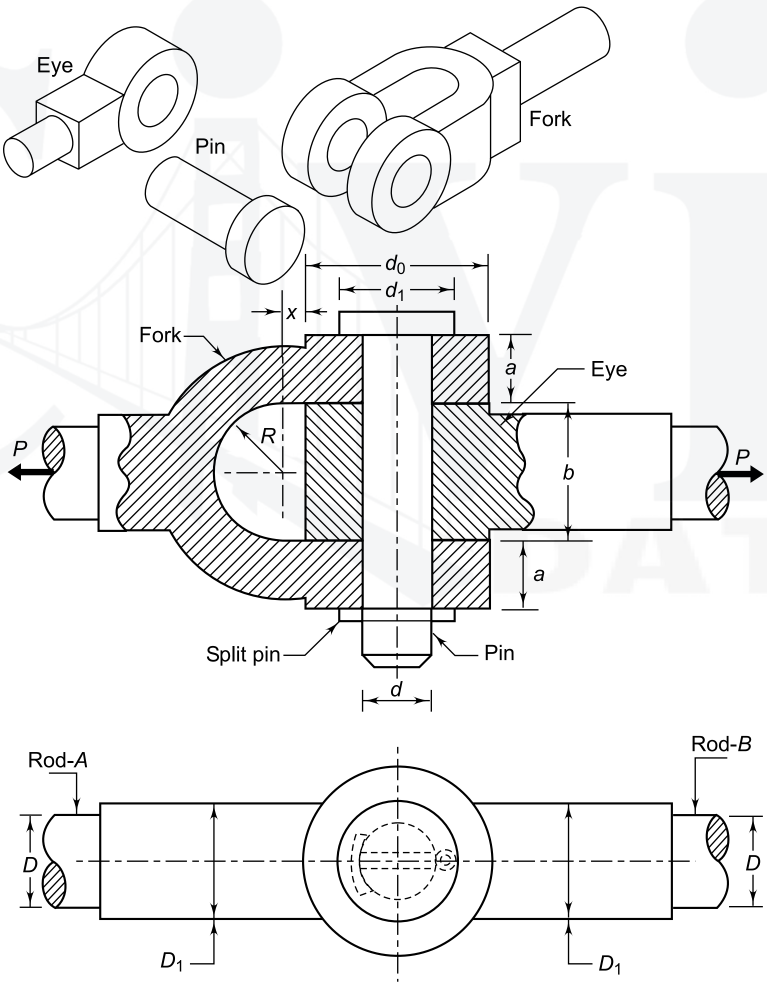


Fig. 4.18 *Knuckle Joint*

The knuckle joint offers the following advantages:

(i) The joint is simple to design and manufacture.

(ii) There are a few parts in the knuckle joint, which reduces cost and improves reliability.

(iii) The assembly or dismantling of the parts of a knuckle joint is quick and simple. The assembly consists of inserting the eye of one rod inside the fork of the other rod and putting the pin in their common hole and finally putting the split-pin to hold the pin. Dismantling consists of removing the split-pin and taking the pin out of the eye and the fork.

In Fig. 4.18, the following notations are used.

D = diameter of each rod (mm)
D_1 = enlarged diameter of each rod (mm)
d = diameter of knuckle pin (mm)
d_0 = outside diameter of eye or fork (mm)
a = thickness of each eye of fork (mm)
b = thickness of eye end of rod-B (mm)
d_1 = diameter of pin head (mm)
x = distance of the centre of fork radius R from the eye (mm)

For the purpose of stress analysis of a knuckle joint, the following assumptions are made:

(i) The rods are subjected to axial tensile force.

(ii) The effect of stress concentration due to holes is neglected.

(iii) The force is uniformly distributed in various parts.

Free body diagram of forces acting on three components of the knuckle joint, viz., fork, pin and eye is shown in Fig. 4.19. This diagram is constructed by using the principle that actions and reactions are equal and opposite. The forces are determined in the following way,

(i) Consider rod-A with the fork end. The rod is subjected to a horizontal force P to the left. The sum of all horizontal forces acting on rod-A must be equal to zero. Therefore, there should be a force P to the right acting on the fork end. The force P is divided into two parts, each equal to (P/2) on the fork end.

(ii) Consider rod-B with the eye end. The rod is subjected to a horizontal force P to the right side. The sum of all horizontal forces acting on rod-B must be equal to zero. Therefore, there should be a force P to the left acting on the eye end.

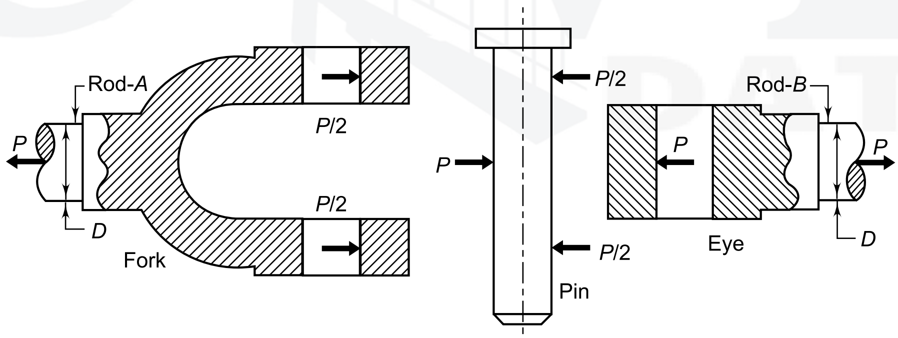


Fig. 4.19 *Free Body Diagram of Forces*

(iii) The forces shown on the pin are equal and opposite reactions of forces acting on the fork end of rod-A and the eye end of rod-B.

In order to find out various dimensions of the parts of a knuckle joint, failures in different parts and at different cross-sections are considered. For each type of failure, one strength equation is written. Finally, these strength equations are used to find out various dimensions of the knuckle joint.

(i) Tensile Failure of Rods Each rod is subjected to a tensile force P. The tensile stress in the rod is given by,

$$\sigma_t = \frac{P}{\left(\frac{\pi}{4}D^2\right)} \quad \text{or} \quad D = \sqrt{\frac{4P}{\pi\,\sigma_t}} \qquad (4.26a)$$

where σ_t is the permissible tensile stress for the rods. The enlarged diameter D_1 of the rod near the joint is determined by the following empirical relationship,

$$D_1 = 1.1\,D \qquad (4.26b)$$

(ii) Shear Failure of Pin The pin is subjected to double shear as shown in Fig. 4.20. The area of each of the two planes that resist shear failure is $\left(\frac{\pi}{4}d^2\right)$. Therefore, shear stress in the pin is given by,

$$\tau = \frac{P}{2\left(\frac{\pi}{4}d^2\right)} \quad \text{or} \quad d = \sqrt{\frac{2P}{\pi\,\tau}} \qquad (4.26c)$$

where τ is the permissible shear stress for the pin. The standard proportion for the diameter of the pin is as follows,

$$d = D \qquad (4.26d)$$

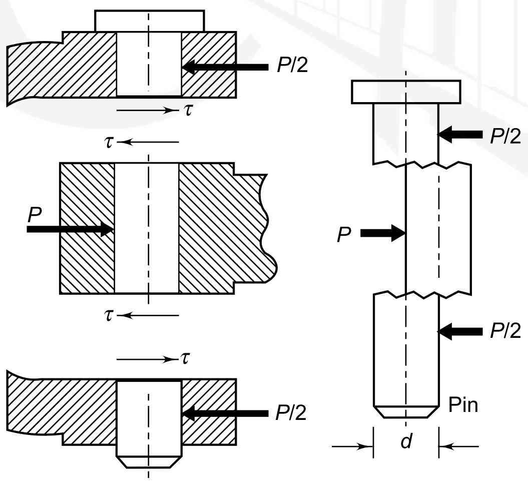


Fig. 4.20 *Shear Failure of Pin*

(iii) Crushing Failure of Pin in Eye When a cylindrical surface such as a pin is subjected to a force along its periphery, its projected area is taken into consideration to find out the stress. As shown in Fig. 4.21, the projected area of the cylindrical surface is $(l \times d)$ and the compressive stress is given by,

$$\sigma_c = \frac{\text{force}}{\text{projected area}} = \frac{P}{(l \times d)}$$

As shown in Fig. 4.18, the projected area of the pin in the eye is (bd) and the compressive stress between the pin and the eye is given by,

$$\sigma_c = \frac{P}{bd} \qquad (4.26e)$$

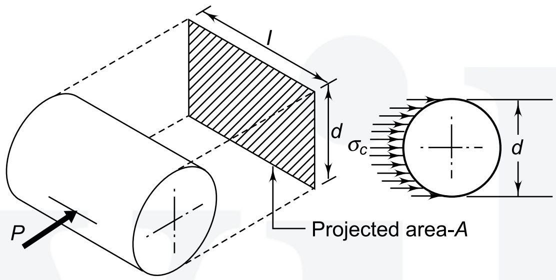


Fig. 4.21 *Projected Area of Cylindrical Surface*

(iv) Crushing Failure of Pin in Fork As shown in Fig. 4.18, the total projected area of the pin in the fork is $(2ad)$ and the compressive stress between the pin and the fork is given by,

$$\sigma_c = \frac{P}{2ad} \qquad (4.26f)$$

(v) Bending Failure of Pin When the pin is tight in the eye and the fork, failure occurs due to shear. On the other hand, when the pin is loose, it is subjected to bending moment as shown in Fig. 4.22. It is assumed that the load acting on the pin is uniformly distributed in the eye, but uniformly varying in two parts of the fork. For triangular distribution of load between the pin and the fork,

$$x = \frac{1}{3}a \qquad \text{also,} \qquad z = \frac{1}{2}\left(\frac{1}{2}b\right) = \frac{1}{4}b$$

The bending moment is maximum at the centre. It is given by,

$$M_b = \frac{P}{2}\left[\frac{b}{2} + x\right] - \frac{P}{2}(z)$$

$$= \frac{P}{2}\left[\frac{b}{2} + \frac{a}{3}\right] - \frac{P}{2}\left[\frac{b}{4}\right] = \frac{P}{2}\left[\frac{b}{4} + \frac{a}{3}\right]$$

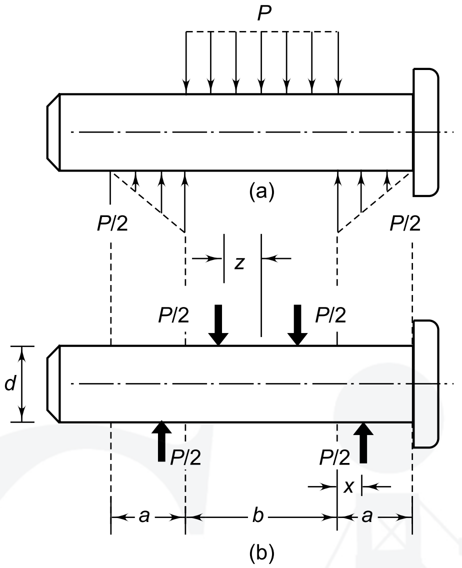


Fig. 4.22 *Pin Treated as Beam (a) Actual Distribution of Forces (b) Simplified Diagram of Forces*

Also, $I = \dfrac{\pi d^4}{64}$ and $y = \dfrac{d}{2}$

From Eq. (4.12),

$$\sigma_b = \frac{M_b y}{I} = \frac{\frac{P}{2}\left[\frac{b}{4}+\frac{a}{3}\right]\frac{d}{2}}{\frac{\pi d^4}{64}}$$

or
$$\sigma_b = \frac{32}{\pi d^3} \times \frac{P}{2}\left[\frac{b}{4}+\frac{a}{3}\right] \tag{4.26g}$$

(vi) Tensile Failure of Eye Section *XX* shown in Fig. 4.23(a) is the weakest section of the eye. The area of this section is given by,

$$\text{area} = b\,(d_0 - d)$$

The tensile stress at section *XX* is given by,

$$\sigma_t = \frac{P}{\text{area}} \quad \text{or} \quad \sigma_t = \frac{P}{b\,(d_0 - d)} \tag{4.26h}$$

(vii) Shear Failure of Eye The eye is subjected to double shear as shown in Fig. 4.23(b). The area of each of the two planes resisting the shear failure is

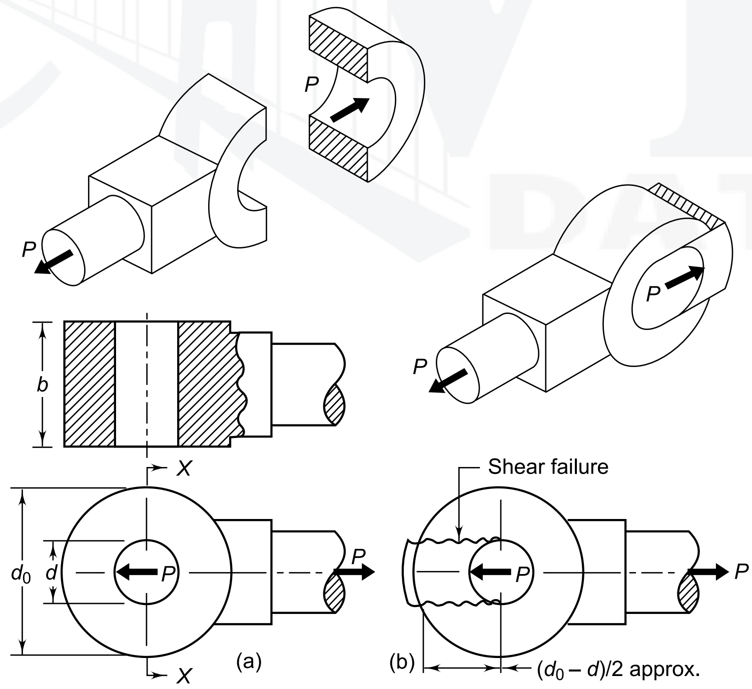


Fig. 4.23 *(a) Tensile Failure of Eye (b) Shear Failure of Eye*

[b ($d_0 - d$)/2] approximately. Therefore, shear stress is given by,

$$\tau = \frac{P}{2[b(d_0 - d)/2]}$$

or
$$\tau = \frac{P}{b\,(d_0 - d)} \tag{4.26i}$$

Standard proportion for outside diameter of the eye or the fork is given by the following relationship,

$$d_0 = 2d \tag{4.26j}$$

(viii) Tensile Failure of Fork Fork is a double eye and as such, Fig. 4.23 is applicable to a fork except for dimension b which can be modified as $2a$ in case of a fork. The area of the weakest section resisting tensile failure is given by

$$\text{area} = 2a\,(d_0 - d)$$

Tensile stress in the fork is given by

$$\sigma_t = \frac{P}{2a\,(d_0 - d)} \tag{4.26k}$$

(ix) Shear Failure of Fork Each of the two parts of the fork is subjected to double shear. Modifying Eq. (4.26i),

$$\tau = \frac{P}{2a\,(d_0 - d)} \tag{4.26l}$$

Standard proportions for the dimensions a and b are as follows,

$$a = 0.75\,D \tag{4.26m}$$
$$b = 1.25\,D \tag{4.26n}$$

The diameter of the pinhead is taken as,

$$d_1 = 1.5\,d \tag{4.26o}$$

The gap x shown in Fig. 4.18 is usually taken as 10 mm.

$$\therefore \quad x = 10 \text{ mm} \tag{4.26p}$$

The applications of strength equations from (4.26a) to (4.26l) in finding out the dimensions of the knuckle joint are illustrated in the next example. The eye and the fork are usually made by the forging process and the pin is machined from rolled steel bars.

4.12 DESIGN PROCEDURE FOR KNUCKLE JOINT

The basic procedure to determine the dimensions of the knuckle joint consists of the following steps:

(i) Calculate the diameter of each rod by Eq. (4.26a).

$$D = \sqrt{\frac{4P}{\pi\sigma_t}}$$

(ii) Calculate the enlarged diameter of each rod by empirical relationship using Eq. (4.26b).

$$D_1 = 1.1\,D$$

(iii) Calculate the dimensions a and b by empirical relationship using Eqs (4.26m) and (4.26n).

$$a = 0.75\,D \qquad b = 1.25\,D$$

(iv) Calculate the diameters of the pin by shear consideration using Eq. (4.26c) and bending consideration using Eq. (4.26g) and select the diameter, whichever is maximum.

$$d = \sqrt{\frac{2P}{\pi\tau}} \quad \text{or} \quad d = \sqrt[3]{\frac{32}{\pi\sigma_b} \times \frac{P}{2}\left[\frac{b}{4} + \frac{a}{3}\right]}$$

(whichever is maximum)

(v) Calculate the dimensions d_o and d_1 by empirical relationships using Eqs (4.26j) and (4.26o) respectively.

$$d_o = 2d \qquad d_1 = 1.5d$$

(vi) Check the tensile, crushing and shear stresses in the eye by Eqs (4.26h), (4.26e) and (4.26i) respectively.

$$\sigma_t = \frac{P}{b\,(d_0 - d)}$$

$$\sigma_c = \frac{P}{bd}$$

$$\tau = \frac{P}{b\,(d_0 - d)}$$

(vii) Check the tensile, crushing and shear stresses in the fork by Eqs (4.26k), (4.26f) and (4.26l) respectively.

$$\sigma_t = \frac{P}{2a(d_0 - d)}$$

$$\sigma_c = \frac{P}{2ad}$$

$$\tau = \frac{P}{2a(d_0 - d)}$$

The application of the above mentioned procedure is illustrated in the next example.

Example 4.5 *It is required to design a knuckle joint to connect two circular rods subjected to an axial tensile force of 50 kN. The rods are co-axial and a small amount of angular movement between their axes is permissible. Design the joint and specify the dimensions of its components. Select suitable materials for the parts.*

Solution

Given $P = (50 \times 10^3)$ N

Part I *Selection of material*

The rods are subjected to tensile force. Therefore, yield strength is the criterion for the selection of material for the rods. The pin is subjected to shear stress and bending stresses. Therefore, strength is also the criterion of material selection for the pin. On strength basis, the material for two rods and pin is selected as plain carbon steel of Grade 30C8 (S_{yt} = 400 N/mm²). It is further assumed that the yield strength in compression is equal to yield strength in tension. In practice, the compressive strength of steel is much higher than its tensile strength.

Part II *Selection of factor of safety*

In stress analysis of knuckle joint, the effect of stress concentration is neglected. To account for this effect, a higher factor of safety of 5 is assumed in the present design.

Part III *Calculation of permissible stresses*

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{400}{5} = 80 \text{ N/mm}^2$$

$$\sigma_c = \frac{S_{yc}}{(fs)} = \frac{S_{yt}}{(fs)} = \frac{400}{5} = 80 \text{ N/mm}^2$$

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.5\, S_{yt}}{(fs)} = \frac{0.5(400)}{5} = 40 \text{ N/mm}^2$$

Part IV *Calculation of dimensions*

The dimensions of the knuckle joint are calculated by the procedure outlined in Section 4.10.

Step I *Diameter of rods*

$$D = \sqrt{\frac{4P}{\pi \sigma_t}} = \sqrt{\frac{4(50 \times 10^3)}{\pi\,(80)}} = 28.21 \text{ or } 30 \text{ mm}$$

Step II *Enlarged diameter of rods (D_1)*

$$D_1 = 1.1\, D = 1.1(30) = 33 \text{ or } 35 \text{ mm}$$

Step III *Dimensions a and b*

$$a = 0.75\, D = 0.75(30) = 22.5 \text{ or } 25 \text{ mm}$$
$$b = 1.25\, D = 1.25(30) = 37.5 \text{ or } 40 \text{ mm}$$

Step IV *Diameter of pin*

$$d = \sqrt{\frac{2P}{\pi \tau}} = \sqrt{\frac{2\,(50 \times 10^3)}{\pi\,(40)}} = 28.21 \text{ or } 30 \text{ mm}$$

Also,

$$d = \sqrt[3]{\frac{32}{\pi \sigma_b} \times \frac{P}{2}\left[\frac{b}{4} + \frac{a}{3}\right]}$$

$$= \sqrt[3]{\frac{32}{\pi\,(80)} \times \frac{(50 \times 10^3)}{2}\left[\frac{40}{4} + \frac{25}{3}\right]}$$

$$= 38.79 \text{ or } 40 \text{ mm}$$

$\therefore \quad d = 40$ mm

Step V *Dimensions d_0 and d_1*

$$d_0 = 2d = 2(40) = 80 \text{ mm}$$
$$d_1 = 1.5d = 1.5(40) = 60 \text{ mm}$$

Step VI *Check for stresses in eye*

$$\sigma_t = \frac{P}{b\,(d_0 - d)} = \frac{(50 \times 10^3)}{40\,(80 - 40)} = 31.25 \text{ N/mm}^2$$

$\therefore \quad \sigma_t < 80$ N/mm²

$$\sigma_c = \frac{P}{b\, d} = \frac{(50 \times 10^3)}{40\,(40)} = 31.25 \text{ N/mm}^2$$

$\therefore \quad \sigma_c < 80$ N/mm²

$$\tau = \frac{P}{b\,(d_0 - d)} = \frac{(50 \times 10^3)}{40(80 - 40)} = 31.25 \text{ N/mm}^2$$

$\therefore \quad \tau < 40$ N/mm²

Step VII *Check for stresses in fork*

$$\sigma_t = \frac{P}{2a\,(d_0 - d)} = \frac{(50 \times 10^3)}{2(25)(80 - 40)} = 25 \text{ N/mm}^2$$

$\therefore \quad \sigma_t < 80$ N/mm²

$$\sigma_c = \frac{P}{2ad} = \frac{(50 \times 10^3)}{2(25)\,(40)} = 25 \text{N/mm}^2$$

$\therefore \quad \sigma_c < 80$ N/mm²

$$\tau = \frac{P}{2a\,(d_0 - d)}$$

$$= \frac{(50 \times 10^3)}{2(25)\,(80 - 40)} = 25 \text{N/mm}^2$$

$\therefore \quad \tau < 40$ N/mm²

It is observed that stresses are within limits.

Part V *Dimensioned sketch of knuckle joint*
Main dimensions of the knuckle joint are shown in Fig. 4.24.

Example 4.6 *A wall-rack, used to store round steel bars, consists of two I-section cantilever beams fixed in the wall. The bars are stacked in a triangular fashion as shown in Fig. 4.25(a). The total weight of the bars is 75 kN. The permissible bending stress for the cantilevers is 165 N/mm². Select a standard rolled I-section beam from the following table:*

Designation	*b (mm)*	*h (mm)*	I_{xx} *(mm⁴)*
ISLB 150	80	150	688.2×10^4
ISLB 175	90	175	1096.2×10^4
ISLB 200	100	200	1696.6×10^4
ISLB 225	100	225	2501.9×10^4
ISLB 250	125	250	3717.8×10^4

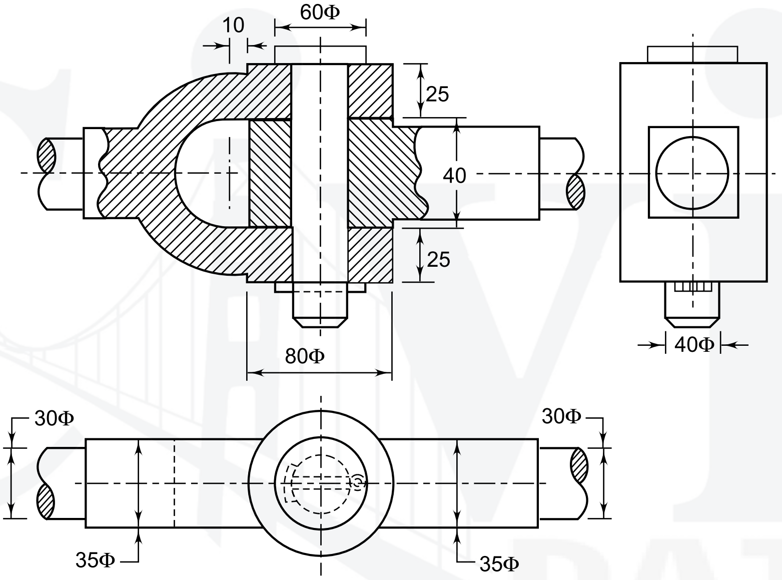


Fig 4.24 *Dimensions of Knuckle Joint*

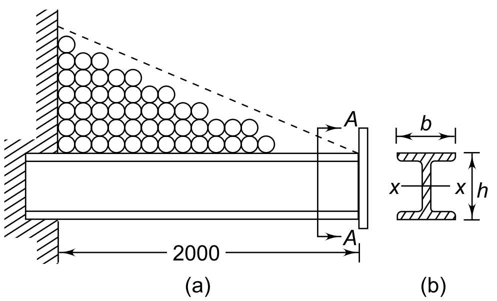


Fig 4.25

Solution

Given $W = 75$ kN $\quad \sigma_b = 165$ N/mm²

Step I *Calculation of bending moment*
There are two cantilever beams and the load supported by each beam is (75/2) or 37.5 kN. For a triangular load distribution, the centre of gravity of the resultant load is at a distance of (2000/3) mm from the wall. Therefore,

$$M_b = (37.5 \times 10^3)\left(\frac{2000}{3}\right) = 25 \times 10^6 \text{ N-mm}$$

Step II *Calculation of* (I_{xx}/y)
From Eq. (4.12),

$$\frac{I_{xx}}{y} = \frac{M_b}{\sigma_b} = \frac{25 \times 10^6}{165} = 151.51 \times 10^3 \text{ mm}^3$$

Step III *Selection of beam*
The cross-section of the beam is shown in Fig. 4.25 (b), ($y = h/2$)

Trial I Suppose beam (ISLB 175) is suitable for the application. For this beam,

$$\frac{I_{xx}}{y} = \frac{1096.2 \times 10^4}{(175/2)} = 125.28 \times 10^3 \text{ mm}^3$$

Since the required (I_{xx}/y) is (151.51×10^3) mm^2, beam (ISLB 175) is not suitable.

Trial II Suppose beam (ISLB 200) is suitable for the application. For this beam,

$$\frac{I_{xx}}{y} = \frac{1696.6 \times 10^4}{(200/2)} = 169.66 \times 10^3 \text{ mm}^3$$

$$(I_{xx}/y) > (151.51 \times 10^3) \text{ mm}^3$$

Therefore, the cantilever beams of standard cross-section ISLB 200 are suitable for this application.

Example 4.7 *The frame of a hacksaw is shown in Fig. 4.26(a). The initial tension P in the blade should be 300 N. The frame is made of plain carbon steel 30C8 with a tensile yield strength of 400 N/mm^2 and the factor of safety is 2.5. The cross-section of the frame is rectangular with a ratio of depth to width as 3, as shown in Fig. 4.26(b). Determine the dimensions of the cross-section.*

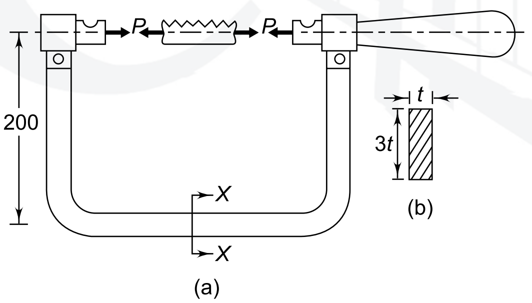


Fig. 4.26 *(a) Frame of Hacksaw (b) Section at XX*

Solution

Given $P = 300$ N $S_{yt} = 400$ N/mm^2 $(fs) = 2.5$ (depth/width) = 3

Step I *Calculation of permissible tensile stress*

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{400}{2.5} = 160 \text{ N/mm}^2$$

Step II *Calculation of direct and bending stresses*

The stresses at section *XX* consist of direct compressive stress and bending stresses. The tensile stress is maximum at the lower fibre. At the lower fibre,

$$\sigma_c = \frac{P}{A} = \frac{300}{(t)(3t)} = \frac{100}{t^2} \text{ N/mm}^2 \qquad \text{(i)}$$

$$\sigma_b = \frac{M_b\, y}{I}$$

$$= \frac{(300 \times 200)\ (1.5\, t)}{\left[\frac{1}{12}(t)(3t)^3\right]} = \frac{40000}{t^3} \text{ N/mm}^2 \qquad \text{(ii)}$$

Step III *Calculation of dimensions of cross-section*

Superimposing the two stresses and equating it to permissible stress,

$$\frac{40000}{t^3} - \frac{100}{t^2} = 160$$

or $160\, t^3 + 100\, t - 40000 = 0$

Solving the above equation by trail and error method,

$$t \cong 6.3 \text{ mm}$$

Example 4.8 *An offset link subjected to a force of 25 kN is shown in Fig. 4.27. It is made of grey cast iron FG300 and the factor of safety is 3. Determine the dimensions of the cross-section of the link.*

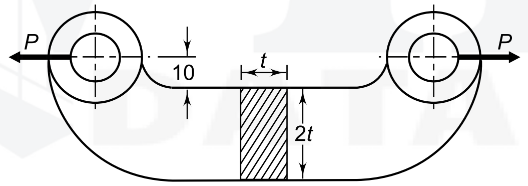


Fig. 4.27 *Offset Link*

Solution

Given $P = 25$ kN $S_{ut} = 300$ N/mm^2 $(fs) = 3$

Step I *Calculation of permissible tensile stress for the link*

$$\sigma_t = \frac{S_{ut}}{(fs)} = \frac{300}{3} = 100 \text{ N/mm}^2 \qquad \text{(a)}$$

Step II *Calculation of direct tensile and bending stresses*

The cross-section is subjected to direct tensile stress and bending stresses. The stresses are maximum at the top fibre. At the top fibre,

$$\sigma_t = \frac{P}{A} + \frac{M_b\, y}{I}$$

$$= \frac{25\times10^3}{t(2t)} + \frac{25\times10^3(10+t)(t)}{\left[\frac{1}{12}t(2t)^3\right]} \qquad \text{(b)}$$

Step III *Calculation of dimensions of cross-section*

Equating (a) and (b),

$$\frac{12500}{t^2} + \frac{37500\,(10+t)}{t^3} = 100$$

or, $t^3 - 500\,t - 3750 = 0$

Solving the above equation by trial and error method,

$$t \cong 25.5 \text{ mm}$$

Example 4.9 *The frame of a hydraulic press consisting of two identical steel plates is shown in Fig. 4.28. The maximum force P acting on the frame is 20 kN. The plates are made of steel 45C8 with tensile yield strength of 380 N/mm². The factor of safety is 2.5. Determine the plate thickness.*

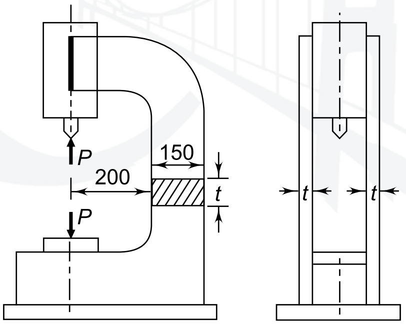


Fig. 4.28 *Frame of Hydraulic Press*

Solution

Given $P = 20$ kN $S_{yt} = 380$ N/mm²
$(fs) = 2.5$

Step I *Calculation of permissible tensile stress for the plates*

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{380}{2.5} = 152 \text{ N/mm}^2 \qquad \text{(i)}$$

Step II *Calculation of direct tensile and bending stresses in plates*

Since the plates are identical, the force acting on each plate is (20/2) or 10 kN. The plates are subjected to direct tensile stress and bending stresses. The stresses are maximum at the inner fibre.

At the inner fibre,

$$\sigma_t = \frac{P}{A} + \frac{M_b y}{I}$$

$$= \frac{10000}{(150\,t)} + \frac{10000\ (200+75)\,(75)}{\left[\frac{1}{12}t\,(150)^3\right]}$$

or $$\sigma_t = \frac{800}{t} \qquad \text{(ii)}$$

Step III *Calculation of plate thickness*

From (i) and (ii),

$$\frac{800}{t} = 152 \qquad \text{or} \qquad t = 5.26 \text{ mm}$$

Example 4.10 *A hollow circular column carries a projecting bracket, which supports a load of 25 kN as shown in Fig. 4.29. The distance between the axis of the column and the load is 500 mm. The inner diameter of the column is 0.8 times of the outer diameter. The column is made of steel FeE 200 (S_{yt} = 200 N/mm²) and the factor of safety is 4. The column is to be designed on the basis of maximum tensile stress and compression is not the criterion of failure. Determine the dimensions of the cross-section of the column.*

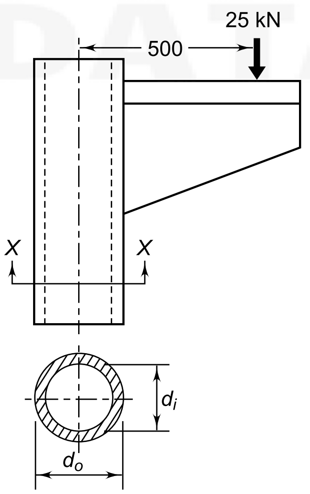


Fig. 4.29

Solution

Given $P = 25$ kN $e = 500$ mm

$S_{yt} = 200$ N/mm^2 $(fs) = 4$ $d_i = 0.8\ d_o$

Step I *Calculation of permissible tensile stress*

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{200}{4} = 50\ \text{N/mm}^2$$

Step II *Calculation of direct compressive and bending stresses*

$$d_i = 0.8\ d_0$$

The direct compressive stress is given by,

$$\frac{P}{A} = \frac{25 \times 10^3}{\frac{\pi}{4}(d_o^2 - d_i^2)}$$

$$= \frac{25 \times 10^3}{\frac{\pi}{4}[d_0^2 - (0.8\ d_0)^2]} = \left(\frac{88419.41}{d_0^2}\right) \text{N/mm}^2 \qquad \text{(i)}$$

The bending stresses are tensile on the left side and compressive on the right side of the cross-section. The tensile stress due to bending moment is given by,

$$\frac{Pey}{I} = \frac{25 \times 10^3 (500)(0.5 d_0)}{\frac{\pi}{64}\left[d_0^4 - (0.8 d_0)^4\right]}$$

$$= \left(\frac{215\,657\,104.5}{d_0^3}\right) \text{N/mm}^2 \qquad \text{(ii)}$$

Step III *Calculation of outer and inner diameters*

The resultant tensile stress is obtained by subtracting (i) from (ii). Equating the resultant stress to permissible tensile stress,

$$\left(\frac{215\,657\,104{\cdot}5}{d_0^3}\right) - \left(\frac{88\,419.41}{d_0^2}\right) = 50$$

or, $$(d_0^3 + 1768.39\ d_0) = 4\,313\,142{\cdot}09$$

The above expression indicates cubic equation and it is solved by trial and error method. The trial values of d_o and corresponding values of left hand side are tabulated as follows:

d_0	$(d_0^3 + 1768.39\ d_0)$
150	3 640 258.5
160	4 378 942.4
158	4 223 717.6
159	4 300 853.0
160	4 378 942.4

From the above table, it is observed that the value of d_o is between 159 and 160 mm

$\therefore$ $d_0 = 160$ mm

$d_i = 0.8\ d_o = 0.8\ (160) = 128$ mm

4.13 PRINCIPAL STRESSES

In previous articles and examples, mechanical components, which are subjected to only one type of load, are considered. There are many components, which are subjected to several types of load simultaneously. A transmission shaft is subjected to bending as well as torsional moment at the same time. In design, it is necessary to determine the state of stresses under these conditions. An element of a plate subjected to two-dimensional stresses is shown is Fig. 4.30(a). In this analysis, the stresses are classified into two groups—normal stresses and shear stresses. The *normal stress* is perpendicular to the area under consideration, while the *shear stress* acts over the area.

There is a particular system of notation for these stresses. The normal stresses are denoted by σ_x, σ_y and σ_z in the X, Y and Z directions respectively. Tensile stresses are considered to be positive, while compressive stresses as negative. The shear stresses are denoted by two subscripts, viz. τ_{xy} or τ_{yz}, as shown in Fig. 4.30(a). The first subscript denotes the area over which it acts and the second indicates the direction of shear force. As an example, consider the shear stress denoted by τ_{xy}. The subscript x indicates that the shear stress is acting on the area, which is perpendicular to the X-axis. The subscript y indicates that the shear stress is acting in the Y-direction. The shear stresses are positive if they act in the positive direction of the reference axis. It can be proved that,

$$\tau_{xy} = \tau_{yx} \qquad (4.27)$$

Figure 4.30(b) shows the stresses acting on an oblique plane. The normal to the plane makes an angle θ with the *X*-axis. σ and τ are normal

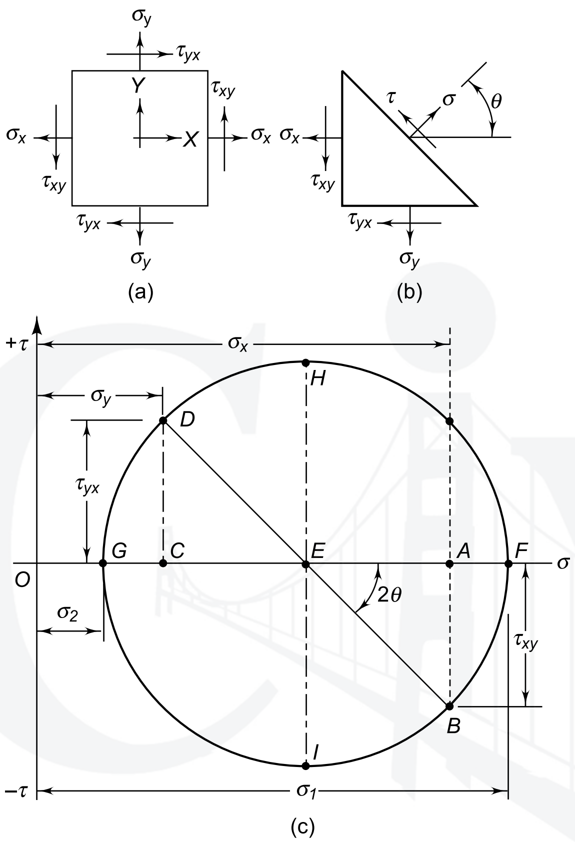


Fig. 4.30 *(a) Two-Dimensional State of Stress (b) Stresses on Oblique Plane (c) Mohr's Circle Diagram*

and shear stresses associated with this plane. Considering equilibrium of forces, it can be proved that,[2, 3]

$$\sigma = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta \tag{4.28}$$

and

$$\tau = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta \tag{4.29}$$

Differentiating Eq. (4.28) with respect to θ and setting the result to zero, we have

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \tag{4.30}$$

Equation (4.30) defines two values of (2θ), one giving the maximum value of normal stress and other the minimum value. If σ_1 and σ_2 are the maximum and minimum values of normal stress, then substituting Eq. (4.30) in Eq. (4.28), we get

$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} \tag{4.31}$$

$$\sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} \tag{4.32}$$

σ_1 and σ_2 are called the principal stresses.

Similarly, Eq. (4.29) is differentiated with respect to θ and the result is equated to zero. This gives the following condition:

$$\tan 2\theta = -\left(\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right) \tag{4.33}$$

Substituting Eq. (4.33) in Eq. (4.29),

$$\tau_{max.} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} \tag{4.34}$$

$\tau_{max.}$ is called the principal shear stress.

One of the most effective methods to determine the principal stresses and the principal shear stress is the construction of *Mohr's circle diagram* as shown in Fig. 4.30 (c). It is a graphical method for the representation of stresses. The following conventions are used to construct the *Mohr's circle*:

(i) The normal stresses σ_x, σ_y, and the principal stresses σ_1, σ_2 are plotted on the abscissa. The tensile stress, considered as positive, is plotted to the right of the origin and the compressive stress, considered as negative, to its left.

[2] JE Shigley —*Applied Mechanics of Materials*—McGraw-Hill Kogakusha Ltd.
[3] FR Shanley —*Strength of Materials*—McGraw-Hill Book Company Inc.

(ii) The shear stresses τ_{xy}, τ_{yx} and the principal shear stress τ_{max} are plotted on the ordinate. A pair of shear stresses is considered as positive if they tend to rotate the element clockwise, and negative if they tend to rotate it anticlockwise.

The Mohr's circle in Fig. 4.30 (c) is constructed by the following method:

(i) Plot the following points:

$$\overline{OA} = \sigma_x \qquad \overline{OC} = \sigma_y$$

$$\overline{AB} = \tau_{xy} \qquad \overline{CD} = \tau_{yx}$$

(ii) Join $\overline{DB}$. The point of intersection of $\overline{DB}$ and $\overline{OA}$ is E.

(iii) Construct Mohr's circle with E as centre and $\overline{DB}$ as the diameter.

It can be proved that points F and G represent the maximum and minimum principal stresses σ_1 and σ_2 respectively. The two principal shear stresses $\pm\tau_{max}$ are denoted by points H and I respectively.

4.14 THEORIES OF ELASTIC FAILURE

There are number of machine components, which are subjected to several types of loads simultaneously. For example, a power screw is subjected to torsional moment as well as axial force. Similarly, an overhang crank is subjected to combined bending and torsional moments. The bolts of the bracket are subjected to forces that cause tensile stress and shear stress. Crankshaft, propeller shaft and connecting rod are examples of components subjected to complex loads. When the component is subjected to several types of loads, combined stresses are induced. For example, torsional moment induces torsional shear stress, while bending moment causes bending stresses in the transmission shaft. The failures of such components are broadly classified into two groups—elastic failure and yielding and fracture. Elastic failure results in excessive elastic deformation, which makes the machine component unfit to perform its function satisfactorily. Yielding results in excessive plastic deformation after the yield point stress is reached, while fracture results in breaking the component into two or more pieces. *Theories of failure discussed in this article are applicable to elastic failure of machine parts.*

The design of machine parts subjected to combined loads should be related to experimentally determined properties of material under 'similar' conditions. However, it is not possible to conduct such tests for different possible combinations of loads and obtain mechanical properties. In practice, the mechanical properties are obtained from a simple tension test. They include yield strength, ultimate tensile strength and percentage elongation. In the tension test, the specimen is axially loaded in tension. It is not subjected to either bending moment or torsional moment or a combination of loads. Theories of elastic failure provide a relationship between the strength of machine component subjected to complex state of stresses with the mechanical properties obtained in tension test. With the help of these theories, the data obtained in the tension test can be used to determine the dimensions of the component, irrespective of the nature of stresses induced in the component due to complex loads.

Several theories have been proposed, each assuming a different hypothesis of failure. The principal theories of elastic failure are as follows:

(i) Maximum principal stress theory (Rankine's theory)
(ii) Maximum shear stress theory (Coulomb, Tresca and Guest's theory)
(iii) Distortion energy theory (Huber von Mises and Hencky's theory)
(iv) Maximum strain theory (St. Venant's theory)
(v) Maximum total strain energy theory (Haigh's theory)

We will discuss the first three theories in this chapter. Let us assume σ_1, σ_2 and σ_3 as the principal stresses induced at a point on the machine part as a result of several types of loads. We will apply the theories of failure to obtain the relationship between σ_1, σ_2 and σ_3 on one hand and the properties of material such as S_{yt} or S_{ut} on the other.

4.15 MAXIMUM PRINCIPAL STRESS THEORY

This criterion of failure is accredited to the British engineer WJM Rankine (1850). *The theory states that the failure of the mechanical component subjected to bi-axial or tri-axial stresses occurs when the maximum principal stress reaches the yield or ultimate strength of the material.*

If σ_1, σ_2 and σ_3 are the three principal stresses at a point on the component and

$$\sigma_1 > \sigma_2 > \sigma_3$$

then according to this theory, the failure occurs whenever

$$\sigma_1 = S_{yt} \quad \text{or} \quad \sigma_1 = S_{ut} \tag{4.35}$$

whichever is applicable.

The theory considers only the maximum of principal stresses and disregards the influence of the other principal stresses. The dimensions of the component are determined by using a factor of safety.

For tensile stresses,

$$\sigma_1 = \frac{S_{yt}}{(fs)} \quad \text{or} \quad \sigma_1 = \frac{S_{ut}}{(fs)} \tag{4.36}$$

For compressive stresses,

$$\sigma_1 = \frac{S_{yc}}{(fs)} \quad \text{or} \quad \sigma_1 = \frac{S_{uc}}{(fs)} \tag{4.37}$$

Region of Safety

The construction of region of safety for bi-axial stresses is illustrated in Fig. 4.31. The two principal stresses σ_1 and σ_2 are plotted on X and Y axes respectively. Tensile stresses are considered as positive,

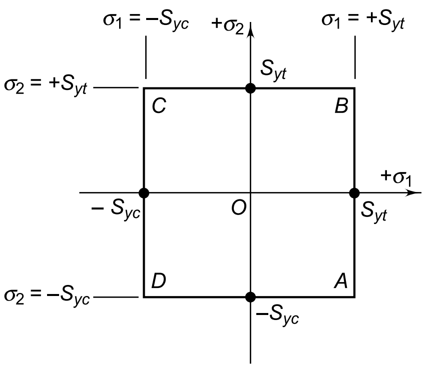


Fig. 4.31 *Boundary for Maximum Principal Stress Theory under Bi-axial Stresses*

while compressive stresses as negative. It is further assumed that

$$S_{yc} = S_{yt}$$

It should be noted that,

(i) The equation of vertical line to the positive side of X-axis is ($x = + a$)

(ii) The equation of vertical line to the negative side of X-axis is ($x = - a$)

(iii) The equation of horizontal line to the positive side of Y-axis is ($y = + b$)

(iv) The equation of horizontal line to the negative side of Y-axis is ($y = - b$)

The borderline for the region of safety for this theory can be constructed in the following way:

Step 1: Suppose $\sigma_1 > \sigma_2$ As per this theory, we will consider only the maximum of principal stresses (σ_1) and disregard the other principal stress (σ_2).

Suppose (σ_1) is the tensile stress. The limiting value of (σ_1) is yield stress (S_{yt}). Therefore, the boundary line will be,

$$\sigma_1 = + S_{yt}$$

A vertical line $\overline{AB}$ is constructed such that $\sigma_1 = + S_{yt}$.

Step 2: Suppose $\sigma_1 > \sigma_2$ and (σ_1) is compressive stress. The limiting value of (σ_1) is compressive yield stress ($- S_{yc}$). Therefore, the boundary line will be,

$$\sigma_1 = - S_{yc}$$

A vertical line $\overline{DC}$ is constructed such that $\sigma_1 = - S_{yc}$.

Step 3: Suppose $\sigma_2 > \sigma_1$ As per this theory, we will consider only the maximum of principal stresses (σ_2) and disregard the other principal stress (σ_1).

Suppose (σ_2) is the tensile stress. The limiting value of (σ_2) is the yield stress (S_{yt}). Therefore, the boundary line will be,

$$\sigma_2 = + S_{yt}$$

A horizontal line $\overline{CB}$ is constructed such that $\sigma_2 = + S_{yt}$

Step 4: Suppose $\sigma_2 > \sigma_1$ and (σ_2) is the compressive stress. The limiting value of (σ_2) is compressive yield stress ($-S_{yc}$). Therefore, the boundary line will be,

$$\sigma_2 = -S_{yc}$$

A horizontal line $\overline{DA}$ is constructed such that $\sigma_2 = -S_{yc}$.

The complete region of safety is the area *ABCD*. Since we have assumed ($S_{yc} = S_{yt}$), *ABCD* is a square.

According to the maximum principal stress theory of failure, if a point with co-ordinates (σ_1, σ_2) falls outside this square then it indicates the failure condition. On the other hand, if the point falls inside the square, the design is safe and the failure may not occur.

Experimental investigations suggest that the maximum principal stress theory gives good predictions for brittle materials. However, it is not recommended for ductile materials.

4.16 MAXIMUM SHEAR STRESS THEORY

This criterion of failure is accredited to CA Coulomb, H Tresca and JJ Guest. *The theory states that the failure of a mechanical component subjected to bi-axial or tri-axial stresses occurs when the maximum shear stress at any point in the component becomes equal to the maximum shear stress in the standard specimen of the tension test, when yielding starts*. In the tension test, the specimen is subjected to uni-axial stress (σ_1) and ($\sigma_2 = 0$). The stress in the specimen of tension test and the corresponding Mohr's circle diagram are shown in Fig. 4.32. From the figure,

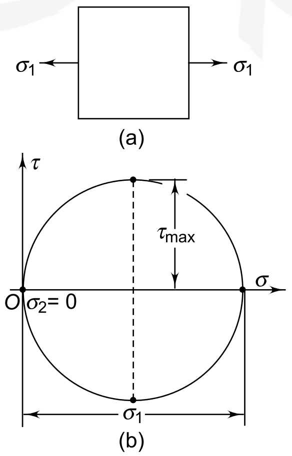


Fig. 4.32 *(a) Stresses in Simple Tension Test (b) Mohr's Circle for Stresses*

$$\tau_{max} = \frac{\sigma_1}{2}$$

When the specimen starts yielding ($\sigma_1 = S_{yt}$), the above equation is written as

$$\tau_{max} = \frac{S_{yt}}{2}$$

Therefore, the maximum shear stress theory predicts that the yield strength in shear is half of the yield strength in tension, i.e.,

$$S_{sy} = 0.5\, S_{yt} \tag{4.38}$$

Suppose σ_1, σ_2 and σ_3 are the three principal stresses at a point on the component, the shear stresses on three different planes are given by,

$$\tau_{12} = \left(\frac{\sigma_1 - \sigma_2}{2}\right) \quad \tau_{23} = \left(\frac{\sigma_2 - \sigma_3}{2}\right)$$

$$\tau_{31} = \left(\frac{\sigma_3 - \sigma_1}{2}\right) \tag{a}$$

The largest of these stresses is equated to (τ_{max}) or ($S_{yt}/2$).

Considering factor of safety,

$$\left(\frac{\sigma_1 - \sigma_2}{2}\right) = \frac{S_{yt}}{2\,(fs)}$$

or $$(\sigma_1 - \sigma_2) = \frac{S_{yt}}{(fs)}$$

$$(\sigma_2 - \sigma_3) = \frac{S_{yt}}{(fs)}$$

$$(\sigma_3 - \sigma_1) = \frac{S_{yt}}{(fs)} \tag{4.39}$$

The above relationships are used to determine the dimensions of the component. Refer to expression (a) again and equating the largest shear stress (τ_{max}) to ($S_{yt}/2$),

$$\left(\frac{\sigma_1 - \sigma_2}{2}\right) = \frac{S_{yt}}{2}$$

or $$\sigma_1 - \sigma_2 = S_{yt} \quad \text{(b)}$$

Similarly,

$$\sigma_2 - \sigma_3 = S_{yt} \quad \text{(c)}$$

$$\sigma_3 - \sigma_1 = S_{yt} \quad \text{(d)}$$

For compressive stresses,

$$\sigma_1 - \sigma_2 = -S_{yc} \quad \text{(e)}$$

$$\sigma_2 - \sigma_3 = -S_{yc} \quad \text{(f)}$$

$$\sigma_3 - \sigma_1 = -S_{yc} \quad \text{(g)}$$

The above equations can be written as,

$$\sigma_1 - \sigma_2 = \pm S_{yt} \qquad [\text{Assuming } S_{yc} = S_{yt}]$$

$$\sigma_2 - \sigma_3 = \pm S_{yc}$$

$$\sigma_3 - \sigma_1 = \pm S_{yt}$$

Region of Safety For bi-axial stresses,

$$\sigma_3 = 0$$

The above equations can be written as,

$$\sigma_1 - \sigma_2 = \pm S_{yt} \quad \text{(h)}$$

$$\sigma_2 = \pm S_{yt} \quad \text{(i)}$$

$$\sigma_1 = \pm S_{yt} \quad \text{(j)}$$

It will be observed at a later stage that Eq. (h) are applicable in second and fourth quadrants, while Eqs (i) and (j) are applicable in the first and third quadrants of the diagram.

The construction of region of safety is illustrated in Fig. 4.33. The two principal stresses σ_1 and σ_2 are plotted on the X and Y axes respectively. Tensile stresses are considered as positive, while compressive stresses as negative.

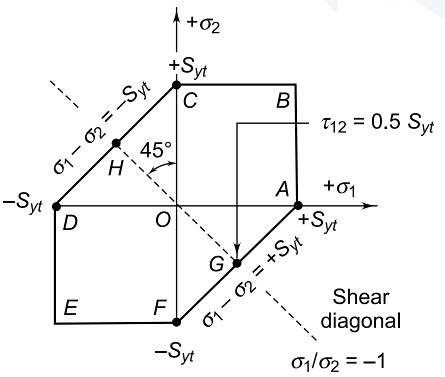


Fig. 4.33 *Boundary for Maximum Shear Stress Theory under Bi-axial Stresses*

It should be noted that,

(i) The equation $(x - y = -a)$ indicates a straight line in the second quadrant with $(-a)$ and $(+a)$ as intercepts on the X and Y axes respectively.

(ii) The equation $(x - y = +a)$ indicates a straight line in the fourth quadrant with $(+a)$ and $(-a)$ as intercepts on the X and Y axes respectively.

The borderline for the region of safety for this theory can be constructed in the following way:

Step 1: In the first quadrant, both (σ_1) and (σ_2) are positive or tensile stresses. The yielding will depend upon where (σ_1) or (σ_2) is greater in magnitude.

Suppose $\sigma_1 > \sigma_2$

The boundary line will be,

$$\sigma_1 = +S_{yt}$$

A vertical line $\overline{AB}$ is constructed such that

$$\sigma_1 = +S_{yt}.$$

Suppose $\sigma_2 > \sigma_1$

The boundary line will be,

$$\sigma_2 = +S_{yt}$$

A horizontal line $\overline{CB}$ is constructed such that $\sigma_2 = +S_{yt}$

Step 2: In the third quadrant, both (σ_1) and (σ_2) are negative or compressive stresses. The yielding will depend upon whether (σ_1) or (σ_2) is greater in magnitude.

Suppose $\sigma_1 > \sigma_2$

The boundary line will be,

$$\sigma_1 = -S_{yt}$$

A vertical line $\overline{DE}$ is constructed such that

$$\sigma_1 = -S_{yt}$$

Suppose $\sigma_2 > \sigma_1$

The boundary line will be,

$$\sigma_2 = -S_{yt}$$

A horizontal line $\overline{EF}$ is constructed such that

$$\sigma_2 = -S_{yt}$$

Step 3: In the second and fourth quadrants, (σ_1) and (σ_2) are of opposite sign. One stress is tensile while the other is compressive. The yielding will occur when,

$$\sigma_1 - \sigma_2 = \pm S_{yt}$$

In the second quadrant, line $\overline{DC}$ is constructed such that,

$$\sigma_1 - \sigma_2 = -S_{yt}$$

It is observed that the intercept of the above line on the X-axis $(\sigma_2 = 0)$ is $(-S_{yt})$ and intercept on the Y-axis $(\sigma_1 = 0)$ is $(+S_{yt})$.

Step 4: In the fourth quadrant, line $\overline{FA}$ is constructed such that,

$$\sigma_1 - \sigma_2 = + S_{yt}$$

It is observed that the intercept of the above line on the X-axis ($\sigma_2 = 0$) is ($+ S_{yt}$) and intercept on the Y-axis ($\sigma_1 = 0$) is ($- S_{yt}$).

The complete region of safety is the hexagon *ABCDEFA*.

In case of bi-axial stress, if a point with coordinates (σ_1, σ_2) falls outside this hexagon region, then it indicates the failure condition. On the other hand, if the point falls inside the hexagon, the design is safe and the failure may not occur.

Shear Diagonal *Shear diagonal or line of pure shear is the locus of all points, corresponding to pure shear stress*. It will be proved at a later stage (Fig. 4.35) that for pure shear stress,

$$\sigma_1 = - \sigma_2 = \tau_{12}$$

The above equation can be written as,

$$\frac{\sigma_1}{\sigma_2} = -1 = -\tan(45°)$$

A line $\overline{GH}$ is constructed in such a way that it passes through the origin O and makes an angle of $-45°$ with the Y-axis. This line is called shear diagonal or line of pure shear. This line intersects the hexagon at two points G and H. The point of intersection of lines $\overline{FA}$ ($\sigma_1 - \sigma_2 = + S_{yt}$) and $\overline{GH}$ $\left[\dfrac{\sigma_1}{\sigma_2} = -1\right]$ is G.

Solving two equations simultaneously,

$$\sigma_1 = - \sigma_2 = + S_{yt}/2$$

Since $\sigma_1 = - \sigma_2 = \tau_{12}$

$$\tau_{12} = \frac{1}{2} S_{yt}$$

Since the point G is on the borderline, this is the limiting value for shear stress.

or $$S_{sy} = \frac{1}{2} S_{yt}$$

The maximum shear stress theory of failure is widely used by designers for predicting the failure of components, which are made of ductile materials, like transmission shaft.

4.17 DISTORTION-ENERGY THEORY

This theory was advanced by MT Huber in Poland (1904) and independently by R von Mises in Germany (1913) and H Hencky (1925). It is known as the *Huber von Mises and Hencky's theory. The theory states that the failure of the mechanical component subjected to bi-axial or tri-axial stresses occurs when the strain energy of distortion per unit volume at any point in the component, becomes equal to the strain energy of distortion per unit volume in the standard specimen of tension-test, when yielding starts.*

A unit cube subjected to the three principal stresses σ_1, σ_2 and σ_3 is shown in Fig. 4.34(a). The total strain energy U of the cube is given by,

$$U = \frac{1}{2}\sigma_1\varepsilon_1 + \frac{1}{2}\sigma_2\varepsilon_2 + \frac{1}{2}\sigma_3\varepsilon_3 \qquad \text{(a)}$$

where ε_1, ε_2 and ε_3 are strains in respective directions.

Also,
$$\varepsilon_1 = \frac{1}{E}[\sigma_1 - \mu(\sigma_2 + \sigma_3)]$$
$$\varepsilon_2 = \frac{1}{E}[\sigma_2 - \mu(\sigma_1 + \sigma_3)]$$
$$\varepsilon_3 = \frac{1}{E}[\sigma_3 - \mu(\sigma_1 + \sigma_2)] \qquad \text{(b)}$$

Substituting the above expressions in Eq. (a),

$$U = \frac{1}{2E}[(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)] \qquad \text{(c)}$$

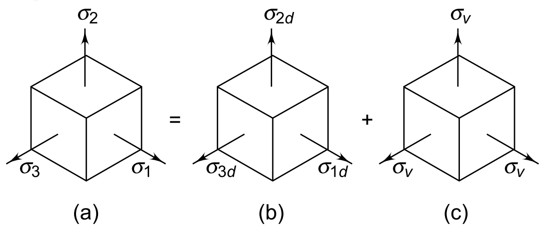


Fig. 4.34 *(a) Element with Tri-axial Stresses (b) Stress Components due to Distortion of Element (c) Stress Components due to Change of Volume*

The total strain energy U is resolved into two components—first U_v corresponding to the change of volume with no distortion of the element and the second U_d corresponding to the distortion of the element with no change of volume. Therefore,

$$U = U_v + U_d \tag{d}$$

The corresponding stresses are also resolved into two components as shown in Fig. 4.34 (b) and (c). From the figure,

$$\sigma_1 = \sigma_{1d} + \sigma_v$$
$$\sigma_2 = \sigma_{2d} = \sigma_v$$
$$\sigma_3 = \sigma_{3d} = \sigma_v \tag{e}$$

The components σ_{1d}, σ_{2d} and σ_{3d} cause distortion of the cube, while the component σ_v results in volumetric change. Since the components σ_{1d}, σ_{2d} and σ_{3d} do not change the volume of the cube,

$$\varepsilon_{1d} + \varepsilon_{2d} + \varepsilon_{3d} = 0 \tag{f}$$

Also,

$$\varepsilon_{1d} = \frac{1}{E}[\sigma_{1d} - \mu(\sigma_{2d} + \sigma_{3d})]$$

$$\varepsilon_{2d} = \frac{1}{E}[\sigma_{2d} - \mu(\sigma_{1d} + \sigma_{3d})]$$

$$\varepsilon_{3d} = \frac{1}{E}[\sigma_{3d} - \mu(\sigma_{1d} + \sigma_{2d})] \tag{g}$$

Substituting Eq. (g) in Eq. (f),

$$(1 - 2\mu)(\sigma_{1d} + \sigma_{2d} + \sigma_{3d}) = 0$$

Since $(1 - 2\mu) \neq 0$

$$\therefore \quad (\sigma_{1d} + \sigma_{2d} + \sigma_{3d}) = 0 \tag{h}$$

From Eq. (h) in Eq. (e),

$$\sigma_v = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) \tag{j}$$

The strain energy U_v corresponding to the change of volume for the cube is given by,

$$U_v = 3\left[\frac{\sigma_v \varepsilon_v}{2}\right] \tag{k}$$

Also $$\varepsilon_v = \frac{1}{E}[\sigma_v - \mu(\sigma_v + \sigma_v)]$$

or $$\varepsilon_v = \frac{(1 - 2\mu)\sigma_v}{E} \tag{l}$$

From expressions (k) and (l),

$$U_v = \frac{3(1 - 2\mu)\sigma_v^2}{2E} \tag{m}$$

Substituting expression (j) in the Eq. (m),

$$U_v = \frac{(1 - 2\mu)(\sigma_1 + \sigma_2 + \sigma_3)^2}{6E} \tag{n}$$

From expressions (c) and (n),

$$U_d = U - U_v$$

or $$U_d = \left(\frac{1 + \mu}{6E}\right)[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] \tag{4.40}$$

In simple tension test, when the specimen starts yielding,

$$\sigma_1 = S_{yt} \quad \text{and} \quad \sigma_2 = \sigma_3 = 0$$

Therefore, $$U_d = \left(\frac{1 + \mu}{3E}\right)S_{yt}^2 \tag{4.41}$$

From Eqs (4.40) and (4.41), the criterion of failure for the distortion energy theory is expressed as

$$2S_{yt}^2 = \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]$$

or $$S_{yt} = \sqrt{\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]} \tag{4.42}$$

Considering the factor of safety,

$$\frac{S_{yt}}{(fs)} = \sqrt{\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]} \tag{4.43}$$

For bi-axial stresses ($\sigma_3 = 0$),

$$\frac{S_{yt}}{(fs)} = \sqrt{(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)} \tag{4.44}$$

A component subjected to pure shear stresses and the corresponding Mohr's circle diagram is shown in Fig. 4.35.

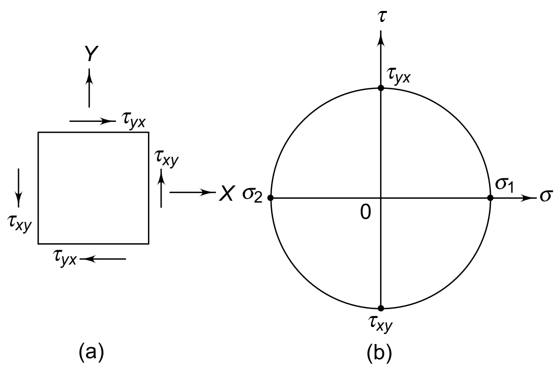


Fig. 4.35 *(a) Element subjected to Pure Shear Stresses (b) Mohr's Circle for Shear Stresses*

From the figure,

$$\sigma_1 = -\sigma_2 = \tau_{xy} \quad \text{and} \quad \sigma_3 = 0$$

Substituting these values in Eq. (4.42),

$$S_{yt} = \sqrt{3}\,\tau_{xy}$$

Replacing (τ_{xy}) by S_{sy},

$$S_{sy} = \frac{S_{yt}}{\sqrt{3}} = 0.577\, S_{yt} \qquad (4.45)$$

Therefore, according to the distortion-energy theory, the yield strength in shear is 0.577 times the yield strength in tension.

Region of Safety The construction of the region of safety is illustrated in Fig. 4.36. The two principal stresses σ_1 and σ_2 are plotted on the X and Y axes respectively. Tensile stresses are considered as positive, while compressive stresses as negative.

It should be noted that,

$$x^2 - xy + y^2 = a^2$$

is an equation of an ellipse whose semi-major axis is $(\sqrt{2}a)$ and semi-minor axis is $(\sqrt{2/3}\,a)$.

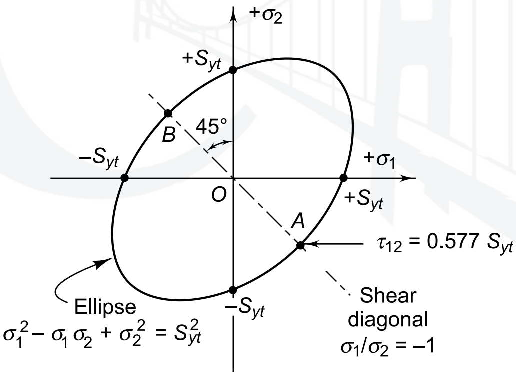


Fig. 4.36 *Boundary for Distortion Energy Theory under Bi-axial Stresses*

For bi-axial stresses,

$$\sigma_3 = 0$$

Substituting this value in Eq. (4.42),

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = S_{yt}^2$$

The above equation indicates an ellipse whose semi-major axis is $(\sqrt{2}\,S_{yt})$ and semi-minor axis is $\left[\sqrt{\frac{2}{3}}S_{yt}\right]$.

If a point with coordinates (σ_1, σ_2) falls outside this ellipse, then it indicates the failure condition. On the other hand, if the point falls inside the ellipse, the design is safe and the failure may not occur.

Shear Diagonal As mentioned in the previous section, shear diagonal or line of pure shear is the locus of all points, corresponding to pure shear stress. The condition for the line of shear is,

$$\sigma_1 = -\sigma_2 = \tau_{12}$$

The above equation can be written as

$$\frac{\sigma_1}{\sigma_2} = -1 = -\tan(45°)$$

A line $\overline{AB}$ is constructed in such a way that it passes through the origin O and makes an angle of – 45° with the Y-axis. This line is called shear diagonal or line of pure shear. This line intersects the ellipse at two points A and B.

A is the point of intersection of the ellipse and the line AB. The coordinates of the point A are obtained by solving the following two equations simultaneously,

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = S_{yt}^2$$

$$\sigma_1/\sigma_2 = -1$$

Solving two equations simultaneously,

$$\sigma_1 = -\sigma_2 = +\frac{1}{\sqrt{3}}S_{yt}$$

Since $\qquad \sigma_1 = -\sigma_2 = \tau_{12}$

$$\tau_{12} = \frac{1}{\sqrt{3}}S_{yt} = 0.577 S_{yt}$$

Since the point A is on the borderline, this is the limiting value for shear stress.

or $\qquad S_{sy} = 0.577\ S_{yt}$

Experiments have shown that the distortion-energy theory is in better agreement for predicting the failure of a ductile component than any other theory of failure.

4.18 SELECTION AND USE OF FAILURE THEORIES

The plots of three theories of failure on the σ_1, σ_2 coordinate system are shown in Fig. 4.37. While selecting theories of failure, the following points should be noted.

(i) Ductile materials typically have the same tensile strength and compressive strength. Also, yielding is the criterion of failure in ductile materials. In maximum shear stress theory and distortion energy theory, it is assumed that the yield strength in tension (S_{yt}) is equal to the yield strength in compression (S_{yc}). Also, the criterion of failure is yielding. Therefore, maximum shear stress theory and distortion energy theory are used for ductile materials.

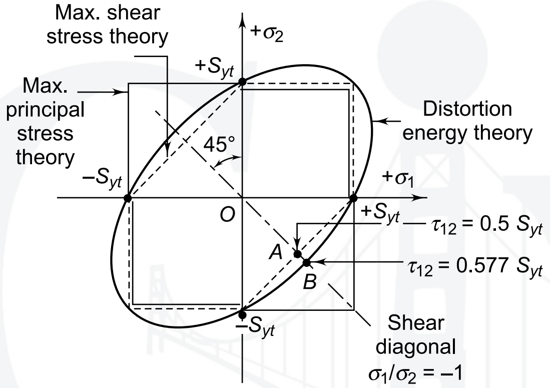


Fig. 4.37 *Comparison of Theories of Failure*

(ii) Distortion energy theory predicts yielding with precise accuracy in all four quadrants. The design calculations involved in this theory are slightly complicated as compared with those of maximum shear stress theory.

(iii) The hexagonal diagram of maximum shear stress theory is inside the ellipse of distortion energy theory. Therefore, maximum shear stress theory gives results on the conservative side. On the other hand, distortion energy theory is slightly liberal.

(iv) The graph of maximum principal stress theory is the same as that of maximum shear stress theory in the first and third quadrants. However, the graph of maximum principal stress theory is outside the ellipse of distortion energy theory in the second and fourth quadrants. Thus, it would be dangerous to apply maximum principal stress theory in these regions, since it might predict safety, when in fact no safety exists.

(v) Maximum shear stress theory is used for ductile materials, if dimensions need not be held too close and a generous factor of safety is used. The calculations involved in this theory are easier than those of distortion energy theory.

(vi) Distortion energy theory is used when the factor of safety is to be held in close limits and the cause of failure of the component is being investigated. This theory predicts the failure most accurately.

(vii) The compressive strength of brittle materials is much higher than their tensile strength. Therefore, the failure criterion should show a difference in tensile and compressive strength. On this account, maximum principal stress theory is used for brittle materials. Also, brittle materials do not yield and they fail by fracture.

To summarise, the maximum principal stress theory is the proper choice for brittle materials. For ductile materials, the choice of theory depends on the level of accuracy required and the degree of computational difficulty the designer is ready to face. For ductile materials, the most accurate way to design is to use distortion energy theory of failure and the easiest way to design is to apply maximum shear stress theory

Example 4.11 *A cantilever beam of rectangular cross-section is used to support a pulley as shown in Fig. 4.38 (a). The tension in the wire rope is 5 kN. The beam is made of cast iron FG 200 and the factor of safety is 2.5. The ratio of depth to width of the cross-section is 2. Determine the dimensions of the cross-section of the beam.*

Solution

Given $P = 5$ kN $\qquad S_{ut} = 200$ N/mm^2

$(fs) = 2.5 \quad d/w = 2$

Step I *Calculation of permissible bending stress*

$$\sigma_b = \frac{S_{ut}}{(fs)} = \frac{200}{2.5} = 80 \text{ N/mm}^2$$

Step II *Calculation of bending moments*

The forces acting on the beam are shown in Fig. 4.38(b). Referring to the figure,

$$(M_b)_{\text{at B}} = 5000 \times 500 = 2500 \times 10^3 \text{ N-mm}$$

$$(M_b)_{\text{at A}} = 5000 \times 500 + 5000 \times 1500$$

$$= 10000 \times 10^3 \text{ N-mm}$$

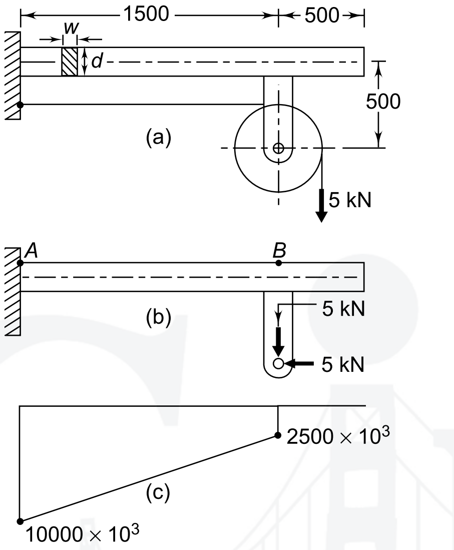


Fig. 4.38

Step III *Calculation of dimensions of cross-section*

The bending moment diagram is shown in Fig. 4.38(c). The cross-section at A is subjected to maximum bending stress. For this cross-section,

$$y = \frac{d}{2} = w \qquad I = \frac{1}{12}[(w)(2w)^3] = \frac{2}{3}w^4 \text{ mm}^4$$

$$\sigma_b = \frac{M_b y}{I} \quad \text{or} \quad 80 = \frac{(10000 \times 10^3)(w)}{\left(\frac{2}{3}w^4\right)}$$

Therefore,

$$w = 57.24 \text{ mm or } 60 \text{ mm} \qquad d = 2w = 120 \text{ mm}$$

Example 4.12 *A wall bracket with a rectangular cross-section is shown in Fig. 4.39. The depth of the cross-section is twice of the width. The force P acting on the bracket at 60⁰ to the vertical is 5 kN. The material of the bracket is grey cast iron FG 200 and the factor of safety is 3.5. Determine the dimensions of the cross-section of the bracket. Assume maximum normal stress theory of failure.*

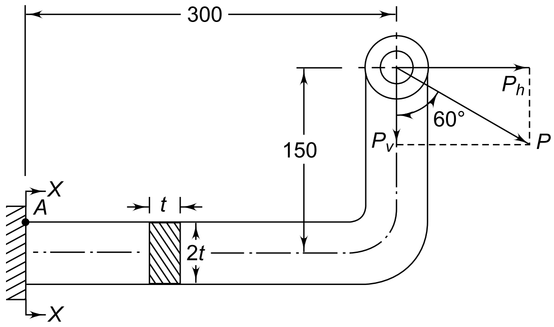


Fig. 4.39 *Wall Bracket*

Solution

Given $P = 5$ kN

$S_{ut} = 200$ N/ mm^2 $\quad (fs) = 3.5 \quad d/w = 2$

Step I *Calculation of permissible stress*

$$\sigma_{\max} = \frac{S_{ut}}{(fs)} = \frac{200}{3.5} = 57.14 \text{ N/mm}^2 \qquad \text{(i)}$$

Step II *Calculation of direct and bending tensile stresses*

The stress is maximum at the point A in the section XX. The point is subjected to combined bending and direct tensile stresses. The force P is resolved into two components—horizontal component P_h and vertical component P_v.

$$P_h = P \sin 60° = 5000 \sin 60° = 4330.13 \text{ N}$$

$$P_v = P \cos 60° = 5000 \cos 60° = 2500 \text{ N}$$

The bending moment at the section XX is given by

$$M_b = P_h \times 150 + P_v \times 300$$

$$= 4330.13 \times 150 + 2500 \times 300$$

$$= 1399.52 \times 10^3 \text{ N-mm}$$

$$\sigma_b = \frac{M_b y}{I}$$

$$= \frac{(1399.52 \times 10^3)(t)}{\left[\frac{1}{12}(t)(2t)^3\right]} = \frac{2099.28 \times 10^3}{t^3} \text{ N/mm}^2$$

The direct tensile stress due to component P_h is given by,

$$\sigma_t = \frac{P_h}{A} = \frac{4330.13}{2t^2} = \frac{2165.07}{t^2} \text{ N/mm}^2$$

The vertical component P_v induces shear stress at the section XX. It is however small and neglected.

Step III *Calculation of dimensions of cross-section*

The resultant tensile stress $\sigma_{max.}$ at the point A is given by,

$$\sigma_{max.} = \sigma_b + \sigma_t = \frac{2099.28 \times 10^3}{t^3} + \frac{2165.07}{t^2} \qquad \text{(ii)}$$

Equating (i) and (ii),

$$\frac{2099.28 \times 10^3}{t^3} + \frac{2165.07}{t^2} = 57.14$$

or

$$t^3 - 37.89\, t - 36739.24 = 0$$

Solving the above cubic equation by trial and error method,

$$t = 33.65 \text{ mm} \cong 35 \text{ mm}$$

The dimensions of the cross-section are 35 × 70 mm

Example 4.13 *The shaft of an overhang crank subjected to a force P of 1 kN is shown in Fig. 4.40(a). The shaft is made of plain carbon steel 45C8 and the tensile yield strength is 380 N/mm². The factor of safety is 2. Determine the diameter of the shaft using the maximum shear stress theory.*

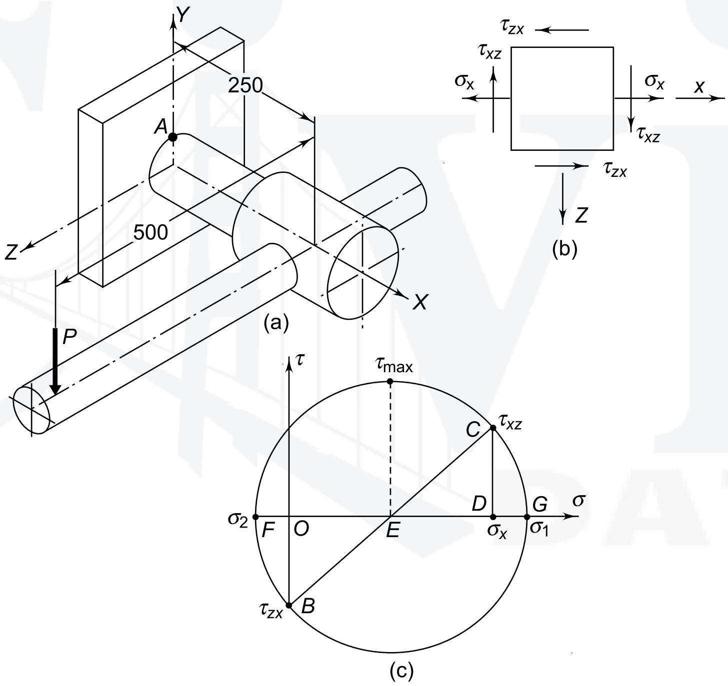


Fig. 4.40

Solution

Given $P = 1$ kN $\quad S_{yt} = 380$ N/mm² $\quad (fs) = 2$

Step I *Calculation of permissible shear stress*

According to maximum shear stress theory,

$$S_{sy} = 0.5\, S_{yt} = 0.5\,(380) = 190 \text{ N/mm}^2$$

The permissible shear stress is given by,

$$\tau_{max.} = \frac{S_{sy}}{(fs)} = \frac{190}{2} = 95 \text{ N/mm}^2 \qquad \text{(i)}$$

Step II *Calculation of bending and torsional shear stresses*

The stresses are critical at the point A, which is subjected to combined bending and torsional moments. At the point A,

$M_b = P \times (250) = (1000)(250) = 250 \times 10^3$ N-mm
$M_t = P \times (500) = (1000)(500) = 500 \times 10^3$ N-mm

$$\sigma_b = \frac{M_b y}{I} = \frac{(250\times10^3)(d/2)}{(\pi d^4/64)}$$

$$= \left(\frac{2546.48\times10^3}{d^3}\right) \text{N/mm}^2$$

$$\tau = \frac{M_t r}{J} = \frac{(500\times10^3)(d/2)}{(\pi d^4/32)}$$

$$= \left(\frac{2546.48\times10^3}{d^3}\right) \text{N/mm}^2$$

Step III *Calculation of maximum shear stress*
The stresses at point *A* and corresponding Mohr's circle are shown in Fig. 4.40(b) and (c) respectively. In these figures,

$$\sigma_x = \sigma_b = \left(\frac{2546.48\times10^3}{d^3}\right) \text{N/mm}^2 \qquad \sigma_z = 0$$

$$\tau = \tau_{xz} = \tau_{zx} = \left(\frac{2546.48\times10^3}{d^3}\right) \text{N/mm}^2$$

From Mohr's circle,

$$\tau_{max.} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xz})^2}$$

$$= \left[\sqrt{\left(\frac{2546.48}{2d^3}\right)^2 + \left(\frac{2546.48}{d^3}\right)^2}\right] \times 10^3$$

$$= \frac{2847.05\times10^3}{d^3} \qquad \text{(ii)}$$

Step IV *Calculation of shaft diameter*
Equating (i) and (ii),

$$\frac{2847.05\times10^3}{d^3} = 95 \qquad \therefore d = 31.06 \text{ mm}$$

Example 4.14 *The dimensions of an overhang crank are given in Fig. 4.41. The force P acting at the crankpin is 1 kN. The crank is made of steel 30C8 (S_{yt} = 400 N/mm²) and the factor of safety is 2. Using maximum shear stress theory of failure, determine the diameter d at the section - XX.*

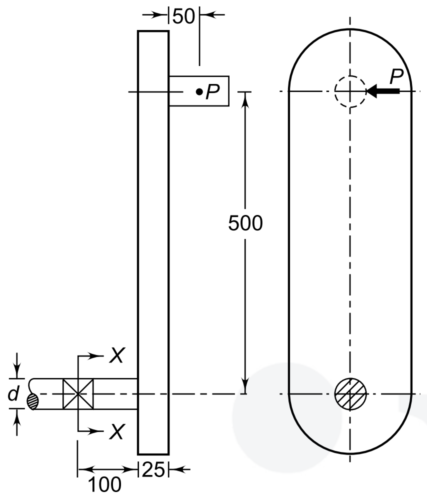


Fig. 4.41 *Overhang Crank*

Solution

Given $P = 1$ kN $\quad S_{yt} = 400$ N/mm² $\quad (fs) = 2$

Step I *Calculation of permissible shear stress*
According to maximum shear stress theory,

$$S_{sy} = 0.5\, S_{yt} = 0.5\,(400) = 200 \text{ N/mm}^2$$

The permissible shear stress is given by,

$$\tau_{max.} = \frac{S_{sy}}{(fs)} = \frac{200}{2} = 100 \text{ N/mm}^2 \qquad \text{(i)}$$

Step II *Calculation of bending and torsional shear stresses*
The section of the crankpin at *XX* is subjected to combined bending and torsional moments. At the section *XX*,

$M_b = 1000 \times (50 + 25 + 100) = 175 \times 10^3$ N-mm
$M_t = 1000 \times 500 = 500 \times 10^3$ N-mm

$$\sigma_x = \sigma_b = \frac{M_b y}{I} = \frac{(175\times10^3)(d/2)}{(\pi d^4/64)}$$

$$= \left(\frac{1782.54\times10^3}{d^3}\right) \text{N/mm}^2$$

$$\sigma_y = 0$$

$$\tau = \frac{M_t r}{J} = \frac{(500\times10^3)(d/2)}{(\pi d^4/32)}$$

$$= \left(\frac{2546.48\times10^3}{d^3}\right) \text{N/mm}^2$$

Step III *Calculation of maximum shear stress*

The problem is similar to the previous one and the maximum shear stress is given by,

$$\tau_{max.} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau)^2}$$

$$= \sqrt{\left(\frac{1782.54 \times 10^3}{2d^3}\right)^2 + \left(\frac{2546.48 \times 10^3}{d^3}\right)^2}$$

$$= \frac{2697.95 \times 10^3}{d^3} \text{ N/mm}^2 \qquad \text{(ii)}$$

Step IV *Calculation of diameter at section-XX*

Equating (i) and (ii),

$$\frac{2697.95 \times 10^3}{d^3} = 100 \quad \therefore \quad d = 29.99 \text{ or } 30 \text{ mm}$$

4.19 LEVERS

A lever is defined as a mechanical device in the form of a rigid bar pivoted about the fulcrum to multiply or transfer the force. The construction of a simple lever is shown in Fig. 4.42. F is the force produced by the lever and P is the effort required to produce that force. The force F is often called '*load*'. The perpendicular distance of the line of action of any force from the fulcrum is called the *arm* of the lever. Therefore l_1 and l_2 are the effort arm and load arm respectively. Taking moment of forces about the fulcrum,

$$F \times l_2 = P \times l_1$$

or

$$\frac{F}{P} = \frac{l_1}{l_2} \qquad \text{(a)}$$

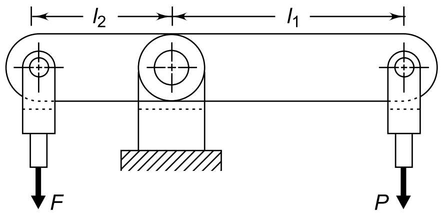


Fig. 4.42 *Construction of Lever*

The ratio of load to effort, i.e., (F/P) is called the '*mechanical advantage*' of the lever. The ratio of the effort arm to the load arm, i.e., (l_1/l_2) is called the '*leverage*'. Therefore, mechanical advantage is equal to the leverage. It is seen by Eq. (a), that a large force can be exerted by a small effort by increasing leverage, i.e., increasing l_1 and reducing l_2. In many applications, it is not possible to increase effort arm l_1 due to space restrictions. In such applications, compound levers are used to obtain more leverage.

There are three types of levers, based on the relative positions of the effort point, the load point and the fulcrum as illustrated in Fig. 4.43. They are as follows:

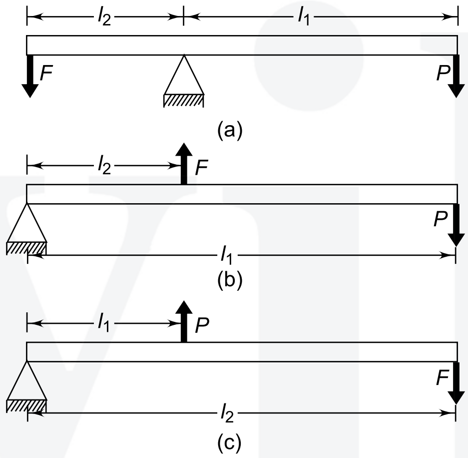


Fig. 4.43 *Types of Lever*

(i) In the '*first*' type of lever, the fulcrum is located between the load and the effort, as shown in Fig. 4.43(a). In this case, the effort arm can be kept less than the load arm or equal to the load arm or more than the load arm. Accordingly, the mechanical advantages will vary in the following way:

When $l_1 < l_2$,
mechanical advantage < 1

When $l_1 = l_2$,
mechanical advantage = 1

When $l_1 > l_2$,
mechanical advantage > 1

Usually, the effort arm is kept more than the load arm to get mechanical advantage. This type of lever is used in applications like the rocker arm for the overhead valves of internal combustion engine, bell crank levers in railway signal mechanisms and levers of hand pumps.

(ii) In the '*second*' type of lever, the load is located between the fulcrum and the effort, as shown in Fig. 4.43(b). In this case, the effort arm is always more than the load arm and the mechanical advantage is more than 1. This type of lever is used in lever-loaded safety valves mounted on the boilers.

(iii) In the '*third*' type of lever, the effort is located between the load and the fulcrum, as shown in Fig. 4.43(c). In this case, the load arm is always greater than the effort arm and the mechanical advantage is less than 1. This type of lever is not recommended in engineering applications. A picking fork is an example of this type of lever.

Levers have wide applications, ranging from simple nutcrackers and paper punching machines to complex lever systems in scales and weighing machines.

4.20 DESIGN OF LEVERS

Lever design is easy compared to design of other machine elements. The length of the lever is decided on the basis of leverage required to exert a given load F by means of an effort P. The cross-section of the lever is designed on the basis of bending stresses. The design of a lever consists of the following steps:

Step 1: Force Analysis

In any application, the load or the force F to be exerted by the lever is given. The effort required to produce this force is calculated by taking moments about the fulcrum. Therefore,

$$F \times l_2 = P \times l_1$$

or,

$$P = F\left(\frac{l_2}{l_1}\right) \tag{4.46}$$

The free body diagram of forces acting on the 'first' type of lever is shown in Fig. 4.44. R is the reaction at the fulcrum pin. Since the sum of vertical forces acting on the lever must be equal to zero,

$$R = F + P \tag{4.47}$$

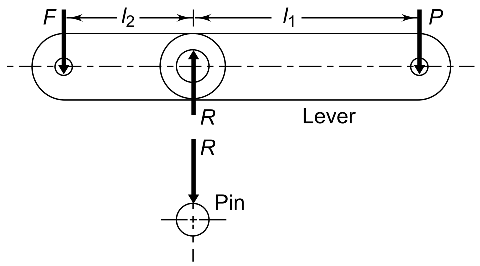


Fig. 4.44 *Free body Diagram of Forces acting on First type of Lever*

The free body diagram of forces acting on the 'second' type of lever is shown in Fig 4.45. In this case, the load and the effort act in opposite directions. Considering equilibrium of forces in a vertical direction,

$$F = R + P$$

or,

$$R = F - P \tag{4.48}$$

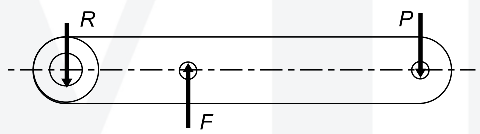


Fig. 4.45 *Free body Diagram of Forces acting on Second type of Lever*

In the above two cases, the forces are assumed to be parallel. Sometimes, the forces F and P act along lines that are inclined to one another as shown in Fig. 4.46. In such cases, l_1 is the perpendicular distance from the fulcrum to the line of action of the force P. Similarly, l_2 is the perpendicular distance from the fulcrum to the line of action of the force F. The following rules from statics apply to the reaction R at the fulcrum:

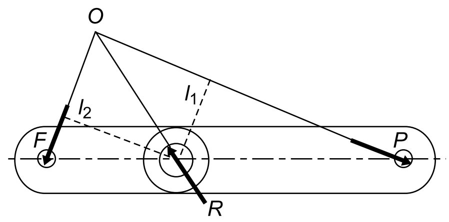


Fig. 4.46

(i) The magnitude of the reaction R is equal to the resultant of the load F and the effort P. It is determined by the *parallelogram law* of forces.

(ii) The line of action of the reaction R passes through the intersection of P and F, i.e., the point O in Fig. 4.46 and also through the fulcrum.

Figure 4.47 illustrates a bell-crank lever with the arms that are inclined at angle θ with one another. The load F and the effort P act at right angles to their respective arms. The reaction R at the fulcrum is given by

$$R = \sqrt{F^2 + P^2 - 2FP\cos\theta} \tag{4.49}$$

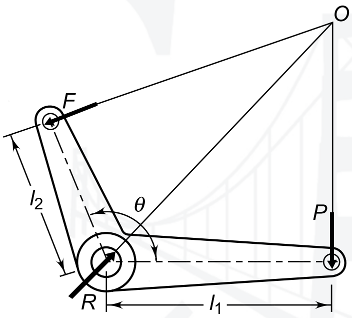


Fig. 4.47

When the arms of the bell-crank lever are at right angles to one another,

$$\theta = 90° \text{ and } \cos\theta = 0$$

Therefore,

$$R = \sqrt{F^2 + P^2} \tag{4.50}$$

Step 2: Design of Lever Arm

When the forces acting on the lever are determined, the next step in lever design is to find out the dimensions of the cross-section of the lever. The cross-section of the lever is subjected to bending moment. In case of a two-arm lever, as shown in Fig. 4.48(a), the bending moment is zero at the point of application of P or F and maximum at the boss of the lever. The cross-section at which the bending moment is maximum can be determined by constructing a bending-moment diagram. In Fig. 4.48(b), the bending moment is maximum at section XX and it is given by,

$$M_b = P\,(l_1 - d_1)$$

The cross-section of the lever can be rectangular, elliptical or I-section.

For a rectangular cross-section,

$$I = \frac{bd^3}{12} \quad \text{and} \quad y = \frac{d}{2}$$

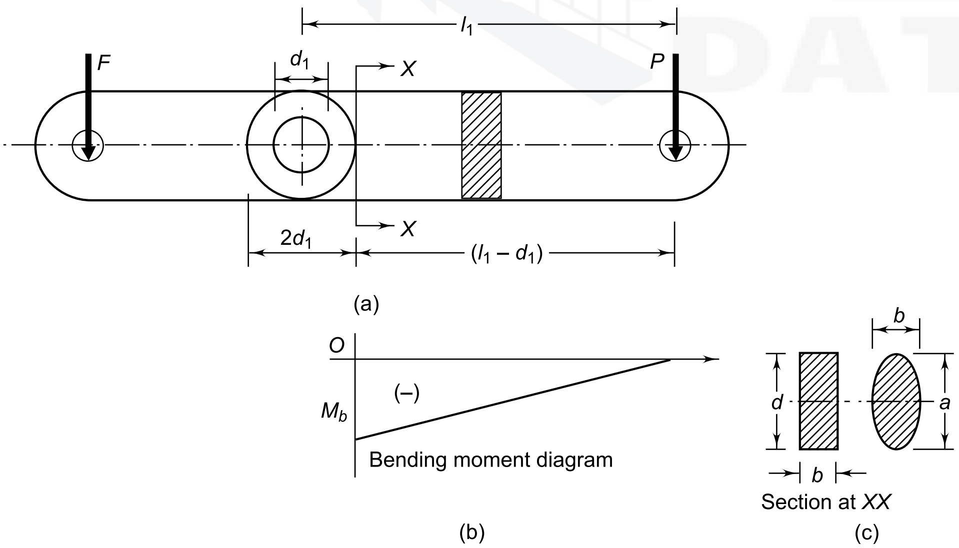


Fig. 4.48

where b is the distance parallel to the neutral axis, and d is the distance perpendicular to the neutral axis. The dimension d is usually taken as twice of b.

or, $$d = 2b$$

For an elliptical cross-section,

$$I = \frac{\pi b a^3}{64} \quad \text{and} \quad y = \frac{a}{2}$$

where a and b are major and minor axes of the section. Usually, the major axis is taken as twice the minor axis.

or, $$a = 2b$$

Using the above mentioned proportions, the dimensions of the cross-section of the lever can be determined by,

$$\sigma_b = \frac{M_b y}{I}$$

Figure 4.48(b) shows the variation of bending moment. It varies from a maximum value M_b at the section XX to zero at the point of application of P. Therefore, the cross-section of the arm is usually tapered from the boss of the fulcrum to the end.

Step 3: Design of Fulcrum Pin

The fulcrum pin is subjected to reaction R as shown in Fig. 4.49. The forces acting on the boss of lever and the pin are equal and opposite. The dimensions of the pin, viz., diameter d_1 and length l_1 in lever boss are determined by bearing consideration and then checked for shear consideration. There is relative motion between the pin and the boss of lever and bearing pressure becomes the design criterion. The projected area of the pin is $(d_1 \times l_1)$. Therefore,

$$R = p\,(d_1 \times l_1) \tag{4.51}$$

where p is the the permissible bearing pressure.

For the fulcrum pin, the ratio of length to diameter (l_1/d_1) is usually taken from 1 to 2. The outside diameter of the boss in the lever is taken as twice of the diameter of the pin, i.e., $(2d_1)$. A phosphor bronze bush, usually 3 mm thick, is fitted inside the boss to reduce the friction. The permissible bearing pressure for a phosphor bronze bush is 5 to 10 N/mm^2. A lubricant is provided between the pin and the bush to reduce the friction.

It can be observed that expressions for bearing pressure and compressive or crushing stress are same. Rearranging Eq. (4.51), the bearing pressure is given by,

$$p = \frac{R}{(d_1 \times l_1)} \tag{a}$$

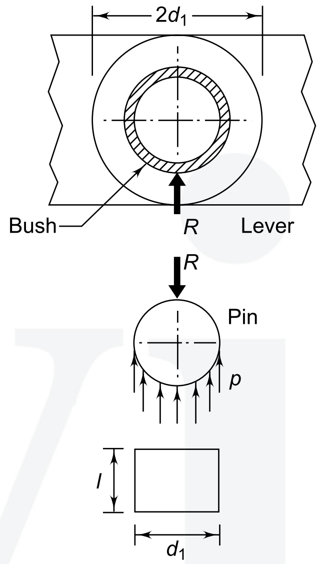


Fig. 4.49

There is a similar example of the pin in a knuckle joint illustrated in Fig. 4.21. For this pin, the compressive stress is given by,

$$\sigma_c = \frac{\text{force}}{\text{projected area}}$$

or

$$\sigma_c = \frac{P}{(d \times l)} \tag{b}$$

Although the expressions (a) and (b) are same, there is a basic difference between bearing pressure and crushing or compressive stress. The bearing pressure is considered when there is relative motion between two surfaces such as surfaces of the pin and the bushing. On the other hand, crushing stress is considered when there is no relative motion between the surfaces under consideration. The bearing pressure is always low such as 10 N/mm^2, while the magnitude of compressive stress is high such as

150 N/mm^2. A rotating shaft in the bearing, a fulcrum pin of an oscillating lever, a power screw rotating inside the nut are examples where bearing pressure is the design consideration. The contact area between a cotter and spigot end, and cotter and socket end; between knuckle pin and eye or knuckle pin and fork are the examples where crushing stress is the criterion of design.

Example 4.15 *A lever-loaded safety valve is mounted on the boiler to blow off at a pressure of 1.5 MPa gauge. The effective diameter of the opening of the valve is 50 mm. The distance between the fulcrum and the dead weights on the lever is 1000 mm. The distance between the fulcrum and the pin connecting the valve spindle to the lever is 100 mm. The lever and the pin are made of plain carbon steel 30C8 (S_{yt} = 400 N/mm^2) and the factor of safety is 5. The permissible bearing pressure at the pins in the lever is 25 N/mm^2. The lever has a rectangular cross-section and the ratio of width to thickness is 3:1. Design a suitable lever for the safety valve.*

Solution

Given $S_{yt} = 400$ N/mm^2 $(fs) = 5$
For valve, $d = 50$ mm $p = 1.5$ MPa
For lever, $l_1 = 1000$ mm $l_2 = 100$ mm $d/b = 3$
For pin, $p = 25$ N/mm^2

The construction of the lever-loaded safety valve is shown in Fig. 4.50. It is mounted on steam boilers to limit the maximum steam pressure. When the pressure inside the boiler exceeds this limiting value, the valve automatically opens due to excess of steam pressure and steam blows out through the valve. Consequently, the steam pressure inside the boiler is reduced.

Step I *Calculation of permissible stresses for lever and pin*

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{400}{5} = 80 \text{ N/mm}^2$$

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.5S_{yt}}{(fs)} = \frac{0.5(400)}{5} = 40 \text{ N/mm}^2$$

Step II *Calculation of forces acting on lever*

The valve is held tight on the valve seat against the upward steam force F by the dead weights P attached at the end of the lever. The distance l_1 and the dead weights P are adjusted in such a way, that when the steam pressure inside the boiler reaches the limiting value, the moment $(F \times l_2)$ overcomes the moment $(P \times l_1)$. As a result, the valve opens and steam blows out until the pressure falls to the required limiting value and then the valve is automatically closed.

The maximum steam load F, at which the valve blows off is given by,

$$F = \frac{\pi}{4} d^2 p = \frac{\pi}{4} (50)^2 (1.5) = 2945.24 \text{ N} \qquad \text{(a)}$$

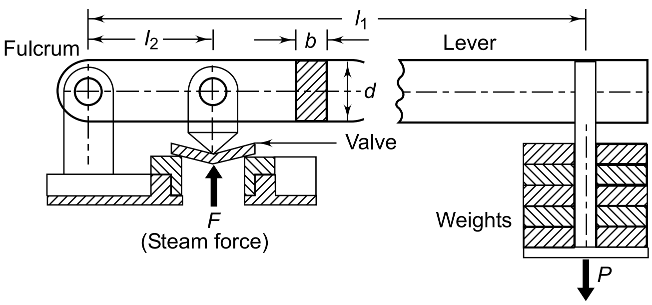


Fig. 4.50 *Lever Loaded Safety Valve*

Taking moment of forces F and P about the fulcrum,

$$F \times l_2 = P \times l_1 \quad \text{or} \quad 2945.24 \times 100 = P \times 1000$$

$$\therefore \quad P = 294.52 \text{ N} \qquad \text{(b)}$$

The forces acting on the lever are shown in Fig. 4.51(a). Considering equilibrium of vertical forces,

$$F = R + P$$

or $\quad R = F - P = 2945.24 - 294.52 = 2650.72 \text{ N} \quad$ (c)

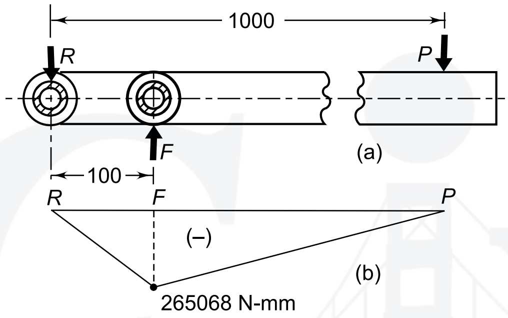


Fig. 4.51 *Bending Moment Diagram of Lever*

Step III *Diameter and length of pin*

From (a), (b) and (c), the pin at the point of application of the force F is subjected to maximum force and as such, it is to be designed from bearing consideration. Suppose, d_1 and l_1 are the diameter and the length of the pin at F and assume,

$$l_1 = d_1$$

From Eq. (4.51),

$$F = p\,(d_1 \times l_1) \quad \text{or} \quad 2945.24 = 25\,(d_1 \times d_1)$$

$$\therefore \quad d_1 = 10.85 \text{ or } 12 \text{ mm}$$

$$l_1 = d_1 = 12 \text{ mm} \qquad \text{(i)}$$

The pin is subjected to double shear stress, which is given by,

$$\tau = \frac{F}{2\left[\frac{\pi}{4} d_1^2\right]} = \frac{2945.24}{2\left[\frac{\pi}{4}(12)^2\right]} = 13.02 \text{ N/mm}^2$$

$$\therefore \quad \tau < 40 \text{ N/mm}^2$$

The force on the fulcrum pin (R) is comparatively less than the force acting on the spindle pin (F). Therefore, the dimensions d_1 and l_1 of the pin at the fulcrum will be slightly less. However, we will assume both pins of the same diameter and length to facilitate interchangeability of parts and variety reduction.

Step IV *Width and thickness of lever*

A gunmetal bush of 2-mm thickness is press fitted at both pin holes to reduce friction. Therefore, the inside diameter of the boss will be ($d_1 + 2 \times 2$) or ($12 + 2 \times 2$) or 16 mm. The outside diameter of the boss is kept twice of the inside diameter, i.e., 32 mm.

The bending moment diagram for the lever is shown in Fig. 4.51(b). The bending moment is maximum at the valve spindle axis. It is given by,

$$M_b = \text{P}\,(1000 - 100) = 294.52\,(1000 - 100)$$
$$= 265\,068 \text{ N-mm}$$

For a lever,

$$d = 3b$$

$$\sigma_b = \frac{M_b y}{I} \qquad 80 = \frac{(265\,068)\,(1.5b)}{\left[\frac{1}{12} b\,(3b)^3\right]}$$

$$\therefore \quad b = 13.02 \text{ or } 15 \text{ mm} \qquad d = 3b = 45 \text{ mm}$$

The lever becomes weak due to the pinhole at the valve spindle axis and it is necessary to check bending stresses at this critical section. The cross-section of the lever at the valve spindle axis is shown in Fig. 4.52. In this case, the length of the pin is increased from 12 mm to 20 mm to get practical proportions for the boss. For this cross-section,

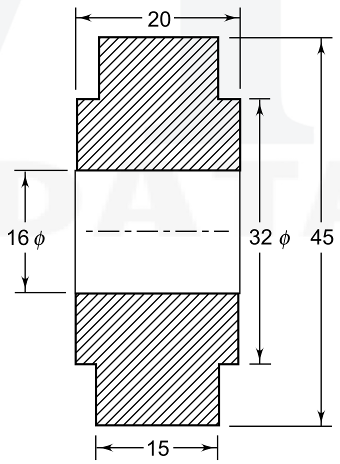


Fig. 4.52 *Cross-section of Lever*

$$M_b = 265\,068 \text{ N-mm} \qquad y = 22.5 \text{ mm}$$

$$I = \frac{1}{12}\left[15(45)^3 + 5(32)^3 - 20(16)^3\right]$$

$$= 120\,732.92 \text{ mm}^4$$

Therefore,

$$\sigma_b = \frac{M_b y}{I} = \frac{(265\,068)(22.5)}{(120\,732.92)} = 49.40 \text{ N/mm}^2$$

Since, $\sigma_b < 80$ N/mm^2

the design is safe.

Example 4.16 *A right angled bell-crank lever is to be designed to raise a load of 5 kN at the short arm end. The lengths of short and long arms are 100 and 450 mm respectively. The lever and the pins are made of steel 30C8 (S_{yt} = 400 N/mm^2) and the factor of safety is 5. The permissible bearing pressure on the pin is 10 N/mm^2. The lever has a rectangular cross-section and the ratio of width to thickness is 3:1. The length to diameter ratio of the fulcrum pin is 1.25:1. Calculate*

(i) The diameter and the length of the fulcrum pin

(ii) The shear stress in the pin

(ii) The dimensions of the boss of the lever at the fulcrum

(iii) The dimensions of the cross-section of the lever

Assume that the arm of the bending moment on the lever extends up to the axis of the fulcrum.

Solution

Given S_{yt} = 400 N/mm^2 $(fs) = 5$ $F = 5$ kN

For lever, long arm = 450 mm short arm = 100 mm

$d/b = 3$

For pin $p = 10$ N/mm^2 $l_1/d_1 = 1.25$

Step I *Calculation of permissible stresses for the pin and lever*

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{400}{5} = 80 \text{ N/mm}^2$$

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.5S_{yt}}{(fs)} = \frac{0.5(400)}{5} = 40 \text{ N/mm}^2$$

Step II *Calculation of forces acting on the lever*

The forces acting on the lever are shown in Fig. 4.53. Taking moment of forces about the axis of the fulcrum,

$$(5 \times 10^3)(100) = P \times 450 \qquad \therefore P = 1111.11 \text{ N}$$

$$R = \sqrt{(5000)^2 + (1111.11)^2} = 5121.97 \text{ N}$$

Step III *Diameter and length of fulcrum pin*

Considering bearing pressure on the fulcrum pin,

$$R = p \text{ (projected area of the pin)} = p\,(d_1 \times l_1)$$

or $$R = p\,(d_1 \times l_1) \qquad \text{(a)}$$

where,

d_1 = diameter of the fulcrum pin (mm)

l_1 = length of the fulcrum pin (mm)

p = permissible bearing pressure (N/mm^2)

Substituting values in Eq. (a),

$$5121.97 = 10\,(d_1 \times 1.25d_1)$$

$$\therefore \quad d_1 = 20.24 \text{ mm}$$

$$l_1 = 1.25\,d_1 = 1.25(20.24) = 25.30 \text{ mm} \qquad \text{(i)}$$

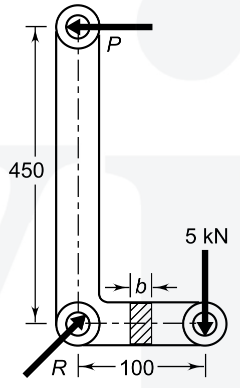


Fig. 4.53

Step IV *Shear stress in pin*

The pin is subjected to double shear. The shear stress in the pin is given by,

$$\tau = \frac{R}{2\left[\frac{\pi}{4}d_1^2\right]} = \frac{5121.97}{2\left[\frac{\pi}{4}(20.24)^2\right]} = 7.96 \text{ N/mm}^2 \quad \text{(ii)}$$

Step V *Dimensions of the boss*

The dimensions of the boss of the lever at the fulcrum are as follows:

inner diameter = 21 mm

outer diameter = 42 mm

length = 26 mm (iii)

Step VI *Dimensions of cross-section of lever*

For the lever,

$$d = 3b \qquad M_b = (5000 \times 100) \text{ N-mm}$$

Therefore,

$$\sigma_b = \frac{M_b y}{I} \quad \text{or} \quad 80 = \frac{(5000 \times 100)(1.5b)}{\left[\frac{1}{12}(b)(3b)^3\right]}$$

$\therefore \quad b = 16.09$ mm

$$d = 3b = 3\,(16.09) = 48.27 \text{ mm} \qquad \text{(iv)}$$

Example 4.17 *A pressure vessel, used in chemical process industries, is shown in Fig. 4.54. It is designed to withstand an internal gauge pressure of 0.25 MPa (or 0.25 N/mm^2). The cover is held tight against the vessel by means of a screw, which is turned down through the tapped hole in the beam, so that the end of the screw presses firmly against the cover. The links L_1 and L_2 are attached to the beam on one side and to the extension cast on the vessel on the other side. The vessel and its cover are made of grey cast iron FG 200. The beam, screw, links and pins are made of steel FeE 250 (S_{yt} = 250 N/mm^2). The factor of safety for all parts is 5. The beam has a rectangular cross-section and the ratio of width to thickness is 2:1 (h = 2b). Assume the following data for screw (ISO Metric threads-Coarse series):*

Size	*Pitch (mm)*	*Stress area (mm^2)*
M 30	*3.5*	*561*
M 36	*4*	*817*
M 42	*4.5*	*1120*
M 48	*5*	*1470*

Determine

(i) Diameter of the screw
(ii) Dimensions of the cross-section of the beam
(iii) Diameter of pins at A, B, C and D
(iv) Diameter d_2 of link L_1 and L_2
(v) Dimensions of the cross-section of the support for pins A and B.

Solution

Given For beam S_{yt} = 250 N/mm^2 $(fs) = 5$
$h/b = 2$

For vessel, S_{ut} = 200 N/mm^2 $(fs) = 5$
p = 0.25 N/mm^2 D = 500 mm

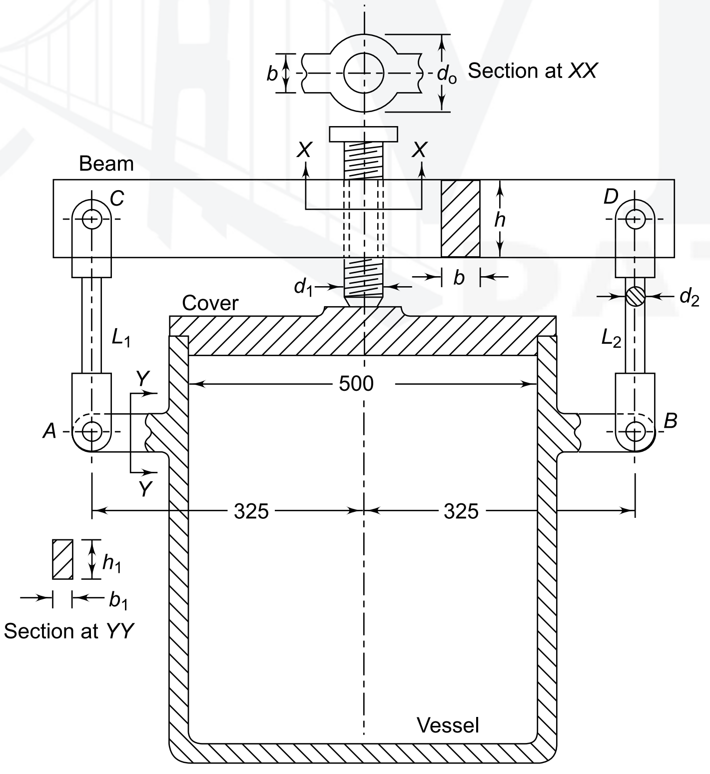


Fig 4.54 *Pressure Vessel*

Step I *Calculation of permissible stresses*

(a) Steel parts

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{250}{5} = 50 \text{ N/mm}^2$$

Assuming

$$S_{yc} = S_{yt}$$

$$\sigma_c = \frac{S_{yc}}{(fs)} = \frac{250}{5} = 50 \text{ N/mm}^2$$

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.5 S_{yt}}{(fs)} = \frac{0.5(250)}{5} = 25 \text{ N/mm}^2$$

(b) Cast iron parts

$$\sigma_t = \frac{S_{ut}}{(fs)} = \frac{200}{5} = 40 \text{ N/mm}^2$$

Step II *Free body diagram*

The free body diagram of forces acting on various parts of the pressure vessel is shown in Fig. 4.55. This diagram is constructed starting with the forces acting on the cover and then proceeding to screw, beam,

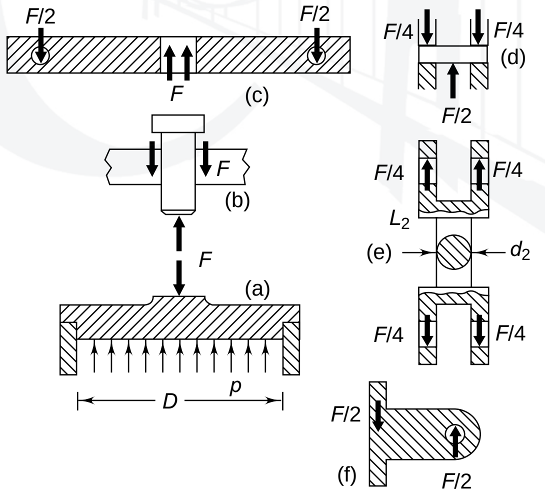


Fig. 4.55 *Free Body Diagram of Forces*

pin, link L_2 and the extension of vessel to support the pin. The direction of forces acting on various parts is decided by using the following two principles:

(i) The sum of vertical forces acting on any part must be zero; and

(ii) Action and reaction are equal and opposite.

Step III *Diameter of screw*

The force acting on the cover, as shown in Fig. 4.55 (a), is given by,

$$F = \frac{\pi}{4} D^2 p = \frac{\pi}{4} (500)^2 \, (0.25) = 49087.39 \text{ N}$$

As shown in Fig. 4.55 (b), the portion of the screw between the beam and the cover is subjected to compressive stress. If a is the stressed area of the screw then the compressive force is given by,

$$F = a\,\sigma_c \quad \text{or} \quad 49087.39 = a\,(50)$$

$$\therefore \qquad a = 981.75 \text{ mm}^2$$

From the given data, a screw of M42 size (stressed area = 1120 mm^2) is suitable. The nominal diameter of the screw is 42 mm and the pitch is 4.5 mm.

Step IV *Cross-section of beam*

As shown in Fig. 4.56(a), the beam is simply supported with a single concentrated load F at the centre of the span length. Due to symmetry of loading, the

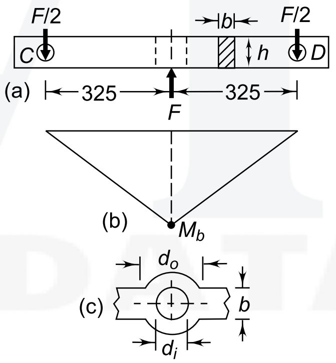


Fig. 4.56

reaction at each of the two pins, C and D, is equal to (F/2). The bending moment is maximum at the midpoint of the beam. It is given by,

$$M_b = 325 \times \left(\frac{F}{2}\right) = 325 \times \left(\frac{49087.39}{2}\right)$$

or,

$$M_b = 7\,976\,700.88 \text{ N-mm}$$

Since,

$$h = 2b \qquad I = \frac{bh^3}{12} = \frac{b(2b)^3}{12} = \left(\frac{2b^4}{3}\right) \text{mm}^4$$

$$y=\frac{h}{2}=b \text{ mm}$$

$$\sigma_b=\frac{M_b y}{I}$$

Substituting,

$$50=\frac{(7\,976\,700.88)(b)}{\left[\frac{2}{3}(b^4)\right]}$$

$\therefore$ $b = 62.08$ or 65 mm $\qquad h = 2b = 130$ mm

As shown in Fig. 4.56(a), the axis of the tapped hole is parallel to h dimension of the section. As shown in Fig 4.56 (c), the solid rectangular section of the beam of thickness b can be split into two halves, each having a width $(b/2)$ at the hole, so that the metallic area in a section through the hole is equal to the area of the solid section $(h \times b)$. In this case, the factor of safety will remain unchanged. The diameter of the hole (d_1) is the nominal diameter of the screw. Therefore,

$$d_1 = 42 \text{ mm}$$

$$d_0=d_1+\frac{b}{2}+\frac{b}{2}=42+\frac{65}{2}+\frac{65}{2}=107 \text{ mm}$$

Step V *Diameter of pins*

As shown in Fig. 4.55(d), the pin is subjected to double shear. The force acting on all four pins at A, B, C and D is same and equal to $(F/2)$. The shear stress in the pin is given by,

$$\tau=\frac{\left(\frac{F}{2}\right)}{2\left[\frac{\pi}{4}d^2\right]} \quad \text{or} \quad 25=\frac{\left(\frac{49087.39}{2}\right)}{2\left[\frac{\pi}{4}d^2\right]}$$

$\therefore$ $d = 25$ mm

Step VI *Diameter of links L_1 and L_2*

As shown in Fig. 4.55(e), the links are subjected to tensile stresses.

$$\frac{\pi}{4}d_2^2\,\sigma_t=\frac{F}{2} \quad \text{or} \quad \frac{\pi}{4}d_2^2\,(50)=\frac{49087.39}{2}$$

$\therefore$ $d_2 = 25$ mm

Step VII *Dimensions of support*

The extensions or brackets on the vessel are part of the casting of the cylinder. They act as cantilevers. As shown in Fig. 4.54, the maximum length of the cantilever can be taken as (325 – 250) or 75 mm, neglecting the thickness of the cylinder.

Therefore,

$$M_b=75\left(\frac{F}{2}\right)=75\left(\frac{49087.39}{2}\right)$$

$$=1\,840\,777\cdot13 \text{ N-mm}$$

Assuming $h_1 = 2b_1$

$$y=\frac{h_1}{2}=b_1 \qquad I=\frac{b_1h_1^3}{12}=\frac{b_1(2b_1)^3}{12}=\frac{2b_1^4}{3}$$

$$\sigma_b=\frac{M_b y}{I}$$

Substituting,

$$40=\frac{(1\,840\,777.13)(b_1)}{\left[\frac{2}{3}(b_1^4)\right]}$$

$\therefore$ $b_1 = 41.02$ or 45 mm $\quad h_1 = 2b_1 = 2\times45 = 90$ mm.

Example 4.18 *The mechanism of a bench-shearing machine is illustrated in Fig. 4.57. It is used to shear mild steel bars up to 6.25 mm diameter. The ultimate shear strength of the material is 350 N/mm². The link, lever and pins at B, C and D are made of steel FeE 250 (S_{yt} = 250 N/mm²) and the factor of safety is 5. The pins at B, C and D are identical and their length to diameter ratio is 1.25. The permissible bearing pressure at the pins is 10 N/mm². The link has circular cross-section. The cross-section of the lever is rectangular and the ratio of width to thickness is 2:1. Calculate*

(i) Diameter of pins at B, C and D;
(ii) Diameter of the link
(iii) Dimensions of the cross-section of the lever

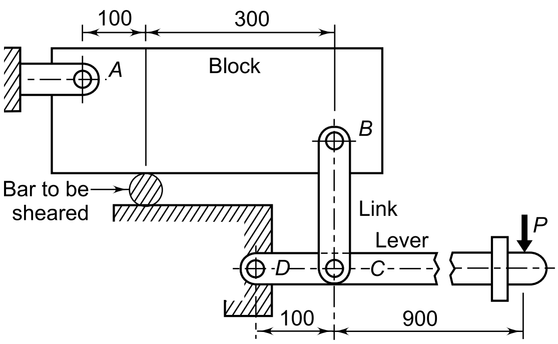


Fig. 4.57 *Bench Shearing Machine*

Solution

Given $S_{yt} = 250$ N/mm^2 $(fs) = 5$

For bars to be sheared $D = 6.25$ mm

$S_{us} = 350$ N/mm^2

For pins, $p = 10$ N/mm^2 $l/d = 1.25$

For lever, $h/b = 2$

Step I *Calculation of permissible stresses*

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{250}{5} = 50 \text{ N/mm}^2$$

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.5S_{yt}}{(fs)} = \frac{0.5(250)}{5} = 25 \text{ N/mm}^2$$

Step II *Calculation of forces*

The maximum force P_s required to shear the bar is given by,

$$P_s = \text{(area of bar)} \times \text{(ultimate shear strength)}$$

$$= \frac{\pi}{4}(6.25)^2 \times 350 = 10737.87 \text{ N}$$

The free body diagram of forces acting on various parts of the shearing machine is shown in Fig. 4.58. This diagram is constructed starting with shear force P_s acting on the bar and then proceeding to the block, link and the lever.

The direction of forces acting on the various parts is decided by using the following two principles:

(i) The action and reaction are equal and opposite.

(ii) The sum of vertical forces acting on any part must be equal to zero.

Taking moment of forces acting on the block, as shown in Fig. 4.58(b) about the fulcrum A,

$P_1 \times 400 = P_s \times 100$ or

$P_1 \times 400 = 10737.87 \times 100$

$\therefore$ $P_1 = 2684.47$ N

Also,

$R_A + P_1 = P_s$ or $R_A = P_s - P_1$

$R_A = 10737.87 - 2684.47 = 8053.4$ N

Taking moment of forces acting on lever, as shown in Fig. 4.58(d), about the fulcrum D,

$P \times 1000 = P_1 \times 100$ or

$P \times 1000 = 2684.47 \times 100$

$\therefore$ $P = 268.45$ N

Also,

$R_D + P = P_1$ or

$R_D = P_1 - P = 2684.47 - 268.45 = 2416.02$ N

Step III *Diameter of pins*

The forces acting on the pins at B, C and D are 2684.47, 2684.47 and 2416.02 N respectively.

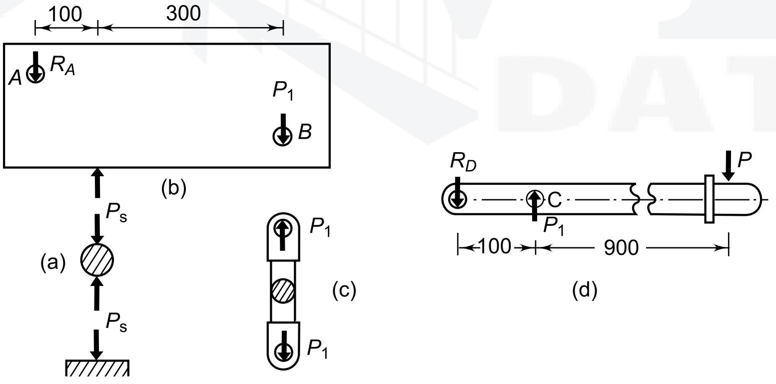


Fig. 4.58 *Free-body Diagram of Forces*

Since the three pins are identical, we will design the pin for maximum force of 2684.47 N. The dimensions of the pins are determined on the basis of bearing consideration and checked for shear consideration.

Considering bearing pressure on the pin,

$R = p$ (projected area of the pin) $= p\,(d_1 \times l_1)$

or $R = p\,(d_1 \times l_1)$ (a)

where,

d_1 = diameter of the pin (mm)

l_1 = length of the pin (mm)

p = permissible bearing pressure (N/mm^2)

Substituting values in Eq. (a),

$$2684.47 = 10\,(d_1 \times 1.25\, d_1)$$

$\therefore$ $d_1 = 14.65$ or 15 mm

$l_1 = 1.25\, d_1 = 1.25(15) = 18.75$ or 20 mm

The pin is subjected to double shear stress, which is given by,

$$\tau = \frac{R}{2\left[\dfrac{\pi}{4} d_1^2\right]} = \frac{2684.47}{2\left[\dfrac{\pi}{4}(15)^2\right]} = 7.60 \text{ N/mm}^2$$

$\therefore$ $\tau < 25$ N/mm^2

Step IV *Diameter of link*

The link is subjected to tensile stress as shown in Fig. 4.58(c). Therefore,

$$\sigma_t = \frac{P_1}{\left(\dfrac{\pi}{4} d^2\right)} \quad \text{or} \quad 50 = \frac{2684.47}{\left(\dfrac{\pi}{4} d^2\right)}$$

$\therefore$ $d = 8.27$ or 10 mm

Step V *Dimensions of lever*

As shown in Fig. 4.59, the lever is subjected to bending moment. The maximum bending moment occurs at C, which is given by,

$$M_b = P \times 900 = 268.45 \times 900 = 241\,605 \text{ N-mm}$$

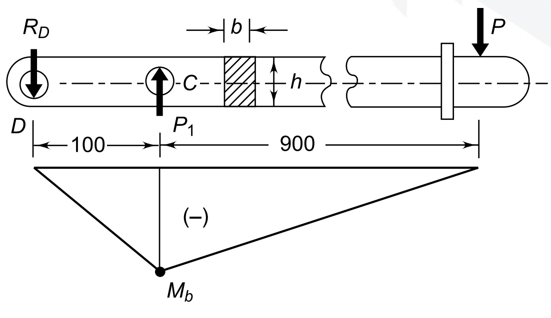


Fig. 4.59 *Bending Moment Diagram for Lever*

Since,

$$h = 2b \quad I = \frac{bh^3}{12} = \frac{b(2b)^3}{12} = \left(\frac{2b^4}{3}\right) \text{mm}^4$$

$$y = \frac{h}{2} = b \text{ mm} \qquad \sigma_b = \frac{M_b y}{I}$$

Substituting,

$$50 = \frac{(241\,605)(b)}{\left(\dfrac{2b^4}{3}\right)}$$

$\therefore$ $b = 19.35$ or 20 mm $\quad h = 2b = 2(20) = 40$ mm

The lever becomes weak due to the pinhole at C and it is necessary to check bending stresses at this critical cross-section, which is shown in Fig. 4.60.

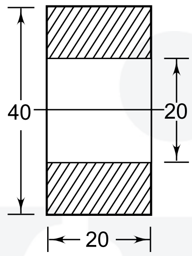


Fig. 4.60 *Cross-section of Lever at Pin-C*

The diameter of the pin is 15 mm, while the length is 20 mm. It is assumed that a gunmetal bush of 2.5 mm thickness is fitted in the hole to reduce friction and wear. Therefore, inner diameter of the boss will be (15 + 2 × 2.5) or 20 mm. The outside diameter of the boss is kept twice of the inner diameter, i.e., 40 mm. Therefore,

$$I = \frac{1}{12}[20(40)^3 - 20(20)^3] = 93\,333.33 \text{ mm}^4$$

$y = 20$ mm

$$\sigma_b = \frac{M_b y}{I} = \frac{(241\,605)(20)}{(93\,333.33)} = 51.77 \text{ N/mm}^2$$

There is slight difference between bending stress (51.77 N/mm^2) and permissible stress (50 N/mm^2). Since the difference is small, it is neglected and the dimensions of the boss of the lever are kept unchanged.

4.21 FRACTURE MECHANICS

There was a series of catastrophic failures of pressure vessels, ships and aircrafts in 10 years between 1940 and 1950. These accidents focused the attention of design engineers to the existence and growth of cracks in machine components. Fracture mechanics has its origin in the work of AA Griffiths, who proved that the fracture strength of a brittle material, like glass, is inversely proportional to the square root of the crack length.

The concept of fracture mechanics begins with the assumption that all components contain microscopic cracks. In case of ductile materials, there is stress concentration in the vicinity of a crack. When the localized stress near the crack reaches the yield point, there is plastic deformation, resulting in redistribution of stresses. Therefore, the effect of crack is not serious in case of components made of ductile materials. However, the effect of crack is much more serious in case of components made of brittle materials due to their inability of plastic deformation. Fracture mechanics is normally concerned with materials that are in the brittle state. They include high-strength, low-alloy steels, high-strength aluminium alloys, titanium alloys and some polymers. They also include 'normally ductile' materials, which under certain conditions of thermal and corrosive environment, behave like brittle materials. For example, low carbon steels at temperatures below 0°C behave like brittle materials.

Fracture mechanics is the science of predicting the influence of cracks and crack like defects on the brittle fracture of components. When a component containing a small microscopic crack is subjected to an external force, there is an almost instantaneous propagation of the crack leading to sudden and total failure. The crack is just like a dropped stitch in knitting. Propagation of a crack is like tearing a cloth. Once you start a little tear, it will propagate rather easily across the full length of the cloth. Less force is required to propagate a crack than to initiate it. Fracture failure occurs at a stress level which is well below the yield point of the material. Therefore, failure due to propagation of crack in components made of brittle materials is catastrophic.

Figure 4.61 shows a rectangular plate subjected to tensile stress in the longitudinal direction. The plate contains a sharp transverse crack located at the centre of the plate. The plate length $2h$ is large compared with the plate width $2b$. Also, the plate width $2b$ is large compared with the crack length $2a$.

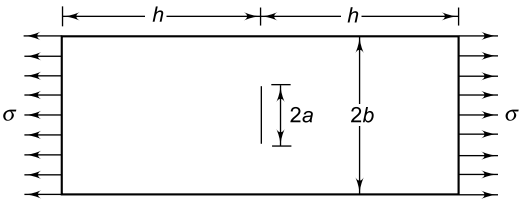


Fig. 4.61 *Plate with Central Crack*

There are two terms in fracture mechanics, viz., the stress intensity factor and fracture toughness. *The stress intensity factor K_0 specifies the stress intensity at the tip of the crack*. It is given by,

$$K_0 = \sigma\sqrt{\pi a} \qquad (4.52)$$

where,

K_0 = stress intensity factor (in units of N/mm^2 $\sqrt{\text{m}}$)

σ = nominal tensile stress at the edge $\left(\dfrac{P}{2bt}\right)$ (N/mm^2)

t = plate thickness (mm)

a = half crack length (m)

The fracture toughness is the critical value of stress intensity at which crack extension occurs. The fracture toughness is denoted by K_I. It is given by,

$$K_I = Y\,K_0 = Y\sigma\sqrt{\pi a} \qquad (4.53)$$

where,

Y = dimensionless correction factor that accounts for the geometry of the part containing the crack

K_I = fracture toughness (in units of N/mm^2 $\sqrt{\text{m}}$)

The variation of the correction factor Y for a plate containing a central crack subjected to tensile stress in longitudinal direction is shown in Fig. 4.62. For example, if h/b = 1.0 and a/b = 0.6 then the value of Y from the graph in Fig. 4.62 is approximately 1.51. Therefore,

$$K_I = Y\,K_0 = 1.51\sigma\sqrt{\pi a}$$

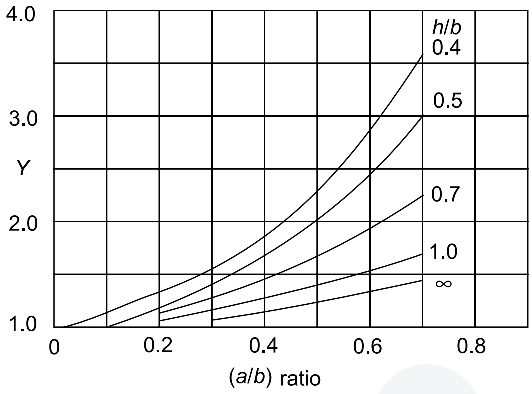


Fig. 4.62 *Y Factor for Plate with Central Crack*

There is a basic difference between stress intensity factor K_0 and fracture toughness K_I, although they have the same units. The stress intensity factor K_0 represents the stress level at the tip of the crack in the machine part. On the other hand, fracture toughness K_I is the highest stress intensity that the part can withstand without fracture at the crack.

There are three basic modes of crack propagation as illustrated in Fig. 4.63. *Mode-I* is called the *opening* or *tensile mode*. It is the most commonly observed mode of crack propagation. In this case, the crack faces separate symmetrically with respect to the crack plane. *Mode-II* is called *sliding* or *in-plane shearing mode*. *Mode-III* is called *tearing mode*. Mode-II and Mode-III are fundamentally shear modes of failures.

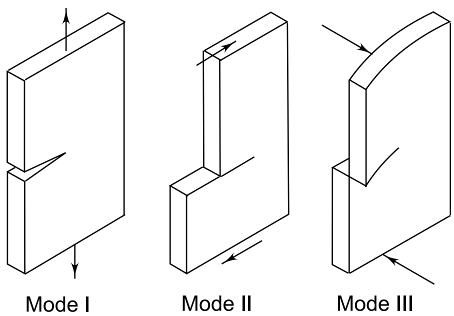


Fig. 4.63 *Deformation Modes*

4.22 CURVED BEAMS

A curved beam is defined as a beam in which the neutral axis in unloaded condition is curved instead of straight. The following assumptions are made in the stress analysis of curved beam:

(i) Plane sections perpendicular to the axis of the beam remain plane after bending.

(ii) The moduli of elasticity in tension and compression are equal.

(iii) The material is homogeneous and obeys Hooke's law.

The distribution of stresses in a curved beam is shown in Fig. 4.64. There are two factors, which distinguish the analysis of straight and curved beams. They are as follows:

(i) The neutral and centroidal axes of the straight beam are coincident. However, in a curved beam the neutral axis is shifted towards the centre of curvature.

(ii) The bending stresses in a straight beam vary linearly with the distance from the neutral axis. This is illustrated in Fig. 4.5. However in curved beams, the stress distribution is hyperbolic.

The following notations are used in Fig. 4.64:

R_o = radius of outer fibre (mm)
R_i = radius of inner fibre (mm)
R = radius of centroidal axis (mm)
R_N = radius of neutral axis (mm)
h_i = distance of inner fibre from neutral axis (mm)
h_o = distance of outer fibre from neutral axis (mm)
M_b = bending moment with respect to centroidal axis (N-mm)
A = area of the cross-section (mm^2)

The eccentricity e between centroidal and neutral axes is given by,

$$e = R - R_N \tag{4.54}$$

The bending stress (σ_b) at a fibre, which is at a distance of y from the neutral axis is given by,

$$\sigma_b = \frac{M_b y}{Ae(R_N - y)} \tag{4.55}$$

The equation indicates the hyperbolic distribution of (σ_b) with respect to y. The maximum stress occurs either at the inner fibre or at the outer fibre. The bending stress at the inner fibre is given by,

$$\sigma_{bi} = \frac{M_b h_i}{AeR_i} \tag{4.56}$$

Similarly, the bending stress at the outer fibre is given by,

$$\sigma_{bo} = \frac{M_b h_o}{AeR_o} \tag{4.57}$$

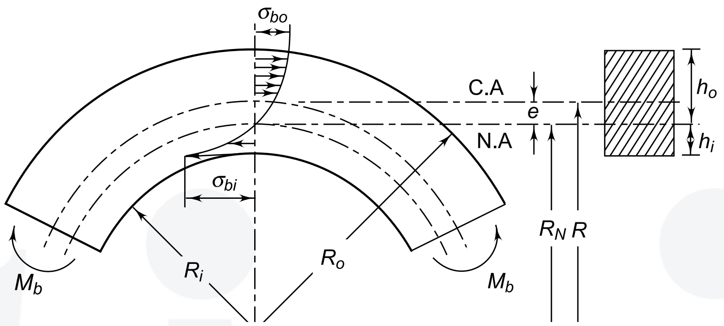


Fig. 4.64 *Stresses in Curved Beam (C.A. = centroidal axis; N.A. = neutral axis)*

In symmetrical cross-sections, such as circular or rectangular, the maximum bending stress always occurs at the inner fibre. In unsymmetrical cross-sections, it is necessary to calculate the stresses at the inner as well as outer fibres to determine the maximum stress. *In most of the engineering problems, the magnitude of e is very small and it should be calculated precisely to avoid a large percentage error in the final results.* The nomenclature for commonly used cross-sections of curved beams is illustrated in Fig. 4.65.

For rectangular cross-section [Fig. 4.65(a)],

$$R_N = \frac{h}{\log_e\left(\dfrac{R_o}{R_i}\right)} \tag{4.58}$$

and

$$R = R_i + \frac{h}{2} \tag{4.59}$$

For circular cross-section [Fig. 4.65(b)],

$$R_N = \frac{\left(\sqrt{R_o} + \sqrt{R_i}\right)^2}{4} \tag{4.60}$$

and

$$R = R_i + \frac{d}{2} \tag{4.61}$$

For trapezoidal cross-section [Fig. 4.65(c)],

$$R_N = \frac{\left(\dfrac{b_i + b_o}{2}\right)h}{\left(\dfrac{b_i R_o - b_o R_i}{h}\right)\log_e\left(\dfrac{R_o}{R_i}\right) - (b_i - b_o)} \tag{4.62}$$

and

$$R = R_i + \frac{h\,(b_i + 2b_o)}{3(b_i + b_o)} \tag{4.63}$$

For an *I*-section beam [Fig. 4.65(d)],

$$R_N = \frac{t_i\,(b_i - t) + t_o\,(b_o - t) + th}{b_i\,\log_e\left(\dfrac{R_i + t_i}{R_i}\right) + t\,\log_e\left(\dfrac{R_o - t_o}{R_i + t_i}\right) + b_o\,\log_e\left(\dfrac{R_o}{R_o - t_o}\right)} \tag{4.64}$$

$$R = R_i + \frac{\frac{1}{2}th^2 + \frac{1}{2}t_i^2(b_i - t) + t_o(b_o - t)(h - t_o/2)}{t_i(b_i - t) + t_o(b_o - t) + th} \tag{4.65}$$

For a *T*-section beam [Fig. 4.65(e)],

$$R_N = \frac{t_i(b_i - t) + th}{(b_i - t)\log_e\left(\dfrac{R_i + t_i}{R_i}\right) + t\log_e\left(\dfrac{R_o}{R_i}\right)} \tag{4.66}$$

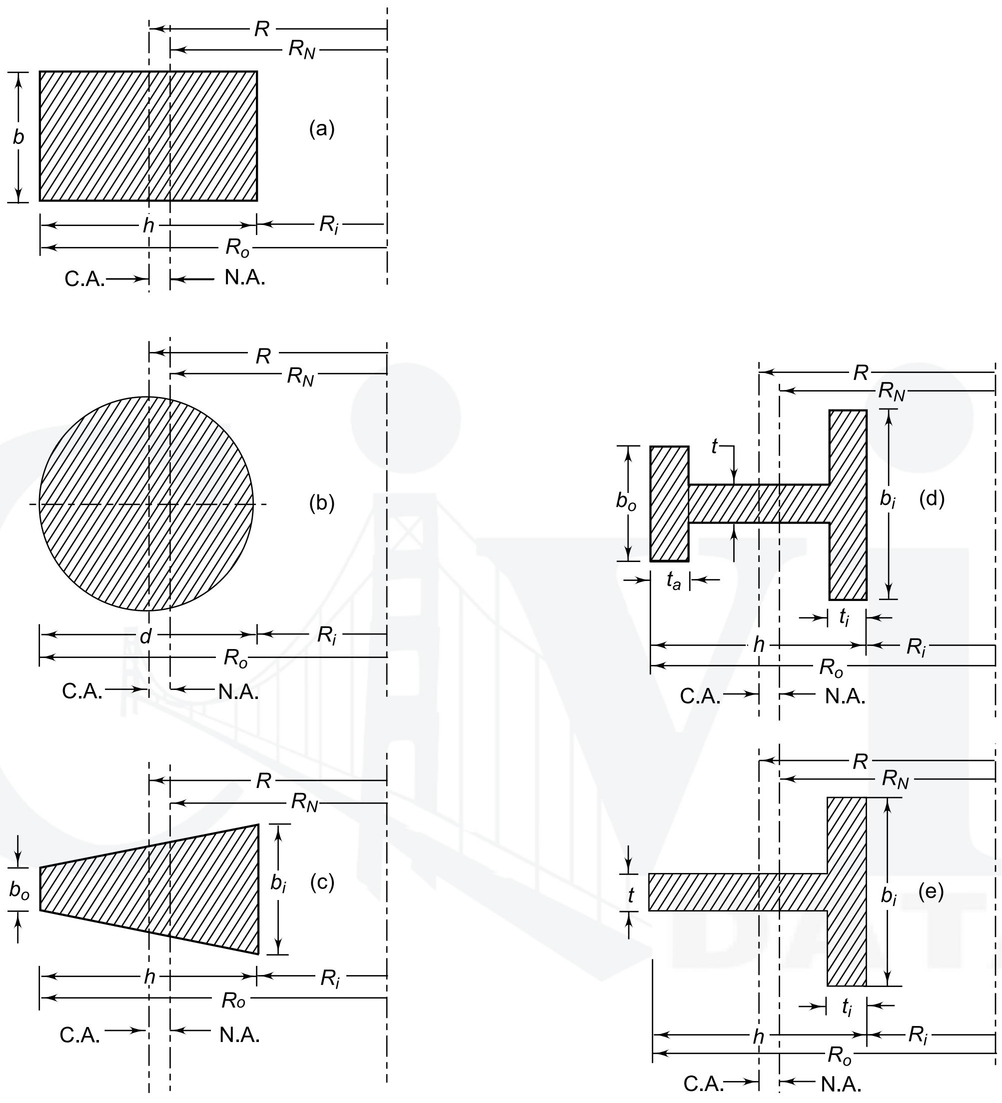


Fig. 4.65 *Nomenclature of Cross-section for Curved Beams*

$$R = R_i + \frac{\frac{1}{2}th^2 + \frac{1}{2}t_i^2(b_i - t)}{th + t_i(b_i - t)} \tag{4.67}$$

Example 4.19 *A crane hook having an approximate trapezoidal cross-section is shown in Fig. 4.66. It is made of plain carbon steel 45C8 (S_{yt} = 380 N/mm²) and the factor of safety is 3.5. Determine the load carrying capacity of the hook.*

Solution

Given $S_{yt} = 380$ N/mm² $(fs) = 3.5$
$b_i = 90$ mm $b_o = 30$ mm $h = 120$ mm
$R_o = 170$ mm $R_i = 50$ mm

Step I *Calculation of permissible tensile stress*

$$\sigma_{max.} = \frac{S_{yt}}{(fs)} = \frac{380}{3.5} = 108.57 \text{ N/mm}^2$$

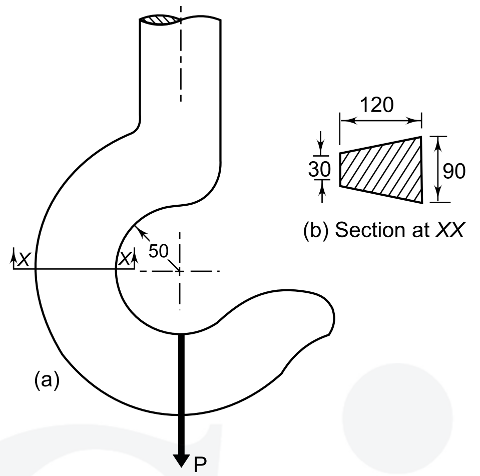


Fig. 4.66

Step II *Calculation of eccentricity (e)*

For the cross-section *XX* [Eqs (4.62) and (4.63)],

$$R_N = \frac{\left(\frac{b_i + b_o}{2}\right)h}{\left(\frac{b_i R_o - b_o R_i}{h}\right)\log_e\left(\frac{R_o}{R_i}\right) - (b_i - b_o)}$$

$$R_N = \frac{\left(\frac{90+30}{2}\right)(120)}{\left(\frac{90\times170 - 30\times50}{120}\right)\log_e\left(\frac{170}{50}\right) - (90-30)}$$

$$= 89.1816 \text{ mm}$$

$$R = R_i + \frac{h(b_i + 2b_o)}{3(b_i + b_o)}$$

$$= 50 + \frac{120(90 + 2\times30)}{3(90+30)} = 100 \text{ mm}$$

$$e = R - R_N = 100 - 89.1816 = 10.8184 \text{ mm}$$

Step III *Calculation of bending stress*

$$h_i = R_N - R_i = 89.1816 - 50 = 39.1816 \text{ mm}$$

$$A = \frac{1}{2}[h(b_i + b_o)] = \frac{1}{2}[(120)(90+30)]$$

$$= 7200 \text{ mm}^2$$

$$M_b = PR = (100\,P) \text{ N-mm}$$

From Eq. (4.56), the bending stress at the inner fibre is given by,

$$\sigma_{bi} = \frac{M_b h_i}{AeR_i} = \frac{(100P)(39.1816)}{(7200)(10.8184)(50)}$$

$$= \frac{(7.2435)P}{(7200)} \text{ N/mm}^2 \qquad \text{(i)}$$

Step IV *Calculation of direct tensile stress*

$$\sigma_t = \frac{P}{A} = \frac{P}{(7200)} \text{ N/mm}^2 \qquad \text{(ii)}$$

Step V *Calculation of load carrying capacity*

Superimposing the two stresses and equating the resultant to permissible stress, we have

$$\sigma_{bi} + \sigma_t = \sigma_{max.}$$

$$\frac{(7.2435)P}{7200} + \frac{P}{7200} = 108.57$$

$$P = 94\,827.95 \text{ N}$$

Example 4.20 *A curved link of the mechanism made from a round steel bar is shown in Fig. 4.67. The material of the link is plain carbon steel 30C8 (S_{yt} = 400 N/mm^2) and the factor of safety is 3.5. Determine the dimensions of the link.*

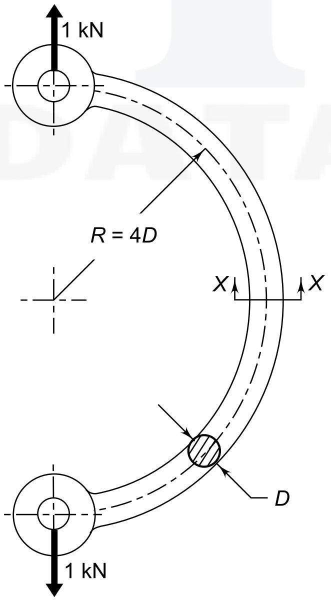


Fig. 4.67

Solution

Given $P = 1$ kN $S_{yt} = 400$ N/mm^2
$(fs) = 3.5$

Step I *Calculation of permissible tensile stress*

$$\sigma_{max.} = \frac{S_{yt}}{(fs)} = \frac{400}{3.5} = 114.29 \text{ N/mm}^2$$

Step II *Calculation of eccentricity (e)*

At the section XX,

$R = 4D$

$R_i = 4D - 0.5D = 3.5D$

$R_o = 4D + 0.5D = 4.5D$

From Eq. (4.60),

$$R_N = \frac{\left(\sqrt{R_o} + \sqrt{R_i}\right)^2}{4}$$

$$= \frac{\left(\sqrt{4.5D} + \sqrt{3.5D}\right)^2}{4} = 3.9843D$$

$e = R - R_N = 4D - 3.9843\,D = 0.0157D$

Step III *Calculation of bending stress*

$h_i = R_N - R_i = 3.9843D - 3.5D = 0.4843D$

$$A = \frac{\pi}{4} D^2 = (0.7854D^2) \text{ mm}^2$$

$M_b = 1000 \times 4D = (4000D)$ N-mm

From Eq. (4.56), the bending stress at the inner fibre is given by,

$$\sigma_{bi} = \frac{M_b h_i}{AeR_i} = \frac{(4000D)(0.4843D)}{(0.7854D^2)(0.0157D)(3.5D)}$$

$$= \left(\frac{44\,886.51}{D^2}\right) \text{N/mm}^2 \quad \text{(i)}$$

Step IV *Calculation of direct tensile stress*

$$\sigma_t = \frac{P}{A} = \frac{1000}{(0.7854D^2)} = \left(\frac{1273.24}{D^2}\right) \text{N/mm}^2 \quad \text{(ii)}$$

Step V *Calculation of dimensions of link*

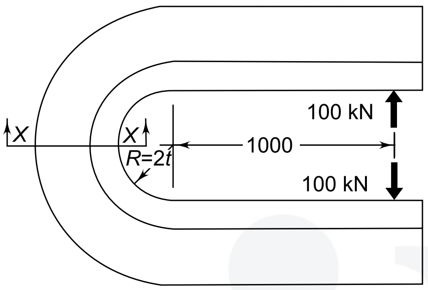
Superimposing the bending and direct tensile stresses and equating the resultant stress to permissible stress, we have

$$\sigma_{bi} + \sigma_t = \sigma_{max.}$$

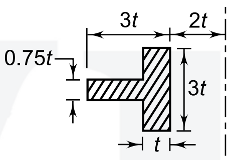
$$\left(\frac{44\,886.51}{D^2}\right) + \left(\frac{1273.24}{D^2}\right) = 114.29$$

$D = 20.10$ mm

Example 4.21 *The C-frame of a 100 kN capacity press is shown in Fig. 4.68(a). The material of the frame is grey cast iron FG 200 and the factor of safety is 3. Determine the dimensions of the frame.*



(a)



(b) Section at XX

Fig. 4.68

Solution

Given $P = 100$ kN $S_{ut} = 200$ N/mm^2
$(fs) = 3$

Step I *Calculation of permissible tensile stress*

$$\sigma_{max.} = \frac{S_{ut}}{(fs)} = \frac{200}{3} = 66.67 \text{ N/mm}^2$$

Step II *Calculation of eccentricity (e)*

Using notations of Eq. (4.66) and Fig. [4.65(e)],

$b_i = 3t$ $h = 3t$ $R_i = 2t$

$R_o = 5t$ $t_i = t$ $t = 0.75t$

From Eq. (4.66),

$$R_N = \frac{t_i(b_i - t) + th}{(b_i - t)\log_e\left(\dfrac{R_i + t_i}{R_i}\right) + t\log_e\left(\dfrac{R_o}{R_i}\right)}$$

$$= \frac{t(3t - 0.75t) + 0.75t(3t)}{(3t - 0.75t)\log_e\left(\dfrac{2t + t}{2t}\right) + 0.75t\log_e\left(\dfrac{5t}{2t}\right)}$$

$= 2.8134t$

From Eq. (4.67),

$$R = R_i + \frac{\frac{1}{2}th^2 + \frac{1}{2}t_i^2(b_i - t)}{th + t_i(b_i - t)}$$

$$= 2t + \frac{\frac{1}{2}(0.75t)(3t)^2 + \frac{1}{2}t^2(3t - 0.75t)}{(0.75t)(3t) + t(3t - 0.75t)} = 3t$$

$$e = R - R_N = 3t - 2.8134t = 0.1866t$$

Step III *Calculation of bending stress*

$$h_i = R_N - R_i = 2.8134t - 2t = 0.8134t$$

$$A = (3t)(t) + (0.75t)(2t) = (4.5t^2) \text{ mm}^2$$

$$M_b = 100 \times 10^3(1000 + R)$$

$$= 100 \times 10^3(1000 + 3t) \text{ N-mm}$$

From Eq. (4.56), the bending stress at the inner fibre is given by,

$$\sigma_{bi} = \frac{M_b h_i}{AeR_i} = \frac{100 \times 10^3(1000 + 3t)(0.8134t)}{(4.5t^2)(0.1866t)(2t)}$$

$$= \frac{100 \times 10^3(1000 + 3t)(2.1795)}{(4.5t^2)} \text{ N/mm}^2$$

Step IV *Calculation of direct tensile stress*

$$\sigma_t = \frac{P}{A} = \frac{100 \times 10^3}{(4.5t^2)} \text{ N/mm}^2$$

Step V *Calculation of dimensions of cross-section*

Adding the two stresses and equating the resultant stress to permissible stress,

$$\sigma_{bi} + \sigma_t = \sigma_{max.}$$

$$\frac{100 \times 10^3(1000 + 3t)(2.1795)}{(4.5t^2)} + \frac{100 \times 10^3}{(4.5t^2)} = 66.67$$

$$t^3 - 2512.83t - 726\,500 = 0$$

Solving the above cubic equation by trial and error method,

$$t = 99.2 \text{ mm} \quad \text{or} \quad t = 100 \text{ mm}$$

4.23 THERMAL STRESSES

When a machine component is subjected to change in temperature, it expands or contracts. If the machine component is allowed to expand or contract freely, no stresses are induced in the component. However, if the expansion or contraction of the component is restricted, stresses are induced in the component. Such stresses, which are caused due to variation in temperature, are called *thermal stresses*.

Consider a rod of length l as shown in Fig. 4.69(a). It is assembled at room temperature and does not have any provision for axial expansion. When the rod is free as shown in Fig. 4.69(b) and the temperature increases by an amount ΔT, the expansion of the rod is given by,

$$\delta = \alpha\, l\, \Delta T \qquad (4.68)$$

where,

δ = expansion of the rod (mm)

l = length of rod (mm)

α = coefficient of thermal expansion (per °C)

ΔT = temperature rise (°C)

The strain ε is given by,

$$\varepsilon = \frac{\delta}{l} = \alpha \Delta T$$

$$\therefore \qquad \varepsilon = \alpha\, \Delta T \qquad (4.69)$$

The thermal stress is given by,

$$\sigma = E\,\varepsilon = E\alpha\,\Delta T$$

$$\therefore \qquad \sigma = -\,\alpha E \Delta T \qquad (4.70)$$

where,

E = modulus of elasticity (N/mm^2)

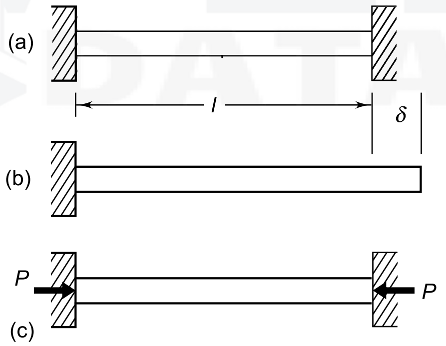


Fig. 4.69 *Thermal Stresses*

Figure 4.69(c) shows the bar which is forced to its original length l by means of an imaginary thermal force P. It shows that when longitudinal expansion is prevented, stress is developed which elastically compresses the rod by the amount $\delta = l\varepsilon$.

Since the stress is compressive, a negative sign is introduced in Eq. (4.70).

We have considered the expansion of the rod in one direction and derived Eq. (4.70). A similar procedure can be extended to a flat plate prevented from expansion in its plane in the x and y directions. It can be proved that for two-directional expansion of the plate, the stress equation is given by,

$$\sigma_x = \sigma_y = -\frac{\alpha E \Delta T}{1-\mu} \qquad (4.71)$$

where μ is Poisson's ratio (0.3).

Similarly, for a three-dimensional box, which is restrained from expansion on all sides, the stress equation is given by,

$$\sigma_x = \sigma_y = \sigma_z = -\frac{\alpha E \Delta T}{1-2\mu} \qquad (4.72)$$

The thermal stresses are important in design of certain components like shrinkage assemblies, compound cylinders, pipelines, parts of internal combustion engines and steam engineering equipment. Such stresses are relieved in certain applications by using expansion joints or sliding supports.

Example 4.22 *A hollow steel tube is assembled at 25°C with fixed ends as shown in Fig. 4.70(a). At this temperature, there is no stress in the tube. The length and cross–sectional area of the tube are 200 mm and 300 mm² respectively. During operating conditions, the temperature of the tube increases to 250°C. It is observed that at this temperature, the fixed ends are separated by 0.15 mm as shown in Fig. 4.70 (b). The modulus of elasticity and coefficient of thermal expansion of steel are 207 000 N/mm² and 10.8 × 10⁻⁶ per °C respectively. Calculate the force acting on the tube and the resultant stress.*

Solution

Given $\Delta T = (250 - 25)$°C $\quad A = 300$ mm²
$l = 200$ mm $\quad E = 207\,000$ N/mm²
$\alpha = 10.8 \times 10^{-6}$ per °C
joint separation = 0.15 mm

Step I *Calculation of expansion of tube*

From Eq. (4.68),

$$\delta = \alpha l \Delta T = (10.8 \times 10^{-6})(200)(250-25) = 0.486 \text{ mm}$$

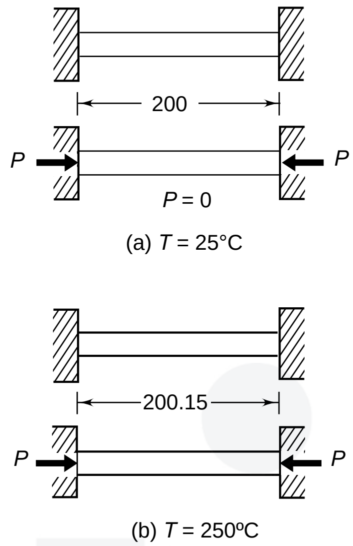


Fig. 4.70

Step II *Calculation of net compression of tube*

When the tube is free to expand, its length will increase by 0.486 mm. However, the fixed ends are separated by 0.15 mm only. Therefore,

Net compression of tube = 0.486–0.15 = 0.336 mm

Step III *Calculation of force*

$$\delta = \frac{PL}{AE} \quad \text{or} \quad 0.336 = \frac{P(200)}{(300)(207000)}$$

$\therefore \quad P = 104\,328$ N

Step IV *Calculation of resultant stress*

$$\sigma = \frac{P}{A} = \frac{104\,328}{300} = 347.76 \text{ N/mm}^2$$

4.24 RESIDUAL STRESSES

Stresses can be classified into two groups, viz., *load stresses* and *residual stresses*. Residual stresses are also called *internal stresses* or *locked-in stresses*. Load stresses are those stresses which remain within the elastic limit and which are induced by external forces. Load stresses return to zero when the external forces are removed. Such stresses can be calculated by equations from Strength of Materials. Residual stresses are those stresses that are independent of external forces. They are usually induced as a result of manufacturing processes and assembly operations. When the machine component with residual stresses is put into service, the load stresses are superimposed on the residual stress. Residual stresses may be harmful or beneficial. If the residual stresses add to the load stresses, they are harmful. On the other hand, if residual stresses are opposite to load stresses and subtract, they are beneficial. The residual stresses are induced due to the following factors:

(i) manufacturing processes like casting and forging,
(ii) machining methods like turning, milling and grinding,
(iii) rolling, extrusion and cold working processes,
(iv) chemical processes like oxidation, corrosion and electroplating,
(v) heat treatment processes like quenching, and
(vi) assembly operations involving some misalignment.

It is observed that it is practically difficult to avoid residual stresses in any component.

It is very important to consider residual stresses when the component is subjected to fluctuating stresses and failure occurs due to fatigue. Residual stresses may either improve the endurance limit of the component or affect it adversely. The growth of fatigue crack is due to tensile stresses. If the residual stresses in the surface of the component are compressive, the growth of a fatigue crack is retarded and the endurance limit is improved. Therefore, residual compressive stresses are purposely induced in the surface of components subjected to fatigue loading. Such parts are subjected to the *shot peening process*, which build compressive stresses into the surface of the component. This improves the endurance limit of the component. Residual stresses are also beneficial in some applications like compound cylinders and press-fitted or shrink-fitted assemblies.

Short-Answer Questions

4.1 What is a static load?
4.2 What is a ductile material? Give its examples.
4.3 What is a brittle material? Give its examples.
4.4 What is elastic limit?
4.5 What is yield point?
4.6 What are the three basic modes of failure of mechanical components?
4.7 Give examples of mechanical components that fail by elastic deflection.
4.8 Give examples of mechanical components that fail by general yielding.
4.9 Give examples of mechanical components that fail by fracture.
4.10 What is factor of safety?
4.11 Why is it necessary to use factor of safety?
4.12 What is allowable stress?
4.13 How will you find out allowable stress for ductile parts using factor of safety?
4.14 How will you find out allowable stress for brittle parts using factor of safety?
4.15 What is the magnitude of factor of safety for cast iron components?
4.16 What is the magnitude of factor of safety for ductile components?
4.17 'When a thick leather belt is bent, cracks appear on the outer surface, while folds on the inside'. Why?
4.18 What is a cotter joint?
4.19 Where do you use a cotter joint? Give practical examples.
4.20 Why is cotter provided with a taper? Why is a taper provided only on one side?
4.21 What are the advantages of a cotter joint?
4.22 What is a knuckle joint?
4.23 Where do you use a knuckle joint? Give practical examples.
4.24 What are the advantages of a knuckle joint?
4.25 What is advantage of using the theories of elastic failures?
4.26 What are the important theories of elastic failures?
4.27 State maximum principal stress theory of failure.
4.28 Where do you use maximum principal stress theory of failure?
4.29 State maximum shear stress theory of failure.
4.30 Where do you use maximum shear stress theory of failure?
4.31 State distortion energy theory of failure.

4.32 Where do you use distortion energy theory of failure?

4.33 What is fracture mechanics?

4.34 What is stress intensity factor in fracture mechanics?

4.35 What is fracture toughness in fracture mechanics?

4.36 What is a curved beam? Give practical examples of machine components made of curved beams.

4.37 Distinguish stress distribution in curved and straight beams.

Problems for Practice

4.1 Two rods are connected by means of a knuckle joint as shown in Fig. 4.18. The axial force *P* acting on the rods is 25 kN. The rods and the pin are made of plain carbon steel 45C8 (S_{yt} = 380 N/mm^2) and the factor of safety is 2.5. The yield strength in shear is 57.7% of the yield strength in tension. Calculate: (i) the diameter of the rods, and (ii) the diameter of the pin.

[*(i) 14.47 mm (ii) 13.47 mm*]

4.2 The force acting on a bolt consists of two components—an axial pull of 12 kN and a transverse shear force of 6 kN. The bolt is made of steel FeE 310 (S_{yt} = 310 N/mm^2) and the factor of safety is 2.5. Determine the diameter of the bolt using the maximum shear stress theory of failure. *(13.2 mm)*

4.3 The layout of a wall crane and the pin-joint connecting the tie-rod to the crane post is shown in Fig. 4.71(a) and (b) respectively. The tension in the tie-rod is maximum, when the load is at a distance of 2 m from the wall. The tie-rod and the pin are made of steel FeE 250 (S_{yt} = 250 N/mm^2) and the factor of safety is 3. Calculate the diameter of the tie-rod and the pin.

(34.96 and 34 .96 mm)

4.4 A *C*-frame subjected to a force of 15 kN is shown in Fig. 4.72. It is made of grey cast iron FG 300 and the factor of safety is 2.5. Determine the dimensions of the cross-section of the frame.

(t = 15.81 mm)

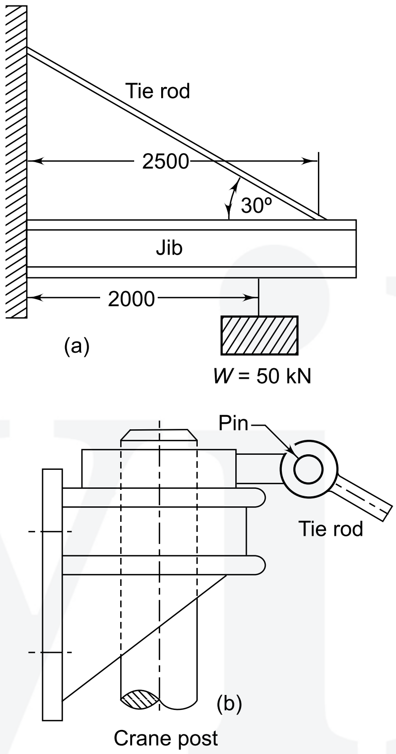


Fig. 4.71

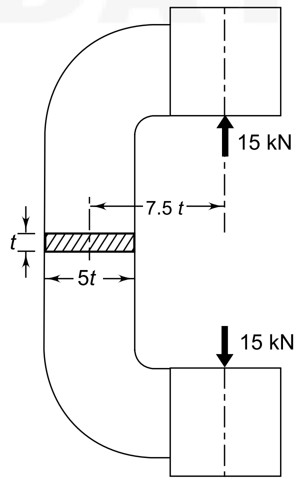


Fig. 4.72

4.5 The principal stresses induced at a point in a machine component made of steel 50C4 (S_{yt} = 460 N/mm^2) are as follows:

σ_1 = 200N/mm^2 σ_2 = 150 N/mm^2 σ_3 = 0

Calculate the factor of safety by (i) the maximum shear stress theory, and (ii) the distortion energy theory.

[*(i) 2.3 (ii) 2.55*]

4.6 The stresses induced at a critical point in a machine component made of steel 45C8 (S_{yt} = 380 N/mm^2) are as follows:

σ_x = 100 N/mm^2 σ_y = 40 N/mm^2

τ_{xy} = 80 N/mm^2

Calculate the factor of safety by (i) the maximum normal stress theory, (ii) the maximum shear stress theory, and (iii) the distortion energy theory.

[*(i) 2.44 (ii) 2.22 (iii) 2.32*]

4.7 A link of *S*-shape made of a round steel bar is shown in Fig. 4.73. It is made of plain carbon steel 45C8 (S_{yt} = 380 N/mm^2) and the factor of safety is 4.5. Calculate the dimensions of the link.

(d = 23.38 mm)

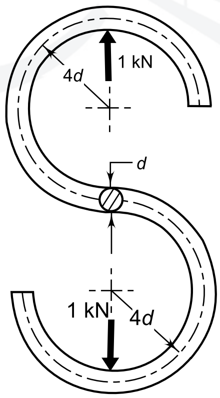


Fig. 4.73

4.8 The frame of a 100 kN capacity press is shown in Fig. 4.74. It is made of grey cast iron FG 300 and the factor of safety is 2.5. Determine the dimensions of the cross-section at *XX*.

(t = 26.62 mm)

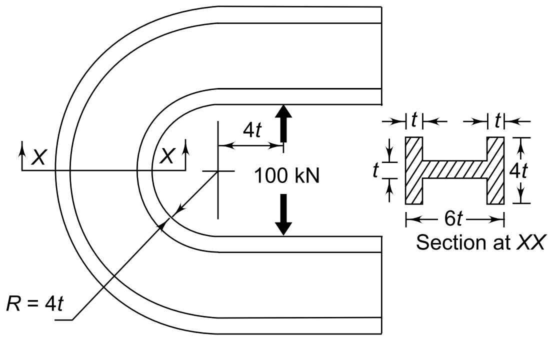


Fig. 4.74

4.9 A bell crank lever is subjected to a force of 7.5 kN at the short arm end. The lengths of the short and long arms are 100 and 500 mm respectively. The arms are at right angles to each other. The lever and the pins are made of steel FeE 300 (S_{yt} = 300 N/mm^2) and the factor of safety is 5. The permissible bearing pressure on the pin is 10 N/mm^2. The lever has a rectangular cross-section and the ratio of width to thickness is 4 : 1. The length to diameter ratio of the fulcrum pin is 1.5 : 1. Calculate:

(i) the diameter and the length of the fulcrum pin

(ii) the shear stress in the pin

(iii) the dimensions of the boss of the lever at the fulcrum pin

(iv) the dimensions of the cross-section of the lever

Assume that the arm of the bending moment on the lever extends up to the axis of the fulcrum.

[*(i) 22.58 and 33.87 mm (ii) 9.55 N/mm^2*
(iii) D_i = 23 mm D_0 = 46 mm
length = 34 mm, (iv) 16.74 × 66.94 mm]

4.10 A bracket, made of steel FeE 200 (S_{yt} = 200 N/mm^2) and subjected to a force of 5 kN acting at an angle of 30° to the vertical, is shown in Fig. 4.75. The factor of safety is 4. Determine the dimensions of the cross-section of the bracket.

[*t = 33.5 mm*]

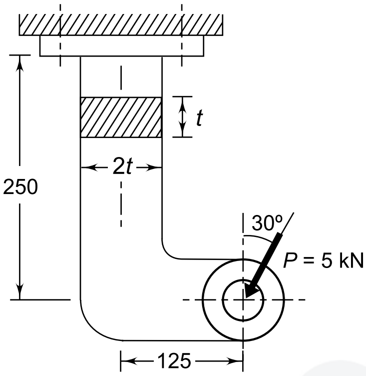


Fig. 4.75

4.11 Figure 4.76 shows a C-clamp, which carries a load P of 25 kN. The cross-section of the clamp is rectangular and the ratio of width to thickness (b/t) is 2 : 1. The clamp is made of cast steel of Grade 20-40 (S_{ut} = 400 N/mm^2) and the factor of safety is 4. Determine the dimensions of the cross-section of the clamp.

[*t = 38.5 mm*]

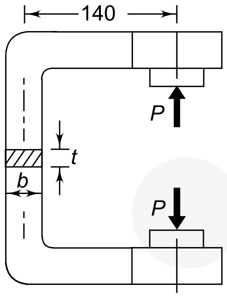


Fig. 4.76

Design against Fluctuating Load

Chapter 5

5.1 STRESS CONCENTRATION

In design of machine elements, the following three fundamental equations are used,

$$\sigma_t = \frac{P}{A}$$

$$\sigma_b = \frac{M_b y}{I}$$

$$\tau = \frac{M_t r}{J}$$

The above equations are called *elementary* equations. These equations are based on a number of assumptions. One of the assumptions is that there are no discontinuities in the cross-section of the component. However, in practice, discontinuities and abrupt changes in cross-section are unavoidable due to certain features of the component such as oil holes and grooves, keyways and splines, screw threads and shoulders. Therefore, it cannot be assumed that the cross-section of the machine component is uniform. Under these circumstances, the 'elementary' equations do not give correct results.

A plate with a small circular hole, subjected to tensile stress is shown in Fig. 5.1. The distribution of stresses near the hole can be observed by using the *Photo-elasticity technique*. In this method, an identical model of the plate is made of epoxy resin. The model is placed in a circular polariscope and loaded at the edges. It is observed that there is a sudden rise in the magnitude of stresses in the vicinity of the hole. The localized stresses in the neighbourhood of the hole are far greater than the stresses obtained by elementary equations.

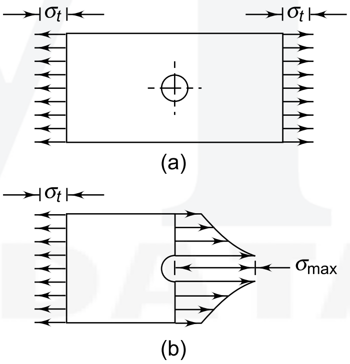


Fig. 5.1 *Stress Concentration*

Stress concentration is defined as the localization of high stresses due to the irregularities present in the component and abrupt changes of the cross-section.

In order to consider the effect of stress concentration and find out localized stresses, a factor called *stress concentration factor* is used. It is denoted by K_t and defined as,

$$K_t = \frac{\text{Highest value of actual stress near discontinuity}}{\text{Nominal stress obtained by elementary equations for minimum croos-section}}$$

or $$K_t = \frac{\sigma_{max.}}{\sigma_0} = \frac{\tau_{max.}}{\tau_0} \qquad (5.1)$$

where σ_0 and τ_0 are stresses determined by elementary equations and $\sigma_{max.}$ and $\tau_{max.}$ are localized stresses at the discontinuities. The subscript t denotes the '*theoretical*' stress concentration factor. The magnitude of stress concentration factor depends upon the geometry of the component.

The causes of stress concentration are as follows:

(i) Variation in Properties of Materials In design of machine components, it is assumed that the material is homogeneous throughout the component. In practice, there is variation in material properties from one end to another due to the following factors:

(a) internal cracks and flaws like blow holes;
(b) cavities in welds;
(c) air holes in steel components; and
(d) nonmetallic or foreign inclusions.

These variations act as discontinuities in the component and cause stress concentration.

(ii) Load Application Machine components are subjected to forces. These forces act either at a point or over a small area on the component. Since the area is small, the pressure at these points is excessive. This results in stress concentration. The examples of these load applications are as follows:

(a) Contact between the meshing teeth of the driving and the driven gear
(b) Contact between the cam and the follower
(c) Contact between the balls and the races of ball bearing
(d) Contact between the rail and the wheel
(e) Contact between the crane hook and the chain

In all these cases, the concentrated load is applied over a very small area resulting in stress concentration.

(iii) Abrupt Changes in Section In order to mount gears, sprockets, pulleys and ball bearings on a transmission shaft, steps are cut on the shaft and shoulders are provided from assembly considerations. Although these features are essential, they create change of the cross-section of the shaft. This results in stress concentration at these cross-sections.

(iv) Discontinuities in the Component Certain features of machine components such as oil holes or oil grooves, keyways and splines, and screw threads result in discontinuities in the cross-section of the component. There is stress concentration in the vicinity of these discontinuities.

(v) Machining Scratches Machining scratches, stamp marks or inspection marks are surface irregularities, which cause stress concentration.

5.2 STRESS CONCENTRATION FACTORS

The stress concentration factors are determined by two methods, viz., the *mathematical method* based on the theory of elasticity and *experimental methods* like photo-elasticity. For simple geometric shapes, the stress concentration factors are determined by photo-elasticity. The charts for stress concentration factors for different geometric shapes and conditions of loading were originally developed by RE Peterson[1]. At present, FEA packages are used to find out the stress concentration factor for any geometric shape.

The chart for the stress concentration factor for a rectangular plate with a transverse hole loaded in tension or compression is shown in Fig. 5.2. The nominal stress σ_o in this case is given by,

$$\sigma_o = \frac{P}{(w-d)t} \qquad (5.2)$$

where t is the plate thickness.

The values of stress concentration factor for a flat plate with a shoulder fillet subjected to tensile or compressive force are determined from Fig. 5.3. The nominal stress σ_o for this case is given by,

$$\sigma_o = \frac{P}{dt} \qquad (5.3)$$

[1] RE Peterson – *Stress concentration design factors*, John Wiley and Sons, Inc.

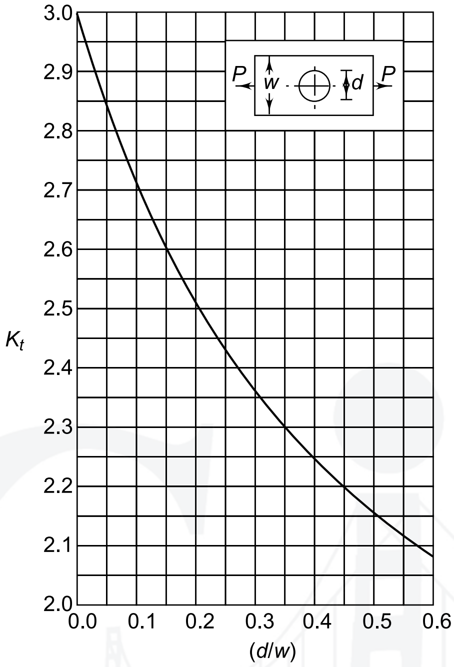


Fig. 5.2 *Stress Concentration Factor (Rectangular Plate with Transverse Hole in Tension or Compression)*

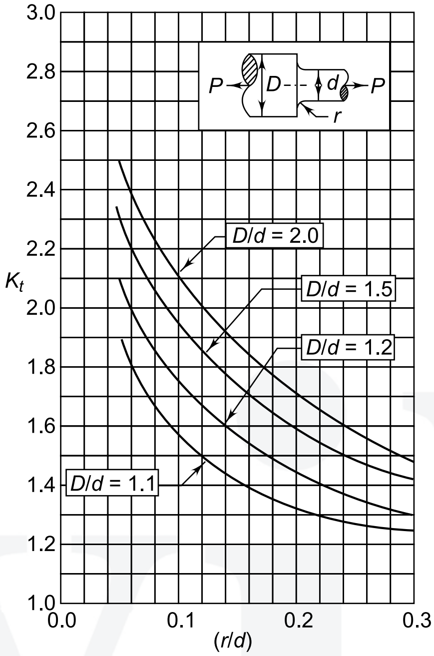


Fig. 5.4 *Stress Concentration Factor (Round Shaft with Shoulder Fillet in Tension)*

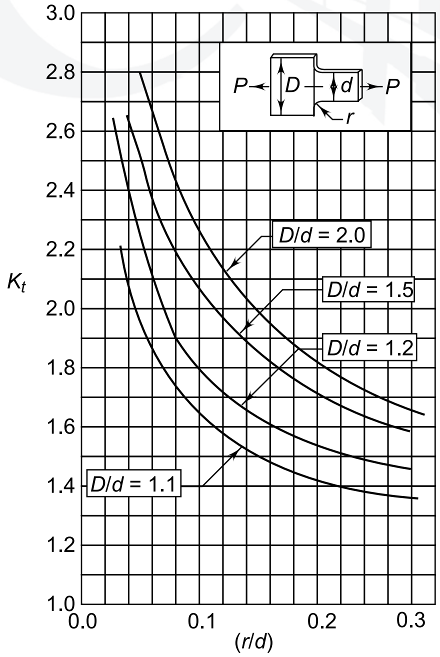


Fig. 5.3 *Stress Concentration Factor (Flat Plate with Shoulder Fillet in Tension or Compression)*

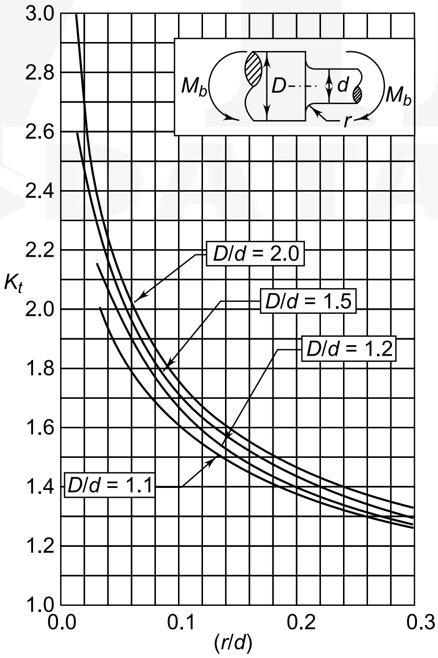


Fig. 5.5 *Stress Concentration Factor (Round Shaft with Shoulder Fillet in Bending)*

The charts for stress concentration factor for a round shaft with shoulder fillet subjected to tensile force, bending moment, and torsional moment are shown in Fig. 5.4, 5.5 and 5.6 respectively. The nominal stresses in these three cases are as follows:

(i) Tensile Force

$$\sigma_o = \frac{P}{\left(\frac{\pi}{4} d^2\right)} \tag{5.4}$$

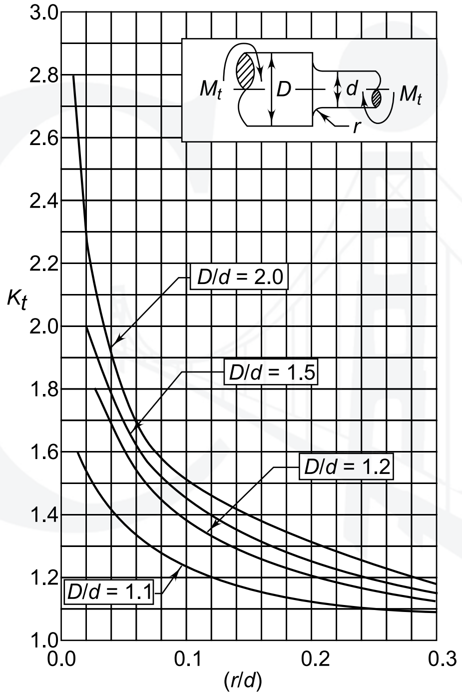


Fig. 5.6 *Stress Concentration Factor (Round Shaft with Shoulder Fillet in Torsion)*

(ii) Bending Moment

$$\sigma_o = \frac{M_b\, y}{I} \tag{5.5}$$

where $\quad I = \dfrac{\pi d^4}{64}$ and $y = \dfrac{d}{2}$

(iii) Torsional Moment

$$\tau_o = \frac{M_t\, r}{J} \tag{5.6}$$

where, $\quad J = \dfrac{\pi d^4}{32}$ and $r = \dfrac{d}{2}$

In practice, there are a number of geometric shapes and conditions of loading. A separate chart for the stress concentration factor should be used for each case.

It is possible to find out the stress concentration factor for some simple geometric shapes using the Theory of elasticity. A flat plate with an elliptical hole and subjected to tensile force, is shown in Fig. 5.7. It can be proved using the Theory of elasticity that the theoretical stress concentration factor at the edge of hole is given by,

$$K_t = 1 + 2\left(\frac{a}{b}\right) \tag{5.7}$$

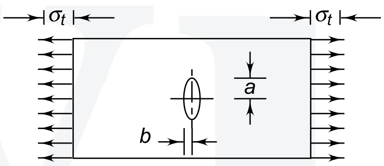


Fig. 5.7 *Stress Concentration due to Elliptical Hole*

where,

a = half width (or semi-major axis) of the ellipse perpendicular to the direction of the load

b = half width (or semi-minor axis) of the ellipse in the direction of the load

As b approaches zero, the ellipse becomes sharper and sharper. A very sharp crack is indicated and the stress at the edge of the crack becomes very large. It is observed from Eq. (5.7) that,

$$K_t = \infty \quad \text{when} \quad b = 0$$

Therefore, as the width of the elliptical hole in the direction of the load approaches zero, the stress concentration factor becomes infinity.

The ellipse becomes a circle when ($a = b$).

From Eq. (5.7),

$$K_t = 1 + 2\left(\frac{a}{b}\right) = 1 + 2 = 3$$

Therefore, the theoretical stress concentration factor due to a small circular hole in a flat plate, which is subjected to tensile force, is 3.

The stress concentration charts are based on either the photo-elastic analysis of the epoxy models using a circular polariscope or theoretical or finite element analysis of the mathematical model. That is why the factor is called *theoretical* stress concentration factor. The model is made of a different material than the actual material of the component. The ductility or brittleness of the material has a pronounced effect on its response to stress concentration. Also, the type of load—whether static or cyclic—affects the severity of stress concentration. Therefore, there is a difference between the stress concentration indicated by the theoretical stress concentration factor and the actual stress concentration in the component. The designer should consider the following guidelines:

(i) Ductile Materials Under Static Load Under a static load, ductile materials are not affected by stress concentration, to the extent that photo-elastic analysis might indicate. When the stress in the vicinity of the discontinuity reaches the yield point, there is plastic deformation, resulting in a redistribution of stresses. This plastic deformation or yielding is local and restricted to a very small area in the component. There is no perceptible damage to the part as a whole. Therefore, it is common practice to ignore the theoretical stress concentration factor for components that are made of ductile materials and subjected to static load.

(ii) Ductile Materials Under Fluctuating Load However, when the load is fluctuating, the stress at the discontinuities may exceed the endurance limit and in that case, the component may fail by fatigue. Therefore, endurance limit of the components made of ductile material is greatly reduced due to stress concentration. This accounts for the use of stress concentration factors for ductile components. However, some materials are more sensitive than others to stress raising notches under a fluctuating load. To account for this effect, a parameter called *notch sensitivity factor* is found for each material. The notch sensitivity factor is used to modify the theoretical stress concentration factor.

(iii) Brittle Materials The effect of stress concentration is more severe in case of brittle materials, due to their inability of plastic deformation. Brittle materials do not yield locally and there is no readjustment of stresses at the discontinuities. Once the local stress at the discontinuity reaches the fracture strength, a crack is formed. This reduces the material available to resist external load and also increases the stress concentration at the crack. The part then quickly fails. Therefore, stress concentration factors are used for components made of brittle materials subjected to both static load as well as fluctuating load.

5.3 REDUCTION OF STRESS CONCENTRATION

Although it is not possible to completely eliminate the effect of stress concentration, there are methods to reduce stress concentrations. This is achieved by providing a specific geometric shape to the component. In order to know what happens at the abrupt change of cross-section or at the discontinuity and reduce the stress concentration, understanding of *flow analogy* is useful. There is a similarity between velocity distribution in fluid flow in a channel and the stress distribution in an axially loaded plate shown in Fig. 5.8. The equations of flow potential in fluid mechanics and stress potential in solid mechanics are same. Therefore, it is perfectly logical to use fluid analogy to understand the phenomena of stress concentration.

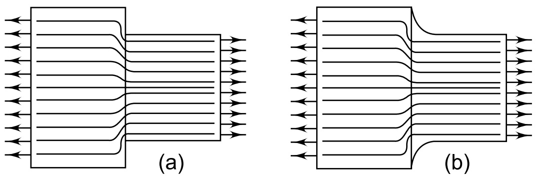


Fig. 5.8 *Force Flow Analogy: (a) Force Flow around Sharp Corner (b) Force Flow around Rounded Corner*

When the cross-section of a channel has uniform dimensions throughout, the velocities are uniform and the streamlines are equally spaced. The flow at any cross-section within the channel is given by,

$$Q = \int u dA \qquad \text{(a)}$$

When the cross-section of the plate has the same dimensions throughout, the stresses are uniform and stress lines are equally spaced. The stress at any section is given by,

$$P = \int \sigma dA \qquad \text{(b)}$$

When the cross-section of the channel is suddenly reduced, the velocity increases in order to maintain the same flow and the streamlines become narrower and narrower and crowd together. A similar phenomenon is observed in a stressed plate. In order to transmit the same force, the stress lines come closer and closer as the cross-section is reduced. At the change of cross-section, the streamlines as well as stress lines bend. When there is sudden change in cross-section, bending of stress lines is very sharp and severe resulting in stress concentration. Therefore, stress concentration can be greatly reduced by reducing the bending by rounding the corners. Streamlined shapes are used in channels to reduce turbulence and resistance to flow. *Streamlining*, or rounding the counters of mechanical components, has similar beneficial effects in reducing stress concentration. There are different methods to reduce the bending of the stress lines at the junction and reduce the stress concentration.

In practice, reduction of stress concentration is achieved by the following methods:

(i) Additional Notches and Holes in Tension Member A flat plate with a V-notch subjected to tensile force is shown in Fig. 5.9(a). It is observed that a single notch results in a high degree of stress concentration. The severity of stress concentration is reduced by three methods: (a) use of multiple notches; (b) drilling additional holes; and (c) removal of undesired material. These methods are illustrated in Fig. 5.9(b), (c) and (d) respectively. The method of removing undesired material is called the *principle of minimization of the material*. In these three methods, the sharp bending of a force flow line is reduced and it follows a smooth curve.

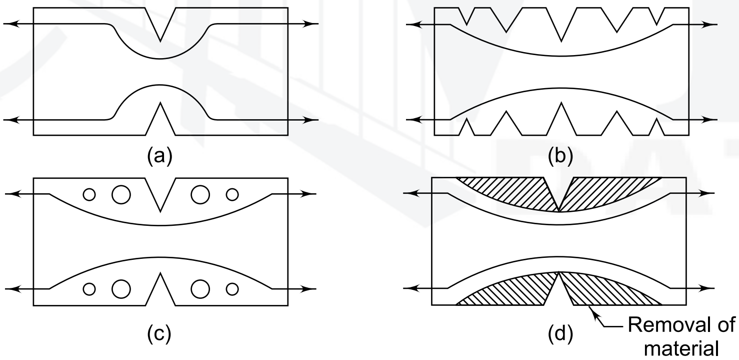


Fig. 5.9 *Reduction of Stress Con*centration due to V-notch: (a) Original Notch (b) Multiple Notches *(c) Drilled Holes (d) Removal of Undesirable Material*

(ii) Fillet Radius, Undercutting and Notch for Member in Bending A bar of circular cross-section with a shoulder and subjected to bending moment is shown in Fig. 5.10(a). Ball bearings, gears or pulleys are seated against this shoulder. The shoulder creates a change in cross-section of the shaft, which results in stress concentration. There are three methods to reduce stress concentration at the base of this shoulder. Figure 5.10(b) shows the shoulder with a fillet radius *r*. This results in gradual transition from small diameter to a large diameter. The fillet radius should be as large as possible in order to reduce stress concentration. In practice, the fillet radius is limited by the design of mating components.

The fillet radius can be increased by undercutting the shoulder as illustrated in Fig. 5.10(c). A notch results in stress concentration. Surprisingly, cutting an additional notch is an effective way to reduce stress concentration. This is illustrated in Fig. 5.10(d).

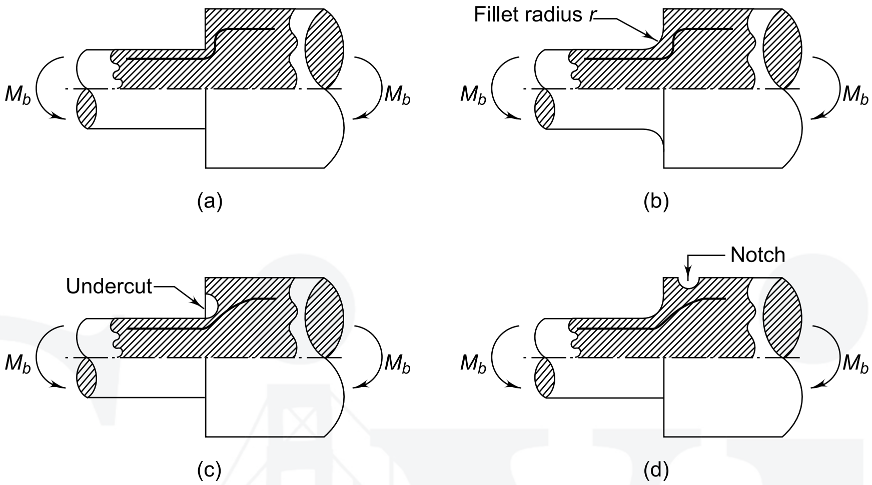


Fig. 5.10 *Reduction of Stress Concentration due to Abrupt Change in Cross-section: (a) Original Component (b) Fillet Radius (c) Undercutting (d) Addition of Notch*

(iii) Drilling Additional Holes for Shaft A transmission shaft with a keyway is shown in Fig. 5.11(a). The keyway is a discontinuity and results in stress concentration at the corners of the keyway and reduces torsional shear strength. An empirical relationship developed by HF Moore for the ratio C of torsional strength of a shaft having a keyway to torsional strength of a same sized shaft without a keyway is given by

$$C = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right) \tag{5.8}$$

where w and h are width and height dimensions of the keyway respectively and d is the shaft diameter. The four corners of the keyway, viz., m_1, m_2, n_1 and n_2 are shown in Fig. 5.11(c). It has been observed that torsional shear stresses at two points, viz. m_1 and m_2 are negligibly small in practice and theoretically equal to zero. On the other hand, the torsional shear stresses at two points, viz., n_1 and n_2 are excessive and theoretically infinite which means even a small torque will produce a permanent set at these points. Rounding corners at two points, viz., n_1 and n_2 by means of a fillet radius can reduce the stress concentration. A stress concentration factor $K_t = 3$ should be used when a shaft with a keyway is subjected to combined bending and torsional moments.

In addition to giving fillet radius at the inner corners of the keyway, there is another method of drilling two symmetrical holes on the sides of the keyway. These holes press the force flow lines and minimise their bending in the vicinity of the keyway. This method is illustrated in Fig. 5.11(b).

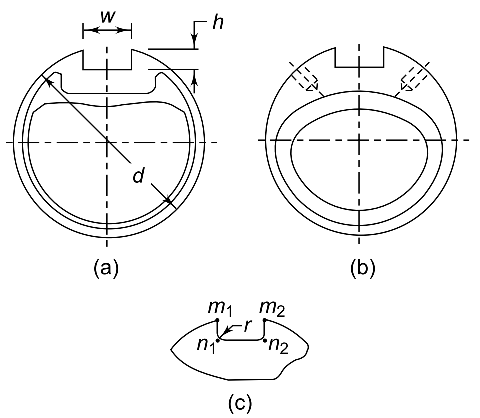


Fig. 5.11 *Reduction of Stress Concentration in Shaft with Keyway: (a) Original Shaft (b) Drilled Holes (c) Fillet Radius*

(iv) Reduction of Stress Concentration in Threaded Members A threaded component is shown in Fig. 5.12 (a). It is observed that the force flow line is bent as it passes from the shank portion to threaded portion of the component. This results in stress concentration in the transition plane. In Fig. 5.12(b), a small undercut is taken between the shank and the threaded portion of the component and a fillet radius is provided for this undercut. This reduces bending of the force flow line and consequently reduces stress concentration. An ideal method to reduce stress concentration is illustrated in Fig. 5.12(c), where the shank diameter is reduced and made equal to the core diameter of the thread. In this case, the force flow line is almost straight and there is no stress concentration.

Many discontinuities found in machine components cannot be avoided. Therefore, stress concentration cannot be totally eliminated. However, it can be greatly reduced by selecting the correct geometric shape by the designer. Many difficult problems involving stress concentration have been solved by removing material instead of adding it. Additional notches, holes and undercuts are the simple means to achieve significant reduction in stress concentration.

Example 5.1 *A flat plate subjected to a tensile force of 5 kN is shown in Fig. 5.13. The plate material is grey cast iron FG 200 and the factor of safety is 2.5. Determine the thickness of the plate.*

Solution

Given $P = 5$ kN $\quad S_{ut} = 200$ N/mm^2 $(fs) = 2.5$

Step I *Calculation of permissible tensile stress*

$$\sigma_{max.} = \frac{S_{ut}}{(fs)} = \frac{200}{2.5} = 80 \text{ N/mm}^2$$

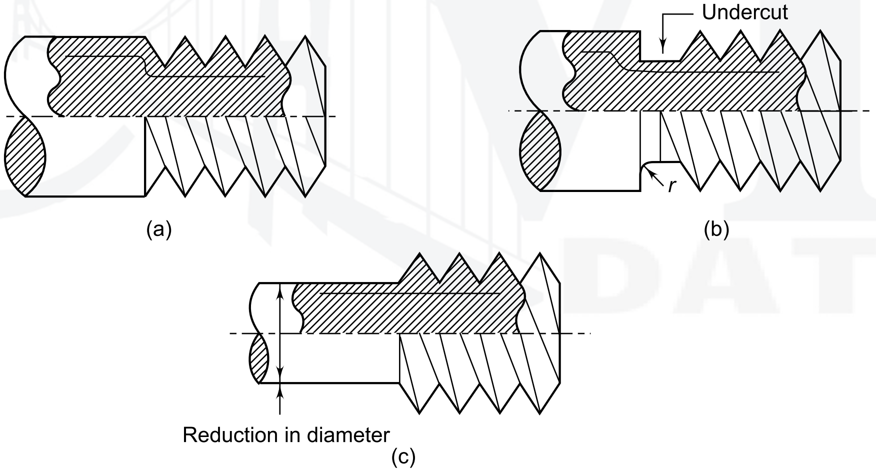


Fig. 5.12 *Reduction of Stress Concentration in Threaded Components: (a) Original Component (b) Undercutting (c) Reduction in Shank Diameter*

Step II *Tensile stress at fillet section*

The stresses are critical at two sections—the fillet section and hole section. At the fillet section,

$$\sigma_o = \frac{P}{dt} = \left(\frac{5000}{30t}\right)$$

$$\frac{D}{d} = \frac{45}{30} = 1.5 \text{ and } \frac{r}{d} = \frac{5}{30} = 0.167$$

From Fig. 5.3, $K_t = 1.8$

$$\therefore \sigma_{max.} = K_t \sigma_o = 1.8\left(\frac{5000}{30t}\right) = \left(\frac{300}{t}\right) \text{N/mm}^2 \quad \text{(i)}$$

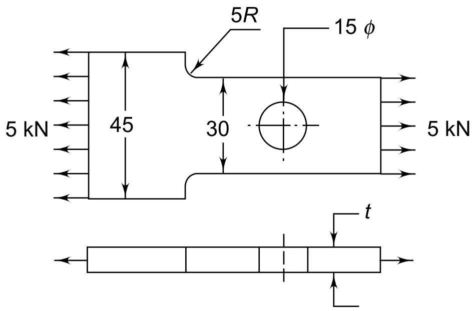


Fig. 5.13

Step III *Tensile stress at hole section*

$$\sigma_o = \frac{P}{(w-d)t} = \frac{5000}{(30-15)t} \text{ N/mm}^2$$

$$\frac{d}{w} = \frac{15}{30} = 0.5$$

From Fig. 5.2,

$$K_t = 2.16$$

$$\sigma_{\max.} = K_t\sigma_o = 2.16\left[\frac{5000}{(30-15)t}\right] = \left(\frac{720}{t}\right) \text{N/mm}^2 \tag{ii}$$

Step IV *Thickness of plate*

From (i) and (ii), it is seen that the maximum stress is induced at the hole section.

Equating it with permissible stress, we get

$$\left(\frac{720}{t}\right) = 80$$

or $\quad t = 9$ mm

Example 5.2 *A non-rotating shaft supporting a load of 2.5 kN is shown in Fig. 5.14. The shaft is made of brittle material, with an ultimate tensile strength of 300 N/mm². The factor of safety is 3. Determine the dimensions of the shaft.*

Solution

Given $P = 2.5$ kN $\quad S_{ut} = 300$ N/mm² $(fs) = 3$

Step I *Calculation of permissible stress*

$$\sigma_{\max.} = \frac{S_{ut}}{(fs)} = \frac{300}{3} = 100 \text{ N/mm}^2$$

Step II *Bending stress at fillet section*

Due to symmetry, the reaction at each bearing is 1250 N. The stresses are critical at two sections—(i) at the centre of span, and (ii) at the fillet. At the fillet section,

$$\sigma_o = \frac{32M_b}{\pi d^3} = \frac{32(1250\times 350)}{\pi d^3} \text{ N/mm}^2$$

$$\frac{D}{d} = 1.1 \text{ and } \frac{r}{d} = 0.1$$

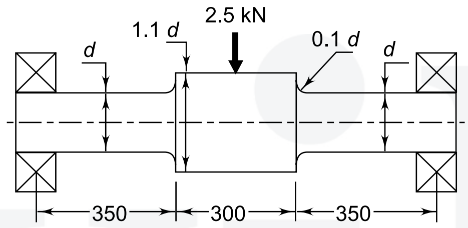


Fig. 5.14

From Fig. 5.5, $K_t = 1.61$

$$\therefore \quad \sigma_{\max.} = K_t\sigma_o = 1.61\left[\frac{32(1250\times 350)}{\pi d^3}\right]$$

$$= \left(\frac{7\,174\,704.8}{d^3}\right)\text{N/mm}^2 \tag{i}$$

Step III *Bending stress at centre of the span*

$$\sigma_o = \frac{32M_b}{\pi d^3} = \frac{32(1250\times 500)}{\pi(1.1d)^3}$$

$$= \left(\frac{4\,783\,018.6}{d^3}\right)\text{N/mm}^2 \tag{ii}$$

Step IV *Diameter of shaft*

From (i) and (ii), it is seen that the stress is maximum at the fillet section. Equating it with permissible stress,

$$\left(\frac{7\,174\,704.8}{d^3}\right) = 100$$

or $\quad d = 41.55$ mm

5.4 FLUCTUATING STRESSES

In the previous chapters, the external forces acting on a machine component were assumed to be static.

In many applications, the components are subjected to forces, which are not static, but vary in magnitude with respect to time. The stresses induced due to such forces are called *fluctuating stresses. It is observed that about 80% of failures of mechanical components are due to 'fatigue failure' resulting from fluctuating stresses*. In practice, the pattern of stress variation is irregular and unpredictable, as in case of stresses due to vibrations. For the purpose of design analysis, simple models for stress–time relationships are used. The most popular model for stress–time relationship is the *sine curve*.

There are three types of mathematical models for cyclic stresses—fluctuating or alternating stresses, repeated stresses and reversed stresses. Stress–time relationships for these models are illustrated in Fig. 5.15. The fluctuating or alternating stress varies in a sinusoidal manner with respect to time. It has some mean value as well as amplitude value. It fluctuates between two limits—maximum and minimum stress. The stress can be tensile or compressive or partly tensile and partly compressive. The repeated stress varies in a sinusoidal manner with respect to time, but the variation is from zero to some maximum value. The minimum stress is zero in this case and therefore, amplitude stress and mean stress are equal. The reversed stress varies in a sinusoidal manner with respect to time, but it has zero mean stress. In this case, half portion of the cycle consists of tensile stress and the remaining half of compressive stress. There is a complete reversal from tension to compression between these two halves and therefore, the mean stress is zero. In Fig. 5.15, $\sigma_{max.}$ and $\sigma_{min.}$ are maximum and minimum stresses, while σ_m and σ_a are called *mean stress* and *stress amplitude* respectively. It can be proved that,

$$\sigma_m = \frac{1}{2}(\sigma_{max.} + \sigma_{min}) \tag{5.9}$$

$$\sigma_a = \frac{1}{2}(\sigma_{max.} - \sigma_{min.}) \tag{5.10}$$

In the analysis of fluctuating stresses, tensile stress is considered as positive, while compressive stress as negative. It can be observed that repeated stress and reversed stress are special cases of fluctuating stress with ($\sigma_{min.} = 0$) and ($\sigma_m = 0$) respectively.

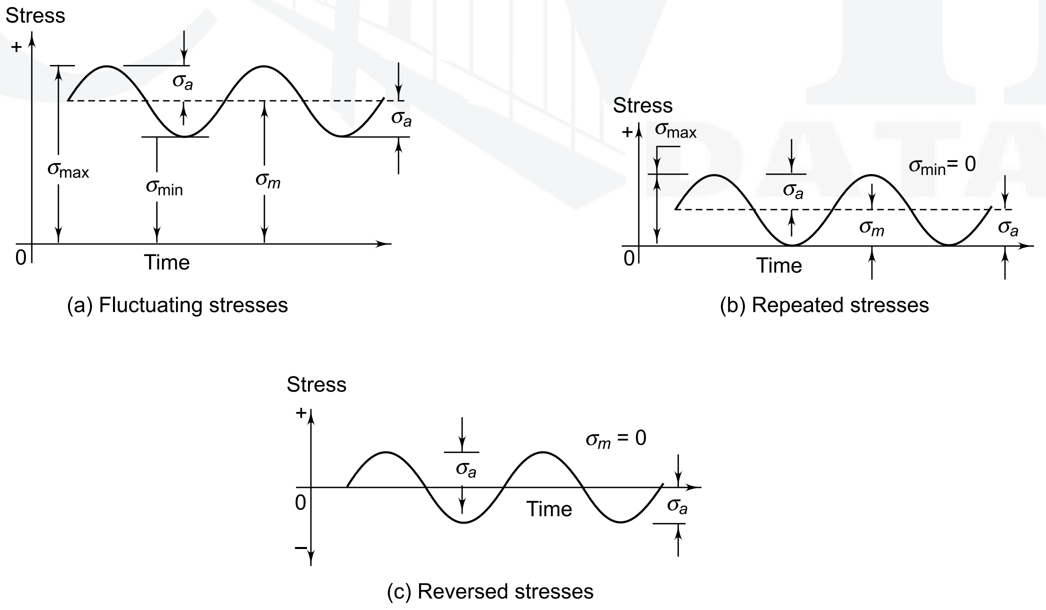


Fig. 5.15 *Types of Cyclic Stresses*

5.5 FATIGUE FAILURE

It has been observed that materials fail under fluctuating stresses at a stress magnitude which is lower than the ultimate tensile strength of the material. Sometimes, the magnitude is even lower than the yield strength. Further, it has been found that the magnitude of the stress causing fatigue failure decreases as the number of stress cycles increase. This phenomenon of decreased resistance of the materials to fluctuating stresses is the main characteristic of fatigue failure.

Let us examine a phenomenon we have experienced in our childhood. Suppose, there is a wire of 2 to 3 mm diameter and we want to cut it into two pieces without any device like a hacksaw. One method is to shear the wire by applying equal and opposite forces P_1 and P_2 by left and right hands

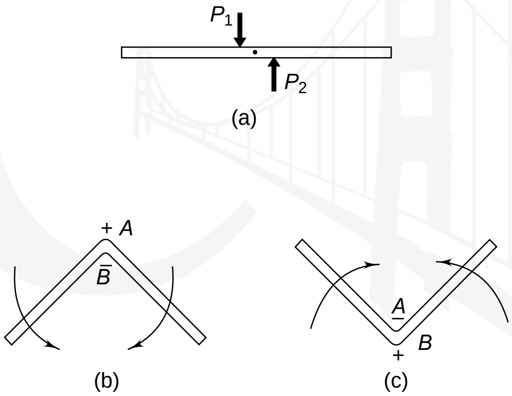


Fig. 5.16 *Shear and Fatigue Failure of Wire: (a) Shearing of Wire (b) Bending of Wire (c) Unbending of Wire*

as illustrated in Fig. 5.16(a). It is difficult to cut the wire by this method. The second method consists of alternatively bending and unbending the wire for few cycles. Let us consider two diametrically opposite points A and B on the surface of the wire. As shown in Fig. 5.16(b), when the wire is bent, A is subjected to tensile stress while B to compressive stress. When the wire is unbent, there is compressive stress at A and tensile stress at B, as shown in Fig. 5.16(c). Therefore, there is complete reversal of stress from tensile stress to compressive stress at the point A due to alternate bending and unbending. Similarly, the point B is subjected to reversal of stress from compressive stress to tensile stress during the same cycle. We have experienced that the wire can be cut very easily in few cycles of bending and unbending. This is a fatigue failure and the magnitude of stress required to fracture is very low. In other words, there is decreased resistance of material to cyclic stresses. *Fatigue failure is defined as time delayed fracture under cyclic loading*. Examples of parts in which fatigue failures are common are transmission shafts, connecting rods, gears, vehicle suspension springs and ball bearings.

There is a basic difference between failure due to static load and that due to fatigue. The failure due to static load is illustrated by the simple tension test. In this case, the load is gradually applied and there is sufficient time for the elongation of fibres. In ductile materials, there is considerable plastic flow prior to fracture. This results in a silky fibrous structure due to the stretching of crystals at the fractured surface. On the other hand, fatigue failure begins with a crack at some point in the material. The crack is more likely to occur in the following regions:

(i) Regions of discontinuity, such as oil holes, keyways, screw threads, etc.
(ii) Regions of irregularities in machining operations, such as scratches on the surface, stamp mark, inspection marks, etc.
(iii) Internal cracks due to defects in materials like blow holes

These regions are subjected to stress concentration due to the crack. The crack spreads due to fluctuating stresses, until the cross-section of the component is so reduced that the remaining portion is subjected to sudden fracture. There are two distinct areas of fatigue failure—(i) region indicating slow growth of crack with a fine fibrous appearance, and (ii) region of sudden fracture with a coarse granular appearance.

In case of failure under static load, there is sufficient plastic deformation prior to failure, which gives a warning well in advance. On the other hand, fatigue cracks are not visible till they reach the surface of the component and by that time, the failure has already taken place. The fatigue failure is sudden and total. It is relatively easy to design a component for a static load. The fatigue failure, however, depends upon a number of factors, such as the number of cycles, mean stress, stress amplitude, stress concentration, residual stresses, corrosion and creep. This makes the design of components subjected to fluctuating stresses more complex.

5.6 ENDURANCE LIMIT

The fatigue or endurance limit of a material is defined as the maximum amplitude of completely reversed stress that the standard specimen can sustain for an unlimited number of cycles without fatigue failure. Since the fatigue test cannot be conducted for unlimited or infinite number of cycles, 10^6 cycles is considered as a sufficient number of cycles to define the endurance limit. There is another term called *fatigue life*, which is frequently used with endurance limit. *The fatigue life is defined as the number of stress cycles that the standard specimen can complete during the test before the appearance of the first fatigue crack*. The dimensions of the standard test specimen (in mm) are shown in Fig. 5.17. The specimen is carefully machined and polished. The final polishing is done

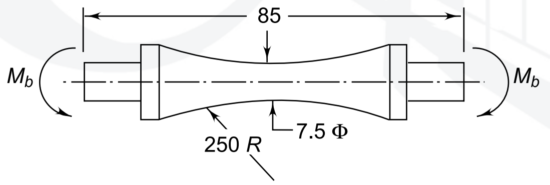


Fig. 5.17 *Specimen for Fatigue Test*

in axial direction in order to avoid circumferential scratches. In the laboratory, the endurance limit is determined by means of a rotating beam machine developed by R R Moore. The principle of a rotating beam is illustrated in Fig. 5.18. A beam of circular cross-section is subjected to bending moment M_b. Under the action of bending moment, tensile stresses are induced in the upper half of the beam and compressive stresses in the lower half. The maximum tensile stress σ_t in the uppermost fibre is equal to the maximum compressive stress σ_c in the lowermost fibre. There is zero stress at all fibres in the central horizontal plane passing through the axis of the beam. Let us consider a point A on the surface of the beam and let us try to find out stresses at this point when the shaft is rotated through one revolution. Initially, the point A occupies position A_1 in the central horizontal plane with zero stress. When the shaft is rotated through 90°, it occupies the position A_2. It is subjected to maximum tensile stress σ_t in this position. When the shaft is further rotated through 90°, the point A will occupy the position A_3 in the central horizontal plane with zero stress. A further rotation of 90° will bring the point A to the position A_4. It is subjected to maximum compressive stress σ_c in this position. The variation of stresses at the point A during one revolution of the beam is shown in Fig. 5.18(b). It is observed that the beam is subjected to completely reversed stresses with tensile stress in the first half and compressive stress in the second half. The distribution is sinusoidal and one stress cycle is completed in one revolution. The amplitude of this cycle (σ_t or σ_c) is given by

$$\sigma_t \text{ or } \sigma_c = \frac{M_b y}{I}$$

The amplitude can be increased or decreased by increasing or decreasing the bending moment respectively.

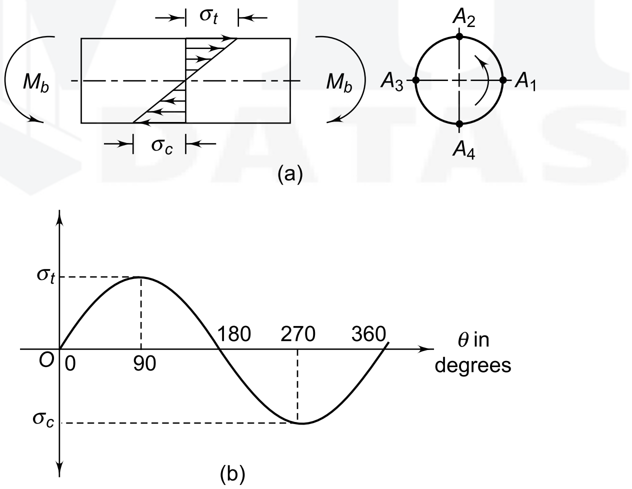


Fig. 5.18 *Rotating Beam Subjected to Bending Moment: (a) Beam, (b) Stress Cycle at Point A*

A schematic diagram of a rotating beam fatigue testing machine is shown in Fig. 5.19. The specimen acts as a 'rotating beam' subjected to a bending moment. Therefore, it is subjected to a completely reversed stress cycle. Changing the bending moment by addition or deletion of weights can vary the stress amplitude. The specimen is rotated by an electric motor. The number of revolutions before the appearance of the first fatigue crack is recorded on a revolution counter. In each test, two readings are taken, viz., stress amplitude (S_f) and number of stress cycles (N). These readings are used as two co-ordinates for plotting a point on the S–N diagram. This point is called *failure point*. To determine the endurance limit of a material, a number of tests are to be carried out.

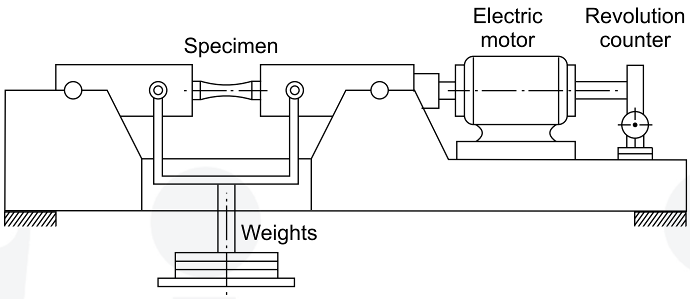


Fig. 5.19 *Rotaing Beam Fatigue Testing Machine*

The results of these tests are plotted by means of an S–N curve. *The S–N curve is the graphical representation of stress amplitude (S_f) versus the number of stress cycles (N) before the fatigue failure on a log-log graph paper*. The S–N curve for steels is illustrated in Fig. 5.20. Each test on the fatigue testing machine gives one failure point on the S–N diagram. In practice, the points are scattered in the figure and an average curve is drawn through them. The S–N diagram is also called *Wöhler diagram*, after August Wöhler, a German engineer who published his fatigue research in 1870. The S–N diagram is a standard method of presenting fatigue data.

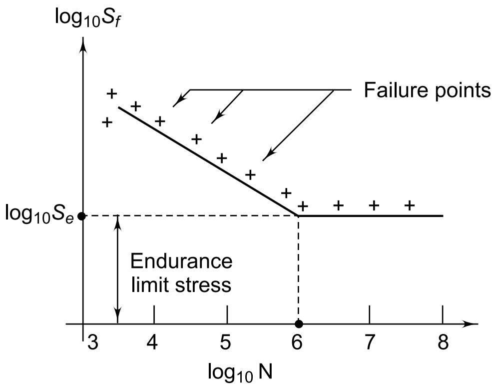


Fig. 5.20 *S-N Curve for Steels*

For ferrous materials like steels, the S–N curve becomes asymptotic at 10^6 cycles, which indicates the stress amplitude corresponding to infinite number of stress cycles. The magnitude of this stress amplitude at 10^6 cycles represents the endurance limit of the material. The S–N curve shown in Fig. 5.20 is valid only for ferrous metals. For non-ferrous metals like aluminium alloys, the S–N curve slopes gradually even after 10^6 cycles. These materials do not exhibit a distinct value of the endurance limit in a true sense. For these materials, endurance limit stress is sometimes expressed as a function of the number of stress cycles.

The endurance limit, in a true sense, is not exactly a property of material like ultimate tensile strength. It is affected by factors such as the size of the component, shape of component, the surface finish, temperature and the notch sensitivity of the material.

5.7 LOW-CYCLE AND HIGH-CYCLE FATIGUE

The S–N curve illustrated in Fig. 5.20 is drawn from 10^3 cycles on a log-log graph paper. The complete S–N curve from 10^0 cycle to 10^8 cycles is shown in Fig. 5.21. There are two regions of this curve namely, low-cycle fatigue and high-cycle fatigue. The difference between these two fatigue failures is as follows:

(i) Any fatigue failure when the number of stress cycles are less than 1000, is called *low-cycle fatigue*. Any fatigue failure when the number of stress cycles are more than 1000, is called *high-cycle fatigue*.

(ii) Failure of studs on truck wheels, failure of setscrews for locating gears on shafts or failures of short-lived devices such as missiles are the examples of low-cycle fatigue. The failure of machine components such as springs, ball bearings or gears that are subjected to fluctuating stresses, are the examples of high-cycle fatigue.

(iii) The low-cycle fatigue involves plastic yielding at localized areas of the components. There are some theories of low-cycle fatigue. However, in many applications, the designers simply ignore the fatigue effect when the number of stress cycles are less than 1000. A greater factor of safety is used to account for this effect. Such components are designed on the basis of ultimate tensile strength or yield strength with a suitable factor of safety.

Components subjected to high-cycle fatigue are designed on the basis of endurance limit stress. *S–N* curves, Soderberg lines, Gerber lines or Goodman diagrams are used in the design of such components.

The discussion in this chapter is restricted to high-cycle fatigue failure of machine elements.

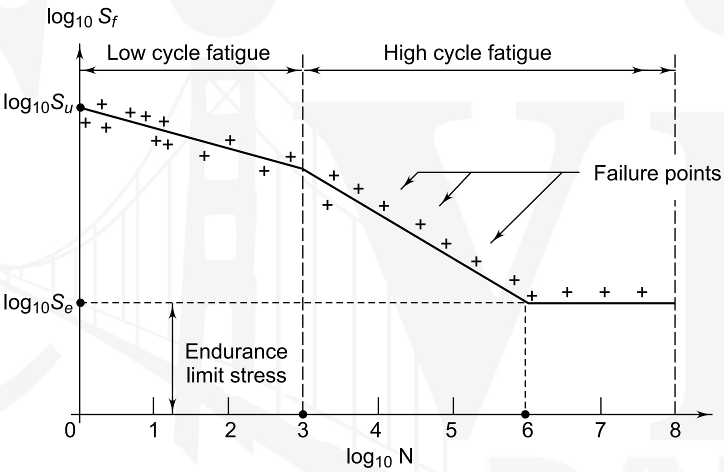


Fig. 5.21 *Low and High Cycle Fatigue*

5.8 NOTCH SENSITIVITY

It is observed that the actual reduction in the endurance limit of a material due to stress concentration is less than the amount indicated by the theoretical stress concentration factor K_t. Therefore, two separate notations, K_t and K_f, are used for stress concentration factors. K_t is the theoretical stress concentration factor, as defined in previous sections, which is applicable to ideal materials that are homogeneous, isotropic and elastic. K_f is the fatigue stress concentration factor, which is defined as follows:

$$K_f = \frac{\text{Endurance limit of the notch free specimen}}{\text{Endurance limit of the notched specimen}}$$

This factor K_f is applicable to actual materials and depends upon the grain size of the material. It is observed that there is a greater reduction in the endurance limit of fine-grained materials as compared to coarse-grained materials, due to stress concentration.

Notch sensitivity is defined as the susceptibility of a material to succumb to the damaging effects of stress raising notches in fatigue loading. The notch sensitivity factor q is defined as

$$q = \frac{\text{Increase of actual stress over nominal stress}}{\text{Increase of theoretical stress over nominal stress}}$$

Since

σ_o = nominal stress as obtained by elementary equations

$\therefore$ actual stress $= K_f\,\sigma_o$

theoretical stress $= K_t\,\sigma_o$

increase of actual stress over nominal stress $= (K_f\,\sigma_o - \sigma_o)$

increase of theoretical stress over nominal stress $= (K_t\,\sigma_o - \sigma_o)$

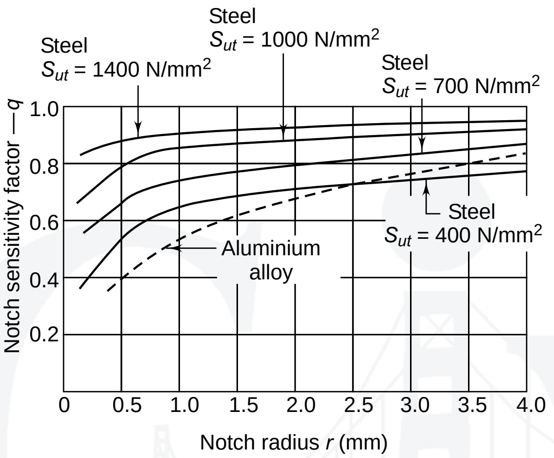


Fig. 5.22 *Notch Sensitivity Charts (for Reversed Bending and Reversed Axial Stresses)*

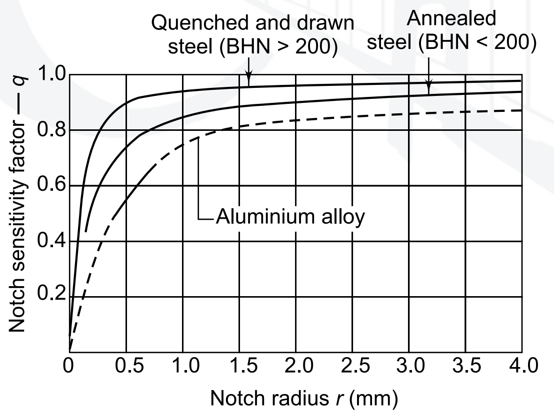


Fig. 5.23 *Notch Sensitivity Charts (for Reversed Torsional Shear Stresses)*

Therefore,

$$q = \frac{(K_f\sigma_o - \sigma_o)}{(K_t\sigma_o - \sigma_o)}$$

or
$$q = \frac{(K_f - 1)}{(K_t - 1)} \qquad (5.11)$$

The above equation can be rearranged in the following form:

$$K_f = 1 + q(K_t - 1) \qquad (5.12)$$

The following conclusions are drawn with the help of Eq. (5.12).

(i) When the material has no sensitivity to notches,

$q = 0$ and $K_f = 1$

(ii) When the material is fully sensitive to notches,

$q = 1$ and $K_f = K_t$

In general, the magnitude of the notch sensitivity factor q varies from 0 to 1. The notch sensitivity factors for various materials for reversed bending or axial stresses and reversed torsional shear stresses[2] are obtained from Fig. 5.22 and 5.23 respectively. In case of doubt, the designer should use (q = 1) or ($K_t = K_f$) and the design will be on the safe side.

5.9 ENDURANCE LIMIT—APPROXIMATE ESTIMATION

The laboratory method for determining the endurance limit of materials, although more precise, is laborious and time consuming. A number of tests are required to prepare one *S–N* curve and each test takes considerable time. It is, therefore, not possible to get the experimental data of each and every material. When the laboratory data regarding the endurance limit of the materials is not available, the procedure discussed in this article should be adopted.

Two separate notations are used for endurance limit, viz, (S'_e) and (S_e) where,

S'_e = endurance limit stress of a rotating beam specimen subjected to reversed bending stress (N/mm²)

[2] George Sines and J L Waisman, *Metal Fatigue*—McGraw-Hill Book Co., Inc., 1959 and University of California (Engineering Extension Series), Article No. 13, Notch Sensitivity by R E Peterson.

S_e = endurance limit stress of a particular mechanical component subjected to reversed bending stress (N/mm^2)

There is an approximate relationship between the endurance limit and the ultimate tensile strength (S_{ut}) of the material.

For steels,

$$S'_e = 0.5\ S_{ut} \tag{5.13}$$

For cast iron and cast steels,

$$S'_e = 0.4\ S_{ut} \tag{5.14}$$

For wrought aluminium alloys,

$$S'_e = 0.4\ S_{ut} \tag{5.15}$$

For cast aluminium alloys,

$$S'_e = 0.3\ S_{ut} \tag{5.16}$$

These relationships are based on 50% reliability.

The endurance limit of a component is different from the endurance limit of a rotating beam specimen due to a number of factors. The difference arises due to the fact that there are standard specifications and working conditions for the rotating beam specimen, while the actual components have different specifications and work under different conditions. Different modifying factors are used in practice to account for this difference. These factors are, sometimes, called *derating factors*. The purpose of derating factors is to 'derate' or reduce the endurance limit of a rotating beam specimen to suit the actual component. In this article, only four factors that normally require attention are discussed.

The relationship between (S_e) and (S'_e) is as follows[3]:

$$S_e = K_a K_b K_c K_d S'_e \tag{5.17}$$

where,

K_a = surface finish factor
K_b = size factor
K_c = reliability factor
K_d = modifying factor to account for stress concentration.

(i) Surface finish Factor The surface of the rotating beam specimen is polished to mirror finish. The final polishing is carried out in the axial direction to smooth out any circumferential scratches. This makes the specimen almost free from surface scratches and imperfections. It is impractical to provide such an expensive surface finish for the actual component. The actual component may not even require such a surface finish. When the surface finish is poor, there are scratches and geometric irregularities on the surface. These surface scratches serve as stress raisers and result in stress concentration. The endurance limit is reduced due to introduction of stress concentration at these scratches. The surface finish factor takes into account the reduction in endurance limit due to the variation in the surface finish between the specimen and the actual component. Figure 5.24 shows the surface finish factor for steel components[4]. It should be noted that ultimate tensile strength is also a parameter affecting the surface finish factor. High strength materials are more sensitive to stress concentration introduced by surface irregularities. Therefore, as the ultimate tensile strength increases, the surface finish factor decreases.

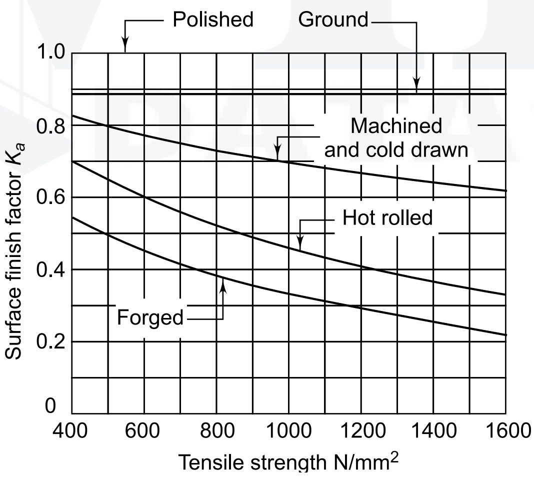


Fig. 5.24 *Surface Finish Factor*

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[3] Joseph E Shigley – *Mechanical Engineering Design* – McGraw-Hill Kogakusha Ltd.

[4] Robert C Juvinall – *Engineering considerations of Stress, Strain and Strength* – McGraw Hill Book Company, New York,1967.

Shigley and Mischke have suggested an exponential equation for the surface finish factor. This equation is based on experimental data points obtained by Noll and Lipson. This equation is in the following form,

$$K_a = a(S_{ut})^b \quad [\text{if } K_a > 1, \text{ set } K_a = 1] \tag{5.18}$$

The values of coefficients a and b are given in Table 5.1.

Table 5.1 *Values of coefficients a and b in surface finish factor*

Surface finish	*a*	*b*
Ground	1.58	–0.085
Machined or cold-drawn	4.51	–0.265
Hot-rolled	57.7	–0.718
As forged	272	–0.995

The above mentioned values of surface finish factors are developed only for steel components. They should not be applied to components made of other ductile materials like aluminium alloys.

The surface finish factor for ordinary grey cast iron components is taken as 1, irrespective of their surface finish. It is observed that even mirror-finished samples of grey cast iron parts have surface discontinuities because of graphite flakes in the cast iron matrix. Adding some more surface scratches does not make any difference. *Therefore, whatever is the machining method; the value of surface finish factor for cast iron parts is always taken as 1.*

In this chapter, the values of surface finish factors are obtained from Fig. 5.24 instead of Eq. (5.18).

(ii) Size Factor The rotating beam specimen is small with 7.5 mm diameter. The larger the machine part, the greater the probability that a flaw exists somewhere in the component. The chances of fatigue failure originating at any one of these flaws are more. The endurance limit, therefore, reduces with increasing the size of the component. The size factor K_b takes into account the reduction in endurance limit due to increase in the size of the component. For bending and torsion, the values of size factor (K_b) are given in Table 5.2.

Table 5.2 *Values of size factor*

Diameter (d) (mm)	K_b
$d \leq 7.5$	1.00
$7.5 < d \leq 50$	0.85
$d > 50$	0.75

Shigley and Mischke have suggested an exponential equation for the size factor. For bending and torsion, the equation is in the following form:

For 2.79 mm $\leq d <$ 51 mm

$$K_b = 1.24\, d^{-0.107} \tag{5.19}$$

For 51 mm $< d \leq$ 254 mm

$$K_b = 0.859 - 0.000\,873\, d \tag{5.20}$$

For axial loading, $K_b = 1$

In this chapter, Table 5.2 is used instead of Eqs (5.19) and (5.20) to find out the value of size factor.

Table 5.2 as well as Eqs (5.19) and (5.20) can be used only for cylindrical components. It is difficult to determine the size factor for components having a non-circular cross-section. However, since the endurance limit is reduced in such components, it is necessary to define *effective diameter* based on an equivalent circular cross-section. In this case, *Kuguel's equality* is widely used. This equality is based on the concept that fatigue failure is related to the probability of high stress interacting with a discontinuity. When the volume of material subjected to high stress is large, the probability of fatigue failure originating from any flaw in that volume is more. Kuguel assumes a volume of material that is stressed to 95% of the maximum stress or above as high stress volume. *According to Kuguel's equality, the effective diameter is obtained by equating the volume of the material stressed at and above 95% of the maximum stress to the equivalent volume in the rotating beam specimen.* When these two volumes are equated, the lengths of the component and specimen cancel out and only areas need be considered. This concept is illustrated in Fig. 5.25. The rotating beam specimen is subjected to bending stresses. The bending stress is linearly proportional to the distance from the centre of the cross-section. There is maximum stress at the outer fibre. Therefore, the area (A_{95}) stressed above 95% of the maximum stress is the area of a ring, having an inside diameter of (0.95d) and an outside diameter of (1.0d).

$$A_{95} = \pi\left[\frac{d^2 - (0.95d)^2}{4}\right] = 0.0766 d^2 \tag{5.21}$$

The above equation is also valid for a hollow rotating shaft.

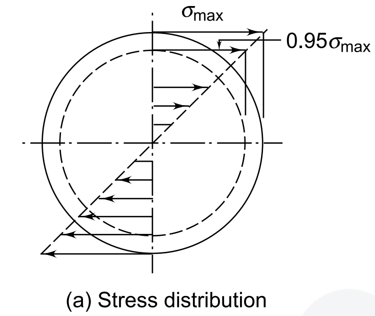

Fig. 5.25

The 'effective' diameter of any non-circular cross-section is then given by,

$$d_e = \sqrt{\frac{A_{95}}{0.0766}} \tag{5.22}$$

where,

A_{95} = portion of cross-sectional area of the non-cylindrical part that is stressed between 95% and 100% of the maximum stress

d_e = effective diameter of the non-cylindrical part

Formulae for areas that are stressed between 95% and 100% of maximum stress for commonly used cross-sections loaded in bending, are given in Fig. 5.26.

For a non-rotating solid shaft,

$$A_{95} = 0.0105d^2 \tag{5.23}$$

From Eqs (5.22) and (5.23),

$$d_e = \sqrt{\frac{0.0105d^2}{0.0766}} = 0.37d \tag{5.24}$$

The above effective diameter d_e is used to find out the size factor for the non-rotating cylindrical component.

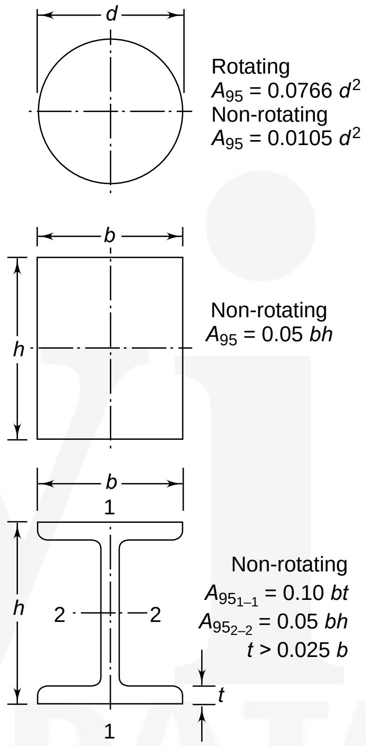

Fig. 5.26 *Area above 95% of Maximum Stress*

For a rectangular cross-section having width b and depth h,

$$A_{95} = 0.05\ bh \tag{5.25}$$

From Eqs (5.22) and (5.25),

$$d_e = \sqrt{\frac{0.05bh}{0.0766}} = 0.808\sqrt{bh} \tag{5.26}$$

The above effective diameter d_e is used to find out the size factor from Table 5.2 or Eqs (5.19) and (5.20).

A similar procedure is followed for I-section beam.

(iii) Reliability Factor The laboratory values of endurance limit are usually mean values. There is considerable dispersion of the data when a number of tests are conducted even using the same material and same conditions. The standard deviation of endurance limit tests is 8% of the mean value. The reliability factor K_c depends upon the reliability that is used in the design of the component. The greater the likelihood that a part will survive, the more is the reliability and lower is the reliability factor. The reliability factor is one for 50% reliability. This means that 50% of the components will survive in the given set of conditions. To ensure that more than 50% of the parts will survive, the stress amplitude on the component should be lower than the tabulated value of the endurance limit. The reliability factor is used to achieve this reduction. The reliability factors based on a standard deviation of 8% are given in Table 5.3[3].

Table 5.3 *Reliability factor*

Reliability R (%)	K_c
50	1.000
90	0.897
95	0.868
99	0.814
99.9	0.753
99.99	0.702
99.999	0.659

(iv) Modifying Factor to Account for Stress Concentration The endurance limit is reduced due to stress concentration. The stress concentration factor used for cyclic loading is less than the theoretical stress concentration factor due to the notch sensitivity of the material. To apply the effect of stress concentration, the designer can either reduce the endurance limit by (K_d) or increase the stress amplitude by (K_f). We will use the first approach. The modifying factor K_d to account for the effect of stress concentration is defined as,

$$K_d = \frac{1}{K_f} \tag{5.27}$$

The above mentioned four factors are used to find out the endurance limit of the actual component.

The endurance limit (S_{se}) of a component subjected to fluctuating torsional shear stresses is obtained from the endurance limit in reversed bending (S_e) using theories of failures.

According to the maximum shear stress theory,

$$S_{se} = 0.5\, S_e \tag{5.28}$$

According to distortion energy theory,

$$S_{se} = 0.577\, S_e \tag{5.29}$$

When the component is subjected to an axial fluctuating load, the conditions are different. In axial loading, the entire cross-section is uniformly stressed to the maximum value. In the rotating beam test, the specimen is subjected to bending stress. The bending stress is zero at the centre of cross-section and negligible in the vicinity of centre. It is only the outer region near the surface, which is subjected to maximum stress. There is more likelihood of a microcrack being present in the much higher high-stress field of axial loading than in the smaller volume outer region of the rotating beam specimen. Therefore, endurance limit in axial loading is lower than the rotating beam test.

For axial loading,

$$(S_e)_a = 0.8\, S_e \tag{5.30}$$

5.10 REVERSED STRESSES—DESIGN FOR FINITE AND INFINITE LIFE

There are two types of problems in fatigue design—(i) components subjected to completely reversed stresses, and (ii) components subjected to fluctuating stresses. As shown in Fig. 5.15, the mean stress is zero in case of completely reversed stresses. The stress distribution consists of tensile stresses for the first half cycle and compressive stresses for the remaining half cycle and the stress cycle passes through zero. In case of fluctuating stresses, there is always a mean stress, and the stresses can be purely tensile, purely compressive or mixed depending upon the magnitude of the mean stress. Such problems are solved with the help of the modified Goodman diagram, which will be discussed in Section 5.13.

The design problems for completely reversed stresses are further divided into two groups—(i) design for infinite life, and (ii) design for finite life.

Case I: When the component is to be designed for infinite life, the endurance limit becomes the criterion of failure. The amplitude stress induced in such components should be lower than the endurance limit in order to withstand the infinite number of cycles. Such components are designed with the help of the following equations:

$$\sigma_a = \frac{S_e}{(fs)} \tag{5.31}$$

$$\tau_a = \frac{S_{se}}{(fs)} \tag{5.32}$$

where (σ_a) and (τ_a) are stress amplitudes in the component and S_e and S_{se} are corrected endurance limits in reversed bending and torsion respectively.

Case II: When the component is to be designed for finite life, the *S–N* curve as shown in Fig. 5.27 can be used. The curve is valid for steels. **It consists of a straight line *AB* drawn from (0.9 S_{ut}) at 10^3 cycles to (S_e) at 10^6 cycles on a log-log paper**. The design procedure for such problems is as follows:

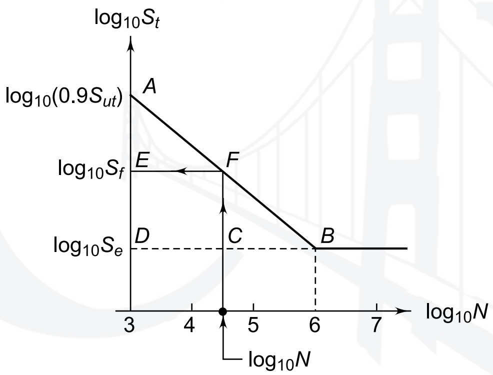

Fig. 5.27 *S-N Curve*

(i) Locate the point *A* with coordinates [3, $\log_{10}$ (0.9S_{ut})] since $\log_{10}(10^3) = 3$

(ii) Locate the point *B* with coordinates [6, $\log_{10}$ (S_e)] since $\log_{10}(10^6) = 6$

(iii) Join $\overline{AB}$, which is used as a criterion of failure for finite-life problems

(iv) Depending upon the life *N* of the component, draw a vertical line passing through $\log_{10}$ (*N*) on the abscissa. This line intersects $\overline{AB}$ at point *F*.

(v) Draw a line $\overline{FE}$ parallel to the abscissa. The ordinate at the point *E*, i.e. $\log_{10}$ (S_f), gives the fatigue strength corresponding to *N* cycles.

The value of the fatigue strength (S_f) obtained by the above procedure is used for the design calculations.

Infinite-life Problems (Reversed Load)

Example 5.3 *A plate made of steel 20C8 (S_{ut} = 440 N/mm²) in hot rolled and normalised condition is shown in Fig. 5.28. It is subjected to a completely reversed axial load of 30 kN. The notch sensitivity factor q can be taken as 0.8 and the expected reliability is 90%. The size factor is 0.85. The factor of safety is 2. Determine the plate thickness for infinite life.*

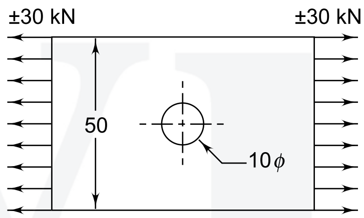

Fig. 5.28

Solution

Given $P = \pm 30$ kN $\quad S_{ut} = 440$ N/mm² $\quad (fs) = 2$
$R = 90\% \quad q = 0.8 \quad K_b = 0.85$

Step I *Endurance limit stress for plate*

$S'_e = 0.5\, S_{ut} = 0.5(440) = 220$ N/mm²

From Fig. 5.24 (hot rolled steel and S_{ut} = 440 N/mm²),

$K_a = 0.67$

$K_b = 0.85$

For 90% reliability,

$K_c = 0.897$

$$\frac{d}{w} = \frac{10}{50} = 0.2$$

From Fig. 5.2, $\quad K_t = 2.51$

From Eq. (5.12),

$$K_f = 1 + q\,(K_t - 1) = 1 + 0.8\,(2.51 - 1) = 2.208$$

$$K_d = \frac{1}{K_f} = \frac{1}{2.208} = 0.4529$$

$$S_e = K_a K_b K_c K_d S'_e$$
$$= 0.67(0.85)(0.897)(0.4529)(220)$$
$$= 50.9 \text{ N/mm}^2$$

For axial load, (Eq. 5.30)

$$(S_e)_a = 0.8\, S_e = 0.8(50.9) = 40.72 \text{ N/mm}^2$$

Step II *Permissible stress amplitude*

$$\sigma_a = \frac{(S_e)_a}{(fs)} = \frac{40.72}{2} = 20.36 \text{ N/mm}^2 \qquad \text{(a)}$$

Step III *Plate thickness*

$$\sigma_a = \frac{P}{(w-d)t} = \frac{(30)(10^3)}{(50-10)t} \text{ N/mm}^2 \qquad \text{(b)}$$

From (a) and (b),

$$20.36 = \frac{(30)(10^3)}{(50-10)t}$$

$$\therefore \qquad t = 36.84 \text{ mm}$$

Example 5.4 *A rod of a linkage mechanism made of steel 40Cr1 (S_{ut} = 550 N/mm²) is subjected to a completely reversed axial load of 100 kN. The rod is machined on a lathe and the expected reliability is 95%. There is no stress concentration. Determine the diameter of the rod using a factor of safety of 2 for an infinite life condition.*

Solution

Given $P_a = \pm 100$ kN $\quad S_{ut} = 550$ N/mm²
$(fs) = 2 \quad R = 95\%$

Step I *Endurance limit stress for rod*

$S'_e = 0.5 S_{ut} = 0.5(550) = 275$ N/mm²

From Fig. 5.24, (machined surface and S_{ut} = 550 N/mm²),

$$K_a = 0.78$$

Assuming $7.5 < d < 50$ mm

$$K_b = 0.85$$

For 95% reliability, $K_c = 0.868$

$S_e = K_a K_b K_c S'_e = 0.78\,(0.85)\,(0.868)\,(275)$
$= 158.26$ N/mm²

Step II *Permissible stress amplitude*

From Eq. (5.30),

$(S_e)_a = 0.8 S_e = 0.8(158.26) = 126.6$ N/mm²

$$\sigma_a = \frac{(S_e)_a}{(fs)} = \frac{126.6}{2} = 63.5 \text{ N/mm}^2$$

Step III *Diameter of rod*

For rod, $$P_a = \left(\frac{\pi}{4} d^2\right)\sigma_a$$

or $$100 \times 10^3 = \left(\frac{\pi}{4} d^2\right)(63.5)$$

$$\therefore \qquad d = 44.78 \text{ mm}$$

Example 5.5 *A component machined from a plate made of steel 45C8 (S_{ut} = 630 N/mm²) is shown in Fig. 5.29. It is subjected to a completely reversed axial force of 50 kN. The expected reliability is 90% and the factor of safety is 2. The size factor is 0.85. Determine the plate thickness t for infinite life, if the notch sensitivity factor is 0.8.*

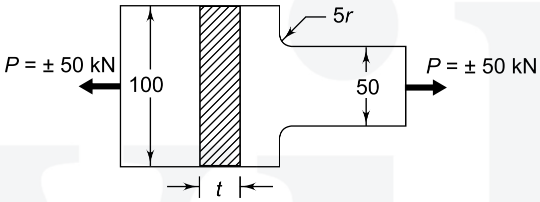

Fig. 5.29

Solution

Given $P = \pm 50$ kN $\quad S_{ut} = 630$ N/mm² $\quad (fs) = 2$
$R = 90\% \quad q = 0.8 \quad K_b = 0.85$

Step I *Endurance limit stress for plate*

$S'_e = 0.5 S_{ut} = 0.5(630) = 315$ N/mm²

From Fig. 5.24 (machined surface and S_{ut} = 630 N/mm²),

$$K_a = 0.76$$
$$K_b = 0.85$$

For 90% reliability, $K_c = 0.897$

$$\left(\frac{D}{d}\right) = \frac{100}{50} = 2$$

and $$\left(\frac{r}{d}\right) = \frac{5}{50} = 0.1$$

From Fig. 5.3, $K_t = 2.27$

From Eq. (5.12),

$K_f = 1 + q\,(K_t - 1) = 1 + 0.8\,(2.27 - 1) = 2.016$

$$K_d = \frac{1}{K_f} = \frac{1}{2.016} = 0.496$$

$S_e = K_a K_b K_c K_d S'_e$
$= 0.76\,(0.85)(0.897)(0.496)(315)$
$= 90.54$ N/mm²

Step II *Permissible stress amplitude*

From Eq. (5.30),

$(S_e)_a = 0.8S_e = 0.8(90.54) = 72.43 \text{ N/mm}^2$

$$\sigma_a = \frac{(S_e)_a}{(fs)} = \frac{72.43}{2} = 36.22 \text{ N/mm}^2$$

Step III *Plate thickness*

Since $\sigma_a = \dfrac{P}{(50t)}$

$$\therefore \quad t = \frac{P}{50\sigma_a} = \frac{(50 \times 10^3)}{50(36.22)} = 27.61 \text{ mm}$$

Finite-life Problems (Reversed Load)

Example 5.6 *A rotating bar made of steel 45C8 (S_{ut} = 630 N/mm²) is subjected to a completely reversed bending stress. The corrected endurance limit of the bar is 315 N/mm². Calculate the fatigue strength of the bar for a life of 90,000 cycles.*

Solution

Given $S_{ut} = 630 \text{ N/mm}^2$ $S_e = 315 \text{ N/mm}^2$
$N = 90000$ cycles

Step I *Construction of S–N diagram*

$0.9S_{ut} = 0.9\ (630) = 567 \text{ N/mm}^2$
$\log_{10}(0.9S_{ut}) = \log_{10}(567) = 2.7536$
$\log_{10}(S_e) = \log_{10}(315) = 2.4983$
$\log_{10}(90\,000) = 4.9542$
Also, $\log_{10}(10^3) = 3$ and $\log_{10}(10)^6 = 6$
Figure 5.30 shows the S–N curve for the bar.

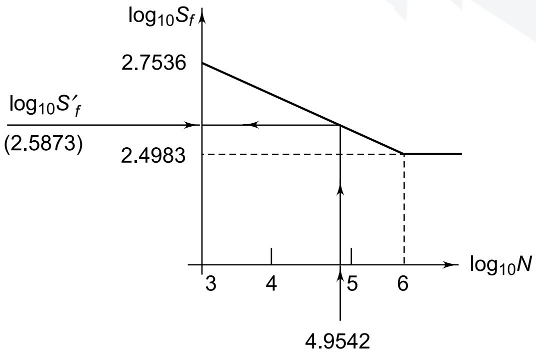

Fig. 5.30

Step II *Fatigue strength for 90000 cycles*
Referring to Fig. 5.30,

$$\log 10\,(S_f') = 2.7536 - \frac{(2.7536 - 2.4983)}{(6-3)} \times (4.9542 - 3) = 2.5873$$

$$S_f' = 386.63 \text{ N/mm}^2$$

Example 5.7 *A forged steel bar, 50 mm in diameter, is subjected to a reversed bending stress of 250 N/mm². The bar is made of steel 40C8 (S_{ut} = 600 N/mm²). Calculate the life of the bar for a reliability of 90%.*

Solution

Given $S_f = \sigma_b = 250 \text{ N/mm}^2$
$S_{ut} = 600 \text{ N/mm}^2$ $R = 90\%$

Step I *Construction of S–N diagram*

$S_e' = 0.5S_{ut} = 0.5(600) = 300 \text{ N/mm}^2$
From Fig. 5.24, (S_{ut} = 600 N/mm² and forged bar),
$K_a = 0.44$
For 50 mm diameter, $K_b = 0.85$
For 90% reliability, $K_c = 0.897$
$S_e = K_a K_b K_c S_e' = 0.44(0.85)\ (0.897)\ (300)$
$= 100.64 \text{ N/mm}^2$
$0.9S_{ut} = 0.9(600) = 540 \text{ N/mm}^2$
$\log_{10}(0.9S_{ut}) = \log_{10}(540) = 2.7324$
$\log_{10}(S_e) = \log_{10}(100.64) = 2.0028$
$\log 10(S_f) = \log_{10}(250) = 2.3979$
Also, $\log_{10}(10^3) = 3$ and $\log_{10}(10^6) = 6$
The S–N curve for the bar is shown in Fig. 5.31.

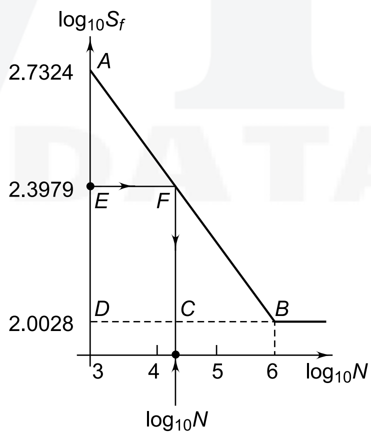

Fig. 5.31

Step II *Fatigue life of bar*
From Fig. 5.31,

$$\overline{EF} = \frac{\overline{DB} \times \overline{AE}}{\overline{AD}} = \frac{(6-3)(2.7324 - 2.3979)}{(2.7324 - 2.0028)}$$

$$= 1.3754$$

Therefore,

$\log_{10} N = 3 + \overline{EF} = 3 + 1.3754$

$\log_{10} N = 4.3754$

$N = 23\,736.2$ cycles

Example 5.8 *A rotating shaft, subjected to a non-rotating force of 5 kN and simply supported between two bearings A and E is shown in Fig. 5.32(a). The shaft is machined from plain carbon steel 30C8 (S_{ut} = 500 N/mm²) and the expected reliability is 90%. The equivalent notch radius at the fillet section can be taken as 3 mm. What is the life of the shaft?*

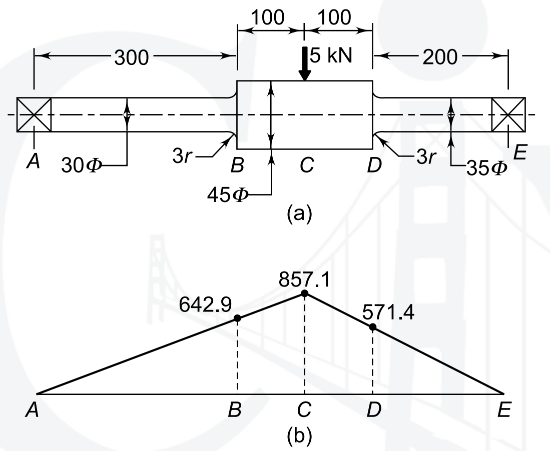

Fig. 5.32

Solution

Given $P = 5$ kN $\quad S_{ut} = 500$ N/mm²

$R = 90\%$ $\quad r = 3$ mm

Step I *Selection of failure-section*

Taking the moment of the forces about bearings A and E, the reactions at A and E are 2143 and 2857 N respectively. The bending moment diagram is shown in Fig. 5.32(b). The values of the bending moment shown in the figure are in N-m. The possibility of a failure will be at the three sections B, C and D. The failure will probably occur at the section B rather than at C or D. At the section C, although the bending moment is maximum, the diameter is more and there is no stress concentration. At the section D, the diameter is more and the bending moment is less compared with that of section B. Therefore, it is concluded that failure will occur at the section B.

Step II *Construction of S–N diagram*

At the section B,

$$S_f = \sigma_b = \frac{32 M_b}{\pi d^3} = \frac{32(642.9 \times 10^3)}{\pi (30)^3}$$

$$= 242.54 \text{ N/mm}^2$$

$$S_e' = 0.5 S_{ut} = 0.5(500) = 250 \text{ N/mm}^2$$

From Fig. 5.24 (machined surface and S_{ut} = 500 N/mm²),

$$K_a = 0.79$$

For 30 mm diameter, $K_b = 0.85$

For 90% reliability, $\quad K_c = 0.897$

Since $\dfrac{r}{d} = \dfrac{3}{30} = 0.1$ and $\dfrac{D}{d} = \dfrac{45}{30} = 1.5$

From Fig. 5.5, $K_t = 1.72$

From Fig. 5.22 (r = 3 mm and S_{ut} = 500 N/mm²),

$$q \simeq 0.78$$

From Eq. (5.12),

$K_f = 1 + q\,(K_t - 1) = 1 + 0.78\,(1.72 - 1)$

$= 1.5616$

$$K_d = \frac{1}{K_f} = \frac{1}{1.5616} = 0.64$$

$S_e = K_a K_b K_c K_d S_e'$

$= 0.79\,(0.85)(0.897)(0.64)(250) = 96.37$ N/mm²

$0.9 S_{ut} = 0.9\,(500) = 450$ N/mm²

$\log_{10}(0.9 S_{ut}) = \log_{10}(450) = 2.6532$

$\log_{10}(S_e) = \log_{10}(96.37) = 1.9839$

$\log_{10}(S_f) = \log_{10}(242.54) = 2.3848$

Also, $\log_{10}(10^3) = 3$ and $\log_{10}(10^6) = 6$

The S–N curve for the shaft is shown in Fig. 5.33.

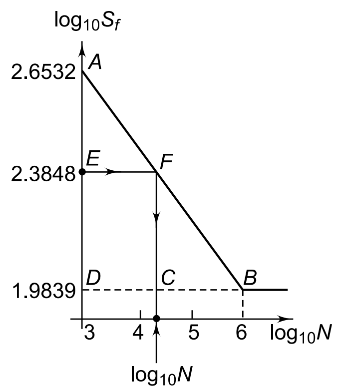

Fig. 5.33

Step III *Fatigue life of shaft*

From Fig. 5.33,

$$\overline{EF} = \frac{\overline{DB} \times \overline{AE}}{\overline{AD}} = \frac{(6-3)(2.6532-2.3848)}{(2.6532-1.9839)}$$

$$= 1.2030$$

Therefore,

$\log_{10} N = 3 + \overline{EF} = 3 + 1.2030$

$\log_{10} N = 4.2030$

$N = 15958.79$ cycles

Example 5.9 *The section of a steel shaft is shown in Fig. 5.34. The shaft is machined by a turning process. The section at XX is subjected to a constant bending moment of 500 kN-m. The shaft material has ultimate tensile strength of 500 MN/m², yield point of 350 MN/m² and endurance limit in bending for a 7.5 mm diameter specimen of 210 MN/m². The notch sensitivity factor can be taken as 0.8. The theoretical stress concentration factor may be interpolated from following tabulated values:*

$\left(\frac{r_f}{d}\right)$	0.025	0.05	0.1
K_t	2.6	2.05	1.66

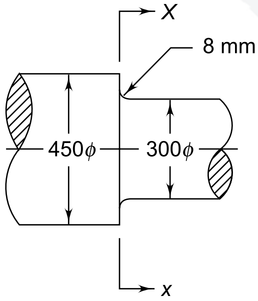

Fig. 5.34

where r_f is the fillet radius and d is the shaft diameter. The reliability is 90%.

Determine the life of the shaft.

Solution

Given $M_b = 500$ kN-m $S_{ut} = 500$ MN/m²
$S_{yt} = 350$ MN/m² $S'_e = 210$ MN/m² $q = 0.8$
$R = 90\%$

Step I *Construction of S–N diagram*

$S'_e = 210 \text{ MN/m}^2 = (210 \times 10^6) \text{ N/m}^2$
$= (210 \times 10^6 \times 10^{-6}) \text{ N/mm}^2 = 210 \text{ N/mm}^2$

From Fig. 5.24 (machined surface and S_{ut} = 500 N/mm²),

$K_a = 0.79$

For 300 mm diameter shaft, $K_b = 0.75$

For 90% reliability, $K_c = 0.897$

Since, $\left(\frac{r_f}{d}\right) = \left(\frac{8}{300}\right) = 0.02667$

$$K_t = 2.05 + \frac{(2.6-2.05)}{(0.05-0.025)}(0.05-0.02667)$$

$$= 2.5633$$

$q = 0.8$

From Eq. (5.12),

$K_f = 1 + q\,(K_t - 1) = 1 + 0.8\,(2.5633 - 1) = 2.25$

$$K_d = \frac{1}{K_f} = \frac{1}{2.25} = 0.4443$$

$S_e = K_a K_b K_c K_d S'_e$
$= 0.79\,(0.75)\,(0.897)\,(0.4443)\,(210)$
$= 49.59 \text{ N/mm}^2$

$M_b = 500 \text{ kN-m} = (500 \times 10^3) \text{ N-m}$
$= (500 \times 10^3 \times 10^3) \text{ N-mm}$

$$\sigma_b = \frac{32M_b}{\pi d^3} = \frac{32(500 \times 10^6)}{\pi(300)^3} = 188.63 \text{ N/mm}^2$$

$0.9S_{ut} = 0.9(500) = 450 \text{ N/mm}^2$

$\log_{10}(0.9S_{ut}) = \log_{10}(450) = 2.6532$

$\log_{10}(S_e) = \log_{10}(49.59) = 1.6954$

$\log_{10}(\sigma_b) = \log_{10}(188.63) = 2.2756$

The S–N curve for the shaft is shown in Fig. 5.35.

Step II *Fatigue life of shaft*

From Fig. 5.35,

$$\overline{EF} = \frac{\overline{DB} \times \overline{AE}}{\overline{AD}} = \frac{(6-3)(2.6532-2.2756)}{(2.6532-1.6954)}$$

$$= 1.1827$$

Therefore,

$\log_{10} N = 3 + \overline{EF} = 3 + 1.1827 = 4.1827$

$N = 15\,230$ cycles

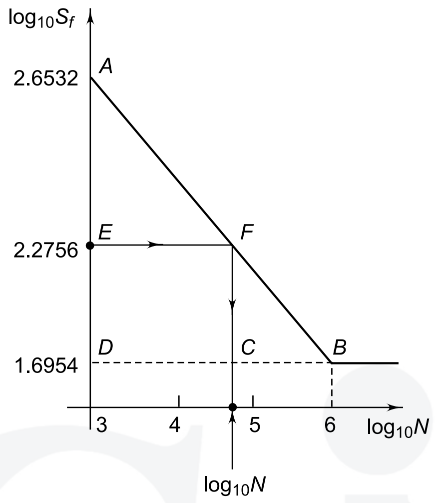

Fig. 5.35

Example 5.10 *A cantilever beam made of cold drawn steel 20C8 (S_{ut} = 540 N/mm^2) is subjected to a completely reversed load of 1000 N as shown in Fig. 5.36. The notch sensitivity factor q at the fillet can be taken as 0.85 and the expected reliability is 90%. Determine the diameter d of the beam for a life of 10000 cycles.*

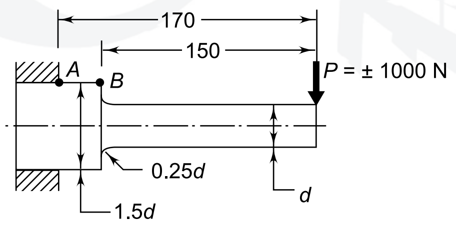

Fig. 5.36

Solution

Given $P = \pm 1000$ N $S_{ut} = 540$ N/mm^2
$q = 0.85$ $R = 90\%$ $N = 10\,000$ cycles

Step I *Selection of failure section*

The failure will occur either at the section A or at the section B. At section A, although the bending moment is maximum, there is no stress concentration and the diameter is also more compared with that of the section B. It is, therefore, assumed that the failure will occur at the section B.

Step II *Construction of S–N diagram*

$$S'_e = 0.5\, S_{ut} = 0.5(540) = 270 \text{ N/mm}^2$$

From Fig. 5.24 (cold drawn steel and S_{ut} = 540 N/mm^2),

$$K_a = 0.78$$

Assuming, $7.5 < d < 50$ mm,

$$K_b = 0.85$$

For 90% reliability, $K_c = 0.897$

At the section B,

$$\left(\frac{D}{d}\right) = 1.5 \text{ and } \left(\frac{r}{d}\right) = 0.25$$

From Fig. 5.5, $K_t = 1.35$

From Eq. (5.12),

$$K_f = 1 + q\,(K_t - 1) = 1 + 0.85\,(1.35 - 1) = 1.2975$$

$$K_d = \frac{1}{K_f} = \frac{1}{1.2975} = 0.771$$

$S_e = K_a\, K_b\, K_c\, K_d\, S'_e$
$= 0.78(0.85)(0.897)(0.771)(270) = 123.8$ N/mm^2
$0.9S_{ut} = 0.9\,(540) = 486$ N/mm^2
$\log_{10}(0.9S_{ut}) = \log_{10}(486) = 2.6866$
$\log_{10}(S_e) = \log_{10}(123.8) = 2.0927$
$\log_{10}(10\,000) = 4$

The S–N curve for this problem is shown in Fig. 5.37.

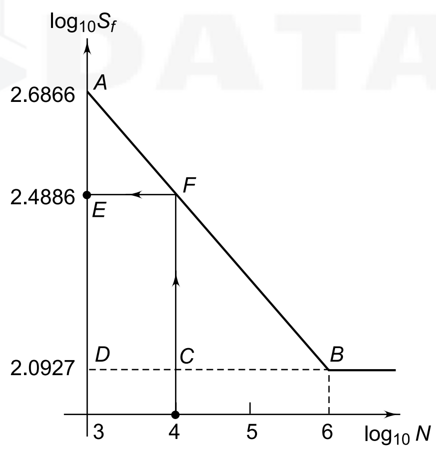

Fig. 5.37

Step III *Diameter of beam*

From Fig. 5.37,

$$\overline{AE} = \frac{\overline{AD} \times \overline{EF}}{\overline{DB}} = \frac{(2.6866 - 2.0927)(4-3)}{(6-3)} = 0.198$$

Therefore,

$$\log_{10} S_f = 2.6866 - \overline{AE} = 2.6866 - 0.198 = 2.4886$$

$$S_f = 308.03 \text{ N/mm}^2$$

$$S_f = \sigma_b = \frac{32M_b}{\pi d^3}$$

$$d^3 = \frac{32M_b}{\pi S_f} = \frac{32(1000 \times 150)}{\pi(308.03)}$$

$$d = 17.05 \text{ mm}$$

5.11 CUMULATIVE DAMAGE IN FATIGUE

In certain applications, the mechanical component is subjected to different stress levels for different parts of the work cycle. The life of such a component is determined by *Miner's equation*. Suppose that a component is subjected to completely reversed stresses (σ_1) for (n_1) cycles, (σ_2) for (n_2) cycles, and so on. Let N_1 be the number of stress cycles before fatigue failure, if only the alternating stress (σ_1) is acting. One stress cycle will consume ($1/N_1$) of the fatigue life and since there are n_1 such cycles at this stress level, the proportionate damage of fatigue life will be $[(1/N_1)n_1]$ or (n_1/N_1). Similarly, the proportionate damage at stress level (σ_2) will be (n_2/N_2). Adding these quantities, we get

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \ldots + \frac{n_x}{N_x} = 1 \qquad (5.33)$$

The above equation is known as Miner's equation. Sometimes, the number of cycles $n_1, n_2, \ldots$ at stress levels $\sigma_1, \sigma_2, \ldots$ are unknown. Suppose that $\alpha_1, \alpha_2, \ldots$ are proportions of the total life that will be consumed by the stress levels $\sigma_1, \sigma_2, \ldots$ etc. Let N be the total life of the component. Then,

$$n_1 = \alpha_1 N$$

$$n_2 = \alpha_2 N$$

Substituting these values in Miner's equation,

$$\frac{\alpha_1}{N_1} + \frac{\alpha_2}{N_2} + \ldots + \frac{\alpha_x}{N_x} = \frac{1}{N} \qquad (5.34)$$

Also,

$$\alpha_1 + \alpha_2 + \alpha_3 + \ldots\ldots\ldots + \alpha_x = 1 \qquad (5.35)$$

With the help of the above equations, the life of the component subjected to different stress levels can be determined.

Example 5.11 *The work cycle of a mechanical component subjected to completely reversed bending stresses consists of the following three elements:*

(i) ± 350 N/mm^2 for 85% of time
(ii) ± 400 N/mm^2 for 12% of time
(iii) ± 500 N/mm^2 for 3% of time

The material for the component is 50C4 (S_{ut} = 660 N/mm^2) and the corrected endurance limit of the component is 280 N/mm^2. Determine the life of the component.

Solution

Given $S_{ut} = 660$ N/mm^2 $\quad S_e = 280$ N/mm^2

Step I *Construction of S–N diagram*

$0.9Sut = 0.9(660) = 594$ N/mm^2

$\log10\,(0.9S_{ut}) = \log_{10}(594) = 2.7738$

$\log_{10}(S_e) = \log_{10}(280) = 2.4472$

$\log_{10}(\sigma_1) = \log_{10}(350) = 2.5441$

$\log_{10}(\sigma_2) = \log_{10}(400) = 2.6021$

$\log_{10}(\sigma_3) = \log_{10}(500) = 2.6990$

The S–N curve for this problem is shown in Fig. 5.38.

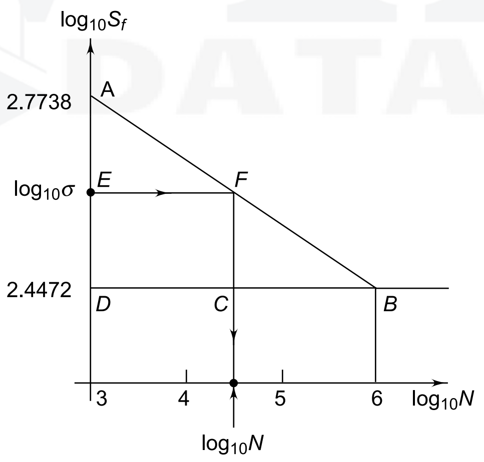

Fig. 5.38

Step II *Calculation of N_1, N_2 and N_3*

From Fig. 5.38,

$$\overline{EF} = \frac{\overline{DB} \times \overline{AE}}{\overline{AD}} = \frac{(6-3)(2.7738 - \log_{10}\sigma)}{(2.7738 - 2.4472)} \quad \text{(a)}$$

and $$\log_{10} N = 3 + \overline{EF} \quad \text{(b)}$$

From (a) and (b),

$\log_{10} N = 3 + 9.1855\ (2.7738 - \log_{10}\sigma)$

Therefore,

$\log_{10} (N_1) = 3 + 9.1855\ (2.7738 - 2.5441)$

or $N_1 = 128\ 798$

$\log_{10} (N_2) = 3 + 9.1855\ (2.7738 - 2.6021)$

or $N_2 = 37\ 770$

$\log_{10} (N_3) = 3 + 9.1855\ (2.7738 - 2.6990)$

or $N_3 = 4865$

Step III *Fatigue life of component*

From Eq. 5.34,

$$\frac{\alpha_1}{N_1} + \frac{\alpha_2}{N_2} + \frac{\alpha_3}{N_3} = \frac{1}{N}$$

$$\frac{0.85}{128\,798} + \frac{0.12}{37\,770} + \frac{0.03}{4865} = \frac{1}{N}$$

or $N = 62\ 723$ cycles

5.12 SODERBERG AND GOODMAN LINES

When a component is subjected to fluctuating stresses as shown in Fig. 5.15 (a), there is mean stress (σ_m) as well as stress amplitude (σ_a). It has been observed that the mean stress component has an effect on fatigue failure when it is present in combination with an alternating component. The fatigue diagram for this general case is shown in Fig. 5.39. In this diagram, the mean stress is plotted on the abscissa. The stress amplitude is plotted on the ordinate. The magnitudes of (σ_m) and (σ_a) depend upon the magnitudes of maximum and minimum force acting on the component. When stress amplitude (σ_a) is zero, the load is purely static and the criterion of failure is S_{ut} or S_{yt}. These limits are plotted on the abscissa. When the mean stress (σ_m) is zero, the stress is completely reversing and the criterion of failure is the endurance limit S_e that is plotted on the ordinate. When the component is subjected to both components of stress, viz., (σ_m) and (σ_a), the actual failure occurs at different scattered points shown in the figure. There exists a border, which divides safe region from unsafe region for various combinations of (σ_m) and (σ_a). Different criterions are proposed to construct the borderline dividing safe zone and failure zone. They include *Gerber line, Soderberg line* and *Goodman line*.

Gerber Line *A parabolic curve joining S_e on the ordinate to S_{ut} on the abscissa is called the Gerber line.*

Soderberg Line *A straight line joining S_e on the ordinate to S_{yt} on the abscissa is called the Soderberg line.*

Goodman Line *A straight line joining S_e on the ordinate to S_{ut} on the abscissa is called the Goodman line.*

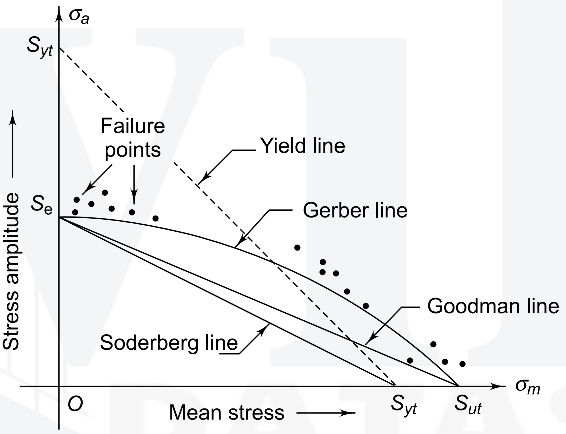

Fig. 5.39 *Soderberg and Goodman Lines*

The Gerber parabola fits the failure points of test data in the best possible way. The Goodman line fits beneath the scatter of this data. Both Gerber parabola and Goodman line intersect at (S_e) on the ordinate to (S_{ut}) on the abscissa. However, the Goodman line is more safe from design considerations because it is completely inside the Gerber parabola and inside the failure points. The Soderberg line is a more conservative failure criterion and there is no need to consider even yielding in this case. A yield line is constructed connecting (S_{yt}) on both axes. It is called the limit on '*first cycle*' of stress. This is because if a part yields, it has failed, regardless of its safety in fatigue.

We will apply following form for the equation of a straight line,

$$\frac{x}{a} + \frac{y}{b} = 1$$

where a and b are the intercepts of the line on the X and Y axes respectively.

Applying the above formula, the equation of the Soderberg line is given by,

$$\frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{S_e} = 1 \tag{5.36}$$

Similarly, the equation of the Goodman line is given by,

$$\frac{\sigma_m}{S_{ut}} + \frac{\sigma_a}{S_e} = 1 \tag{5.37}$$

The Goodman line is widely used as the criterion of fatigue failure when the component is subjected to mean stress as well as stress amplitude. It is because of the following reasons:

(i) The Goodman line is safe from design considerations because it is completely inside the failure points of test data.

(ii) The equation of a straight line is simple compared with the equation of a parabolic curve.

(iii) It is not necessary to construct a scale diagram and a rough sketch is enough to construct fatigue diagram.

5.13 MODIFIED GOODMAN DIAGRAMS

The components, which are subjected to fluctuating stresses, are designed by constructing the modified Goodman diagram. For the purpose of design, the problems are classified into two groups:

(i) components subjected to fluctuating axial or bending stresses; and

(ii) components subjected to fluctuating torsional shear stresses.

Separate diagrams are used in these two cases.

The modified Goodman diagram for fluctuating axial or bending stresses is shown in Fig. 5.40. In this diagram, the Goodman line is 'modified' by combining fatigue failure with failure by yielding. In this diagram, the yield strength S_{yt} is plotted on both the axes—abscissa and ordinate—and a yield line $\overline{CD}$ is constructed to join these two points to define failure by yielding. Obviously, the line $\overline{CD}$ is inclined at 45° to the abscissa. Similarly, a line $\overline{AF}$ is constructed to join S_e on the ordinate with S_{ut} on the abscissa, which is the Goodman line discussed in the previous article. The point of intersection of these two lines is B. The area $OABC$ represents the region of safety for components subjected to fluctuating stresses. The region $OABC$ is called *modified Goodman diagram*. All the points inside the modified Goodman diagram should cause neither fatigue failure nor yielding. The modified Goodman diagram combines fatigue criteria as represented by the Goodman line and yield criteria as represented by yield line. Note that $\overline{AB}$ is the portion of the Goodman line and $\overline{BC}$ is a portion of the yield line.

If the mean component of stress (σ_m) is very large and the alternating component (σ_a) very small, their combination will define a point in the region BCF that would be safely within the Goodman line but would yield on the first cycle. This will result in failure, irrespective of safety in fatigue failure. The portion $\overline{BF}$ of the Goodman line is a vulnerable portion and needs correction. This is the reason to modify the Goodman line.

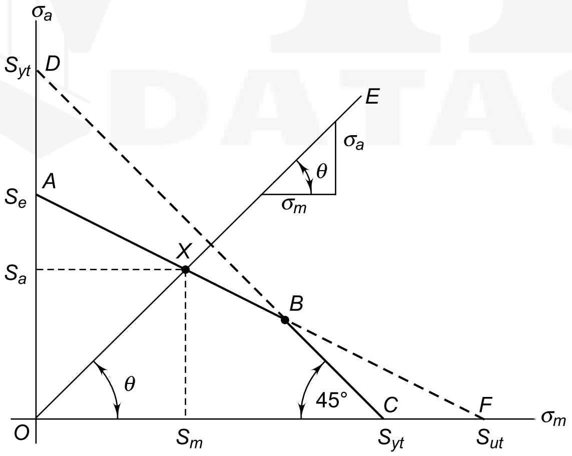

Fig. 5.40 *Modified Goodman Diagram for Axial and Bending Stresses*

While solving a problem, a line $\overline{OE}$ with a slope of tan θ is constructed in such a way that,

$$\tan\theta = \frac{\sigma_a}{\sigma_m} \tag{5.38}$$

Since $$\frac{\sigma_a}{\sigma_m} = \frac{(P_a / A)}{(P_m / A)} = \frac{P_a}{P_m}$$

$\therefore$ $$\tan\theta = \frac{P_a}{P_m} \tag{5.39}$$

The magnitudes of P_a and P_m can be determined from maximum and minimum forces acting on the component.

Similarly, it can be proved that

$$\tan\theta = \frac{(M_b)_a}{(M_b)_m} \tag{5.40}$$

The magnitudes of $(M_b)_a$ and $(M_b)_m$ can be determined from maximum and minimum bending moment acting on the component.

The point of intersection of lines $\overline{AB}$ and $\overline{OE}$ is X. The point X indicates the dividing line between the safe region and the region of failure. The coordinates of the point X (S_m, S_a) represent the limiting values of stresses, which are used to calculate the dimensions of the component. The permissible stresses are as follows:

$$\sigma_a = \frac{S_a}{(fs)} \quad \text{and} \quad \sigma_m = \frac{S_m}{(fs)} \tag{5.41}$$

The modified Goodman diagram for fluctuating torsional shear stresses is shown in Fig. 5.41. In this diagram, the torsional mean stress is plotted on the

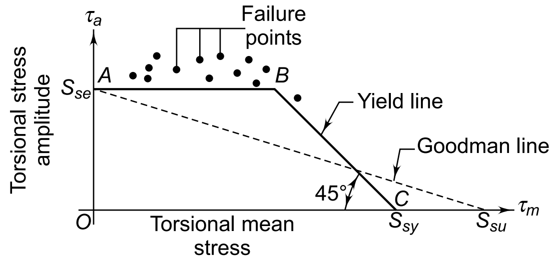

Fig. 5.41 *Modified Goodman Diagram for Torsional Shear Stresses*

abscissa while the torsional stress amplitude on the ordinate. The torsional yield strength S_{sy} is plotted on the abscissa and the yield line is constructed, which is inclined at 45° to the abscissa. It is interesting to note that up to a certain point, the torsional mean stress has no effect on the torsional endurance limit. Therefore, a line is drawn through S_{se} on the ordinate and parallel to the abscissa. The point of intersection of this line and the yield line is B. The area $OABC$ represents the region of safety in this case. It is not necessary to construct a fatigue diagram for fluctuating torsional shear stresses because $\overline{AB}$ is parallel to the X-axis. Instead, a fatigue failure is indicated if,

$$\tau_a = S_{se} \tag{5.42}$$

and a static failure is indicated if,

$$\tau_{\max} = \tau_a + \tau_m = S_{sy} \tag{5.43}$$

The permissible stresses are as follows:

$$\tau_a = \frac{S_{se}}{(fs)} \tag{5.44}$$

and $$\tau_{\max} = \frac{S_{sy}}{(fs)} \tag{5.45}$$

Infinite-life Problems (Fluctuating Load)

Example 5.12 *A cantilever beam made of cold drawn steel 4OC8 (S_{ut} = 600 N/mm^2 and S_{yt} = 380 N/mm^2) is shown in Fig. 5.42. The force P acting at the free end varies from –50 N to +150 N. The expected reliability is 90% and the factor of safety is 2. The notch sensitivity factor at the fillet is 0.9. Determine the diameter 'd' of the beam at the fillet cross-section.*

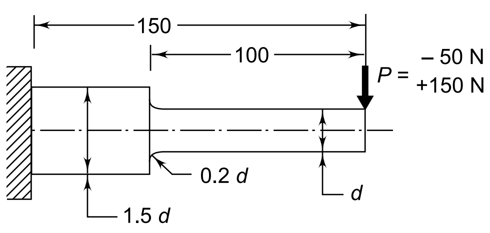

Fig. 5.42

Solution

Given P = –50 N to +150 N S_{ut} = 600 N/mm^2 S_{yt} = 380 N/mm^2 R = 90% (fs) = 2 q = 0.9

Step I *Endurance limit stress for cantilever beam*

$S'_e = 0.5S_{ut} = 0.5\ (600) = 300\ \text{N/mm}^2$

From Fig. 5.24 (cold drawn steel and $S_{ut} = 600$ N/mm^2),

$$K_a = 0.77$$

Assuming $7.5 < d < 50$ mm,

$$K_b = 0.85$$

For 90% reliability, $K_c = 0.897$

Since, $\dfrac{r}{d} = 0.2$ and $\dfrac{D}{d} = 1.5$

From Fig. 5.5, $K_t = 1.44$

From Eq. (5.12),

$K_f = 1 + q\ (K_t - 1) = 1 + 0.9\ (1.44 - 1) = 1.396$

$$K_d = \frac{1}{K_f} = \frac{1}{1.396} = 0.716$$

$$S_e = K_a K_b K_c K_d S'_e$$
$$= 0.77\ (0.85)\ (0.897)\ (0.716)\ (300)$$
$$= 126.11\ \text{N/mm}^2$$

Step II *Construction of modified Goodman diagram*

At the fillet cross-section,

$(M_b)_{\text{max.}} = 150 \times 100 = 15000$ N-mm

$(M_b)_{\text{min.}} = -50 \times 100 = -5000$ N-mm

$$(M_b)_m = \frac{1}{2}[(M_b)_{\text{max.}} + (M_b)_{\text{min.}}]$$
$$= \frac{1}{2}[15000 - 5000] = 5000 \text{ N-mm}$$

$$(M_b)_a = \frac{1}{2}[(M_b)_{\text{max.}} - (M_b)_{\text{min.}}]$$
$$= \frac{1}{2}[15000 + 5000] = 10\,000 \text{ N-mm}$$

$$\tan\theta = \frac{(M_b)_a}{(M_b)_m} = \frac{10000}{5000} = 2$$

$$\theta = 63.435°$$

The modified Goodman diagram for this example is shown in Fig. 5.43.

Step III *Permissible stress amplitude*

Refer to Fig. 5.43. The coordinates of the point X are determined by solving the following two equations simultaneously.

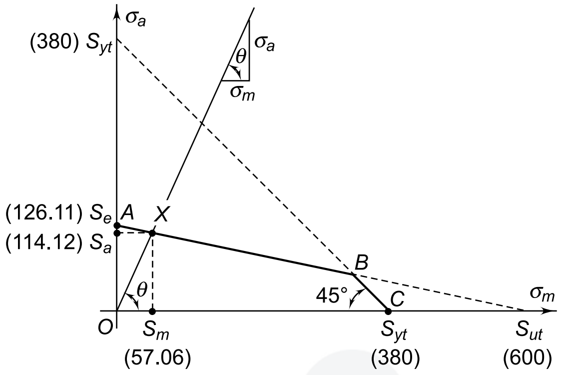

Fig. 5.43

(i) Equation of line AB

$$\frac{S_a}{126.11} + \frac{S_m}{600} = 1 \qquad \text{(a)}$$

(ii) Equation of line OX

$$\frac{S_a}{S_m} = \tan\theta = 2 \qquad \text{(b)}$$

Solving the two equations,

$S_a = 114.12\ \text{N/mm}^2$ and $S_m = 57.06\ \text{N/mm}^2$

Step IV *Diameter of beam*

Since $\sigma_a = \dfrac{S_a}{(fs)}$ $\quad\therefore\quad \dfrac{32(M_b)_a}{\pi d^3} = \dfrac{S_a}{(fs)}$

$$\frac{32(10\,000)}{\pi d^3} = \frac{114.12}{2}$$

$$d = 12.13 \text{ mm}$$

Example 5.13 *A transmission shaft of cold drawn steel 27Mn2 ($S_{ut} = 500$ N/mm^2 and $S_{yt} = 300$ N/mm^2) is subjected to a fluctuating torque which varies from –100 N-m to + 400 N-m. The factor of safety is 2 and the expected reliability is 90%. Neglecting the effect of stress concentration, determine the diameter of the shaft.*

Assume the distortion energy theory of failure.

Solution

Given $M_t = -100$ N-m to $+ 400$ N-m

$S_{ut} = 500\ \text{N/mm}^2$ $S_{yt} = 300\ \text{N/mm}^2$ $R = 90\%$ $(fs) = 2$

Step I *Endurance limit stress for shaft*

$S'_e = 0.5S_{ut} = 0.5\ (500) = 250\ \text{N/mm}^2$

From Fig. 5.24 (cold drawn steel and $S_{ut} = 500\ \text{N/mm}^2$),

$K_a = 0.79$

Assuming $7.5 < d < 50$ mm,

$K_b = 0.85$

For 90% reliability, $K_c = 0.897$

$S_e = K_a K_b K_c S'_e = 0.79\ (0.85)\ (0.897)\ (250)$
$= 150.58\ \text{N/mm}^2$

Step II *Construction of modified Goodman diagram*

Using the distortion energy theory,

$S_{se} = 0.577\ S_e = 0.577\ (150.58) = 86.88\ \text{N/mm}^2$

$S_{sy} = 0.577\ S_{yt} = 0.577\ (300) = 173.1\ \text{N/mm}^2$

$$(M_t)_m = \frac{1}{2}[(M_t)_{\text{max.}} + (M_t)_{\text{min.}}]$$

$$= \frac{1}{2}[400 - 100] = 150 \text{ N-m}$$

$$(M_t)_a = \frac{1}{2}[(M_t)_{\text{max.}} - (M_t)_{\text{min.}}]$$

$$= \frac{1}{2}[400 + 100] = 250 \text{ N-m}$$

$$\tan\theta = \frac{(M_t)_a}{(M_t)_m} = \frac{250}{150} = 1.67$$

$$\theta = 59.04°$$

The modified Goodman diagram for this example is shown in Fig. 5.44.

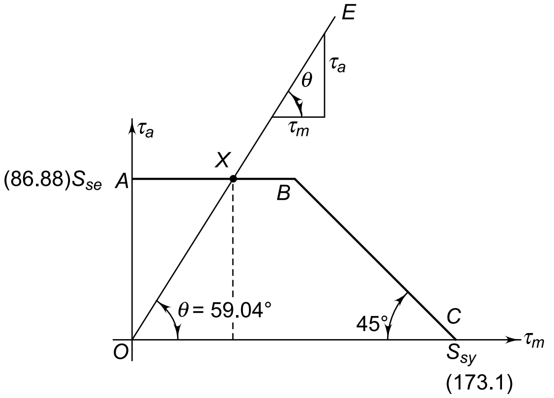

Fig. 5.44

Step III *Permissible shear stress amplitude*

Refer to Fig. 5.44. The ordinate of the point X is S_{se} or $86.88\ \text{N/mm}^2$.

$\therefore \quad S_{sa} = 86.88\ \text{N/mm}^2$

Step IV *Diameter of shaft*

$$\text{Since } \tau_a = \frac{S_{sa}}{(fs)} \qquad \therefore \frac{16(M_t)_a}{\pi d^3} = \frac{S_{sa}}{(fs)}$$

$$\frac{16(250 \times 10^3)}{\pi d^3} = \frac{86.88}{2}$$

$$d = 30.83 \text{ mm}$$

Example 5.14 *A spherical pressure vessel, with a 500 mm inner diameter, is welded from steel plates. The welded joints are sufficiently strong and do not weaken the vessel. The plates are made from cold drawn steel 20C8 (S_{ut} = 440 N/mm^2 and S_{yt} = 242 N/mm^2). The vessel is subjected to internal pressure, which varies from zero to 6 N/mm^2. The expected reliability is 50% and the factor of safety is 3.5. The size factor is 0.85. The vessel is expected to withstand infinite number of stress cycles. Calculate the thickness of the plates.*

Solution

Given For vessel $D_i = 500$ mm

p_i = zero to 6 N/mm^2 $\quad S_{ut} = 440\ \text{N/mm}^2$

$S_{yt} = 242\ \text{N/mm}^2 \quad R = 50\%\ (fs) = 3.5 \quad K_b = 0.85$

Step I *Endurance limit stress for vessel*

$S'_e = 0.5S_{ut} = 0.5\ (440) = 220\ \text{N/mm}^2$

From Fig. 5.24 (cold drawn steel and $S_{ut} = 440\ \text{N/mm}^2$),

$K_a = 0.82$

$K_b = 0.85$

For 50% reliability, $K_c = 1.0$

$S_e = K_a K_b K_c S'_e = 0.82\ (0.85)\ (1.0)\ (220)$
$= 153.34\ \text{N/mm}^2$

Step II *Construction of modified Goodman diagram*

For a spherical pressure vessel,

$$\sigma_t = \frac{p_i D_i}{4t}$$

$$\sigma_{\text{max.}} = \frac{p_{\text{max.}} D_i}{4t} = \frac{6(500)}{4t} = \left(\frac{750}{t}\right) \text{N/mm}^2$$

$$\sigma_{min.} = 0$$

$$\sigma_a = \sigma_m = \frac{1}{2}\sigma_{max.} = \frac{1}{2}\left(\frac{750}{t}\right) = \left(\frac{375}{t}\right) \text{N/mm}^2$$

$$\tan\theta = \frac{\sigma_a}{\sigma_m} = 1 \qquad \text{or} \qquad \theta = 45°$$

The modified Goodman diagram for this example is shown in Fig. 5.45.

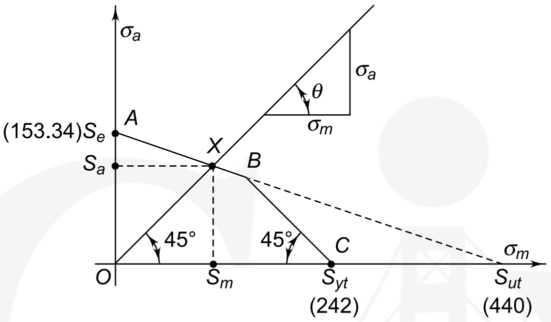

Fig. 5.45

Step III *Permissible stress amplitude*

Refer to Fig. 5.45. The coordinates of the point X are determined by solving the following two equations simultaneously.

(i) Equation of line AB

$$\frac{S_a}{153.34} + \frac{S_m}{440} = 1 \qquad \text{(a)}$$

(ii) Equation of line OX

$$\frac{S_a}{S_m} = \tan\theta = 1 \qquad \text{(b)}$$

Solving the two equations,

$S_a = S_m = 113.71$ N/mm^2

Step IV *Thickness of plate*

$$\text{Since} \quad \sigma_a = \frac{S_a}{(fs)} \qquad \therefore \quad \left(\frac{375}{t}\right) = \frac{113.71}{3.5}$$

$t = 11.54$ mm

Finite-Life Problems (Fluctuating Load)

Example 5.15 *A cantilever spring made of 10 mm diameter wire is shown in Fig. 5.46. The wire is made of stainless steel 4Cr18Ni10 (S_{ut} = 860 N/mm^2 and S_{yt} = 690 N/mm^2). The force P acting at the free end varies from 75 N to 150 N. The surface finish of the wire is equivalent to the machined surface. There is no stress concentration and the expected reliability is 50%. Calculate the number of stress cycles likely to cause fatigue failure.*

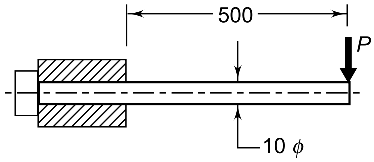

Fig. 5.46

Solution

Given For cantilever spring, d = 10 mm l = 500 mm P = 75 to 150 N S_{ut} = 860 N/mm^2 S_{yt} = 690 N/mm^2 R = 50%

Step I *Endurance limit stress for cantilever beam*

$S'_e = 0.5S_{ut} = 0.5(860) = 430$ N/mm^2

From Fig. 5.24 (machined surface and S_{ut} = 860 N/mm^2),

$K_a = 0.72$

For 10 mm diameter wire, $K_b = 0.85$

For 50% reliability,

$K_c = 1.0$

$S_e = K_a K_b K_c S'_e$

$= 0.72(0.85)(1.0)(430) = 263.16$ N/mm^2

Step II *Construction of modified Goodman diagram*

$(M_b)_{max.} = 150 \times 500 = 75000$ N-mm

$(M_b)_{min.} = 75 \times 500 = 37500$ N-mm

$$(M_b)_m = \frac{1}{2}[(M_b)_{max.} + (M_b)_{min.}]$$

$$= \frac{1}{2}[75\,000 + 37\,500] = 56\,250 \text{ N-mm}$$

$$(M_b)_a = \frac{1}{2}[(M_b)_{max.} - (M_b)_{min.}]$$

$$= \frac{1}{2}[75\,000 - 37\,500] = 18\,750 \text{ N-mm}$$

$$\sigma_m = \frac{32(M_b)_m}{\pi d^3} = \frac{32(56250)}{\pi(10)^3} = 572.96 \text{ N/mm}^2$$

$$\sigma_a = \frac{32(M_b)_a}{\pi d^3} = \frac{32(18750)}{\pi(10)^3} = 190.99 \text{ N/mm}^2$$

The modified Goodman diagram for this example is shown in Fig. 5.47. The point X with coordinates

(572.96, 190.99) falls outside the region of safety. This indicates a finite life for the spring. The value of S_f is obtained by joining C to X and then extending the line to the ordinate. Point A gives the value of S_f. From similar triangles XDC and AOC,

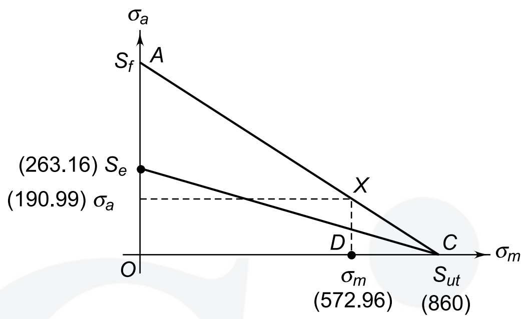

Fig. 5.47

$$\frac{\overline{XD}}{\overline{AO}} = \frac{\overline{DC}}{\overline{OC}}$$

$$S_f = \overline{AO} = \frac{\overline{XD} \times \overline{OC}}{\overline{DC}} = \frac{190.99(860)}{(860 - 572.96)}$$

$$= 572.22 \text{ N/mm}^2$$

Step III *Construction of S–N diagram*

$0.9S_{ut} = 0.9\ (860) = 774 \text{ N/mm}^2$

$\log_{10}(0.9S_{ut}) = \log_{10}(774) = 2.8887$

$\log_{10}(S_e) = \log_{10}(263.16) = 2.4202$

$\log_{10}(S_f) = \log_{10}(572.22) = 2.7576$

The S–N curve for this problem is shown in Fig. 5.48.

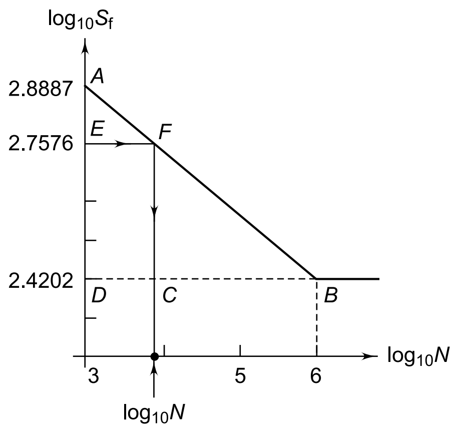

Fig. 5.48

Step IV *Fatigue life of cantilever spring*

From Fig. 5.48,

$$\overline{EF} = \frac{\overline{DB} \times \overline{AE}}{\overline{AD}} = \frac{(6-3)(2.8887 - 2.7576)}{(2.8887 - 2.4202)}$$

$$= 0.8395$$

$$\log_{10} N = 3 + \overline{EF} = 3 + 0.8395 = 3.8395$$

$$N = 6910.35 \text{ cycles}$$

Example 5.16 *A polished steel bar is subjected to axial tensile force that varies from zero to P_{max}. It has a groove 2 mm deep and having a radius of 3 mm. The theoretical stress concentration factor and notch sensitivity factor at the groove are 1.8 and 0.95 respectively. The outer diameter of the bar is 30 mm. The ultimate tensile strength of the bar is 1250 MPa. The endurance limit in reversed bending is 600 MPa. Find the maximum force that the bar can carry for 10^5 cycles with 90% reliability.*

Solution

Given $P = 0$ to $P_{\max}$ $\quad S_{ut} = 1250$ MPa

$S'_e = 600$ MPa $\quad K_t = 1.8 \quad q = 0.95 \quad R = 90\%$

$N = 10^5$ cycles

Step I *Endurance limit stress for bar*

$S'_e = 600 \text{ MPa} = 600 \text{ N/mm}^2$

From Fig. 5.24 (polished surface), $K_a = 1$

For 30 mm diameter wire, $K_b = 0.85$

For 90% reliability, $K_c = 0.897$

From Eq. (5.12),

$K_f = 1 + q\,(K_t - 1) = 1 + 0.95\,(1.8 - 1) = 1.76$

$$K_d = \frac{1}{K_f} = \frac{1}{1.76} = 0.568$$

$S_e = K_a K_b K_c K_d S'_e$

$= 1.0(0.85)(0.897)(0.568)(600) = 259.84 \text{ N/mm}^2$

Step II *Construction of S–N diagram*

$0.9S_{ut} = 0.9\ (1250) = 1125 \text{ N/mm}^2$

$\log_{10}(0.9S_{ut}) = \log_{10}(1125) = 3.0512$

$\log_{10}(S_e) = \log_{10}(259.84) = 2.4147$

$\log_{10}(10^5) = 5$

The S–N curve for this problem is shown in Fig. 5.49.

Step III *Fatigue strength at 10^5 cycles*

From Fig. 5.49,

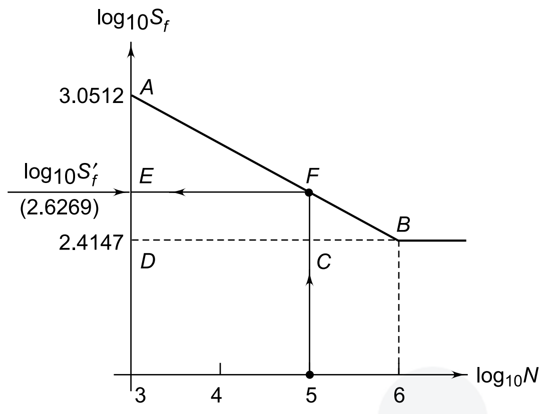

Fig. 5.49

$$\overline{AE} = \frac{\overline{AD} \times \overline{EF}}{\overline{DB}} = \frac{(3.0512 - 2.4147)(5 - 3)}{(6 - 3)}$$
$$= 0.4243$$

$$\log_{10} S_f = 3.0512 - \overline{AE} = 2.6269$$
$$S_f = 423.55 \text{ N/mm}^2$$

Therefore, at 10^5 cycles the fatigue strength is 423.55 N/mm^2.

Step IV *Construction of modified Goodman diagram*

Supose, $P_{\text{max.}} = P$ and $P_{\text{min.}} = 0$

$$P_m = \frac{1}{2}[P_{\text{max.}} + P_{\text{min.}}] = \frac{1}{2}P$$

$$P_a = \frac{1}{2}[P_{\text{max.}} - P_{\text{min.}}] = \frac{1}{2}P$$

$$\tan\theta = \frac{\sigma_a}{\sigma_m} = \frac{P_a}{P_m} = 1$$

$$\theta = 45°$$

The modified Goodman diagram for this example is shown in Fig. 5.50.

Step V *Permissible stress amplitude*

Refer to Fig. 5.50. The coordinates of point X are determined by solving the following two equations simultaneously.

(i) Equation of line AB

$$\frac{S_a}{423.55} + \frac{S_m}{1250} = 1 \qquad \text{(a)}$$

(ii) Equation of line OX

$$\frac{S_a}{S_m} = \tan\theta = 1 \qquad \text{(b)}$$

Solving the two equations,

$$S_a = S_m = 316.36 \text{ N/mm}^2$$

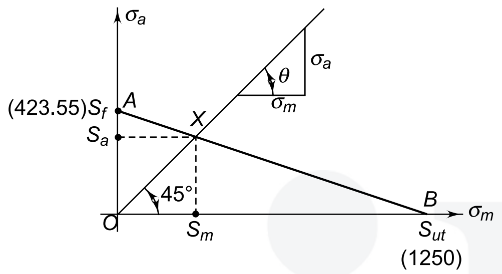

Fig. 5.50

Step VI *Maximum force on bar*

Since $S_a = \left(\frac{P_a}{\text{area}}\right) = \left(\frac{P/2}{\text{area}}\right)$

The minimum cross-section of the bar is shown in Fig. 5.51.

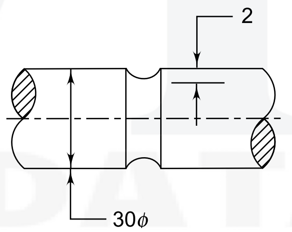

Fig. 5.51

$$S_a = \frac{P/2}{\text{area}} \quad \text{or} \quad 316.36 = \frac{P/2}{\frac{\pi}{4}(30-4)^2}$$

$P = 335\,929.5$ N or 336 kN

5.14 GERBER EQUATION

The Soderberg line and Goodman line illustrated in Fig. 5.39 are straight lines. The theories using such straight lines for predicting fatigue failure are called '*linear*' theories. There are some theories that use parabolic or elliptical curves instead of straight lines. These theories are called '*non-linear*' theories. One of the most popular non-linear theories is the Gerber theory that is based on parabolic curve. The Gerber curve is shown in Fig. 5.52. The equation for the Gerber curve is as follows:

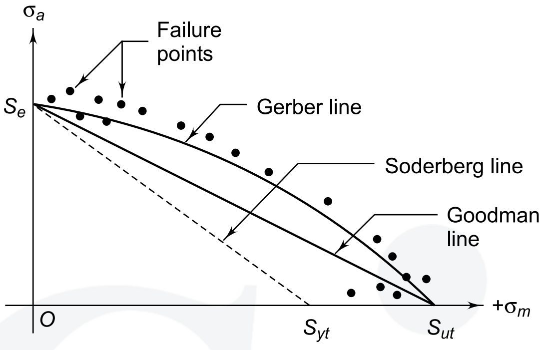

Fig. 5.52 *Gerber line*

$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1 \qquad (5.46)$$

The above equation is called the *Gerber equation*. It can be also written in the following form:

$$S_a = S_e\left[1 - \left(\frac{S_m}{S_{ut}}\right)^2\right] \qquad (5.47)$$

Theories based on the Soderberg line or the Goodman line, as failure criteria are conservative theories. This results in increased dimensions of the component. The Gerber curve takes the mean path through failure points. It is therefore more accurate in predicting fatigue failure.

Example 5.17 *A machine component is subjected to fluctuating stress that varies from 40 to 100 N/mm². The corrected endurance limit stress for the machine component is 270 N/mm². The ultimate tensile strength and yield strength of the material are 600 and 450 N/mm² respectively. Find the factor of safety using*

(i) Gerber theory
(ii) Soderberg line
(iii) Goodman line

Also, find the factor of safety against static failure.

Solution

Given $S_{ut} = 600$ N/mm² $S_{yt} = 450$ N/mm²
$S_e = 270$ N/mm² $\sigma_{max.} = 100$ N/mm²
$\sigma_{min.} = 40$ N/mm²

Step I *Permissible mean and amplitude stresses*

$$\sigma_a = \frac{1}{2}(100 - 40) = 30 \text{ N/mm}^2$$

$$\sigma_m = \frac{1}{2}(100 + 40) = 70 \text{ N/mm}^2$$

$$S_a = n\sigma_a = 30\,n$$
$$S_m = n\sigma_m = 70\,n$$

where n is the factor of safety.

Step II *Factor of safety using Gerber theory*

From Eq. (5.46),

$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$$

or $$\left(\frac{30n}{270}\right) + \left(\frac{70n}{600}\right)^2 = 1$$

$$n^2 + 8.16\,n - 73.47 = 0$$

Solving the above quadratic equation,

$$n = 5.41 \qquad \text{(i)}$$

Step III *Factor of safety using Soderberg line*

The equation of the Soderberg line is as follows,

$$\frac{S_a}{S_e} + \frac{S_m}{S_{yt}} = 1$$

$$\left(\frac{30n}{270}\right) + \left(\frac{70n}{450}\right) = 1$$

$$n = 3.75 \qquad \text{(ii)}$$

Step IV *Factor of safety using Goodman line*

The equation of the Goodman line is as follows:

$$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$$

$$\left(\frac{30n}{270}\right) + \left(\frac{70n}{600}\right) = 1$$

$$n = 4.39 \qquad \text{(iii)}$$

Step V *Factor of safety against static failure*

The factor of safety against static failure is given by,

$$n = \frac{S_{yt}}{\sigma_{max}} = \frac{450}{100} = 4.5 \qquad \text{(iv)}$$

Example 5.18 *A cantilever beam made of cold drawn steel 4OC8 (S_{ut} = 600 N/mm^2 and S_{yt} = 380 N/mm^2) is shown in Fig. 5.53. The force P acting at the free end varies from –50 N to +150 N. The expected reliability is 90% and the factor of safety is 2. The notch sensitivity factor at the fillet is 0.9. Determine the diameter d of the beam at the fillet cross-section using Gerber curve as failure criterion.*

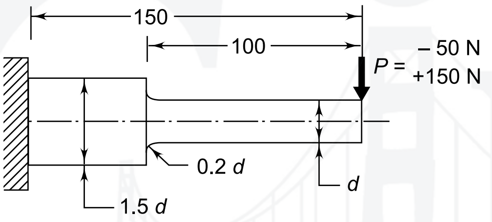

Fig. 5.53

Solution

Given S_{ut} = 600 N/mm^2

S_{yt} = 380 N/mm^2 R = 90% q = 0.9 (fs) = 2

P = –50 N to +150 N

Step I *Endurance limit stress for cantilever beam*

$S'_e = 0.5 S_{ut} = 0.5\,(600) = 300$ N/mm^2

From Fig. 5.24 (cold drawn steel and S_{ut} = 600 N/mm^2),

$K_a = 0.77$

Assuming 7.5 < d <50 mm, $K_b = 0.85$

For 90% reliability, $K_c = 0.897$

Since, $\frac{r}{d} = 0.2$ and $\frac{D}{d} = 1.5$

From Fig. 5.5, $K_t = 1.44$

From Eq. (5.12),

$K_f = 1 + q\,(K_t - 1) = 1 + 0.9\,(1.44 - 1) = 1.396$

$$K_d = \frac{1}{K_f} = \frac{1}{1.396} = 0.716$$

$$S_e = K_a K_b K_c K_d S'_e$$
$$= 0.77(0.85)\,(0.897)\,(0.716)\,(300)$$
$$= 126.11 \text{ N/mm}^2$$

Step II *Construction of Gerber diagram*

At the fillet cross-section,

$(M_b)_{max.} = 150 \times 100 = 15000$ N-mm

$(M_b)_{min.} = -50 \times 100 = -5000$ N-mm

$$(M_b)_m = \frac{1}{2}[(M_b)_{max.} + (M_b)_{min.}]$$
$$= \frac{1}{2}[15000 - 5000] = 5000 \text{ N-mm}$$

$$(M_b)_a = \frac{1}{2}[(M_b)_{max.} - (M_b)_{min.}]$$
$$= \frac{1}{2}[15000 + 5000] = 10\,000 \text{ N-mm}$$

$$\tan\theta = \frac{(M_b)_a}{(M_b)_m} = \frac{10000}{5000} = 2$$

$$\theta = 63.435°$$

The Gerber curve for this example is shown in Fig. 5.54.

Step III *Permissible stress amplitude*

Refer to Fig. 5.54. The co-ordinates of the point X (S_m, S_a) are determined by solving the following two equations simultaneously.

(i) Equation of Gerber curve

$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$$

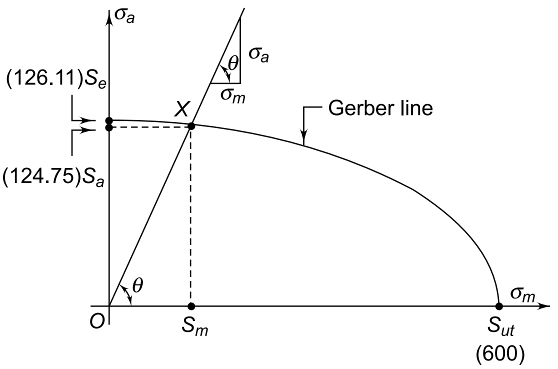

Fig. 5.54

or $$\frac{S_a}{126.11} + \left(\frac{S_m}{600}\right)^2 = 1 \qquad \text{(a)}$$

(ii) Equation of straight line *OX*

$$\frac{S_a}{S_m} = \tan\theta = 2 \qquad \text{(b)}$$

Substituting Eq. (b) in Eq. (a),

$$\frac{S_a}{126.11} + \left(\frac{S_a}{2(600)}\right)^2 = 1 \qquad \text{(c)}$$

$(S_a)^2 + 11418.6\,(S_a) - (1200)^2 = 0$

Solving the above quadratic equation,

$S_a = 124.75$ N/mm^2

Step IV *Diameter of cantilever beam*

Since $$\sigma_a = \frac{S_a}{(fs)} \qquad \therefore \quad \frac{32(M_b)_a}{\pi d^3} = \frac{S_a}{(fs)}$$

$$\frac{32(10\,000)}{\pi d^3} = \frac{124.75}{2}$$

$d = 11.78$ mm

5.15 FATIGUE DESIGN UNDER COMBINED STRESSES

The problems discussed so far are based on the construction of the modified Goodman diagram for the component, which is subjected to either axial load or bending moment or torsional moment. Each type of loading is considered separately. In practice, the problems are more complicated because the component may be subjected to two-dimensional stresses, or to combined bending and torsional moments. In case of two-dimensional stresses, each of the two stresses may have two components—mean and alternating. Similarly, the bending moment as well as torsional moment may have two components—mean and alternating. Such problems involving combination of stresses are solved by the distortion energy theory of failure. The most general equation of the distortion energy theory is as follows:

$$\sigma^2 = \frac{1}{2}[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)] \qquad \text{(a)}$$

where σ_x, σ_y, σ_z are normal stresses in *X*, *Y* and *Z* directions and τ_{xy}, τ_{yz}, τ_{zx} are shear stresses in their respective planes. σ is a stress which is equivalent to these three-dimensional stresses.

In case of two-dimensional stresses, the component is subjected to stresses σ_x and σ_y in *X* and *Y* directions.

Substituting $\sigma_z = \tau_{xy} = \tau_{yz} = \tau_{zx} = 0$ in Eq. (a),

$$\sigma = \sqrt{(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2)} \qquad \text{(b)}$$

The mean and alternating components of σ_x are σ_{xm} and σ_{xa} respectively. Similarly, the mean and alternating components of σ_y are σ_{ym} and σ_{ya} respectively. In this analysis, the mean and alternating components are separately combined by Eq. (b), i.e.,

$$\sigma_m = \sqrt{(\sigma_{xm}^2 - \sigma_{xm}\sigma_{ym} + \sigma_{ym}^2)} \qquad (5.48)$$

Similarly,

$$\sigma_a = \sqrt{(\sigma_{xa}^2 - \sigma_{xa}\sigma_{ya} + \sigma_{ya}^2)} \qquad (5.49)$$

The two stresses σ_m are σ_a obtained by the above equations are used in the modified Goodman diagram to design the component.

In case of combined bending and torsional moments, there is a normal stress σ_x accompanied by the torsional shear stress τ_{xy}.

Substituting $\sigma_y = \sigma_z = \tau_{yz} = \tau_{zx} = 0$ in Eq. (a),

$$\sigma = \sqrt{\sigma_x^2 + 3\tau_{xy}^2} \qquad \text{(c)}$$

The mean and alternating components of σ_x are σ_{xm} and σ_{xa} respectively. Similarly, the mean and alternating components of τ_{xy} are τ_{xym} and τ_{xya} respectively. Combining these components separately by Eq. (c),

$$\sigma_m = \sqrt{\sigma_{xm}^2 + 3\tau_{xym}^2} \qquad (5.50)$$

$$\sigma_a = \sqrt{\sigma_{xa}^2 + 3\tau_{xya}^2} \qquad (5.51)$$

The two stresses σ_m are σ_a obtained by the above equations are used in the modified Goodman diagram to design the component.

Example 5.19 *A machine component is subjected to two-dimensional stresses. The tensile stress in the X direction varies from 40 to 100 N/mm² while the tensile stress in the Y direction varies from 10 to 80 N/mm². The frequency of variation of these stresses is equal. The corrected endurance limit of the component is 270 N/mm². The ultimate tensile strength of the material of the component is 660 N/mm². Determine the factor of safety used by the designer.*

Solution

Given $(\sigma_x)_{max.} = 100$ N/mm²

$(\sigma_x)_{min.} = 40$ N/mm² $(\sigma_y)_{max.} = 80$ N/mm² $(\sigma_y)_{min.} = 10$ N/mm² $S_{ut} = 600$ N/mm² $S_e = 270$ N/mm²

Step I *Mean and amplitude stresses*

$$\sigma_{xm} = \frac{1}{2}(100 + 40) = 70 \text{ N/mm}^2$$

$$\sigma_{xa} = \frac{1}{2}(100 - 40) = 30 \text{ N/mm}^2$$

$$\sigma_{ym} = \frac{1}{2}(80 + 10) = 45 \text{ N/mm}^2$$

$$\sigma_{ya} = \frac{1}{2}(80 - 10) = 35 \text{ N/mm}^2$$

$$\sigma_m = \sqrt{(\sigma_{xm}^2 - \sigma_{xm}\sigma_{ym} + \sigma_{ym}^2)}$$

$$= \sqrt{[(70)^2 - (70)(45) + (45)^2]}$$

$$= 61.44 \text{ N/mm}^2$$

$$\sigma_a = \sqrt{(\sigma_{xa}^2 - \sigma_{xa}\sigma_{ya} + \sigma_{ya}^2)}$$

$$= \sqrt{[(30)^2 - (30)(35) + (35)^2]}$$

$$= 32.79 \text{ N/mm}^2$$

Step II *Construction of modified Goodman diagram*

$$\tan\theta = \frac{\sigma_a}{\sigma_m} = \frac{32.79}{61.44} = 0.534 \text{ or } \theta = 28.09°$$

The modified Goodman diagram for this example is shown in Fig. 5.55.

Step III *Permissible stress amplitude*

Refer to Fig. 5.55. The co-ordinates of the point X are obtained by solving the following two equations simultaneously.

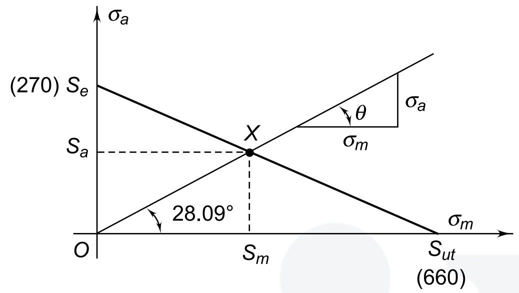

Fig. 5.55

$$\frac{S_a}{270} + \frac{S_m}{660} = 1 \quad \text{(a)}$$

$$\frac{S_a}{S_m} = \tan\theta = 0.534 \quad \text{(b)}$$

$$\therefore \quad S_a = 152.88 \text{ N/mm}^2$$

$$S_m = 286.29 \text{ N/mm}^2$$

Step IV *Factor of safety*

$$(fs) = \frac{S_a}{\sigma_a} = \frac{152.88}{32.79} = 4.66$$

Example 5.20 *A transmission shaft carries a pulley midway between the two bearings. The bending moment at the pulley varies from 200 N-m to 600 N-m, as the torsional moment in the shaft varies from 70 N-m to 200 N-m. The frequencies of variation of bending and torsional moments are equal to the shaft speed. The shaft is made of steel FeE 400 (S_{ut} = 540 N/mm² and S_{yt} = 400 N/mm²). The corrected endurance limit of the shaft is 200 N/mm². Determine the diameter of the shaft using a factor of safety of 2.*

Solution

Given $(M_t)_{max.} = 200$ N-m

$(M_t)_{min.} = 70$ N-m $(M_b)_{max.} = 600$ N-m

$(M_b)_{min.} = 200$ N-m $S_{ut} = 540$ N/mm²

$S_{yt} = 400$ N/mm² $S_e = 200$ N/mm² $(fs) = 2$

Step I *Mean and amplitude stresses*

$$(M_b)_m = \frac{1}{2}[(M_b)_{\text{max.}} + (M_b)_{\text{min.}}]$$

$$= \frac{1}{2}[600 + 200] = 400 \text{ N-m}$$

$$(M_b)_a = \frac{1}{2}[(M_b)_{\text{max.}} - (M_b)_{\text{min.}}]$$

$$= \frac{1}{2}[600 - 200] = 200 \text{ N-m}$$

$$(M_t)_m = \frac{1}{2}[(M_t)_{\text{max.}} + (M_t)_{\text{min.}}]$$

$$= \frac{1}{2}[200 + 70] = 135 \text{ N-m}$$

$$(M_t)_a = \frac{1}{2}[(M_t)_{\text{max.}} - (M_t)_{\text{min.}}]$$

$$= \frac{1}{2}[200 - 70] = 65 \text{ N-m}$$

$$\sigma_{xm} = \frac{32(M_b)_m}{\pi d^3} = \frac{32(400 \times 10^3)}{\pi d^3}$$

$$= \left(\frac{4074.37 \times 10^3}{d^3}\right) \text{N/mm}^2$$

$$\sigma_{xa} = \frac{32(M_b)_a}{\pi d^3} = \frac{32(200 \times 10^3)}{\pi d^3}$$

$$= \left(\frac{2037.18 \times 10^3}{d^3}\right) \text{N/mm}^2$$

$$\tau_{xym} = \frac{16(M_t)_m}{\pi d^3} = \frac{16(135 \times 10^3)}{\pi d^3}$$

$$= \left(\frac{687.55 \times 10^3}{d^3}\right) \text{N/mm}^2$$

$$\tau_{xya} = \frac{16(M_t)_a}{\pi d^3} = \frac{16(65 \times 10^3)}{\pi d^3}$$

$$= \left(\frac{331.04 \times 10^3}{d^3}\right) \text{N/mm}^2$$

$$\sigma_m = \sqrt{\sigma_{xm}^2 + 3\tau_{xym}^2}$$

$$= \sqrt{\left(\frac{4074.37 \times 10^3}{d^3}\right)^2 + 3\left(\frac{687.55 \times 10^3}{d^3}\right)^2}$$

$$= \left(\frac{4244.84 \times 10^3}{d^3}\right) \text{N/mm}^2$$

$$\sigma_a = \sqrt{\sigma_{xa}^2 + 3\tau_{xya}^2}$$

$$= \sqrt{\left(\frac{2037.18 \times 10^3}{d^3}\right)^2 + 3\left(\frac{331.04 \times 10^3}{d^3}\right)^2}$$

$$= \left(\frac{2116.33 \times 10^3}{d^3}\right) \text{N/mm}^2$$

Step II *Construction of modified Goodman diagram*

$$\tan\theta = \frac{\sigma_a}{\sigma_m} = \frac{2116.33}{4244.84} = 0.4986 \quad \text{or} \quad \theta = 26.5°$$

The modified Goodman diagram for this example is shown in Fig. 5.56.

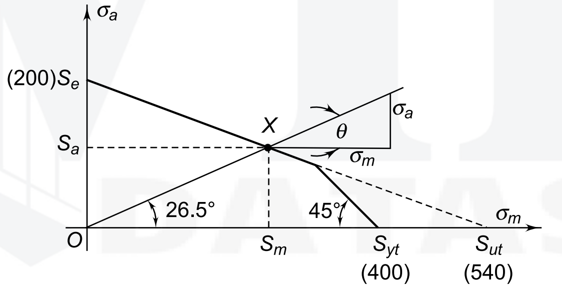

Fig. 5.56

Step III *Permissible stress amplitude*
Refer to Fig. 5.56. The co-ordinates of the point X are obtained by solving the following two equations simultaneously:

$$\frac{S_a}{200} + \frac{S_m}{540} = 1 \tag{a}$$

$$\frac{S_a}{S_m} = \tan\theta = 0.4986 \tag{b}$$

$\therefore$ $S_a = 114.76 \text{ N/mm}^2$

$S_m = 230.16 \text{ N/mm}^2$

Step IV *Diameter of shaft*

$$\text{Since } \sigma_a = \frac{S_a}{(fs)} \quad \therefore \quad \frac{2116.33 \times 10^3}{d^3} = \frac{114.76}{2}$$

$$d = 33.29 \text{ mm}$$

5.16 IMPACT STRESSES

Impact is defined as a collision of one component in motion with a second component, which may be either in motion or at rest. Impact load is the load which is rapidly applied to the machine component. Driving a nail with a hammer or breaking a coconut are examples of impact force. The stress induced in the machine component due to impact load is called *impact stress*. Impact forces are observed in machine components like hoisting ropes, hammers, springs, punches and shears, clutches and brakes.

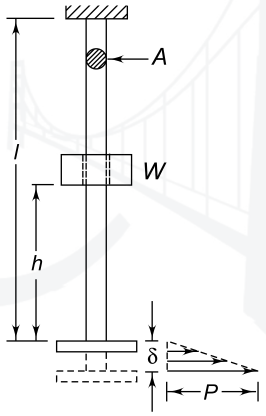

Fig. 5.57 *Impact Load*

We will consider an elastic system loaded by a falling weight W as shown in Fig. 5.57 and investigate the effect of impact. Suppose,

W = falling weight (N)

h = height through which the weight falls (mm)

δ = displacement of the point of load application (mm)

l = length of the bar (mm)

A = cross-sectional area of the bar (mm^2)

P = impact force which produces deflection δ (N)

E = modulus of elasticity of bar material (N/mm^2)

σ_i = impact stress in the bar (N/mm^2)

The weight W falls through the height h and strikes the collar of the bar. In this process, the potential energy released by the falling weight is absorbed by the bar and stored in the form of strain energy.

Energy released by falling weight = potential energy = $W(h + \delta)$

Energy absorbed by the system = strain energy = average load × deflection

$$= \left(\frac{1}{2}P\right)\delta$$

Equating the above two expressions,

$$\left(\frac{1}{2}P\right)\delta = W(h + \delta) \qquad \text{(a)}$$

Also,

$$P = \sigma_i A \qquad \text{(b)}$$

$$\frac{\delta}{l} = \varepsilon = \frac{\sigma_i}{E} \quad \text{or} \quad \delta = \frac{\sigma_i l}{E} \qquad \text{(c)}$$

Substituting (b) and (c) in Eq. (a),

$$(\sigma_i)^2\left(\frac{Al}{2E}\right) - (\sigma_i)\left(\frac{Wl}{E}\right) - Wh = 0 \qquad \text{(d)}$$

The above equation is a quadratic equation. Solving the equation and using positive sign for getting maximum value,

$$\sigma_i = \frac{W}{A}\left[1 + \sqrt{1 + \frac{2hAE}{Wl}}\right] \qquad (5.52)$$

Substituting the above expression for impact stress in Eq. (b),

$$P = W\left[1 + \sqrt{1 + \frac{2hAE}{Wl}}\right] \qquad (5.53)$$

or,
$$\frac{P}{W} = \left[1 + \sqrt{1 + \frac{2hAE}{Wl}}\right] \qquad (5.54)$$

The quantity $\left(\frac{P}{W}\right)$ is called *shock factor*, which indicates the magnification of the load W into the impact force P during impact.

We will consider a special case when the weight is applied instantaneously without any initial velocity,

$$h = 0$$

Substituting in Eq. (5.52),

$$\sigma_i = 2\left(\frac{W}{A}\right)$$

This means that the stress in the bar is double when the load is suddenly applied compared with a gradually applied load.

Example 5.21 *A mass of 50 kg drops through 25 mm at the centre of a 250 mm long simply supported beam. The beam has a square cross-section. It is made of steel 30C8 (S_{yt} = 400 N/mm²) and the factor of safety is 2. The modulus of elasticity is 207 000 N/mm². Determine the dimension of the cross-section of the beam.*

Solution

Given $m = 50$ kg $h = 25$ mm $l = 250$ mm
$S_{yt} = 400$ N/mm² $(fs) = 2$ $E = 207000$ N/mm²

Step I *Impact stress* (σ_i)

From Eq. (5.52),

$$\sigma_i = \frac{W}{A}\left[1+\sqrt{1+\frac{2hAE}{Wl}}\right]$$

In the above equation,

$$\frac{W}{A} = \text{static stress} = \sigma_s \qquad \text{(a)}$$

$$\frac{Wl}{AE} = \text{static deflection} = \delta_s \qquad \text{(b)}$$

Substituting (a) and (b) in Eq. (5.52),

$$\sigma_i = \sigma_s\left[1+\sqrt{1+\frac{2h}{\delta_s}}\right] \qquad \text{(c)}$$

Step II *Static stress* (σ_s)

For a simply supported beam,

$$W = mg = 50(9.81) = 490.5 \text{ N}$$

$$M_b = \frac{Wl}{4} = \frac{490.5(250)}{4} = 30\,656.25 \text{ N-mm}$$

$$I = \frac{bd^3}{12} = \frac{a(a)^3}{12} = \frac{a^4}{12}\text{ mm}^4 \qquad y = \frac{a}{2}$$

where a is the side of the square cross-section. Therefore,

$$\sigma_s = \sigma_b = \frac{M_b y}{I} = \frac{(30\,656.25)\left(\frac{a}{2}\right)}{\left(\frac{a^4}{12}\right)}$$

$$= \frac{183\,973.5}{a^3}\text{ N/mm}^2 \qquad \text{(d)}$$

Step III *Static deflection*

$$\delta_s = \frac{Wl^3}{48EI} = \frac{(490.5)(250)^3(12)}{48(207\,000)a^4} = \frac{9256.11}{a^4}\text{ mm} \qquad \text{(e)}$$

Step IV *Cross-section of beam*

Equating impact stress to permissible stress,

$$\sigma_i = \frac{S_{yt}}{(fs)} = \frac{400}{2} = 200\text{ N/mm}^2 \qquad \text{(f)}$$

Substituting (d), (e) and (f) in Eq. (c),

$$200 = \frac{183\,973.5}{a^3}\left[1+\sqrt{1+\frac{2(25)a^4}{9256.11}}\right]$$

or

$$\frac{a^3}{919.87} = \left[1+\sqrt{1+\frac{2(25)a^4}{9256.11}}\right]$$

$$\left(\frac{a^3}{919.87}-1\right)^2 = 1+\frac{a^4}{185.12}$$

Simplifying,

$$\frac{a^3}{846160.82} - \frac{1}{459.94} = \frac{a}{185.12}$$

The term (1/459.94) is very small and neglected. Therefore, $a = 67.6$ or 70 mm.

The cross-section of the beam is 70 × 70 mm.

Step V *Check for impact stresses*

$$\sigma_s = \frac{183\,973.5}{a^3} = \frac{183\,973.5}{(70)^3} = 0.5363\text{ N/mm}^2$$

$$\delta_s = \frac{9256.11}{a^4} = \frac{9256.11}{(70)^4} = 3.855\times10^{-4}\text{ mm}$$

$$\sigma_i = \sigma_s\left[1+\sqrt{1+\frac{2h}{\delta_s}}\right]$$

$$= 0.5363\left[1 + \sqrt{1 + \frac{2(25)}{(3.855 \times 10^{-4})}}\right]$$

$$= 193.68 \text{ N/mm}^2$$

$$\therefore \quad \sigma_i < 200 \text{ N/mm}^2$$

Short-Answer Questions

5.1 What is stress concentration?
5.2 How will you account for stress concentration in design of machine parts?
5.3 What is stress concentration factor?
5.4 What are the causes of stress concentration?
5.5 What are the methods of reducing stress concentration?
5.6 What is fluctuating stress? Draw a stress–time curve for fluctuating stress.
5.7 What is repeated stress? Draw a stress–time curve for repeated stress.
5.8 What is reversed stress? Draw a stress–time curve for reversed stress.
5.9 What is fatigue failure?
5.10 What are the machine components that fail by fatigue?
5.11 What is the difference between failure due to static load and fatigue failure?
5.12 What is endurance limit?
5.13 What is fatigue life?
5.14 What is *S–N* curve?
5.15 What is low-cycle fatigue?
5.16 Give practical examples of low-cycle fatigue failure.
5.17 What is high-cycle fatigue?
5.18 Give practical examples of high-cycle fatigue failure.
5.19 What is fatigue stress concentration factor?
5.20 What is notch sensitivity?
5.21 What is notch sensitivity factor?
5.22 What are the factors that affect endurance limit of a machine part?
5.23 What is surface finish factor?
5.24 What is size factor?
5.25 What is reliability factor?
5.26 What is modifying factor to account for stress concentration?
5.27 What is Miner's equation?
5.28 Where do you use Miner's equation?
5.29 What is the Goodman line?
5.30 What is the Soderberg line?
5.31 Explain the modified Goodman diagram for bending stresses.
5.32 Explain the modified Goodman diagram for torsional shear stresses.
5.33 What is the Gerber curve?
5.34 What is the difference between the Gerber curve and Soderberg and Goodman lines?

Problems for Practice

5.1 A rectangular plate, 15 mm thick, made of a brittle material is shown in Fig. 5.58. Calculate the stresses at each of three holes of 3, 5 and 10 mm diameter.

[*161.82, 167.33 and 200 N/mm²*]

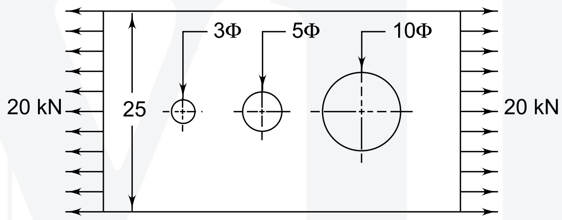

Fig. 5.58

5.2 A round shaft made of a brittle material and subjected to a bending moment of 15 N-m is shown in Fig. 5.59. The stress concentration factor at the fillet is 1.5 and the ultimate tensile strength of the shaft material is 200 N/mm². Determine the diameter *d*, the magnitude of stress at the fillet and the factor of safety.

[*11.76 mm, 140.91 N/mm², and 1.42*]

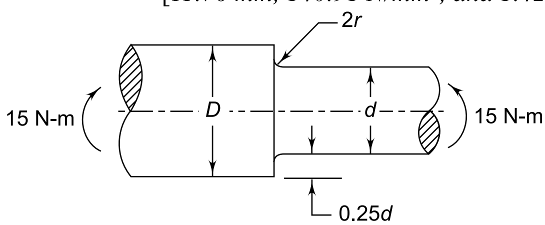

Fig. 5.59

5.3 A shaft carrying a load of 5 kN midway between two bearings is shown in Fig. 5.60. Determine the maximum bending stress at the fillet section. Assume the shaft material to be brittle. [*20.39 N/mm²*]

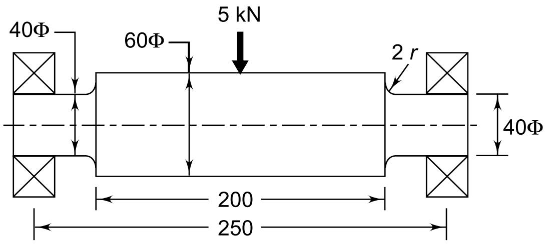

Fig. 5.60

5.4 A plate, 10 mm thick, subjected to a tensile load of 20 kN is shown in Fig. 5.61. The plate is made of cast iron (S_{ut} = 350 N/mm^2) and the factor of safety is 2.5. Determine the fillet radius. [*2.85 or 3 mm*]

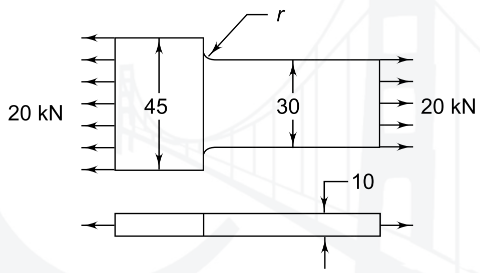

Fig. 5.61

5.5 A 25 mm diameter shaft is made of forged steel 30C8 (S_{ut} = 600 N/mm^2). There is a step in the shaft and the theoretical stress concentration factor at the step is 2.1. The notch sensitivity factor is 0.84. Determine the endurance limit of the shaft if it is subjected to a reversed bending moment. [*59.67 N/mm²*]

5.6 A 40 mm diameter shaft is made of steel 50C4 (S_{ut} = 660 N/mm^2) and has a machined surface. The expected reliability is 99%. The theoretical stress concentration factor for the shape of the shaft is 1.6 and the notch sensitivity factor is 0.9. Determine the endurance limit of the shaft. [*112.62 N/mm²*]

5.7 A cantilever beam made of steel Fe 540 (S_{ut} = 540 N/mm^2 and S_{yt} = 320 N/mm^2) and subjected to a completely reversed load (P) of 5 kN is shown in Fig. 5.62. The beam is machined and the reliability is 50%. The factor of safety is 2 and the notch sensitivity factor is 0.9. Calculate

(i) endurance limit at the fillet section; and
(ii) diameter *d* of the beam for infinite life.

[*(i) 109.20 N/mm² (ii) 45.35 mm*]

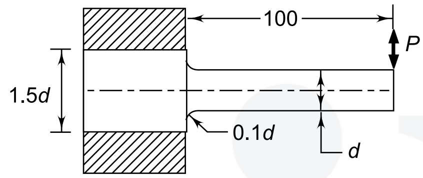

Fig. 5.62

5.8 A solid circular shaft made of steel Fe 620 (S_{ut} = 620 N/mm^2 and S_{yt} = 380 N/mm^2) is subjected to an alternating torsional moment, which varies from –200 N-m to + 400 N-m. The shaft is ground and the expected reliability is 90%. Neglecting stress concentration, calculate the shaft diameter for infinite life. The factor of safety is 2. Use the distortion-energy theory of failure. [*29.31 mm*]

5.9 A solid circular shaft, 15 mm in diameter, is subjected to torsional shear stress, which varies from 0 to 35 N/mm^2 and at the same time, is subjected to an axial stress that varies from –15 to +30 N/mm^2. The frequency of variation of these stresses is equal to the shaft speed. The shaft is made of steel FeE 400 (S_{ut} = 540 N/mm^2 and S_{yt} = 400 N/mm^2) and the corrected endurance limit of the shaft is 200 N/mm^2. Determine the factor of safety. [*4.05*]

5.10 A bar of steel has an ultimate tensile strength of 700 MPa, a yield point stress of 400 MPa and fully corrected endurance limit (S_e) of 220 MPa. The bar is subjected to a mean bending stress of 60 MPa and a stress amplitude of 80 MPa. Superimposed on it is a mean torsional stress and torsional stress amplitude of 70 and 35 MPa respectively. Find the factor of safety. [*1.54*]

Power Screws

Chapter 6

6.1 POWER SCREWS

A power screw is a mechanical device used for converting rotary motion into linear motion and transmitting power. A power screw is also called a *translation screw*. It uses helical translatory motion of the screw thread in transmitting power rather than clamping the machine components. The main applications of power screws are as follows:

(i) to raise the load, e.g., screw-jack;
(ii) to obtain accurate motion in machining operations, e.g., lead-screw of lathe;
(iii) to clamp a work piece, e.g., a vice; and
(iv) to load a specimen, e.g., universal testing machine.

There are three essential parts of the power screw, viz., screw, nut and a part to hold either the screw or the nut in its place. Depending upon the holding arrangement, power screws operate in two different ways. In some cases, the screw rotates in its bearing, while the nut has axial motion. The lead screw of the lathe is an example of this category. In other applications, the nut is kept stationary and the screw moves in an axial direction. A screw jack and machine vice are the examples of this category. A power screw offers the following advantages:

(i) A power screw has large load carrying capacity.
(ii) The overall dimensions of the power screw are small, resulting in compact construction.
(iii) A power screw is simple to design.
(iv) The manufacturing of a power screw is easy without requiring specilised machinery. Square threads are turned on the lathe. Trapezoidal threads are manufactured on a thread milling machine.
(v) A power screw provides large mechanical advantage. A load of 15 kN can be raised by applying an effort as small as 400 N. Therefore, most of the power screws used in various applications like screw-jacks, clamps, valves and vices are manually operated.
(vi) A power screw provides precisely controlled and highly accurate linear motion required in machine tool applications.
(vii) A power screw gives smooth and noiseless service without any maintenance.
(viii) There are few parts in a power screw. This reduces cost and increases reliability.
(ix) A power screw can be designed with self-locking property. In screw-jack application, self-locking characteristic is required to prevent the load from descending on its own.

The disadvantages of a power screw are as follows:

(i) A power screw has very poor efficiency, as low as 40%. Therefore, it is not used in continuous power transmission in machine tools, with the exception of the lead screw. Power screws are mainly used for intermittent motion that is occasionally required for lifting the load or actuating the mechanism.

(ii) High friction in threads causes rapid wear of the screw or the nut. In case of square threads, the nut is usually made of soft material and replaced when worn out. In trapezoidal threads, a split-type of nut is used to compensate for the wear. Therefore, wear is a serious problem in power screws.

There are two types of applications of power screws—applications where high efficiency is desired and applications where low efficiency is desired. The applications where high efficiency is expected are power transmission applications such as lead screw and presses. The applications where low efficiency is desired for the purpose of self-locking, are screw jacks, clamps and vices.

The efficiency of a power screw is increased if sliding friction is replaced by rolling friction. This principle is used in recirculating ball screw.

6.2 FORMS OF THREADS

The threads used for fastening purposes, such as V threads are not suitable for power screws. The purpose of fastening threads is to provide high frictional force, which lessens the possibility of loosening the parts assembled by threaded joint. On the other hand, the purpose of power transmission threads is to reduce friction between the screw and nut. Therefore, V threads are not suitable for power screws. Screws with smaller angle of thread, such as trapezoidal threads, are preferred for power transmission.

There are two popular types of threads used for power screws, viz., *square* and *ISO metric trapezoidal*, as shown in Fig. 6.1. The advantages of square threads over trapezoidal threads are as follows:

(i) The efficiency of square threads is more than that of trapezoidal threads.

(ii) There is no radial pressure or side thrust on the nut. This radial pressure is called '*bursting*' pressure on the nut. Since there is no side thrust, the motion of the nut is uniform. The life of the nut is also increased.

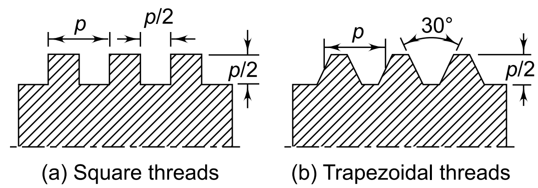

Fig. 6.1 *Forms of Thread*

The disadvantages of square threads are as follows:

(i) Square threads are difficult to manufacture. They are usually turned on a lathe with a single-point cutting tool. Machining with a single-point cutting tool is an expensive operation compared with machining with a multi-point cutting tool.

(ii) The strength of a screw depends upon the thread thickness at the core diameter. As shown in Fig. 6.1, square threads have less thickness at the core diameter than trapezoidal threads. This reduces the load carrying capacity of the screw.

(iii) The wear of the thread surface becomes a serious problem in the service life of the power screw. It is not possible to compensate for wear in square threads. Therefore, when worn out, the nut or the screw requires replacement.

The advantages of trapezoidal threads over square threads are as follows:

(i) Trapezoidal threads are manufactured on a thread milling machine. It employs a multi-point cutting tool. Machining with a multi-point cutting tool is an economic operation compared with machining with a single-point cutting tool. Therefore, trapezoidal threads are economical to manufacture.

(ii) A trapezoidal thread has more thickness at the core diameter than a square thread. Therefore, a screw with trapezoidal threads is stronger than an equivalent screw with square threads. Such a screw has a large load carrying capacity.

(iii) The axial wear on the surface of trapezoidal threads can be compensated by means of a split-type of nut. The nut is cut into two parts along the diameter. When the threads get worn out, the two halves of the nut are tightened together. The split-type nut can be used only for trapezoidal threads. It is used in lead-screw of a lathe to compensate wear at periodic intervals by tightening the two halves.

The disadvantages of trapezoidal threads are as follows:

(i) The efficiency of trapezoidal threads is less than that of square threads.

(ii) Trapezoidal threads result in side thrust or radial pressure on the nut. The radial pressure or bursting pressure on the nut affects its performance.

There is a special type of trapezoidal thread called *acme thread*. It is shown in Fig. 6.2. Trapezoidal and acme threads are identical in all respects except the thread angle. In an acme thread, the thread angle is 29° instead of 30°. The relative advantages and disadvantages of acme threads are same as those of trapezoidal threads.

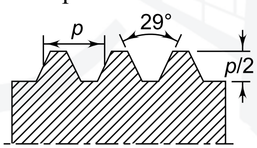

Fig. 6.2 *Acme Threads*

There is another type of thread called *buttress thread*. It is shown in Fig. 6.3. It combines the advantages of square and trapezoidal threads. Buttress threads are used where a heavy axial force acts along the screw axis in one direction only. The advantages of buttress threads are as follows:

(i) It has higher efficiency compared with trapezoidal threads.

(ii) It can be economically manufactured on a thread milling machine.

(iii) The axial wear at the thread surface can be compensated by means of a split-type nut.

(iv) A screw with buttress threads is stronger than an equivalent screw with either square threads or trapezoidal threads. This is because of greater thickness at the base of the thread.

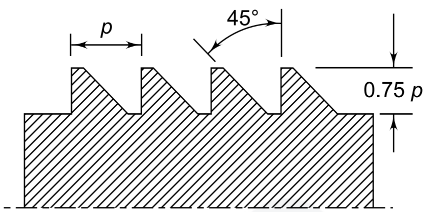

Fig. 6.3 *Buttress Threads*

The buttress threads have one disadvantage. It can transmit power and motion only in one direction. On the other hand, square and trapezoidal threads can transmit force and motion in both directions.

Square threads are used for screw jacks, presses and clamping devices. Trapezoidal and acme threads are used for lead-screw and other power transmission devices in machine tools. Buttress threads are used in vices, where force is applied only in one direction. Buttress threads are ideally suited for connecting tubular components that must carry large forces such as connecting the barrel to the housing in anti-aircraft guns. The standard proportions of square and ISO metric trapezoidal threads are given in Tables 6.1 and 6.2 respectively.[1, 2]

Table 6.1 *Proportions of square threads (normal series)*

Nominal diameter, d (mm)	*Pitch, p (mm)*
22, 24, 26, 28	5
30, 32, 36	6
40, 44	7
48, 50, 52	8
55, 60	9
65, 70, 75, 80	10
85, 90, 95, 100	12

[1] IS 4694–1968: Basic dimensions for square threads.
[2] IS 7008–1988: ISO metric trapezoidal screw threads (in four parts).

Table 6.2 *Proportions of ISO metric trapezoidal threads*

Nominal diameter, d (mm)	*Pitch, p (mm)*
24, 28	5
32, 36	6
40, 44	7
48, 52	8
60	9
70, 80	10
90, 100	12

There is a particular method of designation for square and trapezoidal threads. A power screw with single-start square threads is designated by the letters 'Sq' followed by the nominal diameter and the pitch expressed in millimetres and separated by the sign '×'. For example,

Sq 30 × 6

It indicates single-start square threads with 30 mm nominal diameter and 6 mm pitch.

Similarly, single-start ISO metric trapezoidal threads are designated by the letters 'Tr' followed by the nominal diameter and the pitch expressed in millimetres and separated by the sign '×'. For example,

Tr 40 × 7

It indicates single-start trapezoidal threads with 40 mm nominal diameter and 7 mm pitch.

Multiple-start trapezoidal threads are designated by the letters 'Tr' followed by the nominal diameter and the lead, separated by the sign '×' and in brackets the letter '*P*' followed by the pitch expressed in millimetres. For example,

Tr 40 × 14 (*P* 7)

In the above designation,

lead = 14 mm   pitch = 7 mm

∴ No. of starts = 14/7 = 2

Therefore, the above designation indicates a two-start trapezoidal thread with 40 mm nominal diameter and a 7 mm pitch. In case of left-hand threads, the letter *LH* is added to the thread designation. For example,

Tr 40 × 14 (*P* 7) *LH*

6.3 MULTIPLE THREADED SCREWS

Multiple threaded power screws are used in certain applications where higher travelling speed is required. They are also called multiple start screws such as double-start or triple-start screws. These screws have two or more threads cut side by side, around the rod. Such a screw can be imagined by winding two or more coloured strings side by side around the pencil. A multiple threaded screw has the following advantages:

(i) It provides large axial motion per revolution of the screw. This increases the travelling speed of the sliding member. For a double-start screw, the travelling speed is twice that of the single-start screw.

(ii) The efficiency of a multi-threaded screw is more than a single-threaded screw due to increase in helix angle.

The disadvantages of a multiple threaded screw are as follows:

(i) The mechanical advantage obtained with multiple threaded screws is lower than that of a single-threaded screw. Therefore, the effort required to raise a particular load or apply a particular force is more.

(ii) It is likely that the self-locking property may be lost in a multi-threaded screw. The resulting condition is dangerous in certain applications like screw jacks, where the load may descend on its own.

Multiple threaded screws are used in high speed actuators and sluice valves.

6.4 TERMINOLOGY OF POWER SCREW

The main dimensions of a double-threaded screw are shown in Fig. 6.4. The terminology of the screw thread is as follows:

(i) Pitch The pitch is defined as the distance measured parallel to the axis of the screw from a point on one thread to the corresponding point on the adjacent thread. It is denoted by the letter *p*.

(ii) Lead The lead is defined as the distance measured parallel to the axis of the screw which the nut will advance in one revolution of the screw. It is denoted by the letter l. For a single-threaded screw, the lead is same as the pitch. For a double-threaded screw, the lead is twice of the pitch, and so on.

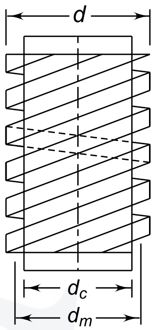

Fig. 6.4 *Terminology of Power Screw*

(iii) Nominal Diameter Nominal diameter is the largest diameter of the screw. It is also called *major diameter*. It is denoted by the letter d.

(iv) Core Diameter The core diameter is the smallest diameter of the screw thread. It is also called *minor diameter*. It is denoted by the letters d_c.

(v) Helix Angle The helix angle is defined as the angle made by the helix of the thread with a plane perpendicular to the axis of the screw. The helix angle is related to the lead and the mean diameter of the screw. It is also called *lead* angle. The helix angle is denoted by α.

From Fig. 6.4,

$$d_c = d - \left[\frac{p}{2} + \frac{p}{2}\right] \quad \text{or} \quad d_c = (d - p) \tag{6.1}$$

In Fig. 6.4, d_m is the mean diameter of the screw. It is given by,

$$d_m = \frac{1}{2}[d + d_c] = \frac{1}{2}[d + (d - p)]$$

or

$$d_m = (d - 0.5p) \tag{6.2}$$

Let us imagine that one thread of the screw is unwound and developed for one complete turn. This development of the thread is illustrated in Fig. 6.5. The thread will become the hypotenuse of a right-angle triangle, whose base is (πd_m) and whose height is the lead (l). This can be imagined by cutting the paper in the form of the right-angle triangle, with base equal to (πd_m) and height equal to (l). Wrap this paper around a rod with diameter d_m. The hypotenuse of the triangle will become the thread around the rod. Considering this right-angle triangle, the relationship between the helix angle, mean diameter and lead can be expressed in the following form:

$$\tan \alpha = \frac{l}{\pi d_m} \tag{6.3}$$

where α is the helix angle of the thread.

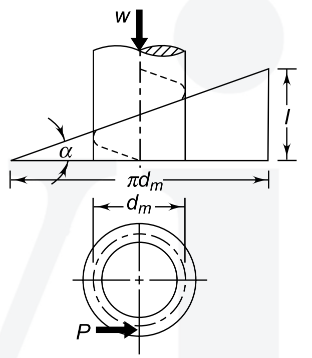

Fig. 6.5 *Development of Thread*

Figure 6.5 is used as basis for determining the torque required to raise or lower the load. The following conclusions can be drawn on the basis of development of thread,

(i) The screw can be considered as an inclined plane with α as the inclination.

(ii) The load W always acts in a vertically downward direction. When the load W is raised, it moves up the inclined plane. When the load W is lowered, it moves down the inclined plane.

(iii) The load W is raised or lowered by means of an imaginary force P acting at the mean radius of the screw. The force P multiplied by the mean radius $(d_m/2)$ gives the torque required to raise or lower the load. Remember P is perpendicular to the load W.

We will consider two separate cases to find out the torque required to raise or lower the load in the following articles.

6.5 TORQUE REQUIREMENT—LIFTING LOAD

The screw is considered as an inclined plane with inclination α as shown in Fig. 6.6. When the load is being raised, the following forces act at a point on this inclined plane:

(i) Load W It always acts in a vertically downward direction.

(ii) Normal Reaction N It acts perpendicular (normal) to the inclined plane.

(iii) Frictional Force μN Frictional force acts opposite to the motion. Since the load is moving up the inclined plane, frictional force acts along the inclined plane in the downward direction.

(iv) Effort P It is explained in the previous article that the effort P acts perpendicular to the load W. It may act towards the right or towards the left. It should act towards the right to overcome the friction and raise the load.

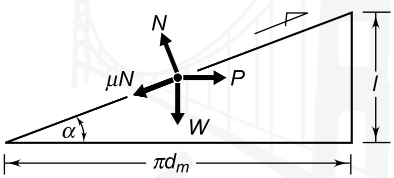

Fig. 6.6 *Force Diagram for Lifting Load*

Considering equilibrium of horizontal forces,

$$P = \mu N \cos\alpha + N \sin\alpha \qquad \text{(a)}$$

Considering equilibrium of vertical forces,

$$W = N \cos\alpha - \mu N \sin\alpha \qquad \text{(b)}$$

Dividing expression (a) by (b),

$$P = \frac{W(\mu\cos\alpha + \sin\alpha)}{(\cos\alpha - \mu\sin\alpha)}$$

Dividing the numerator and denominator of the right-hand side by cos α,

$$P = \frac{W(\mu + \tan\alpha)}{(1 - \mu\tan\alpha)} \qquad \text{(c)}$$

The coefficient of friction μ is expressed as

$$\mu = \tan\phi \qquad (6.4)$$

where ϕ is the friction angle.

Substituting $\mu = \tan\phi$ in Eq. (c), we have

$$P = \frac{W(\tan\phi + \tan\alpha)}{(1 - \tan\phi\ \tan\alpha)}$$

or

$$P = W\tan(\phi + \alpha) \qquad (6.5)$$

The torque M_t required to raise the load is given by,

$$M_t = \frac{Pd_m}{2}$$

$$M_t = \frac{Wd_m}{2}\tan(\phi + \alpha) \qquad (6.6)$$

6.6 TORQUE REQUIREMENT—LOWERING LOAD

When the load is being lowered, the forces acting at a point on the inclined plane are shown in Fig. 6.7. They are as follows:

(i) Load W It acts in a vertically downward direction.

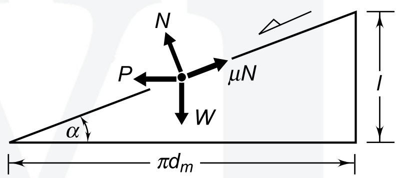

Fig. 6.7 *Force Diagram for Lowering Load*

(ii) Normal Reaction N It is perpendicular (normal) to the inclined plane.

(iii) Frictional Force μN Frictional force always acts opposite to the motion. Since the load is moving down the inclined plane, frictional force acts along the inclined plane in the upward direction.

(iv) Effort P Effort P is perpendicular to the load W. It should act towards the left to overcome the friction and lower the load.

Considering the equilibrium of horizontal and vertical forces,

$$P = \mu N \cos\alpha - N \sin\alpha \qquad \text{(a)}$$

$$W = N \cos\alpha + \mu N \sin\alpha \qquad \text{(b)}$$

Dividing expression (a) by (b),

$$P = \frac{W(\mu\cos\alpha - \sin\alpha)}{(\cos\alpha + \mu\sin\alpha)}$$

Dividing the numerator and denominator of the right-hand side by cos α, we have

$$P = \frac{W(\mu - \tan\alpha)}{(1 + \mu \tan\alpha)} \qquad (c)$$

Substituting $\mu = \tan\phi$ in Eq. (c),

$$P = \frac{W(\tan\phi - \tan\alpha)}{(1 + \tan\phi \tan\alpha)}$$

or $$P = W \tan(\phi - \alpha) \qquad (6.7)$$

and $$M_t = \frac{Wd_m}{2} \tan(\phi - \alpha) \qquad (6.8)$$

6.7 SELF-LOCKING SCREW

The torque required to lower the load can be obtained by Eq. (6.8). Rewriting the equation,

$$M_t = \frac{Wd_m}{2} \tan(\phi - \alpha)$$

it can be seen that when; $\phi < \alpha$ the torque required to lower the load is negative. It indicates a condition that no force is required to lower the load. The load itself will begin to turn the screw and descend down, unless a restraining torque is applied. This condition is called *overhauling* of the screw. This condition is also called *back driving* of screw. This property is not useful in screw-jack applications. However, it is useful in some other applications like a Yankee screwdriver. In this type of screwdriver, there is a high-lead thread on the barrel and the handle is a nut. As the worker pushes the handle axially down, the barrel turns and drives the wood screw into place.

When $\phi \geq \alpha$ a positive torque is required to lower the load. Under this condition, the load will not turn the screw and will not descend on its own unless an effort P is applied. In this case, the screw is said to be '*self-locking*'. A self-locking screw will hold the load in place without a brake. This is a very useful property in screw-jack application. For example, the driver can jack up the car and leave the jack handle and carry out the work. The car will not descend on its own during the work and the driver can do the work without any tension.

Neglecting collar friction, the rule for a self-locking screw is as follows,

"*A screw will be self-locking if the coefficient of friction is equal to or greater than the tangent of the helix angle*".

For a self-locking screw,

$$\phi > \alpha$$

$$\tan\phi > \tan\alpha$$

$$\mu > \frac{l}{\pi d_m} \qquad (6.9)$$

The following conclusions are drawn by examination of Eq. (6.9).

(i) Self-locking of screw is not possible when the coefficient of friction (μ) is low. The coefficient of friction between the surfaces of the screw and the nut is reduced by lubrication. Excessive lubrication may cause the load to descend on its own.

(ii) Self-locking property of the screw is lost when the lead is large. The lead increases with a number of starts. For a double-start thread, the lead is twice of the pitch and for a triple-threaded screw, it is three times of the pitch. Therefore, single-threaded screw is better than multiple-threaded screws from self-locking considerations.

Self-locking condition is essential in applications like screw jacks.

6.8 EFFICIENCY OF SQUARE THREADED SCREW

Let us refer to the force diagram for lifting the load, illustrated in Fig. 6.6. Suppose the load W moves from the lower end to the upper end of the inclined plane. The output consists of raising the load. Therefore,

Work output = force × distance travelled in the direction of force = (Wl)

The input consists of rotating the screw by means of an effort P.

Work input = force × distance traveled in the direction of force = $P(\pi d_m)$

The efficiency η of the screw is given by,

$$\eta = \frac{\text{work output}}{\text{work input}} = \frac{Wl}{P\pi d_m} = \frac{W}{P}\tan\alpha \qquad (a)$$

Substituting Eq. (6.5) in the above equation,

$$\eta = \eta = \frac{\tan\alpha}{\tan(\phi + \alpha)} \qquad (6.10)$$

The efficiency of a square threaded power screw depends upon (α) and (ϕ). Also, (α) depends upon the lead (l) and mean diameter of screw (d_m), since ($\tan \alpha = l/\pi d_m$).

Conclusions The efficiency of a square threaded power screw depends upon the following three factors:

(i) Mean diameter of screw
(ii) Lead of the screw
(iii) Coefficient of friction

The above expression of efficiency has certain limitations. It considers frictional loss only at the contacting surfaces between the screw and the nut. It does not take into consideration other frictional losses, such as collar-friction loss. Therefore, the equation does not give the overall efficiency of the screw mechanism.

From Eq. (6.10), it is evident that the efficiency of the square threaded screw depends upon the helix angle α and the friction angle ϕ. Figure 6.8 shows variation of the efficiency of a square threaded screw against the helix angle for various values of coefficient of friction. The graph is applicable when the load is lifted. The following conclusions can be derived from the observation of these graphs:

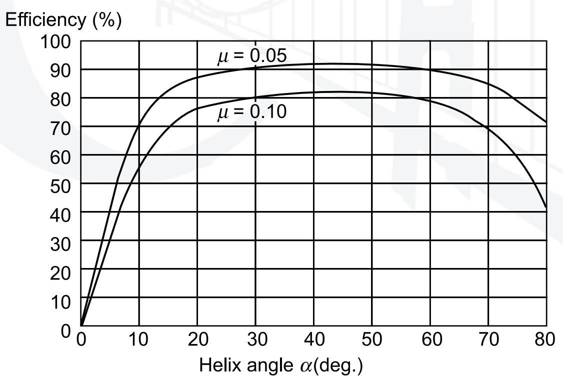

Fig. 6.8 *Efficiency of Square Threaded Screw*

(i) The efficiency of a square threaded screw increases rapidly up to helix angle of 20°.
(ii) The efficiency is maximum, when the helix angle is between 40 to 45°.
(iii) The efficiency decreases after the maximum value is reached.
(iv) The efficiency decreases rapidly when the helix angle exceeds 60°.
(v) The efficiency decreases as the coefficient of friction increases.

There are two ways to increase the efficiency of a square threaded screw. They are as follows:

(i) Reduce the coefficient of friction between the screw and the nut by proper lubrication; and
(ii) Increase the helix angle up to 40° to 45° by using multiple-start threads.

However, a screw with such a helix angle has other disadvantages like loss of self-locking property.

Rewriting Eq. (6.10),

$$\eta = \frac{\tan\alpha}{\tan(\phi+\alpha)} = \frac{\tan\alpha}{\tan(\alpha+\phi)}$$

$$= \frac{\sin\alpha/\cos\alpha}{\sin(\alpha+\phi)/\cos(\alpha+\phi)}$$

$$= \frac{\sin\alpha\cos(\alpha+\phi)}{\cos\alpha\sin(\alpha+\phi)}$$

$$\eta = \frac{2\sin\alpha\cos(\alpha+\phi)}{2\cos\alpha\sin(\alpha+\phi)} \quad \text{(b)}$$

We will use the following trigonometric relation,

$$2\sin A\cos B = \sin(A+B) + \sin(A-B) \quad \text{(c)}$$

If we interchange A and B in the above expression,

$$2\sin B\cos A = \sin(B+A) + \sin(B-A)$$

Rearranging the terms,

$$2\cos A\sin B = \sin(A+B) - \sin(A-B) \quad \text{(d)}$$

Substituting (c) and (d) in (b),

$$\eta = \frac{\sin[\alpha+(\alpha+\phi)] + \sin[\alpha-(\alpha+\phi)]}{\sin[\alpha+(\alpha+\phi)] - \sin[\alpha-(\alpha+\phi)]}$$

$$= \frac{\sin(2\alpha+\phi) + \sin(-\phi)}{\sin(2\alpha+\phi) - \sin(-\phi)}$$

$$\eta = \frac{\sin(2\alpha+\phi) - \sin\phi}{\sin(2\alpha+\phi) + \sin\phi} \quad \text{(e)}$$

The coefficient of friction is constant and the only variable is (α). For efficiency (η) to be maximum, the term [sin ($2\alpha + \phi$)] should be maximum.

The maximum value of the 'sin' term is 1 when the angle is 90°.

$$[\sin(2\alpha + \phi)] = 1 \quad \text{or} \quad 2\alpha + \phi = 90°$$

$$\therefore \qquad \alpha = \left(45 - \frac{\phi}{2}\right) \text{ degrees} \qquad \text{(f)}$$

Substituting $[2\alpha + \phi = 90°]$ in the expression (e),

$$\eta_{max} = \frac{\sin(90°) - \sin\phi}{\sin(90°) + \sin\phi}$$

or
$$\eta_{max} = \frac{1 - \sin\phi}{1 + \sin\phi} \qquad \text{(g)}$$

Expressions (f) and (g) give the condition for maximum power.

Suppose friction angle = 30°

$$\eta_{max} = \frac{1 - \sin 30°}{1 + \sin 30°} = \frac{1 - 1/2}{1 + 1/2} = \frac{1/2}{3/2} = \frac{1}{3} = 33\%$$

Conclusions (i) The efficiency of a square threaded power screw is maximum when,

$$\alpha = \left(45 - \frac{\phi}{2}\right) \text{ degrees}$$

(ii) Maximum efficiency of a square threaded power screw depends only on friction angle or coefficient of friction.

(iii) Maximum efficiency of a square threaded power screw is given by,

$$\eta_{max} = \frac{1 - \sin\phi}{1 + \sin\phi}$$

6.9 EFFICIENCY OF SELF-LOCKING SCREW

The efficiency of a square threaded screw is given by,

$$\eta = \frac{\tan\alpha}{\tan(\phi + \alpha)} \qquad \text{(a)}$$

For a self-locking screw,

$$\phi \geq \alpha$$

Substituting the limiting value $(\phi = \alpha)$ in Eq. (a),

$$\eta \leq \frac{\tan\phi}{\tan(\phi + \phi)} \quad \text{or} \quad \eta \leq \frac{\tan\phi}{\tan(2\phi)}$$

Substituting, $\tan(2\phi) = \dfrac{2\tan\phi}{1 - \tan^2\phi}$

in the above expression,

$$\eta \leq \frac{\tan\phi\,(1 - \tan^2\phi)}{2\tan\phi}$$

or,
$$\eta \leq \left[\frac{1}{2} - \frac{\tan^2\phi}{2}\right] \qquad (6.11)$$

Therefore, efficiency of a self-locking square threaded power screw is less than 1/2 or 50%.

6.10 TRAPEZOIDAL AND ACME THREADS

The force acting on the surface of the trapezoidal thread is shown in Fig. 6.9. The thread angle is (2θ).

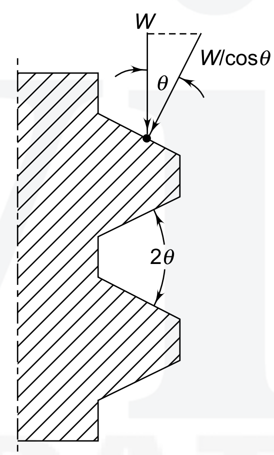

Fig. 6.9 *Force Diagram for Trapezoidal Thread*

For ISO Metric trapezoidal thread, $2\theta = 30°$

For acme thread, $2\theta = 29°$

There is a basic difference between the force acting on the thread of the square and trapezoidal threads. In case of square threads, W is the axial load raised by the screw. It is also the normal force acting on the thread surface. In case of trapezoidal or acme threads, these two forces are different. As shown in Fig. 6.9, the axial force on the screw is W, while the normal force on the thread surface is $(W/\cos\theta)$ or $(W \sec\theta)$. The frictional force depends upon the normal force. Therefore, the effect of the thread angle is to increase the frictional force by a term $(\sec\theta)$. To account for this effect, the coefficient of friction is taken as $(\mu \sec\theta)$ instead of (μ) in case of trapezoidal threads. The equations derived for a square threaded screw are used for trapezoidal or acme threads by making this change.

Case I *Lifting load*
Modifying Eq. (c) of Section 6.5,

$$P = \frac{W(\mu \sec\theta + \tan\alpha)}{(1 - \mu \sec\theta \tan\alpha)} \tag{6.12}$$

and $$M_t = P\left(\frac{d_m}{2}\right) = \frac{Wd_m}{2} \cdot \frac{(\mu \sec\theta + \tan\alpha)}{(1 - \mu \sec\theta \tan\alpha)} \tag{6.13}$$

Case II *Lowering load*
From Eq. (c) of Section 6.6

$$P = \frac{W(\mu \sec\theta - \tan\alpha)}{(1 + \mu \sec\theta \tan\alpha)} \tag{6.14}$$

and $$M_t = P\left(\frac{d_m}{2}\right) = \frac{Wd_m}{2} \cdot \frac{(\mu \sec\theta - \tan\alpha)}{(1 + \mu \sec\theta \tan\alpha)} \tag{6.15}$$

Efficiency of Screw From Eq. (a) of Section 6.8, the efficiency of a square threaded screw is given by,

$$\eta = \frac{W}{P}\tan\alpha$$

Substituting Eq. (c) of Section 6.5 in the above expression,

$$\eta = \frac{\tan\alpha(1 - \mu \tan\alpha)}{(\mu + \tan\alpha)}$$

Substituting (μ sec θ) in place of (μ), the efficiency of trapezoidal threads is given by,

$$\eta = \frac{\tan\alpha(1 - \mu \sec\theta \tan\alpha)}{(\mu \sec\theta + \tan\alpha)} \tag{6.16}$$

6.11 COLLAR FRICTION TORQUE

In many applications of the power screw, there is collar friction in addition to the friction at the thread surface. The principle of collar friction can be explained with the help of Fig. 6.10(a). The cup remains stationary under the action of the load W, while the collar that is integral with the screw rotates when the load is being raised or lowered. Therefore, there is relative motion between the cup and the collar at the annular interface from diameter D_i to D_o. This relative motion results in friction called *collar friction*. The torque required to overcome this friction or collar friction torque $(M_t)_c$ can be determined by using uniform pressure theory or uniform wear theory.

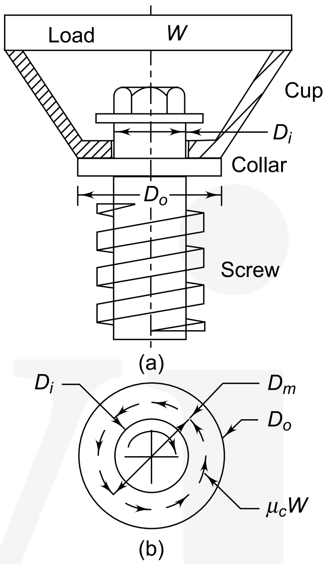

Fig. 6.10 *Collar Friction*

According to the uniform pressure theory,

$$(M_t)_c = \frac{\mu_c W}{3} \cdot \frac{D_0^3 - D_i^3}{D_0^2 - D_i^2} \tag{6.17}$$

According to the uniform wear theory,

$$(M_t)_c = \frac{\mu_c W}{4} .(D_0 + D_i) \tag{6.18}$$

where

μ_c = coefficient of friction at the collar
D_o = outer diameter of the collar (mm)
D_i = inner diameter of the collar (mm)
$(M_t)_c$ = collar friction torque (N-mm)

Uniform pressure theory is applicable only when the collar surface is new. Uniform wear theory is applicable to the collar surface after the initial wear. Therefore, it is logical to use uniform wear theory for collar friction.

There is a simple method to derive Eq. (6.18) applicable to uniform wear condition. Since the normal force on the collar surface is W, the frictional force over the collar surface is $\mu_c W$. It is assumed that frictional force is concentrated at the mean collar diameter D_m as shown in Fig. 6.10(b). This force acts opposite to the rotation of the collar surface. The collar friction torque is given by,

$$(M_t)_c = \mu_c W \left(\frac{D_m}{2}\right) \tag{a}$$

where $$D_m = \left(\frac{D_0 + D_i}{2}\right) \tag{b}$$

Substituting Eq. (b) in Eq (a), we get,

$$(M_t)_c = \frac{\mu_c W}{4}.(D_0 + D_i)$$

In certain applications, the collar between the cup and the screw is replaced by thrust ball bearing to reduce friction. The advantage of using thrust ball bearing at the collar is that the sliding friction is replaced by rolling friction. The collar friction torque becomes almost negligible in these cases. Eqs (6.17) and (6.18) should not be used where rolling contact bearings are employed to take the thrust reaction.

6.12 OVERALL EFFICIENCY

The total external torque required to raise the load consists of two factors—the torque required to overcome friction at the thread surface and the collar friction torque. Therefore,

$$(M_t)_t = M_t + (M_t)_c \tag{6.19}$$

where

$(M_t)_t$ = external torque required to raise the load (N-mm);

M_t = torque required to overcome friction at the thread surface (N-mm); and

$(M_t)_c$ = collar friction torque (N-mm)

Let us again refer to the force diagram for lifting the load shown in Fig. 6.6. Suppose the load W moves from the lower end to the upper end of the inclined plane. The output consists of raising the load.

Work output = force × distance traveled in the direction of force = $(W l)$

The input consists of torque applied to the screw $(M_t)_t$.

Work input = torque × angle turned through
$= [(M_t)_t\,(2\pi)]$

The overall efficiency η_0 of the power screw is given by,

$$\eta_0 = \frac{\text{work output}}{\text{work input}} = \frac{W \times l}{(M_t)_t \times 2\pi}$$

or, $$\eta_0 = \frac{Wl}{2\pi (M_t)_t} \tag{6.20}$$

6.13 COEFFICIENT OF FRICTION

It has been found that the coefficient of friction (μ) at the thread surface depends upon the workmanship in cutting the threads and on the type of the lubricant. It is practically independent of the load, rubbing velocity or materials. An average value of 0.15 can be taken for the coefficient of friction at the thread surface, when the screw is lubricated with mineral oil. The values of the coefficient of friction (μ_c) for the thrust collar with sliding contact are given in Table 6.3. When thrust ball bearing is used at the collar surface, its coefficient of friction is about 1/10th of plain sliding surface. It varies from 0.01 to 0.02.

Table 6.3 *Coefficient of friction for thrust collars*

Material combination	μ_c	
	Starting	*Running*
Soft-steel – cast iron	0.17	0.12
Hardened steel – cast iron	0.15	0.09
Soft steel – bronze	0.10	0.08
Hardened steel – bronze	0.08	0.06

6.14 DESIGN OF SCREW AND NUT

There are three basic components of a power screw, viz., screw, nut and frame. The desirable properties of screw material are as follows:

(i) It should have sufficient strength to withstand stresses due to external load and applied torque.

(ii) It should possess high wear resistance.

(iii) It should have good machinability.

Screws are made of plain carbon steel such as 30C8, 40C8 and 45C8 or alloy steels like 40Cr1. The screws are case hardened, e.g., the lead screw of a lathe is case hardened by the nitriding process.

There is a relative motion between the screw and the nut and wear is inevitable. The material for the nut should be soft and conformable to that of the screw. The wear is always restricted to a softer surface. Therefore, if at all a component is to be replaced, it should be the nut, which is less costly compared to the screw. Bronze is the ideal material for nut. Mainly tin bronze and phosphor bronze are used for nut.

The frame of a power screw is usually made of grey cast iron of grade FG 200. Cast iron is cheap, can be given any complex shape and possesses high compressive strength.

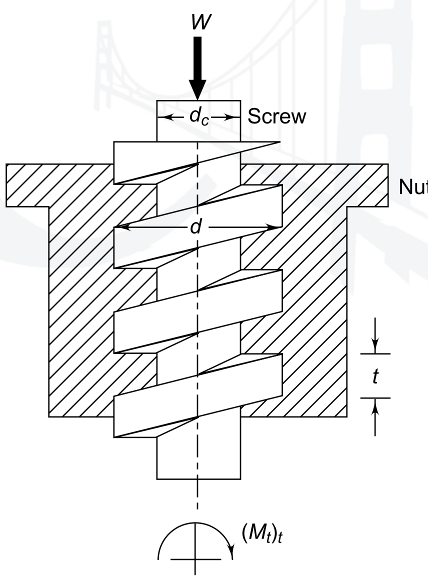

Fig. 6.11

The body of the screw is subjected to an axial force W and torsional moment $(M_t)_t$, as shown in Fig. 6.11. The direct compressive stress σ_c is given by,

$$\sigma_c = \frac{W}{\left(\dfrac{\pi}{4} d_c^2\right)} \tag{a}$$

The torsional shear stress is given by,

$$\tau = \frac{16(M_t)_t}{\pi d_c^3} \tag{b}$$

The principal shear stress is given by,

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_c}{2}\right)^2 + (\tau)^2} \tag{c}$$

The threads of the screw, which are engaged with the nut, are subjected to transverse shear stress. The screw will tend to shear off the threads at the core diameter under the action of the load W. The shear area of one thread is $(\pi d_c t)$. The transverse shear stress in the screw is given by

$$\tau_s = \frac{W}{\pi d_c\, tz} \tag{6.21}$$

where

τ_s = transverse shear stress at the root of the screw (N/mm^2)

t = thread thickness at the core diameter (mm)

z = number of threads in engagement with the nut

The transverse shear stresses in the nut are determined in a similar way. Under the action of the load W, the thread of the nut will tend to shear off at the nominal diameter. The shear area of one thread is (πdt). Therefore,

$$\tau_n = \frac{W}{\pi d t z} \tag{6.22}$$

where

τ_n = transverse shear stress at the root of the nut (N/mm)^2

t = thread thickness at the root of the nut (mm)

The bearing pressure between the contacting surfaces of the screw and the nut is an important consideration in design. The bearing area between the screw and the nut for one thread is $\left[\frac{\pi}{4}(d^2 - d_c^2)\right]$. Therefore,

$$S_b = \frac{W}{\left[\dfrac{\pi}{4}(d^2 - d_c^2)z\right]}$$

or
$$S_b = \frac{4W}{\pi z(d^2 - d_c^2)} \quad (6.23)$$

where

S_b = unit bearing pressure (N/mm^2)

The permissible bearing pressure depends upon the materials of the screw and the nut and the rubbing velocity. The permissible values of the unit bearing pressures are given in Table 6.4.

Table 6.4 *Unit bearing pressure for power screws*

Type of application	*Material*		*S_b (N/mm^2)*	*Rubbing speed*
	Screw	*Nut*		
Hand press	Steel	Bronze	18–24	Low speed
Screw-jack	Steel	Cast iron	13–17	Low speed < 2.5 m/min
Screw-jack	Steel	Bronze	11–17	Low speed < 3 m/min
Hoisting screw	Steel	Cast iron	4–7	Medium speed 6–12 m/min
Hoisting screw	Steel	Bronze	5–10	Medium speed 6–12 m/min
Lead screw	Steel	Bronze	1–1.5	High speed > 15 m/min

Example 6.1 *The nominal diameter of a triple-threaded square screw is 50 mm, while the pitch is 8 mm. It is used with a collar having an outer diameter of 100 mm and inner diameter as 65 mm. The coefficient of friction at the thread surface as well as at the collar surface can be taken as 0.15. The screw is used to raise a load of 15 kN. Using the uniform wear theory for collar friction, calculate:*

(i) torque required to raise the load;
(ii) torque required to lower the load; and
(iii) the force required to raise the load, if applied at a radius of 500 mm.

Solution

Given $W = 15$ kN

For screw, $d = 50$ mm $p = 8$ mm $\mu = 0.15$

No. of starts = 3

For collar, $D_o = 100$ mm $D_i = 65$ mm

$\mu = 0.15$

Step I *Torque required to raise the load*

$l = 3p = 3(8) = 24$ mm

From Eq. (6.2),

$d_m = d - 0.5p = 50 - 0.5(8) = 46$ mm.

$$\tan\alpha = \frac{l}{\pi d_m} = \frac{24}{\pi(46)} = 0.166 \text{ or } \alpha = 9.429°$$

$$\tan\phi = \mu = 0.15 \quad \text{or} \quad \phi = 8.531°$$

From Eq. (6.6),

$$M_t = \frac{Wd_m}{2}\tan(\phi + \alpha)$$

$$= \frac{(15\times10^3)(46)}{2}\tan(8.531 + 9.429)$$

$$= 111\,831.06 \text{ N-mm}$$

From Eq. (6.18),

$$(M_t)_c = \frac{\mu_c W}{4}(D_0 + D_i)$$

$$= \frac{(0.15)(15\times10^3)}{4}(100+65)$$

$$= 92\,812.5 \text{ N-mm}$$

$$\therefore \ (M_t)_t = M_t + (M_t)_c = 111\,831.06 + 92\,812.5$$
$$= 204\,643.56 \text{ N-mm or } 204.64 \text{ N-m} \quad \text{(i)}$$

Step II *Torque required to lower the load*

From Eq. (6.8),

$$M_t = \frac{Wd_m}{2}\tan(\phi - \alpha)$$

$$= \frac{(15\times10^3)(46)}{2}\tan(8.531 - 9.429)$$

$$= -5407.63 \text{ N-mm}$$

$$\therefore \ (M_t)_t = M_t + (M_t)_c = -5407.63 + 92\,812.5$$
$$= 87\,404.87 \text{ N-mm or } 87.4 \text{ N-m} \quad \text{(ii)}$$

The negative sign indicates that the screw alone is not self-locking. However, due to the restraining torque of collar friction, the screw is self-locking.

Step III *Force required to raise the load*

The force P_i at the radius of 500 mm is given by,

$$P_i = \frac{(M_t)_t}{\text{radius}} = \frac{204\,643.56}{500} = 409.3 \text{ N} \qquad \text{(iii)}$$

Example 6.2 *A double-threaded power screw, with ISO metric trapezoidal threads is used to raise a load of 300 kN. The nominal diameter is 100 mm and the pitch is 12 mm. The coefficient of friction at the screw threads is 0.15. Neglecting collar friction, calculate*

(i) torque required to raise the load;
(ii) torque required to lower the load; and
(iii) efficiency of the screw.

Solution

Given $W = 300$ kN

For screw, $d = 100$ mm $p = 12$ mm $\mu = 0.15$

No. of starts = 2

Step I *Torque required to raise the load*

For ISO metric trapezoidal threads,

$\theta = 15°$ $l = 2p = 2(12) = 24$ mm

$d_m = d - 0.5p = 100 - 0.5(12) = 94$ mm

$$\tan\alpha = \frac{l}{\pi d_m} = \frac{24}{\pi(94)} = 0.0813$$

$$\mu\sec\theta = \frac{\mu}{\cos\theta} = \frac{0.15}{\cos(15)} = 0.1553$$

$$M_t = \frac{Wd_m}{2}\,\frac{(\mu\sec\theta + \tan\alpha)}{(1 - \mu\sec\theta\tan\alpha)}$$

$$= \frac{(300\times10^3)(94)}{2} \times \frac{(0.1553 + 0.0813)}{(1 - 0.1553 + 0.0813)}$$

$$= 3378.72 \times 10^3 \text{ N-mm or } 3378.72 \text{ N-m} \qquad \text{(i)}$$

Step II *Torque required to lower the load*

$$M_t = \frac{Wd_m}{2}\,\frac{(\mu\sec\theta - \tan\alpha)}{(1 + \sec\theta\tan\alpha)}$$

$$= \frac{(300\times10^3)(94)}{2}\,\frac{(0.1553 - 0.0813)}{(1 + 0.1553\times0.0813)}$$

$$= 1030.39 \times 10^3 \text{ N-mm or } 1030.39 \text{ N-m} \qquad \text{(ii)}$$

Step III *Efficiency of screw*

$$\eta = \frac{\tan\alpha(1 - \mu\sec\theta\tan\alpha)}{(\mu\sec\theta + \tan\alpha)}$$

$$= \frac{0.0813(1 - 0.1553\times0.0813)}{(0.1553 + 0.0813)}$$

$$= 0.3393 \quad \text{or} \quad 33.93\% \qquad \text{(iii)}$$

Example 6.3 *A machine vice, as shown in Fig. 6.12, has single-start, square threads with 22 mm nominal diameter and 5 mm pitch. The outer and inner diameters of the friction collar are 55 and 45 mm respectively. The coefficients of friction for thread and collar are 0.15 and 0.17 respectively. The machinist can comfortably exert a force of 125 N on the handle at a mean radius of 150 mm. Assuming uniform wear for the collar, calculate*

(i) the clamping force developed between the jaws; and
(ii) the overall efficiency of the clamp.

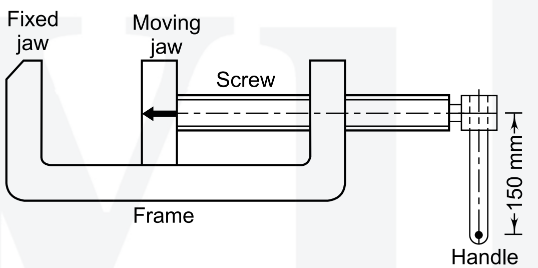

Fig. 6.12

Solution

Given For screw, $d = 22$ mm $l = p = 5$ mm
$\mu = 0.15$

For collar, $D_o = 55$ mm

$D_i = 45$ mm $\mu = 0.17$

For handle $P = 125$ N

length = 150 mm

Step I *Screw and collar friction torques*

$d_m = d - 0.5\,p = 22 - 0.5(5) = 19.5$ mm

$$\tan\alpha = \frac{l}{\pi d_m} = \frac{5}{\pi(19.5)} \quad \text{or} \quad \alpha = 4.666°$$

$\tan\phi = \mu = 0.15$ or $\phi = 8.531°$

From Eq. (6.6),

$$M_t = \frac{Wd_m}{2}\tan(\phi + \alpha)$$

$$= \frac{W(19.5)}{2}\tan(8.531 + 4.666)$$

$$= (2.286W) \text{ N-mm} \qquad \text{(a)}$$

From Eq. (6.18),

$$(M_t)_c = \frac{\mu_c W}{4}(D_0 + D_i)$$

$$= \frac{(0.17)W}{4}(55 + 45)$$

$$= (4.25\ W)\text{ N-mm} \qquad \text{(b)}$$

Step II *Clamping force developed between jaws (W)*

The total external torque applied to the handle is (125×150) N-mm. Therefore,

$$(M_t)_t = M_t + (M_t)_c$$

or $$125 \times 150 = 2.286\ W + 4.25\ W$$

$$W = 2868.73\text{ N} \qquad \text{(i)}$$

Step III *Overall efficiency of clamp*

From Eq. (6.20),

$$\eta_o = \frac{Wl}{2\pi (M_t)_t} = \frac{(2868.73)(5)}{2\pi(125 \times 150)} = 0.1218$$

or $$12.18\% \qquad \text{(ii)}$$

Example 6.4 *The construction of a gate valve used in high-pressure pipeline is shown in Fig. 6.13. The screw is rotated in its place by means of the handle. The nut is fixed to the gate. When the screw rotates, the nut along with the gate moves downward or upward depending upon the direction of rotation of the screw. The screw has single-start square threads of 40 mm outer diameter and 7 mm pitch. The weight of the gate is 5 kN. The water pressure in the pipeline induces frictional resistance between the gate and its seat. The resultant frictional resistance in the axial direction is 2 kN. The inner and outer diameters of thrust washer are 40 and 80 mm respectively. The values of coefficient of friction at the threads and at the washer are 0.15 and 0.12 respectively. The handle is rotated by the two arms, each exerting equal force at a radius of 500 mm from the axis of the screw. Calculate*

(i) the maximum force exerted by each arm when the gate is being raised;

(ii) the maximum force exerted by each arm when the gate is being lowered;

(iii) the efficiency of the gate mechanism; and

(iv) the length of the nut, if the permissible bearing pressure is 5 N/mm².

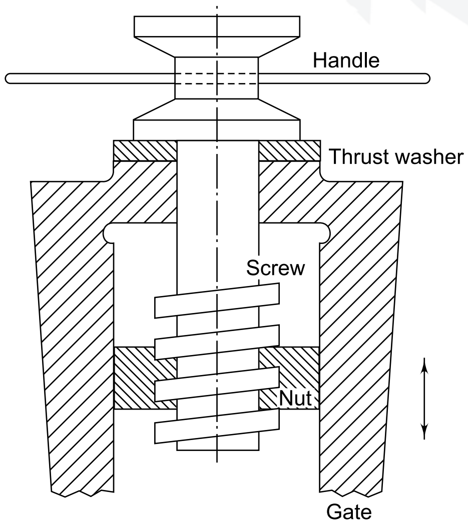

Fig. 6.13 *Gate Valve*

Solution

Given For screw, $d = 40$ mm $\quad l = p = 7$ mm

$\mu = 0.15$

For collar, $D_o = 80$ mm $\quad D_i = 40$ mm

$\mu = 0.12$

For handle, radius = 500 mm

For nut $S_b = 5\text{ N/mm}^2$

For gate, weight = 5 kN

frictional resistant = 2 kN

Step I *Force exerted by each arm to raise the gate*

From Eq. (6.2),

$$d_m = d - 0.5\,p = 40 - 0.5(7) = 36.5\text{ mm}$$

$$\tan\alpha = \frac{l}{\pi d_m} = \frac{7}{\pi\ (36.5)}$$

$$= 0.061 \quad \text{or} \quad \alpha = 3.493°$$

$$\tan\phi = \mu = 0.15 \quad \text{or} \quad \phi = 8.531°.$$

Frictional resistance acts opposite to the motion. When the gate is being raised the frictional force acts in downward direction. Therefore, axial force on the screw consists of addition of the weight of the gate plus the frictional resistance. Or,

$$W = 5000 + 2000 = 7000\text{ N}$$

From Eq. (6.6),

$$M_t = \frac{Wd_m}{2}\tan(\phi + \alpha)$$

$$= \frac{(7000)(36.5)}{2} \tan(8.531 + 3.493)$$

$= 27\,210.04$ N-mm.

From Eq. (6.18),

$$(M_t)_c = \frac{\mu_c W}{4} \cdot (D_0 + D_i)$$

$$= \frac{(0.12)(7000)(80+40)}{4}$$

$= 25\,200$ N-mm

$(M_t)_t = M_t + (M_t)_c = 27\,210.04 + 25\,200$
$= 52\,410.04$ N-mm.

There are two arms, each exerting a force P at a radius of 500 mm. Therefore,

$(M_t)_t = 2P \times 500$

$$P = \frac{(M_t)_t}{1000} = \frac{52\,410.04}{1000} = 52.41 \text{ N} \qquad \text{(i)}$$

Step II *Force exerted by each arm to lower the gate*

When the gate is being lowered, the frictional resistance acts in a vertically upward direction, while the weight acts in a downward direction. Therefore, the net axial force consists of the difference between the two.

or $\quad W = 5000 - 2000 = 3000$ N

From Eq. (6.8),

$$M_t = \frac{W d_m}{2} \tan(\phi - \alpha)$$

$$= \frac{(3000)(36.5)}{2} \tan(8.531 - 3.493)$$

$= 4826.60$ N-mm

From Eq. (6.18),

$$(M_t)_c = \frac{\mu_c W}{4} (D_o + D_i)$$

$$= \frac{(0.12)(3000)}{4} (80 + 40)$$

$= 10\,800$ N-mm

$(M_t)_t = M_t + (M_t)_c = 4826.60 + 10\,800$
$= 15\,626.6$ N-mm

The force P exerted by each arm on the handle is given by,

$(M_t)_t = (2P) \times 500$

or $$P = \frac{(M_t)_t}{1000} = \frac{15\,626.6}{1000} = 15.63 \text{ N} \qquad \text{(ii)}$$

Step III *Efficiency of gate mechanism*

From Eq. (6.20),

$$\eta_o = \frac{W\, l}{2\pi\, (M_t)_t}$$

Substituting values of W and $(M_t)_t$ obtained in case of raising the gate,

$$\eta_o = \frac{(7000)\,(7)}{2\pi\,(52\,410.04)} = 0.1488 \text{ or } 14.88\% \qquad \text{(iii)}$$

Step IV *Length of the nut*

From Eq. (6.11),

$d_c = d - p = 40 - 7 = 33$ mm

From Eq. (6.23),

$$z = \frac{4W}{\pi S_b (d^2 - d_c^2)} = \frac{4(7000)}{\pi(5)(40^2 - 33^2)}$$

$= 3.49$ or 4 threads

$l = zp = 4 \times 7 = 28$ mm $\qquad$ (iv)

Example 6.5 *The construction of a shaft straightener used on the shop floor is shown in Fig. 6.14. The screw has single-start square threads of 80 mm nominal diameter and 10 mm pitch. The screw is required to exert a maximum axial force of 10 kN. The mean radius of the friction collar is 30 mm. The axial length of the nut is 40 mm. The coefficient of friction at the threads and the collar is 0.12. The mean diameter of the rim of the hand wheel is 500 mm. Calculate*

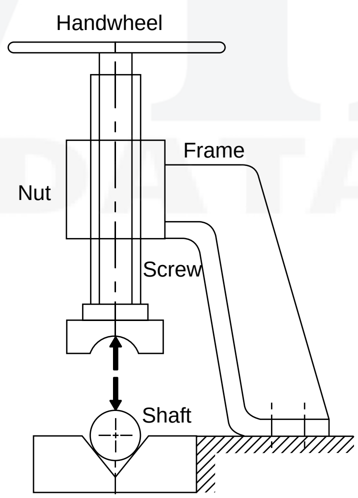

Fig. 6.14 *Shaft Straightener*

(i) the force exerted at the rim to drive the screw;
(ii) the efficiency of the straightener; and
(iii) the bearing pressure on the threads in the nut.

Solution

Given $W = 10$ kN

For screw, $d = 80$ mm $l = p = 10$ mm $\mu = 0.12$

For collar, $r_m = 30$ mm $\mu = 0.12$

For nut, $l = 40$ mm

For hand wheel, $D = 500$ mm

Step I *Force exerted at rim to drive screw*

From Eq. (6.2),

$$d_m = d - 0.5p = 80 - 0.5\,(10) = 75 \text{ mm}$$

$$\tan\alpha = \frac{l}{\pi d_m} = \frac{10}{\pi\ (75)}$$

$$= 0.0424 \text{ or } \alpha = 2.43°$$

$\tan\phi = \mu = 0.12$ or $\varphi = 6.843°$

From Eq. (6.6),

$$M_t = \frac{Wd_m}{2}\tan\ (\phi + \alpha)$$

$$= \frac{(10\times10^3)\ (75)}{2}\tan\ (6.843 + 2.43)$$

$$= 61\,227.17 \text{ N-mm.}$$

The mean radius r_m of friction collar is given by,

$$r_m = \frac{1}{2}\left(\frac{D_0}{2} + \frac{D_i}{2}\right) = \left(\frac{D_0 + D_i}{4}\right) \qquad \text{(a)}$$

From Eq. (6.18),

$$(M_t)_c = \frac{\mu_c W}{4}\,.\,(D_0 + D_i)$$

Substituting Eq. (a) in the above expression,

$$(M_t)_c = \mu_c\, W\, r_m = 0.12\ (10\times10^3)\ (30)$$
$$= 36\,000 \text{ N-mm.}$$
$$(M_t)_t = M_t + (M_t)_c = 61\,227.17 + 36\,000$$
$$= 97\,227.17 \text{ N-mm.}$$

The mean diameter of the rim is 500 mm. Therefore, hand force P is exerted at a radius of 250 mm from the axis of the screw. The torque exerted on the screw is $(250 \times P)$ N-mm. Equating,

$$(M_t)_t = 250 \times P$$

or
$$P = \frac{(M_t)_t}{250} = \frac{97\,227.17}{250} = 388.91 \text{ N} \qquad \text{(i)}$$

Step II *Efficiency of the straightener*

From Eq. (6.20),

$$\eta_o = \frac{W\,l}{2\pi\,(M_t)_t} = \frac{(10\times10^3)(10)}{2\pi(97\,227.17)}$$

$$= 0.1637 \text{ or } 16.37\% \qquad \text{(ii)}$$

Step III *Bearing pressure on threads in nut*

The length of the nut is 40 mm. Therefore,

$$z = \frac{40}{p} = \frac{40}{10} = 4 \text{ threads}$$

From Eq. (6.1),

$$d_c = d - p = 80 - 10 = 70 \text{ mm}$$

From Eq. (6.23),

$$S_b = \frac{4W}{\pi z(d^2 - d_c^2)} = \frac{4(10\times10^3)}{\pi(4)(80^2 - 70^2)}$$

$$= 2.122 \text{ N/mm}^2 \qquad \text{(iii)}$$

Example 6.6 *The lead screw of a lathe has single-start ISO metric trapezoidal threads of 52 mm nominal diameter and 8 mm pitch. The screw is required to exert an axial force of 2 kN in order to drive the tool carriage during turning operation. The thrust is carried on a collar of 100 mm outer diameter and 60 mm inner diameter. The values of coefficient of friction at the screw threads and the collar are 0.15 and 0.12 respectively. The lead screw rotates at 30 rpm. Calculate*

(i) the power required to drive the lead screw; and
(ii) the efficiency of the screw.

Solution

Given $W = 2$ kN $n = 30$ rpm

For screw, $d = 52$ mm $l = p = 8$ mm $\mu = 0.15$

For collar, $D_o = 100$ mm $D_i = 60$ mm $\mu = 0.12$

Step I *Power required to drive the lead screw*

$$\theta = 15°$$

From Eq. (6.2),

$$d_m = d - 0.5p = 52 - 0.5\ (8) = 48 \text{ mm}$$

$$\tan \alpha = \frac{l}{\pi d_m} = \frac{8}{\pi (48)} = 0.0531$$

$$\mu \sec \theta = \frac{\mu}{\cos \theta} = \frac{0.15}{\cos (15)} = 0.1553$$

The torque required to overcome friction at the thread surface is given by,

$$M_t = \frac{W d_m}{2} \cdot \frac{(\mu \sec \theta + \tan \alpha)}{(1 - \mu \sec \theta \tan \alpha)}$$

$$= \frac{(2 \times 10^3)(48)}{2} \frac{(0.1553 + 0.0531)}{(1 - 0.1553 \times 0.0531)}$$

$= 10\,086.38$ N-mm.

From Eq. (6.18),

$$(M_t)_c = \frac{\mu_c W}{4} \cdot (D_0 + D_i)$$

$$= \frac{(0.12)\,(2000)\,(100 + 60)}{4}$$

$= 9600$ N-mm

The total torque required to drive the lead screw is given by,

$$(M_t)_t = M_t + (M_t)_c = 10\,086.38 + 9600$$

$$= 19\,686.38 \text{ N-mm}$$

The power required to drive the lead screw is given by,

$$\text{kW} = \frac{2\pi n (M_t)_t}{60 \times 10^6} = \frac{2\pi\,(30)\,(19\,686.38)}{60 \times 10^6}$$

$$= 0.0618 \qquad \text{(i)}$$

Step II *Efficiency of screw*

From Eq. (6.20),

$$\eta_o = \frac{Wl}{2\pi (M_t)_t} = \frac{(2 \times 10^3)(8)}{2\pi (19\,686.38)}$$

$$= 0.1294 \quad \text{or} \quad 12.94\% \qquad \text{(ii)}$$

Example 6.7 *In a machine tool application, the tool holder is pulled by means of an operating nut mounted on a screw. The tool holder travels at a speed of 5 m/min. The screw has single-start square threads of 48 mm nominal diameter and 8 mm pitch. The operating nut exerts a force of 500 N to drive the tool holder. The mean radius of the friction collar is 40 mm. The coefficient of friction at thread and collar surfaces is 0.15. Calculate*

(i) power required to drive the screw; and

(ii) the efficiency of the mechanism.

Solution

Given $W = 500$ N

For screw, $d = 48$ mm $\quad l = p = 8$ mm $\quad \mu = 0.15$

For collar, $r_m = 40$ mm $\quad \mu = 0.15$

Tool holder speed = 5 m/min

Step I *Power required to drive the screw*

From Eq. (6.2),

$$d_m = d - 0.5p = 48 - 0.5\,(8) = 44 \text{ mm}$$

$$\tan \alpha = \frac{l}{\pi d_m} = \frac{8}{\pi(44)} = 0.0579 \text{ or } \alpha = 3.312°$$

$$\tan \phi = \mu = 0.15 \text{ or } \phi = 8.531°$$

From Eq. (6.6),

$$M_t = \frac{W d_m}{2} \tan (\phi + \alpha)$$

$$= \frac{(500)\,(44)}{2} \tan\,(8.531 + 3.312)$$

$= 2306.64$ N-mm.

From Eq. (6.18),

$$(M_t)_c = \frac{\mu_c W}{4} \cdot (D_0 + D_i) = \mu_c W r_m$$

or $(M_t)_c = \mu_c W r_m = (0.15)(500)(40) = 3000$ N-mm.

$$(M_t)_t = M_t + (M_t)_c = 2306.64 + 3000$$

$$= 5306.64 \text{ N-mm.}$$

The nut travels at a speed of 5 m/min along with the tool holder. If n is the rpm of the screw, the relationship between the pitch, speed and rpm can be written as,

$$n = \frac{(5 \times 10^3)}{p} = \frac{(5 \times 10^3)}{8} = 625 \text{ rpm}$$

Power required to drive the screw is given by,

$$kW = \frac{2\pi n (M_t)_t}{60 \times 10^6} = \frac{2\pi(625)(5306.64)}{60 \times 10^6} = 0.35 \quad \text{(i)}$$

Step II *Efficiency of mechanism*

From Eq. (6.20),

$$\eta_o = \frac{Wl}{2\pi (M_t)_t} = \frac{(500)(8)}{2\pi(5306.64)} = 0.12 \text{ or } 12\% \text{(ii)}$$

Example 6.8 *It is required to design a double-start screw with square threads for the C-clamp shown in Fig. 6.15(a). The maxi-*

mum force exerted by the clamp is 5 kN. It is assumed that the operator will exert a force of 250 N at the ball handle of the hand wheel. The screw is made of plain carbon steel 45C8 (S_{yt} = 330 N/mm^2), while the nut is made of grey cast iron FG 200. The dimensions of the collar are given in Fig. 6.15(b). The factor of safety is 2. Determine the dimensions of the screw and the nut and calculate the radius R_m of the ball handle.

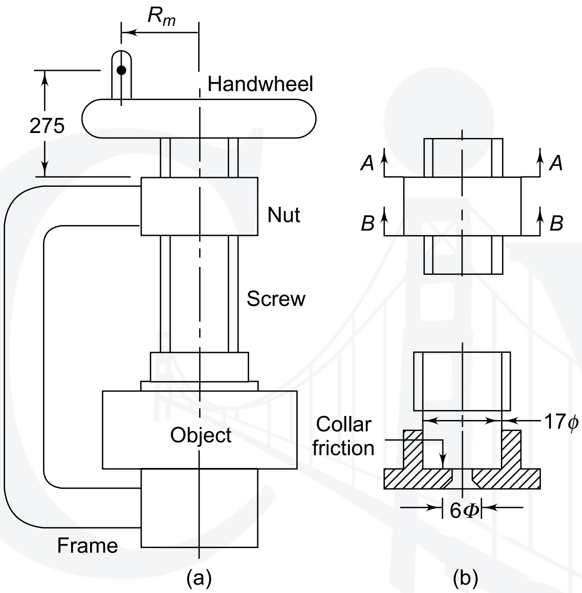

Fig. 6.15 *C-clamp*

Solution

Given W = 5 kN operator force = 250 N
For screw, No. of starts = 2 S_{yt} = 330 N/mm^2
$(fs) = 2$
For collar, D_o = 17 mm D_i = 6 mm
For nut, S_{ut} = 200 N/mm^2

Step I *Selection of diameter and pitch of screw*
The lower portion of the screw between the nut and the object is subjected to compressive stress. Assuming,

$$S_{yc} = S_{yt} = 330 \text{ N/mm}^2$$

$$\sigma_c = \frac{S_{yc}}{(fs)} = \frac{330}{2} = 165 \text{ N/mm}^2$$

Since

$$\sigma_c = \frac{W}{\left(\frac{\pi}{4} d_c^{\ 2}\right)} \quad \therefore \quad 165 = \frac{5000}{\left(\frac{\pi}{4} d_c^2\right)}$$

or $\quad d_c = 6.21 \quad$ or $\quad$ 7 mm

There are additional stresses due to the collar friction torque. At this stage, it is not possible to calculate the additional stresses in the lower and upper parts of the screw. To account for these additional stresses, the diameter should be increased. As a first trial, a screw with a 22 mm nominal diameter and 5 mm pitch is selected for this clamp.

Step II *Screw and collar friction torques*

$$d_m = d - 0.5p = 22 - 0.5(5) = 19.5 \text{ mm}$$
$$l = 2p = 2(5) = 10 \text{ mm}$$

$$\tan \alpha = \frac{l}{\pi d_m} = \frac{10}{\pi(19.5)} \quad \text{or} \quad \alpha = 9.271°$$

Assuming the coefficient of friction to be 0.15,

$\tan \phi = \mu = 0.15 \quad$ or $\quad \phi = 8.531°$

$$\therefore \quad M_t = \frac{Wd_m}{2} \tan(\phi + \alpha)$$

$$= \frac{(5000)\ (19.5)}{2} \tan(8.531 + 9.271)$$

$$= 15\,653.79 \text{ N-mm}$$

The coefficient of friction at the collar is taken as 0.17. Thus,

$$(M_t)_c = \frac{\mu_c W}{4} \cdot (D_0 + D_i)$$

$$= \frac{(0.17)\ (5000)}{4} (17 + 6)$$

$$= 4887.5 \text{ N-mm.}$$

Step III *Check for stresses in the screw*

$$d_c = d - p = 22.5 = 17 \text{ mm}$$

Stresses at the section AA

At the section *AA*, the body of the screw is subjected to two types of stresses:

(i) torsional shear stress due to the total torque $(M_t)_t$; and
(ii) bending stress due to a hand force of 250 N.

$$\tau = \frac{16\ (M_t)_t}{\pi d_c^3} = \frac{16(15\,653.79 + 4887.5)}{\pi (17)^3}$$

$$= 21.29 \text{ N/mm}^2$$

$$\sigma_b = \frac{32M_b}{\pi d_c^3} = \frac{32(250\times 275)}{\pi(17)^3}$$
$$= 142.54 \text{ N/mm}^2$$
$$\tau_{max.} = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + (\tau)^2}$$
$$= \sqrt{\left(\frac{142.54}{2}\right)^2 + (21.29)^2}$$
$$= 74.38 \text{ N/mm}^2$$
$$\therefore \quad (fs) = \frac{S_{sy}}{\tau_{max.}} = \frac{0.5\,S_{yt}}{\tau_{max.}} = \frac{0.5\ (330)}{74.38}$$
$$= 2.22$$

The recommended factor of safety is 2. Therefore, the screw with a 22 mm nominal diameter is justified.

Stresses at the section BB

At the section *BB*, the body of the screw is subjected to two types of stresses:

(i) torsional shear stress due to the collar friction torque; and

(ii) direct compressive stress due to the clamping force.

$$\tau = \frac{16(M_t)_c}{\pi d_c^3} = \frac{16(4887.5)}{\pi(17)^3} = 5.07 \text{ N/mm}^2$$
$$\sigma_c = \frac{W}{\left(\frac{\pi}{4}d_c^2\right)} = \frac{5000}{\left(\frac{\pi}{4}(17)^2\right)} = 22.03 \text{ N/mm}^2$$
$$\tau_{max.} = \sqrt{\left(\frac{\sigma_c}{2}\right)^2 + (\tau)^2}$$
$$= \sqrt{\left(\frac{22.03}{2}\right)^2 + (5.07)^2} = 12.13 \text{ N/mm}^2$$
$$\therefore \quad (fs) = \frac{S_{sy}}{\tau_{max.}} = \frac{0.5\ (330)}{12.13} = 13.6$$

Therefore, the stresses at the section *BB* are not critical.

Step IV *Design of the nut*

The unit bearing pressure for the cast iron nut and the soft steel screw is 13 to 17 N/mm². Using the mean value of 15 N/mm²,

$$z = \frac{4W}{\pi S_b(d^2 - d_c^2)} = \frac{4(5000)}{\pi(15)\left[(22)^2 - (17)^2\right]}$$
$$= 2.18 \quad \text{or} \quad 3$$

$\therefore$ length of the nut $= zp = 3(5) = 15$ mm.

Step V *Radius of ball handle*

The total torque is given by,

$$(M_t)_t = M_t + (M_t)_c = 15\,653.79 + 4887.5 \qquad \text{(i)}$$

The operator exerts a force of 250 N on the ball handle. Therefore,

$$(M_t)_t = 250 \times R_m \qquad \text{(ii)}$$

Equating (i) and (ii),

$$R_m = 82.17 \text{ mm}$$

Example 6.9 *A screw clamp used on the shop floor is shown in Fig. 6.16. The screw has single-start square threads of 22 mm nominal diameter and 5 mm pitch. The coefficient of friction at the threads and the collar is 0.15. The mean radius of the friction collar is 15 mm. The capacity of the clamp is 750 N. The handle is made of steel 30C8 (S_{yt} = 400 N/mm²). It can be assumed that the operator exerts a force of 20 N on the handle.*

(i) What torque is required to tighten the clamp to full capacity?

(ii) Determine the length and the diameter of the handle such that it will bend with a permanent set, when the rated capacity of the clamp is exceeded.

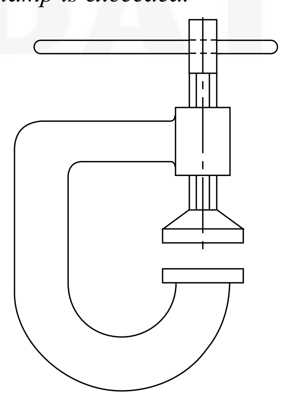

Fig. 6.16 *C-clamp*

Solution

Given $W = 750$ N

For screw, $d = 22$ mm $\quad l = p = 5$ mm $\quad \mu = 0.15$

For collar, $r_m = 15$ mm $\mu = 0.15$
For handle, $S_{yt} = 400$ N/mm^2
Operator force = 20 N

Step I *Torque required to tighten the clamp to full capacity*

From Eq. (6.2),

$$d_m = d - 0.5p = 22 - 0.5\ (5) = 19.5 \text{ mm}$$

$$\tan\alpha = \frac{l}{\pi d_m} = \frac{5}{\pi(19.5)}$$

$$= 0.0816 \quad \text{or} \quad \alpha = 4.666°$$

$$\tan\phi = \mu = 0.15 \quad \text{or} \quad \phi = 8.531°$$

From Eq. (6.6),

$$M_t = \frac{Wd_m}{2}\tan(\phi + \alpha)$$

$$= \frac{750\,(19.5)}{2}\tan(8.531 + 4.666)$$

$$= 1714.73 \text{ N-mm}$$

From Eq. (6.18),

$$(M_t)_c = \frac{\mu_c W}{4}\cdot(D_0 + D_i) = \mu_c W r_m$$

or $\quad (M_t)_c = \mu_c W r_m = 0.15(750)(15)$
$\quad = 1687.5$ N-mm.

The torque required to tighten the clamp to full capacity is given by,

$$(M_t)_t = M_t + (M_t)_c = 1714.73 + 1687.5$$
$$= 3402.23 \text{ N-mm} \qquad \text{(i)}$$

Step II *Length and diameter of handle*

The length '*a*' of the handle from the axis of the screw to the point of application of hand force is given by,

$$(M_t)_t = P \times a \qquad \text{(a)}$$

or $\quad 3402.23 = 20 \times a \qquad \therefore\ a = 170.11$ mm

The handle is subjected to bending moment and the maximum bending moment near the screw is approximately given by,

$$M_b = P \times a \qquad \text{(b)}$$

From (a) and (b),

$$M_b = (M_t)_t = 3402.23 \text{ N-mm}$$

The handle will bend with a permanent set when the bending stress reaches the yield strength of the material,

or, $\quad \sigma_b = S_{yt} = 400$ N/mm^2

The bending stress is given by,

$$\sigma_b = \frac{32\,M_b}{\pi d^3} \quad \text{or} \quad 400 = \frac{32\ (3402.23)}{\pi d^3}$$

$$\therefore \quad d = 4.42 \text{ mm} \qquad \text{(ii)}$$

Example 6.10 *It is required to design a fly-press, as shown in Fig. 6.17, which is capable of punching 50 mm diameter circles from a 1.5 mm thick mild steel sheet. The ultimate shear strength of the sheet metal is 375 N/mm^2 and it can be assumed that shearing will be complete when the punch penetrates through half the thickness of the sheet. The screw, with square threads, is made of steel 30C8 (S_{yt} = 400 N/mm^2), while the nut is made of bronze. The factor of safety is 3. The operator is expected to sit in front of the fly-press, insert the sheet by his left hand and operate the handle by the right hand. The total working stroke consists of a one-quarter revolution, 45° in front of the press and 45° behind the press. During the return stroke, the punch is raised by 5 mm to provide clearance to insert the sheet. The forward or working stroke is completed in 1 second. The balls are made of cast iron, with a mass density of 7280 kg/m^3 and the radius R_m is 500 mm. Neglecting collar friction, calculate*

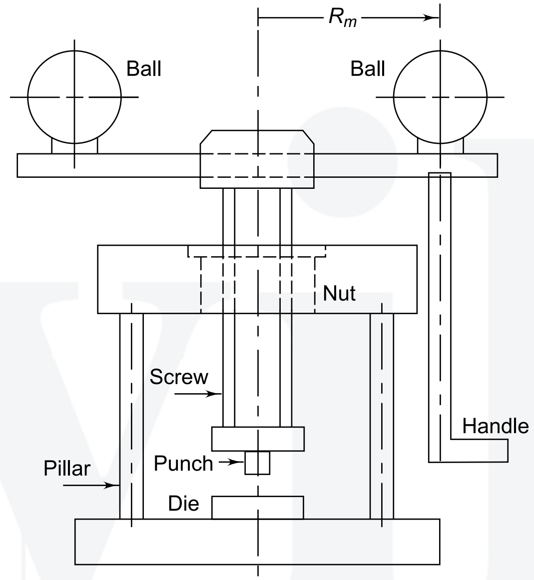

Fig. 6.17 *Fly-press*

(i) the dimensions of the screw;

(ii) the length of the nut; and

(iii) the size of the balls.

Solution

Given For screw, $S_{yt} = 400 \text{ N/mm}^2$ $(fs) = 3$

For punched holes, $d = 50$ mm $t = 1.5$ mm

$S_{us} = 375 \text{ N/mm}^2$

Mass density of balls $= 7280 \text{ kg/m}^3$ $R_m = 500$ mm

Step I *Diameter, pitch and number of starts for screw*

The force W required to shear the sheet is given by

$$W = \pi dtS_{us} = \pi(50)(1.5)(375) = 88\,357.29 \text{ N}$$

The lower portion of the screw is subjected to compressive stress. Assuming,

$$S_{yc} = S_{yt} = 400 \text{ N/mm}^2$$

$$\sigma_c = \frac{S_{yc}}{(fs)} = \frac{400}{3} = 133.33 \text{ N/mm}^2$$

The core diameter d_c of the screw is given by

$$d_c = \sqrt{\frac{4W}{\pi\sigma_c}} = \sqrt{\frac{4(88357.29)}{\pi(133.33)}} = 29.05 \text{ mm}$$

In addition to compressive stress, the screw is subjected to torsional shear stress. Therefore, the core diameter is to be increased. As a first trial value, a nominal diameter of 50 mm with 8 mm pitch is recommended for the screw. Since the punch is to be raised by 5 mm during the return stroke of one-quarter revolution, the minimum lead should be 20 mm. The screw is therefore triple-threaded.

$$l = 3p = 3(8) = 24 \text{ mm}$$

$$d_m = d - 0.5p = 50 - 0.5(8) = 46 \text{ mm}$$

Step II *Check for stresses in screw*

$$\tan\alpha = \frac{l}{\pi d_m} = \frac{24}{\pi(46)} \quad \text{or} \quad \alpha = 9.429°$$

Assuming the coefficient of friction as 0.15,

$$\tan\phi = \mu = 0.15 \quad \text{or} \quad \phi = 8.531°$$

$$M_t = \frac{Wd_m}{2}\tan(\phi + \alpha)$$

$$= \frac{(88\,357.29)(46)}{2}\tan(8.531 + 9.429)$$

$$= 658\,739.29 \text{ N-mm}$$

Stresses in screw body

$$d_c = d - p = 50 - 8 = 42 \text{ mm}$$

$$\sigma_c = \frac{W}{\frac{\pi}{4}d_c^2} = \frac{88\,357.29}{\frac{\pi}{4}(42)^2} = 63.78 \text{ N/mm}^2$$

$$\tau = \frac{16M_t}{\pi d_c^3} = \frac{16(658\,739.29)}{\pi(42)^3} = 45.28 \text{ N/mm}^2$$

$$\tau_{\text{max.}} = \sqrt{\left(\frac{\sigma_c}{2}\right)^2 + (\tau)^2}$$

$$= \sqrt{\left(\frac{63.78}{2}\right)^2 + (45.28)^2} = 55.38 \text{ N/mm}^2$$

According to the maximum shear stress theory,

$$S_{sy} = 0.5\,S_{yt} = 0.5(400) = 200 \text{ N/mm}^2$$

$$(fs) = \frac{S_{sy}}{\tau_{\text{max.}}} = \frac{200}{55.38} = 3.61$$

Therefore, the factor of safety is more than 3. The efficiency of the screw is given by

$$\eta = \frac{\tan\alpha}{\tan(\phi + \alpha)} = \frac{\tan(9.429)}{\tan(8.531 + 9.429)}$$

$$= 0.5123 \quad \text{or} \quad 51.23\%$$

Step III *Length of nut*

For steel screw with a bronze nut, the unit bearing pressure (S_b) is taken as 18 N/mm^2.

From Eq. (6.23),

$$z = \frac{4W}{\pi S_b(d^2 - d_c^2)} = \frac{4(88\,357.29)}{\pi(18)(50^2 - 42^2)}$$

$$= 8.49 \quad \text{or} \quad 9 \text{ threads}$$

Length of nut $= zp = 9(8) = 72$ mm

The shear stresses in the threads of the screw and the nut are as follows:

$$t = p/2 = 8/2 = 4 \text{ mm}$$

$$\tau_s = \frac{W}{\pi d_c tz} = \frac{88\,357.29}{\pi(42)(4)(9)} = 18.6 \text{ N/mm}^2$$

$$\tau_n = \frac{W}{\pi dtz} = \frac{88\,357.29}{\pi(50)(4)(9)} = 15.62 \text{ N/mm}^2$$

Step IV *Size of balls*

Since the process of shearing is complete when the punch penetrates through half the thickness of the sheet, the work done by the punch is given by,

work done = punching force × 0.5 (sheet thickness)

$$= 88\,357.29 \times 0.5\,(1.5) = 66\,268 \text{ N-mm}$$

$$= 66.268 \text{ N-m or J}$$

The efficiency of the screw is 51.23% and the collar friction is to be neglected. Therefore, the input or the work done by the balls is given by,

$$\text{Work done by balls} = \frac{66.268}{0.5123} = 129.35\ \text{J} \qquad \text{(a)}$$

This is achieved by the kinetic energy of the moving balls. The forward stroke consists of $\left(\frac{\pi}{2}\right)$ rotation and takes 1 second

$$\omega_{ave} = \frac{\pi/2}{1} = \frac{\pi}{2}\ \text{rad/s}$$

The initial velocity of the balls is zero.

$$\omega_{ave} = \frac{\omega_{max.} + 0}{2}$$

or $$\omega_{max.} = 2\omega_{ave} = 2\left(\frac{\pi}{2}\right) = \pi\ \text{rad/s}$$

The kinetic energy of a rotating body of mass m concentrated at a radius of gyration k and rotating with angular velocity $\omega_{max.}$ is given by,

$$KE = \frac{mk^2\omega^2_{max.}}{2} \qquad \text{(i)}$$

$k = R_m = 500$ mm $= 0.5$ m

Equating the work done by the balls with the kinetic energy stored in them, we get

$$129.35 = \frac{m(0.5)^2\,(\pi)^2}{2}$$

or $$m = 104.85\ \text{kg}$$

This is the mass of the two balls. Hence, the mass of each ball is 52.425 kg.

mass = volume × density

$$\therefore \qquad 52.425 = \left(\frac{\pi d^3}{6}\right)(7280)$$

or $d = 0.2396$ m or 239.6 mm

The diameter of the balls is 240 mm.

6.15 DESIGN OF SCREW JACK

A screw jack is a portable device consisting of a screw mechanism used to raise or lower the load. There are two types of jacks—hydraulic and mechanical. A *hydraulic jack* consists of a cylinder and piston mechanism. The movement of the piston rod is used to raise or lower the load. *Mechanical jacks* can be either hand operated or power driven. Although a jack is simple and widely used device, the use of any lifting device is subject to certain hazards. In screw-jack applications, the hazards are dropping, tipping or slipping of machines or their parts during the operation. These hazards may result in serious accidents. The main reasons of such accidents are as follows:

(i) The load is improperly secured on the jack.

(ii) The screw jack is overloaded.

(iii) The centre of gravity of the load is off-centre with respect to the axis of the jack.

(iv) The screw jack is not placed on hard and level surface.

(v) The screw jack is used for a purpose for which it is not designed.

Proper size, strength and stability are the essential requirements for the design of the screw jack from safety considerations.

The construction of screw jack is shown in Fig. 6.18. It consists of a screw and a nut. The nut is fixed in a cast iron frame and remains stationary.

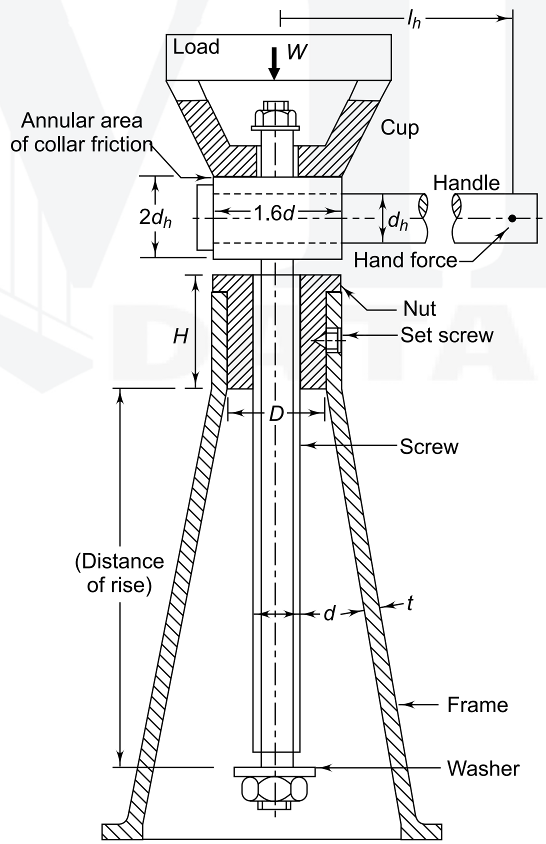

Fig. 6.18 *Screw Jack*

The rotation of the nut inside the frame is prevented by pressing a setscrew against it. The screw is rotated in the nut by means of a handle, which passes through a hole in the head of the screw. The head carries a cup, which supports the load and remains stationary while the screw is being rotated. There is a collar friction at the annular contacting surface between the cup and the head of the screw. A washer is fixed to the other end of the screw inside the frame, which prevents the screw to be completely turned out of the nut.

Step I *Problem Specification*

It is required to design a screw jack for supporting the machine parts during their repair and maintenance on the shop floor. It should be a general-purpose jack with a load carrying capacity of 100 kN and a maximum lifting height of 0.5 m. The jack is to be manually operated.

Step II *Selection of Materials*

(i) The frame of the screw jack has complex shape. It is subjected to compressive stress. Grey cast iron of grade FG 200 (S_{ut} = 200 N/mm^2) is selected as the material for the frame. Cast iron is cheap and it can be given any complex shape without involving costly machining operations. Cast iron has higher compressive strength compared with steel. Therefore, it is technically and economically advantageous to use cast iron for the frame.

(ii) The screw is subjected to torsional moment, compressive force and bending moment. From strength consideration, plain carbon steel of grade 30C8 (S_{yt} = 400 N/mm^2 and E = 207 000 N/mm^2) is selected as material for the screw.

(iii) There is relative motion between the screw and the nut, which results in friction. The friction causes wear at the contacting surfaces. When the same material is used for these two components, the surfaces of both components get worn out, requiring replacement. This is undesirable. The size and shape of the screw make it costly compared with the nut. Therefore, if at all a component is to be replaced due to wear, it should be the nut, which is less costly compared with the screw. The wear is always restricted to a softer surface. Therefore, the nut should be made of softer material. This protects the screw against wear. Cast variety of phosphor bronze of Grade-1 (S_{ut} = 190 N/mm^2) is selected as the material for the nut. Phosphor bronze is soft compared with hardened steel screw. In addition to this consideration, phosphor bronze has low coefficient of friction, which reduces the torque to overcome friction at the thread surface. It has excellent conformability and machinability. Conformability is the ability of the material to yield and adopt its shape to that of the screw. Cost is the main limitation of phosphor bronze. For steel screw and phosphor bronze nut, the permissible bearing pressure (S_b) and coefficient of friction (μ) are taken as 10 N/mm^2 and 0.1 respectively.

(iv) The handle is subjected to bending moment. The yield strength is the criterion for the selection of material. Plain carbon steel of grade 30C8 (S_{yt} = 400 N/mm^2) is selected as the material for the handle.

(v) The shape and dimensions of the cup are such that it is easier and economical to make it by the casting process. Grey cast iron of Grade FG 200 is used for the cup.

The complete part list of the screw jack is given in the following table.

Sr. No.	*Name of component*	*Quantity*	*Material*
1	frame	1	Grey cast iron FG 200 (IS: 210 – 1978)
2	screw	1	Steel 30C8 (IS: 1570 – 1978)
3	nut	1	Phosphor bronze Grade-1 (IS: 28 – 1975)
4	handle	1	Steel 30C8 (IS: 1570 – 1978)
5	cup	1	Grey cast iron FG 200 (IS: 210 – 1978)
6	set screw	1	Commercial steel
7	washer	1	Commercial steel

***Step III** General Considerations*

(i) The screw jack is manually operated. According to ergonomists, hand force should not exceed 130 N. However, this value is recommended for prolonged work. The jack is never operated continuously and, as such, a higher value of 400 N is assumed for hand force in this analysis. It is further assumed that two workers are required to raise the load of 100 kN. When two workers are at work, there is inconvenience and the resultant force is less than twice the individual force. A coefficient of 0.9 is assumed in this case to account for reduction in force due to inconvenience. Therefore, total hand force exerted on the handle by two workers (P) is given by

$$P = (0.9 \times 2 \times 400) \text{ N}$$

(ii) A screw-jack is a lifting device and subject to certain hazards. Breakdown of the jack has serious consequences such as injury to the operator and damage to machine parts. The jack should be robust and '*idiot*' proof. To account for this safety aspect, a higher factor of safety of 5 is used for the components of the screw jack.

***Step IV** Design of Screw*

The screw jack is an intermittently used device and wear of the threads is not an important consideration. Therefore, instead of trapezoidal threads, the screw is provided with square threads. Square threads have higher efficiency and provision can be made for self-locking arrangement. When the condition of self-locking is fulfilled, the load itself will not turn the screw and descend down, unless the handle is rotated in reverse direction with some effort.

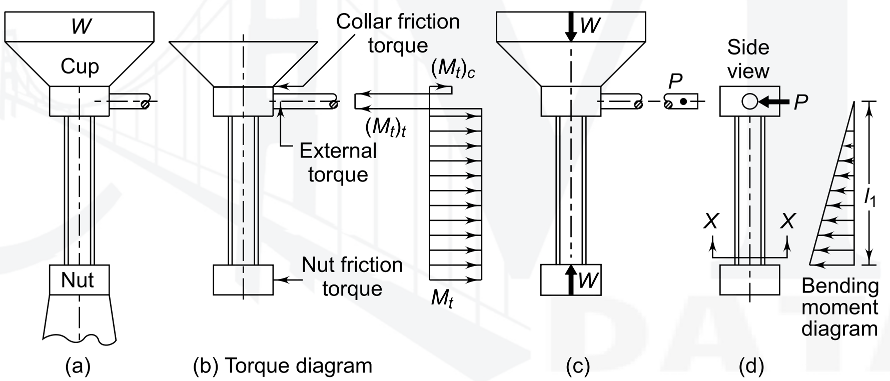

Fig. 6.19 *(a) Load in Raised Position (b) Torque Diagram (c) Compression of Screw (d) Bending Moment Diagram*

The portion of the screw between the handle and the nut is subjected to maximum stress, when the load is being raised. The screw is subjected to torsional moment, compressive force and bending moment which is illustrated in Fig. 6.19. The screw is made of plain carbon steel ($S_{yt} = 400 \text{ N/mm}^2$). Assuming

$$S_{yc} = S_{yt} = 400 \text{ N/mm}^2$$

and the factor of safety of 5,

$$\sigma_c = \frac{S_{yc}}{(fs)} = \frac{400}{5} = 80 \text{ N/mm}^2$$

From Fig. 6.19(c),

$$\sigma_c = \frac{W}{\left(\frac{\pi}{4} d_c^2\right)}$$

where d_c is the core diameter of the screw. Substituting the values,

$$80 = \frac{100 \times 10^3}{\left(\frac{\pi}{4} d_c^2\right)}$$

or, $\quad d_c = 39.89$ or 40 mm

There are additional stresses due to torsional and bending moments. The diameter should be increased to account for these stresses. As a first trial, a square threaded screw with 60 mm nominal diameter and 9 mm pitch (Table 6.1) is selected.

Trial No. 1

$d = 60$ mm $\quad p = 9$ mm

From Eqs (6.1) and (6.2),

$$d_c = d - p = 60 - 9 = 51 \text{ mm}$$

$$d_m = d - 0.5p = 60 - 0.5(9) = 55.5 \text{ mm}$$

It is assumed that the screw has single-start threads.

$$\therefore \quad l = p = 9 \text{ mm}$$

$$\tan\alpha = \frac{l}{\pi d_m} = \frac{9}{\pi(55.5)} \quad \text{or} \quad \alpha = 2.95°$$

The coefficient of friction between the steel screw and bronze nut is normally taken as 0.1. The maximum possible value of the coefficient of friction is 0.18. This occurs when the friction is maximum on account of poor lubrication. We will consider the worst case where the operator is careless about the lubrication of the screw.

$\tan\phi = \mu = 0.18 \quad$ or $\quad \phi = 10.20°$

Since $\phi > \alpha$ the screw is self locking.

From Eq. (6.6),

$$M_t = \frac{W d_m}{2} \tan(\phi + \alpha)$$

$$= \frac{(100 \times 10^3)(55.5)}{2} \tan(10.20 + 2.95)$$

$$= 648\,316.03 \text{ N-mm}$$

The torque diagram for the screw is shown in Fig. 6.19(b). It is important to note the following points with respect to this diagram,

(i) The portion of the screw between the nut and the axis of the handle is subjected to torque M_t only.

(ii) The portion of the screw between the cup and the axis of the handle is subjected to torque $(M_t)_c$ only.

(iii) The external torque $(P \times l_h)$ exerted at the axis of the handle consists of addition of M_t plus $(M_t)_c$.

(iv) No cross-section of the screw is subjected to addition of M_t plus $(M_t)_c$.

At the section-XX,

$$\tau = \frac{16 M_t}{\pi d_c^3} = \frac{16(648\,316.03)}{\pi(51)^3}$$

$$= 24.89 \text{ N/mm}^2 \qquad \text{(i)}$$

$$\sigma_c = \frac{W}{\left(\frac{\pi}{4} d_c^2\right)} = \frac{100 \times 10^3}{\frac{\pi}{4}(51)^2} = 48.95 \text{ N/mm}^2 \qquad \text{(ii)}$$

The portion of the screw with the side view of the handle is shown in Fig. 6.19(d). The hand force P acting on the handle causes a bending moment at the section-XX. The bending moment is given by,

$$M_b = P \times l_1$$

The lifting height of the jack is 500 mm and the distance l_1 can be assumed as,

$$l_1 = 500 + 50 + 20 = 570 \text{ mm}$$

$$M_b = P \times l_1 = (0.9 \times 2 \times 400)(570)$$

$$= 410\,400 \text{ N-mm}$$

Therefore,

$$\sigma_b = \frac{32 M_b}{\pi d_c^3} = \frac{32(410\,400)}{\pi(51)^3} = 31.51 \text{ N/mm}^2 \qquad \text{(iii)}$$

Figure 6.20(c) shows the superimposition of direct compressive stress and bending stresses as determined by equations (ii) and (iii). In this figure, tensile and compressive stresses are shown as positive and negative respectively. The resultant stresses are compressive. A compressive stress

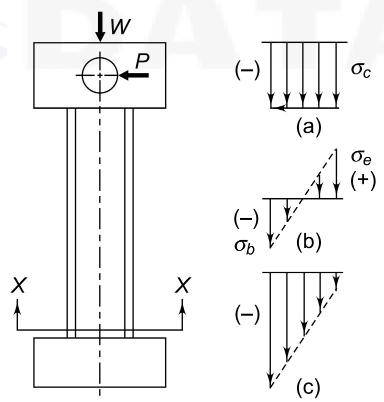

Fig. 6.20 *Stresses at Section XX: (a) Direct Compressive stress (b) Bending Stresses (c) Resultant Stresses*

closes the crack, while tensile stress opens the crack. In general, failure is associated with tensile stress rather than compressive stress. To be on the safe side, we will neglect the compressive stress as determined by the equation (ii) and consider the effect of combination of torsional shear stress and bending stresses as determined by equations (i) and (iii) respectively. The principal shear stress at the section-*XX* is given by,

$$\tau_{max.} = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + (\tau)^2}$$

$$= \sqrt{\left(\frac{31.51}{2}\right)^2 + (24.89)^2} = 29.46 \text{ N/mm}^2$$

The factor of safety is given by,

$$(fs) = \frac{S_{sy}}{\tau_{max.}} = \frac{0.5\,S_{yt}}{\tau_{max.}} = \frac{0.5(400)}{29.46} = 6.79$$

Since the factor of safety is more than 5, the design is safe. Therefore, the screw with single-start square threads of 60 mm nominal diameter and 9 mm pitch is suitable for the screw jack.

Step V *Buckling Consideration*

The buckling of columns is discussed in Chapter 23. Section 23.5 on 'Buckling of Columns' explains Johnson's equation and Euler's equation. When the load is raised through a distance of 500 mm, the portion of the screw between the nut and the handle acts as a column. For the purpose of buckling, the length of the column (l) is taken as,

$$l = 500 + 50 = 550 \text{ mm}$$

For a circular cross-section of diameter d_c,

$$I = \frac{\pi d_c^4}{64} \quad \text{and} \quad A = \frac{\pi d_c^2}{4}$$

From Eq. (23.6),

$$k = \sqrt{\frac{I}{A}}$$

Substituting values of I and A,

$$k = \frac{d_c}{4} = \frac{51}{4} = 12.75 \text{ mm}$$

The slenderness ratio of the screw is given by,

$$(l/k) = (550 / 12.75) = 43.14 \qquad \text{(i)}$$

Since one end of the screw is fixed in the nut and the other end is free, the end fixity coefficient is 0.25. The borderline between the short and long columns is given by,

$$\frac{S_{yt}}{2} = \frac{n\pi^2 E}{(l/k)^2}$$

Substituting the values,

$$\frac{400}{2} = \frac{(0.25)\pi^2 (207\ 000)}{(l/k)^2}$$

or $$(l/k) = 50.53 \qquad \text{(ii)}$$

The critical slenderness ratio is 50.53. The slenderness ratio of the screw (43.14) is less than the critical slenderness ratio (50.53). Therefore, the screw should be treated as short column and Johnson's equation is applied (refer to Fig. 23.11).

From Eq. (23.8),

$$P_{cr} = S_{yt} A \left[1 - \frac{S_{yt}}{4n\pi^2 E} \times \left(\frac{l}{k}\right)^2\right]$$

$$= (400)\left(\frac{\pi}{4}\right)(51)^2 \left[1 - \frac{400\ (43.14)^2}{4(0.25)\pi^2 (207\ 000)}\right]$$

$$= 519\ 386.04 \text{ N}$$

The factor of safety from buckling consideration is given by,

$$(fs) = \frac{P_{cr}}{W} = \frac{519\ 386.04}{100 \times 10^3} = 5.19$$

Therefore, the screw is safe against buckling.

Step VI *Design of Nut*

The permissible bearing pressure between the steel screw and the bronze nut is 10 N/mm^2. The number of threads required to support the load is z.

From Eq. (6.23),

$$z = \frac{4W}{\pi S_b (d^2 - d_c^{\,2})} = \frac{4(100 \times 10^3)}{\pi(10)(60^2 - 51^2)}$$

$$= 12.75 \quad \text{or} \quad 13$$

The axial length of the nut (H) is given by,

$$H = z\,p = (13)\,(9) = 117 \text{ mm}$$

The transverse shear stress at the root of the threads in the nut is given by,

$$\tau_n = \frac{W}{\pi d t z} = \frac{(100 \times 10^3)}{\pi(60)(4.5)(13)} = 9.07 \text{ N/mm}^2$$

$$(fs) = \frac{S_{su}}{\tau_n} = \frac{0.5 S_{ut}}{\tau_n} = \frac{0.5(190)}{9.07} = 10.47$$

The dimensions of the nut are shown in Fig. 6.21. The outer diameter of the nut is assumed to be twice of the nominal diameter of the thread.

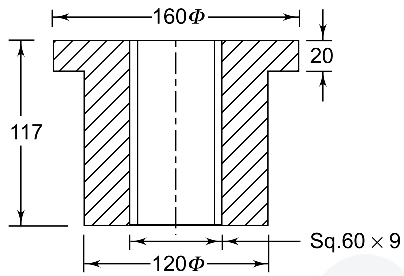

Fig. 6.21 *Dimensions of Nut*

Step VII *Design of Cup*

As shown in Fig. 6.18, the annular area of collar friction has an outer diameter of 1.6d. The inner diameter is assumed as 0.8d.

$$D_o = 1.6d = 1.6(60) = 96 \text{ mm}$$

$$D_i = 0.8d = 0.8(60) = 48 \text{ mm}$$

The collar friction torque $(M_t)_c$ is given by,

$$(M_t)_c = \frac{\mu_c W}{4}(D_o + D_i)$$

$$= \frac{(0.2)(100 \times 10^3)}{4}(96 + 48) = 720\,000 \text{ N-mm}$$

The total torque $(M_t)_t$ required to raise the load is given by,

$$(M_t)_t = M_t + (M_t)_c = 648\,316.03 + 720\,000$$

$$= 1\,368\,316.03 \text{ N-mm.}$$

The external torque, which is exerted by two workers is given by,

$$(M_t)_t = (0.9 \times 2 \times 400) \times l_h \qquad \text{(ii)}$$

From (i) and (ii),

$$1\,368\,316.03 = 0.9 \times 2 \times 400 \times l_h$$

$$l_h = 1900.43$$

The length of the handle (l_h) is too large and impractical. It is, therefore, necessary to change the design of the cup and replace the sliding friction by rolling friction by using thrust ball bearing. In thrust ball bearing, the friction torque $(M_t)_c$ is so small, that it can be neglected.

Thrust ball bearing shown in Fig. 6.22 is suitable for a purely axial load. It is a single-direction thrust ball bearing, because it can support axial load in one direction only, i.e., vertically downward. This ball bearing should not be subjected to radial load. Single-direction thrust ball bearings are separable and the mounting is simple as the components can be mounted individually. There are three separable parts of this bearing known as a shaft washer, a housing washer and the ball and cage assembly. The mounting of thrust bearing is shown in Fig. 6.23. The inner diameter of the shaft washer is press fitted in the screw body. The outer diameter of the housing washer is press fitted in the cup. These two components are separately mounted before final assembly.

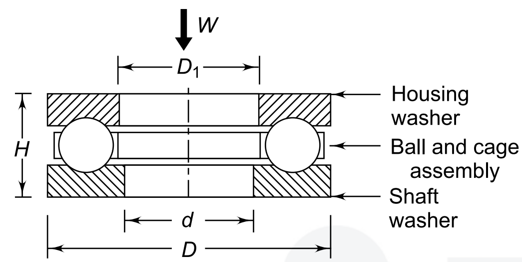

Fig. 6.22 *Thrust Ball Bearing*

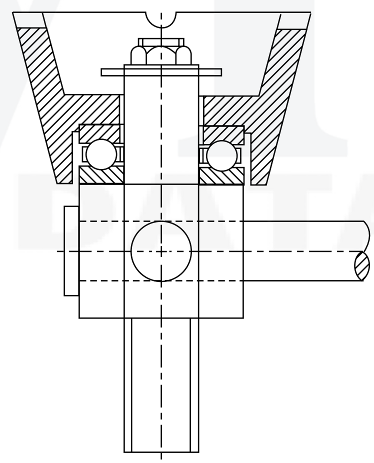

Fig. 6.23 *Mounting of Thrust Bearing*

The procedure for selection of the ball bearing from the manufacturer's catalogue is explained in Section 15.12. The screw jack is intermittently used and, as such, the life of the thrust bearing is assumed to be 3000 hours. The handle is rotated manually and it is not possible to find out the speed of rotation accurately. For the purpose of bearing selection, it is assumed that the handle rotates at 10 rpm. Therefore, the life of the bearing in million revolutions is given by,

$$L = \frac{60\, nL_h}{10^6} = \frac{60(10)(3000)}{10^6} = 1.8 \text{ million rev.}$$

also $P = W = 100 \times 10^3$ N

The dynamic load capacity of the bearing is given by,

$$C = P\, L^{1/3} = 100 \times 10^3\, (1.8)^{1/3} = 121\,644.04 \text{ N}$$

It is assumed that the bore diameter of the bearing is 50 mm. For this diameter, the following four bearings are available[3].

Sr. No.	d (mm)	D (mm)	H (mm)	C (N)	C_o (N)	D_1 (mm)	Designation
1	50	70	14	25 500	50 000	52	51110
2	50	78	22	41 600	73 500	52	51210
3	50	95	31	97 500	160 000	52	51310
4	50	110	43	159 000	250 000	52	51410

C = dynamic load capacity C_o = static load capacity

From the above table, Bearing No. 51410 with dynamic load carrying capacity of 159 000 N is selected for the jack. The dimensions of the bearing are as follows:

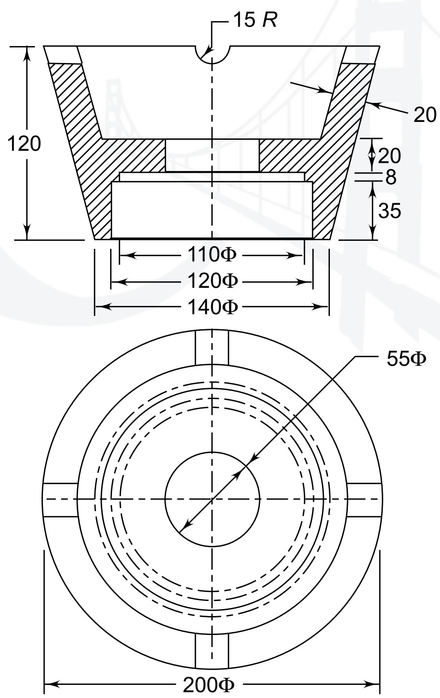

Fig. 6.24 *Dimensions of Cup*

$d = 50$ mm $D = 110$ mm $H = 43$ mm $D_1 = 52$ mm

The dimensions of the cup are shown in Fig. 6.24. The section thickness is kept 20 mm throughout as far as possible.

Step VIII *Design of Handle*

The handle is subjected to bending moment. The force exerted by two workers on the handle is given by,

$$P = (0.9 \times 2 \times 400) \text{ N}$$

The handle is made of steel 30C8 (S_{yt} = 400 N/mm^2). There is no collar friction torque. Therefore,

$$M_b = P \times l_h = (M_t)_t$$

or $$P \times l_h = M_t$$

$$(0.9 \times 2 \times 400)\, l_h = 648\,316.03$$

$\therefore$ $l_h = 900.43$ or 910 mm

Since,

$$\sigma_b = \frac{32\ M_b}{\pi\, d_h^3}$$

$$\therefore \quad \left(\frac{400}{5}\right) = \frac{32\ (0.9 \times 2 \times 400)\ (910)}{\pi\, d_h^3}$$

or

$$d_h = 43.69 \quad \text{or} \quad 45 \text{ mm}$$

The handle is inserted through a hole in the head of the screw as shown in Fig. 6.23. Two holes are provided, at right angles to each other, for changing the position of the handle after a quarter revolution.

[3] SKF General Catalogue No. 3200E

The dimensions of the cast iron frame are shown in Fig. 6.25.

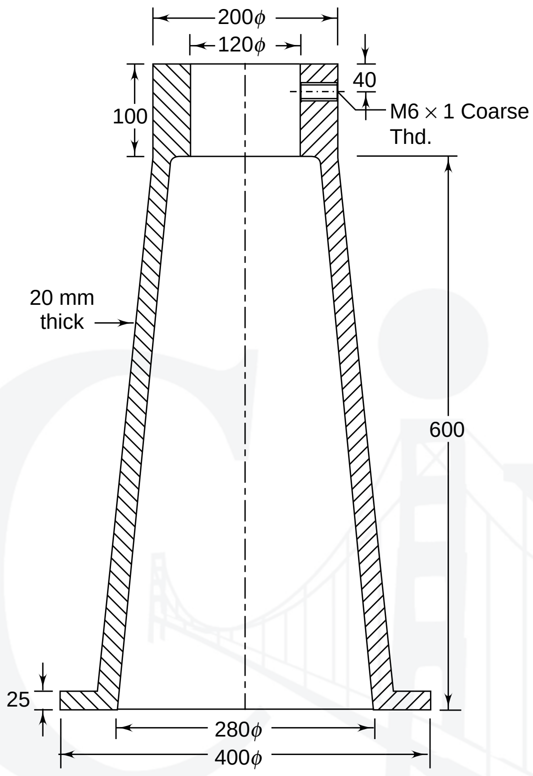

Fig. 6.25 *Dimensions of Frame*

Step IX *Safety Aspect*

To guard against injury to the workers and prevent damage to the machine parts, the following safety measures should be taken:

(i) After fabrication, the mechanical jack should be proof tested. In a proof test, the jack is loaded to 150% of its rated load with the lifting member at approximately 90% of full extension. After this test, the jack should be functional for full extension under 100% of the lifting rated load[4, 5].

(ii) The jack should be provided with a warning. A sample of the warning is as follows: "WARNING: DO NOT OVERLOAD JACK. PLACE LOAD ON CENTRE OF CUP ONLY. PLACE THE FRAME OF JACK ON HARD LEVEL SURFACE. LOAD AND STAND SHALL BE STABLE. STUDY, UNDERSTAND AND FOLLOW ALL INSTRUCTIONS. FAILURE TO HEED THIS WARNING MAY RESULT IN PERSONAL INJURY AND/OR PROPERTY DAMAGE".

(iii) The following operational instructions should be given in the manual or leaflet of the jack:

(a) The jack shall be visually examined for general condition before each shift or each use, whichever is less frequent.

(b) A determination of the load shall be made to assure that it is within the load rating of the jack.

(c) The jack shall be firmly supported at the base such that it is stable under load.

(d) Operators shall be instructed in proper use of the jack.

(e) Remove the handle when not in use to avoid accidental dislocation of the jack and reduce tripping hazard.

(f) Take precautions to ensure that all personnel are clear of the load before lowering.

(g) Ensure that there is sufficient swing area for the handle.

(h) Off-centre loading of jacks should be avoided.

(iv) The rated load should be legibly and durably marked in a prominent position on the jack.

(v) The nut and the thrust ball bearing of the jack should be regularly lubricated with grease.

The strength and stability considerations in design are no doubt essential for safety. However, they are not enough to prevent an accident. It is also essential to have personnel involved in the use and operation of jack to be careful, competent,

[4] ASME B 30.1—1992: *Jacks—The American Society of Mechanical Engineers.*

[5] ASME PALD—1993—*Portable Automotive Lifting Devices—The American Society of Mechanical Engineers.*

trained and qualified in safe operation of the screw jack and its proper use.

6.16 DIFFERENTIAL AND COMPOUND SCREWS

There are certain applications where very slow advance of the screw is required for fine adjustment, whereas in some applications very fast advance of the screw is desirable. One of the methods to achieve small advance of the screw is to reduce the pitch. The small pitch results in weak threads. The rapid movement of the screw is usually obtained by using multi-start threads. Increasing number of starts affects the self-locking property. To overcome these difficulties, differential and compound screws are used.

A differential screw is defined as a mechanical device consisting of two screws in series, which are arranged in such a way that the resultant motion is the difference of individual motions of the two screws. The principle of differential screw is explained with the help of Fig. 6.26(a). The composite screw consists of two parts—the larger part S_1 with a pitch of 4 mm and the smaller part S_2 with a 3 mm pitch. The hand of helix for the two threads is same, both being right-handed. The larger part S_1 moves through the frame F. There is a square nut N on the smaller part S_2. The rotation of the nut is prevented as illustrated in Fig. 6.26(c) and it can only slide in an axial direction with respect to the frame.

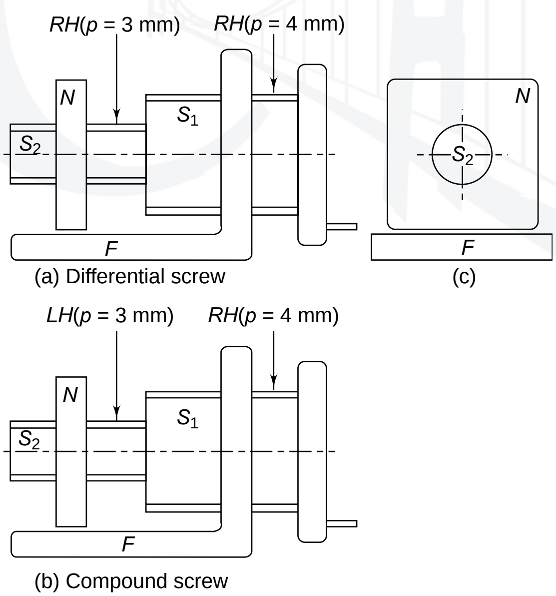

Fig. 6.26

The handle is turned through one revolution in the clockwise direction when viewed from the right side. According to the right-hand thumb rule, when the fingers are kept in the direction of rotation, the thumb indicates the direction of movement of the screw. The direction of movement of the nut is opposite to that of the screw. The screw S_1 will move through 4 mm to the left with respect to the frame. The nut N will move through 3 mm to the right relative to the frame. The resultant motion of the nut with respect to the frame will be (4-3) mm to the left. In general, if p_1 and p_2 are the pitches for two screws, the resultant motion is equal to $(p_1 - p_2)$ or the difference of the individual motions of the two screws.

A compound screw is defined as a mechanical device consisting of two screws in series, which are arranged in such a way that the resultant motion is the sum of individual motions of the two screws. The compound screw is shown in of Fig. 6.26(b). The arrangement is similar to that of a differential screw, except for the hand of helix for the two screws. The threads on the larger part S_1 are right-handed while those on the smaller part S_2 are left handed. It can be proved that the resultant movement of the nut with respect to the frame is (4 + 3) mm to the left. In general, if p_1 and p_2 are the pitches for two screws, the resultant motion is equal to $(p_1 + p_2)$ or the sum of the individual motions of the two screws.

Example 6.11 *A differential type of screw jack is shown in Fig. 6.27. In this construction, the two screws do not rotate and the nut is rotated by the operator by applying a force of 100 N at a mean radius of 500 mm. The coefficient of friction at the threads is 0.15. Calculate*

(i) the load that can be raised; and

(ii) the efficiency of the screw jack.

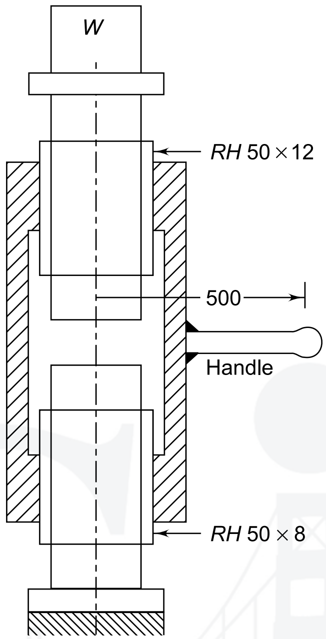

Fig. 6.27 *Differential Screw Jack*

Solution

Given For handle, $R = 500$ mm $P = 100$ N
$\mu = 0.15$
Upper screw = RH 50 × 12
Lower screw = RH 50 × 8

Step I *Load capacity of screw jack*

In one revolution of the nut, the load is raised through a distance equal to the difference in the pitch of the two screws.

Output = work done on $W = W(p_1 - p_2)$
$= W(12 - 8) = (4\ W)$ N-mm

For the upper screw,

$$d_m = d - 0.5p = 50 - 0.5(12) = 44 \text{ mm}$$

$$\tan\alpha = \frac{l}{\pi d_m} = \frac{12}{\pi(44)} \quad \text{or} \quad \alpha = 4.962°$$

$$\tan\phi = \mu = 0.15 \quad \text{or} \quad \phi = 8.531°$$

$$M_{t1} = \frac{Wd_m}{2}\tan(\phi + \alpha)$$

$$= \frac{W(44)}{2}\tan(8.531 + 4.962)$$

$$= (5.279\ \text{W}) \text{ N-mm}$$

For the lower screw,

$$d_m = d - 0.5\,p = 50 - 0.5(8) = 46 \text{ mm}$$

$$\tan\alpha = \frac{l}{\pi d_m} = \frac{8}{\pi(46)} \quad \text{or} \quad \alpha = 3.169°$$

When the nut is rotated, the upper screw moves in the upward direction. As far as the lower screw is concerned, it is the nut, which moves in the upward direction. In other words, the lower screw is moving in a downward direction relative to the nut and the load. Therefore,

$$M_{t2} = \frac{Wd_m}{2}\tan(\phi - \alpha)$$

$$= \frac{W(46)}{2}\tan(8.531 - 3.169)$$

$$= (2.159\ \text{W}) \text{ N-mm}$$

Adding the torques of upper and lower screws and equating with external torque,

$$100(500) = 5.279\ \text{W} + 2.159\ \text{W}$$

$$\therefore \quad W = 6722 \text{ N} \qquad \text{(i)}$$

Step II *Efficiency of the screw jack*

$$\eta_o = \frac{4W}{2\pi(100 \times 500)} = \frac{4(6722)}{2\pi(100 \times 500)}$$

$$= 0.0856 \quad \text{or} \quad 8.56\% \qquad \text{(ii)}$$

6.17 RECIRCULATING BALL SCREW

A recirculating ball screw, as shown in Fig. 6.28, consists of a screw and a nut, the surfaces of which are separated by a series of balls. The screw and the nut have approximately semi-circular thread profiles instead of conventional square or trapezoidal shape. As the screw is rotated, the balls advance in the grooves in the nut and the screw. They are collected at the end of the nut and returned back. The recirculating ball screw is also called *ball bearing screw* or simply *ball screw*. Such screws are preloaded and give accurate motion due to elimination of the backlash. There is no heat generation due to negligible friction. Recirculating ball screws can be used for high speeds even up to 10 m/min. The balls, screw and nut are subjected to contact stresses. They are usually made of nickel chromium steel and heat treated to a surface hardness of 58 to 65 HRC.

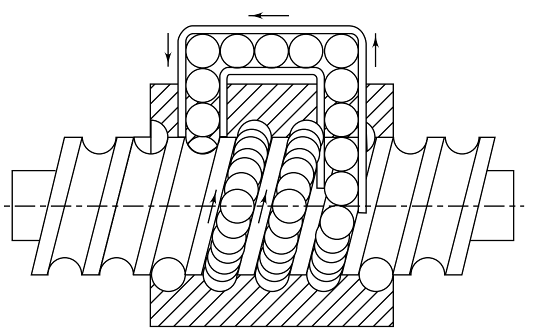

Fig. 6.28 *Recirculating Ball Screw*

Compared with conventional power screws, recirculating ball screws offer the following advantages:

(i) In conventional power screw, there is sliding friction between the screw and nut threads. In recirculating ball screw, there is rolling friction between the balls and the grooves in the screw and nut. This reduces friction drastically. The efficiency of a conventional power screw is as low as 40%. The efficiency of ball screws is as high as 90%, because sliding friction is replaced by rolling friction.

(ii) In conventional power screw, 'stick-slip' phenomenon is observed due to difference between the values of coefficient of static friction and coefficient of sliding friction. It is a serious drawback. In recirculating ball screw, it is nearly eliminated and the operation is smooth.

(iii) A conventional power screw must be adjusted periodically to compensate for wear on the surfaces of the screw and the nut. A recirculating ball screw is virtually wear-free due to presence of lubricant film between the contacting surfaces and protection from contamination by dirt particles.

(iv) The load carrying capacity of recirculating ball screw is more than that of conventional power screw. For the same load carrying capacity, recirculating ball screw is more compact and lightweight than conventional screw jack.

Compared with conventional power screws, recirculating ball screws has the following disadvantages:

(i) A recirculating ball screw is much more costly than conventional power screw.

(ii) Recirculating ball screws are usually overhauling due to low friction. Therefore, a separate brake is required to hold the load in its place.

(iii) Buckling of screw and critical speed are serious problems with recirculating ball screw.

(iv) Recirculating ball screws require a high degree of cleanliness compared with conventional power screws. They are completely enclosed to prevent the entry of foreign particles.

(v) Recirculating ball screws require a thin film of lubricant between the balls and grooves in the nut and the screw for satisfactory operation. Conventional power screws can be easily lubricated by grease.

Recirculating ball screws are used in the following applications:

(i) Automobile steering gears
(ii) Power actuators
(iii) X-Y recorders of CNC machines
(iv) Aircraft landing gear retractors
(v) Hospital bed adjustors
(vi) Machine tool controls

Short-Answer Questions

6.1 What is power screw?

6.2 What are the applications of power screws?

6.3 What are the advantages of power screws?

6.4 What are the disadvantages of power screws?

6.5 What are the types of threads for power screw?

6.6 Why are *V* threads not used in power screws?

6.7 What are the advantages of square threads over trapezoidal threads?

6.8 What are the disadvantages of square threads compared with trapezoidal threads?

6.9 What are the applications of square threads?

6.10 What are the applications of trapezoidal threads?

6.11 How will you designate square threads?

6.12 How will you designate trapezoidal threads?

6.13 How will you designate multiple-start trapezoidal threads?

6.14 What are the advantages of multiple-start screws?

6.15 What are the disadvantages of multiple-start screws?

6.16 What are the applications of multiple-start screws?

6.17 What is 'overhauling' of power screw? What is the condition for overhauling?

6.18 What is 'self-locking' of power screw? What is the condition for self-locking?

6.19 State the applications where self-locking is essential.

6.20 What are the two methods to increase the efficiency of a square threaded screw?

6.21 Why is the efficiency of self-locking square threaded screw less than 50%?

6.22 What is collar friction?

6.23 What is differential screw?

6.24 What is compound screw?

6.25 Where do you use differential and compound screws?

6.26 What is recirculating ball screw?

6.27 What are the advantages of recirculating ball screw?

6.28 What are the disadvantages of recirculating ball screw?

6.29 What are the applications of recirculating ball screw?

Problems for Practice

6.1 A double-threaded power screw, used for lifting a load, has a nominal diameter of 30 mm and a pitch of 6 mm. The coefficient of friction at the screw threads is 0.1. Neglecting collar friction, calculate:

(i) efficiency of the screw with square threads; and

(ii) efficiency with Acme threads ($2\theta = 29°$).

[*(i) 57.76%, (ii) 56.96%*]

6.2 A sluice valve, used in a water-pipeline, consists of a gate raised by the spindle, which is rotated by the hand wheel. The spindle has single-start square threads. The nominal diameter is 36 mm and the pitch is 6 mm. The inner and outer diameters of the friction collar are 30 mm and 50 mm respectively. The coefficients of friction at the threads and the collar are 0.15 and 0.20 respectively. The weight of the gate is 7.5 kN and the frictional resistance to open the valve due to water pressure is 2.5 kN. Using the uniform wear theory for collar friction, calculate:

(i) the torque required to raise the gate; and

(ii) the overall efficiency of the mechanism.

[*(i) 74.6 N-m, (ii) 12.8%*]

6.3 A double-threaded power screw is used to raise a load of 5 kN. The nominal diameter is 60 mm and the pitch is 9 mm. The threads are Acme type ($2\theta = 29°$) and the coefficient of friction at the screw threads is 0.15. Neglecting collar friction, calculate:

(i) the torque required to raise the load;

(ii) the torque required to lower the load; and

(iii) the efficiency of the screw for lifting load.

[*(i) 36.39 N-m, (ii) 7.06 N-m, (iii) 39.35%*]

6.4 A 50 kN capacity screw jack consists of a square-threaded steel screw meshing with a bronze nut. The nominal diameter is 60 mm and the pitch is 9 mm. The permissible bearing pressure at the threads is 10 N/mm^2. Calculate:

(i) the length of the nut; and

(ii) the transverse shear stress in the nut.

[*(i) 63 mm (ii) 8.42 N/mm^2*]

6.5 A triple-threaded power screw, used in a screw jack, has a nominal diameter of 50 mm and a pitch of 8 mm. The threads are square and the length of the nut is 48 mm. The screw jack is used to lift a load of 7.5 kN. The coefficient of friction at the threads is 0.12 and the collar friction is negligible. Calculate:

(i) the principal shear stress in the screw body;
(ii) the transverse shear stresses in the screw and the nut; and
(iii) the unit bearing pressure.

State whether the screw is self-locking.
[*(i) 4.39, (ii) 2.37 and 1.99, (iii) 2.16 N/mm²*]

6.6 Plot a graph of efficiency v/s helix angle, which varies from 0 to 60°, for a square-threaded screw. The coefficient of friction at the threads is 0.15 and the collar friction is negligible.

$\alpha(^o)$	5	10	15	20	25	30	35	40	45	50	55	60
$\eta(\%)$	36.35	52.60	61.53	66.95	70.37	72.50	73.71	74.16	73.49	72.94	71.11	68.12

6.7 The following data is given for a machinist's clamp:

type of thread = single-start square
nominal diameter = 20 mm
pitch = 5 mm
collar friction radius = 8 mm.

The coefficient of friction at the threads and the collar is 0.15. The operator exerts a force of 50 N on the handle at a distance of 150 mm from the axis of the screw. Determine the maximum clamping force that can be developed.

[*2247.12 N*]

Threaded Joints

Chapter 7

7.1 THREADED JOINTS

Threaded joint is defined as a separable joint of two or more machine parts that are held together by means of a threaded fastening such as a bolt and a nut. The salient features of this definition are as follows:

(i) Threaded joints are used to hold two or more machine parts together. These parts can be dismantled, if required, without any damage to machine parts or fastening. Therefore, threaded joints are *detachable joints*, unlike welded joints.

(ii) Thread is the basic element of these joints. The thread is formed by cutting a helical groove on the surface of a cylindrical rod or cylindrical hole. The threaded element can take the shape of bolt and nut, screw or stud. Sometimes, threads are cut on the parts to be joined.

Threaded joints are extensively used in mechanical assemblies. It has been observed that over 60% of the parts have threads. The popularity of threaded joints is due to certain advantages offered by them. The advantages of threaded joints are as follows:

(i) The parts are held together by means of a large clamping force. There is wedge action at the threads, which increases the clamping force. There is no loosening of the parts. Therefore, threaded joints are '*reliable*' joints.

(ii) The parts are assembled by means of a spanner. The length of the spanner is large compared with the radius of the thread. Therefore, the mechanical advantage is more and force required to tighten the joint is small.

(iii) Threaded joints have small overall dimensions resulting in compact construction.

(iv) The threads are self-locking. Therefore, threaded joints can be placed in any position—vertical, horizontal or inclined.

(v) Threaded fasteners are economical to manufacture. Their manufacturing is simple. High accuracy can be maintained for the threaded components.

(vi) The parts joined together by threaded joints can be detached as and when required. This requirement is essential in certain applications for the purpose of inspection, repair or replacement.

(vii) Threaded fasteners are standardised and a wide variety is available for different operating conditions and applications.

There are certain disadvantages of threaded joints. They are as follows:

(i) Threaded joints require holes in the machine parts that are to be clamped. This results in stress concentration near the threaded portion of the parts. Such areas are vulnerable to fatigue failure.

(ii) Threaded joints loosen when subjected to vibrations.

(iii) Threaded fasteners are considered as a major obstacle for efficient assembly. *In manual assembly, the cost of tightening a screw can be six to ten times the cost of the screw itself.*[1] Therefore, *Design for Manufacture and Assembly* (DFMA) recommends minimum number of threaded fasteners.

7.2 BASIC TYPES OF SCREW FASTENING

There are three parts of a threaded fastening, viz., a bolt or screw, a nut and a washer. There is a basic difference between the bolt and the screw. A bolt is a fastener with a head and straight threaded shank and intended to be used with a nut to clamp two or more parts. The same bolt can be called screw when it is threaded into a tapped hole in one of the parts and not into the nut. Although bolt and screw are similar, there is a fundamental difference in their assembly. A bolt is held stationary, while torque is applied to the nut to make threaded joint, whereas the torque is applied to the screw to turn it into matching threads in one of the parts. A nut is a small symmetrical part, usually having hexagonal or square shape, containing matching internal threads.

Simple washers are thin annular shaped metallic disks. The functions of a washer are as follows:

(i) It distributes the load over a large area on the surface of clamped parts.

(ii) It prevents marring of clamped parts during assembly.

(iii) It prevents marring of the bolt head and nut surface during assembly.

(iv) It provides bearing surface over large clearance holes.

Threaded fastenings are classified according to their shape and the purpose for which they are used. Common types of threaded fastenings are as follows:

(i) Through Bolts A through bolt is simply called a '*bolt*' or a '*bolt and nut*'. It is shown in Fig. 7.1(a). The bolt consists of a cylindrical rod with head at one end and threads at the other. The cylindrical portion between the head and the threads is called *shank*. The shank passes through the holes in the parts to be fastened. The threaded portion of the bolt is screwed into the nut. The head of the bolt and the nut are either hexagonal or square. Hexagonal head bolt and nut are popular in the machine building industry. Square head and nut are used mostly with

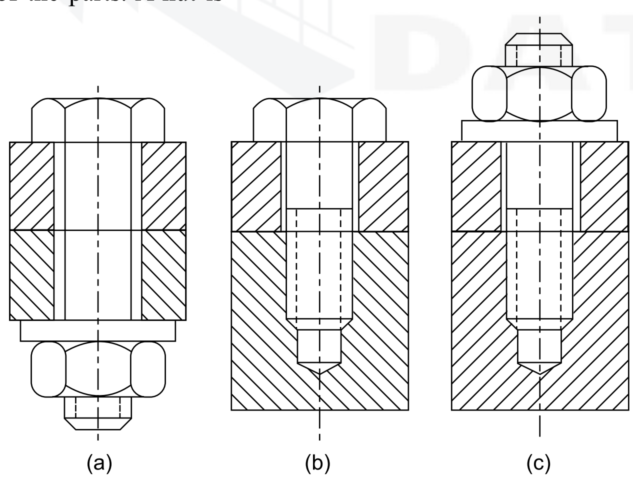

Fig. 7.1 *Types of Screw Fastening: (a) Through Bolt (b) Tap Bolt (c) Stud*

[1] Bart Huthwite—*Checklist for DFM—Machine Design*—Vol. 62, No. 3—February 1990.

rough type of bolts in construction work. Through bolts are used under the following conditions:

(a) The parts that are fastened have medium thickness, e.g., plates, flanges or beams and space is available to accommodate the bolt head and the nut. Space should also be available to accommodate the spanner to tighten the nut.

(b) The parts that are fastened are made of materials, which are too weak to make durable threads.

(c) The parts that are fastened require frequent dismantling and reassembly.

The shank of the bolt may or may not have finished surface. It depends upon the type of force induced in the bolt. When the nut is tightened, the shank of the bolt is subjected to pure tensile force in axial direction. In this case, the shank of the bolt can be rough. The only requirement is that it should pass easily through the holes in the parts. In some applications like rigid coupling, the shank is subjected to shear force, which is perpendicular to its axis and which tends to slide one part with respect to the other. In such cases, the shank is machined and passes through rimmed holes. Therefore, the bolt is finger tight in the holes. Depending upon the usage, through bolts are called *machine bolts, automobile bolts, eyebolts* or *carriage bolts*. A machine bolt has rough shank. The head and the nut of the machine bolt may be rough or finished as per requirement. Commercial machine bolts are available in sizes from 5 mm to 75 mm nominal diameter. An automobile bolt has fine threads. It is finished all over. The thickness of the hexagonal head and the nut is small in this case. Such bolts are available in sizes from 5 mm to 40 mm nominal diameter. The head is often provided with a slot for the purpose of tightening by means of a screwdriver. An automobile bolt can have ordinary hexagonal nut or castle nut. A carriage bolt is used when the head must rest against a wooden surface. The part of the shank near the head has square cross-section. This prevents the bolt from turning when the nut is tightened.

Through bolts do not require tapping of threads in the parts that are fastened. However, they are inconvenient in assembly operation. Projections of head or nut give a bad appearance to the product.

(ii) Tap Bolts and Cap Screws There is a basic difference between through bolt and tap bolt. The tap bolt is turned into a threaded (tapped) hole in one of the parts being connected and not into a nut. On the other hand, the through bolt is turned into the nut. Tap bolt is shown in Fig. 7.1(b). Cap screws are similar to tap bolts. However, they are available in small sizes from 5 mm to 30 mm nominal diameter and they have a variety of shapes for their head. Tap bolts or cap screws are used under the following three conditions:

(a) one of the parts is thick enough to accommodate a threaded hole;

(b) the material of the part with threaded hole has sufficient strength to ensure durable threads; and

(c) there is no place to accommodate the nut.

The relative advantages and disadvantages of tap bolt compared with through bolt are as follows:

(a) Tap bolt and cap screw are cheaper than through bolt.

(b) When a tap bolt or cap screw is subjected to shear force perpendicular to its axis, it carries the load to the root area of the thread. On the other hand, a through bolt carries a corresponding load to larger shank area.

(c) When the tap bolt or cap screw is removed frequently, the threads in the part get worn out resulting in costly repairs. In case of through bolts, there are no threads in any of the parts being fastened.

Tap bolts and cap screws are good for fastening the parts that are seldom dismantled.

(iii) Studs A stud is a cylindrical rod threaded at both ends. One end of the stud is screwed into the tapped hole in one of the connecting parts. The other end of the stud receives a nut. A stud joint is shown in Fig. 7.1(c). Stud joints are used under the following conditions:

(a) One of the parts is thick enough to accommodate a threaded hole.

(b) The material of the part with threaded hole has sufficient strength to ensure durable threads.

(c) The material of the other part, without tapped hole, cannot ensure sufficient durability of the threads, e.g., light alloy or cast iron.

(d) The parts that are connected require frequent dismantling and reassembly.

Studs are particularly used for connecting cylinder with the cylinder head.

7.3 CAP SCREWS

As mentioned in the previous article, cap screws belong to the category of tap bolts. Cap screws differ from tap bolt in the following respects:

(i) Cap screws are small compared with tap bolt.

(ii) A wide variety of shapes are available for the head of cap screw. On the other hand, tap bolt has hexagonal or square head.

Cap screws with different types of heads are illustrated in Fig. 7.2.

Depending upon the shape of the head, cap screws are divided into the following two groups:

(i) cap screws in which the head is engaged externally by a spanner; and

(ii) cap screws in which the head is engaged internally and from the end face.

A cap screw with hexagonal head, shown in Fig. 7.2(a), is tightened externally by means of a spanner. On the other hand, a cap screw with a hexagonal socket head is tightened by a socket wrench in a counter bored hole. It belongs to the second category.

Hexagonal and square heads are used in ordinary applications. The normal height of hexagonal or square head is ($0.7d$) where d is the major diameter of the thread on the screw. Head of increased height is used for the screws, which are frequently turned in and out. Head with decreased height up to ($0.5d$) is used when the screw is infrequently turned in and out. Hexagonal head differs from square head in the following respects:

(i) Hexagonal and square heads require space for turning the spanner. However, hexagonal head requires a relatively small angle of rotation for the spanner than a square head. Theoretically, the angle of rotation for hexagonal head is one-sixth of a revolution to enable the next pair of flats to be engaged. In case of square head, the angle of rotation is one-fourth of a revolution at a time. Therefore, square head requires more space than hexagonal head for the rotation of the spanner.

(ii) For the same overall dimensions, square head has wider faces than hexagonal head, enabling a higher tightening torque to be transmitted. Alternatively, for the same torque, wider faces have a longer life.

Square head cap screws are used in jigs and fixtures, where the screw is often turned in and out and there is sufficient space to turn the spanner through large angle.

Cap screw heads, which are tightened internally and from the end face are shown in Fig. 7.2(b),

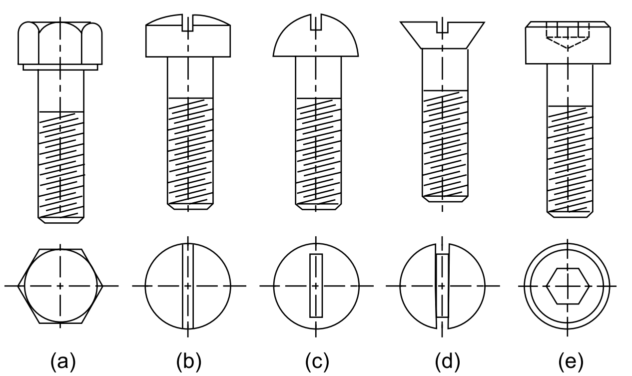

Fig. 7.2 *Cap Screws with Different Heads: (a) Hexagonal (b) Filister (c) Button (d) Flat (e) Fexagonal Socket Head*

(c), (d) and (e). They are called *fillister, button, flat,* and *hexagonal socket heads*. A fillister head cap screw has cylindrical head with a slot for the screwdriver. A button head cap screw has spherical head with similar slot for the screwdriver. A flat head cap screw has conical head with a slot. Heads with the slot for the screwdriver are used for cap screws of small sizes, which require small torque for tightening. Fillister heads are used for cap screws with the head sunk in a counterbore. If the head is to be flush with the surface and the part being fastened has insufficient thickness to accommodate a fillister head, a cap screw with flat head is employed. When it is not possible to sink the head and a better appearance is required, cap screws with button or round head are used.

Cap screw with hexagonal socket head is shown in Fig. 7.2(e). These cap screws are tightened by a simple wrench made of hexagonal bar bent at right angles. The important advantage of cap screw with hexagonal socket head is that the maximum tightening torque permitted by the socket wrench corresponds to the strength of the screw. Therefore, the head cannot be twisted off during tightening. Cap screws with heads engaged internally and from the end face have the following advantages:

(i) Such cap screws can be installed in confined spaces and in counter bored holes.
(ii) They result in better appearance for the product.
(iii) They reduce the overall dimensions of the product resulting in compact construction.
(iv) There is convenience in cleaning or wiping off the surface of the product.

7.4 SETSCREWS

Setscrew is used to prevent relative motion between two parts. The threaded portion of the setscrew passes through a tapped hole in one of the parts and the end of the screw presses against the other part. The end of the screw is called the *point* of the screw. The friction between the point and the part to be held prevents relative motion of that part with respect to the part through which the setscrew is screwed. Setscrew can be used instead of key to prevent relative motion between the hub and the shaft in small power transmission. Setscrews differ from cap screws in the following respects:

(i) Setscrews are subjected to compressive force only. Cap screws are subjected to tensile and shear forces.
(ii) Setscrew transmits force from threaded component to the other mating component by means of screw point. In cap screw, the force is transmitted by the head.
(iii) Setscrews are short and threaded over full length of the shank compared with the cap screws.

Setscrews with different types of points and heads are illustrated in Fig. 7.3. Some setscrews have hexagonal or square head as shown in Fig. 7.3(b) and (c), but the majority do not have heads. Usually, they have a slot for the screwdriver as shown in Fig. 7.3(a) and (e) or a hexagonal

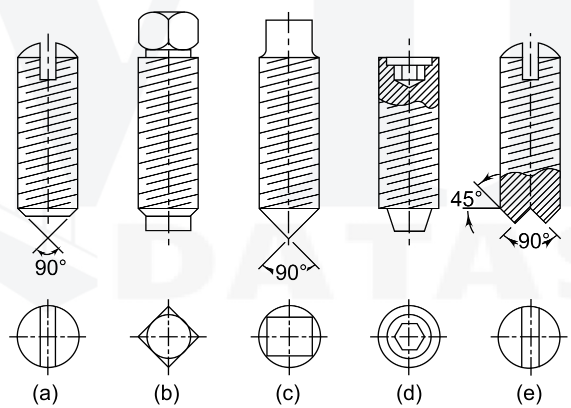

Fig. 7.3 *Types of Set Screws*

socket shown in Fig. 7.3(d). Setscrew used in rotating parts has no head. On the other hand, setscrew for stationary parts may have hexagonal or square heads. The types of point used in setscrew are as follows:

(i) Flat Point (Fig. 7.3 a) Flat point is used when the lateral force, which tends to displace one part with respect to another, is randomly applied. It is also used when the part with tapped hole into which the setscrew is screwed, does not have sufficient thickness.

(ii) Dog Point (Fig. 7.3 b) Dog point is used when the lateral force, which tends to displace one part with respect to other, is large. Also the part, which is held, should have sufficient thickness to accommodate a cylindrical hole for the dog point.

(iii) Cone Point (Fig. 7.3 c) Cone point is used when the lateral force is small. Also, the part with the tapped hole into which the setscrew is screwed, does not have sufficient thickness. The part being held is provided with conical hole.

(iv) Hanger Point (Fig. 7.3 d) Hanger point has a small taper. It is used when the lateral force is large. Also the part, which is held, should have sufficient thickness to accommodate a cylindrical hole. Hanger point ensures good location of the part.

(v) Cut Point (Fig. 7.3 e) Cut point is used when the part being held cannot be drilled or hardened. It is also used to transmit force to steel balls or spherical parts.

The point of the setscrew is generally hardened.

7.5 BOLT OF UNIFORM STRENGTH

Bolts are subjected to shock and impact loads in certain applications. The bolts of cylinder head of an internal combustion engine or the bolts of connecting rod are the examples of such applications. In such cases, resilience of the bolt is important design consideration to prevent breakage at the threads. *Resilience is defined as the ability of the material to absorb energy when deformed elastically and to release this energy when unloaded.* A resilient bolt absorbs energy within elastic range without any permanent deformation and releases this energy when unloaded. It can be called *spring* property of the bolt. A resilient bolt absorbs shocks and vibrations like leaf springs of the vehicle. In other words, the bolt acts like a spring.

It can be shown that the energy absorbed during elastic deformation is proportional to the square of the stress induced in the material and the volume of the material under the stress. Figure 7.4(a) shows an ordinary bolt with usual shape. The major diameter of the thread as well as the diameter of the shank is d. The core diameter of the threads is d_c. When this bolt is subjected to tensile force, there are two distinct regions of stress. They are as follows:

(i) The diameter of threaded portion d_c is less than the shank diameter d. The threaded portion is also subjected to stress concentration. Therefore, stress induced in the threaded portion is more than the stress in the shank portion. The energy absorbed by each unit volume of bolt material is proportional to the square of the stress. Hence, a large part of the energy is absorbed in the threaded portion of the bolt.

(ii) The diameter of the shank is more than the core diameter of the threaded portion. There is no stress concentration in the shank. Therefore, when the bolt is subjected to tensile force, the stress in the shank portion is less than the stress in the threaded portion. The energy absorbed in the shank, which is proportional to the square of the stress, is less than the energy absorbed in the threaded part.

The shock absorbing capacity of bolt can be increased if the shank of bolt is turned down to a diameter equal to the root diameter of threads or even less. In this case, the shank is subjected to higher stress and hence absorbs a greater proportion of strain energy and relieves the thread portion of high stress. The resilience of the bolt can also be increased by increasing its length. The strain energy absorbed by the shank is linearly proportional to its length. Therefore, there are two methods for increasing the shock absorbing capacity of bolts. They are as follows:

(i) Reduce the shank diameter to core diameter of threads or even less.

(ii) Increase the length of the shank portion of the bolt.

The threaded portion of the bolt is the weakest part and maximum amount of elastic energy is absorbed in this region. The ideal bolt will be one which is subjected to same stress level at different cross-sections in the bolt. It is called the bolt of *uniform strength*. In a bolt of uniform strength, the entire bolt is stressed to the same limiting value, thus resulting in maximum energy absorption.

There are two ways to reduce the cross-sectional area of the shank and convert an ordinary bolt into a bolt of uniform strength. They are illustrated in Fig. 7.4(b) and (c). One method is to reduce the diameter of the shank as shown in Fig. 7.4(b). In another method, the cross-sectional area of the shank is reduced by drilling a hole, as illustrated in Fig. 7.4(c). Both methods reduce cross-sectional area of the shank and increase stress and energy absorption. In the first method, the diameter of

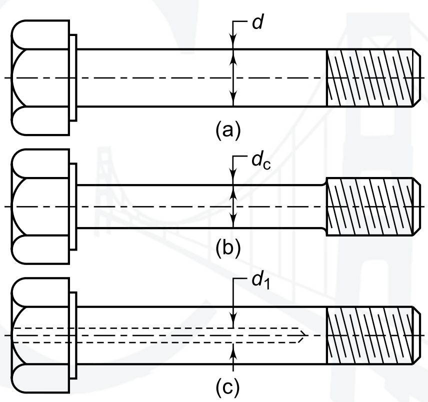

Fig. 7.4 *Bolts of Uniform Strength*

the shank is usually reduced to the core diameter of the threads. Therefore, the cross-sectional area of the shank is equal to the cross-sectional area of the threaded portion. When this bolt is subjected to tensile force, the stress in the shank and the stress in the threaded portion are equal. In the second method, the diameter of the hole (d_1) is obtained by equating the cross-sectional area of the shank to that of the threaded part. Therefore,

$$\frac{\pi}{4} d^2 - \frac{\pi}{4} d_1^2 = \frac{\pi}{4} d_c^2$$

or
$$d_1 = \sqrt{d^2 - d_c^2} \tag{7.1}$$

where,

d = nominal or major diameter of the bolt

d_c = core or minor diameter of the threads

d_1 = diameter of the hole

Machining a long hole is difficult operation compared with turning down the shank diameter. Drilled hole results in stress concentration. Therefore, a bolt with reduced shank diameter is preferred over a bolt with an axial hole.

7.6 LOCKING DEVICES

The threads used on all screw fastenings satisfy the condition of self-locking. The helix angle for metric threads with 6 to 68 mm major diameter varies in the range from 3°30′ to 1°40′. The helix angle is considerably less than the angle of friction. In addition, there is friction at the bearing surfaces of the nut and the head of the bolt. Therefore, loosening of the threads between the bolt and nut should not occur under normal working conditions. Ordinary threaded fastenings remain tight under the action of static loads. However, many of these fastenings become loose when subjected to cyclic and impact loads. Loosening is due to the reduction of friction force in the threads due to consecutive expansion and contraction of the bolt resulting from fluctuating axial load. It is also due to elastic vibrations along the axis of the screw. In many applications, vibrations are the main cause of loosening of the threads. Locking devices are used to prevent the loosening of the threads between the nut and the screw. Locking can be accomplished by the following three methods:

(i) by creating supplementary friction;
(ii) by using special locking devices like split-pin; and
(iii) by plastic deformation.

A large number of locking devices are available in practice. Only some of them are described in this article.

The most common method to lock the threads is to use a *jam nut*, i.e., a second nut as shown in Fig. 7.5. It is also called a *locknut*. The procedure of locking the threads consists of the following steps:

(i) The lower nut is first tightened down with normal force.

(ii) The upper nut is then tightened down upon the lower nut.

(iii) The upper nut is held with spanner and the lower nut is slackened back against it by another spanner.

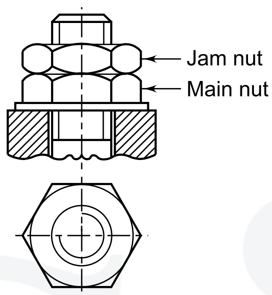

Fig. 7.5 *Jam Nut*

The tightening of lower nut against the surface of the upper nut creates additional friction at the contacting interface. This force of friction does not depend on the external force applied to the joint. After the assembly, the upper nut, called the jam nut, carries the main axial load. The tightening force and friction force in the threads of the lower nut, called the *main nut*, are reduced. The joint remains tight because of the friction between the surfaces of the two nuts, even when the axial force on the bolt is relieved.

Another common method to obtain positive locking of threads is to use a *castle nut* with a split pin. It is shown in Fig. 7.6. The castle nut consists of the usual hexagonal nut but with an addition of upper cylindrical part. This cylindrical portion has six slots located at the centre of each face of the hexagonal part. Due to these slots, castle nut is sometimes called a *slotted nut*. A split pin is passed through diametrically opposite slots in the castle nut and a hole in the bolt. This ensures positive locking unless the pin gets sheared. The split pin consists of a wire of semicircular cross-section. It is bent over in such a way that its flat sides are in contact. The split pin has a looped head on one side. The two ends at the other side can be separated and bent back over the cylindrical portion of the castle nut. A castle nut is extensively used for joints that are subjected to vibrations. They are popular in the automobile industry.

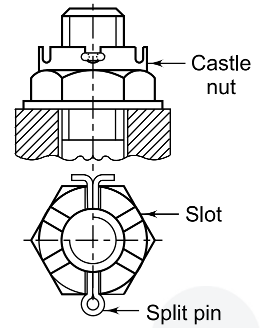

Fig. 7.6 *Castle Nut*

A split nut is shown in Fig. 7.7. It is similar to ordinary hexagonal nut but with a saw cut. A cap screw is provided to tighten or loosen the two parts of the nut separated by the saw cut. Initially, the nut is tightened on the screw and then the slot is opened by means of the cap screw. Opening of the slot results in deformation of nut and introduces additional friction at the threads. This prevents loosening of the nut.

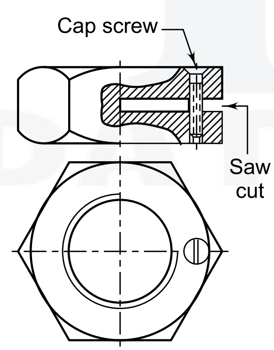

Fig. 7.7 *Split nut*

A positive locking arrangement with the help of setscrew is shown in Fig. 7.8. The elastic piece is made of soft material like copper or lead. It prevents damage to the threads when the setscrew is tightened in the nut. This arrangement results in additional friction between the threads of the bolt and the elastic piece and prevents loosening.

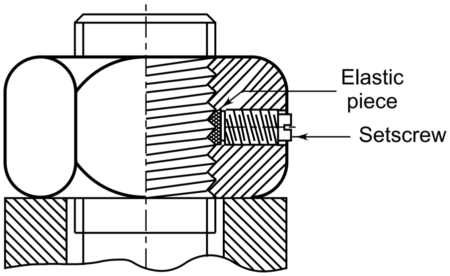

Fig. 7.8 *Locking with Setscrew*

Locking by spring washer is common method to prevent loosening of threads. The spring washer consists of a ring made of hardened steel with a cut at an angle of 15° with the ring axis as shown in Fig. 7.9(b). The ends of the washer at the cut are slightly separated and provided with sharp teeth. During tightening, the spring washer is compressed and its teeth bite into the contacting surface of the nut on one side and the base on the other. Thus, the nut gets fixed in the base. The principle of locking by means of spring washer is illustrated in Fig. 7.9(c). The action of the spring washer is more effective when the contacting surfaces are soft. The cut in the spring washer has '*left-hand*' inclination for right-hand threads and '*right-hand*' inclination for left-hand threads.

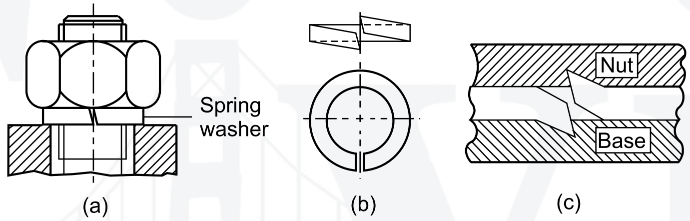

Fig. 7.9 *Locking with Spring Washer*

7.7 TERMINOLOGY OF SCREW THREADS

The right-hand threads are always used unless there is special reason for requiring left-hand thread. Unless and otherwise stated, specifications for threads imply right-hand threads. When the screw is vertical, the thread lines slope upward from left to right in case of right-hand threads. On the other hand, the thread lines slope downward from left to right in case of left-hand threads.

Various dimensions of external and internal threads are illustrated in Fig. 7.10. They are as follows:

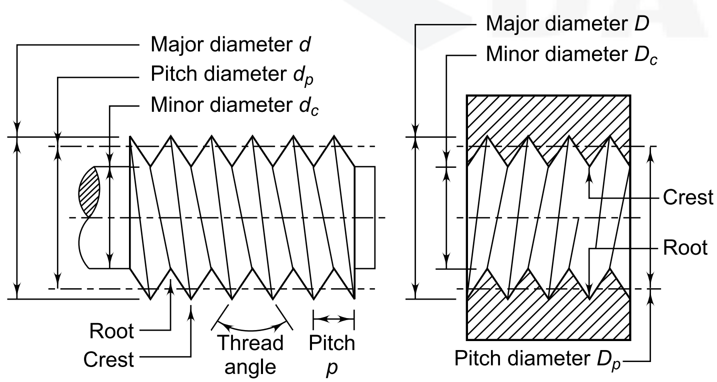

Fig. 7.10 *Terminology of Threads*

(i) Major Diameter The major diameter is the diameter of an imaginary cylinder that bounds the crest of an external thread (d) or the root of an internal thread (D). The major diameter is the largest diameter of the screw thread. It is also called the *nominal diameter* of the thread.

(ii) Minor Diameter The minor diameter is the diameter of an imaginary cylinder that bounds the roots of an external thread (d_c) or the crest of an internal thread (D_c). The minor diameter is the smallest diameter of the screw thread. It is also called *core* or *root diameter* of the thread.

(iii) Pitch Diameter The pitch diameter is the diameter of an imaginary cylinder, the surface of which would pass through the threads at such points as to make the width of the threads equal to the width of spaces cut by the surface of the cylinder. It is also called the *effective diameter* of the thread. Pitch diameter is denoted by d_p for external threads and D_p for internal threads.

(iv) Pitch Pitch is the distance between two similar points on adjacent threads measured parallel to the axis of the thread. It is denoted by the letter p.

(v) Lead Lead is the distance that the nut moves parallel to the axis of the screw, when the nut is given one turn.

(vi) Thread Angle Thread angle is the angle included between the sides of the thread measured in an axial plane. Thread angle is 60° for ISO metric threads.

(vii) Tensile Stress Area It has been observed during testing of the threaded rods that an unthreaded rod, having a diameter equal to the mean of the pitch diameter and the minor diameter [i.e., $(d_p + d_c)/2$] has the same tensile strength as the threaded rod. The cross-sectional area of this unthreaded rod is called the 'tensile-stress area'. This area is used for the purpose of calculating the tensile strength of the bolts.

7.8 ISO METRIC SCREW THREADS

Fastening threads are usually vee threads. They offer the following advantages:

(i) Vee threads result in higher friction, which lessen the possibility of loosening.

(ii) Vee threads have higher strength due to increased thread thickness at the core diameter.

(iii) Vee threads are more convenient to manufacture.

The profile of an ISO metric screw thread is illustrated in Fig. 7.11[2, 3]. It consists of an equilateral triangle with a thread angle of 60°. The base of this triangle is equal to the pitch. The

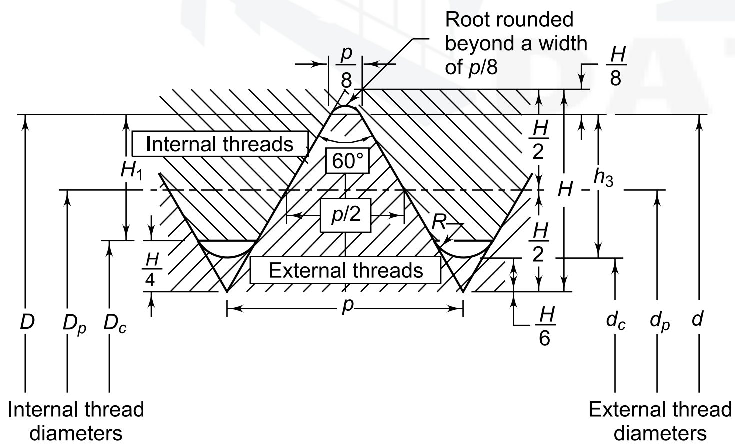

Fig. 7.11 *Profile of Internal and External Threads*

[2] IS 4218–1976: ISO Metric screw threads (in four parts)
[3] ISO 68–ISO General purpose screw threads—Basic profile

dimensions of the standard profile are given in Tables 7.1 and 7.2. It is observed from the tooth profile that the crests and the roots of the threads are either flattened or rounded to a circular arc. This is essential in fastening threads. It has the following advantages:

(i) It reduces the stress concentration in threads.
(ii) It increases the tool life of the thread cutting tool.
(iii) It reduces the possibility of damaging the threads by denting.
(iv) In fluid tight threads, it prevents leakage because there is engagement in the threads on crests and roots.

Metric threads are divided into coarse and fine series. The thread profiles in these two cases are generally similar. The coarse thread is considered as the basic series. Coarse threads offer the following advantages:

(i) The static load carrying capacity of coarse threads is higher.
(ii) Coarse threads are easier to cut than fine threads.
(iii) The errors in manufacturing and wear have less effect on the strength of coarse threads than that of fine threads.
(iv) Coarse threads are less likely to seize during tightening.
(v) Coarse threads have more even stress distribution.

Fine threads offer the following advantages:

(i) Fine threads have greater strength when subjected to fluctuating loads.
(ii) Fine threads have greater resistance to unscrewing as a result of lower helix angle. Therefore, threads with fine pitch are more dependable than threads with coarse pitch in respect of self-unscrewing.

Table 7.1 *Basic dimensions for ISO metric screw threads (coarse series)*

Designation	*Nominal or major dia d/D (mm)*	*Pitch (p) (mm)*	*Pitch diameter d_p/D_P (mm)*	*Minor diameter (mm)*		*Tensile stress area (mm^2)*
				d_c	D_c	
M 4	4	0.70	3.545	3.141	3.242	8.78
M 5	5	0.80	4.480	4.019	4.134	14.20
M 6	6	1.00	5.350	4.773	4.917	20.10
M 8	8	1.25	7.188	6.466	6.647	36.60
M 10	10	1.50	9.026	8.160	8.376	58.00
M 12	12	1.75	10.863	9.853	10.106	84.30
M 16	16	2.00	14.701	13.546	13.835	157
M 20	20	2.50	18.376	16.933	17.294	245
M 24	24	3.00	22.051	20.319	20.752	353
M 30	30	3.50	27.727	25.706	26.211	561
M 36	36	4.00	33.402	31.093	31.670	817
M 42	42	4.50	39.077	36.479	37.129	1120
M 48	48	5.00	44.752	41.866	42.587	1470
M 56	56	5.50	52.428	49.252	50.046	2030
M 64	64	6.00	60.103	56.639	57.505	2680
M 72	72	6.00	68.103	64.639	65.505	3460
M 80	80	6.00	76.103	72.639	73.505	4340
M 90	90	6.00	86.103	82.639	83.505	5590
M 100	100	6.00	96.103	92.639	93.505	7000

Coarse threads are recommended for general industrial applications, which are free from vibrations. Fine threads are used in the following applications:

(i) The parts subjected to dynamic loads and vibrations, e.g., automobile applications.

(ii) Hollow thin walled parts, where coarse threads are liable to weaken the wall considerably.

(iii) The parts in which the thread is used for the purpose of adjustment.

A screw thread of coarse series is designated by the letter 'M' followed by the value of the nominal diameter in mm. For example, M 12

A screw thread of fine series is specified by the letter 'M', followed by the values of the nominal diameter and the pitch in mm and separated by the symbol '×'. For example, M 12 × 1.25

Table 7.2 *Basic dimensions for ISO metric screw threads (fine series)*

Designation	Nominal or major dia d/D (mm)	Pitch (p) (mm)	Pitch diameter d_p/D_p (mm)	Minor diameter (mm) d_c	D_c	Tensile stress area (mm^2)
M 6 × 1	6	1.00	5.350	4.773	4.917	20.1
M 6 × 0.75	6	0.75	5.513	5.080	5.188	22.0
M 8 × 1.25	8	1.25	7.188	6.466	6.647	36.6
M 8 × 1	8	1.00	7.350	6.773	6.917	39.2
M 10 × 1.25	10	1.25	9.188	8.466	8.647	61.2
M 10 × 1	10	1.00	9.350	8.773	8.917	64.5
M 12 × 1.5	12	1.50	11.026	10.160	10.376	88.1
M 12 × 1.25	12	1.25	11.188	10.466	10.647	92.1
M 16 × 1.5	16	1.50	15.026	14.160	14.376	167
M 16 × 1	16	1.00	15.350	14.773	14.917	178
M 20 × 2	20	2.00	18.701	17.546	17.835	258
M 20 × 1.5	20	1.50	19.026	18.160	18.376	272
M 24 × 2	24	2.00	22.701	21.546	21.835	384
M 24 × 1.5	24	1.50	23.026	22.160	22.376	401
M 30 × 3	30	3.00	28.051	26.319	26.752	581
M 30 × 2	30	2.00	28.701	27.546	27.835	621
M 36 × 3	36	3.00	34.051	32.319	32.752	865
M 36 × 2	36	2.00	34.701	33.546	33.835	915
M 42 × 4	42	4.00	39.402	37.093	37.670	1150
M 42 × 3	42	3.00	40.051	38.319	38.752	1210
M 48 × 4	48	4.00	45.402	43.093	43.670	1540
M 48 × 3	48	3.00	46.051	44.319	44.752	1600

7.9 MATERIALS AND MANUFACTURE

Lightly loaded small bolts, studs and nuts are made of free cutting steels. High strength bolts often fail in fatigue. They are made of plain carbon steels like 40C8 or 45C8 or alloy steels like 35Mn6Mo3, 40Cr4Mo2, 40Ni14 or 40Ni10Cr3Mo6. Stainless steel is used for threaded fastener where corrosion resistance is required.

The head of the bolt or screw is made by the upsetting process. This is done on automatic forging machines, which give finished shape with practically no scrap. The head is cold formed for diameters up to 20 mm. For larger diameters, hot forming is employed.

There are two methods for making threads, viz., *thread cutting* and *thread rolling*. Thread cutting is done on automatic machines called '*screw*' machines. In the thread rolling method, threads are formed by rolling the bar stock between dies which depress part of the material to form the root of the thread and which force the remaining material up the top to form the crest of threads. Therefore, the outside diameter of the thread is more than the bar stock on which it was rolled. Thread rolling is a superior method of making threads. The advantages of thread rolling over thread cutting are as follows:

(i) Cold forming induces residual compressive stresses on the thread surface, which improve fatigue strength of the bolt.
(ii) Cold forming creates radii at the root and the crest and reduces stress concentration.
(iii) Thread cutting results in cutting the fibre lines of the original bar stock. In thread rolling, the fibre lines are rearranged to suit the thread shape.
(iv) Compared with cut threads, rolled threads have less waste as no material is removed.

All these factors contribute to increased popularity of rolled threads as compared to cut threads.

7.10 BOLTED JOINT—SIMPLE ANALYSIS

A bolted joint subjected to tensile force P is shown in Fig. 7.12. The cross-section at the core diameter d_c is the weakest section. The maximum tensile stress in the bolt at this cross-section is given by,

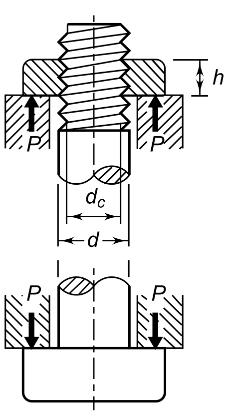

Fig. 7.12 *Bolt in Tension*

$$\sigma_t = \frac{P}{\left(\frac{\pi}{4} d_c^2\right)} \tag{7.2}$$

The height of the nut h can be determined by equating the strength of the bolt in tension with the strength in shear. The procedure is based on the following assumptions:

(i) Each turn of the thread in contact with the nut supports an equal amount of load.
(ii) There is no stress concentration in the threads.
(iii) The yield strength in shear is equal to half of the yield strength in tension ($S_{sy} = 0.5 S_{yt}$).
(iv) Failure occurs in the threads of the bolt and not in the threads of the nut.

Substituting,

$$\sigma_t = \frac{S_{yt}}{(fs)}$$

in Eq. (7.2), the strength of the bolt in tension is given by,

$$P = \frac{\pi}{4} d_c^2 \left(\frac{S_{yt}}{fs}\right) \tag{a}$$

The threads of the bolt in contact with the nut are sheared at the core diameter d_c. The shear area is equal to $(\pi d_c h)$, where h is the height of the nut. The strength of the bolt in shear is given by,

$$P = (\pi d_c h)\left(\frac{S_{sy}}{fs}\right)$$

$$= (\pi d_c h)\left(\frac{S_{yt}}{2 fs}\right) \tag{b}$$

Equating (a) and (b),

$$h = 0.5 d_c$$

Assuming ($d_c = 0.8d$),

$$h = 0.4d$$

Therefore, for standard coarse threads, the threads are equally strong in failure by shear and failure by tension, if the height of the nut is approximately 0.4 times of the nominal diameter of the bolt. The height of the standard hexagonal nut is $(0.8d)$. Hence, the threads of the bolt in the standard nut will not fail by shear.

Rewriting the height of the standard nut,

$$h = 0.8d \tag{7.3}$$

The design of the bolt consists of determination of correct size of the bolt. The size of the bolt is given by the nominal diameter d and pitch p. In design calculations, many times the core diameter d_c is determined. Therefore, it is necessary to convert the core diameter d_c into the nominal diameter d. This can be easily done when the tables like (7.1) and (7.2) are available. Knowing the minor or core diameter, the corresponding designation of the thread can be obtained from these tables. However, when the tables for threads are not available, some relationship between d_c and d has to be used. The correct relationship for ISO metric screw threads is as follows[4],

$$d_c = d - 1.226\,87p$$

Since there are two unknowns on the right hand side, it is not possible to find out the value of d by knowing the value of d_c. Therefore, the following approximate relationship can be used,

$$d_c = 0.8d \tag{7.4}$$

Preference should be given to use values given in Tables 7.1 and 7.2.

Example 7.1 *An electric motor weighing 10 kN is lifted by means of an eye bolt as shown in Fig. 7.13. The eye bolt is screwed into the frame of the motor. The eye bolt has coarse threads. It is made of plain carbon steel 30C8 (S_{yt} = 400 N/mm^2) and the factor of safety is 6. Determine the size of the bolt.*

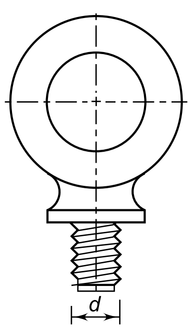

Fig. 7.13 *Eye Bolt*

Solution

Given $P = 10$ kN $\quad S_{yt} = 400$ N/mm^2 $\quad (fs) = 6$

Eye bolt is used for lifting and transporting heavy machinery on the shop floor. It consists of a ring of circular cross-section at the top end and threaded portion at the lower end. The threaded portion is screwed inside a threaded hole on the top surface of the machine to be lifted. A crane hook or chain is inserted in the circular ring. The circular ring is called the eye. The threaded portion of the eye bolt is subjected to tensile stress due to the weight being lifted.

Step I *Permissible tensile stress*

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{400}{6} = 66.67 \text{ N/mm}^2$$

Step II *Size of bolt*

From Eq. (7.2),

$$\sigma_t = \frac{P}{\frac{\pi}{4} d_c^2} \qquad \therefore\ 66.67 = \frac{(10 \times 10^3)}{\frac{\pi}{4} d_c^2}$$

or $\quad d_c = 13.82$ mm

From Eq. (7.4),

$$d = \frac{d_c}{0.8} = \frac{13.82}{0.8} = 17.27 \text{ or } 18 \text{ mm}$$

From Table 7.1, the standard size of the bolt is M20.

Example 7.2 *Two plates are fastened by means of two bolts as shown in Fig. 7.14. The bolts are made of plain carbon steel 30C8 (S_{yt} = 400 N/mm^2) and the factor of safety is 5. Determine the size of the bolts if,*

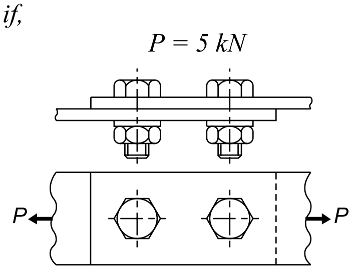

Fig. 7.14

[4] IS 4218 (Part 1)—1976: ISO Metric screw threads—Part 1: Basic and design profile.

Solution

Given $P = 5$ kN $S_{yt} = 400$ N/mm^2 $(fs) = 5$

Step I *Permissible shear stress*

$$S_{sy} = 0.5\, S_{yt} = 0.5\,(400) = 200 \text{ N/mm}^2$$

$$\tau = \frac{S_{sy}}{(fs)} = \frac{200}{5} = 40 \text{ N/mm}^2$$

Step II *Size of bolt*

The shank portion of the bolts is subjected to direct shear stress. Let the diameter of the shank be denoted by d.

$$\text{Shear area of 2 bolts} = 2\left(\frac{\pi}{4}d^2\right) \text{ mm}^2$$

Therefore,

$$P = 2\left(\frac{\pi}{4}d^2\right)\tau$$

$$5 \times 10^3 = 2\left(\frac{\pi}{4}d^2\right)(40)$$

or $\quad d = 8.92$ or 9 mm

From Table 7.1, the standard size of the bolt is M10.

7.11 ECCENTRICALLY LOADED BOLTED JOINTS IN SHEAR

In structural connections, a group of bolts is frequently employed, as shown in Fig. 7.15. Let $A_1, A_2, \ldots, A_5$ be the cross-sectional areas of the bolts. The co-ordinates $(x_1, y_1), (x_2, y_2), \ldots, (x_5, y_5)$ indicate the position of bolt-centres with respect to

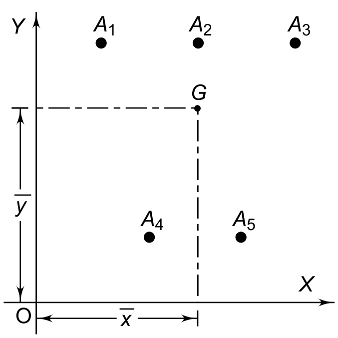

Fig. 7.15

the origin. G is the centre of gravity of the group of bolts. The co-ordinates $(\bar{x}, \bar{y})$ indicate the location of the centre of gravity.

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + \ldots + A_5x_5}{A_1 + A_2 + \ldots + A_5}$$

$$\bar{x} = \frac{\sum A_i x_i}{\sum A_i} \tag{7.5}$$

Similarly,

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i} \tag{7.6}$$

Quite often, the centre of gravity is located by symmetry.

An eccentrically loaded bolted connection is shown in Fig. 7.16. The eccentricity of the external force P is e from the centre of gravity. This

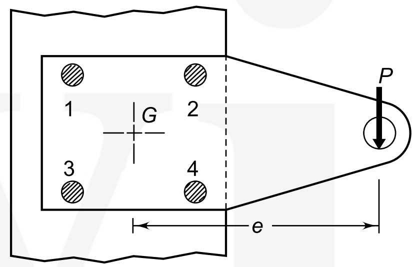

Fig. 7.16

eccentric force can be considered as equivalent to an imaginary force P at the centre of gravity and a moment $(P \times e)$ about the same point. This is illustrated in Fig. 7.17. The imaginary force P at the centre of gravity results in primary shear forces $P_1', P_2', \ldots$, etc., given by the following equation,

$$P_1' = P_2' = P_3' = P_4' = \frac{P}{(\text{No. of bolts})} \tag{7.7}$$

The moment $(P \times e)$ about the centre of gravity results in secondary shear forces $P_1'', P_2'', \ldots$, etc. If $r_1, r_2, \ldots$, etc., are the radial distances of the bolt-centres from the centre of gravity, then

$$P \times e = P_1''r_1 + P_2''r_2 + P_3''r_3 + P_4''r_4 \tag{a}$$

It is assumed that the secondary shear force at any bolt is proportional to its distance from the centre of gravity. Therefore,

$$\left.\begin{aligned} P_1'' &= Cr_1 \\ P_2'' &= Cr_2 \\ P_3'' &= Cr_3 \\ P_4'' &= Cr_4 \end{aligned}\right\} \tag{b}$$

Substituting (b) in (a), we get

$$C = \frac{Pe}{(r_1^2 + r_2^2 + r_3^2 + r_4^2)} \qquad \text{(c)}$$

From (b) and (c),

$$P_1'' = \frac{Pe r_1}{(r_1^2 + r_2^2 + r_3^2 + r_4^2)}$$

$$P_2'' = \frac{Pe r_2}{(r_1^2 + r_2^2 + r_3^2 + r_4^2)} \qquad (7.8)$$

and so on.

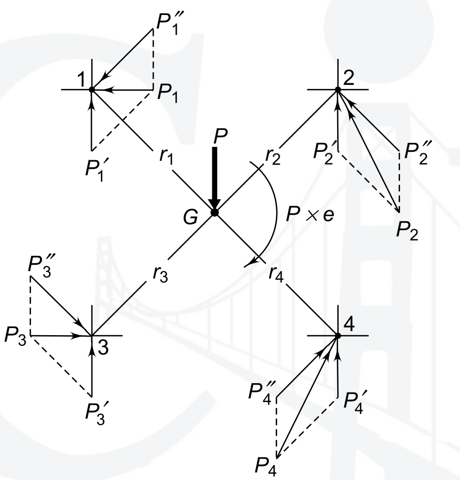

Fig. 7.17 *Primary and Secondary Shear Forces*

The primary and secondary shear forces are added by vector addition method to get the resultant shear forces P_1, P_2, P_3, and P_4. In this analysis, it is assumed that the components connected by the bolts are rigid and the bolts have the same cross-sectional area.

Example 7.3 *The structural connection shown in Fig. 7.16 is subjected to an eccentric force P of 10 kN with an eccentricity of 500 mm from the CG of the bolts. The centre distance between bolts 1 and 2 is 200 mm, and the centre distance between bolts 1 and 3 is 150 mm. All the bolts are identical. The bolts are made from plain carbon steel 30C8 (S_{yt} = 400 N/mm²) and the factor of safety is 2.5. Determine the size of the bolts.*

Solution

Given $P = 10$ kN $\quad S_{yt} = 400$ N/mm² $\quad (fs) = 2.5$
$e = 500$ mm

Step I *Permissible shear stress*

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.5 S_{yt}}{(fs)} = \frac{0.5(400)}{2.5} = 80 \text{ N/mm}^2$$

Step II *Primary and secondary shear forces*

By symmetry, the centre of gravity G is located at a distance of 100 mm to the right of bolts 1 and 3 and 75 mm below bolts 1 and 2. Thus,

$$r_1 = r_2 = r_3 = r_4 = r$$

and $\quad r = \sqrt{(100)^2 + (75)^2} = 125$ mm

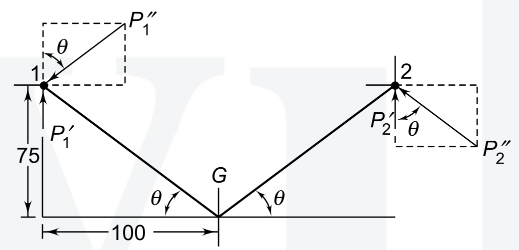

Fig. 7.18 *Vector Addition of Shear Forces*

The primary and secondary shear forces are shown in Fig. 7.18. Thus,

$$P_1' = P_2' = P_3' = P_4' = \frac{10\,000}{4} = 2500 \text{ N} \qquad \text{(i)}$$

$$P_1'' = \frac{(Pe)r_1}{(r_1^2 + r_2^2 + r_3^2 + r_4^2)} = \frac{(Pe)r}{4r^2}$$

$$= \frac{Pe}{4r} = \frac{(10\,000)(500)}{4(125)} = 10\,000 \text{ N}$$

Similarly, it can be proved that

$$P_1'' = P_2'' = P_3'' = P_4'' = 10\,000 \text{ N}$$

Step III *Resultant shear force*

Referring to Fig. 7.18,

$$\tan\theta = \frac{75}{100} = 0.75 \quad \text{or} \quad \theta = 36.87°$$

$$P_1 = \sqrt{(P_1''\cos\theta - P_1')^2 + (P_1''\sin\theta)^2}$$

$$= \sqrt{[10\,000\cos(36.87) - 2500]^2 + [10\,000\sin(36.87)]^2}$$

$$= 8139.41 \text{ N}$$

$$P_2 = \sqrt{(P_2'' \cos\theta + P_2')^2 + (P_2'' \sin\theta)^2}$$

$$= \sqrt{[10\,000 \cos(36.87) + 2500]^2 + [10\,000 \sin(36.87)]^2}$$

$$= 12093.38 \text{ N}$$

Bolts 2 and 4 are subjected to maximum shear forces.

Step IV *Size of bolts*

$$\tau = \frac{P_2}{A} \qquad 80 = \frac{12093.38}{\frac{\pi}{4} d_c^2}$$

$\therefore \quad d_c = 13.87$ mm

From Eq. (7.4)

$$d = \frac{d_c}{0.8} = \frac{13.87}{0.8} = 17.34 \text{ or } 18 \text{ mm}$$

From Table 7.1, the standard size of the bolts is M 20.

Example 7.4 *A steel plate subjected to a force of 5 kN and fixed to a channel by means of three identical bolts is shown in Fig. 7.19(a). The bolts are made from plain carbon steel 45C8 (S_{yt} = 380 N/mm²) and the factor of safety is 3. Specify the size of bolts.*

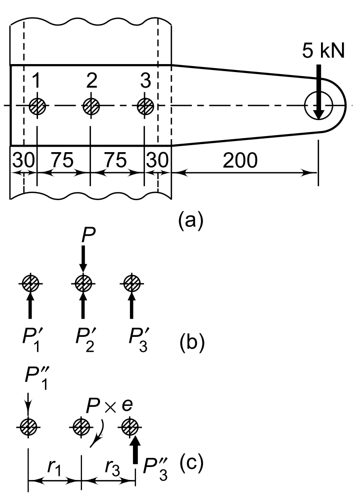

Fig. 7.19

Solution

Given $P = 5$ kN $\quad S_{yt} = 380$ N/mm² $\quad (fs) = 3$

Step I *Permissible shear stress*

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.5\ S_{yt}}{(fs)} = \frac{0.5\ (380)}{3} = 63.33 \text{ N/mm}^2$$

Step II *Primary and secondary shear forces*

The centre of gravity of three bolts will be at the centre of bolt-2.

The primary and secondary shear forces are shown in Fig. 7.19(b) and (c).

$$P_1' = P_2' = P_3' = \frac{P}{3} = \frac{5000}{3} = 1\,666.67 \text{ N}$$

$$P_1'' = P_3'' = \frac{(Pe)(r_1)}{(r_1^2 + r_3^2)} = \frac{(5000 \times 305)(75)}{(75^2 + 75^2)}$$

$$= 10\,166.67 \text{ N}$$

Step III *Resultant shear force*

The resultant shear force on the bolt 3 is maximum.

$$P_3 = P_3' + P_3'' = 1666.67 + 10\,166.67 = 11\,833.34 \text{ N}$$

Step IV *Size of bolts*

$$\tau = \frac{P_3}{A} \quad \text{or} \quad 63.33 = \frac{11\,833.34}{\frac{\pi}{4} d_c^2}$$

$\therefore \quad d_c = 15.42$ mm

From Eq. (7.4),

$$d = \frac{d_c}{0.8} = \frac{15.42}{0.8} = 19.28 \quad \text{or} \quad 20 \text{ mm}$$

The standard size of the bolts is M20.

7.12 ECCENTRIC LOAD PERPENDICULAR TO AXIS OF BOLT

A bracket, fixed to the steel structure by means of four bolts, is shown in Fig. 7.20(a). It is subjected to eccentric force P at a distance e from the structure. The force P is perpendicular to the axis of each bolt. The lower two bolts are denoted by 2, while the upper two bolts by 1. In this analysis, the following assumptions are made:

(i) The bracket and the steel structure are rigid.

(ii) The bolts are fitted in reamed and ground holes.

(iii) The bolts are not preloaded and there are no tensile stresses due to initial tightening.

(iv) The stress concentration in threads is neglected.

(v) All bolts are identical.

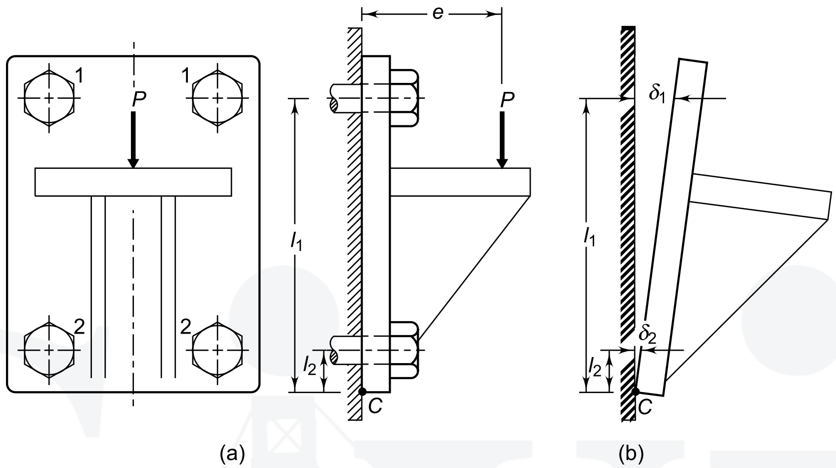

Fig. 7.20

The force P results in direct shear force on the bolts. Since the bolts are identical, the shear force on each bolt is given by,

$$P_1' = P_2' = \frac{P}{(\text{No. of bolts})} \tag{7.9}$$

The moment ($P \times e$) tends to tilt the bracket about the edge C. As shown in Fig. 7.20(b), each bolt is stretched by an amount (δ) which is proportional to its vertical distance from the point C. Or,

$$\delta_1 \propto l_1 \quad \text{and} \quad \delta_2 \propto l_2$$

Also,

force ∝ stress because $[P = \sigma A]$

stress ∝ strain because $[\sigma = E\epsilon]$

strain ∝ stretch because $[\epsilon = \delta/l]$

Therefore, it can be concluded that the resisting force induced in any bolt, due to the tendency of the bracket to tilt under the moment ($P \times e$), is proportional to its distance from the tilting edge. If P_1'', P_2'' are the resisting forces induced in the bolts,

$$P_1'' \propto l_1 \quad \text{and} \quad P_2'' \propto l_2$$

or,

$$P_1'' = Cl_1$$

$$P_2'' = Cl_2 \tag{a}$$

where C is the constant of proportionality. Equating the moment due to resisting forces with the moment due to external force P about the edge C,

$$Pe = 2P_1'' l_1 + 2P_2'' l_2 \tag{b}$$

Substituting (a) in (b),

$$Pe = 2(C\, l_1)\, l_1 + 2(C\, l_2)\, l_2$$

$$\therefore \quad C = \frac{Pe}{2(l_1^2 + l_2^2)} \tag{c}$$

From (a) and (c),

$$P_1'' = \frac{Pel_1}{2(l_1^2 + l_2^2)}$$

$$P_2'' = \frac{Pel_2}{2(l_1^2 + l_2^2)} \tag{7.10}$$

The bolts denoted by 1 are subjected to maximum force. In general, a bolt, which is located at the farthest distance from the tilting edge C, is subjected to maximum force.

Equations (7.9) and (7.10) give shear and tensile forces that act on the bolt due to eccentric load perpendicular to the axis of the bolts. The direct shear stress in the bolt is given by,

$$\tau = \frac{P_1'}{A} \quad (7.11)$$

The tensile stress in the bolt is given by,

$$\sigma_t = \frac{P_1''}{A} \quad (7.12)$$

where A is the cross-sectional area of the bolt at the minor or core diameter.

The bolts can be designed on the basis of principal stress theory or principal shear stress theory.

The principal stress σ_1 is given by,

$$\sigma_1 = \frac{\sigma_t}{2} + \sqrt{\left(\frac{\sigma_t}{2}\right)^2 + \tau^2} \quad (7.13)$$

The principal shear stress is given by,

$$\tau_{max.} = \sqrt{\left(\frac{\sigma_t}{2}\right)^2 + \tau^2} \quad (7.14)$$

Using the following relationships,

$$\sigma_1 = \frac{S_{yt}}{(fs)} \quad \text{and} \quad \tau_{max.} = \frac{S_{sy}}{(fs)} = \frac{0.5S_{yt}}{(fs)}$$

the cross-sectional area of the bolts and their size can be determined. The bolt material is usually ductile. Therefore, it is appropriate to use the maximum shear stress theory of failure.

Example 7.5 *The following data is given for the bracket illustrated in Fig. 7.20(a).*

P = 25 kN e = 100 mm

l_1 = 150 mm l_2 = 25 mm

There is no pre-load in the bolts. The bolts are made of plain carbon steel 45C8 (S_{yt} = 380 N/mm²) and the factor of safety is 2.5. Using the maximum shear stress theory, specify the size of the bolts.

Solution

Given P = 25 kN S_{yt} = 380 N/mm²

(fs) = 2.5 e = 100 mm

Step I *Permissible shear stress*

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.5S_{yt}}{(fs)} = \frac{0.5(380)}{2.5} = 76 \text{ N/mm}^2$$

Step II *Direct shear stress in bolt*

$$P_1' = P_2' = \frac{P}{(\text{No. of bolts})} = \frac{25 \times 10^3}{4} = 6250 \text{ N}$$

The direct shear stress is given by,

$$\tau = \left(\frac{6250}{A}\right) \text{N/mm}^2 \quad \text{(a)}$$

where A is area at core cross-section.

Step III *Tensile stress in bolt*

From Eq. (7.10),

$$P_1'' = \frac{Pel_1}{2(l_1^2 + l_2^2)} = \frac{(25 \times 10^3)(100)(150)}{2[150^2 + 25^2]} = 8108.11 \text{ N}$$

The bolts 1 are subjected to maximum forces. The tensile stress in these bolts is given by,

$$\sigma_t = \frac{P_1''}{A} = \frac{8108.11}{A} \text{ N/mm}^2 \quad \text{(b)}$$

Step IV *Principal shear stress in bolt*

$$\tau_{max.} = \sqrt{\left(\frac{\sigma_t}{2}\right)^2 + (\tau)^2}$$

$$= \sqrt{\left(\frac{8108.11}{2A}\right)^2 + \left(\frac{6250}{A}\right)^2}$$

$$= \left(\frac{7449.69}{A}\right) \text{N/mm}^2$$

Step V *Size of bolts*

Equating the value of ($\tau_{max.}$) to permissible shear stress,

$$\left(\frac{7449.69}{A}\right) = 76 \quad \text{or} \quad A = 98.02 \text{ mm}^2$$

From Table 7.1, the standard size of bolts with coarse threads is M16 (A = 157 mm²).

Example 7.6 *A wall bracket is attached to the wall by means of four identical bolts, two at A and two at B, as shown in Fig. 7.21. Assuming that the bracket is held against the wall and prevented*

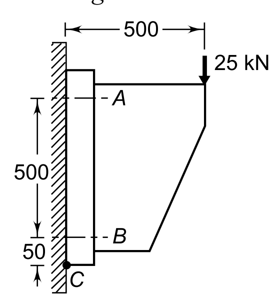

Fig. 7.21

from tipping about the point C by all four bolts and using an allowable tensile stress in the bolts as 35 N/mm², determine the size of the bolts on the basis of maximum principal stress theory.

Solution

Given $P = 25$ kN $e = 500$ mm

$(\sigma_t)_{max.} = 35$ N/mm²

Step I *Direct shear stress in bolt*

Two bolts at A are denoted by 1 and two bolts at B by 2. The direct shear force on each bolt is given by,

$$P_1' = P_2' = \frac{P}{(\text{No. of bolts})}$$

$$\therefore \quad P_1' = P_2' = \frac{25 \times 10^3}{4} = 6250 \text{ N}$$

The direct shear stress in each bolt is given by,

$$\tau = \frac{6250}{A} \text{ N/mm}^2 \qquad \text{(i)}$$

Step II *Tensile stress in bolt*

Since the tendency of the bracket is to tilt about the edge C, the bolts at A denoted by 1, are at the farthest distance from C. Therefore, bolts at A are subjected to maximum tensile force. From Eq. (7.10),

$$P_1'' = \frac{Pel_1}{2(l_1^2 + l_2^2)} = \frac{(25 \times 10^3)(500)(550)}{2(550^2 + 50^2)}$$
$$= 11\,270.49 \text{ N}$$

The tensile stress in bolts at A is given by,

$$\sigma_t = \frac{11270.49}{A} \qquad \text{(ii)}$$

Step III *Principal stress in bolt*

From Eq. (7.13), the principal stress σ_1 in the bolts is given by,

$$\sigma_1 = \frac{\sigma_t}{2} + \sqrt{\left(\frac{\sigma_t}{2}\right)^2 + (\tau)^2}$$

Substituting (i) and (ii) in the above expression,

$$\sigma_1 = \left(\frac{11270.49}{2A}\right) + \sqrt{\left(\frac{11270.49}{2A}\right)^2 + \left(\frac{6250}{A}\right)^2}$$
$$= \left(\frac{14050.62}{A}\right) \text{N/mm}^2$$

Step IV *Size of bolts*

The permissible tensile stress for the bolt is 35 N/mm². Therefore,

$$\frac{14050.62}{A} = 35 \qquad \therefore A = 401.45 \text{ mm}^2$$

From Table 7.1, the standard size of the bolts is M30 ($A = 561$ mm²).

Example 7.7 *A bracket is fastened to the steel structure by means of six identical bolts as shown in Fig. 7.22(a). Assume the following data:*

$l_1 = 300$ mm $l_2 = 200$ mm $l_3 = 100$ mm

$l = 250$ mm $P = 50$ kN

Neglecting shear stress, determine the size of the bolts, if the maximum permissible tensile stress in any bolt is limited to 100 N/mm².

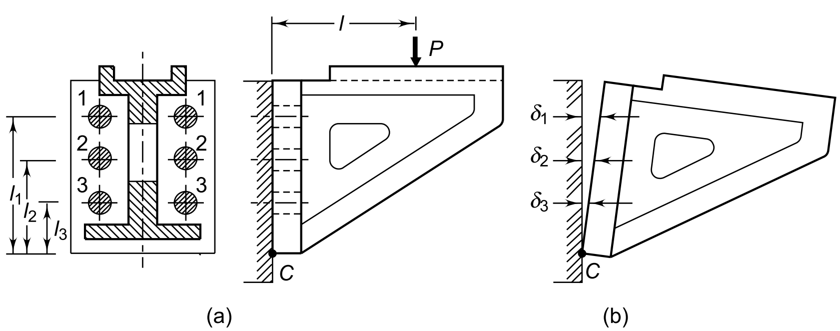

Fig. 7.22

Solution

Given $P = 50$ kN $l = 250$ mm
$(\sigma_1)_{max.} = 100$ N/mm^2

Step I *Maximum tensile force*

The force P tends to tilt the bracket about edge C. Each bolt is stretched by an amount (δ), which is proportional to its distance from the tilting edge as shown in Fig. 7.22 (b).

$\delta_1 \propto l_1$ $\delta_2 \propto l_2$ $\delta_3 \propto l_3$

Also,

force ∝ stress because ($P = \sigma A$)
stress ∝ strain because ($\sigma = E\epsilon$)
strain ∝ stretch because ($\epsilon = \delta/l$)

Therefore, it can be concluded that the force induced in any bolt due to the tendency of the bracket to tilt about the edge C is proportional to its distance from the tilting edge. Therefore,

$$P_1 = Cl_1 \qquad P_2 = Cl_2 \qquad P_3 = Cl_3 \qquad \text{(i)}$$

where C is the constant of proportionality. The bolts, denoted by 1, are subjected to maximum tensile force because of the farthest distance from the tilting edge. Equating the moment of resisting forces to the moment due to external force about C,

$$Pl = 2P_1 l_1 + 2P_2 l_2 + 2P_3 l_3 \qquad \text{(ii)}$$

Substituting (i) in (ii),

$$Pl = 2C\,(l_1^2 + l_2^2 + l_3^2)$$

$$C = \frac{Pl}{2(l_1^2 + l_2^2 + l_3^2)} \qquad \text{(iii)}$$

From (i) and (iii), $P_1 = \dfrac{P\,l\,l_1}{2\,(l_1^2 + l_2^2 + l_3^2)}$

Substituting numerical values,

$$P_1 = \frac{(50 \times 10^3)\ (250)\ (300)}{2\ (300^2 + 200^2 + 100^2)} = 13\,392.86 \text{ N}$$

The shear stress is to be neglected.

Step II *Size of bolts*

$$P_1 = A\,(\sigma_1)_{max.} \quad \text{or} \quad 13\,392.86 = A\,(100)$$

$\therefore$ $A = 133.93$ mm^2

From Table 7.1, the standard size of the bolts is $M16$ ($A = 157$ mm^2).

Example 7.8 *A crane-runway bracket is fastened to the roof truss by means of two identical bolts as shown in Fig. 7.23. Determine the size of the bolts, if the permissible tensile stress in the bolts is limited to 75 N/mm^2.*

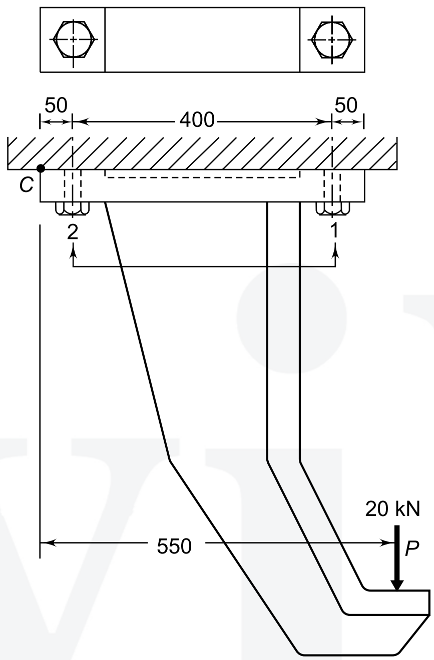

Fig. 7.23

Solution

Given $P = 20$ kN $e = 550$ mm
$(\sigma_t)_{max.} = 75$ N/mm^2

The bolts are subjected to tensile stress on account of the following two factors:

(i) direct tensile stress because the force P is parallel to the axis of the bolts; and
(ii) tensile stress due to the tendency of the bracket to tilt about the edge C due to eccentricity of the force P.

Step I *Direct tensile force*

$$P_1 = P_2 = \frac{P}{(\text{No. of bolts})} = \frac{20 \times 10^3}{2} = 10\,000 \text{ N} \qquad \text{(i)}$$

Step II *Tensile force due to the tendency of bracket to tilt*

The moment ($P \times 550$) tends to tilt the bracket about the edge C. Suppose P_1'' and P_2'' are resisting forces set up in bolts 1 and 2 respectively. As we know,

$$P_1'' \propto l_1 \quad \text{and} \quad P_2'' \propto l_2$$

where l_1 and l_2 are the distances of the axis of bolts from the edge C. Therefore,

$$P_1'' = C\,l_1 \quad \text{and} \quad P_2'' = Cl_2 \qquad \text{(a)}$$

where C is the constant of proportionality. Equating the moment of resisting forces with the moment due to external force about the edge C,

$$P \times 550 = P_1'' l_1 + P_2'' l_2 \qquad \text{(b)}$$

From (a) and (b),

$$P \times 550 = C\,(l_1^2 + l_2^2)$$

$$\therefore \qquad C = \frac{P \times 550}{(l_1^2 + l_2^2)} \qquad \text{(c)}$$

From (a) and (c),

$$P_1'' = \frac{(P \times 550) l_1}{(l_1^2 + l_2^2)}$$

Substituting numerical values,

$$P_1'' = \frac{(20 \times 10^3)(550)(450)}{(450^2 + 50^2)} = 24\ 146.34 \text{ N} \qquad \text{(ii)}$$

Step III *Resultant tensile force*

Bolt 1 is located at the farthest distance from the tilting edge C. Therefore, it is subjected to maximum tensile force. From (i) and (ii), the total tensile force acting on the bolt 1 is (10 000 + 24 146.34) or 34 146.34 N.

Size IV *Size of bolts*

$$A(\sigma_t)_{\text{max.}} = 34146.34 \quad \text{or} \quad A(75) = 34146.34$$

$$\therefore \qquad A = 455.28 \text{ mm}^2$$

From Table 7.1, the standard size of the bolts is M 30 ($A = 561$ mm^2).

Example 7.9 *A cast iron bracket fixed to the steel structure is shown in Fig. 7.24(a). It supports a load P of 25 kN. There are two bolts at A and two bolts at B. The distances are as follows,*

$l_1 = 50$ mm $\quad l_2 = 200$ mm $\quad l = 400$ mm

Determine the size of the bolts, if maximum permissible tensile stress in the bolt is 50 N/mm^2.

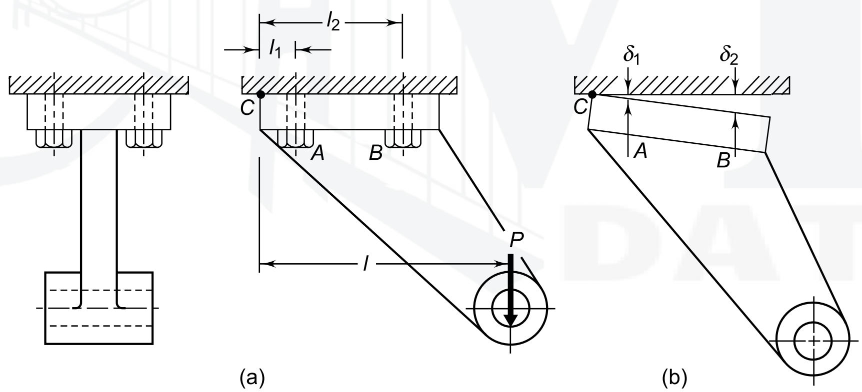

Fig. 7.24

Solution

Given $P = 25$ kN $\quad l = 400$ mm

$(\sigma_t)_{\text{max.}} = 50$ N/mm^2

The bolts are subjected to following stresses:

(i) Direct tensile stress due to load P.

(ii) Tensile stress due to tendency of the bracket to tilt in clockwise direction about the edge C.

Step I *Direct tensile force*

Since the bolts are identical, the direct tensile force on each bolt is given by,

$$P_1' = P_2' = \frac{P}{(\text{No. of bolts})}$$

$$\therefore \qquad P_1' = P_2' = \frac{25 \times 10^3}{4} = 6250 \text{ N} \qquad \text{(i)}$$

Step II *Tensile force due to tendency of bracket to tilt*

The following assumptions are made:

(i) All bolts are identical.

(ii) The bracket and the structure are rigid.

(iii) The bolts are not preloaded and there is no initial tensile stress due to tightening of the bolt.

(iv) As shown in Fig. 7.24(b), when the load tends to tilt the bracket about the edge C, each bolt is stretched by an amount (δ), which is proportional to its distance from the tilting edge. Or,

$$\delta_1 \propto l_1 \qquad \delta_2 \propto l_2$$

Also,

force $\propto$ stress because ($P = \sigma A$)
stress $\propto$ strain because ($\sigma = E \in$)
strain $\propto$ stretch because ($\in = \delta / l$)

Therefore, it can be concluded that the resisting force acting on the bolt due to the tendency of bracket to tilt, is proportional to its distance from the tilting edge. The bolts at A are denoted by 1 and bolts at B by 2.

Suppose C is the load in the bolt per unit distance from the tilting edge, due to the tilting effect of the bracket. Then forces acting on the bolts are given by,

$$P_1'' = C\,l_1 \qquad P_2'' = Cl_2 \qquad \text{(a)}$$

Equating the moments of these resisting forces about the tilting edge to the moment due to the external force,

$$Pl = 2P_1'' l_1 + 2P_2'' l_2 = 2(Cl_1)\,l_1 + 2(Cl_2)\,l_2$$

$$\therefore \qquad C = \frac{Pl}{2(l_1^2 + l_2^2)} \qquad \text{(b)}$$

The maximum force will act on bolts denoted by 2. From (a) and (b),

$$P_2'' = \frac{Pll_2}{2(l_1^2 + l_2^2)}$$

Substituting numerical values,

$$P_2'' = \frac{(25 \times 10^3)(400)(200)}{2(50^2 + 200^2)} = 23\,529.41 \text{ N} \qquad \text{(ii)}$$

Step III *Resultant tensile force*

Adding (i) and (ii), the total tensile force on the bolts denoted by 2, is given by,

$$P_2 = P_2' + P_2'' = 6250 + 23\,529.41 = 29\,779.41 \text{ N}$$

Step IV *Size of bolts*

$$P_2 = A(\sigma_t)_{max.} \qquad 29\,779.41 = A\,(50)$$

$$\therefore \quad A = 595.59 \text{ mm}^2$$

From Table 7.1, the standard size of the bolts is M36 (A = 817 mm^2).

Example 7.10 *A cast iron bracket, supporting the transmission shaft and the belt pulley, is fixed to the steel structure by means of four bolts as shown in Fig. 7.25(a). There are two bolts at A and two bolts at B. The tensions in slack and tight sides of the belt are 5 kN and 10 kN respectively. The belt tensions act in a vertically downward direction. The distances are as follows,*

$$l_1 = 50 \text{ mm} \qquad l_2 = 150 \text{ mm} \qquad l = 200 \text{ mm}$$

The maximum permissible tensile stress in any bolt is 60 N/mm^2. Determine the size of the bolts.

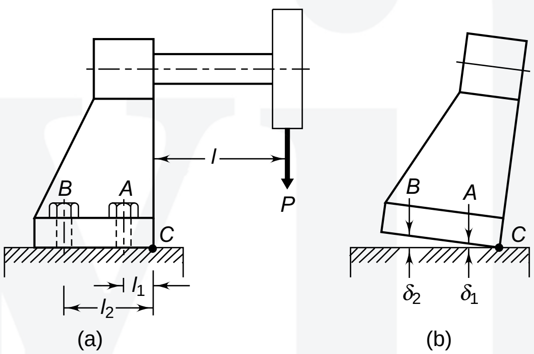

Fig. 7.25

Solution

Given $P = (5 + 10)$ kN $\quad l = 200$ mm
$(\sigma_t)_{max.} = 60$ N/mm^2

Step I *Tensile stress due to the tendency of bracket to tilt*

The bolts are subjected to tensile stress due to the tendency of the bracket to tilt in clockwise direction about the edge C. The following assumptions are made:

(i) All bolts are identical.
(ii) The bracket and structure are rigid.
(iii) The bolts are not preloaded and there is no tensile stress due to initial tightening.
(iv) The effect of stress concentration in the threads is neglected.

As shown in Fig. 7.25(b), when the load tends to tilt the bracket about the edge C, each bolt is stretched by an amount (δ), which is proportional to its distance from the tilting edge. Or,

$$\delta_1 \propto l_1 \qquad \delta_2 \propto l_2$$

Also,

force ∝ stress because $(P = \sigma A)$
stress ∝ strain because $(\sigma = E \in)$
strain ∝ stretch because $(\in = \delta/l)$

Therefore, it can be concluded that the resisting force acting on the bolt due to the tendency of the bracket to tilt, is proportional to its distance from the tilting edge.

The bolts at A are denoted by 1 and the bolts at B by 2. Suppose C is the load in the bolt per unit distance from the tilting edge. Then, forces acting on the bolts are given by,

$$P_1 = Cl_1 \quad P_2 = Cl_2 \tag{a}$$

Equating the moment of these resisting forces about the tilting edge to the moment due to the external force,

$$Pl = 2P_1\, l_1 + 2P_2\, l_2 = 2\,(Cl_1)\, l_1 + 2\,(Cl_2)\, l_2$$

$$\therefore \qquad C = \frac{Pl}{2(l_1^2 + l_2^2)} \tag{b}$$

The maximum force will act on bolts denoted by 2. From (a) and (b),

$$P_2 = \frac{Pl\, l_2}{2(l_1^2 + l_2^2)}$$

Substituting numerical values,

$$P_2 = \frac{(15\times10^3)\ (200)\ (150)}{2\,(50^2 + 150^2)} = 9000 \text{ N}$$

Step II *Size of bolts*

$$P_2 = A\,(\sigma_t)_{max.} \qquad 9000 = A\,(60)$$

$$\therefore \qquad A = 150 \text{ mm}^2$$

From Table 7.1, the standard size of the bolts is M16 ($A = 157$ mm^2).

7.13 ECCENTRIC LOAD ON CIRCULAR BASE

Many times, a machine component is made with a circular base, which is fastened to the structure by means of bolts located on the circumference of a circle. Flanged bearings of the machine tools and the structure of the pillar crane are the examples of this type of loading. A round flange-bearing fastened by means of four bolts is shown in Fig. 7.26(b). It is subjected to an external force P at a distance l from the support. The following assumptions are made:

(i) All bolts are identical.
(ii) The bearing and the structure are rigid.
(iii) The bolts are not preloaded and there is no tensile stress due to initial tightening.
(iv) The stress concentration in the threads is neglected.
(v) The bolts are relieved of shear stresses by using dowel pins.

As shown in Fig. 7.26(a), when the load tends to tilt the bearing about the point C, each bolt is stretched by an amount (δ), which is proportional to its vertical distance from the point C. Or,

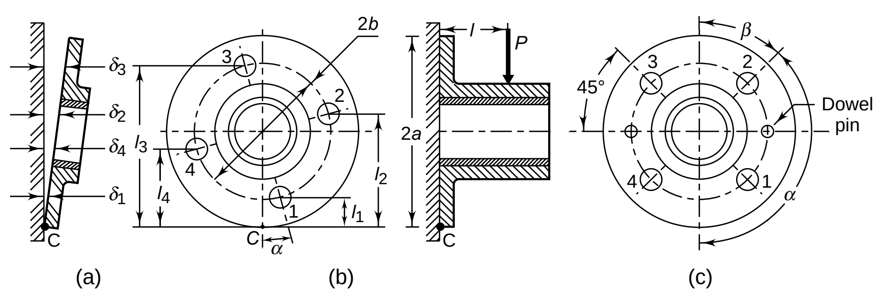

Fig. 7.26

$$\delta_1 \propto l_1 \quad \delta_2 \propto l_2 \quad \text{and so on. Also,}$$

force ∝ stress because $(P = \sigma A)$
stress ∝ strain because $(\sigma = \text{E} \in)$
strain ∝ stretch because $(\in = \delta/l)$

Therefore, it can be concluded that the resisting force acting on any bolt due to the tendency of the bearing to tilt, is proportional to its distance from the tilting edge.

If P_1 P_2 … are the resisting forces induced in the bolts,

$$P_1 \propto l_1$$

or,

$$P_1 = Cl_1 \quad P_2 = Cl_2 \quad P_3 = Cl_3 \quad P_4 = Cl_4 \qquad \text{(a)}$$

where C is the constant of proportionality. Equating the moment due to the external force P about C with the moments due to resisting forces,

$$Pl = P_1 l_1 + P_2 l_2 + P_3 l_3 + P_4 l_4 \qquad \text{(b)}$$

From (a) and (b),

$$Pl = C\,(l_1^2 + l_2^2 + l_3^2 + l_4^2)$$

$$C = \frac{Pl}{(l_1^2 + l_2^2 + l_3^2 + l_4^2)} \qquad \text{(c)}$$

From, (a) and (c), the force acting on the bolt 1 is given by,

$$P_1 = \frac{Pl\,l_1}{(l_1^2 + l_2^2 + l_3^2 + l_4^2)} \qquad \text{(d)}$$

Suppose,

a = radius of the flange

b = radius of pitch circle of the bolts.

From Fig. 7.26(b),

$$l_1 = a - b\cos\alpha$$

$$l_2 = a + b\sin\alpha$$

$$l_3 = a + b\cos\alpha$$

$$l_4 = a - b\sin\alpha$$

Therefore,

$$(l_1^2 + l_2^2 + l_3^2 + l_4^2) = 4a^2 + 2b^2 \qquad \text{(e)}$$

From (d) and (e),

$$P_1 = \frac{Pl(a - b\cos\alpha)}{2(2a^2 + b^2)} = \frac{2Pl(a - b\cos\alpha)}{4(2a^2 + b^2)} \qquad \text{(f)}$$

Four bolts are considered in the above analysis. If the procedure is repeated for n equally spaced bolts, we get the general expression in the following form:

$$P_1 = \frac{2\,Pl(a - b\cos\alpha)}{n(2a^2 + b^2)} \qquad (7.15)$$

The force P_1 has maximum value when the term (cos α) has minimum value. The minimum value of (cos α) is (–1), when (α = 180°). With reference to Fig. 7.26 (b), the bolt 1 will occupy the topmost position, at the farthest distance form C, when α = 180°, Substituting α = 180° in Eq. (7.15),

$$P_{\text{max.}} = \frac{2Pl(a + b)}{n(2a^2 + b^2)} \qquad (7.16)$$

Equation (7.16) gives absolute maximum value of the force acting on any of the bolts. It should be used for finding out the size of the bolts, when the direction of the external force P can change with respect to the bolts, as in case of the base of a vertical pillar crane. When the direction of the external force P is fixed and known, the maximum load on the bolts can be reduced, so that the two of them can be equally stressed as shown in Fig. 7.26(c). In this particular case, the number of bolts is 4 and the angle α made by the centre line of bolt 2 is 135°. For a general case with n as number of bolts,

$$\alpha = 180 - \beta \qquad \text{(i)}$$

$$\beta = \frac{1}{2}\,\frac{360}{n} \qquad \text{(ii)}$$

$$\therefore \qquad \alpha = 180 - \frac{360}{2n} \qquad (7.17)$$

Therefore,

$$\cos\alpha = \cos\left[180 - \frac{360}{2n}\right] = -\cos\left(\frac{360}{2n}\right)$$

$$= -\cos\left(\frac{180}{n}\right)$$

Substituting the above value of (cos α) in Eq. (7.15),

$$P_1 = \frac{2Pl\left[a + b\cos\left(\frac{180}{n}\right)\right]}{n(2a^2 + b^2)} \qquad (7.18)$$

The above equation is applicable only when two bolts are equally stressed. This condition can be satisfied under the following circumstances:

(i) the direction of the external force P is fixed with respect to the bolts;

(ii) the number of bolts is even; and

(iii) two bolts at the top are symmetrically spaced with angle β on either side of the vertical line.

The bolts are usually relieved of shear stresses by using dowel pins as shown in Fig. 7.26(c).

Example 7.11 *A round flange bearing, as shown in Fig. 7.26(b), is fastened to the machine frame by means of four cap screws spaced equally on a 300 mm pitch circle diameter. The diameter of the flange is 400 mm. The external force P is 25 kN, which is located at a distance of 150 mm from the machine frame. There are two dowel pins to take shear load. The cap screws are relieved of all shear force. Determine the size of the cap screws, if the maximum permissible tensile stress in the cap screw is limited to 50 N/mm².*

Solution

Given $2a = 400$ mm $\quad 2b = 300$ mm $\quad P = 25$ kN
$l = 150$ mm $\quad (\sigma_t)_{max.} = 50$ N/mm^2

Step I *Maximum force on cap screw*

It is assumed that the direction of the external force P is fixed and cap screws are located in such a way that two of them are equally stressed. From Eq. (7.18),

$$P_1 = \frac{2\,P\,l\left[a + b\cos\left(\frac{180}{n}\right)\right]}{n\,(2a^2 + b^2)}$$

$$= \frac{2\,(25 \times 10^3)(150)\left[200 + 150\cos\left(\frac{180}{4}\right)\right]}{4\,(2 \times 200^2 + 150^2)}$$

$$= 5598.77 \text{ N}$$

Step II *Size of cap screw*

The cap screws are subjected to only tensile stress. Therefore,

$$\frac{\pi}{4} d_c^2 (\sigma_t)_{max.} = P_1 \qquad \frac{\pi}{4} d_c^2 (50) = 5598.77$$

$$\therefore \quad d_c = 11.94 \text{ mm}$$

From Eq. (7.4),

$$d = \frac{d_c}{0.8} = \frac{11.94}{0.8} = 14.93 \text{ or } 15 \text{ mm}$$

Example 7.12 *A pillar crane, shown in Fig. 7.27, is fastened to the foundation by means of 16 identical bolts spaced equally on 2 m pitch circle diameter. The diameter of the pillar flange is 2.25 m. Determine the size of the bolts if a load of 50 kN acts at a radius of 7.5 m from the axis of the crane. The maximum permissible tensile stress in the bolt is limited to 75 N/mm².*

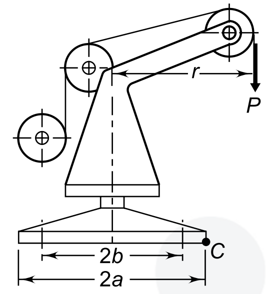

Fig. 7.27 *Pillar Crane*

Solution

Given $2a = 2250$ mm $\quad 2b = 2000$ mm $\quad P = 50$ kN
$r = 7500$ mm $\quad (\sigma_t)_{max.} = 75$ N/mm^2

Step I *Absolute maximum force on bolt*

As shown in Fig. 7.27, the tendency of the pillar crane is to tilt about the point C under the action of the eccentric load P. Therefore, the moment arm l for load P about the point C is $(r–a)$. Or,

$$l = r - a = 7500 - 1125 = 6375 \text{ mm}$$

The pillar crane can rotate about its axis and the location of P will change with respect to the bolts. Therefore, the bolts are to be designed for absolute maximum value of force.

From Eq. (7.16),

$$P_{max.} = \frac{2\,Pl\,(a+b)}{n\,(2\,a^2 + b^2)}$$

$$= \frac{2\,(50 \times 10^3)(6375)(1125 + 1000)}{16\,(2 \times 1125^2 + 1000^2)}$$

$$= 23\,976.77 \text{ N}$$

Step II *Size of cap screw*

The bolts are subjected to tensile stress. Therefore,

$$A(\sigma_t)_{max.} = P_{max.} \qquad A(75) = 23976.77$$

$$\therefore \quad A = 319.69 \text{ mm}^2$$

From Table 7.1, the standard size of the bolts is M24 ($A = 353$ mm^2).

Example 7.13 *Figure 7.28 shows the bracket used in a jib crane to connect the tie rod. The maximum force in the tie rod is 5 kN, which is inclined at an angle of 30° with the horizontal. The bracket is fastened by means of four identical bolts, two at A and two at B. The bolts are made of plain carbon steel 30C8 (S_{yt} = 400 N/mm²) and the factor of safety is 5. Assume maximum shear stress theory and determine the size of the bolts.*

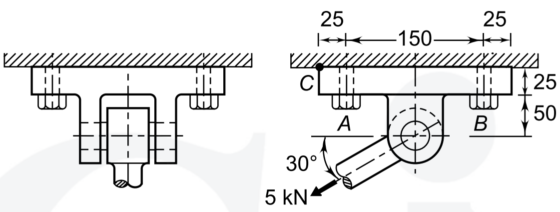

Fig. 7.28

Solution

Given $P = 5$ kN $\quad S_{yt} = 400$ N/mm² $\quad (fs) = 5$

Step I *Permissible shear stress*

$$\tau_{max.} = \frac{S_{sy}}{(fs)} = \frac{0.5S_{yt}}{(fs)} = \frac{0.5(400)}{5} = 40 \text{ N/mm}^2$$

Step II *Tensile and shear stresses in bolts*

The axial force in the tie rod (P) is resolved into vertical and horizontal components.

Vertical component $= P \sin\theta = 5000 \sin(30) = 2500$ N

Horizontal component $= P\cos\theta = 5000\cos(30) = 4330.13$ N

The vertical component produces direct tensile force on each bolt. It is given by,

$$P_1 = P_2 = \frac{2500}{(\text{No. of bolts})} = \frac{2500}{4} = 625 \text{ N} \qquad \text{(i)}$$

The horizontal component produces direct shear force on each bolt. It is given by,

$$P_1' = P_2' = \frac{P}{(\text{No. of bolts})} = \frac{4330.13}{4} = 1082.53 \text{ N} \qquad \text{(ii)}$$

In addition, the horizontal component produces tensile force on each bolt due to its moment about the tilting edge C. The bolts at A are denoted by 2 and the bolts at B by 1. The bolts at B are located at the farthest distance from the tilting edge C. From Eq. (7.10),

$$P_1'' = \frac{Pel_1}{2(l_1^2 + l_2^2)}$$

$$= \frac{(4330.13)(50+25)(175)}{2(25^2 + 175^2)}$$

$$= 909.33 \text{ N} \qquad \text{(iii)}$$

From (i) and (iii),

$$\sigma_t = \frac{(P_1 + P_1'')}{A} = \frac{625 + 909.33}{A} = \frac{1534.33}{A} \text{ N/mm}^2$$

From (ii),

$$\tau = \frac{P_1'}{A} = \frac{1082.53}{A} \text{ N/mm}^2$$

Step III *Maximum shear stress in bolts*

From Eq. (7.14),

$$\tau_{max.} = \sqrt{\left(\frac{\sigma_t}{2}\right)^2 + (\tau)^2}$$

$$= \sqrt{\left(\frac{1534.33}{2A}\right)^2 + \left(\frac{1082.53}{A}\right)^2}$$

$$= \left(\frac{1326.81}{A}\right) \text{N/mm}^2$$

Step IV *Size of bolts*

$$\tau_{max.} = \frac{1326.81}{A} \quad \text{or} \quad 40 = \frac{1326.81}{A}$$

$\therefore \qquad A = 33.17$ mm²

From Table 7.1, the standard size of the bolts is M8 ($A = 36.6$ mm²).

Example 7.14 *A bracket, subjected to a force of 5 kN inclined at an angle of 60° with the vertical, is shown in Fig. 7.29. The bracket is fastened by means of four identical bolts to the structure. The bolts are made of plain carbon steel 30C8 (S_{yt} = 400 N/mm²) and the factor of safety is 5 based on maximum shear stress. Assume maximum shear stress theory and determine the size of the bolts.*

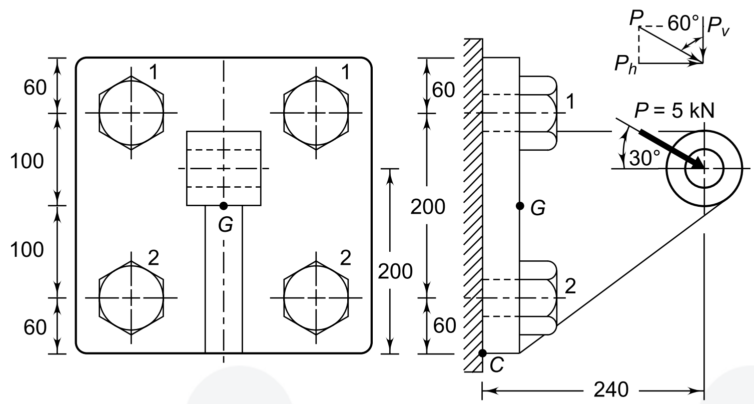

Fig. 7.29

Solution

Given $P = 5$ kN $S_{yt} = 400$ N/mm^2 $(fs) = 5$

Step I *Permissible shear stress*

$$\tau_{max.} = \frac{S_{sy}}{(fs)} = \frac{0.5\,S_{yt}}{(fs)} = \frac{0.5\,(400)}{5} = 40 \text{ N/mm}^2$$

Step II *Tensile and shear stresses in bolts*

The force P is resolved into horizontal component P_h and vertical component P_v. These components are given by,

$P_h = P \sin 60° = 5000 \sin 60° = 4330.13$ N

$P_v = P \cos 60° = 5000 \cos 60° = 2500$ N

Effect of P_h

The horizontal component P_h is parallel to the axis of the bolts. It has following two effects:

(i) a direct tensile force on each bolt; and

(ii) a turning moment about the centre of gravity of four bolts (G) in clockwise direction.

The direct tensile force on each bolt is given by,

$$P_1 = P_2 = \frac{P_h}{(\text{No. of bolts})} = \frac{4330.13}{4}$$
$$= 1082.53 \text{ N} \qquad \text{(i)}$$

As shown in Fig. 7.29, the centre of gravity of four bolts is at a distance (60 + 100) or 160 mm from the lower edge. The point of application of the force P_h is at a distance of 200 mm from the lower edge. Therefore, the eccentricity of the force P_h is (200–160) or 40 mm about the centre of gravity. The turning moment (M_h) due to horizontal component is given by,

$$M_h = P_h \times 40 = 4330.13 \times 40$$
$$= 173\,205.2 \text{ N-mm} \qquad \text{(ii)}$$

The moment M_h acts in the clockwise direction.

Effect of P_v

The vertical component P_v is perpendicular to the axis of the bolts. It has following two effects:

(i) a direct shear force on each bolt; and

(ii) a turning moment about the tilting edge C in the clockwise direction.

The direct shear force on each bolt is given by,

$$P_s = \frac{P_v}{(\text{No. of bolts})} = \frac{2500}{4} = 625 \text{ N} \qquad \text{(iii)}$$

The distance of the vertical component P_v from the tilting edge C is 240 mm. The turning moment (M_v) due to the vertical component is given by,

$$M_v = P_v \times 240 = 2500 \times 240$$
$$= 600\,000 \text{ N-mm} \qquad \text{(iv)}$$

The moment M_v acts in the clockwise direction.

It is observed that turning moments due to horizontal and vertical components are clockwise and the resultant turning moment is obtained by their addition. The resisting forces set up in bolts 1 and 2, are proportional to their distances from the tilting edge C. Two bolts, denoted by 1, are at the farthest distance from the edge C. Therefore, the tensile force due to the tendency of the bracket to tilt about the edge C is maximum for two bolts denoted by 1.

From Eq. (7.10),

$$P_1'' = \frac{(Pe)\,l_1}{2\,(l_1^2 + l_2^2)} = \frac{(M_h + M_v)\,l_1}{2(l_1^2 + l_2^2)}$$

$$= \frac{(173\,205.2 + 600\,0\,00)\,(260)}{2\,(260^2 + 60^2)}$$

$$= 1411.75 \text{ N} \qquad \text{(v)}$$

From (i) and (v), the tensile force acting on each of the two bolts, denoted by 1, is given by,

$$P_t = P_1 + P_1'' = 1082.53 + 1411.75$$

$$= 2494.28 \text{ N} \qquad \text{(vi)}$$

The tensile stress in the bolt is given by,

$$\sigma_t = \frac{P_t}{A} = \frac{2494.28}{A} \text{ N/mm}^2 \qquad \text{(a)}$$

From (iii), the shear stress in the bolt is given by,

$$\tau = \frac{P_s}{A} = \frac{625}{A} \text{ N/mm}^2 \qquad \text{(b)}$$

Step III *Maximum shear stress in bolts*

From Eq. (7.14),

$$t_{\text{max.}} = \sqrt{\left(\frac{\sigma_t}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{2494.28}{2A}\right)^2 + \left(\frac{625}{A}\right)^2}$$

$$= \left(\frac{1394.99}{A}\right) \text{N/mm}^2$$

Step IV *Size of bolts*

$$\tau_{\text{max.}} = \frac{1394.99}{A} \quad \text{or} \quad 40 = \frac{1394.99}{A}$$

$$\therefore \qquad A = 34.87 \text{ mm}^2$$

From Table 7.1, the standard size of the bolts is M 8 ($A = 36.6$ mm^2).

Example 7.15 *A rigid bracket subjected to a vertical force of 10 kN is shown in Fig. 7.30(a). It is fastened to a vertical stanchion by means of four identical bolts. Determine the size of the bolts by maximum shear stress theory. The maximum permissible shear stress in any bolt is limited to 50 N/mm^2.*

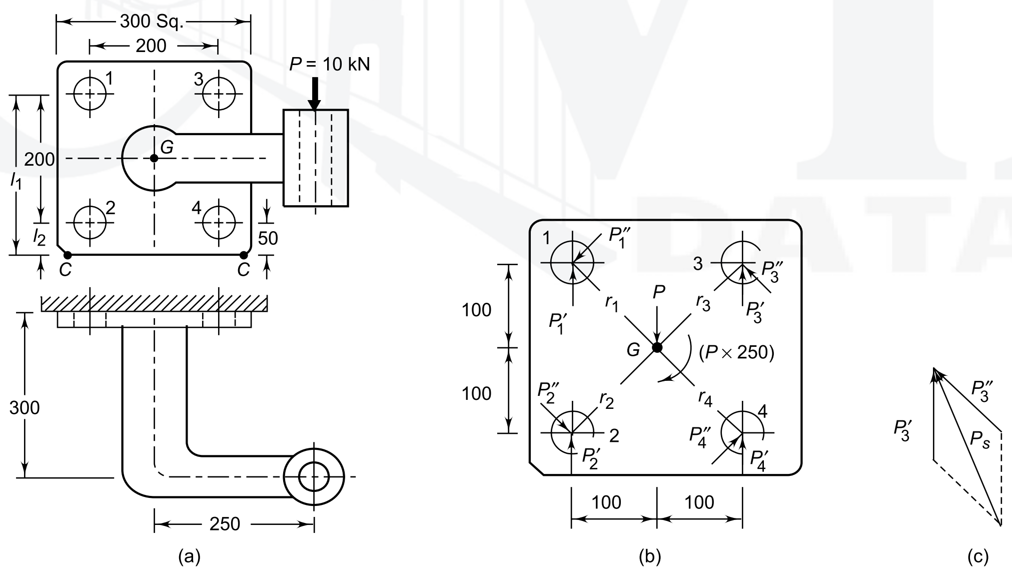

Fig. 7.30

Solution

Given $P = 10$ kN $\qquad \tau_{\text{max.}} = 50$ N/mm^2

Step I *Tensile and shear stresses in bolts*

The bolts are subjected to primary and secondary shear stresses and tensile stress due to the tendency of the bracket to tilt about the edge-CC.

As shown in Fig. 7.30(b), the external force P acting at a distance of 250 mm from the centre of gravity (G) of the bolts can be considered as equivalent to an imaginary force P acting at G and the moment ($P \times 250$) about the same point. The imaginary force P at the centre of gravity G, results in primary shear forces P_1', P_2', P_3' and P_4'. From Eq. (7.7),

$$P_1' = P_2' = P_3' = P_4' = \frac{P}{(\text{No. of bolts})}$$

$$= \frac{10 \times 10^3}{4} = 2500 \text{ N} \quad \text{(i)}$$

The moment ($P \times 250$) about the centre of gravity results in secondary shear forces P_1'', P_2'', P_3'' and P_4''. By symmetry,

$$r_1 = r_2 = r_3 = r_4 = \sqrt{100^2 + 100^2} = 141.42 \text{ mm}$$

The resultant shear force is obtained by addition of primary and secondary shear forces by vector addition method. It is observed from Fig. 7.30(b) that the resultant shear force is maximum for bolts 3 and 4. From Eq. (7.8),

$$P_3'' = \frac{Per_3}{(r_1^2 + r_2^2 + r_3^2 + r_4^2)} = \frac{(P \times 250)\, r}{4\, r^2}$$

$$= \frac{P \times 250}{4\, r} = \frac{(10 \times 10^3) \times 250}{4(141.42)}$$

$$= 4419.46 \text{ N} \quad \text{(ii)}$$

From Fig. 7.30(c), the resultant shear force on the bolt 3 is given by,

$$P_s = \sqrt{(P_3' + P_3'' \sin 45)^2 + (P_3'' \cos 45)^2}$$

$$= \sqrt{(2500 + 4419.46 \sin 45)^2 + (4419.46 \cos 45)^2}$$

$$= 6434.81 \text{ N} \quad \text{(a)}$$

The moment ($P \times 300$) tends to tilt the bracket about the edge-CC. Bolts located at 1 and 3 are at the farthest distance from the edge-CC. Therefore, resisting forces set up in bolts 1 and 3 are maximum. From Eq. (7.10),

$$P_3 = \frac{Pe\, l_3}{2\,(l_1^2 + l_2^2)} = \frac{(10 \times 10^3)(300)(250)}{2\,(250^2 + 50^2)}$$

$$= 5769.23 \text{ N} \quad \text{(b)}$$

From (a) and (b),

$$\sigma_t = \frac{P_3}{A} = \frac{5769.23}{A} \text{ N/mm}^2$$

$$\tau = \frac{P_s}{A} = \frac{6434.81}{A} \text{ N/mm}^2$$

Step II *Maximum shear stress in bolts*

From Eq. (7.14),

$$\tau_{\text{max.}} = \sqrt{\left(\frac{\sigma_t}{2}\right)^2 + \tau^2}$$

$$= \sqrt{\left(\frac{5769.23}{2A}\right)^2 + \left(\frac{6434.81}{A}\right)^2}$$

$$= \frac{7051.79}{A} \text{ N/mm}^2$$

Step III *Size of bolts*

The permissible shear stress in any bolt is 50 N/mm^2.

$$\therefore \quad 50 = \frac{7051.79}{A} \qquad A = 141.04 \text{ mm}^2$$

From Table 7.1, the standard size of the bolts is M16 (A = 157 mm^2).

7.14 TORQUE REQUIREMENT FOR BOLT TIGHTENING

A bolted assembly is tightened by applying force to the wrench handle and rotating the hexagonal nut. In certain applications, as in case of the gasketed joint between the cylinder and the cylinder head of the engine, the bolts are tightened with a specific magnitude of pre-load P_i. It is necessary to determine the magnitude of the torque which will induce this pre-tension. The torque required to tighten the bolt consists of the following two factors:

(i) torque required to overcome thread friction and induce the pre-load, i.e., (M_t); and
(ii) torque required to overcome collar friction between the nut and the washer $(M_t)_c$.

The equations derived for trapezoidal threads are suitably modified for ISO metric screw threads. Replacing W by pretension P_i in Eq. (6.13) (Section 6.10), the torque required to overcome thread friction is given by,

$$M_t = \frac{P_i d_m}{2} \times \frac{(\mu \sec\theta + \tan\alpha)}{(1 - \mu \sec\theta \tan\alpha)} \tag{a}$$

For ISO metric screw threads,

$$\theta = 30° \qquad \alpha \cong 2.5° \qquad d_m \cong 0.9d$$

where d is the nominal or major diameter of the bolt. The coefficient of friction varies from 0.12 to 0.20, depending upon the surface finish and accuracy of the thread profile and lubrication. Assuming,

$$\mu = 0.15$$

and substituting the above values in Eq. (a),

$$M_t = \left(\frac{P_i (0.9)d}{2}\right) \frac{[0.15 \sec(30) + \tan(2.5)]}{[1 - 0.15 \sec(30) \tan(2.5)]}$$

or
$$M_t = 0.098\, P_i\, d \tag{b}$$

According to uniform wear theory, the collar friction torque $(M_t)_c$ is given by,

$$(M_t)_c = \left(\frac{\mu P_i}{2}\right)\left[\frac{D_o + D_i}{2}\right] \tag{c}$$

In the above equation, D_o is the diameter of an imaginary circle across the flats of the hexagonal nut and D_i is the diameter of the hole in the washer. For ISO metric screw threads,

$$\left(\frac{D_o + D_i}{2}\right) \cong 1.4d \quad \text{and} \quad \mu = 0.15$$

Substituting these values in Eq. (c),

$$(M_t)_c = \left(\frac{0.15 P_i}{2}\right)(1.4d)$$

$$(M_t)_c = 0.105\, P_i\, d \tag{d}$$

Adding expressions (b) and (d), the total torque $(M_t)_t$ required to tighten the bolts is given by,

$$(M_t)_t = M_t + (M_t)_c = (0.098 + 0.105)\, P_i\, d$$

or

$$(M_t)_t \cong 0.2\, P_i\, d \tag{7.19}$$

The above equation gives a simple expression to determine the wrench torque $(M_t)_t$ required to create the required pre-load P_i.

7.15 DIMENSIONS OF FASTENERS

Fasteners such as hexagonal head bolts, screws, washers and nuts are frequently used in all design projects. Their standard dimensions are essential to prepare assembly and detail drawings. However, in previous examples the bolts are designed and their standard size such M16 or M20 is specified. Other dimensions of the fasteners are determined by referring to the particular Indian Standard for that fastener. Tables 7.3 to 7.5 give dimensions of commonly used hexagonal head bolts and screws, hexagonal nuts and washers[5, 6]. The dimensions of other fasteners can be similarly obtained from relevant standards or handbooks[7].

It is observed from these tables that the inner diameter of the washer is slightly more than the nut size.

[5] IS 1363–1984: Specification for hexagon head bolts, screws and nuts of Product Grade C. (In three parts).
[6] IS 2016–1967: Specification for plain washers.
[7] SP-29: Handbook on Industrial Fasteners (In three volumes)—Bureau of Indian Standards.

Table 7.3 *Dimensions of hexagonal head bolts and screws*

Thread size, d		M5	M6	M8	M10	M12	M16	M20	M24	M30	M36
Pitch p		0.8	1	1.25	1.5	1.75	2	2.5	3	3.5	4
ds	Max.	5.48	6.48	8.58	10.58	12.7	16.7	20.84	24.84	30.84	37
	Min.	4.52	5.52	7.42	9.42	11.3	15.3	19.16	23.16	29.16	35
e	Min.	8.63	10.89	14.20	17.59	19.85	26.17	32.95	39.55	50.85	60.79
k	Nom.	3.5	4	5.3	6.4	7.5	10	12.5	15	18.7	22.5
	Min.	3.12	3.62	4.92	5.95	7.05	9.25	11.6	14.1	17.65	21.45
	Max.	3.88	4.38	5.68	6.85	7.95	10.75	13.4	15.9	19.75	23.55
r	Min.	0.2	0.25	0.4	0.4	0.6	0.6	0.8	0.8	1	1
s	Max.	8	10	13	16	18	24	30	36	46	55
	Min.	7.64	9.64	12.57	15.57	17.57	23.16	29.16	35	45	53.8

N.B.: (1) All dimensions in millimetres; (2) Refer to Fig. T-7.3.

Fig. *T-7.3*

Table 7.4 *Dimensions of hexagonal nut*

Thread size, d		M5	M6	M8	M10	M12	M16	M20	M24	M30	M36
Pitch p		0.8	1	1.25	1.5	1.75	2	2.5	3	3.5	4
dw	Min.	6.9	8.7	11.5	14.5	16.5	22	27.7	33.2	42.7	51.1
m	Max.	5.6	6.1	7.9	9.5	12.2	15.9	18.7	22.3	26.4	31.5
	Min.	4.4	4.9	6.4	8	10.4	14.1	16.6	20.2	24.3	29.4
s	Max.	8	10	13	16	18	24	30	36	46	55
	Min.	7.64	9.64	12.57	15.57	17.57	23.16	29.16	35	45	53.8
e	Min.	8.63	10.89	14.20	17.59	19.85	26.17	32.95	39.55	50.85	60.79

N.B.: (1) All dimensions in millimetres; (2) Refer to Fig. T-7.4.

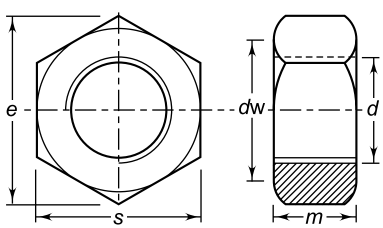

Fig. *T-7.4*

Table 7.5 *Dimensions of punched washer (A-Type) for hexagonal bolts and screws*

Size, d	D	s	For Bolt or Screw Size
5.5	10	1	M5
6.6	12.5	1.6	M6
9	17	1.6	M8
11	21	2	M10
14	24	2.5	M12
18	30	3.15	M16

(*Contd*)

Table 7.5 *(Contd)*

22	37	3.15	M20
26	44	4	M24
33	56	4	M30
39	66	5	M36
45	78	6	M42
52	92	8	M48

N.B.: (1) All dimensions in millimetres; (2) Refer to Fig. T-7.5.

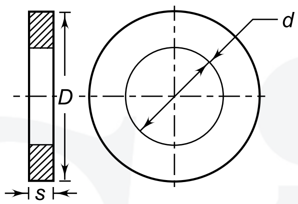

Fig. *T-7.5*

7.16 DESIGN OF TURNBUCKLE

Step I Problem Specification It is required to design a turnbuckle for connecting the tie rods in the roof truss. The maximum pull in the tie rods is 50 kN.

Step II Construction The construction of the turnbuckle is shown in Fig. 7.31. It consists of a central portion called *coupler* and two rods. One rod has right-hand threads while the other rod has left-hand threads. The threaded portions of the rods are screwed to the coupler at the two ends. As the central coupler rotates, the rods are either pulled together or pushed apart depending upon the direction of the rotation of the coupler. The outer portion of the coupler is given hexagonal shape so that it can be rotated by means of a spanner.

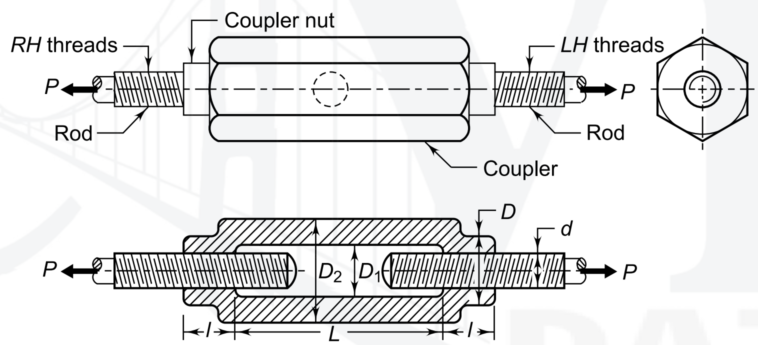

Fig. 7.31 *Turnbuckle*

Sometimes a hole is provided in the coupler as indicated by a dotted circle in the figure. Instead of using a spanner, a tommy bar is inserted in this hole to rotate the coupler. The turnbuckle is used for connecting two rods which are in tension and which require slight adjustment in length during the assembly. Some of its applications are as follows:

(i) to tighten the members of the roof truss;

(ii) to tighten the cables or the stay ropes of electric distribution poles ; and

(iii) to connect the tie rod to the jib in case of jib-cranes.

Step III Selection of Materials The coupler has a relatively complex shape and the economic method to make the coupler is casting. Casting reduces the number of machining operations. Grey cast iron of grade FG 200 (S_{ut} = 200 N/mm^2) is selected as the material for the coupler. The rods are subjected to tensile force and torsional moment. From strength considerations, plain carbon steel of grade 30C8 (S_{yt} = 400 N/mm^2) is selected for the rods.

Step IV General Considerations Many times, turnbuckle is subjected to rough handling in use. Sometimes, workers use a pipe to increase the length of the spanner and tighten the rods. Some workers even use a hammer to apply force on the spanner. To account for this '*miss-use*,' a higher factor of safety of 5 is used in the present design.

The coupler and the rods are provided with ISO metric coarse threads. Coarse threads are preferred because of the following advantages:

(i) Coarse threads are easier to cut than fine threads;

(ii) They are less likely to seize during tightening;

(iii) There is more even stress distribution.

The rods are tightened by applying force on the wrench handle and rotating the hexagonal coupler. The expression for torque required to tighten the rod with specific tension P can be derived by suitable modification of the equation derived for the trapezoidal threads. The torque required to overcome thread friction in case of trapezoidal threads is given by Eq. (6.13) of Chapter 6.

$$M_t = \frac{P\,d_m}{2} \times \frac{(\mu \sec\theta + \tan\alpha)}{(1 - \mu \sec\theta \tan\alpha)} \qquad \text{(a)}$$

For ISO metric screw threads,

$$\theta = 30° \qquad \alpha \cong 2.5° \qquad d_m \cong 0.9\,d$$

where d is the nominal diameter of the threads. The coefficient of friction varies from 0.12 to 0.20 depending upon the surface finish and accuracy of the thread profile and lubrication. Assuming,

$$\mu = 0.15$$

and substituting above values in Eq. (a),

$$M_t = \frac{P\,(0.9\,d)}{2} \times \left[\frac{0.15 \sec(30) + \tan(2.5)}{1 - 0.15 \sec(30) \tan(2.5)}\right]$$

or,

$$M_t = 0.098\,Pd \qquad \text{(b)}$$

The above expression is used to find out torsional moment at each end of the coupler.

Step V Design of Rods The free body diagram of forces acting on the rods and the coupler is shown in Fig. 7.32. Each rod is subjected to a tensile force P and torsional moment M_t. In the initial stages, it is not possible to find out torsional moment. Considering only tensile force,

$$P = A\,\sigma_t \qquad \text{(c)}$$

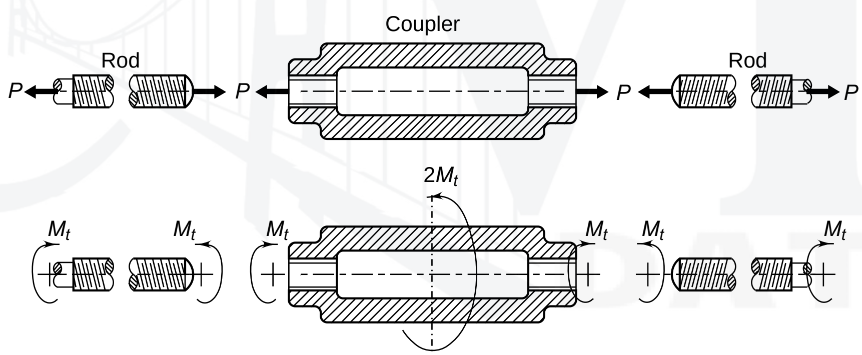

Fig. 7.32 *Free Body Diagram of Forces and Torques (2M_t = Torque Exerted by Spanner)*

where A is the tensile stress area of the threaded portion of the rod and σ_t is the permissible tensile stress. The rod is made of steel 30C8 (S_{yt} = 400 N/mm^2) and factor of safety is 5. Therefore,

$$50 \times 10^3 = A\left(\frac{400}{5}\right).$$

or, $A = 625$ mm^2

From Table 7.1, ISO metric coarse screw thread of M36 designation is suitable for the rod (stress area = 817 mm^2). For M36 size, the core diameter (d_c) as per table is 31.093 mm.

Trial No. 1

$$d = 36 \text{ mm} \quad \text{and} \quad d_c = 31.093 \text{ mm}$$

$$\sigma_t = \frac{P}{\frac{\pi}{4} d_c^2} = \frac{50 \times 10^3}{\frac{\pi}{4}(31.093)^2} = 65.85 \text{ N/mm}^2 \qquad \text{(i)}$$

From Eq. (b),

$$M_t = 0.098\,Pd = 0.098(50 \times 10^3)(36)$$
$$= 176\,400 \text{ N-mm}$$

$$\tau = \frac{16\,M_t}{\pi\,d_c^3} = \frac{16\,(176\,400)}{\pi\,(31.093)^3} = 29.89 \text{ N/mm}^2 \qquad \text{(ii)}$$

The principal shear stress is given by,

$$\tau_{\max.} = \sqrt{\left(\frac{\sigma_t}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{65.85}{2}\right)^2 + (29.89)^2}$$

$$= 44.47 \text{ N/mm}^2$$

and,

$$(fs) = \frac{S_{sy}}{\tau_{\max.}} = \frac{0.5\,S_{yt}}{\tau_{\max.}} = \frac{0.5\,(400)}{44.47} = 4.5$$

The factor of safety is slightly less than the required value of 5. The next size of ISO metric coarse thread is M42 with 36.479 mm as minor diameter (d_c) and 4.5 mm pitch. The stress area is 1120 mm^2.

Trial No. 2

$$d = 42 \text{ mm} \quad \text{and} \quad d_c = 36.479 \text{ mm}$$

$$\sigma_t = \frac{P}{\frac{\pi}{4}d_c^2} = \frac{50\times 10^3}{\frac{\pi}{4}(36.479)^2} = 47.84 \text{ N/mm}^2 \qquad \text{(i)}$$

$$M_t = 0.098\,P\,d = 0.098\,(50 \times 10^3)\,(42)$$
$$= 205\,800 \text{ N-mm}$$

$$\tau = \frac{16M_t}{\pi d_c^3} = \frac{16\,(205\,800)}{\pi(36.479)^3} = 21.59 \text{ N/mm}^2 \qquad \text{(ii)}$$

The principal shear stress is given by,

$$\tau_{\max.} = \sqrt{\left(\frac{\sigma_t}{2}\right)^2 + \tau^2}$$

$$= \sqrt{\left(\frac{47.84}{2}\right)^2 + (21.59)^2} = 32.22 \text{ N/mm}^2$$

$$(fs) = \frac{S_{sy}}{\tau_{\max.}} = \frac{0.5\,S_{yt}}{\tau_{\max.}} = \frac{0.5\,(400)}{32.22} = 6.21$$

The factor of safety is satisfactory. Therefore, the nominal diameter and the pitch of the threaded portion of the rod should be 42 mm and 4.5 mm respectively. In Fig. 7.31, the length of the threaded portion of the rod in contact with coupler threads is denoted by l. It is determined by shearing of the threads at the minor diameter d_c. Equating shear resistance of the threads to the tension in the rod,

$$\pi\, d_c l \tau = P$$

where,

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.5S_{yt}}{5} = \frac{0.5(400)}{5} = 40 \text{ N/mm}^2$$

Therefore,

$$\pi\,(36.479)\,l\,(40) = (50 \times 10^3)$$

$$\therefore \qquad l = 10.91 \text{ mm} \qquad \text{(i)}$$

This length is too small compared with the nominal diameter of 42 mm. In practice, the length l varies from d to 1.25 d. Or,

$$l = d = 42 \text{ mm}$$
$$l = 1.25\,d = 1.25\,(42) = 52.5 \text{ mm} \qquad \text{(ii)}$$

From (i) and (ii), the length of the threaded portion (l) is assumed as 50 mm.

Step VI Design of Coupler The two ends of the coupler are called *coupler nuts*. As shown in Fig. 7.31, the coupler nuts are integral with the coupler. The outer and inner diameters of the coupler nut are D and d respectively and the length is denoted by l. It acts as a hollow rod. Considering tension,

$$P = \frac{\pi}{4}(D^2 - d^2)\sigma_t$$

$$(50 \times 10^3) = \frac{\pi}{4}(D^2 - 42^2)\,\sigma_t \qquad \text{(i)}$$

The coupler is made of cast iron (S_{ut} = 200 N/mm^2) and factor of safety is 5. Therefore,

$$\sigma_t = \frac{S_{ut}}{(fs)} = \frac{200}{5} = 40 \text{ N/mm}^2$$

Substituting the above value in Eq. (i),

$$(50\times 10^3) = \frac{\pi}{4}(D^2 - 42^2)\,(40)$$

or, $\qquad D = 57.93 \text{ or } 60 \text{ mm} \qquad$ (a)

The standard proportion for D is from 1.25 d to 1.5 d. Or,

$$D = 1.25\,d = 1.25\,(42) = 52.5 \text{ mm}$$
$$D = 1.5\,d = 1.5\,(42) = 63 \text{ mm} \qquad \text{(b)}$$

From (a) and (b), it is decided that the dimension D should be 60 mm.

The coupler nut is subjected to direct tensile stress as well as torsional shear stress due to torque M_t.

Step VII Check for Design

$$M_t = 205\,800 \text{ N/mm} \qquad r = \frac{D}{2} = \frac{60}{2} = 30 \text{ mm}$$

$$J = \frac{\pi(D^4 - d^4)}{32} = \frac{\pi(60^4 - 42^4)}{32}$$
$$= 966\,854.98 \text{ mm}^4$$

$$\tau = \frac{M_t r}{J} = \frac{(205\,800)(30)}{(966\,854.98)} = 6.39 \text{ N/mm}^2$$

$$\sigma_t = \frac{P}{\frac{\pi}{4}(D^2 - d^2)} = \frac{50 \times 10^3}{\frac{\pi}{4}(60^2 - 42^2)}$$

$$= 34.67 \text{ N/mm}^2$$

The coupler is made of cast iron. Cast iron is brittle material. For brittle materials, the maximum principal stress theory is applicable. The maximum principal stress is given by,

$$\sigma_{\max.} = \left(\frac{\sigma_t}{2}\right) + \sqrt{\left(\frac{\sigma_t}{2}\right)^2 + (\tau)^2}$$

$$= \left(\frac{34.67}{2}\right) + \sqrt{\left(\frac{34.67}{2}\right)^2 + (6.39)^2}$$

$$= 35.81 \text{ N/mm}^2$$

$$(fs) = \frac{S_{ut}}{\sigma_{\max.}} = \frac{200}{35.81} = 5.59$$

The factor of safety is satisfactory.

Remaining dimensions of the central portion of the coupler, viz., D_1, D_2 and L are decided by the following empirical relationships:

$$D_1 = d + 10 = 42 + 10 = 52 \text{ mm}$$
$$D_2 = 2D_1 = 2(52) = 104 \text{ mm}$$
$$L = 6\,d = 6(42) = 252 \text{ mm}$$

7.17 ELASTIC ANALYSIS OF BOLTED JOINTS

A bolted assembly of two components is shown in Fig. 7.33. Initially, the nut is tightened by means of a spanner, which results in tensile stress in the bolt. It is assumed that the stress is within the elastic limit and obeys Hooke's law. The stiffness of a component k' is given by,

$$k' = \frac{P}{\delta} = \frac{AE}{l} \qquad \text{(a)}$$

The stiffness of the bolt is given by,

$$k'_b = \left(\frac{\pi}{4} d^2\right)\frac{E}{l} \qquad (7.20)$$

where

k'_b = stiffness of the bolt (N/mm)

d = nominal diameter of the bolt (mm)

l = total thickness of parts held together by the bolt (mm)

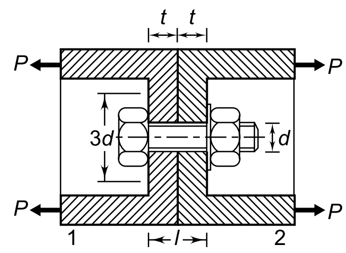

Fig. 7.33 *Bolted Joint in Tension*

There are two components in the grip of the bolt which act as two compression springs in series. Their combined stiffness (k'_c) is given by

$$\frac{1}{k'_c} = \frac{1}{k'_1} + \frac{1}{k'_2} \qquad (7.21)$$

where (k'_1) and (k'_2) are the stiffness of the two parts. It is difficult to predict the area of the two components, which is compressed by the bolt head and the nut. As shown in the figure, it is assumed that an annular area with ($3d$) and (d) as outer and inner diameters respectively, is under the grip of the bolt.

$$A = \frac{\pi}{4}[(3d)^2 - (d)^2] = 2\pi d^2$$

Since $$k'_1 = k'_2 = \frac{AE}{l}$$

$$\therefore \quad k'_1 = k'_2 = \frac{2\pi\, d^2 E}{l} \qquad (7.22)$$

When the nut is initially tightened, the bolt is subjected to an initial tension, which is called the pre-load (P_i). Under the action of pre-load, the bolt is elongated by an amount (δ_b) and the two parts are compressed by an amount (δ_c); thus

$$\delta_b = \frac{P_i}{k'_b} \qquad \text{(b)}$$

$$\delta_c = \frac{P_i}{k'_c} \qquad \text{(c)}$$

The force–deflection diagram is shown in Fig. 7.34. Line $\overline{OA}$ indicates the elongation of the bolt, while the line $\overline{CA}$ shows the compression of the parts. The slope of the line $\overline{CA}$ is negative because of the compressive force.

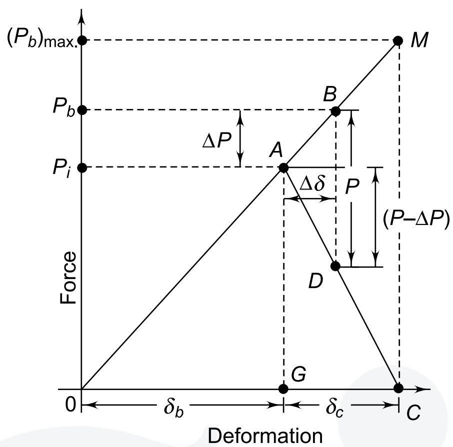

Fig. 7.34 *Force-deformation Diagram of Bolted Joint*

When the parts are clamped and put into service, they are subjected to an external force P. The effects of the external force are as follows:

(i) The bolt is further elongated by an amount ($\Delta\delta$) and, consequently, the bolt load is further increased by an amount (ΔP). This deformation is indicated by the line $\overline{AB}$.

(ii) The compression of the two parts is relieved by the same amount ($\Delta\delta$) and there is a corresponding reduction in load. The reduction in load is $(P - \Delta P)$. This deformation is shown by the line $\overline{AD}$.

$$k_b' = \frac{\Delta P}{\Delta \delta} \qquad \text{(d)}$$

$$k_c' = \frac{(P - \Delta P)}{\Delta \delta} \qquad \text{(e)}$$

Dividing the expression (d) by (e) and rearranging the terms, we have

$$\Delta P = P\left(\frac{k_b'}{k_b' + k_c'}\right) \qquad (7.23)$$

The resultant load on the bolt, i.e., (P_b) is given by,

$$P_b = P_i + \Delta P \qquad (7.24)$$

Referring back to Fig. 7.34, as the external load is further increased, the deformation of the bolt will further continue on line $\overline{OB}$. The limiting point is M, where the compression of the two parts becomes zero. At this point, the joint will begin to open, since the parts can no longer expand to maintain the tight joint. From similar triangles,

$$\frac{\overline{AG}}{\overline{OG}} = \frac{\overline{MC}}{\overline{OC}}$$

or
$$\frac{P_i}{\delta_b} = \frac{(P_b)_{max.}}{\delta_b + \delta_c}$$

or
$$(P_b)_{max.} = P_i\left(\frac{\delta_b + \delta_c}{\delta_b}\right)$$

Substituting expressions (b) and (c) in the above expression,

$$(P_b)_{max.} = P_i\left(\frac{k_b' + k_c'}{k_c'}\right) \qquad (7.25)$$

We will rewrite Eq. (7.23)

$$\Delta P = P\left(\frac{k_b'}{k_b' + k_c'}\right)$$

Dividing numerator and denominator of the right hand side of the above expression by (k_b'),

$$\Delta P = P\left(\frac{k_b'/k_b'}{k_b'/k_b' + k_c'/k_b'}\right) = P\left[\frac{1}{1 + k_c'/k_b'}\right] \qquad \text{(f)}$$

Suppose,

a = ratio of stiffness of connected parts to stiffness of bolt $[k_c'/k_b']$

Substituting in (f),

$$\Delta P = P\left[\frac{1}{1+a}\right] \qquad \text{(g)}$$

From Eq. (7.24),

$$P_b = P_i + \Delta P \qquad \text{(h)}$$

We will consider expressions (g) and (h) with reference to two cases, viz., soft gasket and hard gasket.

Case I Soft Gasket When the gasket is too soft, (k_c') is too small compared with (k_b')

$$a = [k_c'/k_b'] = 0$$

$$\Delta P = P\left[\frac{1}{1+a}\right] = P$$

$$P_b = P_i + \Delta P = P_i + P$$

Therefore, the resultant load on the bolt is the sum of initial tension and external load.

Case II Hard Gasket When the gasket is too hard, (k_c') is too large compared with (k_b')

$$a = [k_c' / k_b'] = \infty$$

$$\Delta P = P\left[\frac{1}{1+a}\right] = 0$$

$$P_b = P_i + \Delta P = P_i + 0 = P_i$$

Therefore, the resultant load on the bolt is initial tension only. However, in rare cases, the external load (P) is more than initial tension (P_i). In such cases, the resultant load on the bolt is taken as the external load only.

Example 7.16 *Two circular plates with (2d) and (d) as outer and inner diameters respectively, are clamped together by means of a bolt as shown in Fig. 7.35. The bolt is made of plain carbon steel 45C8 (S_{yt} = 380 N/mm^2 and E = 207000 N/mm^2), while the plates are made of aluminium (E = 71000 N/mm^2). The initial pre-load in the bolt is 5 kN and the external force acting on the bolted joint is 10 kN. Determine the size of the bolt, if the factor of safety is 2.5.*

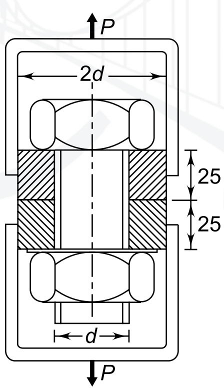

Fig. 7.35

Solution

Given $P_i = 5$ kN $\quad P = 10$ kN $\quad (fs) = 2.5$

For bolts, $S_{yt} = 380$ N/mm^2 $\quad E = 207\,000$ N/mm^2

For plates, $E = 71000$ N/mm^2

Step I *Permissible tensile stress*

$$(\sigma_t)_{\text{max.}} = \frac{S_{yt}}{(fs)} = \frac{380}{2.5} = 152 \text{ N/mm}^2$$

Step II *Stiffness of bolt and plates*

From Eq. (7.20),

$$k_b' = \left(\frac{\pi}{4}d^2\right)\frac{E}{l}$$

$$= \left(\frac{\pi}{4}d^2\right)\frac{207\,000}{50} = (3251.55\,d^2) \text{ N/mm}$$

The area A_c of the two plates is given by,

$$A_c = \frac{\pi}{4}(2d)^2 - \frac{\pi}{4}(d)^2 = (2.356d^2) \text{ mm}^2$$

The combined stiffness (k_c') of the two plates is given by,

$$k_c' = \frac{A_c E_c}{l} = \frac{(2.356d^2)(71000)}{(50)}$$

$$= (3345.52d^2) \text{ N/mm}$$

Step III *Resultant load on bolt*

From Eq. (7.23),

$$\Delta P = P\left(\frac{k_b'}{k_b' + k_c'}\right)$$

$$= (10 \times 10^3)\frac{(3251.55d^2)}{[3251.55d^2 + 3345.52d^2]}$$

$$= 4928.78 \text{ N}$$

$$P_b = P_i + \Delta P = 5000 + 4928.78 = 9928.78 \text{ N}$$

Step IV *Size of bolt*

The bolt is subjected to tensile stress and the tensile stress area A of the bolt is given by,

$$\frac{P_b}{A} = (\sigma_t)_{\text{max.}} \quad \text{or} \quad \frac{9928.78}{A} = 152$$

$$A = 65.32 \text{ mm}^2$$

From Table 7.1, a bolt with threads M12 (A = 84.3 mm^2)is suitable for this joint.

Example 7.17 *A bolted assembly of two components is shown in Fig 7.36. Initially, the nut is tightened by means of a spanner so as to induce a pre-load of 2.5 kN in the bolt. The external force P acting on the assembly is 5 kN. The bolt with coarse threads is made of plain carbon steel 30C8 (S_{yt} = 400 N/mm^2) and the factor of safety is 2.5. The effective stiffness of the parts held together by the bolt is 2.5 times the stiffness of the bolt. Specify the size of the bolt.*

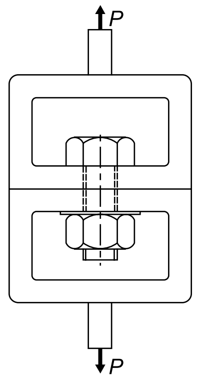

Fig. 7.36

Solution

Given $P_i = 2.5$ kN $P = 5$ kN $(fs) = 2.5$
$S_{yt} = 400$ N/mm^2 $k_c' = 2.5k_b'$

Step I *Permissible tensile stress*

$$(\sigma_t)_{\max.} = \frac{S_{yt}}{(fs)} = \frac{400}{2.5} = 160 \text{ N/mm}^2$$

Step II *Resultant load on bolt*

$$k_c' = 2.5k_b'$$

$$\therefore \quad \Delta P = P\left(\frac{k_b'}{k_b' + k_c'}\right) = (5000)\left(\frac{k_b'}{k_b' + 2.5k_b'}\right)$$

$$= 1428.57 \text{ N}$$

$$P_b = P_i + \Delta P = 2500 + 1428.57 = 3928.57 \text{ N}$$

Step III *Size of bolt*

The tensile stress area A of the bolt is given by,

$$\frac{P_b}{A} = (\sigma_t)_{\max.} \quad \text{or} \quad \frac{3928.57}{A} = 160$$

$$A = 24.55 \text{ mm}^2$$

From Table 7.1, a bolt with threads M8 (A = 36.6 mm^2) is suitable for this application.

7.18 BOLTED JOINT UNDER FLUCTUATING LOAD

In many applications, the external force acting on the bolted joint fluctuates between two limits. The endurance limit is the criterion of failure in these applications. The endurance limit of the bolt is determined by the procedure discussed in Chapter 5. This involves the use of the surface finish factor, the size factor, the reliability factor and the modifying factor to account for stress concentration. The fatigue stress concentration factors (K_f) for the threaded parts are given in Table 7.6. According to SAE and metric specifications, bolt grades are numbered as per tensile strength. Higher the tensile strength, more is the grade number. Cutting is the simplest method of making threads. However, rolled threads have smoother thread finish than cut threads. Rolling also provides an unbroken flow of material grain in the thread region. Therefore, rolled threads are preferred for applications subjected to fluctuating loads. After determining the endurance strength, Goodman diagram is constructed. The following points should be noted in finding endurance limit of threaded fasteners:

(i) It is not necessary to consider surface finish factor separately. It is incorporated in fatigue stress concentration factor.

(ii) The size factor is taken as 1 for axial loading.

(iii) The reliability is assumed as 90% for finding reliability factor.

A typical analysis of bolt failures indicate that

(i) 15% failures of bolt occur at the fillet under the head

(ii) 20% failures of bolt occur at the end of threads on the shank

(iii) 65% failures of bolt occur in the threads that are in contact with the nut

Table 7.6 *Fatigue stress concentration factors (K_f) for threaded parts*

SAE grade	*Metric grade*	*Rolled threads*	*Cut threads*	*Fillet*
0 to 2	3.6 to 5.8	2.2	2.8	2.1
4 to 8	6.6 to 10.9	3.0	3.8	2.3

From Eq. (7.24),

$$P_b = P_i + \Delta P \tag{a}$$

From Eq. (7.23)

$$\Delta P = \left(\frac{k_b'}{k_b' + k_c'}\right) P \tag{b}$$

From (a) and (b),

$$P_b = P_i + \left(\frac{k_b'}{k_b' + k_c'}\right) P \tag{c}$$

We will define a factor C such that,

$$C = \left(\frac{k_b'}{k_b' + k_c'}\right) \tag{7.26}$$

Substituting the above expression in (c),

$$P_b = P_i + CP \tag{d}$$

In many cases, the external applied load acting on the bolt fluctuates from zero to some maximum value (P). The maximum and minimum bolt load is given by,

$$(P_b)_{\text{max.}} = P_i + CP$$

$$(P_b)_{\text{min.}} = P_i$$

The maximum and minimum stresses in the bolt are given by,

$$\sigma_{\text{max.}} = \frac{P_i}{A} + \frac{CP}{A} \tag{e}$$

$$\sigma_{\text{min.}} = \frac{P_i}{A} \tag{f}$$

where A is tensile stress area of the bolt.

The mean and alternating components of stresses are given by,

$$\sigma_m = \frac{1}{2}(\sigma_{\text{max.}} + \sigma_{\text{min.}}) = \frac{1}{2}\left[\left(\frac{P_i}{A} + \frac{CP}{A}\right) + \left(\frac{P_i}{A}\right)\right]$$

$$\sigma_m = \frac{P_i}{A} + \frac{CP}{2A} \tag{7.27}$$

$$\sigma_a = \frac{1}{2}(\sigma_{\text{max.}} - \sigma_{\text{min.}}) = \frac{1}{2}\left[\left(\frac{P_i}{A} + \frac{CP}{A}\right) - \left(\frac{P_i}{A}\right)\right]$$

$$\sigma_a = \frac{CP}{2A} \tag{7.28}$$

From Eqs (7.27) and (7.28),

$$\sigma_m = \frac{P_i}{A} + \sigma_a \tag{7.29}$$

The above equation indicates a straight line illustrated in Fig. 7.37(a) in the form of $x = a + y$

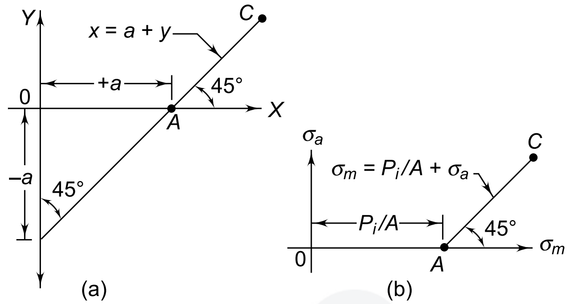

Fig. 7.37

On the X-axis,

$$y = 0 \quad \text{and} \quad x = a$$

On the Y-axis,

$$x = 0 \quad \text{and} \quad y = -a$$

Therefore, $(x = a + y)$ is a straight line having the same intercepts on X and Y axes. Obviously, it is inclined at 45° to X and Y axes. It is a line passing through the point A on the X-axis at a distance of $(+a)$ from the origin and inclined at 45° to the X-axis.

Similarly, $\left[\sigma_m = \frac{P_i}{A} + \sigma_a\right]$ is a straight line $\overline{AC}$ passing through the point A on abscissa at a distance of $\left(\frac{P_i}{A}\right)$ from the origin and inclined at 45° to the abscissa. This line is called the *Kimmelmann line* or simply the *load line*.

The fatigue diagram for bolted joint is shown in Fig. 7.38. $\overline{EF}$ is the Goodman line and $\overline{AC}$ is the Kimmelmann load line. The failure point C is the point of intersection of two straight lines $\overline{EF}$ and $\overline{AC}$. The co-ordinates of the point C are (S_m, S_a). Since C is point on the line $\overline{EF}$,

$$\frac{S_m}{S_{ut}} + \frac{S_a}{S_e} = 1 \tag{g}$$

Since C is the point on the line $\overline{AC}$,

$$S_m = \frac{P_i}{A} + S_a \tag{h}$$

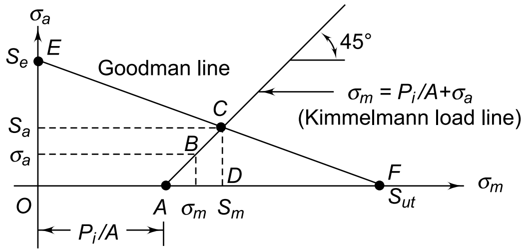

Fig. 7.38 *Fatigue Diagram for Bolted Joint*

Solving the Eqs (g) and (h) simultaneously, the co-ordinates (S_m, S_a) can be obtained. The problem is then solved by using a factor of safety.

$$\sigma_a = \frac{S_a}{(fs)} \qquad \text{and} \qquad \sigma_m = \frac{S_m}{(fs)}$$

This is illustrated by the point B in Fig. 7.38.

$$(fs) = \frac{S_a}{\sigma_a} = \frac{S_m}{\sigma_m} = \frac{\overline{AC}}{\overline{AB}}$$

Alternatively, from Eq. (g),

$$S_m = S_{ut}\left[1 - \frac{S_a}{S_e}\right]$$

Substituting the above value in Eq. (h),

$$\frac{P_i}{A} + S_a = S_{ut}\left[1 - \frac{S_a}{S_e}\right] = S_{ut} - (S_{ut}/S_e)S_a$$

$$S_a + (S_{ut}/S_e)\,S_a = S_{ut} - (P_i/A)$$

$$S_a\,[1 + (S_{ut}/S_e) = S_{ut} - (P_i/A)$$

$$S_a = \frac{S_{ut} - (P_i/A)}{1 + (S_{ut}/S_e)} \qquad (7.30)$$

and $$\sigma_a = \frac{S_a}{(fs)}$$

The factor of safety is taken as 2.

Connecting Rod Bolts We will consider a special case of connecting rod bolts. The bolts are subjected to initial pre-load plus fluctuating external load. Combining the expressions (a) and (b) mentioned above,

$$P_b = P_i + P\left[\frac{k_b'}{k_b' + k_c'}\right]$$

or $$P_b = P_i + P\left[\frac{1}{1 + (k_c'/k_b')}\right] \qquad (j)$$

Suppose the external load varies from 0 to P.

$$P_{max.} = P \qquad \text{and} \qquad P_{min.} = 0$$

The maximum external load on the bolt is P. Therefore,

$$(P_b)_{max.} = P_i + P\left[\frac{1}{1 + (k_c'/k_b')}\right]$$

The minimum external load on the bolt is zero. Therefore,

$$(P_b)_{min.} = P_i$$

The mean and alternating components are given by,

$$(P_b)_m = \frac{1}{2}[(P_b)_{max.} + (P_b)_{min.}]$$

$$= P_i + \frac{1}{2}\left[\frac{1}{1 + (k_c'/k_b')}\right]P \qquad (k)$$

$$(P_b)_a = \frac{1}{2}[(P_b)_{max.} - (P_b)_{min.}]$$

$$= \frac{1}{2}\left[\frac{1}{1 + (k_c'/k_b')}\right]P \qquad (l)$$

It is observed that the designer has a control over the influence of resultant load on the bolt. The designer can select dimensions of the bolt and the connected parts in such a way that a specific ratio a or $[k_c'/k_b']$ is obtained. Suppose, the combined stiffness of parts held together by bolt is ten times the stiffness of the bolt ($k_c' = 10k_b'$).

From expression (k) and (l),

$$(P_b)_m = P_i + \frac{1}{2}\left[\frac{1}{1 + (k_c'/k_b')}\right]P$$

$$= P_i + \frac{1}{2}\left[\frac{1}{1 + (10)}\right]P = P_i + \left(\frac{1}{22}\right)P$$

$$(P_b)_a = \frac{1}{2}\left[\frac{1}{1 + (k_c'/k_b')}\right]P$$

$$= \frac{1}{2}\left[\frac{1}{1 + (10)}\right]P = \left(\frac{1}{22}\right)P$$

Connecting rod bolts are tightened with initial tension greater than external load due to the following reasons:

(i) When the connecting rod bolts are tightened up with initial pre-load greater than external load, the term (P_i) is much more than the term [1/22)P]. The fluctuating term [(1/22)P] is small and neglected. The total load on the bolt will be almost static and the bolt can be designed on the basis of static failure.

(ii) If the initial pre-load is not high enough, the term [(1/22)P] is comparable with the term (P_i). The resultant load on the bolt will be affected by the external load, which is fluctuating and it is required to design the bolt on the basis of endurance limit to avoid fatigue failure.

It is desirable to design the bolts on the basis of static strength rather than on the basis of endurance limit.

Therefore, in automotive applications, the connecting rod bolts are tightened with very high pre-load with the stress approaching the yield point.

Example 7.18 *The assembly of two circular plates clamped together by means of a bolt, which is shown in Fig. 7.35, is subjected to a variable force P varying from 0 to 10 kN. The bolt is made of plain carbon steel 45C8 (S_{ut} = 630 N/mm^2; S_{yt} = 380 N/mm^2 and E = 207000 N/mm^2). The two circular plates are made of aluminium (E = 71000 N/mm^2). The fatigue stress concentration factor is 2.2 and the expected reliability is 90%. The initial pre-load in the bolt is 5 kN. Determine the size of the bolt if the factor of safety is 2.*

Solution

Given $P_i = 5$ kN $P = 0$ to 10 kN $(fs) = 2$

For bolts, $S_{ut} = 630$ N/mm^2 $S_{yt} = 380$ N/mm^2

$E = 207000$ N/mm^2 $K_f = 2.2$ $R = 90\%$

For plates, $E = 71000$ N/mm^2

Step I *Endurance limit stress for bolt*

$$S'_e = 0.5\ S_{ut} = 0.5(630) = 315 \text{ N/mm}^2$$

The surface finish factor is incorporated in the fatigue stress concentration factor. The size factor is 1 for axial load.

For 90% reliability,

$$K_c = 0.897$$

$$K_d = \frac{1}{K_f} = \frac{1}{2.2} = 0.4545$$

$$S_e = K_b K_c K_d S'_e = (1.0)(0.897)(0.4545)(315)$$
$$= 128.42 \text{ N/mm}^2$$

Step II *Construction of Goodman diagram*

As determined in Example 7.16,

$$\left(\frac{k'_b}{k'_b + k'_c}\right) = \frac{(3251.55d^2)}{[3251.55d^2 + 3345.52d^2]} = 0.4929$$

The maximum and minimum forces in the bolt are given by

$$P_{\text{max.}} = P_i + (0.4929)\,P$$
$$= 5000 + (0.4929)(10000) = 9929 \text{ N}$$
$$P_{\text{min.}} = P_i + (0.4929)\,P$$
$$= 5000 + (0.4929)(0) = 5000 \text{ N}$$
$$P_m = ½\,(P_{\text{max.}} + P_{\text{min.}}) = ½\,(9929 + 5000)$$
$$= 7464.5 \text{ N}$$
$$P_a = ½\,(P_{\text{max.}} - P_{\text{min.}}) = ½\,(9929 - 5000)$$
$$= 2464.5 \text{ N}$$

The Goodman diagram for this example is shown in Fig. 7.39.

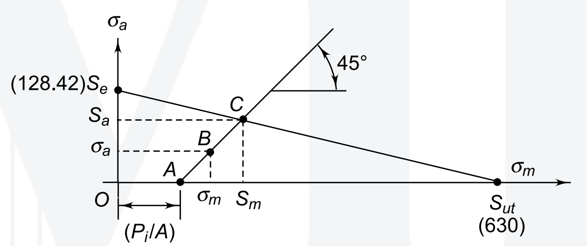

Fig. 7.39

Step III *Permissible stress amplitude*

The coordinates of the point C (S_m, S_a) are obtained by solving the following two equations simultaneously:

$$\frac{S_m}{630} + \frac{S_a}{128.42} = 1 \qquad \text{(a)}$$

$$S_m = \frac{P_i}{A} + S_a = \frac{5000}{A} + S_a \qquad \text{(b)}$$

where A is the tensile stress area of the bolt.

The solution is obtained by Eq. (7.30).

From Eq. (7.30),

$$S_a = \frac{S_{ut} - (P_i / A)}{1 + (S_{ut} / S_e)}$$

$$= \frac{630 - (5000 / A)}{1 + (630 / 128.42)} = \frac{630 - (5000 / A)}{5.9}$$

Step IV *Size of bolt*

Since

$$\sigma_a = \frac{S_a}{(fs)} \quad \text{or} \quad \frac{P_a}{A} = \frac{S_a}{(fs)}$$

or $$\frac{2464.5}{A} = \frac{630 - (5000/A)}{5.9(2)}$$

$$\frac{2464.5}{A} = 53.39 - \frac{423.73}{A} \text{ or } \frac{2888.23}{A} = 53.39$$

$$A = 54.1 \text{ mm}^2$$

From Table 7.2, a bolt with fine threads M10 × 1.25 (A = 61.2 mm^2) is suitable for this application.

Example 7.19 *A bolted assembly is subjected to an external force, which varies from 0 to 10 kN. The combined stiffness of the parts, held together by the bolt, is three times the stiffness of the bolt. The bolt is initially so tightened that at 50% overload condition, the parts held together by the bolt are just about to separate. The bolt is made of plain carbon steel 5OC4 (S_{ut} = 660 N/mm^2 and S_{yt} = 460 N/mm^2). The fatigue stress concentration factor is 2.2 and the expected reliability is 90%. The factor of safety is 2. Determine the size of the bolt with fine threads.*

Solution

Given P = 0 to 10 kN $(fs) = 2$

For bolts, S_{ut} = 660 N/mm^2 S_{yt} = 460 N/mm^2

$K_f = 2.2$ $R = 90\%$ $k'_c = 3k'_b$

Step I *Endurance limit stress for bolt*

$$S'_e = 0.5\ S_{ut} = 0.5(660) = 330 \text{ N/mm}^2$$

The surface finish factor is incorporated in the fatigue stress concentration factor. The size factor is 1 for axial load.

For 90% reliability,

$$K_c = 0.897$$

$$K_d = \frac{1}{K_f} = \frac{1}{2.2} = 0.4545$$

$$S_e = K_b K_c K_d S'_e = (1.0)(0.897)(0.4545)(330) = 134.54 \text{ N/mm}^2$$

Step II *Construction of Goodman diagram*

At 50% overload condition, the external force will be (1.5 × 10) or 15 kN. From Eq. (7.25),

$$(P_b)_{\text{max.}} = P_i \left(\frac{k'_b + k'_c}{k'_c} \right)$$

or $$(15 \times 10^3) = P_i \left(\frac{k'_b + 3k'_b}{3k'_b} \right)$$

$$P_i = 11\,250 \text{ N}$$

Similarly,

$$\left(\frac{k'_b}{k'_b + k'_c} \right) = \left(\frac{k'_b}{k'_b + 3k'_b} \right) = \frac{1}{4} = 0.25$$

The maximum and minimum forces in the bolt are given by

$$P_{\text{max.}} = P_i + (0.25)\,P = 11\,250 + (0.25)(10\,000) = 13\,750 \text{ N}$$

$$P_{\text{min.}} = P_i + (0.25)(0) = 11\,250 + (0.25)(0) = 11\,250 \text{ N}$$

$$P_m = \tfrac{1}{2}(P_{\text{max.}} + P_{\text{min.}}) = \tfrac{1}{2}(13\,750 + 11\,250) = 12\,250 \text{ N}$$

$$P_a = \tfrac{1}{2}(P_{\text{max.}} - P_{\text{min.}}) = \tfrac{1}{2}(13\,750 - 11\,250) = 1250 \text{ N}$$

The Goodman diagram for this example is shown in Fig. 7.40.

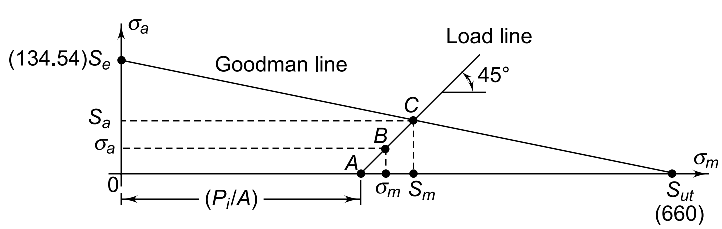

Fig. 7.40

Step III *Permissible stress amplitude*

The co-ordinates of the point C (S_m, S_a) are obtained by solving the following two equations simultaneously:

$$\frac{S_m}{660} + \frac{S_a}{134.54} = 1 \qquad \text{(a)}$$

$$S_m = \frac{P_i}{A} + S_a = \frac{11250}{A} + S_a \qquad \text{(b)}$$

where A is the tensile stress area of the bolt.

The solution is obtained by Eq. (7.30).

From Eq. (7.30),

$$S_a = \frac{S_{ut} - (P_i / A)}{1 + (S_{ut} / S_e)}$$

$$= \frac{660 - (11250 / A)}{1 + (660 / 134.54)} = \frac{660 - (11250 / A)}{5.9}$$

Step IV *Size of bolt*

Since $\sigma_a = \dfrac{S_a}{(fs)}$ $\qquad \therefore \quad \dfrac{P_a}{A} = \dfrac{S_a}{(fs)}$

or $\dfrac{1250}{A} = \dfrac{660 - (11250/A)}{5.9(2)}$

$$\frac{1250}{A} = 55.93 - \frac{953.39}{A} \quad \text{or} \quad \frac{2203.39}{A} = 55.93$$

$$A = 39.4 \text{ mm}^2$$

From Table 7.2, a bolt with fine threads M8 × 1 ($A = 39.2$ mm^2) is suitable for this application.

Example 7.20 *The following data is given for a four-stroke diesel engine;*

Engine speed = 2000 rpm

Length of stroke = 100 mm

Length of connecting rod = 200 mm

Mass of reciprocating parts = 5 kg

At the big end, the cap is fastened to the connecting rod by means of two bolts. It can be assumed that the bolts are subjected to inertia forces only. The bolts are initially so tightened that at 50% overspeed condition, the cap is just about to separate. The combined stiffness of the parts held together by the bolts is four times the stiffness of the bolt. The bolts are made of chromium–molybdenum steel (S_{ut} = 600 N/mm^2 and S_{yt} = 450 N/mm^2). The bolts have rolled threads and the fatigue stress concentration factor is 3.0. The expected reliability is 90%. Determine the size of the bolts using a factor of safety of 2.

Solution

Given $S_{ut} = 600$ N/mm^2 $\quad S_{yt} = 450$ N/mm^2

$(fs) = 2 \quad K_f = 3 \quad R = 90\% \quad k_c' = 4k_b'$

Step I *Endurance limit stress for bolt*

$$S_e' = 0.5 S_{ut} = 0.5(600) = 300 \text{ N/mm}^2$$

The surface finish factor is incorporated in the fatigue stress concentration factor. The size factor is 1 for the axial load.

For 90% reliability,

$K_c = 0.897$

$$K_d = \frac{1}{K_f} = \frac{1}{3}$$

$$S_e = K_b K_c K_d S_e' = (1.0)(0.897)(1/3)(300)$$
$$= 89.7 \text{ N/mm}^2$$

Step II *Inertia force on bolt*

The analysis of inertia forces is explained in the books on 'Theory of Machines'. The inertia force I is given by the following expression:

$$I = m\omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n_1} \right]$$

where

m = mass of reciprocating parts (5 kg)

r = crank radius (0.5 × length of stroke = 0.05 m)

n_1 = ratio of length of connecting rod to crank radius (200/50 = 4), and

$$\omega = \left(\frac{2\pi n}{60} \right)$$

The inertia force is maximum at the dead centre position ($\theta = 0$). Substituting the values, we have

$$I_{\text{max.}} = (5)\left(\frac{2\pi n}{60} \right)^2 (0.05)\left[1 + \frac{1}{4} \right]$$

$$= (3.4269 \times 10^{-3}) n^2$$

In normal running condition (n = 2000 rpm),

$I_{\text{max.}} = (3.4269 \times 10^{-3})(2000)^2 = 13707.6$ N

At 50% overspeed condition (n = 3000 rpm),

$I_{\text{max.}} = (3.4269 \times 10^{-3})(3000)^2 = 30842.1$ N

Since there are two bolts, the force acting on each bolt is one half of the above values.

Step III *Construction of Goodman diagram*

In normal running condition,

$$P = \frac{13\,707.6}{2} = 6853.8 \text{ N}$$

At 50% overspeed condition,

$$(P_b)_{\text{max.}} = \frac{30\,842.1}{2} = 15\,421.1 \text{ N}$$

From Eq. (7.25),

$$(P_b)_{\text{max.}} = P_i\left(\frac{k_b' + k_c'}{k_c'}\right)$$

or $$15\,421.1 = P_i\left(\frac{k_b' + 4k_b'}{4k_b'}\right)$$

$P_i = 12\,336.88$ N

Similarly,

$$\left(\frac{k_b'}{k_b' + k_c'}\right) = \left(\frac{k_b'}{k_b' + 4k_b'}\right) = \frac{1}{5} = 0.2$$

The maximum and minimum forces in the bolt are given by

$$P_{\text{max.}} = P_i + (0.2)\,P = 12\,336.88 + (0.2)(6853.8) = 13\,707.64 \text{ N}$$

$$P_{\text{min.}} = P_i = 12\,336.88 \text{ N}$$

$$P_m = \frac{1}{2}\,(P_{\text{max.}} + P_{\text{min.}}) = \frac{1}{2}\,(13\,707.64 + 12\,336.88) = 13\,022.26 \text{ N}$$

$$P_a = \frac{1}{2}\,(\text{P}_{\text{max.}} - P_{\text{min.}}) = \frac{1}{2}\,(13\,707.64 - 12\,336.88) = 685.38 \text{ N}$$

The Goodman diagram for this example is shown in Fig. 7.41.

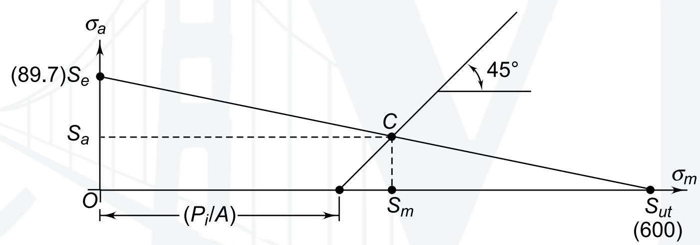

Fig. 7.41

Step IV *Permissible stress amplitude*

The co-ordinates of the point C (S_m, S_a) are obtained by solving the following two equations simultaneously:

$$\frac{S_m}{600} + \frac{S_a}{89.7} = 1 \qquad \text{(a)}$$

$$S_m = \frac{P_i}{A} + S_a = \frac{12336.88}{A} + S_a \qquad \text{(b)}$$

where A is the tensile stress area of the bolt.

The solution is obtained by Eq. (7.30).

From Eq. (7.30),

$$S_a = \frac{S_{ut} - (P_i/A)}{1 + (S_{ut}/S_e)}$$

$$= \frac{600 - (12336.88/A)}{1 + (600/89.7)} = \frac{600 - (12336.88/A)}{7.689}$$

Step V *Size of bolt*

Since $\sigma_a = \dfrac{S_a}{(fs)}$ $\quad\therefore\ \dfrac{P_a}{A} = \dfrac{S_a}{(fs)}$

or $$\frac{685.38}{A} = \frac{600 - (12336.88/A)}{7.689(2)}$$

$$\frac{685.38}{A} = 39.02 - \frac{802.24}{A} \text{ or } \frac{1487.62}{A} = 39.02$$

$A = 38.12$ mm^2

From Table 7.2, a bolt with fine threads M8 × 1 ($A = 39.2$ mm^2) is suitable for this application.

Example 7.21 *Bolts are used to hold the cover plate on a pressure vessel, which is subjected to an internal pressure varying from zero to 2 MPa. The area over which the pressure acts may be taken to correspond to a 400 mm diameter circle. The bolts are preloaded to the extent of 1.3 times the* *maximum force exerted by the fluid on the cover plate. The combined stiffness of the parts, held together by the bolt (including the copper gasket), is four times the stiffness of the bolt. The following data is given for the bolts:*

Ultimate tensile strength = 900 N/mm²
Yield strength = 700 N/mm²
Endurance limit in bending = 300 N/mm²
Fatigue stress concentration factor = 2.2
Factor of safety = 1.5
Number of bolts = 8

Determine the size of the bolts assuming fine threads.

Solution

Given $S_{ut} = 900$ N/mm² $S_{yt} = 700$ N/mm²
$S_e' = 300$ N/mm² $(fs) = 1.5$ $K_f = 2.2$ $k_c' = 4k_b'$
For vessel, $D = 400$ mm $p = 0$ to 2 MPa

Step I *Endurance limit stress for bolt*

$$S_e' = 300 \text{ N/mm}^2$$

The surface finish factor is incorporated in the fatigue stress concentration factor. The size factor is 1 for the axial load.

$$K_d = \frac{1}{K_f} = \frac{1}{2.2}$$

$$S_e = K_b K_d S_e' = (1.0)(1/2.2)(300) = 136.36 \text{ N/mm}^2$$

Step II *Construction of Goodman diagram*

Maximum force of fluid on cover = P

$$P = \frac{\pi}{4} D^2 p = \frac{\pi}{4}(400)^2(2) = 251\,327.41 \text{ N}$$

$$P_i = 1.3\,P = 1.3(251\,327.41) = 326\,725.64 \text{ N}$$

$$\left(\frac{k_b'}{k_b' + k_c'}\right) = \left(\frac{k_b'}{k_b' + 4k_b'}\right) = \frac{1}{5} = 0.2$$

From Eq. (7.23),

$$\Delta P = P\left(\frac{k_b'}{k_b' + k_c'}\right) = 0.2P$$

From Eq. (7.24), the resultant load on the bolt, i.e., (P_b) is given by,

$$P_b = P_i + \Delta p = P_i + 0.2P$$

Therefore, the maximum and minimum forces in the bolt are given by,

$$P_{\max.} = P_i + (0.20)\,P = 326\,725.64 + (0.20)(251\,327.41) = 376\,991.12 \text{ N}$$

$$P_{\min.} = P_i + (0.20)(0) = 326\,725.64 \text{ N}$$

$$P_m = \frac{1}{2}(P_{\max.} + P_{\min.}) = \frac{1}{2}(376\,991.12 + 326\,725.64) = 351\,858.38 \text{ N}$$

$$P_a = \frac{1}{2}(P_{\max.} - P_{\min.}) = \frac{1}{2}(376\,991.12 - 326\,725.64) = 25\,132.74 \text{ N}$$

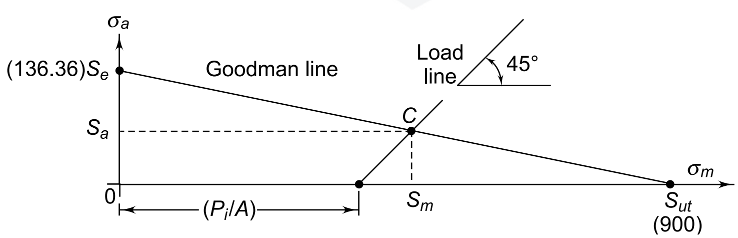

Fig. 7.42

The Goodman diagram for this example is shown in Fig. 7.42.

Step III *Permissible stress amplitude*

The co-ordinates of point $C(S_m, S_a)$ are obtained by solving the following two equations simultaneously,

$$\frac{S_m}{900} + \frac{S_a}{136.36} = 1 \qquad \text{(a)}$$

$$S_m = \frac{P_i}{A} + S_a = \frac{326\,725.64}{A} + S_a \qquad \text{(b)}$$

where A is the tensile stress area of the bolt.

The solution is obtained by Eq. (7.30).

From Eq. (7.30),

$$S_a = \frac{S_{ut} - (P_i/A)}{1 + (S_{ut}/S_e)} = \frac{900 - (326\,725.64/A)}{1 + (900/136.36)}$$

$$= \frac{900 - (326\,725.64/A)}{7.6}$$

Step VI *Size of bolt*

Since $\sigma_a = \frac{S_a}{(fs)}$ $\quad \therefore \frac{P_a}{A} = \frac{S_a}{(fs)}$

or $\frac{25\,132.74}{A} = \frac{900 - (326\,725.64/A)}{7.6(1.5)}$

$$\frac{25\,132.74}{A} = 78.95 - \frac{28\,660.14}{A}$$

or $\frac{53\,792.88}{A} = 78.95$

$A = 681.35 \text{ mm}^2$

There are 8 bolts. Therefore, the area of each bolt is given by,

$$A = \frac{681.35}{8} = 85.17 \text{ mm}^2$$

From Table 7.2, bolts with fine threads M12 × 1.5 ($A = 88.1 \text{ mm}^2$) are suitable for this application.

Example 7.22 *Figure 7.43 shows the arrangement of supporting a machine weighing 200 kg at a distance of 1 m from the nearest point of support. The operation of the machine creates a rotating unbalanced force of 2000 N in the plane of the figure and at the position of the machine. The speed of rotation is 14 rpm. The weight of the channel is 20 kg/m. Two bolts, denoted by 1 and 2, hold the channel to the main frame. The bolts are located at 35 and 270 mm from the nearest point of support. The following data is given for the bolts:*

Ultimate tensile strength = 960 MPa

Yield point strength = 850 MPa

Endurance limit in bending = 500 MPa

Fatigue stress concentration factor = 3.0

Factor of safety = 2

The initial preload in each bolt is 55 kN. The ratio of stiffness of the parts held together by the bolts to the stiffness of the bolts is 3.

Assume Goodman line as the criterion of failure. Determine the size of the bolts.

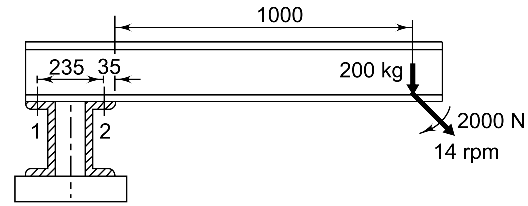

Fig. 7.43

Solution

Given $S_{ut} = 960 \text{ N/mm}^2$ $\quad S_{yt} = 850 \text{ N/mm}^2$

$S'_e = 500 \text{ N/mm}^2$ $\quad (fs) = 2$ $\quad K_f = 3$

$k'_c = 3k'_b$ $\quad P_i = 55 \text{ kN}$

Step I *Endurance limit stress for bolt*

$S'_e = 500\text{MPa}$ or 500 N/mm^2

$$K_d = \frac{1}{K_f} = \frac{1}{3}$$

$$S_e = K_d S'_e = (1/3)(500) = 166.67 \text{ N/mm}^2$$

Step II *Analysis of forces*

As shown in Fig 7.44, when the load tends to tilt the bracket about the nearest point of support *C*, each bolt is stretched by an amount (δ), which is proportional to its distance from the tilting edge.

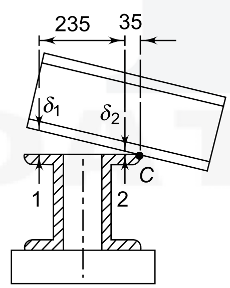

Fig. 7.44

or, $\delta_1 \propto l_1$ and $\delta_2 \propto l_2$

$$\frac{\delta_1}{\delta_2} = \frac{l_1}{l_2} \qquad \text{(a)}$$

It is assumed that the bolts are identical. Therefore,

$\delta_1 = \frac{P_1 L}{AE}$ and $\delta_2 = \frac{P_2 L}{AE}$

$$\frac{\delta_1}{\delta_2} = \frac{P_1}{P_2} \qquad \text{(b)}$$

From (a) and (b),

$$\frac{P_1}{P_2} = \frac{\delta_1}{\delta_2} = \frac{l_1}{l_2} = \frac{270}{35}$$

$$P_2 = \left(\frac{35}{270}\right) P_1 = 0.1296 P_1 \qquad \text{(c)}$$

The length of the channel from the nearest point of support is 1 m. The weight of the channel is 20 kg/m.

Weight of channel = 20 kg = 20(9.81) = 196.2 N

Weight of machine = 200 kg = 200(9.81) = 1962 N

The forces acting on the channel are shown in Fig. 7.45.

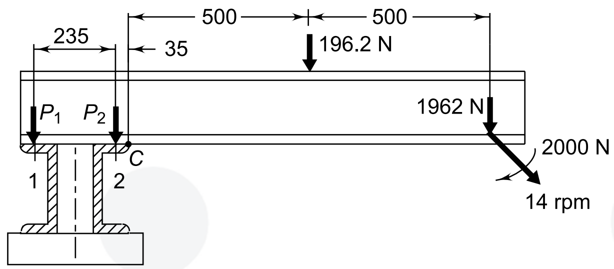

Fig. 7.45

Case 1 *When the rotating force is acting downward*

Taking moment of forces about the point *C*,

$$196.2(500) + (1962 + 2000)(1000) = P_1(270) + P_2(35)$$

$$196.2(500) + (1962 + 2000)(1000) = P_1(270) + (0.1296)\, P_1(35)$$

$\therefore \quad P_1 = 14\,788.95$ N and $P_2 = 1916.65$ N

Case 2 *When the rotating force is acting upward*

Taking moment of forces about the point *C*,

$$196.2(500) + (1962 - 2000)(1000) = P_1(270) + P_2(35)$$

$$196.2(500) + (1962 - 2000)(1000) = P_1(270) + (0.1296)\, P_1(35)$$

$\therefore \quad P_1 = 218.91$ N and $P_2 = 28.37$ N

Therefore, the force acting on the bolt 1 is critical and it fluctuates from 218.91 to 14 788.95 N.

Step III *Construction of Goodman diagram*

$P_i = 55\,000$ N

$$\left(\frac{k_b'}{k_b' + k_c'}\right) = \left(\frac{k_b'}{k_b' + 3k_b'}\right) = \frac{1}{4} = 0.25$$

From Eq. (7.23),

$$\Delta P = P\left(\frac{k_b'}{k_b' + k_c'}\right) = 0.25\,P$$

From Eq. (7.24), the resultant load on the bolt, i.e., (P_b) is given by,

$$P_b = P_i + \Delta P = P_i + 0.25\,P$$

Therefore, the maximum and minimum forces in the bolt 1 are given by,

$$P_{max.} = P_i + (0.25)\,P = 55\,000 + (0.25)(14\,788.95) = 58\,697.24 \text{ N}$$

$$P_{min.} = P_i + (0.25)(P) = 55\,000 + (0.25)(218.91) = 55\,054.73 \text{ N}$$

$$P_m = \frac{1}{2}(P_{max.} + P_{min.}) = \frac{1}{2}(58\,697.24 + 55\,054.73) = 56\,875.99 \text{ N}$$

$$P_a = \frac{1}{2}(P_{max.} - P_{min.}) = \frac{1}{2}(58\,697.24 - 55\,054.73) = 1821.26 \text{ N}$$

$$\tan\theta = \frac{P_a}{P_m} = \frac{1821.26}{56\,875.99} = 0.032$$

or $\theta = 1.834°$

It should be noted that the external force (P) does not vary from zero to some maximum value in this example. On the contrary, it varies from 218.91 N to 14 788.95 N in every cycle. Therefore, the load line will not be inclined at 45° to the *X*-axis. The load line will be inclined at 1.834° with the *X*-axis.

The Goodman line for this example is shown in Fig. 7.46.

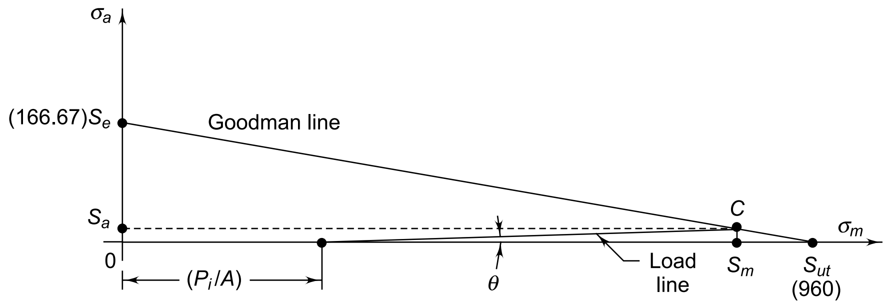

Fig. 7.46

Step IV *Permissible stress amplitude*

The co-ordinates of the point C (S_m, S_a) are obtained by solving the following two equations simultaneously:

$$\frac{S_m}{960} + \frac{S_a}{166.67} = 1 \qquad \text{(a)}$$

$$S_m = \frac{P_i}{A} + x = \frac{P_i}{A} + \frac{S_a}{\tan\theta}$$

$$S_m = \frac{55\,000}{A} + \frac{S_a}{0.032} \qquad \text{(b)}$$

where A is the tensile stress area of the bolt.

Substituting (b) in (a),

$$\frac{55\,000}{960A} + \frac{S_a}{960(0.032)} + \frac{S_a}{166.67} = 1$$

$$S_a\left[\frac{1}{960(0.032)} + \frac{1}{166.67}\right] = 1 - \frac{55\,000}{960\,A}$$

$$\frac{S_a}{25.939} = 1 - \frac{57.29}{A}$$

$$S_a = 25.939 - \frac{1486.05}{A}$$

Step V *Size of bolt*

$$\text{Since} \quad \sigma_a = \frac{S_a}{(fs)} \qquad \therefore \frac{P_a}{A} = \frac{S_a}{(fs)}$$

$$\text{or} \quad \frac{1821.26}{A} = \frac{1}{2}\left[25.939 - \frac{1486.05}{A}\right]$$

$$\frac{2564.29}{A} = 12.97$$

$$A = 197.71 \text{ mm}^2$$

From Table 7.2, the bolts with fine threads M20 $\times$ 2 (A = 258 mm^2) are suitable for this application.

Step V *Check for static design*

$$P_{max.} = 58697.24 \text{ N}$$

$$\sigma_t = \frac{P_{max.}}{A} = \frac{58\,697.24}{258} = 227.51 \text{ N/mm}^2$$

$$(fs) = \frac{S_{yt}}{\sigma_t} = \frac{850}{227.51} = 3.74$$

Example 7.23 *A copper reinforced asbestos gasket is used between the cover plate and the flanged end of a pressure vessel as shown in Fig. 7.47. The pressure inside the vessel varies from 0 to 1 MPa during the operation. The gasket requires a seating pressure of 5 MPa to make the joint leakproof. The total number of bolts used is 8. The ratio of stiffness of the parts held together by the bolts to the stiffness of the bolts is 4. The effective sealing area may be taken up to the mean radius of the gasket. The factor of safety is to be 2 for the fluctuating load. The following data is given for the bolts:*

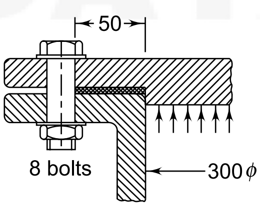

Fig. 7.47

Bolt material = 40Ni3
Ultimate tensile strength = 780 MPa
Yield point strength = 580 MPa
Endurance limit in bending = 260 MPa
Fatigue stress concentration factor = 3.0
Determine the size of the bolts.

Solution

Given $S_{ut} = 780$ N/mm^2 $S_{yt} = 580$ N/mm^2

$S_e' = 260$ N/mm^2 $(fs) = 2$ $K_f = 3$ $k_c' = 4k_b'$

For vessel, $p = 0$ to 1 MPa seating pressure of gasket = 5 MPa

Step I *Endurance limit stress for bolt*

$$S_e' = 260 \text{ MPa} \quad \text{or} \quad 260 \text{ N/mm}^2$$

$$K_d = \frac{1}{K_f} = \frac{1}{3}$$

$$S_e = K_d S_e' = (1/3)(260) = 86.67 \text{ N/mm}^2$$

Step II *Construction of Goodman diagram*

The seating pressure is 5 MPa (or 5 N/mm^2). Therefore, the initial preload on eight bolts is given by,

$$\text{Preload} = \text{seating pressure} \times \text{gasket area}$$

$$= (5)\left[\frac{\pi}{4}(400^2 - 300^2)\right] = 274\,889.36 \text{ N}$$

$$P_i = \text{Preload per bolt} = \left(\frac{1}{8}\right)(274\,889.36)$$

$$= 34\,361.17 \text{ N}$$

The effective sealing area is up to the mean radius of the gasket. Therefore, the total external load is given by,

$$\text{External load} = \left(\frac{\pi}{4}\right)(300 + 50)^2(1) = 96\,211.28 \text{ N}$$

$$\text{External load per bolt} = \left(\frac{1}{8}\right)(96\,211.28)$$

$$= 12\,026.41 \text{ N}$$

$$\left(\frac{k_b'}{k_b' + k_c'}\right) = \left(\frac{k_b'}{k_b' + 4k_b'}\right) = \frac{1}{5} = 0.20$$

From Eq. (7.23),

$$\Delta P = P\left(\frac{k_b'}{k_b' + k_c'}\right) = 0.20\,P$$

From Eq. (7.24), the resultant load on the bolt, i.e., (P_b) is given by,

$$P_b = P_i + \Delta P = P_i + 0.20P$$

Therefore, the maximum and minimum forces in the bolt are given by,

$$P_{max.} = P_i + (0.20)\,P$$
$$= 34\,361.17 + (0.20)(12\,026.41)$$
$$= 36\,766.45 \text{ N}$$
$$P_{min.} = P_i + (0.20)(P)$$
$$= 34\,361.17 + (0.20)(0) = 34\,361.17 \text{ N}$$

$$P_m = \frac{1}{2}(P_{max.} + P_{min.})$$
$$= \frac{1}{2}(36\,766.45 + 34\,361.17)$$
$$= 35\,563.81 \text{ N}$$
$$P_a = \frac{1}{2}(P_{max.} - P_{min.})$$
$$= \frac{1}{2}(36\,766.45 - 34\,361.17)$$
$$= 1202.64 \text{ N}$$

The Goodman line for this example is shown in Fig. 7.48.

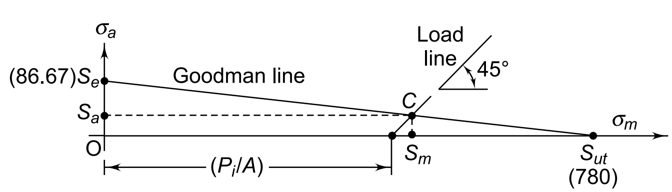

Fig. 7.48

Step III *Permissible stress amplitude*

The co-ordinates of the point C (S_m, S_a) are obtained by solving the following two equations simultaneously:

$$\frac{S_m}{780} + \frac{S_a}{86.67} = 1 \qquad \text{(a)}$$

$$S_m = \frac{P_i}{A} + S_a = \frac{34\,361.17}{A} + S_a \qquad \text{(b)}$$

where A is tensile stress area of the bolt.

The solution is obtained by Eq. (7.30).

From Eq. (7.30),

$$S_a = \frac{S_{ut} - (P_i/A)}{1 + (S_{ut}/S_e)} = \frac{780 - (34\,361.17/A)}{1 + (780/86.67)}$$

$$= \frac{780 - (34\,361.17\,/\,A)}{10}$$

Step IV *Size of bolt*

Since $\sigma_a = \dfrac{S_a}{(fs)} \qquad \therefore \dfrac{P_a}{A} = \dfrac{S_a}{(fs)}$

or $\dfrac{1202.64}{A} = \dfrac{1}{2}\left[\dfrac{780 - (34361.17/A)}{10}\right]$

$$\frac{1202.64}{A} = 39 - \frac{1718.06}{A}$$

or $\dfrac{2920.7}{A} = 39$

$A = 74.89 \text{ mm}^2$

From Table 7.2, bolts with fine threads M12 × 1.5 ($A = 88.1 \text{ mm}^2$) are suitable for this application.

Short-Answer Questions

7.1 What is threaded joint?
7.2 What are the advantages of threaded joints?
7.3 What are the disadvantages of threaded joints?
7.4 What is a through bolt?
7.5 What is a machine bolt?
7.6 What is an automobile bolt?
7.7 What is a tap bolt?
7.8 What is a cap screw?
7.9 When do you use tap bolts and cap screw?
7.10 What is a stud?
7.11 Why is hexagonal head preferred for cap screw instead of square head?
7.12 What is a setscrew?
7.13 What is bolt of uniform strength?
7.14 What are the methods of preventing loosening of threads between the nut and the screw?
7.15 What is lock nut? What is the principle of lock nut?
7.16 What is a castle nut? Why is it called castle nut?
7.17 What is a split pin?
7.18 How is locking of threads obtained in castle nut?
7.19 What is a split nut?
7.20 How is locking of threads obtained in split nut?
7.21 What is nominal diameter of screw thread?
7.22 What is root diameter of screw thread?
7.23 What is pitch diameter of screw thread?
7.24 What is pitch of screw thread?
7.25 What is lead of screw thread?
7.26 What is thread angle of screw thread?
7.27 What is magnitude of thread angle of ISO metric thread?
7.28 What is tensile stress area of screw thread?
7.29 What are the advantages of coarse threads?
7.30 What are the advantages of fine threads?
7.31 What are the applications of coarse threads?
7.32 What are the applications of fine threads?
7.33 How will you designate ISO metric coarse threads?
7.34 How will you designate ISO metric fine threads?
7.35 What do you understand by 'hard' and 'soft' gaskets?

Problems for Practice

7.1 A gearbox weighing 7.5 kN is provided with a steel eye bolt for lifting and transporting on the shop-floor. The eyebolt is made of plain carbon steel 30C8 ($S_{yt} = 400 \text{ N/mm}^2$) and the factor of safety is 5. Determine the nominal diameter of the eye bolt having coarse threads if,

$$d_c = 0.8d$$

where d_c and d are core and major diameters respectively.

[*13.66 mm*]

7.2 A steam engine cylinder has an effective diameter of 250 mm. It is subjected to a maximum steam pressure of 1.5 MPa (1.5 N/mm^2). The cylinder cover is fixed to the cylinder flange by means of 12 studs. The pitch circle diameter of the studs is 400 mm. The permissible tensile stress in the studs is limited to 30 N/mm^2.

(i) Determine the nominal diameter of the studs if $d_c = 0.84d$.

(ii) Calculate the circumferential pitch of the studs. Is it satisfactory?

[*(i) 19.21 mm (ii) 104.72 mm, between 5d to 10d*]

7.3 A steel plate subjected to a force of 3 kN and fixed to a vertical channel by means of four identical bolts is shown in Fig. 7.49. The bolts are made of plain carbon steel 45C8 (S_{yt} = 380 N/mm^2) and the factor of safety is 2. Determine the diameter of the shank.

[*6.58 mm*]

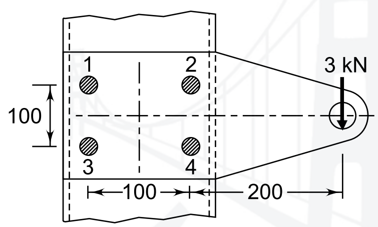

Fig. 7.49

7.4 A steel plate subjected to a force of 5 kN and fixed to a channel by means of three identical bolts is shown in Fig. 7.50. The bolts are made of plain carbon steel 30C8 (S_{yt} = 400 N/mm^2) and the factor of safety is 3. Determine the diameter of the shank.

[*12.74 mm*]

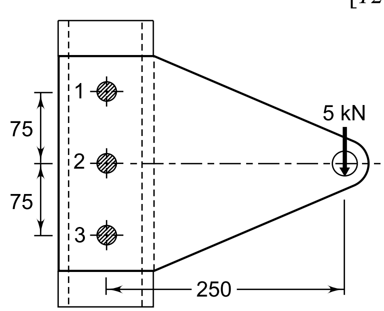

Fig. 7.50

7.5 A bracket for supporting the travelling crane is shown in Fig. 7.51. The bracket is fixed to the steel column by means of four identical bolts, two at *A* and two at *B*. The maximum load that comes on the bracket is 5 kN acting vertically downward at a distance of 250 mm from the face of the column. The bolts are made of steel 40C8 (S_{yt} = 380 N/mm^2) and the factor of safety is 5. Determine the major diameter of the bolts on the basis of maximum principal stress. Assume (d_c = 0.8d)

[*7.74 mm*]

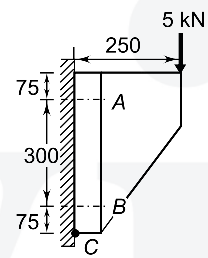

Fig. 7.51

7.6 A cast iron bracket, as shown in Fig. 7.52, supports a load of 10 kN. It is fixed to the horizontal channel by means of four identical bolts, two at *A* and two at *B*. The bolts are made of steel 30C8 (S_{yt} = 400 N/mm^2) and the factor of safety is 6. Determine the major diameter of the bolts if (d_c = 0.8d).

[*17.73 mm*]

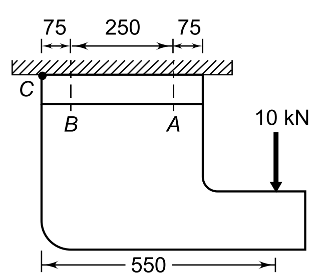

Fig. 7.52

7.7 Assume the following data for the cast iron bracket shown in Fig. 7.25(a).

l_1 = 75 mm $\quad$ l_2 = 225 mm

l = 300 mm $\quad$ P = 10 kN

The bracket is fixed to the horizontal column by means of four identical bolts, two at *A* and two at *B*. The bolts are made of steel 30C8 (S_{yt} = 400 N/mm^2) and the factor of safety is 5. Determine the nominal diameter of the bolts if (d_c = 0.8d).

[*12.22 mm*]

7.8 A pillar crane has a circular base of 750 mm diameter, which is fixed to the concrete base by four identical bolts, equally spaced on the pitch circle of 600 mm diameter as shown in Fig. 7.53. The bolts are made of steel 40C8 (S_{yt} = 380 N/mm^2) and the factor of safety is 5. The maximum load is 50 kN which acts at a radius of 750 mm from the axis of the crane. Determine the size of the bolts if,

(i) the load acts in plane-*XX* at the point P_1; and

(ii) the load acts in plane-*YY* at Point P_2.

Assume (d_c = 0.8*d*)

[*(i) 19.7 mm (ii) 21.12 mm*]

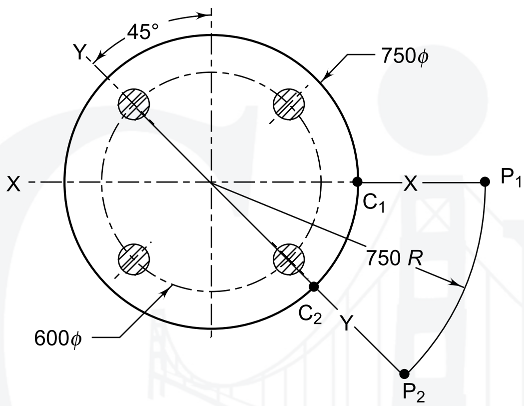

Fig. 7.53

7.9 The maximum pull in the tie rods of a turnbuckle used in the roof truss is 4.5 kN. The tie rods are made of steel 40C8 (S_{yt} = 380 N/mm^2) and the factor of safety is 5. Determine the nominal diameter of the threads on the tie rod on the basis of maximum principal stress theory. Assume d_c = 0.8*d*.

[*11.92 mm*]

7.10 A bolted joint is used to connect two components. The combined stiffness of the two components is twice the stiffness of the bolt. The initial tightening of the nut results in a preload of 10 kN in the bolt. The external force of 7.5 kN creates further tension in the bolt. The bolt is made of plain carbon steel 30C8 (S_{yt} = 400 N/mm^2) and the factor of safety is 3. There are coarse threads on the bolt. Calculate the tensile stress area of the bolt and specify a suitable size for the bolt.

[*93.75 mm^2 and M 16*]

7.11 A through bolt is used to connect two plates with a gasket in between. The combined stiffness of the parts held together by the bolt is three times the stiffness of the bolt. The bolted assembly is subjected to an external tensile force, which fluctuates from 0 to 5 kN. The initial preload in the bolt is 4.5 kN. The bolt has M8 coarse threads with a stress area of 36.6 mm^2. The bolt is made of plain carbon steel 45C8 (S_{ut} = 630 N/mm^2 and S_{yt} = 380 N/mm^2). The fatigue stress concentration factor is 3. Determine the factor of safety used in the design for a reliability of 50%.

[*4.24*]

Welded and Riveted Joints

Chapter 8

8.1 WELDED JOINTS

Welding can be defined as a process of joining metallic parts by heating to a suitable temperature with or without the application of pressure. Welding is an economical and efficient method for obtaining a permanent joint of metallic parts.

There are two distinct applications of welded joints—a welded joint can be used as a substitute for a riveted joint and a welded structure as an alternative method for casting or forging. Welded joints offer the following advantages compared with riveted joints:

(i) Riveted joints require additional cover plates, gusset plates, straps, clip angles and a large number of rivets, which increase the weight. Since there are no such additional parts, welded assembly results in lightweight construction. Welded steel structures are lighter than the corresponding iron castings by 50% and steel castings by 30%.

(ii) Due to the elimination of these components, the cost of welded assembly is lower than that of riveted joints.

(iii) The design of welded assemblies can be easily and economically modified to meet the changing product requirements. Alterations and additions can be easily made in the existing structure by welding.

(iv) Welded assemblies are tight and leakproof as compared with riveted assemblies.

(v) The production time is less for welded assemblies.

(vi) When two parts are joined by the riveting method, holes are drilled in the parts to accommodate the rivets. The holes reduce the cross-sectional area of the members and result in stress concentration. There is no such problem in welded connections.

(vii) A welded structure has smooth and pleasant appearance. The projection of rivet head adversely affects the appearance of the riveted structure.

(viii) The strength of welded joint is high. Very often, the strength of the weld is more than the strength of the plates that are joined together.

(ix) Machine components of certain shape, such as circular steel pipes, find difficulty in riveting. However, they can be easily welded.

Welded structures offer the following advantages over cast iron structures:

(i) Welded structures made of mild steel plates, angles and rods are lighter in weight than cast iron or cast steel structures.

(ii) In welded structures, the metal is put exactly where it is required. Heavy plates can be used where strength is required and thin plates can be used at other places. The rules of uniform cross-section and minimum section thickness required for casting process are not necessary for a welded design. The designer has more freedom and flexibility in the design of welded assemblies.

(iii) Welded assemblies are more easily machined than castings.

(iv) The capital investment for a welding shop is considerably lower than that for a foundry shop. Here, the cost of pattern making and storing is eliminated which reduces the cost of welded structures.

Welded joints have the following disadvantages:

(i) As compared with cast iron structures, the capacity of welded structure to damp vibrations is poor.

(ii) Welding results in a thermal distortion of the parts, thereby inducing residual stresses. In many cases, stress-relieving heat treatment is required to relieve residual stresses. Riveted or cast structures do not require such stress-relieving treatment.

(iii) The quality and the strength of the welded joint depend upon the skill of the welder. It is difficult to control the quality when a number of welders are involved.

(iv) The inspection of the welded joint is more specialised and costly compared with the inspection of riveted or cast structures.

Today, riveting has been superseded by welding in metal working industries, ship building industries and boiler manufacture, except for certain special cases. Welding has also become the chief method to make joints in steel structures in civil engineering.

8.2 WELDING PROCESSES

Welding processes are broadly classified into the following two groups:

(i) Welding processes that use heat alone to join the two parts.

(ii) Welding processes that use a combination of heat and pressure to join the two parts.

The welding process that uses heat alone is called the *fusion* welding process. In this method, the parts to be joined are held in position and molten metal is supplied to the joint. The molten metal can come either from the parts themselves called ‘parent’ metal or an external filler metal is supplied to the joint. The joining surfaces of the two parts become plastic or even molten under the action of heat. When the joint solidifies, the two parts fuse into a single unit. Fusion welding is further classified into the following three groups:

(i) Thermit welding

(ii) Gas welding

(iii) Electric arc welding

(i) Thermit Welding In this method, a mould is prepared around the joint and thermit is placed in the reservoir of the mould. Thermit consists of a mixture of finely divided iron oxide and aluminum. When thermit is ignited, there is chemical reaction, which converts iron oxide into molten steel. This molten steel flows into the mould, melts the parts and forms the joint on solidifying. The advantage of thermit welding is that all parts of the weld section are molten at the same time and cool at a uniform rate. This minimises the residual stresses induced in the joint. Thermit welding is used to weld heavy sections such as rails in the field, where it is uneconomical to transport welding equipment. It is particularly suitable to join parts of large casting or forging that are complicated to make in one piece. Thermit welding is used to repair heavy steel parts such as heavy machinery frames, locomotive frames and ship structures, where it is not possible to relieve the stresses in the joints. Due to uniform rate of cooling, thermit welding is ideally suitable for these assemblies.

(ii) Gas Welding In the gas welding process, oxygen–hydrogen or oxygen–acetylene gas is burned in a torch to create a pointed flame. This flame is directed upon the surfaces to be joined. The intense heat of the flame heats the adjoining parts of the joint to the fusion temperature and simultaneously melts the welding rod to supply the molten metal to the joint. A flux is used to remove the slag. There is a basic difference between gas welding and electric arc welding. In gas welding, the rate of heating is slow compared with electric arc welding. Therefore, the operator has more control over both heating and cooling rates. Gas welding is preferred for joining thin parts due to low rate of heating.

(iii) Electric arc Welding In this method, the heat required for the fusion is generated by an electric arc between the parts to be joined and an electrode. This electrode can be made of filler metal and consumed as the welding progresses or the electrode can be of non-consumable type. Filler metal is separately fed to the joint in case of non-consumable electrodes. In some cases, the consumable electrode is coated with a flux material that vaporizes to form a gaseous shield around the joint and prevents the molten metal from absorbing oxygen or nitrogen from the atmosphere. If not prevented, the absorption of oxygen or nitrogen adversely affects the mechanical properties of the weld. Electric arc welding is popular in machine building industries as well as for structural work because of the consistently high quality of welding.

Welding processes that use a combination of heat and pressure to join the two parts are classified into the following two groups:

(i) Forge welding
(ii) Electric resistance welding

(i) Forge Welding In forge welding, the parts are heated to reach the plastic stage and the joint is prepared by impact force. The impact force is produced either by a hand hammer or a press. Wrought iron or low carbon steel can be forge welded. This process has limited use in present times. It is used in the fabrication of wrought iron pipes.

(ii) Electric Resistance Welding In electric resistance welding, the parts to be connected are clamped together and a high amperage electric current is passed from one part to another. Resistance of metallic parts to electric current creates heat. This heat is utilized for melting the adjoining parts. No filler material is used in this process. There are two terms related to electric resistance welding, viz., spot welding and seam welding. When a lap joint is formed by applying pressure by two electrodes, one on each side of overlapped plates, a *spot weld* is produced. If two rollers are substituted for the point electrodes and the plates are pulled between the rollers, a '*seam*' *weld* is produced. Electric resistance welding can be easily automated and is often used as a mass production technique. Therefore, it is commonly used in automobile industries.

8.3 STRESS RELIEVING OF WELDED JOINTS

Welded joints are subjected to residual stresses due to non-uniform heating of the parts being joined. There is always a possibility that localised thermal stresses may result from uneven heating and cooling during fusion and subsequent cooling. This also results in distortion. The magnitude of residual stresses cannot be predicted with any degree of certainty. This is the major disadvantage of welded joints. The following two methods can reduce the residual stresses:

(i) Preheating of the weld area to retard cooling of the metal in the vicinity of the joint.
(ii) Stress relieving of weld area by using proper heat treatment such as normalising and annealing in temperature range of 550° to 675°.

One of the methods of stress relieving is *hand peening*. It consists of hammering the weld along the length with the peen of the hammer while the joint is hot. It reduces residual stresses and induces residual compressive stresses on the surfaces. This improves the fatigue strength of the joint.

8.4 BUTT JOINTS

Welded joints are divided into two groups—butt joints and fillet joints. *A butt joint can be defined as a joint between two components lying approximately in the same plane*. A butt joint connects the ends of the two plates. The types of butt joints are illustrated in Fig. 8.1. The selection of the types of butt joints depends upon the plate thickness and the reliability. Some guidelines are as follows:

(i) When the thickness of the plates is less than 5 mm, it is not necessary to bevel the edges of the plates. There is no preparation of the edges of the plates before welding. The edges are square with respect to the plates. Therefore, the joint is called *square butt joint*. It is illustrated in Fig. 8.1(a).

(ii) When the thickness of the plates is between 5 to 25 mm, the edges are beveled before the welding operation. The edges of two plates form a V shape. Therefore, the joint is called *V-joint* or *single welded V-joint*. This joint is welded only from one side. It is illustrated in Fig. 8.1(b).

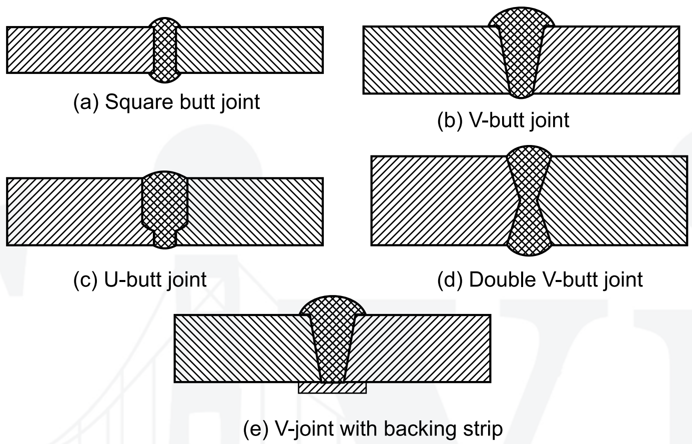

Fig. 8.1 *Types of Butt Joint*

(iii) When the thickness of the plates is more than 20 mm, the edges of the two plates are machined to form a U shape. The joint is welded only from one side. It is called *single welded U-joint*. It is shown in Fig. 8.1(c).

(iv) When the thickness of the plates is more than 30 mm, a double welded V-joint is used. The joint is welded from both sides of the plate. It is shown in Fig. 8.1(d).

When the welding is to be done only from one side, a single welded V-joint with a backing strip is used to avoid the leakage of the molten metal on the other side. There are two types of the backing strip—*permanent steel backing* and *removable copper backing*. This type of joint is shown in Fig. 8.1(e).

In applications like pressure vessels, the reliability of the joint is an important consideration. Single welded V-joint is more reliable than square butt joint. Single welded V-joint with backing strip is more reliable than single welded V-joint without backing strip. Double welded V-joint is more reliable than single welded V-joint with backing strip. The cost also increases with the reliability of the joint.

8.5 FILLET JOINTS

A fillet joint, also called a lap joint, is a joint between two overlapping plates or components. A fillet weld consists of an approximately triangular cross-section joining two surfaces at right angles to each other. There are two types of fillet joints—transverse and parallel, as shown in Fig. 8.2. A fillet weld is called *transverse*, if the direction of the weld is perpendicular to the direction of the force acting on the joint. It is shown in Fig. 8.2(a) and (b). A single transverse fillet joint is not preferred because the edge of the plate, which is not welded, can warp out of shape. In Fig. 8.2(a), the edge of the lower plate is free to deflect. Therefore, a double transverse fillet weld, as shown in Fig. 8.2(b), is preferred. A fillet weld is called *parallel* or *longitudinal*, if the direction of weld is parallel to the direction of the force acting on the joint. It is shown in Fig. 8.2(c).

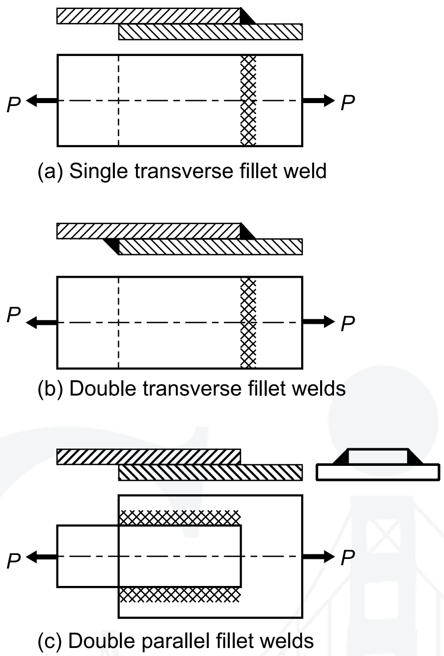

Fig. 8.2 *Types of Fillet Joints*

There are two types of cross-sections for fillet weld—normal and convex, as shown in Fig. 8.3. The *normal weld* consists of an isosceles triangle—a triangle having two equal sides. It is shown in Fig. 8.3(a). A *convex weld* is shown in Fig. 8.3(b). A convex weld requires more filler material and more labour. There is more stress concentration in a convex weld compared to a triangular weld. Therefore, normal weld is preferred over convex weld.

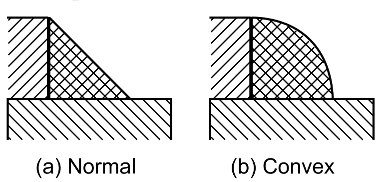

Fig. 8.3 *Cross-section of Fillet Weld*

In addition to butt and fillet joints, there are some other types of welded joints, which are shown in Fig. 8.4. A *tee-joint* is a joint between two components located at right angles to each other in the form of a T. In this case, the end face of one component is welded to the side of the other component by means of a fillet weld as shown in Fig. 8.4(a). A *corner joint* is a joint between two components, which are at right angles to each other in the form of an angle. The adjacent edges are joined by means of a fillet weld as shown in Fig. 8.4(b). An *edge joint* is a joint between the edges of two or more parallel components as shown in Fig. 8.4(c). It is used for thin plates subjected to light loads.

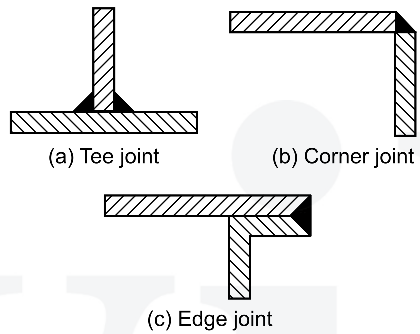

Fig. 8.4 *Types of Welded Joint*

8.6 STRENGTH OF BUTT WELDS

A butt welded joint, subjected to tensile force P, is shown in Fig. 8.5. The average tensile stress in the weld is given by,

$$\sigma_t = \frac{P}{hl} \qquad (8.1)$$

where,

σ_t = tensile stress in the weld (N/mm^2)
P = tensile force on the plates (N)
h = throat of the butt weld (mm)
l = length of the weld (mm)

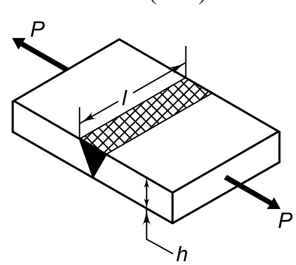

Fig. 8.5 *Butt Weld in Tension*

The throat of the weld does not include the bulge or reinforcement. The reinforcement is provided to compensate for flaws in the weld. Equating the throat of the weld h to the plate thickness t in Eq. (8.1), the strength equation of butt joint can be written as,

$$P = \sigma_t tl \tag{8.2}$$

where,

P = tensile force on plates (N)

σ_t = permissible tensile stress for the weld (N/mm^2)

t = thickness of the plate (mm)

There are certain codes, like code for unfired pressure vessels, which suggest reduction in strength of a butt welded joint by a factor called *efficiency of the joint*. Where the strength is to be reduced, Eq. (8.2) is modified and rewritten in the following way,

$$P = \sigma_t tl\eta \tag{8.3}$$

where,

η = efficiency of the welded joint (in fraction)

Butt welded joint, when properly made, has equal or better strength than the plates and there is no need for determining the stresses in the weld or the size and the length of the weld. All that is required is to match the strength of the weld material to the strength of the plates.

Example 8.1 *A gas tank consists of a cylindrical shell of 2.5 m inner diameter. It is enclosed by hemispherical shells by means of butt welded joint as shown in Fig. 8.6. The thickness of the cylindrical shell as well as the hemispherical cover is 12 mm. Determine the allowable internal pressure to which the tank may be subjected, if the permissible tensile stress in the weld is 85 N/mm^2.*

Assume efficiency of the welded joint as 0.85.

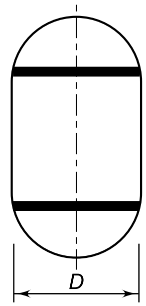

Fig. 8.6

Solution

Given For shell, $D = 2.5$ m $t = 12$ mm

For weld, $\sigma_t = 85$ N/mm^2 $\eta = 0.85$

Step I *Tensile force on plates*

The length of the welded joint is equal to the circumference of the cylindrical shell.

$$l = \pi D = \pi (2.5 \times 10^3) = 7853.98 \text{ mm}$$

From Eq. (8.3),

$$P = \sigma_t tl\eta = (85)(12)(7853.98)(0.85) = (6809.4 \times 10^3) \text{ N}$$

Step II *Allowable internal pressure*

Corresponding pressure inside the tank is given by

$$p = \frac{P}{\frac{\pi}{4}D^2} = \frac{(6809.4 \times 10^3)}{\frac{\pi}{4}(2.5 \times 10^3)^2} = 1.39 \text{ N/mm}^2$$

8.7 STRENGTH OF PARALLEL FILLET WELDS

A parallel fillet weld subjected to a tensile force P is shown in Fig. 8.7(a). The enlarged view of the fillet weld is shown in 8.7(c). There are two terms related to the dimensions of the fillet weld, viz., leg h and throat t. The size of the weld is specified by the leg length. As explained in Section 8.3, the cross-section of the fillet weld consists of a right-angled triangle having two equal sides. The length of each of the two equal sides is called a *leg*. As a rule, the leg length h is equal to the plate thickness. The *throat* is the minimum cross-section of the weld located at 45° to the leg dimension. Therefore,

$$t = h \cos (45°)$$

or

$$t = 0.707\, h \tag{8.4}$$

Failure of the fillet weld occurs due to shear along the minimum cross-section at the throat. It will be proved at a later stage in Section 8.7 that for parallel fillet weld, the inclination of the plane where maximum shear stress is induced, is 45° to the leg dimension. The shear failure of the weld is shown in Fig. 8.7(b). The cross-sectional area at the throat is (tl) or (0.707 hl). The shear stress in the fillet weld is given by,

$$\tau = \frac{P}{0.707\, hl} \tag{8.5}$$

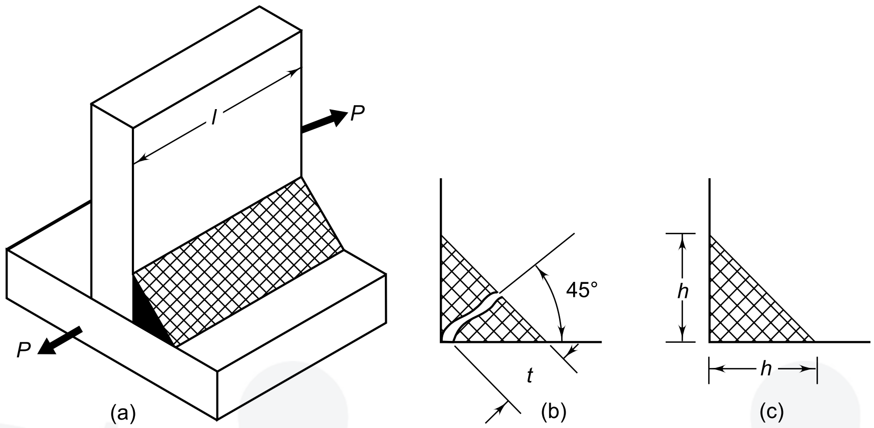

Fig. 8.7 *Parallel Fillet Weld in Shear*

Rearranging the terms of Eq. (8.5), the strength equation of the parallel fillet weld is written in the following form:

$$P = 0.707\, hl\tau \qquad (8.6)$$

where,

P = tensile force on plates (N)
h = leg of the weld (mm)
l = length of the weld (mm)
τ = permissible shear stress for the weld (N/mm^2)

Usually, there are two welds of equal length on two sides of the vertical plate. In that case,

$$P = 2\,(0.707\, hl\tau)$$

or

$$P = 1.414\, hl\tau \qquad (8.7)$$

In determining the required length of the weld, 15 mm should be added to the length of each weld calculated by Eqs (8.6) and (8.7) to allow for starting and stopping of the weld run. In case of a static load, the permissible shear stress for the fillet welds is taken as 94 N/mm^2 as per the code of American Welding Society (AWS).

8.8 STRENGTH OF TRANSVERSE FILLET WELDS

A transverse fillet weld subjected to a tensile force P is shown in Fig. 8.8(a). The transverse fillet welds are subjected to tensile stress. The minimum cross-section of the weld is at the throat. Therefore, the failure due to tensile stress will occur at the throat section. The cross-sectional area at the throat is (tl). The tensile stress in the transverse fillet weld is given by,

$$\sigma_t = \frac{P}{tl}$$

Substituting Eq. (8.4) in the above equation,

$$\sigma_t = \frac{P}{0.707\, hl} \qquad (8.8)$$

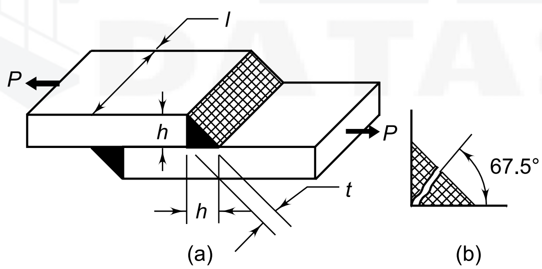

Fig. 8.8 *Failure of Fillet Weld*

Rearranging the terms of Eq. (8.8), the strength equation of the transverse fillet weld is written in the following form,

$$P = 0.707\, hl\sigma_t \qquad (8.9)$$

where,

σ_t = permissible tensile stress for the weld (N/mm^2)

Usually, there are two welds of equal length on two sides of the plate as shown in Fig. 8.8(a). In such cases,

$$P = 2\,(0.707\,hl\sigma_t)$$

or $$P = 1.414hl\sigma_t \qquad (8.10)$$

The nature of stresses in the cross-section of the transverse fillet weld is complex. The weld is subjected to normal stress as well as shear stress. In addition, the throat is subjected to bending moment, which adds to the complications. Theoretically, it can be proved that for transverse fillet weld, the inclination of the plane, where maximum shear stress is induced, is 67.5° to the leg dimension as shown in Fig. 8.8(b). In order to simplify the design of fillet welds, many times shear failure is used as the failure criterion. It is assumed that the stress in the transverse fillet weld is shear stress on the throat area for any direction of applied load. With this assumption, Eqs (8.6) and (8.7) derived for parallel fillet welds are also used for the transverse fillet welds.

Example 8.2 *A steel plate, 100 mm wide and 10 mm thick, is welded to another steel plate by means of double parallel fillet welds as shown in Fig. 8.9. The plates are subjected to a static tensile force of 50 kN. Determine the required length of the welds if the permissible shear stress in the weld is 94 N/mm².*

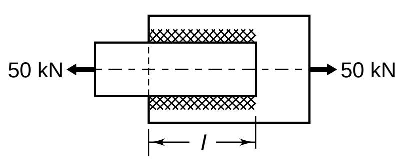

Fig. 8.9

Solution

Given $P = 50$ kN $\tau = 94$ N/mm² $h = 10$ mm

Step I *Length of weld*

From Eq. (8.7),

$$P = 1.414\,hl\tau$$

or $50 \times 10^3 = 1.414\,(10)\,l\,(94)$

$\therefore\quad l = 37.62$ mm

Adding 15 mm of length for starting and stopping of the weld run, the length of the weld is given by,

$$l = 37.62 + 15 = 52.62 \text{ or } 55 \text{ mm}$$

Example 8.3 *Two steel plates, 120 mm wide and 12.5 mm thick, are joined together by means of double transverse fillet welds as shown in Fig. 8.10. The maximum tensile stress for the plates and the welding material should not exceed 110 N/mm². Find the required length of the weld, if the strength of weld is equal to the strength of the plates.*

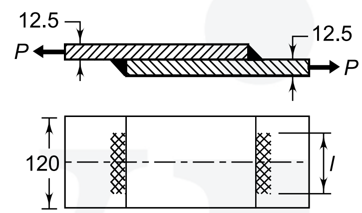

Fig. 8.10

Solution

Given For plates $w = 120$ mm $t = 12.5$ mm

For welds $h = 12.5$ mm $\sigma_t = 110$ N/mm²

Step I *Tensile force on plates*

The plates are subjected to tensile stress. The maximum tensile force acting on the plates is given by,

$$P = (wt)\,\sigma_t = (120 \times 12.5)\,(110) = 165\,000 \text{ N}$$

Step II *Length of the weld*

From Eq. (8.10),

$P = 1.414\,h\,l\,\sigma_t$ or

$165\,000 = 1.414\,(12.5)\,l\,(110)$

$\therefore\quad l = 84.87$ mm

Adding 15 mm for starting and stopping of the weld run, the required length of the weld is given by,

$$l = 84.87 + 15 = 99.87 \text{ or } 100 \text{ mm}$$

Example 8.4 *A plate, 75 mm wide and 10 mm thick, is joined with another steel plate by means of single transverse and double parallel fillet welds, as shown in Fig. 8.11. The joint is subjected to a maximum tensile force of 55 kN. The permissible tensile and shear stresses in the weld material* *are 70 and 50 N/mm² respectively. Determine the required length of each parallel fillet weld.*

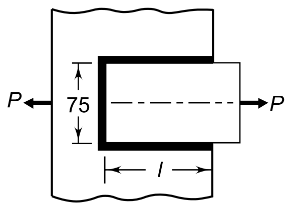

Fig. 8.11

Solution

Given $P = 55$ kN $\tau = 50$ N/mm²
$\sigma_t = 70$ N/mm² $h = 10$ mm

Step I *Strength of transverse and parallel fillet welds*

The strength of the transverse fillet weld is denoted by P_1. From Eq. (8.9),

$$P_1 = 0.707hl\sigma_t = 0.707\ (10)\ (75)\ (70)$$
$$= 37\ 117.5\ \text{N} \qquad \text{(i)}$$

The strength of the double parallel fillet weld is denoted by P_2. From Eq. (8.7),

$$P_2 = 1.414hl\tau = 1.414\ (10)\ l\ (50)$$
$$= (707 \times l)\ \text{N} \qquad \text{(ii)}$$

Step II *Length of parallel fillet weld*

The total strength of the joint should be 55 kN.

From (i) and (ii),

$$37\ 117.5 + 707 \times l = 55 \times 10^3$$
$$\therefore \quad l = 25.29 \text{ mm}$$

Adding 15 mm for starting and stopping of the weld run, the length of the weld is given by,

$$l = 25.29 + 15 = 40.29 \text{ or } 45 \text{ mm}$$

Example 8.5 *A steel plate, 100 mm wide and 10 mm thick, is joined with another steel plate by means of single transverse and double parallel fillet welds, as shown in Fig. 8.12. The strength of the welded joint should be equal to the strength of the plates to be joined. The permissible tensile and shear stresses for the weld material and the plates are 70 and 50 N/mm² respectively. Find the length of each parallel fillet weld. Assume the tensile force acting on the plates as static.*

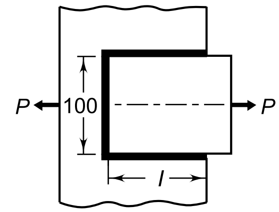

Fig. 8.12

Solution

Given For plates $w = 100$ mm $t = 10$ mm
For welds $h = 10$ mm $\sigma_t = 70$ N/mm²
$\tau = 50$ N/mm²

Step I *Tensile strength of plate*

The tensile strength of the plate is given by,

$$P = (w \times t)\ \sigma_t = (100 \times 10)\ (70) = 70\ 000\ \text{N} \qquad \text{(i)}$$

Step II *Strength of transverse and parallel fillet welds*

The strength of the transverse fillet weld is denoted by P_1. From Eq. (8.9),

$$P_1 = 0.707hl\sigma_t = 0.707\ (10)\ (100)\ (70)$$
$$= 49\ 490\ \text{N} \qquad \text{(ii)}$$

The strength of the double parallel fillet weld is denoted by P_2. From Eq. (8.7),

$$P_2 = 1.414hl\tau = 1.414\ (10)\ l\ (50) = 707 \times l \qquad \text{(iii)}$$

Step III *Length of parallel fillet weld*

The strength of the welded joint is equal to the strength of the plate.

From (i) , (ii) and (iii),

$$70\ 000 = 49\ 490 + 707 \times l$$
$$\therefore \quad l = 29.01 \text{ mm}$$

Adding 15 mm for starting and stopping of the weld run,

$$l = 29.01 + 15 = 44.01 \text{ or } 45 \text{ mm}$$

Example 8.6 *Two plates are joined together by means of single transverse and double parallel fillet welds as shown in Fig. 8.13. The size of the fillet weld is 5 mm and allowable shear load per mm of weld is 330 N. Find the length of each parallel fillet weld.*

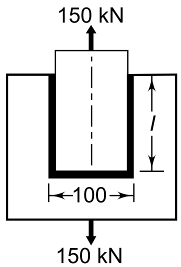

Fig. 8.13

Solution

Given $P = 150$ kN $h = 5$ mm

Allowable shear load = 330 N/mm

Step I *Total length of weld*

It is mentioned earlier that the transverse fillet weld is designed on the basis of shear stress. In such cases, the stress in the fillet weld is considered as shear stress on the throat for any direction of applied load. With this assumption, the equations derived for the parallel fillet weld are also applicable to the transverse fillet weld.

Suppose L is the total length of welds required for the joint. Since the allowable shear load per mm length of weld is 330 N, the required length of weld is given by,

$$L = \frac{150 \times 10^3}{330} = 454.55 \text{ mm} \qquad \text{(i)}$$

Step II *Length of parallel fillet weld*

From Fig. 8.13,

$$L = 2 \times l + 100 \qquad \text{(ii)}$$

From (i) and (ii),

$$2 \times l + 100 = 454.55$$

$$\therefore \quad l = 177.27 \text{ mm}$$

Adding 15 mm for starting and stopping of the weld run,

$$l = 177.27 + 15 = 192.27 \text{ or } 195 \text{ mm}$$

8.9 MAXIMUM SHEAR STRESS IN PARALLEL FILLET WELD

A double parallel fillet weld of equal legs subjected to a force of ($2P$) is shown in Fig. 8.14(a). It is required to find out the inclination (θ) of the plane in the weld, where maximum shear stress is induced and also, the magnitude of the maximum shear stress. The effect of bending is to be neglected. The free body diagram of forces acting on the vertical plate with two welds cut symmetrically is shown in Fig. 8.14(b). The symbol × (cross) indicates a force perpendicular to the plane of paper, which goes away from the observer. The symbol • (dot) indicates a force perpendicular to the plane of paper, which is towards the observer. The welds are cut at an angle θ with the horizontal. t' is the width of plane that is inclined at angle θ with the horizontal.

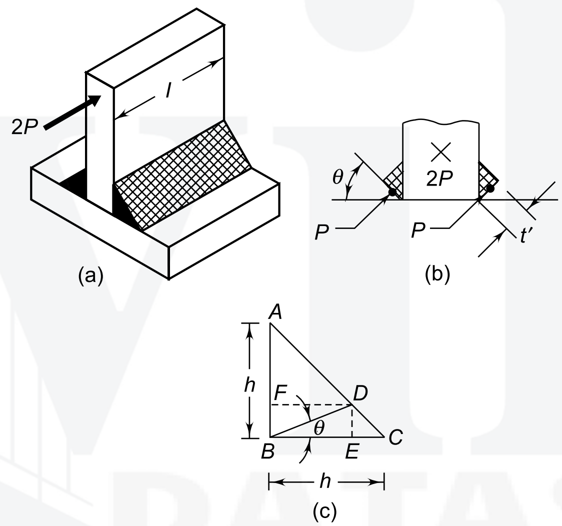

Fig. 8.14

In the triangle ABC (Fig. 8.14c),

$$AB = BC = h$$

$$\therefore \quad \angle ECD = 45°$$

$$DE \perp BC$$

$$BC = BE + EC$$

$$= BE + DE \qquad (DE = EC)$$

$$= BD \cos \theta + BD \sin \theta$$

$$= BD (\cos \theta + \sin \theta)$$

or,

$$h = t' (\sin \theta + \cos \theta)$$

Therefore,

$$t' = \frac{h}{(\sin \theta + \cos \theta)} \qquad (8.11)$$

The area of the weld in the plane inclined at angle θ with horizontal is $(t'l)$. Therefore, the shear stress in this plane is given by,

$$\tau = \frac{P}{(t'l)}$$

Substituting Eq. (8.11) in the above expression,

$$\tau = \frac{P(\sin\theta + \cos\theta)}{hl} \qquad \text{(a)}$$

In order to find out the plane with maximum shear stress, differentiate τ with respect to θ and set the derivative equal to zero.

$$\frac{\partial\tau}{\partial\theta} = \frac{P}{hl}(\cos\theta - \sin\theta) = 0$$

$$\cos\theta - \sin\theta = 0$$

$$\sin\theta = \cos\theta$$

$$\tan\theta = 1$$

Therefore,

$$\theta = 45° \qquad (8.12)$$

The condition for plane with maximum shear stress is $(\theta = 45°)$.

Substituting this value of θ in Eq. (a), the maximum shear stress is given by,

$$\tau_{\text{max.}} = \frac{P(\sin 45° + \cos 45°)}{hl}$$

$$= \frac{1.414\,P}{hl} = \frac{P}{(1/1.414)\,hl} = \frac{P}{0.707hl}$$

or $$\tau_{\text{max.}} = \frac{P}{0.707hl} \qquad (8.13)$$

The above equation is same as Eq. (8.5). Substituting (l = 1 mm) in Eq. (8.13), the allowable load P_{all} per mm length of the weld is given by,

$$P_{\text{all.}} = 0.707h\tau_{\text{max.}} \qquad (8.14)$$

Suppose it is required to find out the allowable load per mm length of parallel fillet weld for the permissible shear stress of 94 N/mm^2 and the leg dimension of 8 mm. From Eq. (8.14),

$$P_{\text{all.}} = 0.707h\tau_{\text{max.}} = 0.707\,(8)\,(94)$$
$$= 531.66 \text{ N/mm}$$

8.10 MAXIMUM SHEAR STRESS IN TRANSVERSE FILLET WELD

A double transverse fillet weld of equal legs is subjected to a force ($2P$) as shown in Fig. 8.15(a). It is required to find out the inclination (θ) of the plane in the weld where the maximum shear stress is induced and also, the magnitude of the maximum shear stress. The effect of bending is to be neglected.

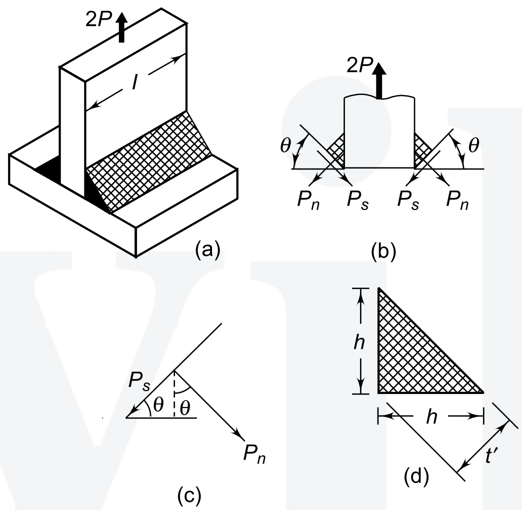

Fig. 8.15

The free body diagram of forces acting on the vertical plate with two symmetrically cut welds is shown in Fig. 8.15(b). The shear force is P_s and the normal force is P_n.

Considering equilibrium of vertical forces, [Fig. 8.15(b) and (c)],

$$2P = 2P_s \sin\theta + 2P_n \cos\theta$$

$$P = P_s \sin\theta + P_n \cos\theta \qquad \text{(a)}$$

Since the resultant of P_s and P_n is vertical, their horizontal components must be equal and opposite. Therefore,

$$P_s \cos\theta = P_n \sin\theta$$

or,

$$P_n = \frac{P_s \cos\theta}{\sin\theta} \qquad \text{(b)}$$

Substituting Eq. (b) in Eq. (a),

$$P = P_s \sin\theta + \frac{P_s \cos\theta \cos\theta}{\sin\theta} \qquad \text{(c)}$$

Multiplying both sides of the above equation by $\sin\theta$,

$$P\sin\theta = P_s\sin^2\theta + P_s\cos^2\theta$$
$$= P_s(\sin^2\theta + \cos^2\theta)$$

or,

$$P_s = P\sin\theta \qquad \text{(d)}$$

From Eq. (8.11), the width t' of the plane in the weld that is inclined at an angle θ with the horizontal is given by,

$$t' = \frac{h}{(\sin\theta + \cos\theta)} \qquad \text{(e)}$$

The area of the weld, in a plane that is inclined at angle θ with horizontal, is $(t'l)$. Therefore, the shear stress in this plane is given by,

$$\tau = \frac{P_s}{t'l}$$

From Eqs (d) and (e),

$$\tau = \frac{P\sin\theta(\sin\theta + \cos\theta)}{hl} \qquad \text{(f)}$$

In order to find out the plane with maximum shear stress, differentiate τ with respect to θ and set the derivative equal to zero.

$$\frac{\partial\tau}{\partial\theta} = 0$$

$$\frac{P}{hl}\frac{\partial}{\partial\theta}[\sin\theta(\sin\theta + \cos\theta)] = 0$$

or,

$$\frac{\partial}{\partial\theta}[\sin\theta(\sin\theta + \cos\theta)] = 0 \qquad \text{(g)}$$

Since,

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$x = \theta \qquad u = \sin\theta \qquad v = (\sin\theta + \cos\theta)$$

$$\frac{du}{d\theta} = \cos\theta \qquad \frac{dv}{d\theta} = (\cos\theta - \sin\theta)$$

Substituting,

$$\frac{\partial}{\partial\theta}[\sin\theta(\sin\theta + \cos\theta)]$$
$$= \sin\theta(\cos\theta - \sin\theta) + (\sin\theta + \cos\theta)\cos\theta$$
$$= 2\sin\theta\cos\theta + (\cos^2\theta - \sin^2\theta)$$
$$= \sin 2\theta + \cos 2\theta \qquad \text{(h)}$$

From (g) and (h),

$$\sin 2\theta + \cos 2\theta = 0$$
$$\sin 2\theta = -\cos 2\theta$$
$$\tan 2\theta = -1$$
$$2\theta = 135°$$

or

$$\theta = 67.5° \qquad (8.15)$$

The condition for the plane with the maximum shear stress is ($\theta = 67.5°$).

Substituting the above value of θ in Eq. (f), the maximum shear stress is given by,

$$\tau_{\text{max.}} = \frac{P\sin(67.5)°[\sin(67.5) + \cos(67.5)°]}{hl}$$

$$\tau_{\text{max.}} = \frac{1.21P}{hl} \qquad (8.16)$$

Substituting (l = 1 mm) in Eq. (8.16), the allowable load $P_{\text{all.}}$ per mm length of transverse fillet weld is given by,

$$P_{\text{all.}} = \frac{h\tau_{\text{max.}}}{1.21}$$

or

$$P_{\text{all.}} = 0.8284h\tau_{\text{max.}} \qquad (8.17)$$

Suppose it is required to find out allowable load per mm length of transverse fillet weld for the permissible shear stress of 94 N/mm^2 and the leg dimension of 8 mm. From Eq. (8.17),

$$P_{\text{all.}} = 0.8284h\tau_{\text{max.}} = 0.8284(8)(94)$$
$$= 622.96 \text{ N/mm}$$

It is observed from Eqs (8.14) and (8.17) that the allowable load for a transverse fillet weld is more than that of a parallel fillet weld. Or,

$$\frac{P_{\text{all.}} \text{ for transverse weld}}{P_{\text{all.}} \text{ for parallel weld}} = \frac{0.8284\,h\tau_{\text{max.}}}{0.707h\tau_{\text{max.}}} = 1.17$$

The strength of transverse fillet weld is 1.17 times of the strength of parallel fillet weld. As mentioned in Section 8.8, many times transverse fillet weld is designed by using the same equations of parallel fillet weld. Such design is on the safer side with the additional advantage of simple calculations.

8.11 AXIALLY LOADED UNSYMMETRICAL WELDED JOINTS

In certain applications, unsymmetrical sections such as angle or T are welded to the steel plates or the beams. Figure 8.16(a) shows an angle section welded to a vertical beam by means of two parallel fillet welds 1 and 2. G is the centre of gravity of the angle section. The external force acting on the joint passes through G. Suppose P_1 and P_2 are the resisting forces set up in the welds 1 and 2 respectively. From Eq. (8.6),

$$P_1 = 0.707 h l_1 \tau \qquad \text{(a)}$$

$$P_2 = 0.707 h l_2 \tau \qquad \text{(b)}$$

The free body diagram of forces acting on the angle section with two welds is shown in Fig. 8.16(b).

Since the sum of horizontal forces is equal to zero,

$$P = P_1 + P_2 \qquad (8.18)$$

Since the moment of forces about the centre of gravity is equal to zero,

$$P_1 y_1 = P_2 y_2 \qquad \text{(d)}$$

Substituting expressions (a) and (b) in the expression (d),

$$l_1 y_1 = l_2 y_2 \qquad (8.19)$$

Assuming total length of welds as l,

$$l_1 + l_2 = 1 \qquad (8.20)$$

Equations (8.18) to (8.20) are used to find out the required lengths l_1 and l_2 of two welds.

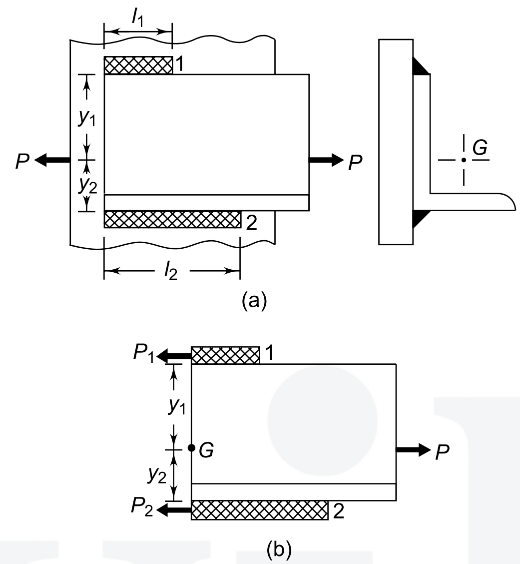

Fig. 8.16

Example 8.7 *An ISA 200 × 100 × 10 angle is welded to a steel plate by means of fillet welds as shown in Fig. 8.17. The angle is subjected to a static force of 150 kN and the permissible shear stress for the weld is 70 N/mm². Determine the lengths of weld at the top and bottom.*

Solution

Given $P = 150$ kN $\quad \tau = 70$ N/mm^2 $\quad h = 10$ mm

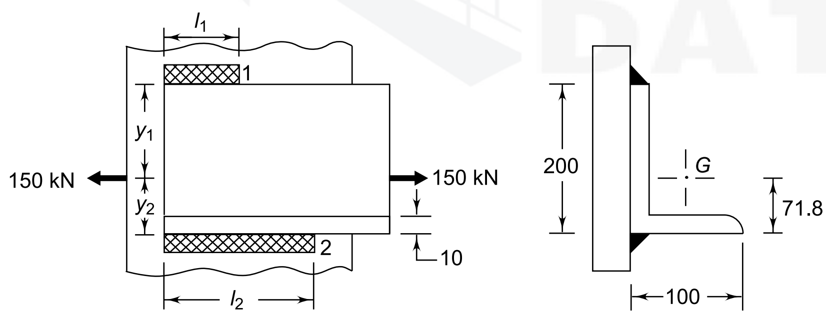

Fig. 8.17

Step I *Total length of weld*

The total length (l) of the weld required to withstand the load of 150 kN is given by [Eq. (8.6)].

$$P = 0.707\, hl\tau \quad \text{or} \quad 150 \times 10^3 = 0.707\,(10)\, l\,(70)$$

$$l = 303.09 \text{ mm} \qquad \text{(i)}$$

Step II *Weld lengths l_1 and l_2*

From Eq. (8.19),

$$l_1 y_1 = l_2 y_2 \quad \text{or} \quad l_1(200 - 71.8) = l_2\,(71.8)$$

$$128.2\, l_1 = 71.8\, l_2 \qquad \text{(ii)}$$

Also,

$$l_1 + l_2 = 1 = 303.09 \text{ mm} \qquad \text{(iii)}$$

From (ii) and (iii),

$$l_1 = 108.81 \text{ mm} \quad \text{and} \quad l_2 = 194.28 \text{ mm}$$

Example 8.8 *How much length of a 10 mm fillet weld is required to weld the long side of an ISA angle 150 × 75 × 10 to a steel plate with side welds only? A static load of 125 kN acts through the centre of gravity of the angle section which is 53.2 mm from the short side. The allowable load per mm of the weld length is 665 N.*

Solution

Given $P = 125$ kN

allowable load = 665 N per mm of weld

Step I *Total length of weld*

The welded joint is shown in Fig. 8.18. The total length (l) of two fillet welds is given by,

$$l = \frac{P}{P_{all}} = \frac{125 \times 10^3}{665} = 187.97 \text{ mm}$$

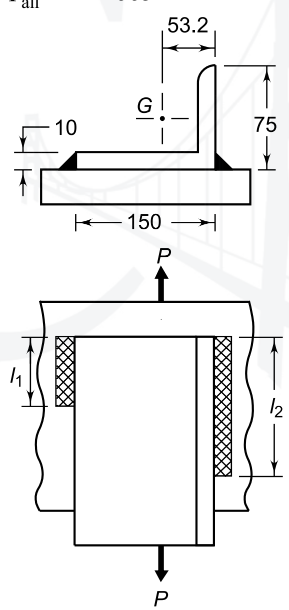

Fig. 8.18

Step II *Weld lengths l_1 and l_2*

$$l_1 + l_2 = l = 187.97 \text{ mm} \quad \text{(i)}$$

From Eq. (8.19),

$$l_1 y_1 = l_2 y_2 \text{ or } l_1 (150 - 53.2) = l_2 (53.2)$$

$$96.8\, l_1 = 53.2\, l_2 \quad \text{(ii)}$$

From (i) and (ii),

$$l_1 = 66.67 \text{ mm} \quad \text{and} \quad l_2 = 121.30 \text{ mm}$$

8.12 ECCENTRIC LOAD IN THE PLANE OF WELDS

The design of welded joint subjected to an eccentric load in the plane of welds, consists of calculations of primary and secondary shear stresses. A bracket subjected to an eccentric force P and attached to the support by means of two fillet welds W_1 and W_2 is shown in Fig. 8.19(a). In such problems, the first step is to determine the centre of gravity of welds, treating the weld as a line. Suppose G is the centre of gravity of two welds and e is the eccentricity between the centre of gravity and the line of action of force P. According to the principle of Applied Mechanics, the eccentric force P can be replaced by an equal and similarly directed force (P) acting through the centre of gravity G, along with a couple ($M = P \times e$) lying in the same plane [Fig. 8.19(b)]. The effects of the force P and the couple M are treated separately as shown in Fig 8.19(c) and (d) respectively.

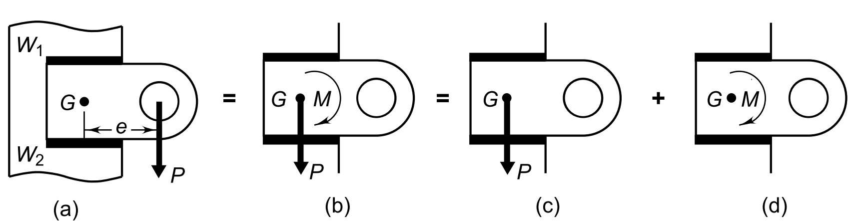

Fig. 8.19 *Analysis of Eccentrically Loaded Welded Joint*

The stresses in this welded joint are shown in Fig. 8.20. The force P acting through the centre of gravity causes direct shear stress in the welds [Fig. 8.20(a)]. It is called the *primary shear stress*. It is assumed that the primary shear stress is uniformly distributed over the throat area of all welds. Therefore,

$$\tau_1 = \frac{P}{A} \tag{8.21}$$

where A is the throat area of all welds.

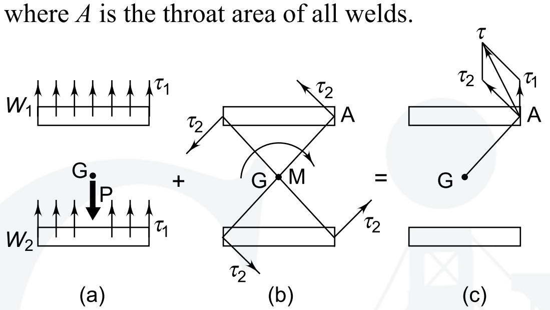

Fig. 8.20 *Primary and Secondary Shear Stresses*

The couple M causes torsional shear stresses in the throat area of welds [Fig. 8.20(b)]. They are called *secondary shear stresses* and given by

$$\tau_2 = \frac{Mr}{J} \tag{8.22}$$

where

r = distance of a point in the weld from G

J = polar moment of inertia of all welds about G

The secondary shear stress at any point in the weld is proportional to its distance from the centre of gravity. Obviously, it is maximum at the farthest point such as A. The resultant shear stress at any point is obtained by vector addition of primary and secondary shear stresses [Fig. 8.20(c)].

Figure 8.21 shows a weld of length l and throat t. G_1 is the centre of gravity of the weld, while G

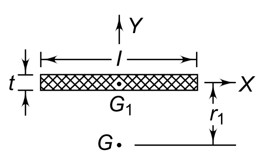

Fig. 8.21

is the centre of gravity of a group of welds. The moment of inertia of this weld about its centre of gravity G_1 is given by,

$$I_{xx} = \frac{lt^3}{12} \quad \text{and} \quad I_{yy} = \frac{tl^3}{12}$$

Since t is very small compared with l, I_{xx} is negligible compared with I_{yy}.

$$\therefore \qquad J_{G_1} = I_{xx} + I_{yy} \cong I_{yy}$$

or

$$J_{G_1} = \frac{tl^3}{12} = \frac{tl(l^2)}{12} = \frac{Al^2}{12}$$

Therefore,

$$J_{G_1} = \frac{Al^2}{12} \tag{8.23}$$

where A is the throat area of the weld and J_{G_1} is the polar moment of inertia of the weld about its centre of gravity. The polar moment of inertia about an axis passing through G is determined by the parallel axis theorem. Thus,

$$J_G = J_{G_1} + Ar_1^2 \tag{8.24}$$

where r_1 is the distance between G and G_1. From Eqs (8.23) and (8.24),

$$J_G = A\left[\frac{l^2}{12} + r_1^2\right] \tag{8.25}$$

Where there are a number of welds, with polar moment of inertias J_1, J_2, J_3,..., etc., about the centre of gravity G, the resultant polar moment of inertia is given by,

$$J = J_1 + J_2 + J_3 + \cdots \tag{8.26}$$

The above value of J is to be used in Eq. (8.22) to determine secondary shear stresses.

Example 8.9 *A welded connection, as shown in Fig. 8.22 is subjected to an eccentric force of 7.5 kN. Determine the size of welds if the permissible shear stress for the weld is 100 N/mm². Assume static conditions.*

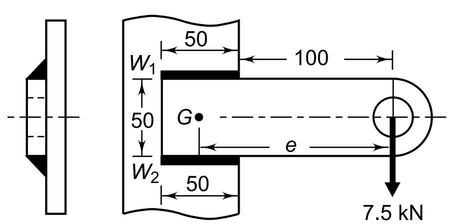

Fig. 8.22

Solution

Given $P = 7.5$ kN $\quad \tau = 100$ N/mm^2

Step I *Primary shear stress*

Suppose t is the throat of each weld. There are two welds W_1 and W_2 and their throat area is given by,

$$A = 2(50t) = (100t) \text{ mm}^2$$

From Eq. (8.21), the primary shear stress is given by,

$$\tau_1 = \frac{P}{A} = \frac{7500}{(100t)} = \left(\frac{75}{t}\right) \text{N/mm}^2 \qquad \text{(a)}$$

Step II *Secondary shear stress*

The two welds are symmetrical and G is the centre of gravity of the two welds.

$e = 25 + 100 = 125$ mm

$M = P \times e = (7500)(125) = 937\ 500$ N-mm $\quad$ (i)

The distance r of the farthest point in the weld from the centre of gravity is given by (Fig. 8.23),

$$r = \sqrt{(25)^2 + (25)^2} = 35.36 \text{ mm} \qquad \text{(ii)}$$

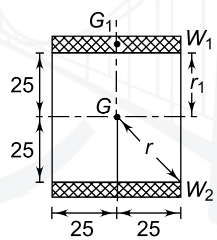

Fig. 8.23

From Eq. (8.25), the polar moment of inertia J_1 of the weld W_1 about G is given by

$$J_1 = A\left[\frac{l^2}{12} + r_1^2\right] = (50t) \times \left[\frac{50^2}{12} + 25^2\right]$$

$= (41\ 667t)$ mm^4

Due to symmetry, the polar moment of inertia of the two welds (J) is given by

$$J = J_1 + J_2 = 2J_1 = 2(41\ 667t) = (83\ 334t) \text{ mm}^4$$

From Eq. (8.22), the secondary shear stress is given by

$$\tau_2 = \frac{Mr}{J} = \frac{(937\ 500)(35.36)}{(83\ 334t)} = \left(\frac{397.8}{t}\right) \text{N/mm}^2$$

Step III *Resultant shear stress*

Figure 8.24 shows the primary and secondary shear stresses. The vertical and horizontal components of these shear stresses are added and the resultant shear stress is determined. Therefore, from the figure,

$$\tau = \left(\frac{454}{t}\right) \text{N/mm}^2$$

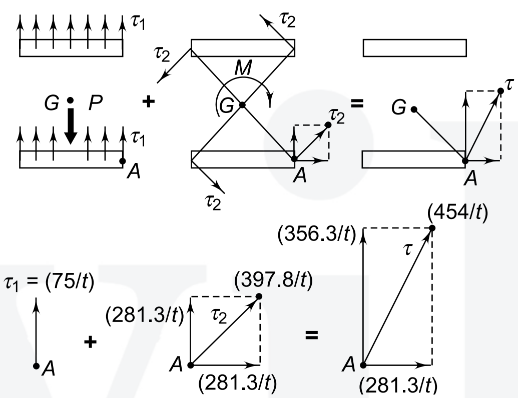

Fig. 8.24

Step IV *Size of weld*

Since the permissible shear stress for the weld material is 100 N/mm^2,

$$\left(\frac{454}{t}\right) = 100 \quad \text{or} \quad t = 4.54 \text{ or } 5 \text{ mm}$$

Example 8.10 *A welded connection, as shown in Fig. 8.25, is subjected to an eccentric force of 60 kN in the plane of the welds. Determine the size of the welds, if the permissible shear stress for the weld is 100 N/mm^2. Assume static conditions.*

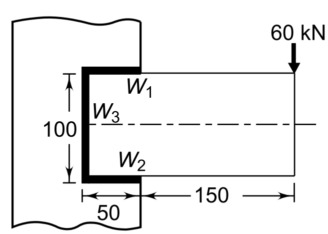

Fig. 8.25

Solution

Given $P = 60$ kN $\tau = 100$ N/mm^2

Step I *Primary shear stress*

There are two horizontal welds W_1 and W_2 and one vertical weld W_3. Refer to Fig. 8.26. By symmetry, the centre of gravity G of the three welds is midway between the two horizontal welds. Therefore,

$$\bar{y} = 50 \text{ mm}$$

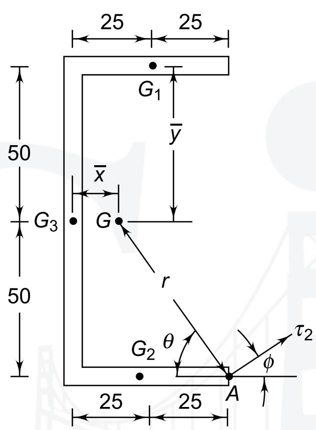

Fig. 8.26

Taking moment about the vertical weld and treating the weld as a line,

$$(50 + 100 + 50)\ \bar{x} = 50 \times 25 + 50 \times 25 + 100 \times 0$$

$$\bar{x} = 12.5 \text{ mm}$$

The areas of three welds are as follows:

$$A_1 = (50\ t) \text{ mm}^2$$
$$A_2 = (50\ t) \text{ mm}^2$$
$$A_3 = (100\ t) \text{ mm}^2$$
$$A = A_1 + A_2 + A_3 = (200\ t) \text{ mm}^2$$

The primary shear stress in the weld is given by,

$$\tau_1 = \frac{P}{A} = \frac{60000}{200t} = \frac{300}{t} \text{ N/mm}^2 \qquad \text{(i)}$$

Step II *Secondary shear stress*

As seen in Fig. 8.26, A is the farthest point from the centre of gravity G and its distance r is given by,

$$r = \sqrt{(50-12.5)^2 + (50)^2} = 62.5 \text{ mm}$$

Also,

$$\tan\theta = \frac{50}{(50-12.5)} \quad \text{or} \quad \theta = 53.13°$$

$$\phi = 90 - \theta = 90 - 53.13 = 36.87°$$

Therefore, the secondary shear stress is inclined at 36.87° with horizontal.

$$e = (50 - \bar{x}) + 150 = (50 - 12.5) + 150 = 187.5 \text{ mm}$$

$$M = P \times e = (60 \times 10^3)(187.5) = (11\,250 \times 10^3) \text{ N-mm}$$

G_1, G_2 and G_3 are the centres of gravity of the three welds and their distances from the common centre of gravity G are as follows,

$$\overline{G_1G} = \overline{G_2G} = \sqrt{(25-12.5)^2 + (50)^2} = 51.54 \text{ mm}$$

or

$$r_1 = r_2 = 51.54 \text{ mm}$$

$$r_3 = \overline{G_3G} = \bar{x} = 12.5 \text{ mm}$$

From Eq. (8.25),

$$J_1 = J_2 = A_1\left[\frac{l_1^2}{12} + r_1^2\right]$$

$$= (50\ t)\left[\frac{(50)^2}{12} + (51.54)^2\right]$$

$$= (143\,235.25\ t) \text{ mm}^4$$

$$J_3 = A_3\left[\frac{l_3^2}{12} + r_3^2\right] = (100\ t)\left[\frac{100^2}{12} + (12.5)^2\right]$$

$$= (98\,958.33\ t) \text{ mm}^4$$

$$J = 2\,J_1 + J_3 = 2(143\,235.25\ t) + (98\,958.33\ t)$$

$$= (385\,428.83\ t) \text{ mm}^4$$

The secondary shear stress at the point A is given by,

$$\tau_2 = \frac{Mr}{J} = \frac{(11\,250 \times 10^3)(62.5)}{(385\,4\,28.83t)}$$

$$= \frac{1824.27}{t} \text{ N/mm}^2 \qquad \text{(ii)}$$

Step III *Resultant shear stress*

The secondary shear stress is inclined at 36.87° with the horizontal. It is resolved into vertical and horizontal components as shown in Fig. 8.27.

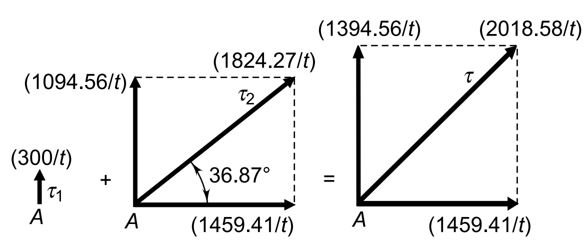

Fig. 8.27

Vertical component = $\tau_2 \sin\phi$

$$= \frac{1824.27}{t}\sin(36.87) = \frac{1094.56}{t}\ \text{N/mm}^2$$

Horizontal component = $\tau_2 \cos\phi$

$$= \frac{1824.27}{t}\cos(36.87) = \frac{1459.41}{t}\ \text{N/mm}^2$$

The primary shear stress $\left(\frac{300}{t}\right)$ is vertically upward. Therefore, the total vertical component is given by,

$$\left(\frac{1094.56}{t} + \frac{300}{t}\right) \text{ or } \left(\frac{1394.56}{t}\right)\text{N/mm}^2$$

The resultant shear stress is given by,

$$\tau = \sqrt{\left(\frac{1394.56}{t}\right)^2 + \left(\frac{1459.41}{t}\right)^2}$$

$$= \frac{2018.58}{t}\ \text{N/mm}^2$$

Step IV *Size of weld*

The permissible shear stress for the weld material is 100 N/mm^2. Therefore,

$$100 = \frac{2018.58}{t} \quad \text{or} \quad t = 20.19 \text{ mm}$$

and $$h = \frac{t}{0.707} = \frac{20.19}{0.707} = 28.56 \text{ mm}$$

Example 8.11 *An eccentrically loaded bracket is welded to the support as shown in Fig. 8.28. The permissible shear stress for the weld material is 55 N/mm^2 and the load is static. Determine the throat and leg dimensions for the welds.*

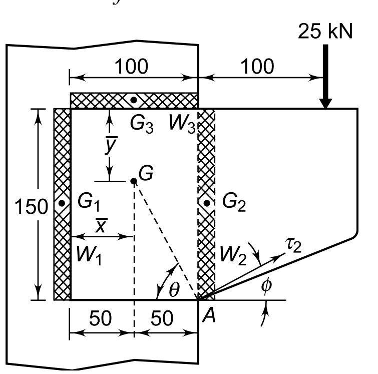

Fig. 8.28

Solution

Given $P = 25$ kN $\quad \tau = 55$ N/mm^2

Step I *Primary shear stress*

There are two vertical welds W_1 and W_2 and one horizontal weld W_3. By symmetry, the centre of gravity G of the welds is midway between the vertical welds.

or $\quad \bar{x} = 50$ mm

Taking moments about the top weld and treating the weld as a line,

$$(150 + 150 + 100)\,\bar{y} = (150)(75) + (150)(75) + (100)(0)$$

$\bar{y} = 56.25$ mm (from the top weld)

The areas of the three welds are as follows:

$A_1 = (150\, t)$ mm^2

$A_2 = (150\, t)$ mm^2

$A_3 = (100\, t)$ mm^2

$A = A_1 + A_2 + A_3 = (400\, t)$ mm^2

The primary shear stress in the weld is given by,

$$\tau_1 = \frac{P}{A} = \frac{25\,000}{400t} = \frac{62.5}{t}\ \text{N/mm}^2 \qquad \text{(i)}$$

Step II *Secondary shear stress*

As seen from Fig. 8.28, A is the farthest point from the centre of gravity and its distance r is given by

$$r = \overline{GA} = \sqrt{(150 - 56.25)^2 + (50)^2} = 106.25 \text{ mm}$$

also, $$\tan\theta = \frac{(150 - 56.25)}{50} \quad \text{or} \quad \theta = 61.93°$$

$$\phi = 90 - \theta = 90 - 61.93 = 28.07°$$

Therefore, the secondary shear stress τ_2 is inclined at 28.07° with the horizontal.

$e = 50 + 100 = 150$ mm

$M = P \times e = (25000)(150) = 3\,750\,000$ N-mm

G_1, G_2 and G_3 are the centres of gravity of the three welds and the distances are as follows:

$$\overline{G_1G} = \overline{G_2G} = \sqrt{(75 - 56.25)^2 + (50)^2} = 53.4 \text{ mm}$$

or $\quad r_1 = r_2 = 53.4$ mm

$$r_3 = \overline{G_3G} = \bar{y} = 56.25 \text{ mm}$$

From Eq. (8.25),

$$J_1 = J_2 = A_1\left[\frac{l^2}{12} + r_1^2\right]$$

$$= (150t) \times \left[\frac{(150)^2}{12} + (53.4)^2 \right]$$

$$= (708\,984t) \text{ mm}^4$$

$$J_3 = A_3 \left[\frac{l^2}{12} + r_3^2 \right]$$

$$= (100t) \times \left[\frac{(100)^2}{12} + (56.25)^2 \right]$$

$$= (399\,740t) \text{ mm}^4$$

$$J = 2J_1 + J_3 = 2\,(708\,984t) + (399\,740t)$$

$$= (1\,817\,708t) \text{ mm}^4$$

The secondary shear stress at the point A is given by

$$\tau_2 = \frac{Mr}{J} = \frac{(3\,750\,000)(106.25)}{(1\,817\,708t)}$$

$$= \left(\frac{219.2}{t} \right) \text{N/mm}^2 \qquad \text{(ii)}$$

Step III *Resultant shear stress*

The secondary shear stress is inclined at an angle of 28.07° with horizontal. It is resolved into vertical

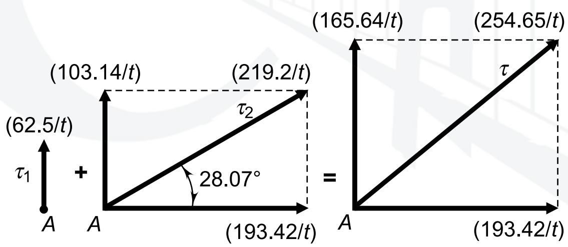

Fig. 8.29

and horizontal components as shown in Fig. 8.29. The components are as follows:

$$\text{Vertical component} = \left(\frac{219.2}{t} \right) \sin (28.07)$$

$$= \left(\frac{103.14}{t} \right) \text{N/mm}^2$$

$$\text{Horizontal component} = \left(\frac{219.2}{t} \right) \cos (28.07)$$

$$= \left(\frac{193.42}{t} \right) \text{N/mm}^2$$

The primary shear stress $\left(\frac{62.5}{t} \right)$ is vertically upward.

Therefore, the total vertical component is

$$\left(\frac{103.14}{t} + \frac{62.5}{t} \right) \quad \text{or} \quad \left(\frac{165.64}{t} \right)$$

and the resultant shear stress is given by

$$\tau = \sqrt{\left(\frac{165.64}{t} \right)^2 + \left(\frac{193.42}{t} \right)^2} = \left(\frac{254.65}{t} \right) \text{N/mm}^2$$

Step IV *Size of weld*

The permissible shear stress for the weld material is 55 N/mm^2. Therefore,

$$55 = \left(\frac{254.65}{t} \right) \quad \text{or} \quad t = 4.63 \text{ mm}$$

$$h = \frac{t}{0.707} = \frac{4.63}{0.707} = 6.55 \cong 7 \text{ mm}$$

8.13 WELDED JOINT SUBJECTED TO BENDING MOMENT

A cantilever beam of rectangular cross-section is welded to a support by means of two fillet welds W_1 and W_2 as shown in Fig. 8.30. According to

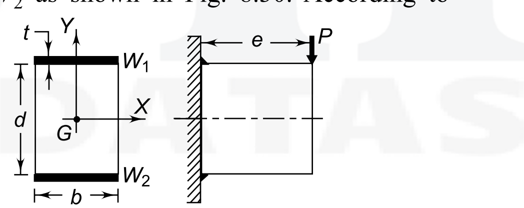

Fig. 8.30 *Welded Joint Subjected to Bending Moment*

the principle of Applied Mechanics, the eccentric force P can be replaced by an equal and similarly directed force P acting through the plane of welds, along with a couple ($M_b = P \times e$) as shown in Fig. 8.31. The force P through the plane of welds causes the primary shear stress τ_1, which is given by

$$\tau_1 = \frac{P}{A} \qquad (8.27)$$

where A is the throat area of all welds. The moment M_b causes bending stresses in the welds. The bending stresses are given by,

$$\sigma_b = \frac{M_b y}{I} \tag{8.28}$$

where

I = moment of inertia of all welds based on the throat area

y = distance of the point in weld from the neutral-axis

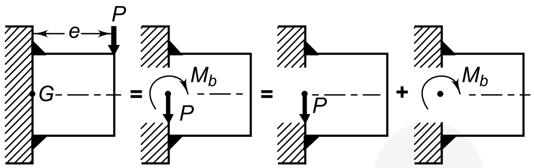

Fig. 8.31

The bending stresses are assumed to act normal to the throat area. The resultant shear stress in the welds is given by,

$$\tau = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + (\tau_1)^2} \tag{8.29}$$

Referring back to Fig. 8.30 and using the parallel axis theorem, the moment of inertia of weld W_1 about the X-axis is given by

$$I_{xx} = \frac{bt^3}{12} + (bt)\left(\frac{d}{2}\right)^2$$

The throat dimension is very small compared to b or d. Therefore, the first term in the above expression is neglected.

or $$I_{xx} = bt\left(\frac{d^2}{4}\right)$$

Since there are two such symmetrical welds,

$$I = 2I_{xx} = t\left(\frac{bd^2}{2}\right) \tag{8.30}$$

The above expression should be used in Eq. (8.28) to determine the bending stresses.

Example 8.12 *A bracket is welded to the vertical column by means of two fillet welds as shown in Fig. 8.32. Determine the size of the welds, if the permissible shear stress in the weld is limited to 70 N/mm².*

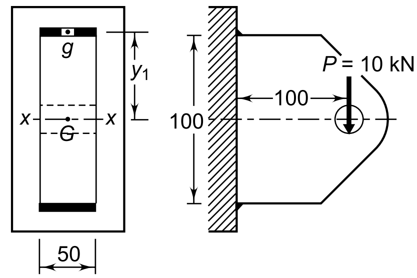

Fig. 8.32

Solution

Given $P = 10$ kN $\quad \tau = 70$ N/mm²

Step I *Primary shear stress*

The area of the two welds is given by,

$$A = 2\,(50\,t) = (100\,t)\ \text{mm}^2$$

The primary shear stress is given by,

$$\tau_1 = \frac{P}{A} = \frac{10 \times 10^3}{(100t)} = \frac{100}{t}\ \text{N/mm}^2 \tag{i}$$

Step II *Bending stress*

The moment of inertia of the top weld about the X-axis passing through its centre of gravity g is $(50t^3/12)$. This moment of inertia is shifted to the centre of gravity of the two welds at G by the parallel axis theorem. It is given by,

$$I_{xx} = \frac{50t^3}{12} + Ay_1^2 = \frac{50t^3}{12} + (50t)(50)^2\ \text{mm}^4$$

The dimension t is very small compared with 50. The term containing t^3 is neglected. Therefore,

$$I_{xx} = (50\,t)\,(50)^2 = (50^3\,t)\ \text{mm}^4$$

Since there are two welds,

$$I = 2I_{xx} = 2(50^3 t) = (250\,000t)\ \text{mm}^4$$

The bending stress in the top weld is given by,

$$\sigma_b = \frac{M_b y}{I} = \frac{(10 \times 10^3 \times 100)(50)}{(250\,000t)}$$

$$= \left(\frac{200}{t}\right)\ \text{N/mm}^2 \tag{ii}$$

Step III *Maximum shear stress*

The maximum principal shear stress in the weld is given by,

$$\tau = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + (\tau_1)^2} = \sqrt{\left(\frac{200}{2t}\right)^2 + \left(\frac{100}{t}\right)^2}$$

$$= \frac{141.42}{t} \text{ N/mm}^2$$

Step IV *Size of weld*

The permissible shear stress in the weld is 70 N/mm^2. Therefore,

$$\frac{141.42}{t} = 70 \quad \text{or} \quad t = 2.02 \text{ mm}$$

$$h = \frac{t}{0.707} = \frac{2.02}{0.707} = 2.86 \quad \text{or} \quad 3 \text{ mm}$$

Example 8.13 *A bracket is welded to the vertical plate by means of two fillet welds as shown in Fig. 8.33. Determine the size of the welds, if the permissible shear stress is limited to 70 N/mm^2.*

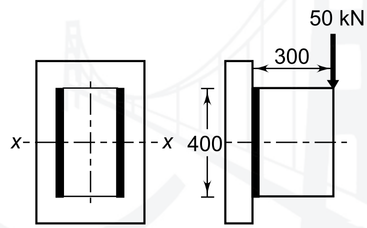

Fig. 8.33

Solution

Given $P = 50$ kN $\tau = 70$ N/mm^2

Step I *Primary shear stress*

The total area of two vertical welds is given by,

$$A = 2(400\,t) = (800\,t) \text{ mm}^2$$

The primary shear stress in the weld is given by,

$$\tau_1 = \frac{P}{A} = \frac{50\,000}{800t} = \frac{62.5}{t} \text{ N/mm}^2 \quad \text{(i)}$$

Step II *Bending stress*

The moment of inertia of two welds about the X-axis is given by,

$$I = 2\left[\frac{t(400)^3}{12}\right] = (10.67 \times 10^6\,t) \text{ mm}^4$$

From Eq. (8.28),

$$\sigma_b = \frac{M_b y}{I} = \frac{(50 \times 10^3 \times 300)(200)}{(10.67 \times 10^6\,t)}$$

$$= \frac{281.16}{t} \text{ N/mm}^2 \quad \text{(ii)}$$

Step III *Maximum shear stress*

The maximum principal shear stress in the weld is given by,

$$\tau = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + (\tau_1)^2} = \sqrt{\left(\frac{281.16}{2t}\right)^2 + \left(\frac{62.5}{t}\right)^2}$$

$$= \frac{153.85}{t} \text{ N/mm}^2$$

Step IV *Size of weld*

The permissible shear stress in the weld is 70 N/mm^2. Therefore,

$$\frac{153.85}{t} = 70 \quad \text{or} \quad t = 2.2 \text{ mm}$$

$$h = \frac{t}{0.707} = \frac{2.2}{0.707} = 3.11 \quad \text{or} \quad 4 \text{ mm}$$

Example 8.14 *A beam of rectangular cross-section is welded to a support by means of fillet welds as shown in Fig. 8.34. Determine the size of the welds, if the permissible shear stress in the weld is limited to 75 N/mm^2.*

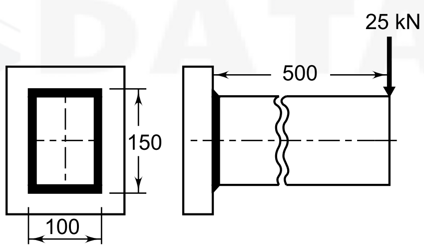

Fig. 8.34

Solution

Given $P = 25$ kN $\tau = 75$ N/mm^2

Step I *Primary shear stress*

The total area of the horizontal and vertical welds is given by,

$$A = 2[100\,t + 150\,t] = (500\,t) \text{ mm}^2$$

The primary shear stress in the welds is given by,

$$\tau_1 = \frac{P}{A} = \frac{25\,000}{(500t)} = \left(\frac{50}{t}\right) \text{N/mm}^2 \qquad \text{(i)}$$

Step II *Bending stress*
Referring to Fig. 8.35, the moment of inertia of four welds about the X-axis is given by

$$I_{xx} = 2\left[\frac{bt^3}{12} + (bt) \times \left(\frac{d}{2}\right)^2\right] + 2\left[\frac{td^3}{12}\right]$$

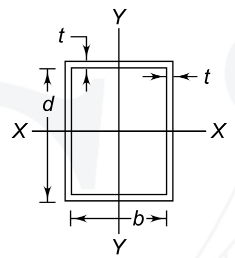

Fig. 8.35

Assuming b and d to be large as compared to the throat dimension t and neglecting the terms containing t^3, we have

$$I_{xx} = t\left[\frac{bd^2}{2} + \frac{d^3}{6}\right]$$

Substituting the values,

$$I_{xx} = t\left[\frac{(100)(150)^2}{2} + \frac{(150)^3}{6}\right] = \left[(75)(150)^2\right] t \text{ mm}^4$$

From Eq. (8.28),

$$\sigma_b = \frac{M_b y}{I} = \frac{(25\,000 \times 500)(75)}{(75)(150)^2 t}$$

$$= \left(\frac{555.55}{t}\right) \text{N/mm}^2 \qquad \text{(ii)}$$

Step III *Maximum shear stress*
From Eq. (8.29), the maximum shear stress in the weld is given by,

$$\tau = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + (\tau_1)^2} = \sqrt{\left(\frac{555.55}{2t}\right)^2 + \left(\frac{50}{t}\right)^2}$$

$$= \frac{282.24}{t} \text{ N/mm}^2$$

Step IV *Size of weld*
Since the permissible shear stress in the weld is 75 N/mm^2,

$$\left(\frac{282.24}{t}\right) = 75 \quad \text{or} \quad t = 3.76 \text{ mm}$$

and $$h = \frac{t}{0.707} = \frac{3.76}{0.707} = 5.32 \cong 6 \text{ mm}$$

Example 8.15 *A circular beam, 50 mm in diameter, is welded to a support by means of a fillet weld as shown in Fig. 8.36. Determine the size of the weld, if the permissible shear stress in the weld is limited to 100 N/mm^2.*

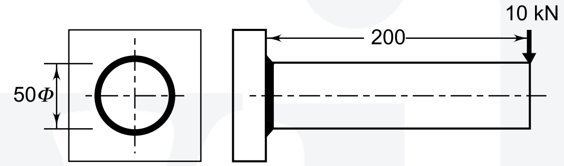

Fig. 8.36

Solution

Given $P = 10$ kN $\quad \tau = 100$ N/mm^2

Step I *Primary shear stress*
From Eq. (8.27), the primary shear stress in the weld is given by

$$\tau_1 = \frac{P}{A} = \frac{P}{\pi Dt} = \frac{10\,000}{\pi(50)(t)} = \left(\frac{63.66}{t}\right) \text{N/mm}^2 \qquad \text{(i)}$$

Step II *Bending stress*
Consider an elemental section of area δA as shown in Fig. 8.37. It is located at an angle θ with X axis and subtends an angle $d\theta$.

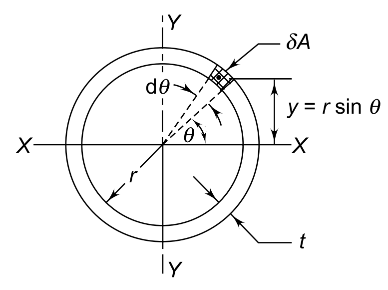

Fig. 8.37

$$\delta A = r\, d\theta\, t$$

and $$\delta(I_{xx}) = (\delta A)\,(y^2) = (r\, d\theta\, t)\,(r \sin\theta)^2 = tr^3 \sin^2\theta\, d\theta$$

The moment of inertia of an annular fillet weld is obtained by integrating the above expression. Thus,

$$I_{xx} = 2\int_0^{\pi} tr^3 \sin^2\theta d\theta = 2tr^3 \int_0^{\pi} \sin^2\theta d\theta$$

$$= 2tr^3 \int_0^{\pi} \left[\frac{1-\cos 2\theta}{2}\right] d\theta = 2tr^3 \left(\frac{\pi}{2}\right)$$

or $$I_{xx} = \pi\, tr^3$$

For the given welded joint,

$$I_{xx} = \pi(t)\,(25)^3 \text{ mm}^4$$

From Eq. (8.28),

$$\sigma_b = \frac{M_b y}{I} = \frac{(10\,000 \times 200)(25)}{\pi t (25)^3}$$

$$= \left(\frac{1018.59}{t}\right) \text{N/mm}^2 \quad \text{(ii)}$$

Step III *Maximum shear stress*

From Eq. (8.29), the maximum shear stress in the weld is given by,

$$\tau = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + (\tau_1)^2} = \sqrt{\left(\frac{1018.59}{2t}\right)^2 + \left(\frac{63.66}{t}\right)^2}$$

$$= \left(\frac{513.26}{t}\right) \text{N/mm}^2$$

Step IV *Size of weld*

Since the permissible shear stress in the weld is 100 N/mm^2,

$$\left(\frac{513.26}{t}\right) = 100 \qquad \therefore t = 5.13 \text{ mm}$$

$$h = \frac{t}{0.707} = \frac{5.13}{0.707} = 7.26 \quad \text{or} \quad 8 \text{ mm}$$

8.14 WELDED JOINT SUBJECTED TO TORSIONAL MOMENT

A shaft of circular cross-section is welded to the plate by means of a circumferential fillet weld as shown in Fig. 8.38. The shaft is subjected to

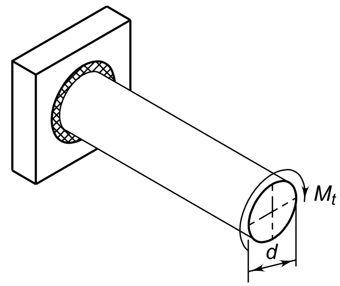

Fig. 8.38

torsional moment M_t that induces torsional shear stresses in the weld. Refer to Fig. 8.39 and consider an elemental section in the weld having an area δA. It is located at an angle θ with X-axis and subtends an angle $d\theta$. The area of the elemental section is given by,

$$\delta A = r\, d\theta \times t$$

and

$$\delta(I_{xx}) = \delta A\, y^2 = (r\, d\theta \times t)\,(r \sin\theta)^2 = tr^3 \sin^2\theta\, d\theta$$

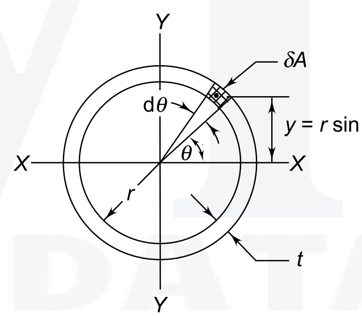

Fig. 8.39

The moment of inertia of the annular fillet weld is obtained by integrating the above expression. Therefore,

$$I_{xx} = 2\int_0^{\pi} tr^3 \sin^2\theta\, d\theta = 2tr^3 \int_0^{\pi} \sin^2\theta\, d\theta$$

$$= 2tr^3 \int_0^{\pi} \left[\frac{1-\cos 2\theta}{2}\right] d\theta = 2tr^3 \left(\frac{\pi}{2}\right)$$

or $$I_{xx} = \pi tr^3 \quad \text{(a)}$$

By symmetry,

$$I_{yy} = \pi tr^3 \quad \text{(b)}$$

The polar moment of inertia is given by

$$J = I_{xx} + I_{yy} = \pi tr^3 + \pi tr^3$$

or,

$$J = 2\,\pi tr^3 \tag{8.31}$$

The torsional shear stress in the weld is given by,

$$\tau = \frac{M_t r}{J} = \frac{M_t r}{2\pi tr^3}$$

or

$$\tau = \frac{M_t}{2\pi tr^2} \tag{8.32}$$

Example 8.16 *A circular shaft, 50 mm in diameter, is welded to the support by means of circumferential fillet weld as shown in Fig. 8.38. It is subjected to torsional moment of 2500 N-m. Determine the size of the weld, if the permissible shear stress in the weld is limited to 140 N/mm².*

Solution

Given $M_t = 2500$ N-m $\quad \tau = 140$ N/mm²

Step I *Torsional shear stress*

From Eq. (8.32),

$$\tau = \frac{M_t}{2\pi tr^2} \quad \text{or} \quad 140 = \frac{2500 \times 10^3}{2\pi t(25)^2}$$

$$\therefore \qquad t = 4.55 \text{ mm}$$

Step II *Size of weld*

$$h = \frac{t}{0.707} = \frac{4.55}{0.707} = 6.43 \quad \text{or} \quad 7 \text{ mm}$$

8.15 STRENGTH OF WELDED JOINTS

The permissible stresses for welded joints in certain structures, such as unfired pressure vessels, framework of buildings, ships or bridges, are covered by national or international codes. While designing such structures or their parts, it is obligatory on the part of the designer to use the values of permissible stresses given in the relevant standards. However, there are a number of welded machine components which do not come under the scope of these standards. Such components can be classified into two groups:

(i) where the strength of the weld is more than the strength of the parts joined together; and

(ii) where the strength of the weld is less than the strength of the connected parts.

In the first category, the failure will occur in the parts joined together by the weld, while, in the second, it will occur in the weld deposit.

The strength of the weld deposit is more than the strength of the connected parts under the following conditions:

(i) The components are made of mild steel with less than 0.3% carbon.

(ii) The welding electrodes contain 0.15% carbon.

(iii) The electrodes are coated, resulting in shielded welding.

During the welding process, the coating on the electrode gives off an inert gas, which acts as a shield, protecting the arc from the surrounding atmosphere. The coating also forms a slag on the molten metal and protects it during the cooling process. The permissible stresses for welded joints under the above three conditions are given in Table 8.1. The values given in the table are recommended by CH Jennings[1]. The failure of fillet welds is usually due to shear at the throat area of the weld. The permissible stresses for fillet weld, given in the table, are therefore permissible shear stresses for any direction of applied load.

Table 8.1 *Weld design stress by Jennings*

Types of weld and stress	*Permissible stresses (N/mm²)*	
	Static load	*Reversed load*
Butt welds		
Tension	110	55
Compression	125	55
Shear	70	35
Fillet welds	95	35

When the components are made of materials, such as high carbon steel or alloy steel, the weld deposit is weaker than the strength of the connected components. In such cases, the properties of weld deposit, such as tensile strength

[1] C H Jennings—'Welding Design'—Trans. ASME—Vol. 58, page 497–1936.

or yield strength, are considered as a criterion for determining permissible stresses. The mechanical properties of weld deposits for the two varieties of electrodes are given in Table 8.2. The suffix *E* indicates solid extrusion of electrode. The electrodes are designated by a suffix letter followed by six digits in numerals and the details can be obtained from the standard[2]. Due to high degree of variation in workmanship and inspection, and because of the complexity of stress distribution and uncertainties in loading, a higher factor of safety of 3 to 4 is used in these cases.

Table 8.2 *Mechanical properties of weld deposits*

Electrode number	*Tensile strength (N/mm^2)*	*Yield strength (minimum) (N/mm^2)*
E XXX – 41	410 – 510	330
E XXX – 51	510 – 610	360

8.16 WELDED JOINTS SUBJECTED TO FLUCTUATING FORCES

When a welded joint is subjected to a static force, the effect of stress concentration is neglected. However, stress concentration is an important consideration while designing a welded joint subjected to fluctuating force. The stress flow lines in the welded joints are shown in Fig. 8.40. There is sudden rise in the magnitude of stresses in the vicinity of welded joints. The values of the fatigue stress concentration factor (K_f) for different types of welds are given in Table 8.3. These factors are recommended by CH Jennings. The welded joints subjected to fluctuating stresses are designed in accordance with the procedure outlined in Chapter 5. This procedure is explained in the problem.

Table 8.3 *Fatigue stress concentration factor*

Type of weld	K_f
Reinforced butt-weld	1.2
Toe of transverse fillet-weld	1.5
End of parallel fillet-weld	2.7
T-butt joint with sharp corners	2.0

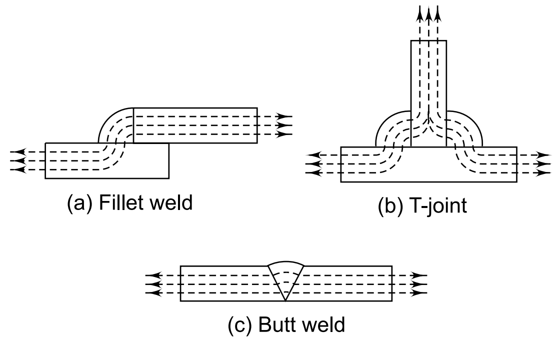

Fig. 8.40 *Stress Concentration in Welds*

Example 8.17 *Two plates, 25 mm thick, are welded together by means of a reinforced butt weld and subjected to a completely reversed axial load of ± 100 kN as shown in Fig. 8.41. The throat of the weld is 25 mm. The ultimate tensile strength of the weld metal is 450 N/mm². The surface finish of the weld is equivalent to that of the forged component and the reliability is 90%. Determine the length of the weld if the factor of safety is 2.*

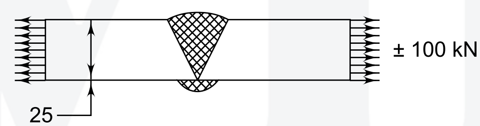

Fig. 8.41

Solution

Given $P = \pm 100$ kN $h = 25$ mm
$S_{ut} = 450$ N/mm² $R = 90\%$ $(fs) = 2$

Step I *Endurance limit stress for weld*

$$S'_e = 0.5\, S_{ut} = 0.5(450) = 225 \text{ N/mm}^2$$

From Fig. 5.24 (forged surface and $S_{ut} = 450$ N/mm²),

$$K_a = 0.52$$

For 25 mm depth of cross-section, $K_b = 0.85$
For 90% reliability, $K_c = 0.897$
For reinforced butt weld, (Table 8.3)

$$K_f = 1.2$$

$$\therefore \qquad K_d = \frac{1}{K_f} = \frac{1}{1.2}$$

[2] IS 815 – 1974: Classification and coding of covered electrodes for metal arc welding of structural steels.

The endurance limit of the butt weld is given by,

$$S_e = K_a K_b K_c K_d S'_e$$

$$= (0.52)(0.85)(0.897)\left(\frac{1}{1.2}\right)(225)$$

$$= 74.34 \text{ N/mm}^2$$

Step II *Permissible stress amplitude*

$$\sigma_t = \frac{S_e}{(fs)} = \frac{74.34}{2} = 37.17 \text{ N/mm}^2$$

Step III *Length of weld*

$$\sigma_t = \frac{P}{lh} \quad \text{or} \quad 37.17 = \frac{100\,000}{l(25)}$$

$$\therefore \quad l = 107.61 \quad \text{or} \quad 110 \text{ mm}$$

8.17 WELDING SYMBOLS

The complete information about the welded joint is conveyed by the designer to the welding operator by placing suitable welding symbols on the drawings. The information includes the type of welded joint, the size of weld, the location of weld and certain special instructions[3]. The basic symbols used to specify the type of weld are shown in Fig. 8.42. The complete weld symbol consists of the following elements:

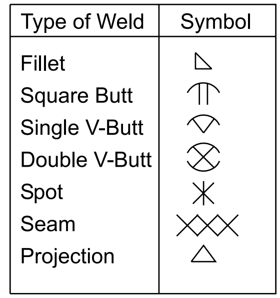

Fig. 8.42 *Basic Weld Symbols*

(i) a basic symbol to specify the type of weld;
(ii) an arrow and a reference line to indicate the location of the weld;
(iii) supplementary symbols to indicate special instructions such as weld-all-round, site weld, finish, etc.; and
(iv) dimensions of the weld in cross-section and length.

The weld symbols are illustrated in Fig. 8.43. The location of the weld is indicated by an arrow and a reference line. The head of the arrow indicates the reference side of the joint. When the weld symbol is below the reference line, the weld is made on the same side of the joint as the arrowhead. When the weld symbol is above the reference line, the weld is made on the other side of the joint opposite the arrowhead. The dimensions given in the fillet weld indicate the leg dimensions. The length of the weld is indicated on the right hand side of the symbol. If nothing is specified, it means that the weld is continuous along the entire length of the joint.

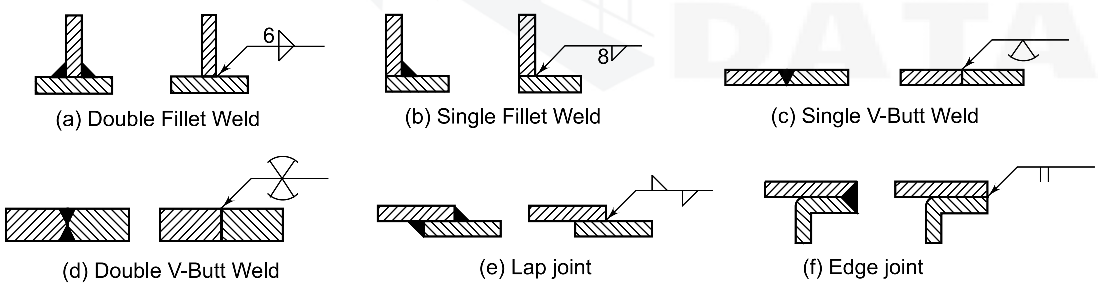

Fig. 8.43 *Weld Symbols*

[3] IS 813–1986: Scheme of symbols for welding.

8.18 WELD INSPECTION

The objective of weld inspection is to ascertain the satisfactoriness of the welding carried out for making a sound joint between the two parts. Welding inspection and testing differ from testing of engineering materials on account of the following two reasons:

(i) It is not possible to cut the portion of the actual welded joint for mechanical testing. Tests are therefore carried out on 'similar welds' prepared specially for testing purposes.

(ii) The quality of welded joint depends upon the skill and ability of the welding operator. Since the work is manual, variations in quality may occur. Therefore, two similar welds may not be exactly similar and this may result in misleading conclusions.

Weld inspections are broadly classified into two groups, namely, destructive and non-destructive examinations. *Destructive examinations* are carried out on 'similar welds' to assess certain characteristics like ultimate tensile strength or hardness of the welded joint. In this case, special specimens are prepared to carry out the tests. *Non-destructive testing* is classified into the following categories:

(i) Visual examination
(ii) Radiographical examination by *X*-rays
(iii) Radiographical examination by γ-rays
(iv) Magnetic crack detection method
(v) Crack detection by ultrasonic vibrations

The objective of non-destructive examination is to detect the discontinuities in the actual welded structure, such as cracks, crater cavities, entrapped slag or flaws. Visual inspection is the simplest and cheapest method of weld inspection. The discontinuities, which are present on the surface of the weld, are detected by this method. However, the defects below the surface of the weld will remain undetected.

The *X-ray method*, which is recommended for class-1 pressure vessels, is expensive. The method consists of exposing the welded joint to *X*-ray radiation. When *X*-rays fall upon the metal, their passage is obstructed by the metal and a part of the radiation is absorbed. The amount of radiation absorbed depends upon the length of metallic path traversed by the rays. Where a void or crack is present, the amount of radiation absorbed in that region is less than that in the rest of the weld. This is revealed on a photographic film by a dark spot. The defects that can be detected by an *X*-ray examination are cracks, cavities due to imperfect fusion between the weld and the surrounding surfaces, gas pockets and cavities due to entrapped non-metallic matter like slag, flux or oxide. The principle and procedure of other non-destructive examinations can be obtained from references.[4]

8.19 RIVETED JOINTS

The joints used in mechanical assemblies are classified into two groups—permanent and separable. *Permanent joints* are those joints which cannot be disassembled without damaging the assembled parts. Riveted and welded joints are permanent joints. *Separable joints* are those joints which permit disassembly and reassembly without damaging the assembled parts. Bolted joints, cotter joints and splined connections are the examples of separable joints. In the past, riveted joints were widely used for making permanent joints in engineering applications like boilers, pressure vessels, reservoirs, ships, trusses, frames and cranes. During the last few decades, rapid development of welding technology has considerably reduced the sphere of applications of riveted joints. Today, riveted joints have almost been replaced by welded joints.

A rivet consists of a cylindrical shank with a head at one end as shown in Fig. 8.44(a). This head is formed on the shank by an upsetting process in a machine called an *automatic header*. The rivet is inserted in the holes of the parts being assembled as shown in Fig. 8.44(b) and the head is firmly held against the back up bar. In the riveting

[4] H B Cary – *Modern Welding Technology* – Prentice-Hall International, 1979.

process, the protruding end of the shank is upset by hammer blows to form the closing head. In rivet terminology, the closing head is called the *point*. The head, shank and point are three main parts of the rivet.

A rivet is specified by the shank diameter of the rivet, e.g., a 20 mm rivet means a rivet having 20 mm as the shank diameter. The standard sizes of rivets are 12, 14, 16, 18, 20, 22, 24, 27, 30, 33,36, 39, 42 and 48 mm.

There are two methods of riveting—hand riveting and machine riveting. In *hand riveting*, a die is placed on the protruding end of the shank as shown in Fig. 8.44(c) and blows are applied by a hammer. In *machine riveting*, the die is a part of the hammer, which is operated by pneumatic, hydraulic or steam pressure. Riveting methods are also classified on the basis of temperature of the shank, viz., *hot riveting* and *cold riveting*. The difference between hot and cold riveting is as follows:

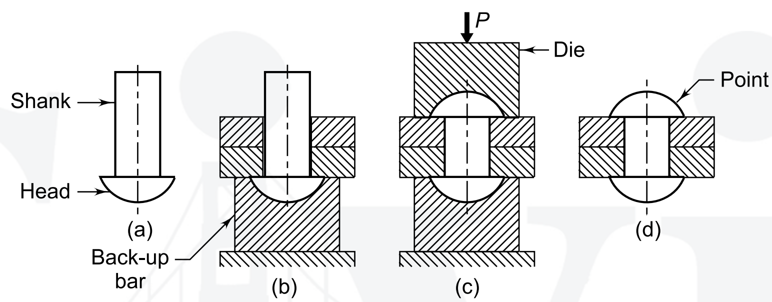

Fig. 8.44 *Riveted Joint*

(i) In hot riveting, the end of the rivet shank is heated to about 1000° to 1100°C till it becomes bright red and then the blows are applied by a hammer. In cold riveting, there is no such heating.

(ii) In hot riveting, when the rivet cools, the reduction in the length of the shank is prevented by the heads resting against the connected members. Therefore, the shank portion of the rivet is subjected to tensile stress while the connected parts are compressed. This is illustrated in Fig. 8.45.

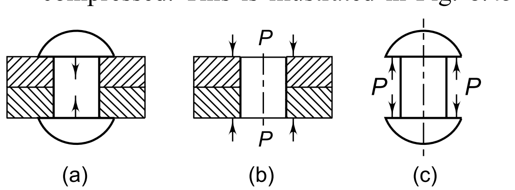

Fig. 8.45 *(a) Tendency of Shank to Contract (b) Parts in Compression (c) Shank in Tension*

The compression of the connected parts causes friction, which resists sliding of one part with respect to another. Cold riveting does not hold the connecting elements together with as great a force as is developed in hot riveting. Therefore, hot riveting is recommended for fluid tight joints in pressure vessels.

(iii) In hot riveting, the shank of the rivet is subjected to tensile stress. In cold riveting, the shank is mainly subjected to shear stress.

(iv) Cold riveting is applicable for steel rivets up to 8 to 10 mm diameter and rivets made of non-ferrous metals like brass, copper and aluminium alloys. Hot riveting is carried out for steel rivets with diameters more than 10 mm.

In riveted structures, there are two methods to make holes in the plates—*punching* and *drilling*. The difference between these two methods is as follows:

(i) Punching is cheaper operation while drilling is costly.

(ii) In punching process, the holes in different plates cannot be located with sufficient accuracy. Drilling results in more accurate location and size of the holes.

(iii) Punching injures the metal in the vicinity of the hole. The drilling operation does not injure metal.

(iv) Punching is feasible only for thin plates up to 25 mm thickness because fine cracks are formed around the periphery of the holes in case of punching of the thick plates. Drilling operation is feasible for any thickness of the plates.

Sometimes, the punching operation is followed by a *reaming* operation. In critical joints, this combines the advantages of punching and drilling.

Traditional mechanical structures involving riveted joints are classified into the following three groups:

(i) boilers, pressure vessels and tanks;
(ii) bridges, trusses, cranes and machinery in general; and
(iii) hulls of ships.

Fluid tightness is a desirable property of the joints in boilers, pressure vessels and ships. *Strength* and *rigidity* are desirable characteristics of joints in bridges, trusses and cranes. The joints in these applications are subjected to external load and strength is necessary to prevent failure of the joint. In application of a ship hull, strength, rigidity, durability and leakproofness are important criteria.

The scope of riveted joints in the above mentioned three groups of traditional applications is encroached upon by welded joints. However, riveted joint still remains the best and simplest type of permanent fastening in many applications. At present, riveted joints are mainly used in the following applications:

(i) In welding process, the parts are heated which deteriorate the metal structure and tamper the heat treated parts. It also results in warping of the components, which have been machined. Riveted joints are used where it is necessary to avoid the thermal after-effects of welding.

(ii) Riveted joints are used for metals with poor weldability like aluminium alloys.

(iii) When the joint is made of heterogeneous materials, such as the joint between steel plate and asbestos friction lining, riveted joints are preferred.

(iv) Welded joints have poor resistance to vibrations and impact load. A riveted joint is ideally suitable in such situations.

(v) Riveted joints are used where thin plates are to be assembled. They are popular especially for aircraft structures where light structures made of aluminium alloys are to be fastened.

A riveted joint has the following advantages over a welded joint:

(i) A riveted joint is more reliable than a welded joint in applications which are subjected to vibrations and impact forces.

(ii) Welded joints are, in general, restricted to steel parts. Riveted joints can be used for non-ferrous metals like aluminium alloy, copper, brass or even non-metals like asbestos or plastics.

(iii) The heat required for welding causes warping and affects the structure of heat treated components. The parts assembled by riveted joints are free from such thermal after-effects.

(iv) The quality of riveted joint can be easily checked, while inspection methods for welded joint, such as radiographic inspection of pressure vessels, are costly and time consuming.

(v) When the riveted joint is dismantled, the connected components are less damaged compared with those of welded joints.

The disadvantages of riveted joints compared with welded joints are as follows:

(i) The material cost of riveted joints is more than the corresponding material cost of welded joints due to high consumption of metal. There are two factors which account for more material consumption. The holes required for rivets weaken the working cross-section of the plate and it is necessary to increase the plate thickness to compensate for this loss. Therefore, the thickness of the plate or part is more in case of riveted joint compared with corresponding thickness of parts in case of welded joint. In addition, the weight of rivet is more than the weight of weld. It is estimated that rivets account for 3.5 to 4 per cent of the weight of the structure, while the weight of weld material comes only to 1 to 1.5 per cent. Increased material required for rivet and additional plate thickness increase the material cost of riveted joints. In addition, overlapping strap-plates are required in some types of riveted joints.

(ii) The labour cost of riveted joints is more than that of welded joints. Riveted joint requires higher labour input due to necessity to perform additional operations like layout and drilling or punching of holes. Besides, the process of riveting is much more complicated and less productive compared with welding operation.

(iii) The overall cost of riveted joint is more than that of welded joint due to increased metal consumption and higher labour input. On the other hand, welding is cheaper compared with riveting.

(iv) Riveted assemblies have more weight than welded assemblies due to strap-plates and rivets. Welded assemblies result in lightweight construction.

(v) Riveting process creates more noise than welding due to hammer blows.

(vi) Holes required to insert rivets cause stress concentration. However, in many applications, plates are made of ductile material like mild steel and the effect of stress concentration is reduced due to plastic flow in the vicinity of the holes. Stress concentration also exists in the rivet at the junction between the shank and head. When riveted joint is subjected to variable external load, vibrations or temperature variation, fatigue failure may occur in the regions of stress concentration.

8.20 TYPES OF RIVET HEADS

There are number of shapes for the head of the rivet. The most popular type of rivet head is *snap head* as shown in Fig. 8.46(a). It is also called *button head*. Riveted joint with a snap head has strength and fluid tightness. It is used in boilers, pressure vessels and general engineering applications. Its main drawback is the protruding head, which is objectionable in some cases. *Pan head* rivet, illustrated in Fig. 8.46(b), consists of frustum of cone attached to the shank. It is also called *cone head rivet*. Pan head rivets are mainly used in boilers and ship hulls and are ideally suited for corrosive atmosphere. Its main drawback is the protruding head. In applications where protruding head is objectionable, *countersunk head rivet* as shown in Fig. 8.46(c), is employed. The riveted joints with snap and countersunk head rivets are illustrated in Fig. 8.47. Countersunk head rivets are used in structural work and ship hulls below the waterline. The countersunk hole weakens the plates or parts that are assembled to a great extent. Therefore, countersunk head rivets should be used under unavoidable circumstances. The *flat head rivet* is shown in Fig. 8.46(d). The height of the protruding head is less than that of snap head rivet or pan head rivet. It does not weaken the plate being assembled. They are used for general engineering applications. Flat head rivets of small sizes are called *tinmen's rivets*, which are used in light sheet metal work such as manufacture of buckets, steel boxes and air conditioning ducts. A combination of countersunk head and snap head is shown in Fig. 8.46(e). It is also called *half countersunk head*. The height of the protruding head is less than that of snap head rivet. It is used for joining steel plates up to 4 mm thickness.

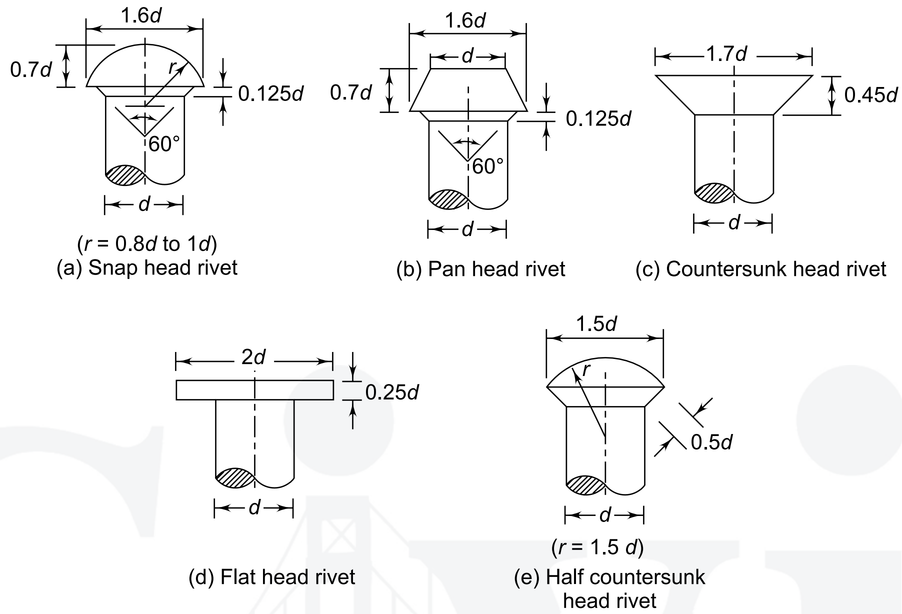

Fig. 8.46 *Types of Rivet Head*

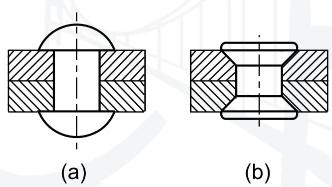

Fig. 8.47 *(a) Shap Head Rivet (b) Counter Sunk Head Rivet*

Depending upon the application, there is slight variation in proportions of rivet heads. There are various standards which give dimensions of various types of rivets.[5–10]

The desirable properties of rivets are as follows:

(i) The rivet should be sound, free from cracks, flaws, burrs, seams, pits and other defects.

(ii) The head of rivet should be concentric with the axis of the shank.

(iii) The end of rivet should be square with respect to the axis.

As shown in Fig. 8.48, the length of rivet shank is given by,

$$l = (t_1 + t_2) + a \tag{8.33}$$

where,

l = length of rivet shank (mm)

t_1, t_2 = thickness of plates (mm)

a = length of shank portion necessary to form the closing head (mm)

Depending upon the shape of the head, the magnitude of a varies from 0.7d to 1.3d. Or,

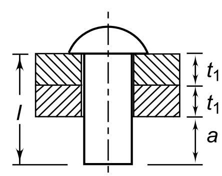

Fig. 8.48 *Length of Shank*

[5] IS 866–1957: Specification for Tinmen's rivets.
[6] IS 1928–1961: Specification for boiler rivets.
[7] IS 1929–1982: Specification for hot forged steel rivets for hot closing.
[8] IS 2155–1982: Specification for cold forged solid steel rivets for hot closing.
[9] IS 2907–1964: Specification for non-ferrous rivets.
[10] IS 2998–1982: Specification for cold forged steel rivets for cold closing.

$$a = 0.7d \text{ to } 1.3d \tag{8.34}$$

where,

d = diameter of the shank of rivet (mm)

8.21 TYPES OF RIVETED JOINTS

Riveted joints used for joining the plates are classified into two groups—lap joint and butt joint. *Lap joint* consists of two overlapping plates, which are held together by one or more rows of rivets as shown in Fig. 8.49. Depending upon the number of rows, the lap joints are further classified into *single-riveted lap joint*, *double-riveted lap joint* or *triple riveted lap joint*. In double or triple riveted lap joints, the rivets can be arranged in chain pattern or zig-zag pattern as shown in Fig. 8.49(b) and (c) respectively. A *chain riveted joint* is a joint in which the rivets are arranged in such a way that rivets in different rows are located opposite to each other. A *zig-zag riveted joint* is a joint in which the rivets are arranged in such a way that every rivet in a row is located in the middle of the two rivets in the adjacent row.

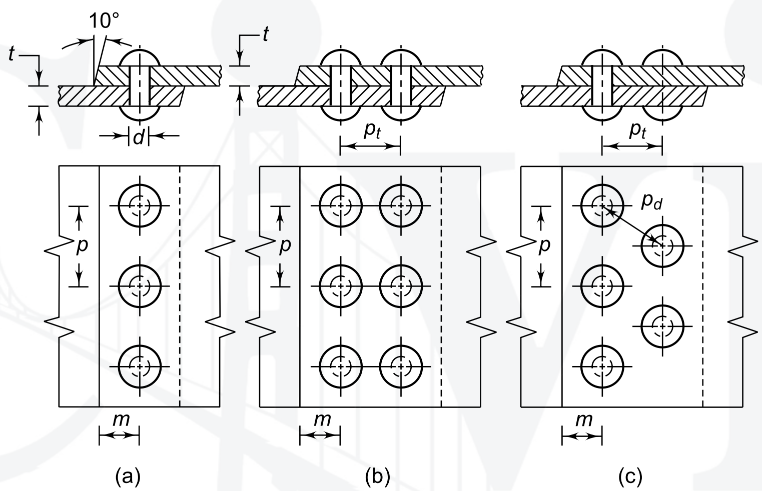

Fig. 8.49

The following terms are used in the terminology of riveted joints:

(i) Pitch (p) The pitch of the rivet is defined as the distance between the centre of one rivet to the centre of the adjacent rivet in the same row. Usually,

$$p = 3d$$

where d is shank diameter of the rivet.

(ii) Margin (m) The margin is the distance between the edge of the plate to the centreline of rivets in the nearest row. Usually,

$$m = 1.5d$$

(iii) Transverse Pitch (p_t) Transverse pitch, also called *back pitch* or *row pitch*, is the distance between two consecutive rows of rivets in the same plate. Usually,

$$p_t = 0.8p \quad \text{(for chain riveting)}$$
$$= 0.6p \quad \text{(for zig-zag riveting)}$$

(iv) Diagonal Pitch (p_d) Diagonal pitch is the distance between the centre of one rivet to the centre of the adjacent rivet located in the adjacent row.

The above terminology is illustrated in Fig. 8.49.

As shown in Fig. 8.50, lap joint is always subjected to bending moment M_b due to the eccentric force P. The line of action of the force P in two plates, that are joined by lap joint, have an eccentricity equal to $(t/2 + t/2)$ or t. The bending moment is given by,

$$M_b = P \times t$$

This bending moment causes distortion of the plates as shown in Fig. 8.50(c). This is the drawback of lap joint.

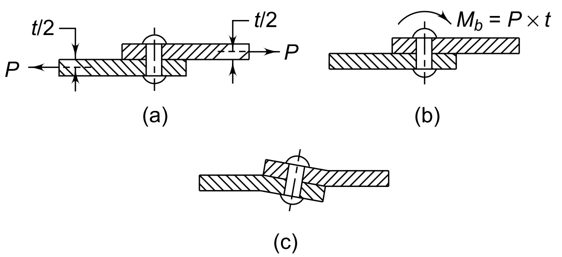

Fig. 8.50 *Bending of Plates in Lap Joint*

The construction of a *butt joint* is shown in Fig. 8.51. It consists of two plates, which are kept in alignment against each other in the same plane and a strap or cover plate is placed over these plates and riveted to each plate. Placing the two plates, which are to be fastened, against each other is called *butting*. Depending upon the number of rows of rivets in each plate, the butt joints are classified as *single-row butt joint* and *double-row butt joint*. Depending upon the number of straps, the butt joints are also classified into *single-strap butt joint* and *double-strap butt joint*. The types of butt joints are illustrated in Fig. 8.51 and 8.52. The line of action of the force acting on two plates, joined by butt joint, lies in the same plane. Therefore, there is no bending moment on the joint and no warping of the plates. This is the main advantage of butt joint compared with lap joint. The disadvantage of butt joint is the requirement of additional strap plates, which increases cost. Therefore, butt joint is costly compared with lap joint.

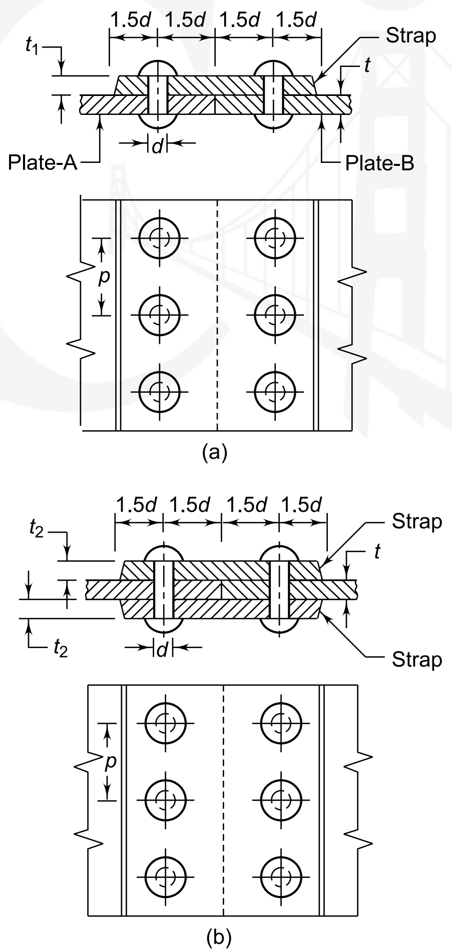

Fig. 8.51 *Types of Single-riveted Butt Joint (a) Single-strap Butt Joint (b) Double-strap Butt Joint*

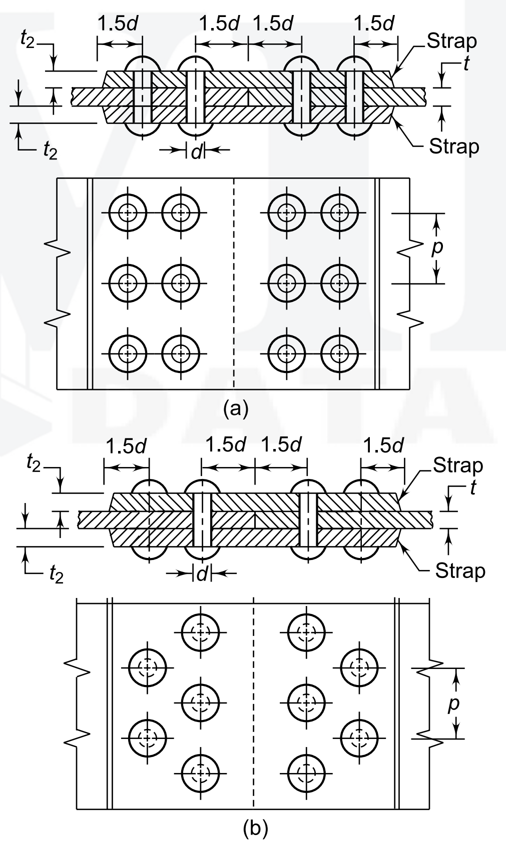

Fig. 8.52 *Types of Double-riveted Double-strap Butt Joint (a) Chain Pattern (b) Zig-zag Pattern*

A typical riveted joint used in construction work such as bridges, trusses and cranes is shown in Fig. 8.53. It is known as *diamond joint* because the rivets are arranged in a diamond shape. It is also called *Lozenge joint*. This type of joint results in economical construction because a plate of smaller width is required for this joint. Lozenge joint is often called 'economical' joint.

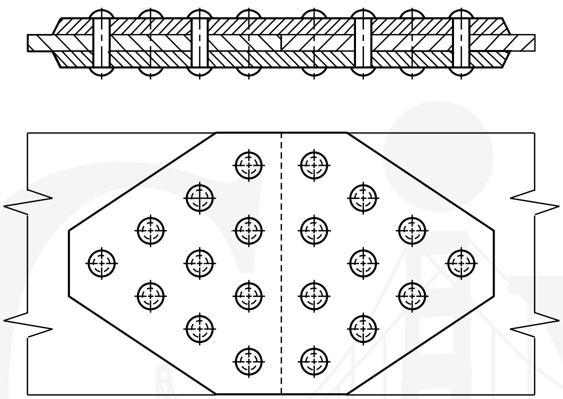

Fig. 8.53 *Diamond or Lozenge Joint*

8.22 RIVET MATERIALS

Rivets used in most of the applications are made of mild steel. There are two varieties of steel rivet bars—*hot rolled steel rivet bar* and *high-tensile steel rivet bar*. Their chemical composition is as follows:

carbon = 0.23% (max)
sulphur = 0.05% (max)
phosphorus = 0.05% (max)

Mechanical properties[11-13] of steel rivet bars are given in Table 8.4. Rivets used in corrosive atmosphere are made of stainless steel. Rivets used for connecting non-ferrous metals and soft materials are made of copper, brass, bronze and aluminium alloys. Structural joints made of aluminium alloy sections employ duralumin rivets. When metal for the parts being joined and rivet metal have different electrochemical potentials, they form galvanic pairs and accelerate the corrosion process. Therefore, many times rivets are made of the same material as the parts being joined.

Table 8.4 *Mechanical properties of steel rivet bars*

Grade	S_{ut} *(N/mm²)*	S_{yt} *(N/mm²)*	S_{su} *(N/mm²)*	*Elongation (%)*
1. Hot rolled steel rivet bar				
6–12 mm dia.	410–530	260	330	23
12–20 mm dia.		250		
20–40 mm dia.		240		
2. High tensile steel rivet bar				
6–12 mm dia.	460	310	370	22
12–20 mm dia.		300		
20–40 mm dia.		280		
3. Steel rivet bar for boiler of Grade St 37 BR				
0–20 mm dia.	360–440	220	–	26
> 20 mm dia.		200		
4. Steel rivet bar for boiler of Grade St 42 BR				
0–20 mm dia.	410–500	250	–	23
> 20 mm dia.		240	240	

S_{su} = ultimate shear strength

[11] IS 1148–1982: Specification for hot rolled steel rivet bars for structural purposes.
[12] IS 1149–1982: Specification for high tensile steel rivet bars for structural purposes.
[13] IS 1990–1973: Specification for steel rivet and stay bars for boilers.

8.23 TYPES OF FAILURE

The types of failure in riveted joints are illustrated in Fig. 8.54. According to conventional theory, the failure of the riveted joint may occur in any one or more of the following ways:

(i) shear failure of the rivet;
(ii) tensile failure of the plate between two consecutive rivets;
(iii) crushing failure of the plate;
(iv) shear failure of the plate in the margin area; and
(v) tearing of the plate in the margin area.

Based upon the above-mentioned criteria of failure, strength equations are written for riveted joints.

8.24 STRENGTH EQUATIONS

The strength of riveted joint is defined as the force that the joint can withstand without causing failure. When the operating force acting on the joint exceeds this force, failure occurs. Strength equations can be written for each type of failure. However, in analysis of riveted joints, mainly three types of failure are considered. They are as follows:

(i) shear failure of the rivet;
(ii) tensile failure of the plate between rivets; and
(iii) crushing failure of the plate.

Based on the above criteria of failure, the strength equations are derived.

(i) Shear Strength of Rivet The shear failure in the rivet of a single-riveted lap joint is illustrated in Fig. 8.55(a). In this case, the rivet is in single shear. The strength equation is written in the following way,

$$P_s = \frac{\pi}{4} d^2 \tau$$

where,

P_s = shear resistance of rivet per pitch length (N)

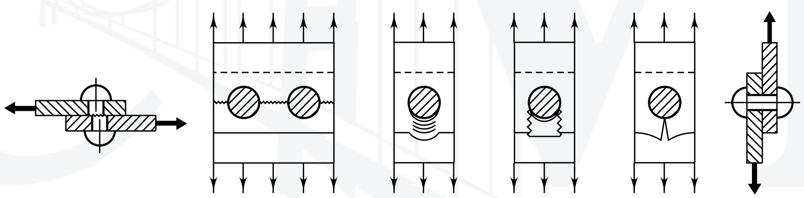

Fig. 8.54 *Types of Failure in Riveted Joint (a) Shear Failure of Rivet (b) Tensile Failure of Plate between Rivets (c) Crushing Failure of Plate by Rivet (d) Shear Failure of Plate by Rivet (e) Tearing of Margin*

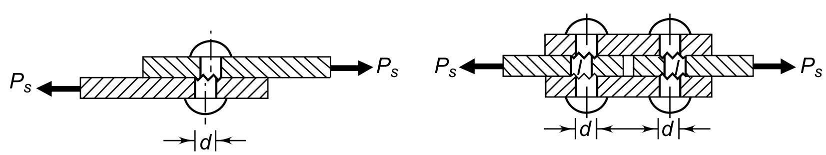

Fig. 8.55 *Shear Failure in Rivets*

d = shank diameter of rivet (mm)

τ = permissible shear stress for rivet material (N/mm²)

In case of double or triple riveted lap joints, there are number of rivets and the above equation is modified and written in the following way:

$$P_s = \frac{\pi}{4} d^2 \tau n \quad \text{(for single shear)} \tag{8.35}$$

where,

n = number of rivets per pitch length.

For double-riveted joint,

$$n = 2$$

For triple-riveted joint,

$$n = 3$$

and so on.

In case of double-strap single-riveted butt joint, the rivets are subjected to double shear as shown in Fig. 8.55(b). The area that resists shear failure is twice the cross-sectional area of the rivet and Eq. (8.35) is modified in the following way:

$$P_s = 2\left[\frac{\pi}{4} d^2 \tau n\right] \quad \text{(for double shear)} \qquad (8.36)$$

(ii) Tensile Strength of Plate between Rivets The tensile failure of the plate between two consecutive rivets in a row is illustrated in Fig. 8.56. The width of plate between the two points A and B is $(p - d/2 - d/2)$ or $(p - d)$ and the thickness is t. Therefore, tensile resistance of the plate between two rivets is given by,

$$P_t = (p - d)\, t\sigma_t \qquad (8.37)$$

where,

P_t = tensile resistance of plate per pitch length (N)

p = pitch of rivets (mm)

t = thickness of plate (mm)

σ_t = permissible tensile stress of plate material (N/mm^2)

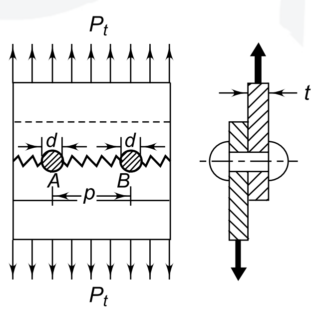

Fig. 8.56 *Tensile Failure of Plate between Rivets*

(iii) Crushing Strength of Plate The crushing failure of the plate is illustrated in Fig. 8.57. This type of failure occurs when the compressive stress between the shank of the rivet and the plate exceeds the yield stress in compression. The failure results in elongating the rivet hole in the plate and loosening of the joint. The crushing resistance of the plate is given by,

$$P_c = dt\sigma_c n \qquad (8.38)$$

where,

P_c = crushing resistance of plate per pitch length (N)

σ_c = permissible compressive stress of plate material (N/mm^2)

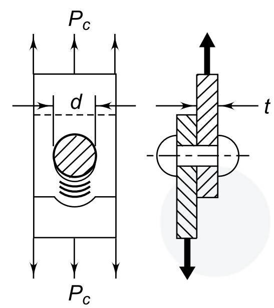

Fig. 8.57 *Crushing Failure of Plate*

8.25 EFFICIENCY OF JOINT

The efficiency of the riveted joint is defined as the ratio of the strength of riveted joint to the strength of unriveted solid plate. The strength of the riveted joint is the lowest value of P_s, P_t and P_c determined from Eqs (8.35) to (8.38). The strength of solid plate of width, equal to the pitch p and thickness t, subjected to tensile stress σ_t is given by,

$$P = pt\sigma_t \qquad (8.39)$$

Therefore, the efficiency is given by,

$$\eta = \frac{\text{Lowest of } P_s, P_t, \text{and } P_c}{P} \qquad (8.40)$$

8.26 CAULKING AND FULLERING

In applications like pressure vessels and boilers, the riveted joint should be leakproof and fluid tight. Caulking and fullering processes are used to obtain such leakproof riveted joints. The caulking process is applied to the edges of plates in a lap joint and the edges of strap plate in a butt joint. These edges are first beveled to approximately 70° to 75° and the caulking tool is hammered on the edge as shown in Fig. 8.58(a). The caulking is done either by hand hammer or by the use of pneumatic or hydraulic hammer. The head of the rivet is also hammered down with the caulking tool. The blows of caulking tool closes the surface asperities and cracks on the contacting surfaces between two plates and also between the rivet and the plates, resulting in leakproof joint. Great care must be exercised to prevent injury to the plate, otherwise caulking will result in opening of the joint instead of closing it. Caulking cannot be applied to plates with less than 6 mm thickness.

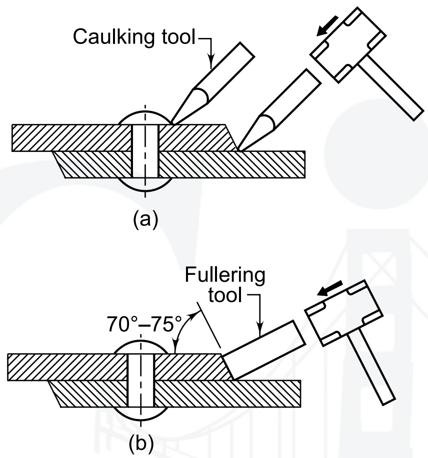

Fig. 8.58 *Caulking and Fullering*

Fullering, as shown in Fig. 8.58(b), is similar to the caulking process except for the shape of the tool. The width of the fullering tool is equal to the thickness of the plate being hammered. The blows of the fullering tool result in simultaneous pressure on the entire edge of the plate.

Example 8.18 *A brake band attached to the hinge by means of a riveted joint is shown in Fig. 8.59. Determine the size of the rivets needed for the load of 10 kN. Also, determine the width of the band. The permissible stresses for the band and rivets in tension, shear and compression are 80, 60 and 120 N/mm² respectively. Assume,*

margin (m) = 1.5d

transverse pitch (p_t) = p

Find the pitch of the rivets.

Solution

Given $P = 10$ kN $t = 3$ mm $\sigma_t = 80$ N/mm²
$\tau = 60$ N/mm² $\sigma_c = 120$ N/mm²

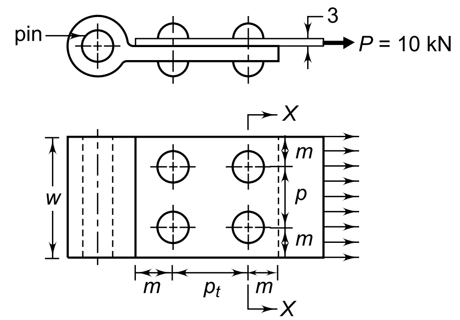

Fig. 8.59

Step I *Diameter of rivets*

There are four rivets in the lap joint, which are subjected to single shear. From shear consideration,

$$4\left[\frac{\pi}{4}d^2\tau\right] = P \quad \text{or} \quad 4\left[\frac{\pi}{4}d^2(60)\right] = 10\times10^3$$

$\therefore \quad d = 7.28$ or 8 mm (a)

From crushing consideration,

$4dt\sigma_c = P$ or $4d(3)(120) = 10 \times 10^3$

$\therefore \quad d = 6.94$ or 7 mm (b)

From (a) and (b), it is observed that shearing becomes the criterion of design. Therefore,

$$d = 8 \text{ mm}$$

Step II *Width of band*

Considering tensile strength of plate along the section *XX*,

$(w - 2d)t\sigma_t = P$ or

$(w - 2 \times 8)(3)(80) = 10 \times 10^3$

$\therefore \quad w = 57.67$ or 60 mm

Step III *Pitch of rivets*

$m = 1.5d = 1.5(8) = 12$ or 15 mm

$p + 2m = w$ or $p + 2(15) = 60$

$p = 30$ mm

$p_t = p = 30$ mm

Example 8.19 *Two flat plates subjected to a tensile force P are connected together by means of double-strap butt joint as shown in Fig. 8.60. The force P is 250 kN and the width of the plate w is 200 mm. The rivets and plates are made of the same steel and the permissible stresses in tension, compression and shear are 70, 100 and 60 N/mm² respectively. Calculate:*

(i) *the diameter of the rivets;*
(ii) *the thickness of the plates;*
(iii) *the dimensions of the seam, viz., p, p_t and m; and*
(iv) *the efficiency of the joint.*

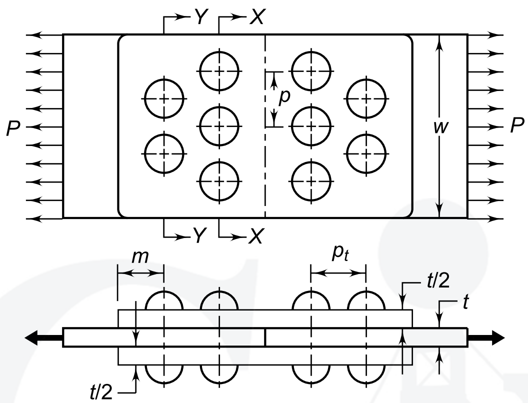

Fig. 8.60

Solution

Given $P = 250$ kN $\quad w = 200$ mm
$\sigma_t = 70$ N/mm^2 $\quad \tau = 60$ N/mm^2 $\quad \sigma_c = 100$ N/mm^2

Step I *Diameter of rivets*

There are five rivets subjected to double shear. From Eq. (8.36),

$$P_s = 2\left[\frac{\pi}{4}d^2\tau n\right]$$

or $$(250 \times 10^3) = 2\left[\frac{\pi}{4}d^2(60)(5)\right]$$

$\therefore \qquad d = 23.03$ or 25 mm $\qquad$ (i)

Step II *Thickness of the plates*

In case of tension in the plate, the first thought is to investigate the section-*XX* passing through the centres of the three holes as shown in Fig. 8.60. This section has minimum cross-sectional area. However, if the main plate is to break at this section, the two-hole section, denoted by *YY*, must also fail before the joint will break. This failure at the section-*YY* may be due to shear failure in two rivets or crushing failure in the plate at two holes. On the other hand, the plate might fail in tension at the two-hole section without affecting the three-hole section. Therefore, strength equations are written for two-hole sections rather than three-hole sections. Or,

$$(w - 2d)\, t\sigma_t = P$$

or $$(200 - 2 \times 25)\, t(70) = 250 \times 10^3$$

$$t = 23.81 \text{ or } 25 \text{ mm} \qquad \text{(ii)}$$

Step III *Pitch of rivets*

The pitch of the rivets is given by,

$$p = \frac{\text{width of plate}}{\text{number of rivets}} = \frac{200}{3} = 66.67 \text{ mm}$$

or $\qquad p = 65$ mm

As mentioned in Section 8.19, the dimensions of the seam are as follows:

$$m = 1.5d = 1.5(25) = 37.5 \text{ or } 40 \text{ mm}$$

$$p_t = 0.6p = 0.6\,(65) = 39 \text{ or } 40 \text{ mm} \qquad \text{(iii)}$$

Step IV *Efficiency of joint*

From Eqs (8.36), (8.37) and (8.38),

$$P_s = 2\left[\frac{\pi}{4}d^2\tau n\right] = 2\left[\frac{\pi}{4}(25)^2(60)(5)\right]$$

$$= 294\,524.31 \text{ N} \qquad \text{(a)}$$

$$P_t = (w - 2d)\, t\sigma_t = (200 - 2 \times 25)(25)(70)$$

$$= 262\,500 \text{ N} \qquad \text{(b)}$$

$$P_c = dt\sigma_c\, n = 25\,(25)(100)(5) = 312\,500 \text{ N} \qquad \text{(c)}$$

From (a), (b) and (c), the lowest strength is 262 500 N. The strength of the solid plate is given by,

$$P = wt\,\sigma_t = 200\,(25)\,(70) = 350\,000 \text{ N}$$

Therefore, the efficiency of the joint is given by,

$$\eta = \frac{262\,500}{350\,000} = 0.75 \quad \text{or} \quad 75\% \qquad \text{(iv)}$$

Example 8.20 *Two tie-bar plates of a bridge structure, 250 mm wide and 20 mm thick, are to be connected by a double-strap butt joint as shown in Fig. 8.61. The rivets and the plates are made of steel. The permissible stresses in tension, shear and compression are 80, 60 and 120 N/mm^2 respectively.*

(i) *Determine the diameter of the rivet by using the following empirical relationship,*

$$d = 6\sqrt{t}$$

where t is the plate thickness.

(ii) *Determine the number of rivets by equating the strength of the plate with the strength of* *the rivets. Assume that shear resistance of one rivet in double shear is 1.875 times its resistance in single shear.*

(iii) Show the arrangement of rivets.

(iv) Determine the efficiency of the joint.

Assume the following relationships:

Margin (m) = 1.5d

Transverse pitch (p_t) = 2d

Thickness of strap = 0.625t

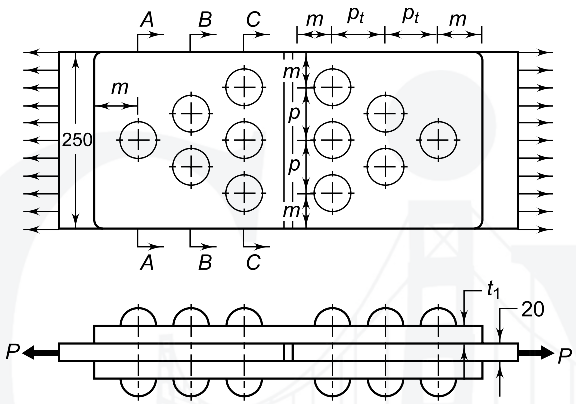

Fig. 8.61

Solution

Given $w = 250$ mm $t = 20$ mm

$\sigma_t = 80$ N/mm^2 $\tau = 60$ N/mm^2 $\sigma_c = 120$ N/mm^2

Step I *Diameter of rivets*

The diameter of the rivet is given by,

$$d = 6\sqrt{t} = 6\sqrt{20} = 26.83 \quad \text{or} \quad 27 \text{ mm} \qquad \text{(i)}$$

Step II *Number of rivets*

The shear resistance of one rivet in double shear is given by,

$$P_s = 1.875\left[\frac{\pi}{4}d^2\tau\right]$$

$$= 1.875\left[\frac{\pi}{4}(27)^2(60)\right] = 64\,412.47 \text{ N} \qquad \text{(a)}$$

Crushing resistance of one rivet is given by,

$$P_c = dt\sigma_c = 27(20)(120) = 64\,800 \text{ N} \qquad \text{(b)}$$

From (a) and (b), $P_s < P_c$

Therefore, shear strength of the rivet is the criterion of design. It is assumed that rivets are so arranged that there is only one rivet in the outer row. The tensile strength of the plate in the outer row is given by,

$$P_t = (w - d)\,t\sigma_t = (250 - 27)(20)(80)$$
$$= 356\,800 \text{ N} \qquad \text{(c)}$$

Equating the strength of the plate with the shear strength of n rivets,

$$356\,800 = n\,(64\,412.47)$$

$$\therefore \quad n = 5.54 \quad \text{or} \quad 6 \text{ rivets} \qquad \text{(ii)}$$

Step III *Arrangement of rivets*

The arrangement of six rivets in three rows is illustrated in Fig. 8.61. This type of arrangement in diamond shape is called Lozenge joint. The dimensions given in the figure have the following numerical values,

$$m = 1.5d = 1.5\,(27) = 40.5 \text{ or } 45 \text{ mm}$$
$$p_t = 2d = 2\,(27) = 54 \text{ or } 55 \text{ mm}$$
$$t_1 = 0.625\,t = 0.625\,(20) = 12.5 \text{ mm}$$

From the figure,

$$m + p + p + m = 250 \quad \text{or} \quad 45 + 2p + 45 = 250$$

$$\therefore \; p = 80 \text{ mm}$$

Step IV *Efficiency of joint*

In order to calculate the efficiency of the joint, we will proceed from the outer row of rivets to the inner row and find out the weakest cross-section of the joint. Let us consider the failure behaviour of three cross-sections denoted by *AA*, *BB* and *CC*.

(i) The joint can fracture along the section-*AA* without affecting the rivets in the middle and inner rows.

(ii) The joint cannot fracture along the section-*BB* without shearing one rivet in double shear. This is the rivet in the outer row.

(iii) The joint cannot fracture along the section-*CC*, without shearing three rivets in double shear. These are the rivets in outer and middle rows.

Based on these assumptions, the strength of the joint along three sections is calculated.

Along the section-*AA*,

$$\text{strength} = (w - d)\,t\sigma_t = (250 - 27)(20)(80)$$
$$= 356\,800 \text{ N} \qquad \text{(a)}$$

Along the section-*BB*,

$$\text{strength} = (w - 2d)\,t\sigma_t + 1 \times P_s$$
$$= (250 - 2 \times 27)(20)(80) + 64\,412.47$$
$$= 378\,012.47 \text{ N} \qquad \text{(b)}$$

Along the section-CC,

$$\text{strength} = (w - 3d)\, t\sigma_t + 3 \times P_s = (250 - 3 \times 27)(20)(80) + 3 \times 64\,412.47 = 463\,637.41 \text{ N} \quad \text{(c)}$$

Shear resistance of all rivets $= 6 \times P_s$

$$= 6 \times 64\,412.47 = 386\,474.8 \text{ N} \quad \text{(d)}$$

From (a), (b), (c), and (d), the lowest strength of the joint is along the section-AA.

Strength of solid plate $= wt\sigma_t = 250\,(20)(80) = 400\,000$ N

Therefore,

$$\eta = \frac{356\,800}{400\,000} = 0.892 \quad \text{or} \quad 89.2\% \quad \text{(iv)}$$

8.27 LONGITUDINAL BUTT JOINT FOR BOILER SHELL

There are two types of riveted joints in a cylindrical boiler shell. They are called longitudinal butt joint and circumferential lap joint. The plate of the boiler shell is bent to form the ring and the two edges of the plate are joined by a *longitudinal butt joint*. This longitudinal joint is usually a double-strap triple-riveted butt joint. The longitudinal joint makes a ring from the steel plate. The circumferential joint is used to get the required length of the boiler shell by connecting one ring to another. For this purpose, one ring is kept overlapping over the adjacent ring and the two rings are joined by a *circumferential lap joint*.

Boilers and pressure vessels are cylindrical vessels. They are subjected to circumferential and longitudinal tensile stresses. It can be proved that circumferential stress is twice the longitudinal stress. Therefore, the longitudinal joint should be stronger than the circumferential joint and butt joint is used for the longitudinal joint.

Boiler joints are subjected to steam pressure. They should withstand the steam pressure and also prevent leakage. Hence, great care must be exercised in their design and a high standard of workmanship should be provided in their manufacture. Further, the boiler shells are subjected to close inspection, which must conform to the Indian Boiler Regulation Act. Most steam boilers are designed, manufactured and afterwards tested according to this act. Indian Boiler Regulations are highly exacting and mandatory. There are two important points before we consider the design of riveted joints for boiler shells. At present, in a number of boiler applications, riveted joints are replaced by welded joints. Also, it is mandatory on the part of the designer to design the boiler joints strictly according to the Indian Boiler Regulations[14]. However, in this chapter, the design of boiler joint is based on fundamental equations to illustrate the theoretical principles. It is meant only for academic exercise. The following procedure is adopted for the design of a longitudinal butt joint for the boiler shell as illustrated in Fig. 8.62.

(i) Thickness of Boiler Shell The thickness of thin cylindrical shell subjected to internal pressure is given by,

$$t = \frac{P_i D_i}{2\sigma_t} \tag{8.41}$$

where,

t = thickness of cylinder wall (mm)

P_i = internal pressure (N/mm^2)

D_i = inner diameter of the cylinder (mm)

σ_t = permissible tensile stress for the cylinder material (N/mm^2)

The plate of the cylinder wall is to be drilled for the rivets. The weaker section passing through the rivet holes will not be as strong as the original solid plate. The ratio of strength of the joint to the strength of the original solid plate is expressed by the efficiency of the joint and denoted by η. Modifying Eq. (8.41) to account for this weakening effect,

$$t = \frac{P_i D_i}{2\sigma_t \eta} \tag{8.42}$$

where,

η = efficiency of the riveted joint (in fraction)

The wall of the boiler shell is subjected to thinning due to corrosion, which reduces the useful life of the shell. Provision has to be made by suitable increase in the wall thickness to compensate for the thinning due to corrosion.

[14] Justice S Malik—'Indian Boiler Regulations 1950'–Law Publishers (India) Pvt. Ltd.—1995.

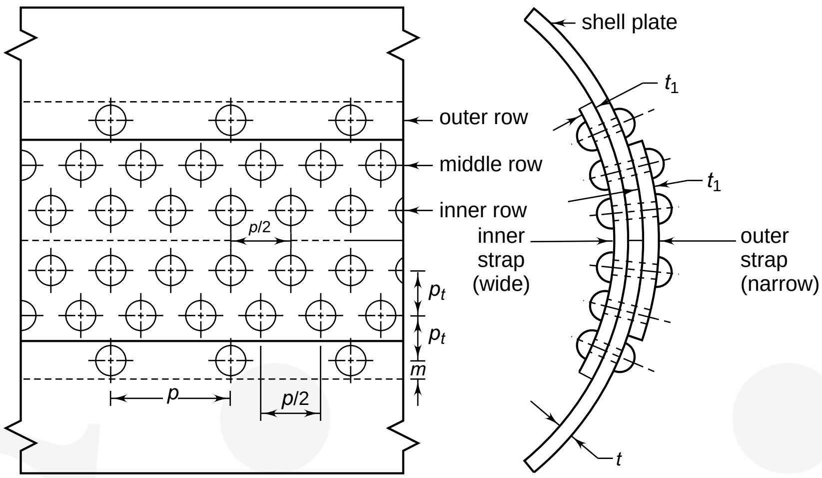

Fig. 8.62 *Triple-riveted Double-strap Butt Joint with Unequal Straps*

Corrosion allowance (CA) is additional metal thickness over and above that required to withstand internal pressure. A minimum corrosion allowance of 1.5 to 2 mm thickness is recommended unless a protective lining is employed. Introducing corrosion allowance in Eq. (8.42),

$$t = \frac{P_i D_i}{2\sigma_t \eta} + \text{CA} \tag{8.43}$$

where,

CA = corrosion allowance (mm)

Equation (8.42) is used to find out the thickness of the boiler shell. The efficiencies of commercial boiler joints are given in Table 8.5.

Table 8.5 *Efficiencies of riveted boiler joints*

Type of joint	*Efficiency (per cent)*
Lap joint	
single-riveted	45–60
double-riveted	63–70
triple-riveted	72–80
Double-strap butt joint	
single-riveted	55–60
double-riveted	70–83
triple-riveted	80–90
quadruple-riveted	85–94

The permissible tensile stress in Eq. (8.43) is obtained by,

$$\sigma_t = \frac{S_{ut}}{(fs)} \tag{8.44}$$

where,

S_{ut} = ultimate tensile strength of the plate material (N/mm^2)

(fs) = factor of safety

The factor of safety in boiler applications varies from 4.5 to 4.75. It is safe practice to assume the factor of safety as 5. There are two popular grades of steel used for boiler shells and boiler rivets. They are designated as Grade-St 37 BR and Grade-St 42 BR. Their ultimate tensile strengths are as follows[15]:

Grade	S_{ut} *(N/mm²)*
St 37 BR	360–440
St 42 BR	410–500

(ii) Diameter of Rivet Indian Boiler Regulations do not specify any formula to calculate the rivet diameter. However, empirical relationships are suggested by design engineers. They are as follows:

(a) When the thickness of plate is more than 8 mm, the rivet diameter is calculated by following Unwin's formula,

[15] IS 1990–1973: Specification for steel rivet and stay bars for boilers.

$$d = 6\sqrt{t} \tag{8.45}$$

where,

d = diameter of the rivet (mm)

t = thickness of the cylinder wall (mm)

(b) When the thickness of the plate is less than 8 mm, the diameter of rivet is obtained by equating shear resistance of rivets to crushing resistance.

(c) In no case, should the diameter of the rivet be less than the plate thickness.

The diameter of the rivet hole is slightly more than the rivet diameter. The standard sizes of a boiler rivet and corresponding rivet holes are given in Table 8.6. Alternatively, the diameter of the rivet hole can be obtained by following approximate relationship:

$$d' = d + (1 \text{ to } 2 \text{ mm}) \tag{8.46}$$

where, d' = diameter of rivet hole

(iii) Pitch of Rivet As shown in Fig. 8.62, the pitch of the rivets in the outer row is maximum.

Table 8.6 *Relationship between rivet diameter and diameter of rivet hole for boiler*

d (mm)	12	14	16	18	20	22	24	27	30	33	36	39	42	48
d' (mm)	13	15	17	19	21	23	25	28.5	31.5	34.5	37.5	41	44	50

The pitch of the rivets in the middle and inner rows is one half of the pitch in outer row, that is, (p/2). The pitch of the rivets in the outer row is obtained by equating the shear strength of the rivets to the tensile strength of the plate. For the purpose of analysis, let us consider the plate of length p with reference to Fig. 8.62. The tensile strength of the plate per pitch length in outer row of the rivets is given by,

$$P_t = (p - d)\, t\sigma_t \tag{a}$$

It is seen in Fig. 8.62, that inner and outer straps are of unequal width. The width of the inner strap is more than that of the outer strap. Therefore, the rivets in the outer row are subjected to single shear. At the same time, the rivets in middle and inner rows are subjected to double shear. Suppose,

n_1 = number of rivets subjected to single shear per pitch length

n_2 = number of rivets subjected to double shear per pitch length

When the straps have equal width, all rivets are subjected to double shear.

For boiler joints, the shear resistance of one rivet in single shear is given by,

$$\left[\frac{\pi}{4} d^2 \tau\right]$$

Also, the shear resistance of one rivet in double shear is assumed as,

$$1.875 \left[\frac{\pi}{4} d^2 \tau\right]$$

Therefore, the shear resistance of rivets in one pitch length is given by,

$$P_s = \left[\frac{\pi}{4} d^2 \tau\right] n_1 + 1.875 \left[\frac{\pi}{4} d^2 \tau\right] n_2$$

or,

$$P_s = (n_1 + 1.875 n_2) \left[\frac{\pi}{4} d^2 \tau\right] \tag{b}$$

Equating expressions (a) and (b),

$$(p - d)\, t\sigma_t = (n_1 + 1.875 n_2) \left[\frac{\pi}{4} d^2 \tau\right]$$

$$p = \frac{(n_1 + 1.875 n_2)\pi d^2 \tau}{4 t \sigma_t} + d \tag{8.47}$$

The pitch obtained by Eq. (8.47) has maximum and minimum limits. According to Indian Boiler Regulations,

(a) The pitch of the rivets should not be less than (2d) to enable the forming of the rivet head,

$$p_{\text{min.}} = 2d \tag{8.48}$$

(b) In order to provide leakproof joint, the maximum pitch is given by,

$$p_{\text{max.}} = Ct + 41.28 \tag{8.49}$$

The values of C are given in Table 8.7.

Table 8.7 *Values of C*

Number of rivets per pitch length	Values of C		
	Lap joint	Single-strap butt joint	Double-strap butt joint
1	1.31	1.53	1.75
2	2.62	3.06	3.5
3	3.47	4.05	4.63
4	4.17	–	5.52
5	–	–	6.0

(iv) Transverse Pitch (p_t) The transverse pitch or distance between the rows of rivets is specified according to Clause 184 of the Indian Boiler Regulations. The provisions of this clause are as follows:

Case I In a lap or butt joint, in which there are more than one row of rivets and in which there is an equal number of rivets in each row, the minimum distance between the rows of rivets is given by,

(For zigzag riveting)

$$p_t = 0.33p + 0.67d \tag{8.50}$$

(For chain riveting)

$$p_t = 2d \tag{8.51}$$

Case II In joints in which the number of rivets in the outer row is one-half of the number of rivets in each of the inner rows and in which the inner rows are zigzag riveted, the minimum distance between the outer row the and next row is given by,

$$p_t = 0.2p + 1.15d \tag{8.52}$$

The minimum distance between the rows in which there are full number of rivets is given by,

$$p_t = 0.165p + 0.67d \tag{8.53}$$

where p is the pitch of rivets in the outer row.

(v) Margin (m) The distance between the centre of the rivet hole from the edge of the plate is called margin. The minimum margin is given by,

$$m = 1.5d \tag{8.54}$$

(vi) Thickness of Straps (t_1) The thickness of straps for a riveted joint is calculated according to Clause 182 of Indian Boiler Regulations. The provisions of this clause are as follows:

Case I When the straps are of unequal width and in which every alternate rivet in the outer row is omitted,

$$t_1 = 0.75t \quad \text{(for wide strap)} \tag{8.55}$$

$$t_1 = 0.625t \quad \text{(for narrow strap)} \tag{8.56}$$

Case II When the straps are of equal width and in which every alternate rivet in outer row is omitted,

$$t_1 = 0.625t\left[\frac{p-d}{p-2d}\right] \tag{8.57}$$

where t_1 is the thickness of straps. The thickness of the strap, in no case, shall be less than 10 mm.

(vii) Permissible Stresses According to Clause 5 of Indian Boiler Regulations, the ultimate tensile strength and shear strength of steel plates and rivets are 26 and 21 tons per square inch respectively. Therefore,

$$S_{ut} = \frac{26 \times 2240 \times 6890}{10^6} = 401.27 \text{ N/mm}^2$$

$$S_{us} = \frac{21 \times 2240 \times 6890}{10^6} = 324.11 \text{ N/mm}^2$$

Assuming a factor of safety of 5, the permissible tensile and shear stresses are given by,

$$\sigma_t = \frac{S_{ut}}{(fs)} = \frac{401.27}{5} = 80.25 \text{ N/mm}^2$$

$$\tau = \frac{S_{us}}{(fs)} = \frac{324.11}{5} = 64.82 \text{ N/mm}^2$$

There is no provision for calculating permissible compressive stress in Boiler Regulations. Assuming,

$$\sigma_c = 1.5\ \sigma_t$$

we get,

$$\sigma_c = 1.5\ (80.25) = 120.38\ \text{N/mm}^2$$

Therefore, the permissible tensile, shear and compressive stresses are assumed as 80, 60 and 120 N/mm^2 respectively in all examples of this chapter.

Example 8.21 *A cylindrical pressure vessel with a 1.5 m inside diameter is subjected to internal steam pressure of 1.5 MPa. It is made from steel plate by triple-riveted double-strap longitudinal butt joint with equal straps. The pitch of the rivets in the outer row is twice of the pitch of the rivets in the inner rows. The rivets are arranged in a zigzag pattern. The efficiency of the riveted joint should be at least 80%. The permissible stresses for the plate and rivets in tension, shear and compression are 80, 60 and 120 N/mm^2 respectively. Assume that the rivet in double shear is 1.875 times stronger than in single shear. Design the joint and calculate:*

(i) thickness of the plate;
(ii) diameter of rivets;
(iii) pitch of rivets;
(iv) distance between the rows of rivets;
(v) margin;
(vi) thickness of the straps; and
(vii) efficiency of the joint.

Draw a neat sketch of the riveted joint showing calculated values of dimensions.

Solution

Given For vessel, $D_i = 1.5$ m $P_i = 1.5$ MPa $\eta = 80\%$ $\sigma_t = 80$ N/mm^2 $\tau = 60$ N/mm^2 $\sigma_c = 120$ N/mm^2

The triple-riveted, double-strap butt joint with equal straps, is shown in Fig. 8.63.

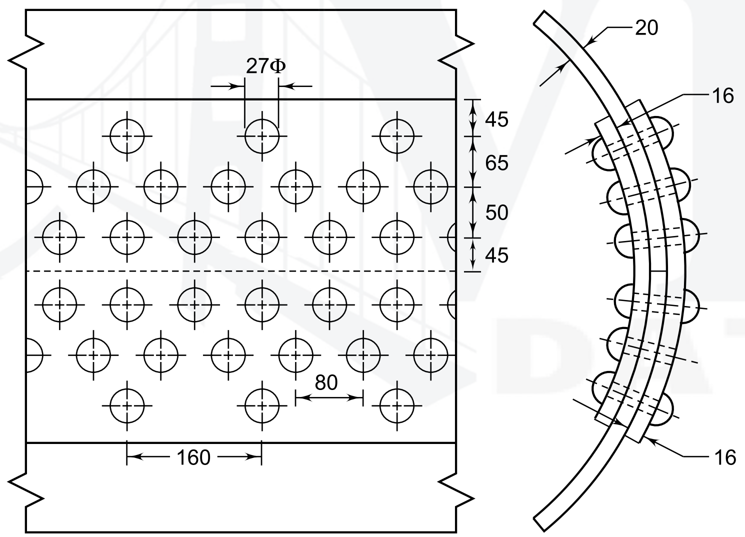

Fig. 8.63 *Triple-riveted Double-strap Butt Joint with Equal Straps*

Step I *Thickness of plate*

From Eq. (8.43),

$$t = \frac{P_i D_i}{2\sigma_t \eta} + \text{CA} = \frac{1.5(1500)}{2(80)(0.8)} + 2$$

$$= 19.58 \text{ or } 20 \text{ mm} \qquad \text{(i)}$$

Step II *Diameter of rivets*

$$t > 8 \text{ mm}$$

From Eq. (8.45),

$$d = 6\sqrt{t} = 6\sqrt{20} = 26.83 \text{ or } 27 \text{ mm} \qquad \text{(ii)}$$

Step III *Pitch of rivets*

The pitch of rivets is obtained by equating the shear strength of rivets with the tensile strength of plate.

From Eq. (8.47),

$$p = \frac{(n_1 + 1.875 n_2)\pi d^2 \tau}{4t\sigma_t} + d$$

As shown in Fig. 8.63, the number of rivets per pitch length in various rows are as follows:

(*a*) *Outer row* There is one half-rivet on the left side and one half-rivet on the right side. These make one rivet per pitch length. It connects the inner strap, the shell plate and the outer strap. This results in double shear in the rivet in the outer row.

(*b*) *Middle row* There are two rivets per pitch length. They connect the inner strap, shell plate and outer strap. This results in double shear in the rivets in the middle row.

(*c*) *Inner row* There is one half-rivet on the left side, one complete rivet in the middle and one half-rivet on the right side. These make two rivets per pitch length. They connect the inner strap, shell plate and outer strap. This results in double shear in the rivets in the inner row.

Therefore, there are five rivets per pitch length for a triple-riveted butt joint. Since the straps are of equal width, all rivets are in double shear. Therefore,

$$n_1 = 0 \qquad n_2 = 5$$

Substituting values in Eq. (8.47),

$$p = \frac{(0 + 1.875 \times 5)\pi(27)^2(60)}{4(20)(80)} + 27$$

$$= 228.29 \text{ or } 230 \text{ mm}$$

From Eqs (8.48) and (8.49),

$$p_{\text{min.}} = 2d = 2\ (27) = 54 \text{ mm}$$

$$p_{\text{max.}} = Ct + 41.28 = 6\ (20) + 41.28 = 161.28 \text{ mm}$$

Since, $p > p_{\text{max.}}$

it is assumed that,

$$p = p_{\text{max.}} = 161.28 \text{ or } 160 \text{ mm} \qquad \text{(iii)}$$

The pitch of rivets in the outer row is 160 mm. The pitch of rivets in the middle and inner rows is (160/2) or 80 mm.

Step IV *Distance between rows of rivets*

The number of rivets in the outer row is one-half of the number of rivets in the middle and inner rows. Also, the rivets have a zig-zag pattern in middle and inner rows. From Eq. (8.52), the distance between the outer and middle rows is given by,

$$p_t = 0.2p + 1.15d = 0.2\ (160) + 1.15\ (27) = 63.05 \text{ or } 65 \text{ mm}$$

From Eq. (8.53), the distance between middle and inner rows is given by,

$$p_t = 0.165p + 0.67d = 0.165\ (160) + 0.67\ (27) = 44.49 \text{ or } 50 \text{ mm} \qquad \text{(iv)}$$

Step V *Margin*

From Eq. (8.54),

$$m = 1.5d = 1.5\ (27) = 40.5 \text{ or } 45 \text{ mm} \qquad \text{(v)}$$

Step VI *Thickness of straps*

The straps are of equal width and every alternate rivet in the outer row is omitted. From Eq. (8.57),

$$t_1 = 0.625t\left[\frac{p-d}{p-2d}\right] = 0.625(20)\left[\frac{160-27}{160-2\times 27}\right]$$

$$= 15.68 \text{ or } 16 \text{ mm} \qquad \text{(vi)}$$

Step VII *Efficiency of joint*

The tensile strength of plate per pitch length in the outer row is given by,

$$P_t = (p-d)\,t\sigma_t = (160-27)(20)(80) = 212\,800 \text{ N} \qquad \text{(a)}$$

The shear strength of rivets per pitch length is given by,

$$P_s = (n_1 + 1.875n_2)\left[\frac{\pi}{4}d^2\tau\right]$$

$$= (0 + 1.875 \times 5)\left[\frac{\pi}{4}(27)^2(60)\right]$$

$$= 322\,062.33 \text{ N} \qquad \text{(b)}$$

The crushing strength of the plate is given by,

$$P_c = (n_1 + n_2)\,dt\sigma_c = (0+5)\ (27)\ (20)\ (120) = 324\,000 \text{ N} \qquad \text{(c)}$$

The tensile strength of the solid plate per pitch length is given by,

$$P = pt\sigma_t = 160\ (20)(80) = 256\,000 \text{ N} \qquad \text{(d)}$$

From (a), (b), (c) and (d),

$$\eta = \frac{212\,800}{256\,000} = 0.8313 \quad \text{or} \quad 83.13\% \qquad \text{(vii)}$$

Figure 8.63 shows the sketch of longitudinal butt joint with calculated values of the dimensions.

Example 8.22 *A cylindrical steam pressure vessel of 1 m inside diameter is subjected to an internal pressure of 2.5 MPa. Design a double-riveted, double-strap longitudinal butt joint for the vessel. The straps are of equal width. The pitch of the rivets in the outer row should be twice of the pitch of the rivets in the inner row. A zigzag pattern is used for rivets in inner and outer rows. The efficiency of the riveted joint should be at least 70%. The permissible tensile stress for the steel plate of the pressure vessel is 80 N/mm^2. The permissible shear stress for the rivet material is 60 N/mm^2. Assume that the rivets in double shear are 1.875 times stronger than in single shear and the joint do not fail by crushing. Calculate:*

(i) thickness of the plate;
(ii) diameter of the rivets;
(iii) pitch of the rivets;
(iv) distance between inner and outer rows of the rivets;
(v) margin;
(vi) thickness of the straps; and
(vii) efficiency of the joint.

Make a neat sketch of the joint showing all calculated values of dimensions.

Solution

Given For vessel, $D_i = 1$ m $P_i = 2.5$ MPa
$\eta = 70\%$ $\sigma_t = 80$ N/mm^2 $\tau = 60$ N/mm^2

A double-riveted double-strap butt joint, with equal straps, is shown in Fig. 8.64.

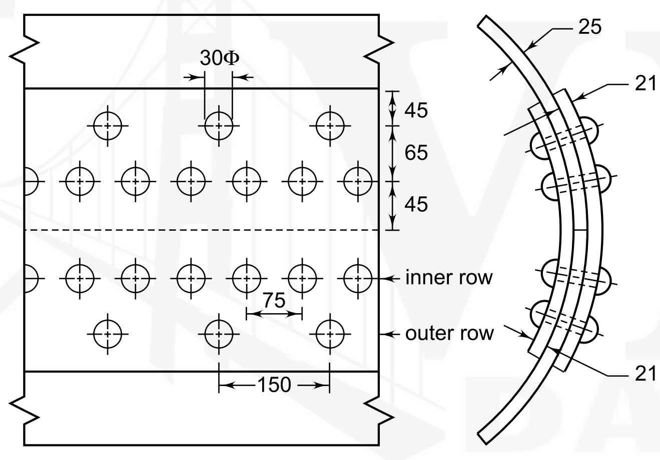

Fig. 8.64 *Double-riveted Double-strap Butt Joint with Equal Straps*

Step I *Thickness of plate*

From Eq. (8.43),

$$t = \frac{P_i D_i}{2\sigma_t \eta} + \text{CA} = \frac{2.5(1000)}{2(80)(0.7)} + 2$$

$$= 24.32 \quad \text{or} \quad 25 \text{ mm} \qquad \text{(i)}$$

Step II *Diameter of rivets*

$$t > 8 \text{ mm}$$

From Eq. (8.45),

$$d = 6\sqrt{t} = 6\sqrt{25} = 30 \text{ mm} \qquad \text{(ii)}$$

Step III *Pitch of rivets*

The pitch of rivets is obtained by equating the shear strength of the rivets to the tensile strength of the plate. From Eq. (8.47),

$$p = \frac{(n_1 + 1.875 n_2)\pi d^2 \tau}{4 t \sigma_t} + d$$

As shown in Fig. 8.64, the number of rivets per pitch length in two rows are as follows:

(*a*) *Outer row* There is one half-rivet on the left side and one half-rivet on the right side. These make one rivet per pitch length.

(*b*) *Inner row* There are two rivets per pitch length.

Adding these values, the total number of rivets per pitch length is 3. Since the straps are of equal width, all rivets connect the inner strap, the shell plate and the outer strap. This results in double shear in all the rivets. Therefore,

$$n_1 = 0 \qquad n_2 = 3$$

Substituting values in Eq. (8.47),

$$p = \frac{(0 + 1.875 \times 3)\pi(30)^2(60)}{4(25)(80)} + 30$$

$$= 149.28 \quad \text{or} \quad 150 \text{ mm}$$

From Eqs (8.48) and (8.49),

$$p_{\text{min.}} = 2d = 2(30) = 60 \text{ mm}$$

$$p_{\text{max.}} = Ct + 41.28 = 4.63\,(25) + 41.28$$

$$= 157.03 \text{ mm}$$

The pitch of 150 mm is within the limits from 60 mm to 157.03 mm. Therefore,

$$p = 150 \text{ mm} \tag{iii}$$

The pitch of rivets in the outer row is 150 mm. The pitch of rivets in the inner row is (150/2) or 75 mm.

Step IV *Distance between inner and outer rows*

The number of rivets in the outer row is one-half of the number of rivets in the inner row. Also, the rivets are arranged in a zig-zag pattern. From Eq. (8.52), the distance between inner and outer rows is given by,

$$p_t = 0.2p + 1.15d = 0.2\,(150) + 1.15\,(30)$$

$$= 64.5 \text{ or } 65 \text{ mm} \tag{iv}$$

Step V *Margin*

From Eq. (8.54),

$$m = 1.5d = 1.5(30) = 45 \text{ mm} \tag{v}$$

Step VI *Thickness of straps*

The straps are of equal width and every alternate rivet in the outer row is omitted. From Eq. (8.57),

$$t_1 = 0.625\, t\left[\frac{p - d}{p - 2d}\right]$$

$$= 0.625\,(25)\left[\frac{150 - 30}{150 - 2 \times 30}\right]$$

$$= 20.83 \text{ or } 21 \text{ mm} \tag{vi}$$

Step VII *Efficiency of joint*

The tensile strength of the plate per pitch length in the outer row of rivets is given by,

$$P_t = (p - d)\, t\sigma_t = (150 - 30)(25)(80)$$

$$= 240\,000 \text{ N} \tag{a}$$

The shear strength of the rivets per pitch length is given by,

$$P_s = (n_1 + 1.875\, n_2)\left[\frac{\pi}{4} d^2 \tau\right]$$

$$= (0 + 1.875 \times 3)\left[\frac{\pi}{4}(30)^2(60)\right]$$

$$= 238\,564.69 \text{ N} \tag{b}$$

It is assumed that the joint does not fail in crushing. Also, the tensile strength of the solid plate per pitch length is given by,

$$P = pt\sigma_t = 150(25)(80) = 300\,000 \text{ N} \tag{c}$$

From (a), (b), and (c),

$$\eta = \frac{238\,564.69}{300\,000} = 0.7952 \quad \text{or} \quad 79.52\% \tag{vii}$$

Figure 8.64 shows the sketch of the longitudinal butt joint with calculated values of dimensions.

8.28 CIRCUMFERENTIAL LAP JOINT FOR BOILER SHELL

The circumferential lap joint is used to connect different cylindrical rings together and form the boiler shell. In this case, one ring is kept overlapping over another ring and the two rings are fastened by circumferential riveted joint. This type of joint is also used to connect the end cover to the cylindrical shell. A single-riveted circumferential lap joint for a cylindrical shell is shown in Fig. 8.65. The design of circumferential lap joint consists of the following steps:

(i) Thickness of Cylindrical Shell The thickness of thin cylindrical shell is obtained by the same equation that is used for longitudinal butt joint. From Eq. (8.43),

$$t = \frac{P_i D_i}{2\sigma_t \eta} + \text{CA}$$

(ii) Diameter of Rivet The diameter of the rivet is obtained by the same procedure that is used for longitudinal butt joint. It is as follows:

(a) When the thickness of the plate is more than 8 mm, the rivet diameter is calculated by Unwin's formula.
From Eq. (8.45),

$$d = 6\sqrt{t}$$

(b) When the thickness of the plate is less than 8 mm, the diameter of the rivet is obtained by equating shear resistance of rivets to crushing resistance.

(c) In no case, should the diameter of the rivet be less than the plate thickness.

(iii) Number of Rivets As shown in Fig. 8.65, the rivets are subjected to single shear. The total shear resistance of all rivets in the joint is given by,

$$P = n\left[\frac{\pi}{4} d^2 \tau\right] \qquad \text{(a)}$$

where, n = number of rivets in the joint

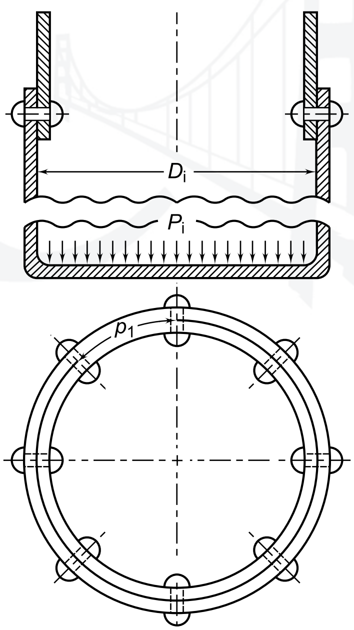

Fig. 8.65 *Single-riveted Circumferential Lap Joint*

The external force acting on the joint is given by,

$$P = \left[\frac{\pi}{4} D_i^2\right] P_i \qquad \text{(b)}$$

where,

D_i = inner diameter of boiler shell (mm)

P_i = internal steam pressure (N/mm^2)

Equating expressions (a) and (b),

$$n\left[\frac{\pi}{4} d^2 \tau\right] = \left[\frac{\pi}{4} D_i^2\right] P_i$$

or

$$n = \left(\frac{D_i}{d}\right)^2 \frac{P_i}{\tau} \qquad (8.58)$$

(iv) Pitch of Rivets The pitch of the rivets p_1 for a circumferential joint is shown in Fig. 8.65. It is obtained by assuming the efficiency of circumferential lap joint. The tensile strength of plate per pitch length of rivets is given by,

$$P_t = (p_1 - d)\, t\sigma_t \qquad \text{(c)}$$

The tensile strength of the solid plate per pitch length is given by,

$$P = (p_1 t\sigma_t) \qquad \text{(d)}$$

From (a) and (b), the efficiency of a circumferential joint is given by,

$$\eta_1 = \frac{P_t}{P} = \frac{(p_1 - d)t\sigma_t}{p_1 t\sigma_t}$$

or

$$\eta_1 = \frac{p_1 - d}{p_1} \qquad (8.59)$$

where,

η_1 = efficiency of circumferential joint (in fraction)

p_1 = pitch of circumferential joint (mm)

The guidelines for the values of efficiency are as follows:

(a) When there are number of circumferential joints in the shell, the efficiency of the intermediate circumferential seams is taken as 62%.

(b) The efficiency of the end circumferential joint is assumed to be 50% of that of the longitudinal joint, but in no case less than 42%. Therefore,

For intermediate joints,

$$\eta_1 = 0.62$$

For end joints

$\eta_1 = 0.5\eta$ or 0.42 (whichever is less)

Knowing efficiency of the joint and rivet diameter, the pitch of rivets is calculated by Eq. (8.59).

(v) Number of Rows Referring to Fig. 8.65, the number of rivets in one row is given by,

$$n_1 = \frac{\pi(D_i + t)}{p_1} \qquad (8.60)$$

where,

n_1 = number of rivets in one row

The number of rows is given by,

$$\text{Number of rows} = \frac{\text{total number of rivets in joint}}{\text{number of rivets in one row}}$$

or

$$\text{Number of rows} = \frac{n}{n_1} \qquad (8.61)$$

After determining the number of rows, the type of joint such as single-riveted lap joint or double-riveted lap joint is decided. The pitch is again readjusted. The pitch p_1 obtained by the above procedure has minimum and maximum limits like the pitch of longitudinal joint.

From Eqs (8.48) and (8.49),

$p_{min.} = 2d$

$p_{max.} = Ct + 41.28$

The minimum limit is set from considerations of manufacturing the rivet head, while maximum limit from considerations of obtaining leakproof joint.

(vi) Transverse Pitch Figure 8.66 shows a double-riveted circumferential lap joint for a cylindrical pressure vessel. The transverse pitch (p_t) is the

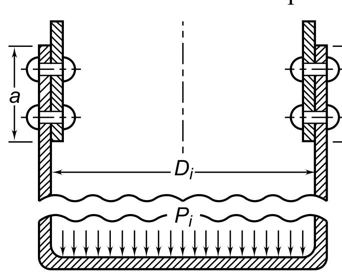

Fig. 8.66 *Double-riveted Circumferential Lap Joint*

distance between two rows of rivets. The overlap of the plate, denoted by a, is given by,

$$a = p_t + 2\text{ m} \qquad (8.62)$$

where,

m = margin

The number of rivets in each row is equal. From Eqs. (8.50) and (8.51), (For zig-zag riveting)

$$p_t = 0.33p + 0.67d$$

(For chain riveting)

$$p_t = 2d$$

The margin m is given by,

$$m = 1.5d$$

Example 8.23 *A cylindrical pressure vessel with 1 m inner diameter is subjected to internal steam pressure of 1.5 MPa. The permissible stresses for the cylinder plate and the rivets in tension, shear and compression are 80, 60 and 120 N/mm² respectively. The efficiency of longitudinal joint can be taken as 80% for the purpose of calculating the plate thickness. The efficiency of circumferential lap joint should be at least 62%. Design the circumferential lap joint and calculate:*

(i) thickness of the plate;
(ii) diameter of the rivets;
(iii) number of rivets;
(iv) pitch of rivets;
(v) number of rows of rivets; and
(vi) overlap of the plates.

Solution

Given For vessel, $D_i = 1$ m $P_i = 1.5$ MPa
$\sigma_t = 80$ N/mm² $\tau = 60$ N/mm² $\sigma_c = 120$ N/mm²
For longitudinal joint, $\eta = 80$ %
For circumferential joint, $\eta_1 = 62$%

Step I *Thickness of plate*

From Eq. (8.43),

$$t = \frac{P_i D_i}{2\sigma_t \eta} + \text{CA} = \frac{1.5(1000)}{2(80)(0.8)} + 2$$

$$= 13.72 \text{ or } 14 \text{ mm} \qquad \text{(i)}$$

Step II *Diameter of rivets*

$$t > 8 \text{ mm}$$

From Eq. (8.45),

$$d = 6\sqrt{t} = 6\sqrt{14} = 22.45 \quad \text{or} \quad 23 \text{ mm}$$

Step III *Number of rivets*

From Eq. (8.58),

$$n = \left(\frac{D_i}{d}\right)^2 \frac{P_i}{\tau} = \left(\frac{1000}{23}\right)^2 \frac{(1.5)}{(60)} = 47.26 \text{ or } 48$$

Step IV *Pitch of rivets*

From Eq. (8.59),

$$\eta_1 = \frac{p_1 - d}{p_1} \quad \text{or} \quad 0.62 = \frac{p_1 - 23}{p_1}$$

$$\therefore \quad p_1 = 60.53 \quad \text{or} \quad 62 \text{ mm} \qquad \text{(a)}$$

From Eqs (8.48) and (8.49),

$$p_{\text{min.}} = 2d = 2\,(23) = 46 \text{ mm} \qquad \text{(b)}$$

$$p_{\text{max.}} = Ct + 41.28 = 1.31\,(14) + 41.28$$
$$= 59.62 \text{ mm} \qquad \text{(c)}$$

From (a) and (c),

$$p_1 > p_{\text{max.}}$$

The pitch of rivets should be from 46 mm to 59.62 mm. We will assume the pitch as 55 mm and recalculate the number of rivets and diameter of rivet.

$$p_1 = 55 \text{ mm} \qquad \text{(iv)}$$

From Eq. (8.60), the number of rivets in one row is given by,

$$n_1 = \frac{\pi(D_i + t)}{p_1} = \frac{\pi(1000 + 14)}{55} = 57.92 \text{ or } 58 \qquad \text{(iii)}$$

Step V *Number of rows of rivets*

It is assumed that the type of joint is single-riveted lap joint. The number of rows of rivets is one. From Eq. (8.58), revised value of rivet diameter is obtained.

$$n = \left(\frac{D_i}{d}\right)^2 \frac{P_i}{\tau} \quad \text{or} \quad 58 = \left(\frac{1000}{d}\right)^2 \frac{(1.5)}{(60)}$$

$$d = 20.76 \text{ or } 21 \text{ mm} \qquad \text{(ii)}$$

Step VI *Overlap of plates*

The margin m is given by,

$$m = 1.5d = 1.5\,(21) = 31.5 \text{ or } 35 \text{ mm}$$

From Eq. (8.62),

$$a = p_t + 2m = 0 + 2\,(35) = 70 \text{ mm} \qquad \text{(vi)}$$

Step VII *Check for design*

From Eq. (8.59),

$$\eta_1 = \frac{p_1 - d}{p_1} = \frac{55 - 21}{55} = 0.6182 \text{ or } 61.82\%$$

The efficiency of the joint is very near to 62% and no changes are required in the calculations.

8.29 ECCENTRICALLY LOADED RIVETED JOINT

Many times, a group of rivets is employed in structural joints. When the line of action of external force does not pass through the center of gravity of these rivets, the joint is called an eccentrically loaded joint. The analysis of eccentrically loaded riveted joint is exactly identical to eccentrically loaded bolted joint in shear explained in Section 7.11 of Chapter 7. The equations in that article are rewritten here.

The primary shear forces (P_1', P_2', P_3', P_4') are given by,

$$P_1' = P_2' = P_3' = P_4' = \frac{P}{(\text{No. of rivets})} \qquad (8.63)$$

The secondary shear forces P_1'' P_2'' P_3'' and P_4'' are given by,

$$P_1'' = Cr_1$$
$$P_2'' = Cr_2$$
$$P_3'' = Cr_3$$
$$P_4'' = Cr_4 \qquad (8.64)$$

where C is the constant of proportionality. It is given by,

$$C = \frac{Pe}{(r_1^2 + r_2^2 + r_3^2 + r_4^2)} \qquad (8.65)$$

From Eqs (8.64) and (8.65),

$$P_1'' = \frac{Per_1}{(r_1^2 + r_2^2 + r_3^2 + r_4^2)}$$

$$P_2'' = \frac{Per_2}{(r_1^2 + r_2^2 + r_3^2 + r_4^2)} \qquad (8.66)$$

and so on.

The primary and secondary shear forces are vectorially added to get the resultant shear force. They are denoted by P_1, P_2, P_3 and P_4. The maximum loaded rivet becomes the criterion of design. The rivets are subjected to direct shear stress. Suppose P_2 or P_4 is the maximum shear force. Equating maximum shear force P_2 or P_4 to the shear strength of the rivet,

$$P_2 = P_4 = \left[\frac{\pi}{4} d^2\right]\tau \qquad (8.67)$$

The above equation is used to find out the diameter of rivets.

Example 8.24 *A bracket, attached to a vertical column by means of four identical rivets, is subjected to an eccentric force of 25 kN as shown in Fig. 8.67(a). Determine the diameter of rivets, if the permissible shear stress is 60 N/mm².*

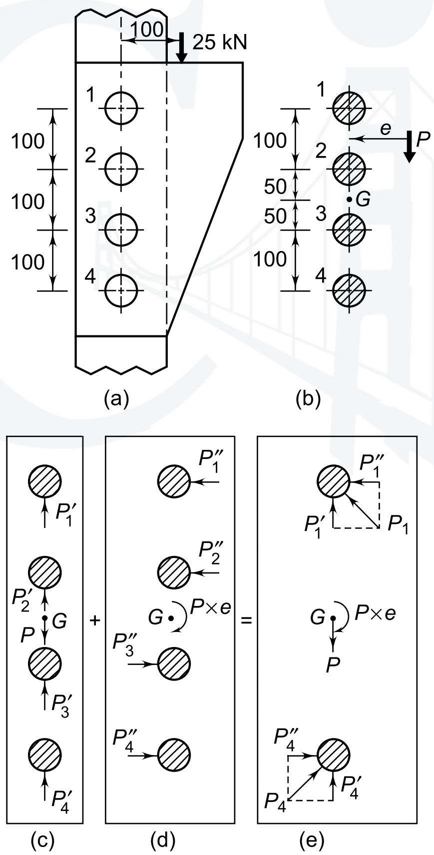

Fig. 8.67

Solution

Given $P = 25$ kN $\quad e = 100$ mm $\quad \tau = 60$ N/mm^2

Step I *Primary shear force*

The primary shear force on each rivet is shown in Fig. 8.67(c). It is given by,

$$P_1' = P_2' = P_3' = P_4' = \frac{P}{4} = \frac{25\times10^3}{4} = 6250 \text{ N}$$

Step II *Secondary shear force*

By symmetry, the centre of gravity of four rivets is located midway between the centres of rivets 2 and 3. The radial distances of the rivet centre from this centre of gravity are as follows:

$$r_1 = r_4 = 150 \text{ mm}$$
$$r_2 = r_3 = 50 \text{ mm}$$

From Eq. (8.65),

$$C = \frac{Pe}{(r_1^2 + r_2^2 + r_3^2 + r_4^2)}$$

$$= \frac{(25\times10^3)(100)}{[2(150)^2 + 2(50)^2]} = 50$$

$$P_1'' = P_4'' = Cr_1 = 50\,(150) = 7500 \text{ N}$$

$$P_2'' = P_3'' = Cr_2 = 50(50) = 2500 \text{ N}$$

Step III *Resultant shear force*

Rivets 1 and 4, which are located at the farthest distance from the centre of gravity, are subjected to maximum shear force. As shown in Fig. 8.67(e), the primary and secondary shear forces acting on rivets 1 or 4 are at right angles to each other. The resultant shear force P_1 is given by,

$$P_1 = \sqrt{(P_1')^2 + (P_1'')^2} = \sqrt{(6250)^2 + (7500)^2}$$

$$= 9762.81 \text{ N}$$

Step IV *Diameter of rivets*

Equating the resultant shear force to the shear strength of rivet,

$$P_1 = \frac{\pi}{4} d^2 \tau \quad \text{or} \quad 9762.81 = \frac{\pi}{4} d^2\,(60)$$

$$\therefore \qquad d = 14.39 \text{ or } 15 \text{ mm}$$

Example 8.25 *A riveted joint, consisting of two identical rivets, is subjected to an eccentric force of 15 kN as shown in Fig. 8.68(a). Determine the diameter of rivets, if the permissible shear stress is 60 N/mm².*

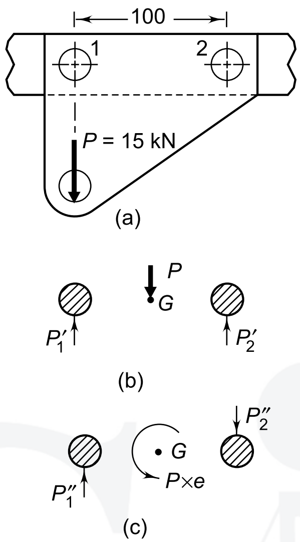

Fig. 8.68

Solution

Given $P = 15$ kN $\quad \tau = 60$ N/mm^2

Step I *Primary shear force*

The primary shear force on each rivet is shown in Fig. 8.68(b). It is given by,

$$P_1' = P_2' = \frac{P}{2} = \frac{15\times 10^3}{2} = 7500 \text{ N}$$

Step II *Secondary shear force*

By symmetry, the centre of gravity is located midway between the centres of two rivets. Therefore,

$$e = 50 \text{ mm}$$
$$r_1 = r_2 = 50 \text{ mm}$$

From Eq. (8.65),

$$C = \frac{Pe}{(r_1^2 + r_2^2)} = \frac{(15\times 10^3)(50)}{\left[(50)^2 + (50)^2\right]} = 150$$

$$P_1'' = P_2'' = Cr_1 = 150(50) = 7500 \text{ N}$$

Step III *Resultant shear force*

As shown in Fig. 8.68(b) and (c), the primary and secondary shear forces at rivet-1 act in the same direction or are additive. On the other hand, the primary and secondary shear forces at rivet-4 act in the opposite direction or are subtractive. Therefore, rivet-1 is subjected to maximum shear force.

$$P_1 = P_1' + P_1'' = 7500 + 7500 = 15\,000 \text{ N}$$

Step IV *Diameter of rivets*

Equating the resultant shear force to the shear strength of the rivet,

$$P_1 = \frac{\pi}{4} d^2 \tau \quad \text{or} \quad 15\,000 = \frac{\pi}{4} d^2 (60)$$

$$\therefore \qquad d = 17.84 \text{ or } 18 \text{ mm}$$

Example 8.26 *A riveted joint, consisting of four identical rivets, is subjected to an eccentric force of 5 kN as shown in Fig. 8.69(a). Determine the diameter of rivets, if the permissible shear stress is 60 N/mm^2.*

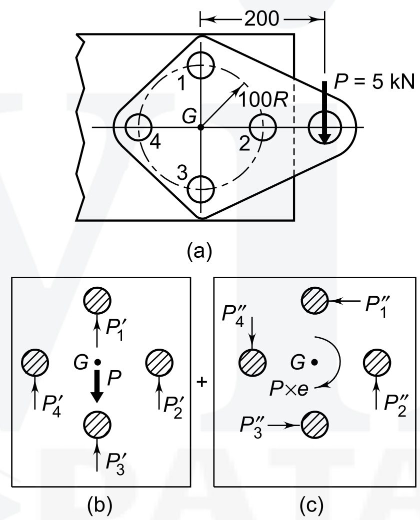

Fig. 8.69

Solution

Given $P = 5$ kN $\quad e = 200$ mm $\quad \tau = 60$ N/mm^2

Step I *Primary shear force*

From Eq. (8.63),

$$P_1' = P_2' = P_3' = P_4' = \frac{P}{4} = \frac{5\times 10^3}{4} = 1250 \text{ N}$$

Step II *Secondary shear force*

$$r_1 = r_2 = r_3 = r_4 = 100 \text{ mm}$$

From Eq. (8.65),

$$C = \frac{Pe}{(r_1^2 + r_2^2 + r_3^2 + r_4^2)} = \frac{(5\times 10^3)(200)}{4(100)^2} = 25$$

Therefore,

$$P_1'' = P_2'' = P_3'' = P_4'' = Cr = 25(100) = 2500 \text{ N}$$

Step III *Resultant shear force*

The primary and secondary shear forces are shown in Fig. 8.69(b) and (c). It is observed from the figure that rivet-2 is subjected to maximum resultant force. At rivet-2, the primary and secondary shear forces are additive. Therefore,

$$P_2 = P_2' + P_2'' = 1250 + 2500 = 3750 \text{ N}$$

Step IV *Diameter of Rivets*

$$P_2 = \frac{\pi}{4} d^2 \tau \quad \text{or} \quad 3750 = \frac{\pi}{4} d^2 (60)$$

$$\therefore \quad d = 8.92 \quad \text{or} \quad 9 \text{ mm}$$

Example 8.27 *A bracket is attached to a steel channel by means of nine identical rivets as shown in Fig. 8.70. Determine the diameter of rivets, if the permissible shear stress is 60 N/mm².*

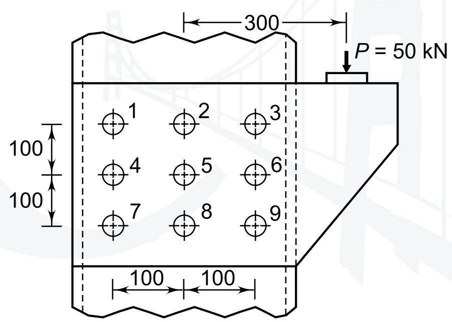

Fig. 8.70

Solution

Given $P = 50$ kN $\quad e = 300$ mm $\quad \tau = 60$ N/mm²

Step I *Primary shear force*

From Eq. (8.63),

$$P_3' = P_6' = P_9' = \frac{P}{(\text{No. of rivets})}$$

$$= \frac{(50 \times 10^3)}{9} = 5555.56 \text{ N}$$

Step II *Secondary shear force*

By symmetry, the centre of gravity G is located at the center of rivet-5. The radial distances of rivet centres from the centre of gravity G are as follows:

$$r_5 = 0$$

$$r_2 = r_6 = r_8 = r_4 = 100 \text{ mm}$$

$$r_3 = r_9 = r_7 = r_1 = \sqrt{100^2 + 100^2} = 141.42 \text{ mm}$$

The primary and secondary shear forces acting on rivets 3, 6 and 9 are shown in Fig. 8.71.

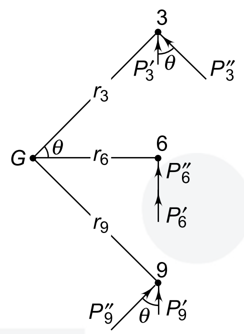

Fig. 8.71

$$\tan\theta = \frac{100}{100} \quad \text{or} \quad \theta = 45°$$

From Eq. (8.65),

$$C = \frac{Pe}{(r_1^2 + r_2^2 + \cdots + r_9^2)}$$

$$= \frac{(50 \times 10^3)(300)}{(4 \times 100^2 + 4 \times 141.42^2 + 1 \times 0^2)} = 125$$

Therefore,

$$P_3'' = Cr_3 = 125\,(141.42) = 17\,677.5 \text{ N}$$

$$P_6'' = Cr_6 = 125(100) = 12\,500 \text{ N}$$

$$P_9'' = Cr_9 = 125(141.42) = 17\,677.5 \text{ N}$$

Step III *Resultant shear force*

The resultant force P_3 is given by,

$$P_3 = \sqrt{(P_3'' \sin\theta)^2 + (P_3'' \cos\theta + P_3')^2}$$

$$= \sqrt{[17\,677.5 \sin(45°)]^2 + [17\,677.5 \cos(45)° + 5555.56]^2}$$

$$= 21\,960.1 \text{ N}$$

The resultant force P_9 is equal to the resultant force P_3.

The resultant force P_6 is given by,

$$P_6 = P_6' + P_6'' = 5555.56 + 12\,500 = 18\,055.56 \text{ N}$$

Therefore, rivet-3 or rivet-9 is subjected to maximum shear force.

Step IV *Diameter of rivets*

Equating the maximum shear force to the shear strength of the rivet,

$$P_3 = \frac{\pi}{4} d^2 \tau \quad \text{or} \quad 21960.1 = \frac{\pi}{4} d^2 (60)$$

$$\therefore \qquad d = 21.59 \quad \text{or} \quad 22 \text{ mm}$$

Short-Answer Questions

8.1 What are the advantages of welded joints compared with riveted joints?

8.2 What are the advantages of welded assemblies compared with cast iron structures?

8.3 What are the disadvantages welded joints?

8.4 What is the cause of residual stresses in welded joint? How are they relieved?

8.5 What is reinforcement in weld? What are its advantages and disadvantages?

8.6 What is butt joint?

8.7 What are the types of butt joints?

8.8 What is fillet joint?

8.9 What is transverse fillet weld?

8.10 What is parallel fillet weld?

8.11 What are the advantages of triangular cross-section normal welds over convex cross-section welds?

8.12 What is leg of fillet weld?

8.13 What is throat of fillet weld?

8.14 What is the relationship between leg and throat of fillet weld?

8.15 Which plane is subjected to maximum shear stress in case of parallel fillet welds?

8.16 Which plane is subjected to maximum shear stress in case of transverse fillet welds?

8.17 What are the four basic elements of weld symbol?

8.18 What are permanent joints? Give their examples.

8.19 What are separable joints? Give their examples.

8.20 Why are riveted joints replaced by welded joints?

8.21 How is rivet specified?

8.22 Distinguish between hot and cold riveting.

8.23 Where do you use riveted joints at present?

8.24 What are the different types of rivet heads? Give their applications.

8.25 What is lap joint?

8.26 What is single-riveted lap joint?

8.27 What is double-riveted lap joint?

8.28 What is chain-riveted lap joint?

8.29 What is zig-zag riveted lap joint?

8.30 What is butt joint?

8.31 What is single-strap butt joint?

8.32 What is double-strap butt joint?

8.33 What is the advantage of butt joint over lap joint?

8.34 What is the disadvantage of butt joint over lap joint?

8.35 What is the common material for rivet?

8.36 What do you understand by efficiency of riveted joints?

8.37 What is caulking? What is its objective?

8.38 What is fullering? What is its objective?

Problems for Practice

8.1 Two plates are joined together by means of fillet welds as shown in Fig. 8.72. The leg dimension of the welds is 10 mm and the permissible shear stress at the throat cross-section is 75 N/mm^2. Determine the length of each weld, if 15 mm weld length is required for starting and stopping of the weld run.

[*48 mm*]

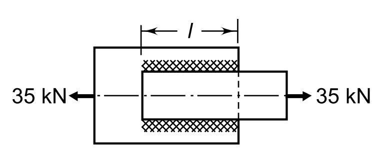

Fig. 8.72

8.2 A steel plate, 80 mm wide and 10 mm thick, is joined to another steel plate by means of a single transverse and double parallel fillet welds, as shown in Fig. 8.73. The strength of the welded joint should be equal to the strength of the plates to be joined. The permissible tensile and shear stresses for the weld material and the plates are 100 and 70 N/mm^2 respectively. Find the length of each parallel fillet weld. Assume that the tensile force passes through the centre of gravity of three welds.

[*23.68 mm*]

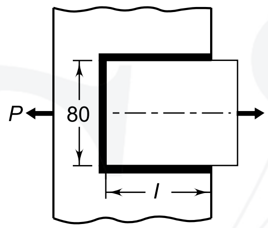

Fig. 8.73

8.3 A bracket, as shown in Fig. 8.74, is welded to a plate. The welds have the same size, and the permissible force per mm of the weld-length is 1 kN. Calculate the lengths l_1 and l_2.

[*40 and 80 mm*]

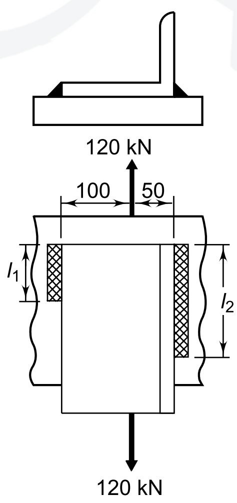

Fig. 8.74

8.4 A welded connection of steel plates, as shown in Fig. 8.75, is subjected to an eccentric force of 10 kN. Determine the throat dimension of the welds, if the permissible shear stress is limited to 95 N/mm^2. Assume static conditions.

[*8.59 mm*]

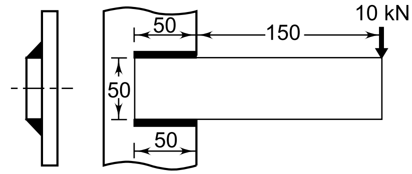

Fig. 8.75

8.5 A welded connection of steel plates is shown in Fig. 8.76. It is subjected to an eccentric force of 50 kN. Determine the size of the weld, if the permissible shear stress in the weld is not to exceed 70 N/mm^2.

[*12.94 mm*]

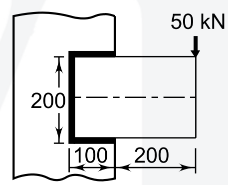

Fig. 8.76

8.6 A welded joint, as shown in Fig. 8.77, is subjected to an eccentric load of 2500 N. Find the size of the weld, if the maximum shear stress in the weld is not to exceed 50 N/mm^2.

[*6.4 mm*]

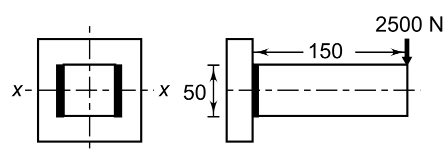

Fig. 8.77

8.7 A circular shaft, 75 mm in diameter, is welded to the support by means of a circumferential fillet weld. It is subjected to a torsional moment of 3000 N- m. Determine the size of weld, if the maximum shear stress in the weld is not to exceed 70 N/mm^2.

[*6.86 mm*]

8.8 A solid circular beam, 25 mm in diameter, is welded to a support by means of a fillet weld as shown in Fig. 8.78. Determine the leg dimension of the weld, if the permissible shear stress is 95 N/mm^2.

[*7.64 mm*]

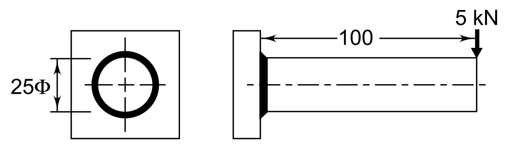

Fig. 8.78

8.9 Prove that the plane, where maximum shear stress is induced, is inclined at 45° to leg dimension in case of parallel fillet weld of equal legs. Also, find the expression for the maximum shear stress.

8.10 Prove that the plane, where maximum shear stress is induced, is inclined at 67.5° to the leg dimension in case of transverse fillet weld of equal legs. Also, find the expression for the maximum shear stress.

8.11 Two plates, each 5 mm thick, are connected by means of four rivets as shown in Fig. 8.79. The permissible stresses for rivets

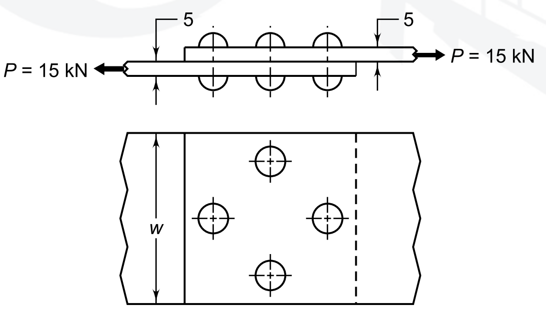

Fig. 8.79

and plates in tension, shear and compression are 80, 60 and 120 N/mm^2 respectively. Calculate:

(i) diameter of the rivets;
(ii) width of the plate; and
(iii) efficiency of the joint.

[*(i) 8.92 or 9 mm (ii) 46.5 or 50 mm (iii) 76.34%*]

8.12 Two plates, each 15 mm thick and carrying an axial load of 175 kN, are connected by means of double-strap butt joint as shown in Fig. 8.80. Assume that rivets in double shear are 1.875 times stronger than in single shear. The permissible stresses for rivets and plates in tension, shear and compression are 80, 60 and 120 N/mm^2 respectively. Calculate:

(i) diameter of the rivets; and
(ii) width of the plate.

Assuming the above values, calculate:

(iii) strength of the joint if failure is to occur along the section-*AA*;
(iv) strength of the joint if failure is to occur along the section-*BB*;
(v) strength of the joint if failure is to occur along the section-*CC*;
(vi) strength of the joint if the failure is to occur due to shearing of rivets;
(vii) strength of the joint if the failure is to occur due to crushing of rivets;
(viii) strength of solid plate; and
(ix) efficiency of the joint.

[*(i) 19.9 or 20 mm (ii) 165.83 or 175 mm (iii) 268.03 kN (iv) 197.34 kN (v) 186 kN (vi) 176.71 kN (vii) 180 kN (viii) 210 kN (ix) 84.15%*]

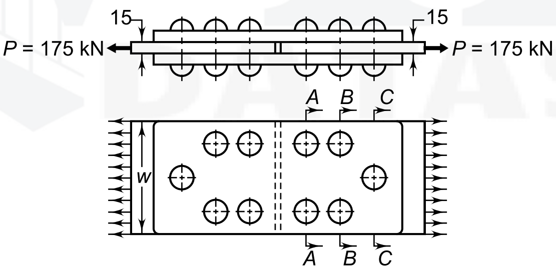

Fig. 8.80

8.13 A double-riveted double-strap butt joint is used to connect two plates; each of 12 mm thickness, by means of 16 mm diameter rivets having a pitch of 48 mm. The rivets and plates are made of steel. The permissible stresses in tension, shear and compression are 80, 60 and 120 N/mm^2 respectively. Determine the efficiency of the joint.

[*66.67%*]

8.14 A pressure vessel of the boiler consists of cylindrical shell of 0.8 m inner diameter. It is subjected to internal steam pressure of 2 MPa. Triple-riveted double-strap longitudinal butt joint is used to make the shell. The straps are of unequal width. The pitch of the rivets in outer row is twice of the pitch of rivets in middle and inner rows. A zig-zag pattern is used for arrangement of rivets. The efficiency of the joint should be at least 80%. The corrosion allowance is 2 mm. The permissible stresses for rivets and shell in tension, shear and compression are 80, 60 and 120 N/mm^2 respectively. Calculate:

(i) thickness of the shell;
(ii) diameter of the rivets;
(iii) pitch of the rivets in outer row;
(iv) distance between outer and middle rows;
(v) distance between middle and inner rows;
(vi) thickness of inner strap;
(vii) thickness of outer strap; and
(viii) efficiency of the joint.

[*(i) 14.5 or 15 mm (ii) 23.24 or 24 mm (iii) 130 mm (iv) 53.6 or 55 mm (v) 37.53 or 40 mm (vi) 11.25 or 12 mm (vii) 9.38 or 10 mm (viii) 81.54%*]

8.15 A cylindrical pressure vessel with a 0.8 m inner diameter is subjected to an internal steam pressure of 2 MPa. The permissible stresses for cylinder plate and rivets in tension, shear and compression are 80, 60 and 120 N/mm^2 respectively. The efficiency of longitudinal joint can be taken as 80% for the purpose of calculating the plate thickness. The corrosion allowance is 2 mm. The efficiency of circumferential lap joint should be at least 62%. Design the circumferential lap joint and calculate:

(i) thickness of the plate;
(ii) diameter of the rivets;
(iii) number of rivets;
(iv) pitch of the rivets;
(v) number of rows of rivets; and
(vi) overlap of the plates.

[*(i) 14.5 or 15 mm (ii) 23 mm (iii) 42 (iv) 60 mm (v) 1 (vi) 70 mm*]

8.16 A bracket is attached to a vertical column by means of six identical rivets as shown in Fig. 8.81. It is subjected to an eccentric force of 60 kN at a distance of 200 mm from the centre of the column. The maximum permissible shear stress for the rivets is 150 N/mm^2.

(i) Which rivet is subjected to maximum shear force?
(ii) What is the magnitude of maximum force?
(iii) Determine the diameter of rivet.

[*(i) Rivet-2 or 4 (ii) 35.38kN (iii) 17.33 mm*]

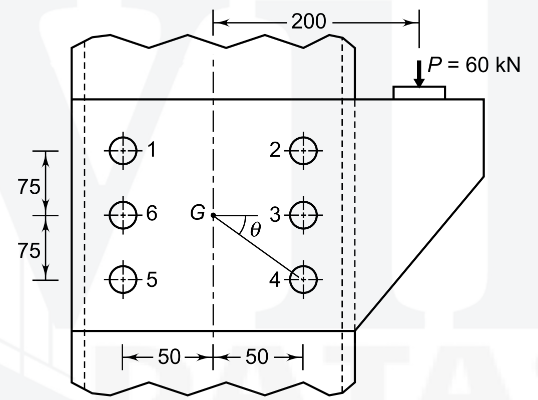

Fig. 8.81

8.17 A bracket is attached to a horizontal column by means of three identical rivets as shown in Fig. 8.82. The maximum permissible shear stress for the rivets is 60 N/mm^2.

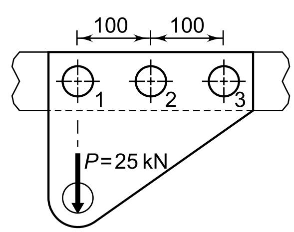

Fig. 8.82

(i) Which rivet is subjected to maximum shear force?

(ii) What is the magnitude of maximum force?

(iii) Determine the diameter of rivet.

[*(i) Rivet-1 (ii) 20.83 kN (iii) 21.03 mm*]

8.18 Prove that the number of rivets (n) required for a circumferential lap-joint of a cylindrical pressure vessel is given by,

$$n = \left(\frac{D_i}{d}\right)^2 \frac{P_i}{\tau}$$

where,

D_i = inner diameter of cylindrical shell

P_i = internal pressure

d = diameter of rivet

τ = permissible shear stress

Shafts, Keys and Couplings

Chapter 9

9.1 TRANSMISSION SHAFTS

The term '*transmission shaft*' usually refers to a rotating machine element, circular in cross-section, which supports transmission elements like gears, pulleys and sprockets and transmits power. A transmission shaft supporting a gear in a speed reducer is shown in Fig. 9.1. The shaft is always stepped with maximum diameter in the middle portion and minimum diameter at the two ends, where bearings are mounted. The steps on the shaft provide shoulders for positioning transmission elements like gears, pulleys and bearings. The rounded-off portion between two cross-sections of different diameters is called *fillet*. The fillet radius is provided to reduce the effect of stress-concentration due to abrupt change in the cross-section.

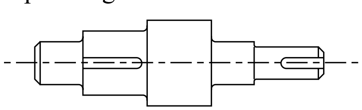

Fig. 9.1 *Transmission Shaft*

Shafts are given specific names in typical applications, although all applications involve transmission of power, motion and torque. Some of the specific categories of transmission shafts are as follows:

(i) Axle The term 'axle' is used for a shaft that supports rotating elements like wheels, hoisting drums or rope sheaves and which is fitted to the housing by means of bearings. In general, an axle is subjected to bending moment due to transverse loads like bearing reactions and does not transmit any useful torque, e.g., rear axle of a railway wagon. Occasionally, the axle also transmits torque, e.g., automobile rear axle. An axle may rotate with the wheel or simply support a rotating wheel.

(ii) Spindle A spindle is a short rotating shaft. The term 'spindle' originates from the round tapering stick on a spinning wheel, on which the thread is twisted. Spindles are used in all machine tools such as the small drive shaft of a lathe or the spindle of a drilling machine.

(iii) Countershaft It is a secondary shaft, which is driven by the main shaft and from which the power is supplied to a machine component. Often, the countershaft is driven from the main shaft by means of a pair of spur or helical gears and thus rotates '*counter*' to the direction of the main shaft. Countershafts are used in multi-stage gearboxes.

(iv) Jackshaft It is an auxiliary or intermediate shaft between two shafts that are used in transmission of power. Its function is same as that of the countershaft.

(v) Line shaft A line shaft consists of a number of shafts, which are connected in axial direction by means of couplings. Line shafts were popular in workshops using group drive. In group drive construction, a single electric motor drives the line shaft. A number of pulleys are mounted on the line shaft and power is transmitted to individual machines by different belts. Therefore, it is possible to drive a number of machines simultaneously by using a single electric motor. However, in recent times, individual drives have replaced group drive, making the line shaft obsolete.

Ordinary transmission shafts are made of medium carbon steels with a carbon content from 0.15 to 0.40 per cent such as 30C8 or 40C8. These steels are commonly called *machinery steels*. Where greater strength is required, high carbon steels such as 45C8 or 50C8 or alloy steels are employed. Alloy steels include nickel, nickel–chromium and molybdenum steels. Common grades of alloy steels used for making transmission shafts are 16Mn5Cr4, 40Cr4Mo2, 16Ni3Cr2, 35Ni5Cr2, 40Ni6Cr4Mo2 and 40Ni10Cr3Mo6. Alloy steels are costly compared with plain carbon steels. However, alloy steels have higher strength, hardness and toughness. Also, high values of hardness and strength can be achieved for components with large section diameters. Alloy steels possess higher resistance to corrosion compared with plain carbon steels. Therefore, in some applications, these advantages justify the higher cost of the alloy steel.

Commercial shafts are made of low carbon steels. They are produced by hot-rolling and finished to size either by cold-drawing or by turning and grinding. Cold-drawing produces a stronger shaft than hot-rolling. However, cold-drawn shafts have certain disadvantages. The tolerances on their diameters and straightness are not very close compared with shafts finished by turning and grinding processes. Also, cold-drawing produces residual stresses at and near the surface of the shaft. During machining operations like slotting and milling, required to make the key-slot, the residual stresses are partially released causing distortion of the shaft. The straightening of distorted and twisted shaft is difficult and expensive operation. Therefore, most of the transmission shafts, after being hot-rolled, are turned and ground. They are further hardened by oil-quenching to achieve the required strength and hardness. Steel bars up to 200 mm in diameter are commercially available. For very large sizes, billets are forged into bars and finished by usual turning and grinding operations. Commercial shafts, used for structural and general engineering purposes, are available in standard sizes.[1] The standard diameters of these shafts are given in Table 9.1.

Table 9.1 *Standard diameters of steel bars used for structural and general engineering purposes*

Diameters (mm)			
5	20	45	90
6	22	50	100
8	25	55	110
10	28	60	120
12	30	65	140
14	32	70	160
16	35	75	180
18	40	80	200

9.2 SHAFT DESIGN ON STRENGTH BASIS

Transmission shafts are subjected to axial tensile force, bending moment or torsional moment or their combinations. Most of the transmission shafts are subjected to combined bending and torsional moments. The design of transmission shaft consists of determining the correct shaft diameter from strength and rigidity considerations. When the shaft is subjected to axial tensile force, the tensile stress is given by,

$$\sigma_t = \frac{P}{\left(\dfrac{\pi d^2}{4}\right)}$$

or,
$$\sigma_t = \frac{4P}{\pi d^2} \tag{9.1}$$

[1] IS 1732–1989: Steel bars, round and square for structural and general engineering purposes—dimensions.

When the shaft is subjected to pure bending moment, the bending stresses are given by,

$$\sigma_b = \frac{M_b y}{I} = \frac{M_b\left(\frac{d}{2}\right)}{\left(\frac{\pi d^4}{64}\right)}$$

or,
$$\sigma_b = \frac{32 M_b}{\pi d^3} \tag{9.2}$$

When the shaft is subjected to pure torsional moment, the torsional shear stress is given by,

$$\tau = \frac{M_t r}{J} = \frac{M_t\left(\frac{d}{2}\right)}{\left(\frac{\pi d^4}{32}\right)}$$

or,
$$\tau = \frac{16 M_t}{\pi d^3} \tag{9.3}$$

When the shaft is subjected to combination of loads, the principal stress and principal shear stress are obtained by constructing Mohr's circle as shown in Fig. 9.2. The normal stress is denoted by σ_x while the shear stress, by τ. We will consider two cases for calculating the value of σ_x.

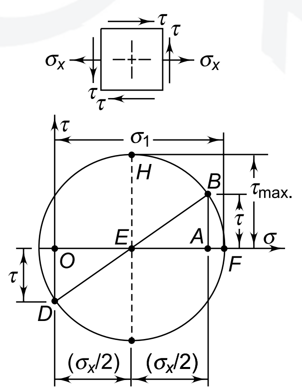

Fig. 9.2 *Mohr's Circle*

Case I In this case, the shaft is subjected to a combination of axial force, bending moment and torsional moment.

$$\sigma_x = \sigma_t + \sigma_b \tag{9.4}$$

Case II In this case, the shaft is subjected to a combination of bending and torsional moments without any axial force.

$$\sigma_x = \sigma_b \tag{9.5}$$

The values of σ_t and σ_b in Eqs (9.4) and (9.5) are obtained from Eqs (9.1) and (9.2) respectively.

The Mohr's circle is constructed by the following steps:

(i) Select the origin O.

(ii) Plot the following points:
$\overline{OA} = \sigma_x \quad \overline{AB} = \tau \quad \overline{OD} = \tau$

(iii) Join $\overline{DB}$. The point of intersection of $\overline{DB}$ and $\overline{OA}$ is E.

(iv) Construct Mohr's circle with E as centre and $\overline{EB}$ as radius.

The principal stress σ_1 is given by,

$$\sigma_1 = \overline{OF} = \overline{OE} + \overline{EF} = \overline{OE} + \overline{EB}$$

or
$$\sigma_1 = \left(\frac{\sigma_x}{2}\right) + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau)^2} \tag{9.6}$$

The principal shear stress τ_{max} is given by,

$$\tau_{max.} = \overline{EH} = \overline{EB}$$

or
$$\tau_{max.} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau)^2} \tag{9.7}$$

Equations (9.1) to (9.7) are fundamental equations for design of shafts. However, every time, it is not necessary to use all these equations. For the design of shafts, simple expressions can be developed by combining the above equations. The shaft can be designed on the basis of maximum principal stress theory or maximum shear stress theory. We will apply these theories to transmission shaft subjected to combined bending and torsional moments.

(i) Maximum Principal Stress Theory The maximum principal stress is σ_1. Since the shaft is subjected to bending and torsional moments without any axial force,

$$\sigma_x = \sigma_b = \frac{32 M_b}{\pi d^3} \tag{a}$$

Also,

$$\tau = \frac{16M_t}{\pi d^3} \qquad \text{(b)}$$

Substituting Eqs (a) and (b) in Eq. (9.6),

$$\sigma_1 = \left(\frac{16M_b}{\pi d^3}\right) + \sqrt{\left(\frac{16M_b}{\pi d^3}\right)^2 + \left(\frac{16M_t}{\pi d^3}\right)^2}$$

or,

$$\sigma_1 = \frac{16}{\pi d^3}\left[M_b + \sqrt{(M_b)^2 + (M_t)^2}\right] \qquad (9.8)$$

The permissible value of maximum principal stress is given by,

$$\sigma_1 = \frac{S_{yt}}{(fs)} \qquad (9.9)$$

Equations (9.8) and (9.9) are used to determine shaft diameter on the basis of principal stress theory.

Experimental investigations suggest that maximum principal stress theory gives good predictions for brittle materials. Shafts are made of ductile material like steel and therefore, this theory is not applicable to shaft design.

(ii) Maximum Shear Stress Theory The principal shear stress is τ_{max}. Substituting Eqs (a) and (b) in Eq. (9.7),

$$\tau_{max.} = \sqrt{\left(\frac{16M_b}{\pi d^3}\right)^2 + \left(\frac{16M_t}{\pi d^3}\right)^2}$$

or,

$$\tau_{max.} = \frac{16}{\pi d^3}\sqrt{(M_b)^2 + (M_t)^2} \qquad (9.10)$$

According to maximum shear stress theory,

$$S_{sy} = 0.5 S_{yt} \qquad (9.11)$$

The permissible value of maximum shear stress is given by,

$$\tau_{max.} = \frac{S_{sy}}{(fs)} = \frac{0.5S_{yt}}{(fs)} \qquad (9.12)$$

Equations (9.10) and (9.12) are used to determine the shaft diameter on the basis of maximum shear stress theory.

The maximum shear stress theory is applicable to ductile materials. Since the shafts are made of ductile materials, it is more logical to apply this theory to shaft design rather than designing the shaft on the basis of principal stress theory.

We will rewrite Eqs (9.10) and (9.8) again.

$$\tau_{max.} = \frac{16}{\pi d^3}\sqrt{(M_b)^2 + (M_t)^2}$$

$$\sigma_1 = \frac{16}{\pi d^3}\left[M_b + \sqrt{(M_b)^2 + (M_t)^2}\right]$$

(i) Equivalent Torsional Moment The expression $\sqrt{(M_b)^2 + (M_t)^2}$ is called '*equivalent*' *torsional moment*. The equivalent torsional moment is defined as the torsional moment, which when acting alone, will produce the same torsional shear stress in the shaft as under the combined action of bending moment (M_b) and torsional moment (M_t).

(i) Equivalent Bending Moment The expression $\left[M_b + \sqrt{(M_b)^2 + (M_t)^2}\right]$ is called '*equivalent*' *bending moment*. The equivalent bending moment is defined as the bending moment, which when acting alone, will produce the same bending stresses (tensile and compressive) in the shaft as under the combined action of bending moment (M_b) and torsional moment (M_t).

The concept of equivalent torsional moment is used in the design of shafts on the basis of maximum shear stress theory of failure. The concept of equivalent bending moment is used in the design of shafts on the basis of maximum principal stress theory of failure.

9.3 SHAFT DESIGN ON TORSIONAL RIGIDITY BASIS

In some applications, the shafts are designed on the basis of either *torsional rigidity* or *lateral rigidity*. A transmission shaft is said to be rigid on the basis of torsional rigidity, if it does not twist too much under the action of an external torque. Similarly, the transmission shaft is said to be rigid on the basis of lateral rigidity, if it does not deflect too much under the action of external forces and bending moment.

In certain applications, like machine tool spindles, it is necessary to design the shaft on the basis of torsional rigidity, i.e., on the basis of permissible angle of twist per metre length of shaft. The angle of twist θ_r (in radians) is given by,

$$\theta_r = \frac{M_t l}{JG}$$

Converting θ_r from radians to degrees (θ),

$$\theta = \frac{180}{\pi} \times \frac{M_t l}{JG} \qquad \text{(a)}$$

For solid circular shaft,

$$J = \frac{\pi d^4}{32} \qquad \text{(b)}$$

Combining Eqs (a) and (b),

$$\theta = \frac{584 M_t l}{G d^4} \qquad (9.13)$$

where,

θ = angle of twist (deg.)
l = length of shaft subjected to twisting moment (mm)
M_t = torsional moment (N-mm)
G = modulus of rigidity (N/mm^2)
d = shaft diameter (mm)

Equation (9.13) is used to design the shaft on the basis of torsional rigidity. The permissible angle of twist for machine tool applications is 0.25° per metre length. For line shafts, 3° per metre length is the limiting value. Modulus of rigidity for steel is 79 300 N/mm^2 or approximately 80 kN/mm^2.

9.4 ASME CODE FOR SHAFT DESIGN

One important approach of designing a transmission shaft is to use the ASME code. According to this code, the permissible shear stress $\tau_{max.}$ for the shaft without keyways is taken as 30% of yield strength in tension or 18% of the ultimate tensile strength of the material, whichever is minimum. Therefore,

$\tau_{max.} = 0.30\, S_{yt}$

or, $\tau_{max.} = 0.18\, S_{ut}$ (whichever is minimum) (9.14)

If keyways are present, the above values are to be reduced by 25 per cent. According to the ASME code, the bending and torsional moments are to be multiplied by factors k_b and k_t respectively, to account for shock and fatigue in operating condition. The ASME code is based on maximum shear stress theory of failure. Therefore, Eq. (9.10) is modified and rewritten as,

$$\tau_{max.} = \frac{16}{\pi d^3} \sqrt{(k_b M_b)^2 + (k_t M_t)^2} \qquad (9.15)$$

where,

k_b = combined shock and fatigue factor applied to bending moment
k_t = combined shock and fatigue factor applied to torsional moment

The values of k_b and k_t for rotating shafts are given in Table 9.2.

Table 9.2 *Values of shock and fatigue factors k_b and k_t*

Application	k_b	k_t
(i) Load gradually applied	1.5	1.0
(ii) Load suddenly applied (minor shock)	1.5–2.0	1.0–1.5
(iii) Load suddenly applied (heavy shock)	2.0–3.0	1.5–3.0

Equations (9.14) and (9.15) are used to design the shaft according to the ASME code.

We will rewrite Eq. (9.15) again.

$$\tau_{max.} = \frac{16}{\pi d^3} \sqrt{(k_b M_b)^2 + (k_t M_t)^2}$$

Similarly, we will write

$$\sigma_1 = \frac{16}{\pi d^3} \left[k_b M_b + \sqrt{(k_b M_b)^2 + (k_t M_t)^2} \right]$$

(i) Equivalent Torsional Moment The expression $\sqrt{(k_b M_b)^2 + (k_t M_t)^2}$ is called '*equivalent' torsional moment* when the shaft is subjected to fluctuating loads. The equivalent torsional moment is defined as the torsional moment, which when acting alone, will produce the same torsional shear stress in the shaft as under the combined action of bending moment (M_b) and torsional moment (M_t) under fluctuating loads.

(ii) Equivalent Bending Moment The expression $\left[k_b M_b + \sqrt{(k_b M_b)^2 + (k_t M_t)^2} \right]$ is called '*equivalent' bending moment* when the shaft is subjected to fluctuating loads. The equivalent bending moment is defined as the bending moment, which when acting alone, will produce the same bending stresses (tensile and compressive) in the shaft as under the combined action of bending moment (M_b) and torsional moment (M_t) under fluctuating loads.

Example 9.1 *The layout of a transmission shaft carrying two pulleys B and C and supported on bearings A and D is shown in Fig. 9.3(a). Power is supplied to the shaft by means of a vertical belt on the pulley B, which is then transmitted to the pulley C carrying a horizontal belt. The maximum tension in the belt on the pulley B is 2.5 kN. The angle of wrap for both the pulleys is 180° and the coefficient of friction is 0.24. The shaft is made of plain carbon steel 30C8 (S_{yt} = 400 N/mm²) and the factor of safety is 3. Determine the shaft diameter on strength basis.*

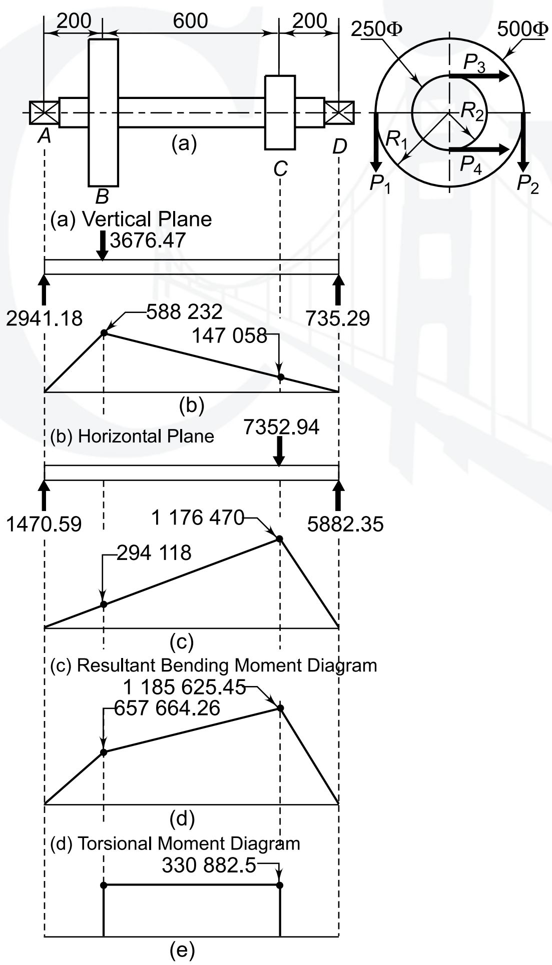

Fig. 9.3

Solution

Given $S_{yt} = 400 \text{ N/mm}^2 \quad (fs) = 3$

For belt drive, maximum tension = 2.5 kN

$\mu = 0.24 \quad \theta = 180°$

Step I *Permissible shear stress*

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.5\ S_{yt}}{(fs)} = \frac{0.5\,(400)}{3} = 66.67 \text{ N/mm}^2$$

Step II *Torsional moment*

$$P_1 = 2.5 \text{ kN} = 2500 \text{ N}$$

$$\frac{P_1}{P_2} = e^{\mu\theta} = e^{0.24\pi} = 2.125$$

$$\therefore \qquad P_2 = \frac{P_1}{2.125} = \frac{2500}{2.125} = 1176.47 \text{ N}$$

The torque supplied to the shaft is given by,

$$M_t = (P_1 - P_2)\,R_1 = (2500 - 1176.47) \times 250$$
$$= 330\,882.5 \text{ N-mm}$$

Step III *Bending moment*

Also, $\qquad (P_3 - P_4)\,R_2 = M_t$

$\therefore \qquad (P_3 - P_4)\,(125) = 330\,882.5$

or, $\qquad (P_3 - P_4) = 2647.06 \text{ N} \qquad$ (a)

Also,

$$\frac{P_3}{P_4} = e^{\mu\theta} = 2.125 \qquad \text{(b)}$$

From Eqs (a) and (b),

$$P_3 = 5000 \text{ N} \quad \text{and} \quad P_4 = 2352.94 \text{ N}$$

Neglecting the weight of the pulley, the downward force at the pulley B is $(P_1 + P_2)$ or 3676.47 N. Similarly, the force in the horizontal plane at the pulley C is $(P_3 + P_4)$ or 7352.94 N. The forces and bending moments in vertical and horizontal planes are shown in Fig. 9.3(a) and (b) respectively. The resultant bending moment is given by,

$$(M_b) \text{ at } B = \sqrt{(588\,232)^2 + (294\,118)^2}$$
$$= 657\,664.26 \text{ N-mm}$$

$$(M_b) \text{ at } C = \sqrt{(147\,058)^2 + (1\,176\,470)^2}$$
$$= 1\,185\,625.45 \text{ N-mm}$$

The resultant bending moment diagram and torsional moment diagram are shown in Fig. 9.3(c) and (d) respectively. The stresses are maximum at the pulley C.

Step IV *Shaft diameter*

From Eq. (9.10),

$$\tau_{max} = \frac{16}{\pi d^3}\sqrt{(M_b)^2 + (M_t)^2}$$

$$66.67 = \frac{16}{\pi d^3}\sqrt{(1\,185\,625.45)^2 + (330\,882.5)^2}$$

$\therefore \quad d = 45.47$ mm.

Example 9.2 *The layout of a shaft carrying two pulleys 1 and 2, and supported on two bearings A and B is shown in Fig. 9.4(a). The shaft transmits 7.5 kW power at 360 rpm from the pulley 1 to the pulley 2. The diameters of pulleys 1 and 2 are 250 mm and 500 mm respectively. The masses of pulleys 1 and 2 are 10 kg and 30 kg respectively. The belt tensions act vertically downward and the ratio of belt tensions on the tight side to slack side for each pulley is 2.5:1. The shaft is made of plain carbon steel 40C8 (S_{yt} = 380 N/mm²) and the factor of safety is 3. Estimate suitable diameter of shaft.*

If the permissible angle of twist is 0.5° per metre length, calculate the shaft diameter on the basis of torsional rigidity. Assume G = 79300 N/mm².

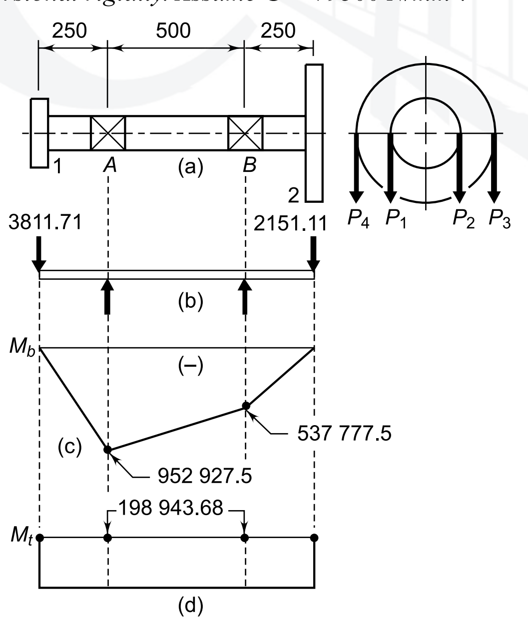

Fig. 9.4

Solution

Given $kW = 7.5 \quad n = 360$ rpm

$S_{yt} = 380$ N/mm² $\quad (fs) = 3$

For belt drive, $\quad P_1/P_2 = 2.5$

$G = 79\,300$ N/mm² $\quad \theta = 0.5°$ per metre length

Step I *Permissible shear stress*

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.5\,S_{yt}}{(fs)} = \frac{0.5(380)}{3} = 63.33 \text{ N/mm}^2$$

Step II *Torsional moment*

$$M_t = \frac{60 \times 10^6 (kW)}{2\pi n} = \frac{60 \times 10^6 (7.5)}{2\pi(360)}$$

$= 198\,943.68$ N-mm.

Step III *Bending moment*

For the pulley 1,

$$(P_1 - P_2) \times 125 = 198\,943.68$$

$$\therefore \quad (P_1 - P_2) = 1591.55 \text{ N} \qquad \text{(a)}$$

$$\text{Also,} \quad \frac{P_1}{P_2} = 2.5 \qquad \text{(b)}$$

From Eqs (a) and (b),

$$P_1 = 2652.58 \text{ N} \quad \text{and} \quad P_2 = 1061.03 \text{ N}$$

The weight of the pulley is given by,

$$W_1 = m_1 g = 10(9.81) = 98.1 \text{ N}$$

The total downward force acting at the centre line of the pulley 1 is given by,

$$(P_1 + P_2 + W_1) = 2652.58 + 1061.03 + 98.1 = 3811.71 \text{ N}$$

The bending moment at the bearing A is given by,

$$(M_b)_{\text{at } A} = 3811.71 \times 250 = 952\,927.5 \text{ N-mm}$$

For the pulley 2,

$$(P_3 - P_4) \times 250 = 198\,943.68$$

$$\therefore \quad (P_3 - P_4) = 795.77 \text{ N} \qquad \text{(c)}$$

$$\text{Also,} \quad \frac{P_3}{P_4} = 2.5 \qquad \text{(d)}$$

From Eqs (c) and (d),

$$P_3 = 1326.29 \text{ N and } P_4 = 530.52 \text{ N}$$

The weight of the pulley is given by,

$$W_2 = m_2 g = 30(9.81) = 294.3 \text{ N}$$

The total downward force acting at the centre line of the pulley 2 is given by,

$$(P_3 + P_4 + W_2) = 1326.29 + 530.52 + 294.3 = 2151.11 \text{ N}$$

The bending moment at the bearing B is given by,

(M_b) at $B = 2151.11 \times 250 = 537\,777.5$ N-mm

Bending and torsional moment diagrams are shown in Fig. 9.4(c) and (d) respectively. It is observed that torsional moment is constant throughout the length of the shaft while bending moment varies. The bending moment is maximum at A.

Step IV *Shaft diameter on strength basis*

From Eq. (9.10),

$$\tau_{\text{max.}} = \frac{16}{\pi d^3}\sqrt{(M_b)^2 + (M_t)^2}$$

$$63.33 = \frac{16}{\pi d^3}\sqrt{(952\,927.5)^2 + (198\,943.68)^2}$$

$$\therefore \quad d = 42.78 \text{ mm} \qquad \text{(i)}$$

Step IV *Shaft diameter on rigity basis*

From Eq. (9.13),

$$d^4 = \frac{584\,M_t l}{G\theta} = \frac{584(198\,943.68)(1000)}{(79\,300)(0.5)}$$

$$\therefore \quad d = 41.37 \text{ mm} \qquad \text{(ii)}$$

Example 9.3 *A line shaft supporting two pulleys A and B is shown in Fig. 9.5(a). Power is supplied to the shaft by means of a vertical belt on the pulley A, which is then transmitted to the pulley B carrying a horizontal belt. The ratio of belt tension on tight and loose sides is 3:1. The limiting value of tension in the belts is 2.7 kN. The shaft is made of plain carbon steel 40C8 (S_{ut} = 650 N/mm^2 and S_{yt} = 380 N/mm^2). The pulleys are keyed to the shaft. Determine the diameter of the shaft according to the ASME code if,*

$$k_b = 1.5 \quad \text{and} \quad k_t = 1.0$$

Solution

Given $S_{ut} = 650$ N/mm^2 $S_{yt} = 380$ N/mm^2

$k_b = 1.5$ $k_t = 1.0$

For belt drive, $P_1/P_2 = 3$

Maximum belt tension = 2.7 kN

Step I *Permissible shear stress*

$0.30\,S_{yt} = 0.30(380) = 114$ N/mm^2

$0.18\,S_{ut} = 0.18(650) = 117$ N/mm^2

The lower of the two values is 114 N/mm^2 and there are keyways on the shaft.

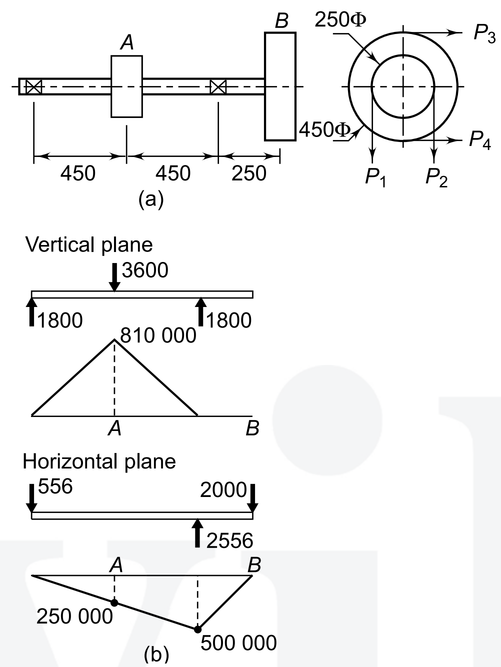

Fig. 9.5

$$\therefore \quad \tau_{\text{max.}} = 0.75(114) = 85.5 \text{ N/mm}^2$$

Step II *Torsional moment*

The maximum belt tension is limited to 2.7 kN. At this stage, it is not known whether P_1 is maximum or P_3 is maximum. The torque transmitted by the pulley A is equal to torque received by the pulley B. Therefore,

$$(P_1 - P_2)\left(\frac{250}{2}\right) = (P_3 - P_4)\left(\frac{450}{2}\right)$$

$$(P_1 - P_2) = 1.8\,(P_3 - P_4) \qquad \text{(a)}$$

Also, $P_2 = \frac{1}{3}P_1$ and $P_4 = \frac{1}{3}P_3$

Substituting the above values in Eq. (a),

$$\left(\frac{2}{3}P_1\right) = 1.8\left(\frac{2}{3}P_3\right) \quad \text{or} \quad P_1 = 1.8\,P_3$$

Therefore, the tension P_1 in the belt on the pulley A is maximum.

$$P_1 = 2700 \text{ N} \quad \text{and} \quad P_2 = 2700/3 = 900 \text{ N}$$

$$P_3 = \frac{P_1}{1.8} = \frac{2700}{1.8} = 1500 \text{ N} \quad P_4 = 1500/3 = 500 \text{ N}$$

A simple way to decide the maximum tension in the belt is the smaller diameter pulley. Smaller the diameter of the pulley, higher will be the belt tension for a given torque.

The torque transmitted by the shaft is given by,

$$M_t = (2700 - 900)\left(\frac{250}{2}\right) = 225\,000 \text{ N-mm}$$

Step III *Bending moment*

The forces and bending moments in vertical and horizontal planes are shown in Fig. 9.5(b). The maximum bending moment is at A. The resultant bending moment at A is given by,

$$M_b = \sqrt{(810\,000)^2 + (250\,000)^2} = 847\,703 \text{ N-mm}$$

Step IV *Shaft diameter*

From Eq. (9.15),

$$d^3 = \frac{16}{\pi\tau_{\max.}}\sqrt{(k_b M_b)^2 + (k_t M_t)^2}$$

$$= \frac{16}{\pi(85.5)}\sqrt{(1.5 \times 847\,703)^2 + (1.0 \times 225\,000)^2}$$

or $d = 42.53$ mm

Example 9.4 *The layout of an intermediate shaft of a gear box supporting two spur gears B and C is shown in Fig. 9.6. The shaft is mounted on two bearings A and D. The pitch circle diameters of gears B and C are 900 and 600 mm respectively. The material of the shaft is steel FeE 580 (S_{ut} = 770 and S_{yt} = 580 N/mm²). The factors k_b and k_t of ASME code are 1.5 and 2.0 respectively. Determine the shaft diameter using the ASME code.*

Assume that the gears are connected to the shaft by means of keys.

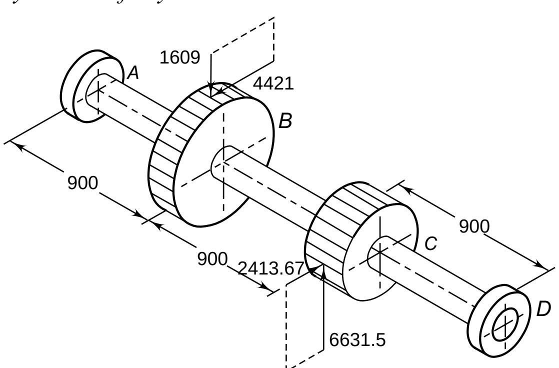

Fig. 9.6

Solution

Given S_{ut} = 770 N/mm² S_{yt} = 580 N/mm²
k_b = 1.5 k_t = 2.0
For gears, $(d'_p)_B = 900$ mm $(d'_p)_C = 600$ mm

Step I *Permissible shear stress*

$$0.30\ S_{yt} = 0.30(580) = 174 \text{ N/mm}^2$$
$$0.18\ S_{ut} = 0.18(770) = 138.6 \text{ N/mm}^2$$

The lower of the two values is 138.6 N/mm² and there are keyways on the shaft.

$$\therefore \qquad \tau_{\max.} = 0.75(138.6) = 103.95 \text{ N/mm}^2$$

Step II *Bending and torsional moment*

The forces and bending moments in vertical and horizontal planes are shown in Fig. 9.7. The maximum bending moment is at C. The resultant bending moment at C is given by,

$$M_b = \sqrt{(3\,496\,203)^2 + (121\,905)^2}$$
$$= 3\,498\,327.4 \text{ N-mm}$$
$$M_t = 4421(450) = 1\,989\,450 \text{ N-mm}$$

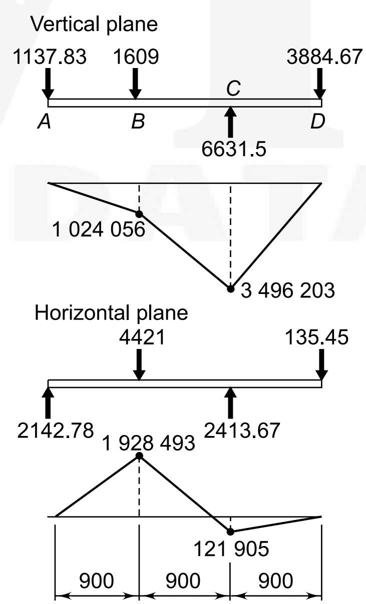

Fig. 9.7

Step III *Shaft diameter*

From Eq. (9.15),

$$d^3 = \frac{16}{\pi\tau_{max.}}\sqrt{(k_b M_b)^2 + (k_t M_t)^2}$$

$$= \frac{16}{\pi(103.95)}\sqrt{(1.5 \times 3\,498\,327.4)^2 + (2.0 \times 1989\,450)^2}$$

or $d = 68.59$ mm

Example 9.5 *A transmission shaft supporting a spur gears B and the pulley D is shown in Fig. 9.8. The shaft is mounted on two bearings A and C. The diameter of the pulley and the pitch circle diameter of the gear are 450 mm and 300 mm respectively. The pulley transmits 20 kW power at 500 rpm to the gear. P_1 and P_2 are belt tensions in the tight and loose sides, while P_t and P_r are tangential and radial components of gear tooth force. Assume,*

$$P_1 = 3P_2 \quad \text{and} \quad P_r = P_t \tan(20°)$$

The gear and pulley are keyed to the shaft. The material of the shaft is steel 50C4 (S_{ut} = 700 and S_{yt} = 460 N/mm²). The factors k_b and k_t of the ASME code are 1.5 each. Determine the shaft diameter using the ASME code.

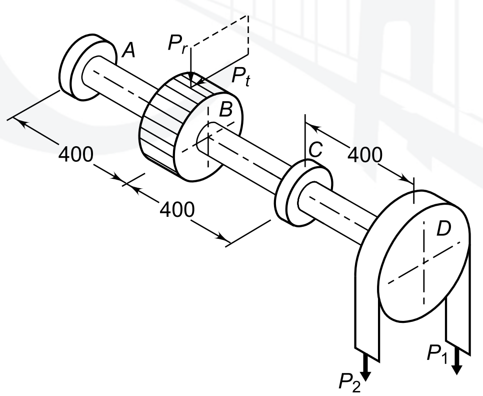

Fig. 9.8

Solution

Given $S_{ut} = 700$ N/mm² $S_{yt} = 460$ N/mm²
$kW = 20$ $n = 500$ rpm
$k_b = k_t = 1.5$ $(d'_p)_B = 300$ mm
Pulley diameter = 450 mm

Step I *Permissible shear stress*

$$0.30\,S_{yt} = 0.30(460) = 138 \text{ N/mm}^2$$
$$0.18\,S_{ut} = 0.18(700) = 126 \text{ N/mm}^2$$

The lower of the two values is 126 N/mm² and there are keyways on the shaft.

$$\therefore \quad \tau_{max.} = 0.75(126) = 94.5 \text{ N/mm}^2$$

Step II *Torsional moment*

The torque transmitted by the shaft is given by,

$$M_t = \frac{60 \times 10^6 (kW)}{2\pi n} = \frac{60 \times 10^6 (20)}{2\pi(500)}$$
$$= 381\,971.86 \text{ N-mm}$$

Step III *Bending moment*

$$(P_1 - P_2) \times 225 = 381\,971.86$$
$$\therefore \quad (P_1 - P_2) = 1697.65 \text{ N} \qquad (a)$$
$$\text{Also,} \quad P_1 = 3P_2 \qquad (b)$$

From Eqs (a) and (b),

$$P_1 = 2546.48 \text{ N} \quad \text{and} \quad P_2 = 848.83 \text{ N}$$
$$\therefore \quad (P_1 + P_2) = 3395.31 \text{ N}$$
$$P_t \times 150 = 381\,971.86$$
$$P_t = 2546.48 \text{ N}$$

$$P_r = P_t \tan(20°) = (2546.48) \tan(20°) = 926.84 \text{ N}$$

The forces and bending moments in vertical and horizontal planes are shown in Fig. 9.9. The maximum bending moment is at C.

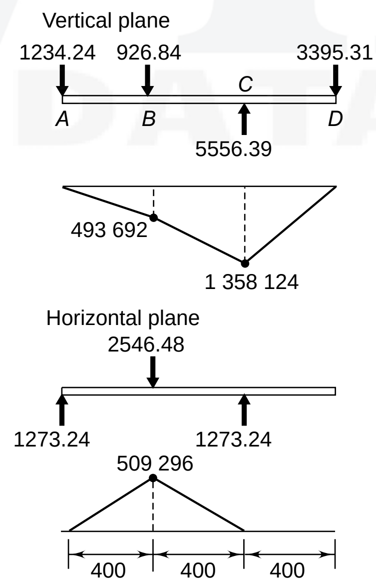

Fig. 9.9

At C,

$$M_b = 1\,358\,124 \text{ N-mm}$$
$$M_t = 381\,971.86 \text{ N-mm}$$

Step IV *Shaft diameter*
From Eq. (9.15),

$$d^3 = \frac{16}{\pi\tau_{\text{max.}}}\sqrt{(k_b M_b)^2 + (k_t M_t)^2}$$
$$\frac{16}{\pi(94.5)}\sqrt{(1.5\times 1\,358\,124)^2 + (1.5\times 381\,971.86)^2}$$

or $\quad d = 48.5$ mm

Example 9.6 *A transmission shaft supporting a helical gear B and an overhung bevel gear D is shown in Fig. 9.10. The shaft is mounted on two bearings, A and C. The pitch circle diameter of the helical gear is 450 mm and the diameter of the bevel gear at the forces is 450 mm. Power is transmitted from the helical gear to the bevel gear. The gears are keyed to the shaft. The material of the shaft is steel 45C8 (S_{ut} = 600 and S_{yt} = 380 N/mm^2). The factors k_b and k_t of ASME code are 2.0 and 1.5 respectively. Determine the shaft diameter using the ASME code.*

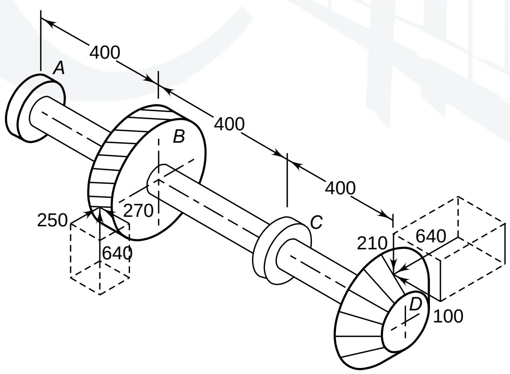

Fig. 9.10

Solution

Given $S_{ut} = 600$ N/mm^2 $\quad S_{yt} = 380$ N/mm^2
$k_b = 2.0 \quad k_t = 1.5$
For gears, $(d'_p)_B = 450$ mm $\quad (d'_p)_D = 450$ mm

Step I *Permissible shear stress*

$$0.30\, S_{yt} = 0.30(380) = 114 \text{ N/mm}^2$$
$$0.18\, S_{ut} = 0.18(600) = 108 \text{ N/mm}^2$$

The lower of the two values is 108 N/mm^2 and there are keyways on the shaft.

$$\therefore \quad \tau_{\text{max.}} = 0.75(108) = 81 \text{ N/mm}^2$$

Step II *Bending moment*
The forces and bending moments in vertical and horizontal planes are shown in Fig. 9.11. The resultant bending moments at B and C are as follows:

At B, $\quad M_b = \sqrt{(158\,752)^2 + (208\,374)^2}$
$= 261\,957.85$ N-mm

At C, $\quad M_b = \sqrt{(61\,500)^2 + (256\,000)^2}$
$= 263\,283.59$ N-mm

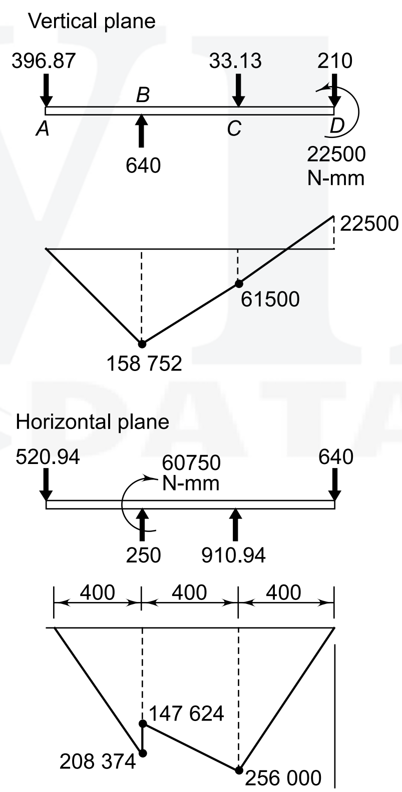

Fig. 9.11

Step III *Torsional moment*

$$M_t = 640 \times 225 = 144\,000 \text{ N-mm}$$

Step IV *Shaft diameter*

From Eq. (9.15),

$$d^3 = \frac{16}{\pi\tau_{max.}}\sqrt{(k_b M_b)^2 + (k_t M_t)^2}$$

$$= \frac{16}{\pi(81)}\sqrt{(2.0\times 263\,283.59)^2 + (1.5\times 144\,000)^2}$$

or $d = 32.95$ mm

Example 9.7 *The armature shaft of a 40 kW, 720 rpm electric motor, mounted on two bearings A and B, is shown in Fig. 9.12. The total magnetic pull on the armature is 7 kN and it can be assumed to be uniformly distributed over a length of 700 mm midway between the bearings. The shaft is made of steel with an ultimate tensile strength of 770 N/mm² and yield strength of 580 N/mm². Determine the shaft diameter using the ASME code if,*

$$k_b = 1.5 \quad \text{and} \quad k_t = 1.0$$

Assume that the pulley is keyed to the shaft.

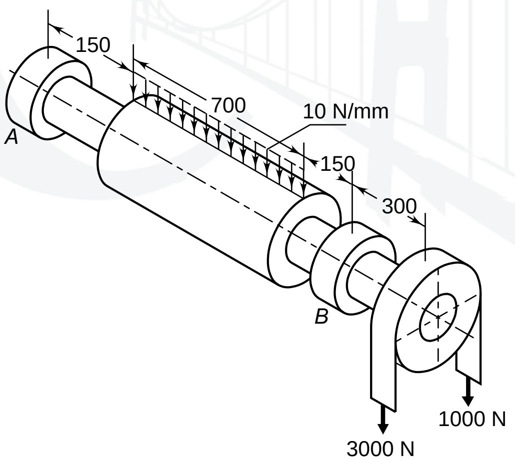

Fig. 9.12

Solution

Given $S_{ut} = 770$ N/mm² $S_{yt} = 580$ N/mm²

$k_b = 1.5$ $k_t = 1.0$

$kW = 40$ $n = 720$ rpm

Step I *Permissible shear stress*

$$0.30\,S_{yt} = 0.30\,(580) = 174 \text{ N/mm}^2$$
$$0.18\,S_{ut} = 0.18\,(770) = 138.6 \text{ N/mm}^2$$

Since the pulley is keyed to the shaft,

$$\tau_{max.} = 0.75\,(138.6) = 103.95 \text{ N/mm}^2$$

Step II *Torsional moment*

$$M_t = \frac{60\times 10^6\,(\text{kW})}{2\pi n} = \frac{60\times 10^6\,(40)}{2\,\pi\,(720)}$$
$$= 530\,516.47 \text{ N-mm}$$

Step III *Bending moment*

The bending moment diagram in the vertical plane is shown in Fig. 9.13. The bending moment, which is maximum at the bearing B, is given by

$$M_b = 1\,200\,000 \text{ N-mm}$$

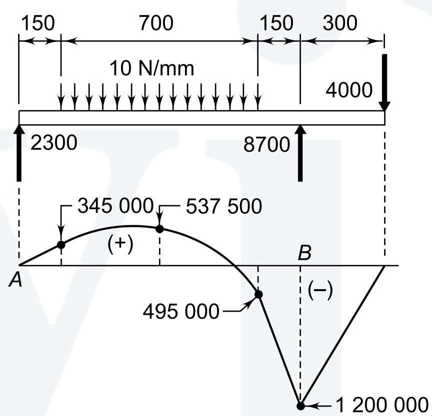

Fig. 9.13

Step IV *Shaft diameter*

From Eq. (9.15),

$$d^3 = \frac{16}{\pi\tau_{max.}}\sqrt{(k_b\,M_b)^2 + (k_t\,M_t)^2}$$

$$= \frac{16}{\pi\,(103.95)}\sqrt{(1.5\times 1\,200\,000)^2 + (530\,516.47)^2}$$

$\therefore$ $d = 45.13$ mm

Example 9.8 *Assume the data of the transmission shaft given in Example 9.5. For this shaft, the permissible angle of twist is 3° per metre length. The modulus of rigidity for the shaft material is 79 300 N/mm². Calculate:*

(i) the permissible angle of twist; and

(ii) the shaft diameter on the basis of torsional rigidity.

Solution

Given $G = 79\ 300\ \text{N/mm}^2$
$\theta = 3°$ per metre length

Step I *Permissible angle of twist for shaft*
Refer to Fig. 9.8. The portion of the shaft between the spur gear B and pulley D is subjected to twisting. The centre distance between the spur gear B and the pulley D is 800 mm (Fig. 9.8). Therefore,

$$\theta = \left(\frac{3}{1000}\right)(800) = 2.4° \qquad \text{(i)}$$

Step II *Shaft diameter*
From Eq. (9.13),

$$d^4 = \frac{584 M_t l}{G\theta} = \frac{584(381\,971.86)(800)}{(79\,300)(2.4)}$$

$$\therefore \qquad d = 31.12 \text{ mm} \qquad \text{(ii)}$$

Example 9.9 *Assume the data of the transmission shaft given in Example 9.6. For this shaft, the permissible angle of twist is 0.25° per metre length. The modulus of rigidity for the shaft material is 79 300 N/mm². Calculate:*

(i) the permissible angle of twist between helical and bevel gears; and
(ii) the shaft diameter on the basis of torsional rigidity.

Compare the results of these two examples and comment on the results.

Solution

Given $G = 79\ 300\ \text{N/mm}^2$ $\theta = 0.25°$ per metre

Step I *Permissible angle of twist for shaft*
Refer to Fig. 9.10. The portion of the shaft between the two gears is subjected to twisting. The distance between the helical gear B and the bevel gear D is 800 mm (Fig. 9.10). Therefore,

$$\theta = 0.25\left(\frac{800}{1000}\right) = 0.2° \qquad \text{(i)}$$

Step II *Shaft diameter*
From Eq. (9.13),

$$d^4 = \frac{584 M_t l}{G\theta} = \frac{584(144\ 000)(800)}{(79\,300)(0.2)}$$

$$\therefore \qquad d = 45.38 \text{ mm} \qquad \text{(ii)}$$

Step III *Criterion of design*
In Example 9.6, the shaft was designed on the basis of its strength and its diameter was calculated as 32.95 mm. On the basis of torsional rigidity, the diameter comes out to be 45.38 mm. Therefore, it is necessary to design the shaft on the basis of strength as well as rigidity.

9.5 DESIGN OF HOLLOW SHAFT ON STRENGTH BASIS

The torsional shear stress and bending stresses in the shaft are given by,

$$\tau = \frac{M_t r}{J} \qquad \sigma_b = \frac{M_b y}{I}$$

The distribution of bending stresses and torsional shear stress is shown in Fig. 9.14. It is observed that the torsional shear stress as well as bending stresses are zero at the shaft centre (r = 0 and y = 0) and negligibly small in the vicinity of the shaft centre, where the radius is small. As the radius increases, the resisting stresses due to external bending and torsional moments increase. Therefore, outer fibres are more effective in resisting the applied moments. In hollow shafts, the material at the centre is removed and spread at large radius. Therefore, hollow shafts are stronger than solid shafts having the same weight.

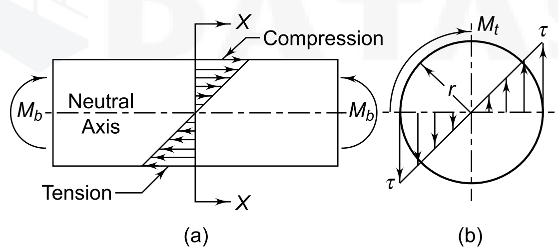

Fig. 9.14 *(a) Distribution of Bending Stresses*
(b) Distribution of Torsional Shear Stress

Compared with solid shaft, hollow shaft offers following advantages:

(i) The stiffness of the hollow shaft is more than that of solid shaft with same weight.
(ii) The strength of hollow shaft is more than that of solid shaft with same weight.

(iii) The natural frequency of hollow shaft is higher than that of solid shaft with same weight.

Compared with solid shaft, hollow shaft has the following disadvantages:

(i) Hollow shaft is costlier than solid shaft.

(ii) The diameter of hollow shaft is more than that of solid shaft and requires more space.

Hollow shafts are used to provide passage for coolants and control cables in case of deep hole drilling and borewell drilling. They are also used as propeller shafts in automobiles and for axles of railway wagons. Their applications include epicyclic gearboxes where one shaft rotates inside another. Hollow shafts are usually made by the extrusion process.

The design of hollow shaft consists of determining the correct inner and outer diameters from strength and rigidity considerations. Such shafts are subjected to axial tensile force, bending moment, torsional moment or combination of these loads. Let us assume,

$$\frac{d_i}{d_o} = C \tag{9.16}$$

where,

d_i = inside diameter of the hollow shaft (mm)
d_o = outside diameter of the hollow shaft (mm)
C = ratio of inside diameter to outside diameter.

When the shaft is subjected to axial tensile force, the tensile stress is given by,

$$\sigma_t = \frac{P}{\frac{\pi}{4}(d_o^2 - d_i^2)} = \frac{P}{\frac{\pi}{4}(d_o^2 - C^2 d_o^2)}$$

or,

$$\sigma_t = \frac{4P}{\pi d_o^2 (1 - C^2)} \tag{9.17}$$

When the shaft is subjected to bending moment, the bending stresses are given by,

$$\sigma_b = \frac{M_b y}{I} \tag{a}$$

For hollow circular cross-section,

$$I = \frac{\pi(d_o^4 - d_i^4)}{64} = \frac{\pi(d_o^4 - C^4 d_o^4)}{64}$$

or,
$$I = \frac{\pi d_o^4 (1 - C^4)}{64} \tag{b}$$

and
$$y = \frac{d_o}{2} \tag{c}$$

Substituting Eqs (b) and (c) in Eq. (a),

$$\sigma_b = \frac{32 M_b}{\pi d_o^3 (1 - C^4)} \tag{9.18}$$

When the shaft is subjected to pure torsional moment, the torsional shear stress is given by,

$$\tau = \frac{M_t r}{J} \tag{d}$$

For a hollow circular cross-section,

$$J = \frac{\pi(d_o^4 - d_i^4)}{32} = \frac{\pi(d_o^4 - C^4 d_o^4)}{32}$$

or,
$$J = \frac{\pi d_o^4 (1 - C^4)}{32} \tag{e}$$

and
$$r = \frac{d_o}{2} \tag{f}$$

Substituting Eqs (e) and (f) in Eq. (d),

$$\tau = \frac{16 M_t}{\pi d_o^3 (1 - C^4)} \tag{9.19}$$

Construction of Mohr's circle diagram for hollow shaft is similar to Mohr's circle for solid shaft. It is illustrated in Fig. 9.2. σ_x is the normal stress while τ is the shear stress. We will consider two cases for calculating the values of σ_x.

Case I In this case, the shaft is subjected to a combination of axial force, bending moment and torsional moment.

$$\sigma_x = \sigma_t + \sigma_b$$

Case II In this case, the shaft is subjected to a combination of bending and torsional moments, without any axial force.

$$\sigma_x = \sigma_b$$

The principal stress σ_1 and principal shear stress τ_{max} are obtained by Eqs (9.6) and (9.7) respectively.

From Eq. (9.6),

$$\sigma_1 = \left(\frac{\sigma_x}{2}\right) + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau)^2} \tag{9.20}$$

From Eq. (9.7),

$$\tau_{max.} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau)^2} \tag{9.21}$$

The hollow shaft can be designed on the basis of maximum principal stress theory or maximum shear stress theory. Let us assume that the hollow shaft is subjected to combined bending and torsional moments without any axial force and apply these theories of failures.

(i) Maximum Principal Stress Theory Substituting Eqs (9.18) and (9.19) in Eq. (9.20),

$$\sigma_1 = \left\{\frac{16M_b}{\pi d_o^3(1-C^4)}\right\} + \sqrt{\left\{\frac{16M_b}{\pi d_o^3(1-C^4)}\right\}^2 + \left\{\frac{16M_t}{\pi d_o^3(1-C^4)}\right\}^2}$$

$$\text{or,} \quad \sigma_1 = \frac{16}{\pi d_o^3(1-C^4)}\left[M_b + \sqrt{(M_b)^2 + (M_t)^2}\right] \tag{9.22}$$

$$\text{Also,} \quad \sigma_1 = \frac{S_{yt}}{(fs)}$$

$$\text{Therefore,} \quad \frac{S_{yt}}{(fs)} = \frac{16}{\pi d_o^3(1-C^4)}\left[M_b + \sqrt{(M_b)^2 + (M_t)^2}\right] \tag{9.23}$$

Equation (9.23) can be used to determine the outer diameter of the hollow shaft on the basis of maximum principal stress theory.

(ii) Maximum Shear Stress Theory Substituting Eqs (9.18) and (9.19) in Eq. (9.21),

$$\tau_{max.} = \sqrt{\left\{\frac{16M_b}{\pi d_o^3(1-C^4)}\right\}^2 + \left\{\frac{16M_t}{\pi d_o^3\ (1-C^4)}\right\}^2}$$

$$\text{or} \quad \tau_{max.} = \frac{16}{\pi d_o^3(1-C^4)}\sqrt{\left[(M_b)^2 + (M_t)^2\right]} \tag{9.24}$$

$$\text{Also,} \quad \tau_{max.} = \frac{S_{sy}}{(fs)} = \frac{0.5S_{yt}}{(fs)}$$

Therefore,

$$\frac{0.5S_{yt}}{(fs)} = \frac{16}{\pi d_o^3(1-C^4)}\sqrt{\left[(M_b)^2 + (M_t)^2\right]} \tag{9.25}$$

It is observed from Eqs (9.8) and (9.22) that expressions for σ_1 for solid and hollow shafts are similar except the term $(1 - C^4)$. The expressions for $\tau_{max.}$ are also similar for solid and hollow shafts except the term $(1 - C^4)$.

9.6 DESIGN OF HOLLOW SHAFT ON TORSIONAL RIGIDITY BASIS

The design of hollow shaft on the basis of torsional rigidity is governed by the permissible angle of twist per metre length of shaft. The angle of twist θ_r (in radians) is given by,

$$\theta_r = \frac{M_t l}{JG}$$

Converting θ_r from radians to degrees (θ),

$$\theta = \left(\frac{180}{\pi}\right)\frac{M_t l}{JG} \tag{a}$$

For hollow circular cross-section,

$$J = \frac{\pi(d_o^4 - d_i^4)}{32} = \frac{\pi(d_o^4 - C^4 d_o^4)}{32}$$

$$\text{or,} \quad J = \frac{\pi d_o^4(1-C^4)}{32} \tag{b}$$

Combining Eqs (a) and (b),

$$\theta = \frac{584\ M_t l}{G d_o^4(1-C^4)} \tag{9.26}$$

Equation (9.26) is used to design the hollow shaft on the basis of torsional rigidity.

Example 9.10 *A propeller shaft is required to transmit 45 kW power at 500 rpm. It is a hollow shaft, having an inside diameter 0.6 times of outside diameter. It is made of plain carbon steel and the permissible shear stress is 84 N/mm². Calculate the inside and outside diameters of the shaft.*

Solution

Given $kW = 45 \quad n = 500$ rpm $\quad \tau = 84$ N/mm^2
$d_i = 0.6\ d_o$

Step I *Torque transmitted by shaft*

$$M_t = \frac{60\times10^6(\text{kW})}{2\pi n} = \frac{60\times10^6(45)}{2\pi(500)}$$

$$= 859\ 436.69 \text{ N-mm.}$$

Step II *Inner and outer diameters of shaft*

$$C = \frac{d_i}{d_o} = 0.6$$

From Eq. (9.19),

$$\tau = \frac{16\, M_t}{\pi\, d_o^3 (1 - C^4)} \quad \text{or} \quad 84 = \frac{16\,(859\ 436.69)}{\pi\, d_o^3 (1 - 0.6^4)}$$

$\therefore \quad d_o = 39.12$ mm

$d_i = 0.6 \quad d_o = 0.6\ (39.12) = 23.47$ mm.

Example 9.11 *A hollow transmission shaft, having inside diameter 0.6 times the outside diameter, is made of plain carbon steel 40C8 (S_{yt} = 380 N/mm²) and the factor of safety is 3. A belt pulley, 1000 mm in diameter, is mounted on the shaft, which overhangs the left hand bearing by 250 mm. The belts are vertical and transmit power to the machine shaft below the pulley. The tension on the tight and slack sides of the belt are 3 kN and 1 kN respectively, while the weight of the pulley is 500 N. The angle of wrap of the belt on the pulley is 180°. Calculate the outside and inside diameters of the shaft.*

Solution

Given $S_{yt} = 380$ N/mm² $\quad (fs) = 3 \quad d_i = 0.6\, d_o$

For belt drive, $\quad P_1 = 3$ kN $\quad P_2 = 1$ kN

For pulley weight = 500 N $\quad$ diameter = 1000 mm

Step I *Permissible shear stress*

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.5\, S_{yt}}{(fs)} = \frac{0.5(380)}{3} = 63.33 \text{ N/mm}^2$$

Step II *Torsional and bending moments*

The layout of the shaft is shown in Fig. 9.15(a).

$$M_t = (P_1 - P_2)\, R = (3000 - 1000)\ (500)$$
$$= (1000 \times 10^3) \text{ N-mm}$$
$$M_b = (P_1 + P_2 + W) \times 250$$
$$= (3000 + 1000 + 500) \times 250$$
$$= (1125 \times 10^3) \text{ N-mm}$$

The bending and torsional moment diagrams are shown in Fig. 9.15(c) and (d) respectively.

Step III *Inner and outer diameters of shaft*

$$C = \frac{d_i}{d_o} = 0.6$$

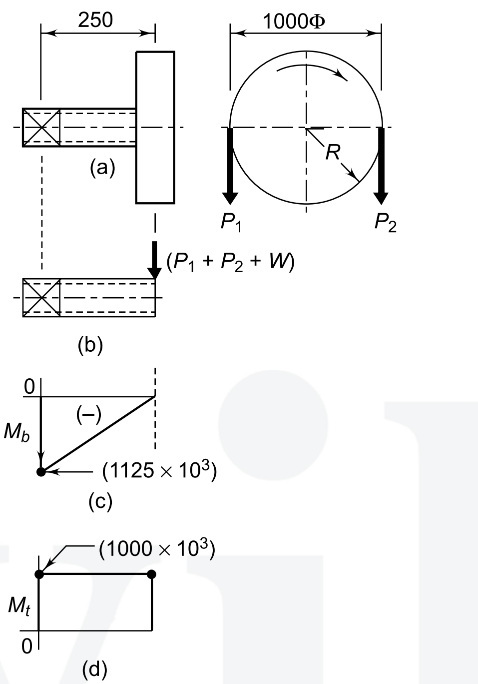

Fig. 9.15

From Eq. (9.24),

$$\tau_{max.} = \frac{16}{\pi d_o^3 (1 - C^4)} \sqrt{(M_b)^2 + (M_t)^2}$$

$$63.33 = \frac{16}{\pi d_o^3 (1 - 0.6^4)} \sqrt{(1125 \times 10^3)^2 + (1000 \times 10^3)^2}$$

$\therefore \ d_o = 51.81$ mm

$d_i = 0.6\, d_o = 0.6\ (51.81) = 31.09$ mm

Example 9.12 *A solid shaft of diameter d is used in power transmission. Due to modification of the existing transmission system, it is required to replace the solid shaft by a hollow shaft of the same material and equally strong in torsion. Further, the weight of the hollow shaft per metre length should be half of the solid shaft. Determine the outer diameter of the hollow shaft in terms of d.*

Solution

Step I *Torque transmitted by solid shaft*

From Eq. (9.3),

$$M_t = \left(\frac{\pi d^3}{16}\right)\tau \qquad \text{(a)}$$

Step II *Torque transmitted by hollow shaft*

$$M_t = \left(\frac{J}{r}\right)\tau \quad \text{where} \quad J = \frac{\pi(d_o^4 - d_i^4)}{32}$$

$$\therefore \quad M_t = \frac{\pi(d_o^4 - d_i^4)}{16 d_o}\tau \qquad \text{(b)}$$

Step III *Outer diameter of hollow shaft*

The solid and hollow shafts are equally strong in torsion. Equating (a) and (b),

$$d^3 = \frac{d_o^4 - d_i^4}{d_o}$$

or

$$d_i^4 = d_o^4 - d^3 d_o \qquad \text{(c)}$$

Since the weight of hollow shaft per metre length is half of the solid shaft,

$$\frac{\pi(d_o^2 - d_i^2)}{4} = \frac{1}{2} \times \frac{\pi d^2}{4}$$

or

$$d_i^2 = d_o^2 - \frac{d^2}{2}$$

$$\therefore \quad d_i^4 = \left(d_o^2 - \frac{d^2}{2}\right)^2 \qquad \text{(d)}$$

Eliminating the term (d_i^4) from Eqs (c) and (d),

$$d_o^4 - d^3 d_o = \left(d_o^2 - \frac{d^2}{2}\right)^2$$

$$\therefore \quad d_o^4 - d^3 d_o = d_o^4 - d_o^2 d^2 + \frac{d^4}{4}$$

or

$$d_o^2 d^2 - d_o d^3 - \frac{d^4}{4} = 0 \qquad \text{(e)}$$

Equation (e) is a quadratic equation and the roots are given by,

$$d_o = \frac{d^3 \pm \sqrt{d^6 - 4(d^2)\left(-\frac{d^4}{4}\right)}}{2d^2} = \frac{d \pm \sqrt{2}\,d}{2}$$

Taking positive root,

$$d_o = \left(\frac{d + \sqrt{2}d}{2}\right)$$

9.7 FLEXIBLE SHAFTS

Flexible shafts are used to transmit torque between machine units, which may change their relative positions during operation. Their main applications include portable power driven tools, speedometer drives, positioning devices, portable grinders, concrete vibrators, remote control devices and servo drives. In these applications, flexibility is required to accommodate large angular and offset misalignment between driving and driven units. Flexible shafts have two important properties:

(i) They have low rigidity in bending, making them flexible.

(ii) They have high rigidity in torsion, making them capable to transmit torque.

A flexible shaft[2] consists of an inner core, outer casing and end attachments. The inner core consists of a wire coil. In effect, it is a helical torsion spring with closely wound coils. The direction of rotation of the shaft should be such that when the torque is transmitted, the spring is further twisted. The casing protects the shaft from dirt, retains the lubricant such as grease on the shaft surface, prevents the formation of loops in operation, and protects the operator from injury. The end fittings are intended to connect the flexible shaft to driving and driven members.

The applications of flexible shafts are restricted to low power drives.

9.8 KEYS

A key can be defined as a machine element which is used to connect the transmission shaft to rotating machine elements like pulleys, gears, sprockets or flywheels. A keyed joint consisting of shaft, hub and key is illustrated in Fig. 9.16. There are two basic functions of the key. They are as follows:

[2] IS 4600–1968: Specification for flexible shafts.

(i) The primary function of the key is to transmit the torque from the shaft to the hub of the mating element and vice versa.

(ii) The second function of the key is to prevent relative rotational motion between the shaft and the joined machine element like gear or pulley. In most of the cases, the key also prevents axial motion between two elements, except in case of feather key or splined connection.

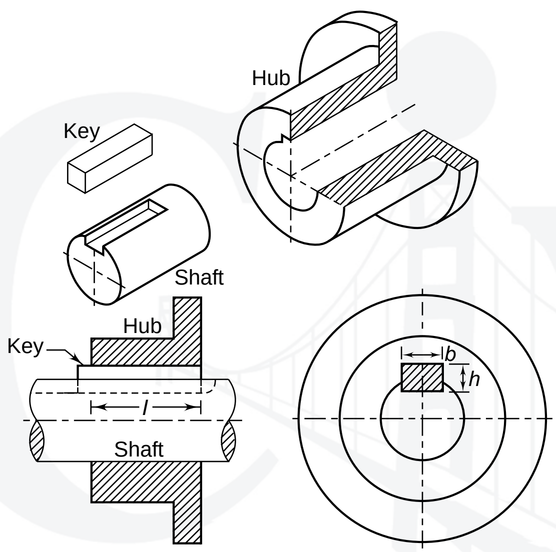

Fig. 9.16 *Key-joint*

A recess or slot machined either on the shaft or in the hub to accommodate the key is called *keyway*. The keyway is usually cut by a vertical or horizontal milling cutter. The keyway results in stress concentration in the shaft and the part becomes weak. This is the main drawback of a keyed joint. Keys are made of plain carbon steels like 45C8 or 50C8 in order to withstand shear and compressive stresses resulting from transmission of torque. According to Indian standards, steel of tensile strength not less than 600 N/mm^2 shall be used as the material for the key.

Many types of keys are available and there are a number of standards, which specify the dimensions of the key[3-6]. There are different ways to classify the keys. Some of them are as follows:

(i) Saddle key and sunk key
(ii) Square key and flat key
(iii) Taper key and parallel key
(iv) Key with and without Gib-head

In addition, there are special types of keys such as Woodruff key, Kennedy key or feather key. The selection of the type of key for a given application depends upon the following factors:

(i) power to be transmitted;
(ii) tightness of fit;
(iii) stability of connection; and
(iv) cost.

In this chapter, only popular types of key are discussed.

9.9 SADDLE KEYS

A saddle key is a key which fits in the keyway of the hub only. In this case, there is no keyway on the shaft. There are two types of saddle keys, namely, *hollow* and *flat*, as shown in Fig. 9.17. A hollow saddle key has a concave surface at the bottom to match the circular surface of the shaft. A flat saddle

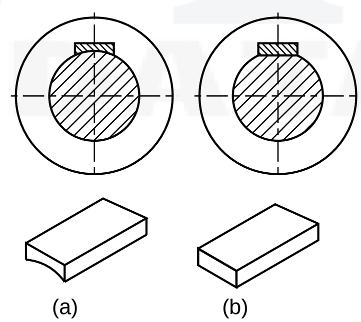

Fig. 9.17 *(a) Hollow Saddle Key (b) Flat Saddle Key*

key has a flat surface at the bottom and it sits on the flat surface machined on the shaft. In both types of saddle keys, friction between the shaft, key and

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[3] IS 2048–1983: Specification for parallel keys and keyways.
[4] IS 2292–1974: Specification for taper keys and keyways.
[5] IS 2293–1974: Specification for Gib-head key and keyways.
[6] IS 2294–1980: Specification for Woodruff keys and keyways.

hub prevents relative motion between the shaft and the hub. The power is transmitted by means of friction. Therefore, saddle keys are suitable for light duty or low power transmission as compared with sunk keys. The resistance to slip in case of flat key is slightly more than that of hollow key with concave surface. Therefore, flat saddle key is slightly superior to hollow saddle key as far as power transmitting capacity is concerned.

Saddle key requires keyway only on the hub. Therefore, cost of the saddle key joint is less than that of sunk key joint. This is the main advantage of the saddle key. The disadvantage of the saddle key is its low power transmitting capacity. Saddle key is liable to slip around the shaft when subjected to heavy torque. Therefore, it cannot be used in medium and heavy duty applications.

9.10 SUNK KEYS

A sunk key is a key in which half the thickness of the key fits into the keyway on the shaft and the remaining half in the keyway on the hub. Therefore, keyways are required both on the shaft as well as the hub of the mating element. This is a standard form of key and may be either of rectangular or square cross-section as shown in Fig. 9.18. The standard dimensions of square and rectangular cross-section sunk keys are given in Table 9.3 given on page 349. In sunk key, power is transmitted due to shear resistance of the key. The relative motion between the shaft and the hub is also prevented by the shear resistance of key. Therefore, sunk key is suitable for heavy duty application, since there is no possibility of the key to slip around the shaft.

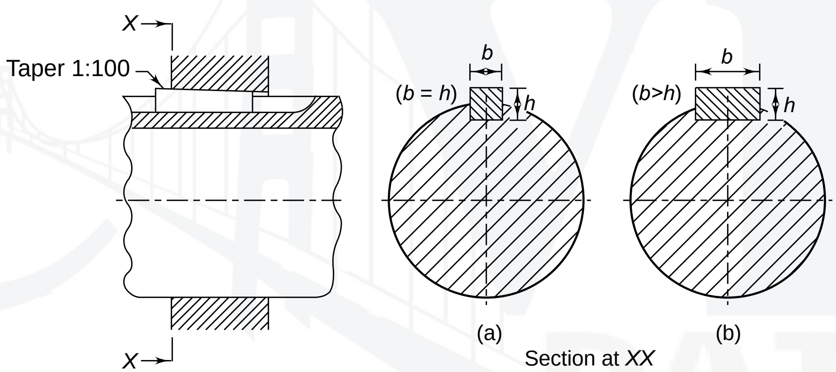


Fig. 9.18 *(a) Square Key (b) Flat Key*

It is a positive drive. This is the main advantage of the sunk key over the saddle key. However, it is necessary to cut keyways both on the shaft and the hub. Therefore, the cost of the sunk key joint is more than that of the saddle key joint.

Sunk keys with square or rectangular cross-sections are widely used in practice. A sunk key with rectangular cross-section is called a *flat key*. The flat key has more stability as compared with square key. Square keys are used in general industrial machinery. Flat keys are more suitable for machine tool applications, where additional stability of the connection is desirable. While selecting the square key without stress analysis, the following rule of thumb may be used. *"The industrial practice is to use a square key with sides equal to one-quarter of the shaft diameter and length at least 1.5 times the shaft diameter"*.

or, $$b = h = \frac{d}{4}$$

and $$l = 1.5\, d$$

where,

b = width of key (mm)

h = height or thickness of key (mm)
l = length of key (mm)
d = diameter of shaft (mm)

Table 9.3 *Dimensions of square and rectangular sunk keys (in mm)*

Shaft diameter		*Key size*	*Keyway depth*
Above	*Up to and including*	$b \times h$	
6	8	2×2	1.2
8	10	3×3	1.8
10	12	4×4	2.5
12	17	5×5	3.0
17	22	6×6	3.5
22	30	8×7	4.0
30	38	10×8	5.0
38	44	12×8	5.0
44	50	14×9	5.5
50	58	16×10	6.0
58	65	18×11	7.0
65	75	20×12	7.5
75	85	22×14	9.0
85	95	25×14	9.0
95	110	28×16	10.0
110	130	32×18	11.0
130	150	36×20	12.0
150	170	40×22	13.0
170	200	45×25	15.0
200	230	50×28	17.0

For a flat key, the thumb-rule dimensions are as follows:

$$b = \frac{d}{4}$$

$$h = \frac{2}{3}\,b = \frac{d}{6}$$

$$l = 1.5\,d$$

Sunk keys with square or rectangular cross-sections are classified into two groups, namely, parallel and taper keys. A *parallel key* is a sunk key which is uniform in width as well as height throughout the length of the key. A *taper key* is uniform in width but tapered in height. The standard taper is 1 in 100. The bottom surface of the key is straight and the top surface is given a taper. The taper is provided for the following two reasons:

(i) When the key is inserted in the keyways of shaft and the hub and pressed by means of hammer, it becomes tight due to wedge action. This insures tightness of joint in operating conditions and prevents loosening of the parts.

(ii) Due to taper, it is easy to remove the key and dismantle the joint.

The taper of the key is on one side. Machining taper on two sides of key is more difficult than making taper on one side. Also, there is no specific advantage of taper on two sides. Tapered keys are often provided with Gib-head to facilitate removal. The Gib-head taper key is shown in Fig. 9.19. The projection of Gib-head is hazardous in rotating parts. As compared with parallel key, taper key has the following advantages:

(i) The taper surface results in wedge action and increases frictional force and the tightness of the joint.

(ii) The taper surface facilitates easy removal of the key, particularly with Gib-head.

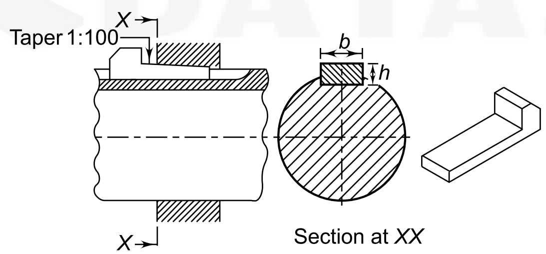


Fig. 9.19 *Gib-head Taper Key*

However, machining taper on the surface increases the cost.

9.11 FEATHER KEY

A feather key is a parallel key which is fixed either to the shaft or to the hub and which permits *relative axial movement between them*. The feather key is a particular type of sunk key with uniform width and height. There are number of methods to fix the key to the shaft or hub. Figure 9.20 shows a feather key, which is fixed to the shaft by means of two cap screws, having countersunk-heads. There is a clearance fit between the key and the keyway in the hub. Therefore, the hub is free to slide over the key. At the same time, there is no relative rotational movement between the shaft and the hub. Therefore, the feather key transmits the torque and at the same time permits some axial movement of the hub. Feather keys are used where the parts mounted on the shaft are required to slide along the shaft such as clutches or gear shifting devices. It is an alternative to splined connection.

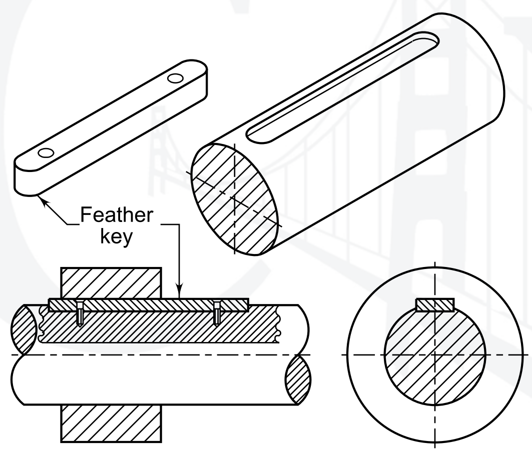


Fig. 9.20 *Feather Key*

9.12 WOODRUFF KEY

A Woodruff key is a sunk key in the form of an almost semicircular disk of uniform thickness as shown in Fig. 9.21. The keyway in the shaft is in the form of a semicircular recess with the same curvature as that of the key. The bottom portion of the Woodruff key fits into the circular keyway in the shaft. The keyway in the hub is made in the usual manner. The projecting part of Woodruff key fits in the keyway in the hub. Once placed in position, the Woodruff key tilts and aligns itself on the shaft. The advantages of Woodruff key are as follows:

(i) The Woodruff key can be used on tapered shaft because it can align by slight rotation in the seat.

(ii) The extra depth of key in the shaft prevents its tendency to slip over the shaft.

The disadvantages of Woodruff key are as follows:

(i) The extra depth of keyway in the shaft increase stress concentration and reduces its strength.

(ii) The key does not permit axial movement between the shaft and the hub.

Woodruff keys are used on tapered shafts in machine tools and automobiles.

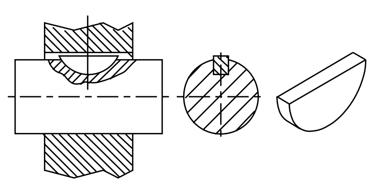


Fig. 9.21 *Woodruff Key*

9.13 DESIGN OF SQUARE AND FLAT KEYS

Although there are many types of keys, only square and flat keys are extensively used in practice. Therefore, the discussion in this chapter is restricted to square and flat keys. A square key is a particular type of flat key, in which the height is equal to the width of the cross-section. Therefore, for the purpose of analysis, a flat key is considered.

The forces acting on a flat key, with width as b and height as h, are shown in Fig. 9.22. The

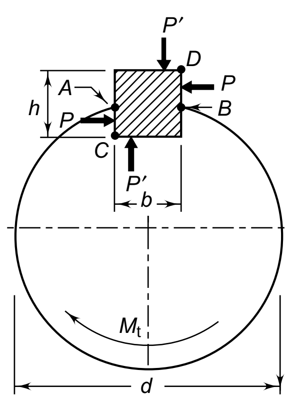


Fig. 9.22 *Forces Acting on Key*

transmission of torque from the shaft to the hub results in two equal and opposite forces denoted by P. The torque M_t is transmitted by means of a force P acting on the left surface AC of the key. The equal and opposite force P, acting on the right surface DB of the key is the reaction of the hub on the key. It is observed that the force P on left surface AC and its equal and opposite reaction P on the right surface DB are not in the same plane. Therefore, forces P^1 ($P^1 = P$) act as resisting couple preventing the key to roll in the keyway.

The exact location of the force P on the surface AC is unknown. In order to simplify the analysis, it is assumed that the force P is tangential to the shaft diameter.

Therefore,

$$P = \frac{M_t}{(d/2)} = \frac{2M_t}{d} \tag{a}$$

where,

M_t = transmitted torque (N-mm)
d = shaft diameter (mm)
P = force on key (N)

The design of square or flat key is based on two criteria, viz., failure due to shear stress and failure due to compressive stress. The shear failure will occur in the plane AB. It is illustrated in Fig. 9.23(a). The shear stress τ in the plane AB is given by,

$$\tau = \frac{P}{\text{area of plane } AB} = \frac{P}{bl} \tag{b}$$

where,

b = width of key (mm)
l = length of key (mm)

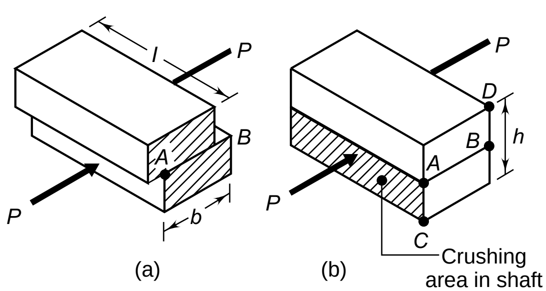


Fig. 9.23 *Failure of Key: (a) Shear Failure (b) Crushing Failure*

From (a) and (b),

$$\tau = \frac{2M_t}{dbl} \tag{9.27}$$

The failure due to compressive stress will occur on surfaces AC or DB. The crushing area between shaft and key is shown in Fig. 9.23(b).

It is assumed that,

$$\overline{AC} \cong \overline{BD} \cong \frac{h}{2}$$

where,

h = height of key (mm)

The compressive stress σ_c in the key is given by,

$$\sigma_c = \frac{P}{\text{area of surface } AC} = \frac{P}{(h/2)\,l} = \frac{2P}{hl} \tag{c}$$

From (a) and (c),

$$\sigma_c = \frac{4M_t}{dhl} \tag{9.28}$$

Equations (9.27) and (9.28) are stress equations of flat key.

For square key,

$$h = b$$

Substituting the above relationship in Eqs (9.27) and (9.28),

$$\tau = \frac{2M_t}{dbl} \tag{a}$$

$$\sigma_c = \frac{4M_t}{dbl} \tag{b}$$

From (a) and (b),

$$\sigma_c = 2\tau \tag{9.29}$$

Therefore, the compressive stress induced in a square key due to the transmitted torque is twice the shear stress.

Example 9.13 *It is required to design a square key for fixing a gear on a shaft of 25 mm diameter. The shaft is transmitting 15 kW power at 720 rpm to the gear. The key is made of steel 50C4 (S_{yt} = 460 N/mm^2) and the factor of safety is 3. For key material, the yield strength in compression can be assumed to be equal to the yield strength in tension. Determine the dimensions of the key.*

Solution

Given kW = 15 $n = 720$ rpm $S_{yt} = 460$ N/mm^2 $(fs) = 3$ $d = 25$ mm

Step I *Permissible compressive and shear stresses*

$$S_{yc} = S_{yt} = 460 \text{ N/mm}^2$$

$$\sigma_c = \frac{S_{yc}}{(fs)} = \frac{460}{3} = 153.33 \text{ N/mm}^2$$

According to maximum shear stress theory of failure,

$$S_{sy} = 0.5\, S_{yt} = 0.5\,(460) = 230 \text{ N/mm}^2$$

$$\tau = \frac{S_{sy}}{(fs)} = \frac{230}{3} = 76.67 \text{ N/mm}^2$$

Step II *Torque transmitted by the shaft*

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n} = \frac{60 \times 10^6 (15)}{2\pi (720)}$$

$$= 198\,943.68 \text{ N-mm}$$

Step III *Key dimensions*

The industrial practice is to use a square key with sides equal to one-quarter of the shaft diameter. Therefore,

$$b = h = \frac{d}{4} = \frac{25}{4} = 6.25 \text{ or } 6 \text{ mm}$$

From Eq. (9.27),

$$l = \frac{2M_t}{\tau db} = \frac{2(198\,943.68)}{(76.67)(25)(6)} = 34.60 \text{ mm} \qquad \text{(a)}$$

From Eq. (9.28),

$$l = \frac{4\,M_t}{\sigma_c\, dh} = \frac{4(198\,943.68)}{(153.33)(25)(6)} = 34.60 \text{ mm} \qquad \text{(b)}$$

From (a) and (b), the length of the key should be 35 mm. The dimensions of the key are 6 × 6 × 35 mm.

Example 9.14 *The standard cross-section for a flat key, which is fitted on a 50 mm diameter shaft, is 16 × 10 mm. The key is transmitting 475 N-m torque from the shaft to the hub. The key is made of commercial steel ($S_{yt} = S_{yc} = 230$ N/mm^2). Determine the length of the key, if the factor of safety is 3.*

Solution

Given $M_t = 475$ N-m $S_{yt} = S_{yc} = 230$ N/mm^2 $(fs) = 3$ $d = 50$ mm $b = 16$ mm $h = 10$ mm

Step I *Permissible compressive and shear stresses*

$$\sigma_c = \frac{S_{yc}}{(fs)} = \frac{230}{3} = 76.67 \text{ N/mm}^2$$

According to maximum shear stress theory of failure,

$$S_{sy} = 0.5\, S_{yt} = 0.5\,(230) = 115 \text{ N/mm}^2$$

$$\tau = \frac{S_{sy}}{(fs)} = \frac{115}{3} = 38.33 \text{ N/mm}^2$$

Step II *Key length*

From Eq. (9.27),

$$l = \frac{2M_t}{\tau db} = \frac{2(475 \times 10^3)}{(38.33)(50)(16)} = 30.98 \text{ mm} \qquad \text{(a)}$$

From Eq. (9.28),

$$l = \frac{4M_t}{\sigma_c dh} = \frac{4(475 \times 10^3)}{(76.67)(50)(10)} = 49.56 \text{ mm} \qquad \text{(b)}$$

From (a) and (b), the length of the key should be 50 mm.

9.14 DESIGN OF KENNEDY KEY

The Kennedy key consists of two square keys as shown in Fig. 9.24. In this case, the hub is bored off the centre and the two keys force the hub and the shaft to a concentric position. Kennedy key is used for heavy duty applications. The analysis of the Kennedy key is similar to that of the flat key.

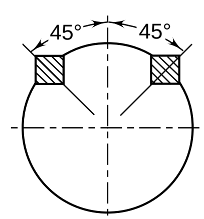


Fig. 9.24 *Kennedy Key*

It is based on two criteria, viz., failure due to shear stress and failure due to compressive stress. The forces acting on one of the two Kennedy keys are shown in Fig. 9.25. Since there are two keys, the torque transmitted by each key is one half of the total torque. The two equal and opposite forces P are due to the transmitted torque. The exact location of the force P is unknown. It is assumed to act tangential to the shaft diameter. Therefore,

$$\frac{M_t}{2} = P\left(\frac{d}{2}\right)$$

or
$$P = \frac{M_t}{d} \qquad \text{(a)}$$

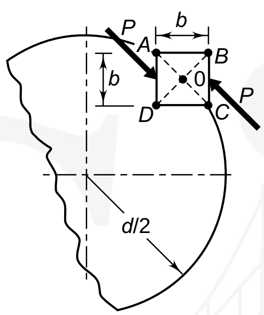


Fig. 9.25 *Forces acting on Kennedy Key*

The failure due to shear stress will occur in the plane AC. The area of the plane AC is $[\overline{AC} \times l]$ or $[\sqrt{2}bl]$. The shear stress is given by,

$$\tau = \frac{P}{\sqrt{2}bl} \qquad \text{(b)}$$

From (a) and (b),

$$\tau = \frac{M_t}{\sqrt{2}dbl} \qquad (9.30)$$

The compressive stress is given by,

$$\sigma_c = \frac{P}{(\text{Projected area})} = \frac{P}{[\overline{OB} \times l]} = \frac{P}{\left(\frac{b}{\sqrt{2}}\right) \times l} \qquad \text{(c)}$$

From (a) and (c),

$$\sigma_c = \frac{\sqrt{2}M_t}{dbl} \qquad (9.31)$$

where l is the length of the key.

Example 9.15 *A shaft, 40 mm in diameter, is transmitting 35 kW power at 300 rpm by means of Kennedy keys of 10 × 10 mm cross-section. The keys are made of steel 45C8 ($S_{yt} = S_{yc} = 380$ N/mm^2) and the factor of safety is 3. Determine the required length of the keys.*

Solution

Given kW = 35 $\quad n = 300$ rpm
$S_{yt} = S_{yc} = 380$ N/mm^2 $\quad (fs) = 3$
$d = 40$ mm $\quad b = h = 10$ mm

Step I *Permissible compressive and shear stresses*

$$\sigma_c = \frac{S_{yc}}{(fs)} = \frac{380}{3} = 126.67 \text{ N/mm}^2$$

According to distortion energy theory of failure,

$$S_{sy} = 0.577\, S_{yt} = 0.577\,(380) = 219.26 \text{ N/mm}^2$$

$$\tau = \frac{S_{sy}}{(fs)} = \frac{219.26}{3} = 73.09 \text{ N/mm}^2$$

Step II *Torque transmitted by shaft*

The torque transmitted by the shaft is given by,

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n} = \frac{60 \times 10^6 (35)}{2\pi (300)}$$
$$= 1\,114\,084.6 \text{ N-mm}$$

Step III *Key length*

From Eq. (9.30),

$$l = \frac{M_t}{\sqrt{2}db\tau} = \frac{(1\,114\,084.6)}{\sqrt{2}(40)(10)(73.09)} = 26.95 \text{ mm}$$

From Eq. (9.31),

$$l = \frac{\sqrt{2}M_t}{db\sigma_c} = \frac{\sqrt{2}(1\,114\,084.6)}{(40)(10)(126.67)} = 31.10 \text{ mm}$$

Example 9.16 *The dimensions of a Woodruff key for a 30 mm diameter shaft are shown in Fig. 9.26(a). The shaft is transmitting 5 kW power*

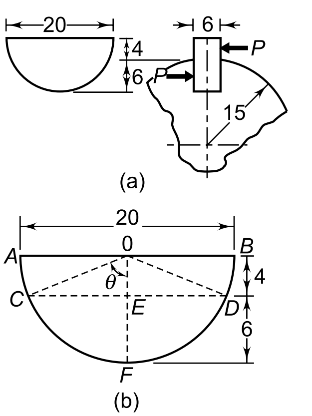


Fig. 9.26 *Woodruff Key*

at 300 rpm The key is made of steel 50C4 ($S_{yt} = S_{yc}$ = 460 N/mm²). Calculate the factor of safety used in design.

Solution

Given kW = 5 $n = 300$ rpm
$S_{yt} = S_{yc} = 460$ N/mm² $d = 30$ mm

Step I *Torque transmitted by the shaft*

The torque transmitted by the shaft is given by,

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n} = \frac{60 \times 10^6 (5)}{2\pi (300)}$$
$$= 159\,154.94 \text{ N-mm}$$

Step II *Force induced by torque*

The torque is transmitted by two, equal and opposite, forces P. It is assumed that the force P acts at the shaft surface. Therefore,

$$P \times 15 = M_t$$
$$P = \frac{M_t}{15} = \frac{159\,154.94}{15} = 10\,610.33 \text{ N}$$

Step III *Key area*

Referring to Fig. 9.26(b),

$$\cos\theta = \frac{\overline{OE}}{\overline{OC}} = \frac{4}{10} \quad \text{or} \quad \theta = 66.42°$$
$$\overline{CD} = 2\,\overline{CE} = 2\,\overline{OC}\sin\theta$$
$$= 2(10)\sin(66.42°) = 18.33 \text{ mm}$$

Total area of the key

$$= \frac{1}{2}\left[\frac{\pi}{4}(20)^2\right] = 157.08 \text{ mm}^2$$

Area of sector $OCFD$

$$= \left(\frac{2\theta}{180}\right)(157.08) = \left(\frac{2(66.42)}{180}\right)(157.08)$$
$$= 115.93 \text{ mm}^2$$

$$\text{Area of } \Delta OCD = \frac{1}{2}\overline{CD} \times \overline{OE}$$
$$= \frac{1}{2}(18.33)(4) = 36.66 \text{ mm}^2$$

Area of key in contact with the shaft = 115.93 – 36.66 = 79.27 mm²

Area of key in contact with the hub = 157.08 – 79.27 = 77.81 mm²

Step IV *Factor of safety against compression failure*

The area of the key in contact with the hub is less and the compressive stress in the area is given by,

$$\sigma_c = \frac{P}{A} = \frac{10\,610.33}{77.81} = 136.36 \text{ N/mm}^2$$
$$(fs) = \frac{S_{yc}}{\sigma_c} = \frac{460}{136.36} = 3.37 \tag{i}$$

Step V *Factor of safety against shear failure*

The shear failure will occur in the plane CD, and its area is $(\overline{CD} \times 6$ or (18.33×6) mm² Therefore,

$$\tau = \frac{P}{A} = \frac{10\,610.33}{(18.33 \times 6)} = 96.48 \text{ N/mm}^2$$
$$(fs) = \frac{S_{sy}}{\tau} = \frac{0.577\,S_{yt}}{\tau}$$
$$= \frac{0.577(460)}{96.48} = 2.75 \tag{ii}$$

9.15 SPLINES

Splines are keys which are made integral with the shaft. They are used when there is a relative axial motion between the shaft and the hub. The gear shifting mechanism in automobile gearboxes requires such type of construction. Splines are cut on the shaft by milling and on the hub by broaching. A splined connection, with straight splines, is shown in Fig. 9.27. The following notations are used:

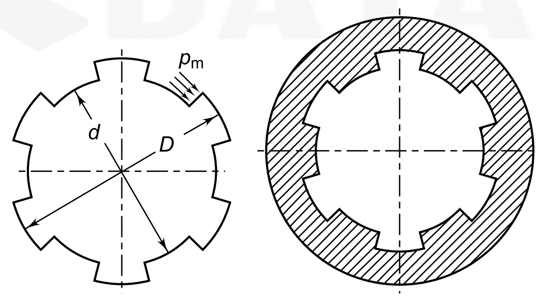


Fig. 9.27 *Splines: (a) Shaft (b) Hub*

D = major diameter of splines (mm)
d = minor diameter of splines (mm)
l = length of hub (mm)
n = number of splines

The torque transmitting capacity of splines is given by,

$$M_t = p_m A R_m \tag{a}$$

where,

M_t = transmitted torque (N-mm)
p_m = permissible pressure on spline (N/mm^2)
A = total area of splines (mm^2)
R_m = mean radius of splines (mm)

The area A is given by,

$$A = \frac{1}{2}(D - d)ln \tag{b}$$

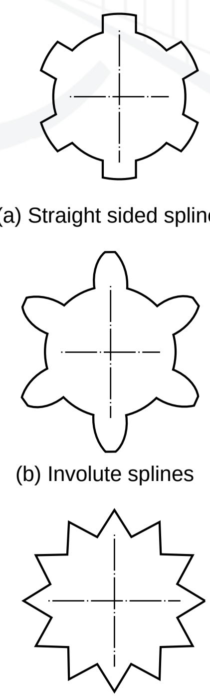
$$R_m = \frac{D + d}{4} \tag{c}$$

Substituting the above values in Eq. (a),

$$M_t = \frac{1}{8} p_m \, ln(D^2 - d^2) \tag{9.32}$$

The permissible pressure on the splines is limited to 6.5 N/mm^2.

The above analysis and Fig. 9.27 refer to straight-sided splines. In addition, there are two other types of splines, namely, involute splines and serrations as shown in Fig. 9.28.



(a) Straight sided splines

(b) Involute splines

(c) Serrations

Fig. 9.28 *Types of Spline Profiles*

(i) Involute Splines Involute splines are in the form of concentric external and internal gear teeth. They are stub teeth with a pressure angle of 30°. These splines are specified by module. Involute splines are more popular than straight splines due to greater strength relative to their size. Involute splines are self centering and tend to adjust to an even distribution of load. However, the cost of involute splines is more than straight-sided splines.

(ii) Serrations Straight-sided serrations are used in applications where it is important to keep the overall size of the assembly as small as possible. They are used as interference joints. Serration joints are also used to obtain small angular relative adjustment between the joined members.

Example 9.17 *A standard splined connection $8 \times 52 \times 60$ mm is used for the gear and the shaft assembly of a gearbox. The splines transmit 20 kW power at 300 rpm. The dimensions of the splines are as follows:*

Major diameter = 60 mm
Minor diameter = 52 mm
Number of splines = 8

Permissible normal pressure on splines is 6.5 N/mm^2. The coefficient of friction is 0.06. Calculate:

(i) The length of hub of the gear
(ii) The force required for shifting the gear

Solution

Given kW = 20 n = 300 rpm $p_m = 6.5\ N/mm^2$
For splines, D = 60 mm d = 52 mm n = 8
$\mu = 0.06$

Step I *Torque transmitted by the shaft*

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n}$$

$$= \frac{60 \times 10^6 (20)}{2\pi (300)} = 636\ 619.76 \text{ N-mm}$$

Step II *Length of hub*

From Eq. (9.32),

$$l = \frac{8M_t}{p_m n(D^2 - d^2)} = \frac{8(636\ 619.76)}{(6.5)(8)(60^2 - 52^2)}$$

$$= 109.31 \text{ or } 110 \text{ mm} \tag{i}$$

Step III *Force required to shift gear*

Due to torque M_t, a normal force P acts on the splines. It is assumed that the force P acts at the mean radius of the splines. Therefore,

$$M_t = P\,R_m \qquad \text{(a)}$$

$$R_m = \frac{D+d}{4} = \frac{60+52}{4} = 28 \text{ mm}$$

Substituting the above value in Eq. (a),

$$P = \frac{M_t}{R_m} = \frac{636\,619.76}{28} = 22\,736.42 \text{ N}$$

Friction force $= \mu P = 0.06(22\,736.42) = 1364.19$ N

The force required to shift the gear is equal and opposite of the friction force. Therefore, the force required to shift the gear is 1364.19 N.

9.16 COUPLINGS

A coupling can be defined as a mechanical device that permanently joins two rotating shafts to each other. The most common application of coupling is joining of shafts of two separately built or purchased units so that a new machine can be formed. For example, a coupling is used to join the output shaft of an engine to the input shaft of a hydraulic pump to raise water from well. A coupling is used to join the output shaft of an electric motor to the input shaft of a gearbox in machine tools. A coupling is also used to join the output shaft of an electric motor to the input shaft of a compressor. There is a basic difference between a coupling and a clutch. Coupling is a permanent connection, while the clutch can connect or disconnect two shafts at the will of the operator.

The shafts to be connected by the coupling may have collinear axes, intersecting axes or parallel axes with a small distance in between. *Oldham coupling* is used to connect two parallel shafts when they are at a small distance apart. *Hooke's coupling* is used to connect two shafts having intersecting axes. When the axes are collinear or in the same line, *rigid* or *flexible couplings* are used. While the flexible coupling is capable of tolerating a small amount of misalignment between the shafts, there is no such provision in rigid coupling. The discussion in this chapter is restricted to rigid and flexible couplings. Oldham and Hooke's couplings are covered in more detail in textbooks on Theory of Machines.

The difference between rigid and flexible couplings is as follows:

(i) A rigid coupling cannot tolerate misalignment between the axes of the shafts. It can be used only when there is precise alignment between two shafts. On the other hand, the flexible coupling, due to provision of flexible elements like bush or disk, can tolerate 0.5° of angular misalignment and 5 mm of axial displacement between the shafts.

(ii) The flexible elements provided in the flexible coupling absorb shocks and vibrations. There is no such provision in rigid coupling. It can be used only where the motion is free from shocks and vibrations.

(iii) Rigid coupling is simple and inexpensive. Flexible coupling is comparatively costlier due to additional parts.

In practice, misalignment always exists due to imperfect workmanship. Therefore, flexible couplings are more popular.

A good coupling, rigid or flexible, should satisfy the following requirements:

(i) The coupling should be capable of transmitting torque from the driving shaft to the driven shaft.

(ii) The coupling should keep the two shafts in proper alignment.

(iii) The coupling should be easy to assemble and disassemble for the purpose of repairs and alterations.

(iv) The failure of revolving bolt heads, nuts, key heads and other projecting parts may cause accidents. They should be covered by giving suitable shape to the flanges or by providing guards.

The couplings are standardized[7, 8] and can be purchased as readymade units.

[7] IS 6196–1971: Dimensions of fitted half coupling.

[8] IS 2693–1964: Specifications for cast iron flexible couplings.

9.17 MUFF COUPLING

Muff coupling is also called *sleeve coupling* or *box coupling*. It is a type of rigid coupling. The construction of the muff coupling is shown in Fig. 9.29. It consists of a sleeve or a hollow cylinder, which is fitted over the ends of input and output shafts by means of a sunk key. The torque is transmitted from the input shaft to the sleeve through the key. It is then transmitted from the sleeve to the output shaft through the key. Muff coupling has following advantages:

(i) It is the simplest form of coupling with only two parts, viz., sleeve and key. It is simple to design and manufacture.

(ii) It has no projecting parts except the key-head. The external surface of the sleeve is smooth. This is an advantage from the standpoint of safety to the operator.

(iii) It has compact construction with small radial dimensions.

(iv) It is cheaper than other types of coupling.

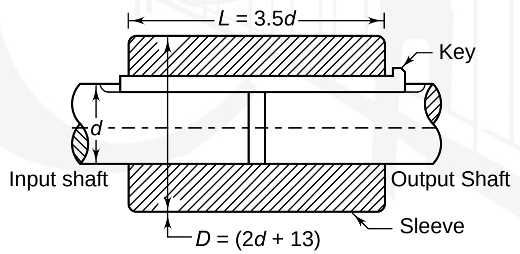


Fig. 9.29 *Muff Coupling*

Muff coupling has following disadvantages:

(i) Muff coupling is difficult to assemble or dismantle. The sleeve has to be either shifted over the shaft by half of its length or the ends of the shafts have to be drawn together or apart by half length of the sleeve.

(ii) It is a rigid type of coupling and requires accurate alignment of shafts. It cannot tolerate misalignment between the axes of two shafts. The misalignment of shafts, caused by inaccurate assembly, induces forces, which tend to bend the shafts.

(iii) Since there is no flexible element in the coupling, it cannot absorb shocks and vibrations. It can be used only where the motion is free from vibrations.

(iv) It requires more axial space compared with flange couplings.

Owing to these shortcomings, muff couplings are less popular compared with flange couplings. They are employed for shafts only up to 70 mm diameter.

Muff coupling is usually designed on shop floors by assuming standard proportions for the dimensions of the sleeve. These empirical relationships were developed by practicing engineers on the basis of their past experience. In general, such proportions result in robust design. For the sleeve of muff coupling, the standard proportions used in practice are as follows:

$$D = (2d + 13) \text{ mm} \tag{9.33}$$

and

$$L = 3.5\,d \tag{9.34}$$

where, D = outer diameter of the sleeve (mm)
L = axial length of the sleeve (mm)
d = diameter of the shaft (mm)

The diameter of the shaft, in the above relationships, is determined by the usual method. The free body diagram of torsional moments acting on various components of the muff coupling is shown in Fig. 9.30. The torsional shear stress in the sleeve is calculated by treating it as a hollow shaft subjected to torsional moment.

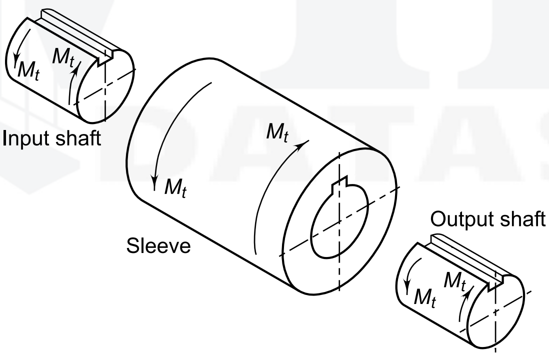


Fig. 9.30 *Free-body Diagram for Torques*

9.18 DESIGN PROCEDURE FOR MUFF COUPLING

The basic procedure for finding out the dimensions of the muff coupling consists of the following steps:

(i) Calculate the diameter of each shaft by the following equations:

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n} \quad \text{and} \quad \tau = \frac{16\, M_t}{\pi\, d^3}$$

(ii) Calculate the dimensions of the sleeve by the following empirical equations,

$$D = (2d + 13) \text{ mm} \quad \text{and} \quad L = 3.5\, d$$

Also, check the torsional shear stress induced in the sleeve by the following equations:

$$\tau = \frac{M_t r}{J} \quad J = \frac{\pi(D^4 - d^4)}{32} \quad r = \frac{D}{2}$$

(iii) Determine the standard cross-section of flat sunk key from Table 9.3. The length of the key in each shaft is one-half of the length of the sleeve. Therefore,

$$l = \frac{L}{2}$$

With these dimensions of the key, check the shear and compressive stresses in the key by Eqs (9.27) and (9.28) respectively.

$$\tau = \frac{2M_t}{dbl} \quad \text{and} \quad \sigma_c = \frac{4M_t}{dhl}$$

The shafts and key are made of plain carbon steel. The sleeve is usually made of grey cast iron of Grade FG 200.

Example 9.18 *Design a muff coupling to connect two steel shafts transmitting 25 kW power at 360 rpm. The shafts and key are made of plain carbon steel 30C8 ($S_{yt} = S_{yc} = 400$ N/mm^2). The sleeve is made of grey cast iron FG 200 ($S_{ut} = 200$ N/mm^2). The factor of safety for the shafts and key is 4. For the sleeve, the factor of safety is 6 based on ultimate strength.*

Solution

Given kW = 25 $n = 360$ rpm

For shafts and key, $S_{yt} = S_{yc} = 400$ N/mm^2 $(fs) = 4$

For sleeve, $S_{ut} = 200$ N/mm^2 $(fs) = 6$

Step I *Permissible stresses*

For the material of shafts and key,

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{400}{4} = 100 \text{ N/mm}^2$$

$$\sigma_c = \frac{S_{yc}}{(fs)} = \frac{400}{4} = 100 \text{ N/mm}^2$$

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.5\, S_{yt}}{(fs)} = \frac{0.5\,(400)}{4} = 50 \text{ N/mm}^2$$

For sleeve material,

$$\tau = \frac{S_{su}}{(fs)} = \frac{0.5\, S_{ut}}{(fs)} = \frac{0.5\,(200)}{6} = 16.67 \text{ N/mm}^2$$

Step II *Diameter of each shaft*

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n} = \frac{60 \times 10^6 (25)}{2\pi (360)}$$

$$= 663\,145.60 \text{ N-mm}$$

$$\tau = \frac{16\, M_t}{\pi\, d^3} \quad \text{or} \quad 50 = \frac{16(663\,145.60)}{\pi\, d^3}$$

$\therefore$ $d = 40.73$ or 45 mm

Step III *Dimensions of sleeve*

$$D = (2d + 13) = 2 \times 45 + 13 = 103 \text{ or } 105 \text{ mm}$$

$$L = 3.5d = 3.5\,(45) = 157.5 \text{ or } 160 \text{ mm}$$

The torsional shear stress in the sleeve is calculated by treating it as a hollow cylinder.

$$J = \frac{\pi(D^4 - d^4)}{32} = \frac{\pi(105^4 - 45^4)}{32}$$

$$= 11\,530\,626.79 \text{ mm}^4$$

$$r = \frac{D}{2} = \frac{105}{2} = 52.5 \text{ mm}$$

$$\tau = \frac{M_t r}{J} = \frac{(663\,145.60)\,(52.5)}{(11\,530\,626.79)}$$

$$= 3.02 \text{ N/mm}^2$$

$\therefore$ $\tau < 16.67$ N/mm^2

Step IV *Dimensions of key*

From Table 9.3, the standard cross-section of flat sunk key for a 45 mm diameter shaft is 14 × 9 mm. The length of key in each shaft is one-half of the length of sleeve. Therefore,

$$l = \frac{L}{2} = \frac{160}{2} = 80 \text{ mm}$$

The dimensions of the key are 14 × 9 × 80 mm.

Step V *Check for stresses in key*

From Eq. (9.27),

$$\tau = \frac{2\,M_t}{d\,b\,l} = \frac{2(663\,145.60)}{(45)(14)(80)} = 26.32 \text{ N/mm}^2$$

$\therefore \quad \tau < 50 \text{ N/mm}^2$

From Eq. (9.28),

$$\sigma_c = \frac{4\,M_t}{dh\,l} = \frac{4(663\,145.60)}{(45)(9)(80)} = 81.87 \text{ N/mm}^2$$

$\therefore \quad \sigma_c < 100 \text{ N/mm}^2$

The design of the key is safe from shear and compression considerations.

9.19 CLAMP COUPLING

The clamp coupling is also called *compression coupling* or *split muff coupling*. It is a rigid type of coupling. In this coupling, the sleeve is made of two halves, which are split along a plane passing through the axes of shafts. The construction of the clamp coupling is shown in Fig. 9.31. The two halves of the sleeve are clamped together by means of bolts. The number of bolts can be four or eight. They are always in multiples of four. The bolts are placed in recesses formed in the sleeve halves. A small clearance is provided in the parting plane between the two halves. Therefore, when the bolts are tightened, a force is exerted between the sleeve halves and the shaft. This force caused by clamping of bolts creates frictional force on the surface of the shaft. The torque is transmitted by means of frictional force on the surface of the shaft. There is also a key between the shafts and sleeve, which also transmits torque. It is not possible to find out what percentage of torque is transmitted by friction or by the key. In design, it is assumed that total torque is transmitted by friction as well as by the key. In other words, the key is designed for total torque. Also, the clamping bolts are designed for total torque. As shown in Fig. 9.31, power is transmitted from the input shaft to the sleeve and from the sleeve to the output shaft by means of the key and friction between the sleeve halves and the shaft. The clamp coupling has the following advantages:

(i) It is easy to assemble and dismantle.

(ii) It can be easily removed without shifting the shaft in axial direction, unlike solid muff coupling.

(iii) As compared with flange coupling, clamp coupling has small diametral dimensions.

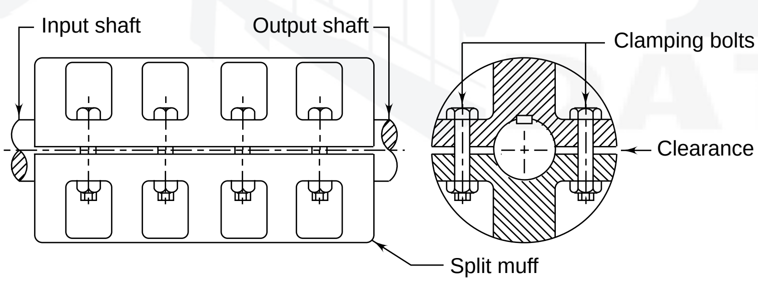


Fig. 9.31 *Split Muff Coupling*

The disadvantages of clamp coupling are as follows:

(i) There is difficulty in dynamic balancing of the coupling. Therefore, it is not possible to use the clamp coupling for high-speed applications.

(ii) Clamp coupling is unsuitable for shock loads.

(iii) It is necessary to provide a guard for the coupling to comply with the factory regulation act.

The main application of clamp coupling is for line shaft in power transmission. Nowadays, the line shaft and the clamp coupling have become obsolete.

There is a fundamental difference between the working of muff and clamp couplings. In muff coupling, torque is transmitted by shear resistance of keys. On the other hand, torque is transmitted partly by means of friction between the sleeve halves and the shaft and partly by shear resistance of key in case of clamp coupling.

Clamp coupling is usually designed on the basis of standard proportions for sleeve halves and clamping bolts.

For sleeve halves,

$$D = 2.5d \tag{9.35}$$

$$L = 3.5d \tag{9.36}$$

where

D = outer diameter of sleeve halves (mm)
L = length of sleeve (mm)
d = diameter of shaft (mm)

For clamping bolts,

$$d_1 = 0.2d + 10 \text{ mm} \tag{9.37}$$

when $d < 55$ mm

and

$$d_1 = 0.15d + 15 \text{ mm} \tag{9.38}$$

when $d > 55$ mm

where,

d_1 = diameter of clamping bolt (mm)

Alternatively, the diameter of the clamping bolts can be calculated from the first principle. Let us assume that even with key, the torque is transmitted only by the friction between the shaft and the coupling halves.

The clamping force of each bolt is given by,

$$P_1 = \frac{\pi}{4} d_1^2 \sigma_t \tag{9.39}$$

where

P_1 = tensile force on each bolt (N)
d_1 = core diameter of clamping bolt (mm)
σ_t = permissible tensile stress (N/mm^2)

It is assumed that half the number of bolts give clamping pressure on input shaft and the remaining half on the output shaft. Therefore, clamping force on each shaft is given by,

$$N = \frac{P_1 n}{2} \tag{a}$$

where,

n = total number of bolts
N = clamping force on each shaft (N)

As shown in Fig. 9.32, the frictional force is (fN) and frictional torque is given by,

$$M_t = fN\left(\frac{d}{2}\right) + fN\left(\frac{d}{2}\right) = fNd \tag{b}$$

where,

f = coefficient of friction

From expression (a) and (b),

$$M_t = \frac{fd P_1 n}{2}$$

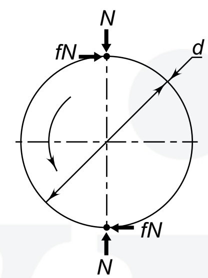


Fig. 9.32 *Forces on Shaft*

or,
$$P_1 = \frac{2M_t}{fdn} \tag{9.40}$$

9.20 DESIGN PROCEDURE FOR CLAMP COUPLING

The basic procedure for finding out the dimensions of clamp coupling consists of the following steps:

(i) Calculate the diameter of each shaft by the following equations:

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n} \quad \text{and} \quad \tau = \frac{16M_t}{\pi d^3}$$

The shaft, key and clamping bolts are usually made of plain carbon steel.

(ii) Calculate the main dimensions of the sleeve halves by using the following empirical equations:

$$D = 2.5\,d \quad \text{and} \quad L = 3.5\,d$$

The sleeve halves are made of grey cast iron of Grade FG 200.

(iii) Determine the standard cross-section of the flat key from Table 9.3. The length of the key in each shaft is one-half of the length of sleeve. Therefore,

$$l = \frac{L}{2}$$

With these dimensions of the key, check the shear and compressive stresses in the key by Eqs (9.27) and (9.28) respectively.

$$\tau = \frac{2M_t}{dbl} \quad \text{and} \quad \sigma_c = \frac{4M_t}{dhl}$$

(iv) Calculate the diameter of clamping bolts by using Eqs (9.40) and (9.39).

$$P_1 = \frac{2M_t}{fdn} \quad \text{and} \quad P_1 = \frac{\pi}{4} d_1^2 \sigma_t$$

Alternatively, standard proportions can be used for calculating the bolt diameter. Equations (9.37) and (9.38) are used for this purpose. The coefficient of friction between the sleeve halves and the shaft is usually taken as 0.3.

Example 9.19 *It is required to design a split muff coupling to transmit 50 kW power at 120 rpm The shafts, key and clamping bolts are made of plain carbon steel 30C8 (S_{yt} = 400 N/mm^2). The yield strength in compression is 150% of the tensile yield strength. The factor of safety for shafts, key and bolts is 5. The number of clamping bolts is 8. The coefficient of friction between sleeve halves and the shaft is 0.3.*

(i) *Calculate the diameter of the input and output shafts.*
(ii) *Specify the length and outer diameter of the sleeve halves.*
(iii) *Find out the diameter of clamping bolts assuming that the power is transmitted by friction.*
(iv) *Specify bolt diameter using standard empirical relations.*
(v) *Specify the size of key and check the dimensions for shear and compression criteria.*

Solution

Given kW = 50 n = 120 rpm μ = 0.3
For shafts, key and bolts S_{yt} = 400 N/mm^2
(fs) = 5 n = 8 $S_{yc} = 1.5\, S_{yt}$

Step I *Permissible stresses*

For the material of shafts, key and bolt,

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{400}{5} = 80 \text{ N/mm}^2$$

$$\sigma_c = \frac{S_{yc}}{(fs)} = \frac{1.5\,(400)}{5} = 120 \text{ N/mm}^2$$

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.5\,S_{yt}}{(fs)} = \frac{0.5\,(400)}{5} = 40 \text{ N/mm}^2$$

Step II *Diameter of each shaft*

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n}$$

$$= \frac{60 \times 10^6 (50)}{2\pi (120)} = 3\,978\,873.58 \text{ N-mm}$$

$$\tau = \frac{16 M_t}{\pi d^3} \quad \therefore\ 40 = \frac{16(3\,978\,873.58)}{\pi d^3}$$

$$\therefore \quad d = 79.72 \text{ or } 80 \text{ mm} \qquad \text{(i)}$$

Step III *Length and outer diameter of sleeve halves*

From Eqs (9.35) and (9.36),

$$D = 2.5\,d = 2.5\,(80) = 200 \text{ mm}$$
$$L = 3.5\,d = 3.5\,(80) = 280 \text{ mm} \qquad \text{(ii)}$$

Step IV *Diameter of clamping bolts – Friction basis*

From Eq. (9.40),

$$P_1 = \frac{2M_t}{fdn} = \frac{2\,(3\,978\,873.58)}{0.3\,(80)\,(8)} = 41\,446.6 \text{ N}$$

From Eq. (9.39),

$$P_1 = \frac{\pi}{4} d_1^{\,2} \sigma_t \quad \text{or} \quad 41\,446.6 = \frac{\pi}{4} d_1^{\,2}\,(80)$$

$$\therefore \quad d_1 = 25.68 \quad \text{or} \quad 26 \text{ mm} \qquad \text{(iii)}$$

Step V *Diameter of clamping bolts – Empirical relation*

From Eq. (9.38),

$$d_1 = 0.15d + 15 = 0.15\,(80) + 15 = 27 \text{ mm} \qquad \text{(iv)}$$

Step VI *Dimensions of key*

From Table 9.3, the standard cross-section of flat key for an 80 mm diameter shaft is 22 × 14 mm. The length of the key in each shaft is one-half of the length of the sleeve half.

Therefore,

$$l = \frac{L}{2} = \frac{280}{2} = 140 \text{ mm}$$

The dimensions of the key are,

$$22 \times 14 \times 140 \text{ mm} \tag{v}$$

Step VII *Check for stresses in key*

From Eq. (9.27),

$$\tau = \frac{2M_t}{d\,b\,l} = \frac{2\,(3\,978\,873.58)}{80\,(22)(140)}$$

$$= 32.3 \text{ N/mm}^2 \quad \therefore \tau < 40 \text{ N/mm}^2$$

From Eq. (9.28),

$$\sigma_c = \frac{4M_t}{dhl} = \frac{4(3\,978\,873.58)}{80(14)(140)}$$

$$= 101.5 \text{ N/mm}^2 \quad \therefore \sigma_c < 120 \text{ N/mm}^2$$

The design of key is safe from shear and compression considerations.

9.21 RIGID FLANGE COUPLINGS

A flange coupling consists of two flanges—one keyed to the driving shaft and the other to the driven shaft as shown in Fig. 9.33. The two flanges are connected together by means of four or six bolts arranged on a circle concentric with the axes of the shafts. Power is transmitted from the driving

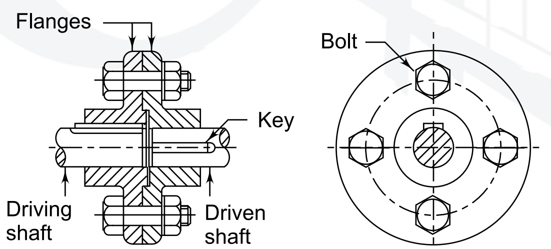


Fig. 9.33 *Unprotected Type Flange Coupling*

shaft to the left side flange through the key. It is then transmitted from the left side flange to the right side flange through the bolts. Finally, power is transmitted from the right side flange to the driven shaft through the key. Since flange coupling is rigid type of coupling, provision should be made for precise location of the axes of two shafts. The mating flanges have spigot and recess for precise location. The left side flange has a cylindrical projection called *spigot* while the right side flange has a corresponding *recess*. The diameters of the spigot and the recess are machined with more accuracy. During the assembly, the spigot fits into the recess and the two flanges are located precisely with respect to each other. This ensures alignment of the axes of the two shafts.

There are two types of rigid flange couplings—unprotected and protected. The flange coupling shown in Fig. 9.33 is the unprotected type of coupling. The revolving bolt heads and nuts are dangerous to the operator and may lead to accident. Protected type flange coupling is shown in Fig. 9.34. In this case, protecting circumferential rims cover the bolt heads and nuts. In case of failure of bolts while the machine is being run, the broken pieces will dash against this rim and eventually fall down. This protects the operator against injuries.

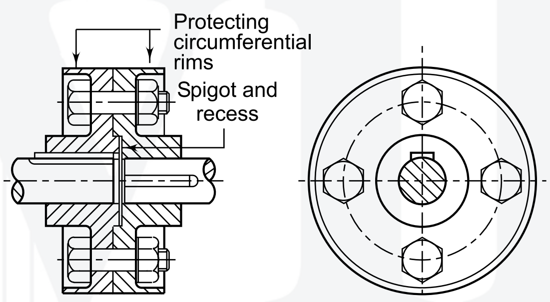


Fig. 9.34 *Protected Type Rigid Coupling*

The rigid flange couplings have the following advantages:

(i) Rigid coupling has high torque transmitting capacity.

(ii) Rigid coupling is easy to assemble and dismantle.

(iii) Rigid coupling has simple construction. It is easy to design and manufacture.

The rigid flange couplings have the following disadvantages:

(i) It is a rigid type of coupling. It cannot tolerate misalignment between the axes of two shafts.

(ii) It can be used only where the motion is free from shocks and vibrations.

(iii) It requires more radial space.

Rigid flange couplings are widely used for transmitting large torques.

The flange of the protected type coupling has three distinct regions—inner hub, central flange with boltholes and peripheral outer rim as shown in Fig. 9.35. The hub is provided with a keyway. The function of the hub is to transmit the torque from the shaft to the central flange and vice versa. The central portion of the flange has holes to accommodate the bolts. Torque is transmitted from one flange to the other by means of these bolts. The outer circumferential rim is for the purpose of safety to cover the projecting bolt heads and nuts. Various dimensions of flanges are shown in Fig. 9.35. Many times, the dimensions of the flanges are calculated by using standard proportions in terms of shaft diameter. Shop-floor engineers have used such empirical formulae for many years without any problem. It is easy to design the coupling using these standard proportions, because no stress analysis is involved. The dimensions calculated by these formulae result in robust design. The standard proportions for various dimensions of the flange shown in Fig. 9.35 are as follows:

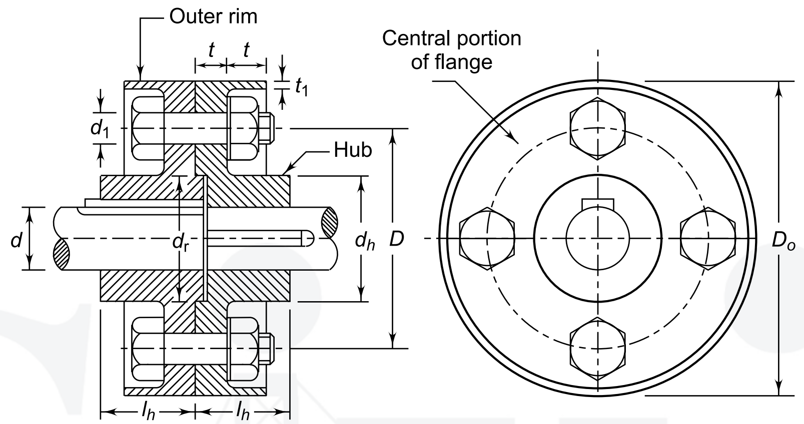


Fig. 9.35 *Proportions of Rigid Coupling*

(i) d_h = outside diameter of hub

$$d_h = 2d$$

(ii) l_h = length of hub or effective length of key

$$l_h = 1.5\ d$$

(iii) D = pitch circle diameter of bolts

$$D = 3d$$

(iv) t = thickness of flanges

$$t = 0.5\ d$$

(v) t_1 = thickness of protecting rim

$$t_1 = 0.25\ d$$

(vi) d_r = diameter of spigot and recess

$$d_r = 1.5\ d$$

(vii) D_0 = outside diameter of flange

$$D_0 = (4d + 2t_1)$$

In the above relationships, d is the shaft diameter.

The number of bolts (N) is also decided from the shaft diameter in the following way:

$N = 3$ for shafts up to 40 mm diameter
$N = 4$ for shafts from 40 to 100 mm diameter
$N = 6$ for shafts from 100 to 180 mm diameter.

The analysis of rigid coupling can be done by two different ways, depending upon the clearance between the bolt and the hole. In the first approach, the bolts are fitted in reamed and ground holes. In this case, there is no clearance and the bolts are finger tight. Therefore, power is transmitted by means of shear resistance of the bolts. In the second approach, the bolts are fitted in large clearance holes. In this case, bolts are tightened with a pre-load and power is transmitted by means of friction between the two flanges.

Case I Bolts Fitted in Reamed and Ground Holes The forces acting on individual bolts due to transmission of the torque are shown in Fig. 9.36. Equating the external torque with the resisting torque,

$$M_t = P \times \frac{D}{2} \times N \qquad \text{(a)}$$

where,

M_t = torque transmitted by the coupling (N-mm)
P = force acting on each bolt (N)
D = pitch circle diameter of bolts (mm)
N = number of bolts.

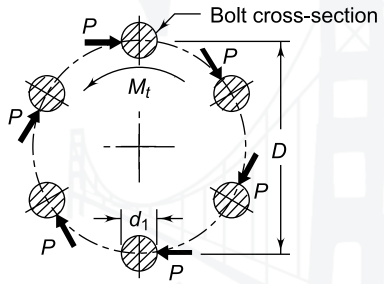


Fig. 9.36 *Shear Resistance of Bolts*

It should be noted that the bolts are subjected to direct shear stress due to the force P and not torsional shear stress. No torque is acting about the axis of the bolt. The force P results in only direct shear stress. The direct shear stress in the bolt is given by,

$$\tau = \frac{P}{\left(\frac{\pi}{4} d_1^2\right)} \qquad \text{(b)}$$

where,

τ = shear stress in the bolt (N/mm^2)
d_1 = nominal diameter of the bolt (mm)
From (a) and (b),

$$\tau = \frac{8M_t}{\pi D N d_1^2} \qquad (9.41)$$

Equation (9.41) is used to determine the nominal diameter of the bolts. The above analysis of the coupling is based on the assumption that the bolts are fitted in rimmed and ground holes. The bolts are finger tight in these holes and there is no clearance between the holes and the nominal diameter of the bolts.

Case II Bolts Fitted in Large Clearance Holes When the bolts are fitted in large clearance holes, the above analysis is not applicable. In this case, the bolts are sufficiently tightened with a pre-load and the torque is transmitted from one flange to the other by means of friction between them. For uniformly distributed pressure, the friction radius R_f is given by,

$$R_f = \frac{2}{3}\frac{(R_o^3 - R_i^3)}{(R_o^2 - R_i^2)} \qquad (9.42)$$

where, (Fig. 9.35)

R_o = outer radius of the flange (D_o /2) (mm)
R_i = radius of the recess (d_r /2) (mm)
Assume that,
P_i = initial tension in each bolt (N)
μ = coefficient of friction between flanges

The friction force will be ($\mu P_i N$) and the torque is given by,

$$M_t = \mu P_i N R_f \qquad (9.43)$$

Flanges have complex shape and the easiest method to make the flanges is casting. Flanges are usually made of grey cast iron of grade FG 200. The bolts, keys and shaft are made of plain carbon steels on strength criterion.

9.22 DESIGN PROCEDURE FOR RIGID FLANGE COUPLING

The basic procedure for finding out the dimensions of the rigid flange coupling consists of the following steps:

(i) Shaft Diameter Calculate the shaft diameter by using the following two equations:

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n}$$

and
$$\tau = \frac{16 M_t}{\pi d^3}$$

(ii) Dimensions of Flanges Calculate the dimensions of the flanges by the following empirical equations:

$$d_h = 2d$$
$$l_h = 1.5d$$
$$D = 3d$$
$$t = 0.5d$$

$$t_1 = 0.25d$$
$$d_r = 1.5d$$
$$D_0 = (4d + 2t_1)$$

The torsional shear stress in the hub can be calculated by considering it as a hollow shaft subjected to torsional moment M_t.

The inner and outer diameters of the hub are d and d_h respectively. The torsional shear stress in the hub is given by,

$$\tau = \frac{M_t r}{J}$$

$$J = \frac{\pi (d_h^4 - d^4)}{32} \tag{9.44}$$

$$r = \frac{d_h}{2}$$

The flange at the junction of the hub is under shear while transmitting the torsional moment M_t. From Fig. 9.35,

area under shear = $(\pi d_h) \times t$

shear force = area × shear stress = $\pi d_h t \tau$

resisting torque = shear force × $\left(\frac{d_h}{2}\right)$

$$\therefore \quad M_t = (\pi d_h t \tau) \times \frac{d_h}{2}$$

or

$$M_t = \frac{1}{2} \pi d_h^2 t \tau \tag{9.45}$$

(iii) Diameter of Bolts Decide the number of bolts using the following guidelines:

$N = 3$ for $d < 40$ mm

$N = 4$ for $40 \leq d < 100$ mm

$N = 6$ for $100 \leq d < 180$ mm

Determine the diameter of the bolt by Eq. (9.41). Rearranging the equation,

$$d_1^2 = \frac{8M_t}{\pi D N \tau} \tag{9.46}$$

where τ is the permissible shear stress for the bolt material.

The compressive stress in the bolt can be determined by referring to Fig. 9.35 again.

crushing area of each bolt = $d_1 t$

crushing area of all bolts = $N d_1 t$

compressive force = $N d_1 t \sigma_c$

torque = $(N d_1 t \sigma_c) \times \frac{D}{2}$

or

$$\sigma_c = \frac{2M_t}{N d_1 t D} \tag{9.47}$$

Equation (9.47) is used to check the compressive stress in the bolt.

(iv) Dimensions of Keys Determine the standard cross-section of flat key from Table 9.3. The length of the key in each shaft is l_h. Therefore,

$$l = l_h$$

With these dimensions of the key, check the shear and compressive stresses in the key by Eqs (9.27) and (9.28) respectively.

$$\tau = \frac{2M_t}{dbl}$$

and

$$\sigma_c = \frac{4M_t}{dhl}$$

Example 9.20 *A rigid coupling is used to transmit 20 kW power at 720 rpm. There are four bolts and the pitch circle diameter of the bolts is 125 mm. The bolts are made of steel 45C8 (S_{yt} = 380 N/mm²) and the factor of safety is 3. Determine the diameter of the bolts.*

Assume that the bolts are finger tight in reamed and ground holes.

Solution

Given kW = 20 n = 720 rpm

For bolts, $S_{yt} = 380$ N/mm² $(fs) = 3$

$D = 125$ mm $N = 4$

Step I *Permissible shear stress*

$$S_{sy} = 0.577\, S_{yt} = 0.577\,(380) = 219.26 \text{ N/mm}^2$$

$$\tau = \frac{S_{sy}}{(fs)} = \frac{219.26}{3} = 73.09 \text{ N/mm}^2$$

Step II *Diameter of bolts*

The torque transmitted by the shaft is given by,

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n} = \frac{60 \times 10^6 (20)}{2\pi (720)}$$

$$= 265\,258.23 \text{ N-mm}$$

From Eq. (9.46),

$$d_1^2 = \frac{8\,M_t}{\pi D N \tau} = \frac{8(265\,258.23)}{\pi(125)(4)(73.09)}$$

$$\therefore \quad d_1 = 4.3 \text{ mm}$$

Example 9.21 *A rigid coupling is used to transmit 50 kW power at 300 rpm. There are six bolts. The outer diameter of the flanges is 200 mm, while the recess diameter is 150 mm. The coefficient of friction between the flanges is 0.15. The bolts are made of steel 45C8 (S_{yt} = 380 N/mm^2) and the factor of safety is 3. Determine the diameter of the bolts. Assume that the bolts are fitted in large clearance holes.*

Solution

Given kW = 50 n = 300 rpm μ = 0.15
For bolts, S_{yt} = 380 N/mm^2 (fs) = 3 N = 6
For flanges, D_o = 200 mm D_i = 150 mm

Step I *Permissible tensile stress*

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{380}{3} = 126.67 \text{ N/mm}^2$$

Step II *Preload in bolts*

The torque transmitted by the shaft is given by,

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n} = \frac{60 \times 10^6 (50)}{2\pi (300)}$$
$$= 1\,591\,549.4 \text{ N-mm}$$

$$R_f = \frac{2}{3}\frac{(R_o^3 - R_i^3)}{(R_o^2 - R_i^2)} = \frac{2}{3}\frac{(100^3 - 75^3)}{(100^2 - 75^2)} = 88.1 \text{ mm}$$

From Eq. (9.43),

$$P_i = \frac{M_t}{\mu N R_f} = \frac{1\,591\,549.4}{0.15(6)(88.1)} = 20\,072.51 \text{ N}$$

Step III *Diameter of bolts*

Due to pre-load of 20 072.51 N, the bolts are subjected to tensile stresses.

$$P_i = \left(\frac{\pi}{4}\right) d_1^2 \sigma_t$$

or
$$d_1^2 = \frac{4P_i}{\pi\sigma_t} = \frac{4(20\,072.51)}{\pi(126.67)}$$

$\therefore$
$$d_1 = 14.2 \text{ mm}$$

Example 9.22 *It is required to design a rigid type of flange coupling to connect two shafts. The input shaft transmits 37.5 kW power at 180 rpm to the output shaft through the coupling. The service factor for the application is 1.5, i.e., the design torque is 1.5 times of the rated torque. Select suitable materials for various parts of the coupling, design the coupling and specify the dimensions of its components.*

Solution

Given kW = 37.5 n = 180 rpm
design torque = 1.5 (rated torque)

Step I *Selection of materials*

(i) The shafts are subjected to torsional shear stress. On the basis of strength, plain carbon steel of grade 40C8 (S_{yt} = 380 N/mm^2) is used for the shaft. The factor of safety for the shafts is assumed to be 2.5.

(ii) The keys and bolts are subjected to shear and compressive stresses. On the basis of strength criterion, plain carbon steel of grade 30C8 (S_{yt} = 400 N/mm^2) is selected for the keys and the bolts. It is assumed that the compressive yield strength is 150% of the tensile yield strength. The factor of safety for the keys and the bolts is taken as 2.5.

(iii) Flanges have complex shape and the easiest method to make the flanges is casting. Grey cast iron FG 200 (S_{ut} = 200 N/mm^2) is selected as the material for the flanges from manufacturing considerations. It is assumed that ultimate shear strength is one half of the ultimate tensile strength. The factor of safety for the flanges is assumed as 6, since the permissible stress is based on the ultimate strength and not on the yield strength.

Step II *Permissible stresses*

(i) Shaft

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.5\,S_{yt}}{(fs)} = \frac{0.5\,(380)}{(2.5)} = 76 \text{ N/mm}^2$$

(ii) Keys and bolts

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.5\,S_{yt}}{(fs)} = \frac{0.5\,(400)}{(2.5)} = 80 \text{ N/mm}^2$$

$$\sigma_c = \frac{S_{yc}}{(fs)} = \frac{1.5\,S_{yt}}{(fs)} = \frac{1.5\,(400)}{(2.5)} = 240 \text{ N/mm}^2$$

(3) Flanges

$$\tau = \frac{S_{su}}{(fs)} = \frac{0.5\,S_{ut}}{(fs)} = \frac{0.5\,(200)}{(6)} = 16.67\text{ N/mm}^2$$

Step III *Diameter of shafts*

Taking into consideration the service factor of 1.5, the design torque is given by,

$$M_t = \frac{60\times 10^6\,(\text{kW})}{2\pi n}\times(1.5)$$

$$= \frac{60\times 10^6\,(37.5)\,(1.5)}{2\pi(180)}$$

$$= 2\,984\,155.18\text{ N-mm}$$

$$\tau = \frac{16M_t}{\pi d^3} \quad\text{or}\quad 76 = \frac{16\,(2\,984\,155.18)}{\pi d^3}$$

$\therefore \quad d = 58.48$ or 60 mm

Step IV *Dimensions of flanges*

The dimensions of the flanges are as follows,

$d_h = 2d = 2\,(60) = 120$ mm

$l_h = 1.5d = 1.5\,(60) = 90$ mm

$D = 3d = 3(60) = 180$ mm

$t = 0.5d = 0.5(60) = 30$ mm

$t_1 = 0.25d = 0.25(60) = 15$ mm

$d_r = 1.5d = 1.5(60) = 90$ mm

$D_0 = (4d + 2t_1) = 4(60) + 2(15) = 270$ mm

The above dimensions of the flange are shown in Fig. 9.37. The thickness of recess is assumed as 5 mm. The hub is treated as a hollow shaft subjected to torsional moment. From Eq. (9.44),

$$J = \frac{\pi(d_h^4 - d^4)}{32} = \frac{\pi(120^4 - 60^4)}{32}$$

$$= 19\,085\,175.37\text{ mm}^4$$

$$r = \frac{d_h}{2} = \frac{120}{2} = 60\text{ mm}$$

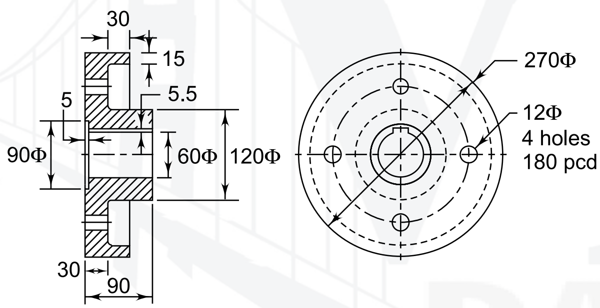


Fig. 9.37 *Dimensions of Flange with Recess*

The torsional shear stress in the hub is given by,

$$\tau = \frac{M_t r}{J} = \frac{(2\,984\,155.18)\,(60)}{(19\,085\,175.37)} = 9.38\text{ N/mm}^2$$

$\therefore \quad \tau < 16.67$ N/mm^2

The shear stress in the flange at the junction of the hub is determined by Eq. (9.45).

$$\tau = \frac{2M_t}{\pi d_h^2\, t} = \frac{2(2\,984\,155.18)}{\pi(120)^2\,(30)}$$

$$= 4.40\text{ N/mm}^2$$

$\therefore \quad \tau < 16.67$ N/mm^2

The stresses in the flange are within limits.

Step V *Diameter of bolts*

The diameter of the shaft is 60 mm.

$\therefore \quad 40 < d < 100$ mm

The number of bolts is 4.

From Eq. (9.46),

$$d_1^2 = \frac{8M_t}{\pi D N \tau} = \frac{8\,(2\,984\,155.18)}{\pi\,(180)\,(4)\,(80)}$$

$\therefore \quad d_1 = 11.49$ or 12 mm

The compressive stress in the bolt is determined by Eq. (9.47).

$$\sigma_c = \frac{2M_t}{N d_1 t D} = \frac{2(2\,984\,155.18)}{(4)(12)(30)(180)}$$

$$= 23.03\text{ N/mm}^2$$

$\therefore \quad \sigma_c < 240 \text{ N/mm}^2$

Step VI *Dimensions of keys*
From Table 9.3, the standard cross-section of the flat key for a 60-mm diameter shaft is 18 × 11 mm. The length of the key is equal to l_h. Or,

$$l = l_h = 90 \text{ mm}$$

The dimensions of the flat key are 18 × 11 × 90 mm.

From Eq. (9.27),

$$\tau = \frac{2M_t}{dbl} = \frac{2(2\,984\,155.18)}{(60)(18)(90)} = 61.4 \text{ N/mm}^2$$

$\therefore \quad \tau < 80 \text{ N/mm}^2$

From Eq. (9.28),

$$\sigma_c = \frac{4M_t}{dhl} = \frac{4(2\,984\,155.18)}{(60)(11)(90)}$$

$$= 200.95 \text{ N/mm}^2$$

$$\sigma_c < 240 \text{ N/mm}^2$$

The shear and compressive stresses induced in the key are within permissible limits.

9.23 BUSHED-PIN FLEXIBLE COUPLING

Rigid coupling can be used only when there is perfect alignment between the axes of two shafts and the motion is free from vibrations and shocks. In practice, it is impossible to obtain perfect alignment of shafts. Misalignment exists due to the following reasons:

(i) deflection of shafts due to lateral forces;
(ii) error in shaft mounting due to manufacturing tolerances;
(iii) use of two separately manufactured units such as an electric motor and a worm gear box; and
(iv) thermal expansion of the parts.

If rigid coupling is used in such circumstances, the misalignment causes excessive bearing reactions resulting in vibrations and wear. To overcome this problem, flexible couplings are used. A flexible coupling employs a flexible element like a rubber bush between the driving and the driven flanges. This flexible rubber bush not only accommodates the misalignment but also absorbs shocks and vibrations. The basic types of misalignment between axes of the input and output shafts are shown in Fig. 9.38. A flexible coupling can tolerate 0.5 mm of lateral or axial misalignment and 1.5° of angular misalignment.

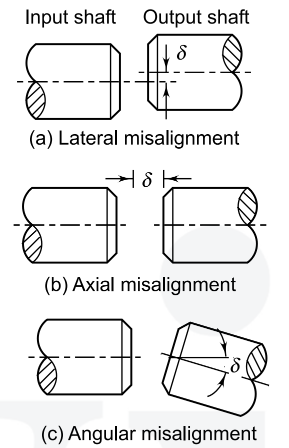


Fig. 9.38 *Types of Misalignment*

The construction of the flexible coupling is shown in Fig. 9.39. It is similar to the rigid type of flange coupling except for the provision of rubber bush and pins in place of bolts. The coupling consists of two flanges, one keyed to the input shaft and the other to the output shaft. The two flanges are connected together by means of four or six pins. At one end, the pin is fixed to the output flange by means of a nut. The diameter of the pin is enlarged in the input flange where a rubber bush is mounted over the pin. The rubber bush is provided with brass lining at the inner surface. The lining reduces the wear of the inner surface of the rubber bush. Power is transmitted from the input shaft to the input flange through the key. It is then transmitted from the input flange to the pin through the rubber bush. The pin then transmits the power to the output flange by shear resistance. Finally, power is transmitted from the output flange to the output shaft through the key.

The bushed-pin type flexible coupling has following advantages:

(i) It can tolerate 0.5 mm of lateral or axial misalignment and 1.5° of angular misalignment.

(ii) It prevents transmission of shock from one shaft to the other and absorbs vibrations.

(iii) It can be used for transmitting high torques.

(iv) It is simple in construction and easy to assemble and dismantle. It is easy to design and manufacture the coupling.

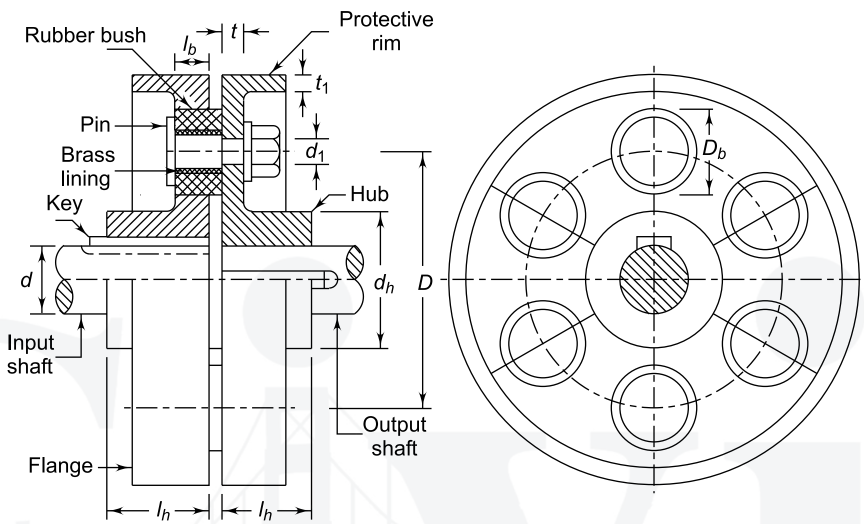


Fig. 9.39 *Flexible Coupling*

The disadvantages of bushed-pin type flexible coupling are as follows:

(i) The cost of flexible coupling is more than that of rigid coupling due to additional parts.

(ii) It requires more radial space compared with other types of coupling.

Flexible bushed-pin type couplings are extensively used in practice due to their advantages. The dimensions of the flanges of flexible bushed-pin coupling are calculated by using standard proportions in terms of shaft diameter. There is a basic difference between the flanges of rigid and flexible couplings. In rigid coupling, two flanges are identical except for the provision of spigot and recess. In flexible coupling, the input flange accommodates the rubber bushes of comparatively large diameter than the diameter of the pins accommodated in the output flange as shown in Fig. 9.39. Therefore, the diameter of holes and the thickness of the two flanges are different. The standard proportions for various dimensions of the flanges shown in Fig. 9.39 are as follows:

(i) d_h = outside diameter of hub

$d_h = 2d$

(ii) l_h = length of hub or effective length of key

$l_h = 1.5\ d$

(iii) D = pitch circle diameter of pins

$D = 3d$ to $4d$

(iv) t = thickness of output flange

$t = 0.5\ d$

(v) t_1 = thickness of protective rim

$t_1 = 0.25\ d$

(vi) d_1 = diameter of pin

$$d_1 = \frac{0.5d}{\sqrt{N}}$$

where N is the number of pins and d is the shaft diameter.

Other dimensions of flanges, viz., outer diameter or thickness of the input flange are calculated after deciding the dimensions of the rubber bushes.

In analysis of flexible coupling, it is assumed that the power is transmitted by the shear resistance of the pins. As the flange on the input shaft rotates, it exerts a force P on each rubber bush. The resisting forces on the rubber bushes are shown in Fig. 9.40. Equating the couples due to resisting force with the torque,

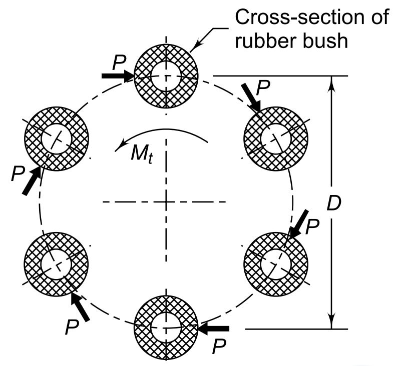


Fig. 9.40 *Resisting Forces in Rubber Bushes*

$$M_t = P \times \frac{D}{2} \times N \tag{a}$$

where,

M_t = torque transmitted by the coupling (N-mm)
P = force acting on each rubber bush or pin (N)
D = pitch circle diameter of bushes or pins (mm)
N = number of bushes or pins

The projected area of the rubber bush is shown in Fig. 9.41. The force P is equal to the product of the projected area and the intensity of pressure. Therefore,

$$P = (D_b\, l_b) \times p_m \tag{b}$$

where,

D_b = outer diameter of the bush (mm)
l_b = effective length of the bush in contact with the input flange (mm)
p_m = permissible intensity of pressure between the flange and the rubber bush (N/mm²)

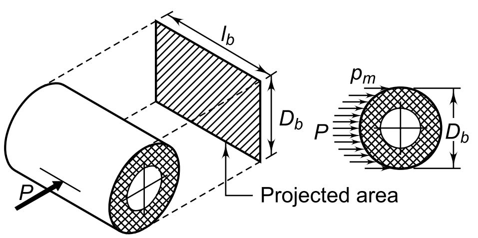


Fig. 9.41 *Projected Area of Rubber Bush*

From, (a) and (b),

$$M_t = \frac{1}{2}\, p_m\, D_b\, l_b\, DN \tag{9.48}$$

The permissible intensity of pressure between the rubber bush and the cast iron flange is usually 1 N/mm². The ratio of length to the outer diameter for the rubber bush is usually assumed as 1.

Therefore,

$$\frac{l_b}{D_b} = 1 \tag{9.49}$$

and $\quad p_m = 1 \text{ N/mm}^2$

Substituting the above relationship in Eq. (9.48),

$$M_t = \frac{1}{2} D_b^2\, DN \tag{9.50}$$

The outer diameter of the rubber bush is obtained by the above equation.

The pin is subjected to direct shear stress due to the force P. Refer to Fig. 9.39 again. The direct shear stress in the pin is given by,

$$\tau = \frac{P}{\left(\frac{\pi}{4} d_1^2\right)} \tag{c}$$

From (a) and (c),

$$\tau = \frac{8 M_t}{\pi d_1^2 DN} \tag{9.51}$$

According to Indian standard[9], the allowable shear stress for pins is 35 N/mm².

The pin is also subjected to bending moment and the calculations of bending stresses are explained in the examples.

The flanges of flexible bush coupling are made of grey cast iron of grade FG 200. The pins are made of carbon steel. The recommended type of fit between the shaft and the hub is H7-j7. The maximum allowable peripheral speed of the coupling is 30 m/s.

There are two important features of flexible bush coupling, which are different than rigid flange coupling. They are as follows:

(i) As shown in Fig. 9.39, there is a gap or clearance between the driving and driven flanges of flexible bush coupling. This gap is essential for taking care of angular misalignment between the two shafts. There is no such clearance between the flanges of rigid coupling. Therefore, rigid coupling cannot tolerate any angular misalignment.

[9] IS 2693–1980: Specification for bush-type flexible coupling for power transmission.

(ii) In case of rigid coupling, the torque is transmitted by means of bolts. These bolts are made of steel and resisting shear or tensile stresses are high. Therefore, the diameter of the bolts or the pitch circle diameter of bolts is comparatively less than that of flexible bush coupling. On the other hand, the torque is transmitted by means of a force passing through a rubber bush in case of flexible bush coupling. The permissible pressure between the rubber bush and cast iron flange is only 1 N/mm^2. Therefore, the diameter of the pin or pitch circle diameter of pins is comparatively large than that of rigid flange coupling. It should be noted that for connecting shafts of a particular size, flexible bush coupling either has greater number of bolts (or pins) than rigid coupling or has larger bolt circle diameter than rigid coupling. This reduces the force acting on the bolts (pins) and lowers bearing pressure on the rubber bush.

Couplings, whether rigid or flexible, should be located very near to the bearing on the shaft. This reduces the bending moment acting on the shafts due to coupling forces. This is the most satisfactory location of coupling.

9.24 DESIGN PROCEDURE FOR FLEXIBLE COUPLING

The basic procedure for finding out the dimensions of bushed pin type flexible coupling consists of the following steps:

(i) Shaft Diameter Calculate the shaft diameter by using the following two equations:

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n} \quad \text{and} \quad \tau = \frac{16 M_t}{\pi d^3}$$

(ii) Dimensions of Flanges Calculate the dimensions of the flanges by the following empirical relationships:

$$d_h = 2d$$
$$l_h = 1.5d$$
$$D = 3d \text{ to } 4d$$
$$t = 0.5d$$
$$t_1 = 0.25d$$

The torsional shear stress in the hub can be calculated by considering it as a hollow shaft subjected to torsional moment M_t. The inner and outer diameters of the hub are d and d_h respectively. The torsional shear stress in the hub is given by,

$$\tau = \frac{M_t r}{J}$$
$$J = \frac{\pi (d_h^4 - d^4)}{32}$$
$$r = \frac{d_h}{2}$$

The shear stress in the flange at the junction with the hub is calculated by Eq. (9.45). It is given by,

$$M_t = \frac{1}{2} \pi d_h^2 t \tau$$

(iii) Diameter of Pins The number of pins is usually 4 or 6. The diameter of the pins is calculated by the following empirical equation,

$$d_1 = \frac{0.5\, d}{\sqrt{N}} \tag{9.52}$$

Determine the shear stress in the pins by Eq. (9.51).

$$\tau = \frac{8 M_t}{\pi d_1^2 D N}$$

The shear stress calculated by the above equation should be less than 35 N/mm^2. Also, determine the bending stresses in the pins and confirm that it is within limit.

(iv) Dimensions of Bushes Calculate the outer diameter of the rubber bush by Eq. (9.50). It is given by,

$$M_t = \frac{1}{2} D_b^2 D N$$

Calculate the effective length of the rubber bush by the following relationship,

$$l_b = D_b$$

(v) Dimensions of Keys Determine the standard cross-section of flat key from Table 9.3. The length of the key in each shaft is l_h. Therefore,

$$l = l_h$$

With the above dimensions of the key, check the shear and compressive stresses in the key by Eqs (9.27) and (9.28) respectively.

$$\tau = \frac{2M_t}{dbl}$$

$$\sigma_c = \frac{4M_t}{dhl}$$

Example 9.23 *A flexible coupling, illustrated in Fig. 9.39, is used to transmit 15 kW power at 100 rpm. There are six pins and their pitch circle diameter is 200 mm. The effective length of the bush (l_b), the gap between two flanges and the length of the pin in contact with the right hand flange are 35, 5 and 23 mm respectively. The permissible shear and bending stresses for the pin are 35 and 152 N/mm² respectively. Calculate:*

(i) pin diameter by shear consideration; and

(ii) pin diameter by bending consideration.

Solution

Given kW = 15 n = 100 rpm

For pins, $\tau = 35$ N/mm² $\sigma_b = 152$ N/mm²

$N = 6$ $D = 200$ mm

Step I *Pin diameter by shear consideration*

$$M_t = \frac{60 \times 10^6\,(\text{kW})}{2\pi n} = \frac{60 \times 10^6 (15)}{2\pi (100)}$$

$$= 1\,432\,394.49 \text{ N-mm}$$

From Eq. (9.51),

$$\tau = \frac{8\,M_t}{\pi\, d_1^{\,2} DN} \quad \text{or} \quad 35 = \frac{8\,(1\,432\,394.49)}{\pi d_1^{\,2}\,(200)\,(6)}$$

$$\therefore \qquad d_1 = 9.32 \text{ mm} \qquad \text{(i)}$$

Step II *Pin diameter by bending consideration*

The force acting on each bush P and torque M_t are related by the following expression,

$$M_t = P \times \frac{D}{2} \times N$$

$$\therefore \qquad P = \frac{2M_t}{DN} = \frac{2(1\,432\,394.49)}{(200)(6)} = 2387.32 \text{ N}$$

It is assumed that the force P is uniformly distributed over the bush length of 35 mm as shown in Fig. 9.42. At the section-XX,

$$M_b = P\left[5 + \frac{35}{2}\right] = (2387.32 \times 22.5) \text{ N-mm}$$

$$\sigma_b = \frac{32M_b}{\pi d_1^3} \quad \text{or} \quad 152 = \frac{32\,(2387.32 \times 22.5)}{\pi d_1^3}$$

$$\therefore \qquad d_1 = 15.33 \text{ mm} \qquad \text{(ii)}$$

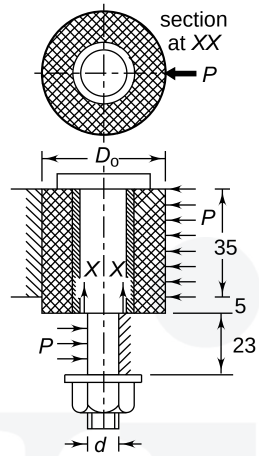


Fig. 9.42

Example 9.24 *It is required to design a bushed-pin type flexible coupling to connect the output shaft of an electric motor to the shaft of a centrifugal pump. The motor delivers 20 kW power at 720 rpm. The starting torque of the motor can be assumed to be 150% of the rated torque. Design the coupling and specify the dimensions of its components.*

Solution

Given kW = 20 n = 720 rpm

design torque = 1.5 (rated torque)

Step I *Selection of materials*

(i) The shafts are subjected to torsional shear stress. On the basis of strength, plain carbon steel of grade 40C8 (S_{yt} = 380 N/mm²) is used for the shafts. The factor of safety for the shaft is assumed as 2.

(ii) The keys are subjected to shear and compressive stresses. The pins are subjected to shear and bending stresses. On the basis of strength criterion, plain carbon steel of grade 30C8 (S_{yt} = 400 N/mm²) is selected for the keys and the pins. The factor of safety for the keys and the pins is taken as 2. It is assumed that the compressive yield strength is 150% of the tensile yield strength.

(iii) Flanges have complicated shape and the economic method to make the flanges is casting. Grey cast iron of grade FG 200 (S_{ut} = 200 N/mm^2) is selected as the material for the flanges from manufacturing considerations. It is assumed that the ultimate shear strength is one-half of the ultimate tensile strength. The factor of safety for the flanges is assumed as 6, since the permissible stress is based on the ultimate strength and not on the yield strength.

Step II *Permissible stresses*

(i) Shaft

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.5 S_{yt}}{(fs)} = \frac{0.5(380)}{(2)} = 95 \text{ N/mm}^2$$

(ii) Keys

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.5 S_{yt}}{(fs)} = \frac{0.5(400)}{(2)} = 100 \text{ N/mm}^2$$

$$\sigma_c = \frac{S_{yc}}{(fs)} = \frac{1.5 S_{yt}}{(fs)} = \frac{1.5(400)}{(2)} = 300 \text{ N/mm}^2$$

(iii) Pins

$\tau = 35$ N/mm^2 (As per IS 2693–1980)

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{400}{2} = 200 \text{ N/mm}^2$$

(iv) Flanges

$$\tau = \frac{S_{su}}{(fs)} = \frac{0.5\, S_{ut}}{(fs)} = \frac{0.5\,(200)}{6} = 16.67 \text{ N/mm}^2$$

Step III *Diameter of shafts*

The starting torque of the motor is 150% of the rated torque. Therefore,

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n} \times (1.5)$$

$$= \frac{60 \times 10^6 (20)\ (1.5)}{2\,\pi\,(720)} = 397\ 887.36 \text{ N-mm}$$

$$\tau = \frac{16 M_t}{\pi d^3} \quad \text{or} \quad 95 = \frac{16(397\ 887.36)}{\pi d^3}$$

$\therefore \quad d = 27.73$ or 30 mm

Step IV *Dimensions of flanges*

The dimensions of flanges are as follows:

$d_h = 2d = 2(30) = 60$ mm

$l_h = 1.5d = 1.5(30) = 45$ mm

$D = 4d = 4(30) = 120$ mm

$t = 0.5d = 0.5(30) = 15$ mm

$t_1 = 0.25d = 0.25(30) = 7.5$ or 8 mm

The hub is treated as a hollow cylinder subjected to torsional moment.

From Eq. (9.44),

$$J = \frac{\pi\,(d_h^4 - d^4)}{32} = \frac{\pi\,(60^4 - 30^4)}{32}$$

$$= 1\ 192\ 823.46 \text{ mm}^4$$

$$r = \frac{d_h}{2} = \frac{60}{2} = 30 \text{ mm}$$

The torsional shear stress in the hub is given by,

$$\tau = \frac{M_t r}{J} = \frac{(397\ 887.36)\,(30)}{(1\,192\,823.46)} = 10.01 \text{ N/mm}^2$$

$\therefore \quad \tau < 16.67$ N/mm^2

The shear stress in the flange at the junction of the hub is determined by Eq. (9.45). It is given by,

$$\tau = \frac{2\,M_t}{\pi\,d_h^2\,t} = \frac{2\,(397\ 887.36)}{\pi\,(60)^2\,(15)} = 4.69 \text{ N/mm}^2$$

$\therefore \quad \tau < 16.67$ N/mm^2

The stresses in the flange are within limit.

Step V *Diameter of pins*

The number of pins is selected as 6.

$$N = 6$$

From Eq. (9.52), the diameter of the pin is given by,

$$d_1 = \frac{0.5\,d}{\sqrt{N}} = \frac{0.5(30)}{\sqrt{6}} = 6.12 \text{ or } 7 \text{ mm}$$

The shear stress in the pin is calculated by Eq. (9.51).

$$\tau = \frac{8\,M_t}{\pi\,d_1^2\,DN} = \frac{8\,(397\ 887.36)}{\pi\,(7)^2\,(120)(6)} = 28.72 \text{ N/mm}^2$$

$\therefore \quad \tau < 35$ N/mm^2

The design is safe on the basis of shear strength. The bending stresses in the pin will be determined at a later stage, after deciding the dimensions of the rubber bushes.

Step VI *Dimensions of bushes*

The permissible intensity of pressure between the rubber bush and the cast iron flange is taken as

1 N/mm^2. The ratio of effective length to diameter for the rubber bush is assumed as 1.

From Eq. (9.50),

$$D_b^2 = \frac{2M_t}{DN} = \frac{2(397\,887.36)}{(120)(6)}$$

or $D_b = 33.25$ or 35 mm

$\therefore \quad D_b = l_b = 35$ mm

Step VII *Bending stresses in pins*

As shown in Fig. 9.40, the force P acting on each pin depends upon the torque and the relationship is given by,

$$M_t = P \times \frac{D}{2} \times N$$

Therefore,

$$P = \frac{2M_t}{DN} = \frac{2(397\,887.36)}{(120)(6)} = 1105.24 \text{ N}$$

The subassembly of the bush and the pin is shown in Fig. 9.43. It is assumed that the force P is uniformly distributed over 35 mm of effective length (l_b) of the bush. At the section-XX,

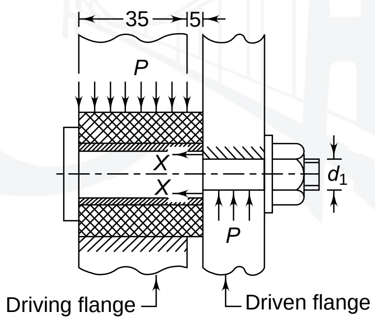


Fig. 9.43 *Bearing Moment at Section XX*

$$M_b = P\left(5 + \frac{35}{2}\right) = 1105.24\left(5 + \frac{35}{2}\right)$$

$$= (1105.24 \times 22.5) \text{ N-mm}$$

$$\sigma_b = \frac{32\,M_b}{\pi d_1^3} \quad \text{or} \quad 200 = \frac{32(1105.24 \times 22.5)}{\pi d_1^3}$$

$\therefore \quad d_1 = 10.82$ or 12 mm

In the initial stages, the diameter of the pin was calculated as 7 mm by using empirical formula. It was sufficient to keep the maximum shear stress within limit. The shear stress induced in the pin was 28.72 N/mm^2 and the limiting stress was 35 N/mm^2. However, it is observed that a 7 mm diameter pin is not sufficient to withstand bending stresses and the minimum diameter of the pin should be 12 mm. Therefore, bending becomes the criterion of design.

As shown in Fig. 9.43, the diameter of the pin should be enlarged at the section-XX in order to fix the pin in the driven flange. This enlarged diameter is taken as (12 + 6) or 18 mm. Other dimensions of the pin and the bush are shown in Fig. 9.44. The thickness of the brass lining is 2 mm. Therefore, the

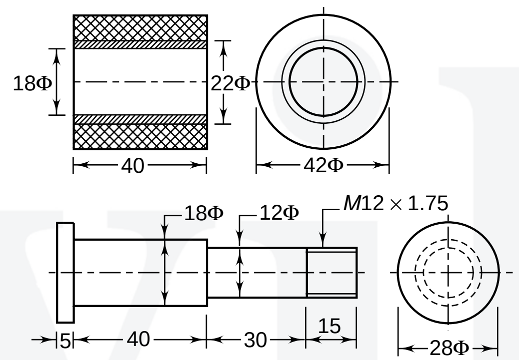


Fig. 9.44 *Dimensions of Bush and Pin*

inner diameter of the rubber bush is (18 + 4) or 22 mm. The minimum thickness of the rubber bush is usually 10 mm. Therefore, the outside diameter of the rubber bush is (22 + 20) or 42 mm. The dimensions of the driving flange are shown in Fig. 9.45.

Step VIII *Dimensions of keys*

From Table 9.3, the standard cross-section of a flat sunk key for 30 mm diameter is 8 × 7 mm. The length of the key is equal to l_h.

$\therefore \quad l = l_h = 45$ mm

The dimensions of the flat key are 8 × 7 × 45 mm.

From Eq. (9.27),

$$\tau = \frac{2M_t}{dhl} = \frac{2(397\,887.36)}{(30)(7)(45)} = 168.42 \text{ N/mm}^2$$

$\therefore \quad \tau < 100$ N/mm^2

From Eq. (9.28),

$$\sigma_c = \frac{2M_t}{dhl} = \frac{2(397\,887.36)}{(30)(7)(45)} = 168.42 \text{ N/mm}^2$$

$\therefore \quad \sigma_c < 300$ N/mm^2

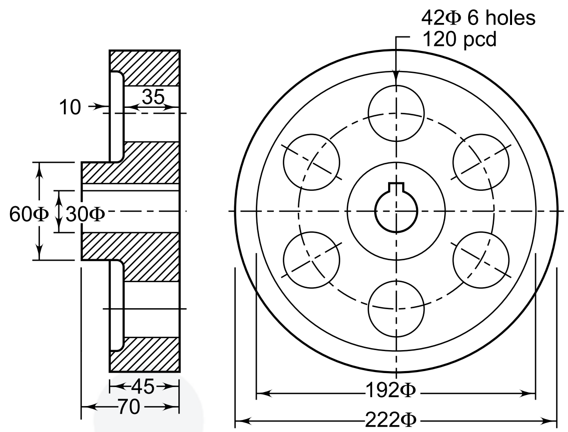


Fig. 9.45 *Dimensions of Driving Flange*

The shear and compressive stresses induced in the key are within permissible limits and the design is safe.

Example 9.25 *A multiflex flexible coupling, transmitting 15 kW power at 720 rpm, is shown in Fig. 9.46. It consists of two flanges keyed to the driving and driven shafts and connected together by means of a number of coils. The coils are placed in the slots on the periphery of the flanges at a distance of 30 mm from the axes of the shafts. The coils have a rectangular cross-section 4 × 2 mm and are made of steel FeE 220 (S_{yt} = 220 N/mm^2). The factor of safety is 2.5. Calculate the required number of coils.*

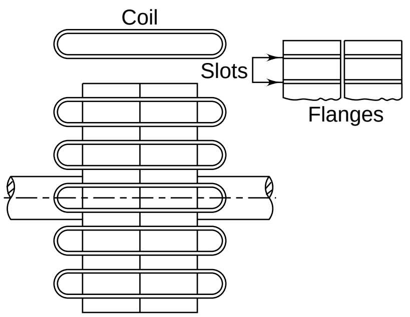


Fig. 9.46 *Multiflex Coupling*

Solution

Given kW = 15 n = 720 rpm

For coils, S_{yt} = 220 N/mm^2 (fs) = 2.5

R = 30 mm

cross-section = 4 × 2 mm

Step I *Permissible shear stress*

$$S_{sy} = 0.577\, S_{yt} = 0.577\,(220) = 126.94 \text{ N/mm}^2$$

$$\tau = \frac{S_{sy}}{(fs)} = \frac{126.94}{2.5} = 50.78 \text{ N/mm}^2$$

Step II *Number of coils*

The torque transmitted by the coupling is given by,

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n} = \frac{60 \times 10^6 (15)}{2\pi\,(720)}$$
$$= 198\,943.68 \text{ N-mm}$$

Each coil is subjected to double shear and the expression for torque is derived in the following way.

Shear resistance of one coil = $2A\tau$

Shear resistance of N coils = $2A\tau N$

Resisting torque = $M_t = 2A\tau NR$ (a)

where,

A = cross-sectional area of wire for the coil

N = number of coils

R = radial distance of coil from the axis of the shaft

Rearranging the terms of Eq. (a),

$$N = \frac{M_t}{2A\tau R} = \frac{198\,943.68}{2(4 \times 2)(50.78)(30)}$$
$$= 8.19 \quad \text{or} \quad 9 \text{ coils}$$

9.25 DESIGN FOR LATERAL RIGIDITY

In certain applications, the components are designed on the basis of lateral rigidity rather than on strength. A component is considered as rigid when it does not deflect or bend too much due to external forces or moments. Permissible deflection is the criterion for design in these cases. When a transmission shaft supporting a gear is deflected, the meshing of the gear teeth is not proper. This results in an uneven distribution of load over the face width of the gear, causing early failure. The shaft deflection also results in a misalignment between the axes of the shaft and the bearing, causing uneven wear of bearing surfaces. The radial clearance in hydrodynamic bearing is affected by the shaft deflection. In design of machine tool components, rigidity governs the accuracy of the machining method.

The maximum permissible deflection for the transmission shaft is generally taken as

$$\delta = (0.001)L \quad \text{to} \quad (0.003)L$$

where L is the span length between the two bearings. The maximum permissible deflection at the gear is taken as

$$\delta = (0.01)m$$

where m is the module of the gear teeth. In practice, the lateral rigidity of the structure is improved by the following methods:

(i) reduce the span length;
(ii) increase the number of supports;
(iii) reduce the number of joints;
(iv) assemble the components with pre-load;
(v) lubricate the contact surfaces with high viscosity oil; and
(vi) select a cross-section in which the cross-sectional area is away from the neutral axis, such as an I or tubular section.

The lateral deflection depends upon the dimensions of the shaft, forces acting on the shaft, and the modulus of elasticity. The modulus of elasticity is practically same for all types of steel, whether plain carbon steel or alloy steel. It is not advisable to use expensive alloy steels where lateral deflection is the design criterion. Plain carbon steel is, therefore, a better choice as a shaft material when the component is designed on the basis of lateral rigidity.

The determination of the deflection of beams is extensively covered in Strength of Materials. Important methods for determining the lateral deflection of beams are as follows:

(i) double integration method;
(ii) area moment method;
(iii) strain energy method; and
(iv) graphical integration method.

In simple cases, the standard formulae from Strength of Materials, which are given in Table 9.4, are used. Sometimes, the principle of superim position of deflection is used. According to this principle, *the deflection at any point in the shaft is equal to the sum of deflections, caused by each load separately.*

Example 9.26 *Consider the forces acting on the intermediate shaft of a gearbox illustrated in Example 9.4. The maximum permissible radial deflection at any gear is limited to 1 mm. The modulus of elasticity of the shaft material is 207000 N/mm². Determine the shaft diameter and compare it with the solution of Example 9.4.*

Table 9.4 *Bending moment and deflection of beams*

Case 1 Cantilever Beam – End load

(A) Bending moments

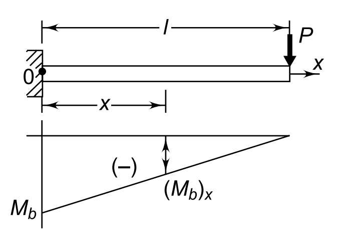
$$(M_b) \text{ at } O = -Pl \quad (1)$$

$$(M_b) \text{ at } x = -P(l-x) \quad (2)$$

(B) Deflections

$$\delta \text{ at } x = \frac{Px^2}{6EI}(x-3l) \quad (3)$$

$$\delta_{\text{max.}} \text{ at } (x=l) = -\frac{Pl^3}{3EI} \quad (4)$$



Case 1

(Contd)

Table 9.4 *(Contd)*

Case 2 Cantilever Beam—Uniformly distributed load

(A) Bending moments

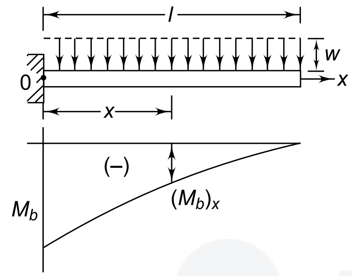
$$(M_b) \text{ at } O = -\frac{wl^2}{2} \qquad (5)$$

$$(M_b) \text{ at } x = -\frac{w(l-x)^2}{2} \qquad (6)$$

(B) Deflections

$$\delta \text{ at } x = \frac{wx^2}{24EI}(4lx - x^2 - 6l^2) \qquad (7)$$

$$\delta_{max.} \text{ at } (x = l) = -\frac{wl^4}{8EI} \qquad (8)$$



Case 2

Case 3 Cantilever Beam—Moment load

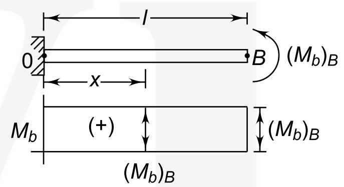
(A) Bending moments

$$(M_b) \text{ at } O = (M_b) \text{ at } x = (M_b)_B \qquad (9)$$

(B) Deflections

$$\delta \text{ at } x = \frac{(M_b)_B x^2}{2EI} \qquad (10)$$

$$\delta_{max.} \text{ at } (x = l) = \frac{(M_b)_B l^2}{2EI} \qquad (11)$$



Case 3

Case 4 Simply supported beam—Centre load

(A) Bending moments

$$(M_b) \text{ at } x = \frac{Px}{2} \qquad (0 < x < l/2) \qquad (12)$$

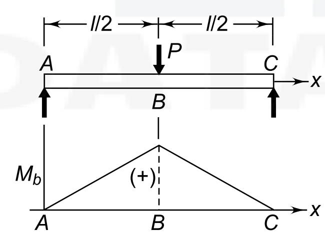
$$(M_b) \text{ at } x = \frac{P(l-x)}{2} \qquad (x > l/2) \qquad (13)$$

$$(M_b) \text{ at } B = \frac{Pl}{4} \qquad (14)$$

(B) Deflections

$$\delta \text{ at } x = \frac{Px(4x^2 - 3l^2)}{48EI} \qquad (0 < x < l/2) \qquad (15)$$

$$\delta_{max.} \text{ at } B = -\frac{Pl^3}{48EI} \qquad (16)$$



Case 4

(Contd)

Table 9.4 *(Contd)*

Case 5 Simply supported Beam—Uniformly distributed load

(A) Bending moments

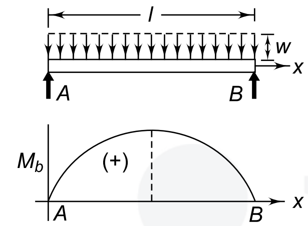
$$(M_b) \text{ at } x = \frac{wx(l-x)}{2} \qquad (17)$$

$$(M_b) \text{ at } l/2 = \frac{wl^2}{8} \qquad (18)$$

(B) Deflections

$$\delta \text{ at } x = \frac{wx(2lx^2 - x^3 - l^3)}{24EI} \qquad (19)$$

$$\delta_{max.} \text{ at } l/2 = -\frac{5wl^4}{384EI} \qquad (20)$$



Case 5

Case 6 Simply supported beam—Moment load

(A) Bending moments

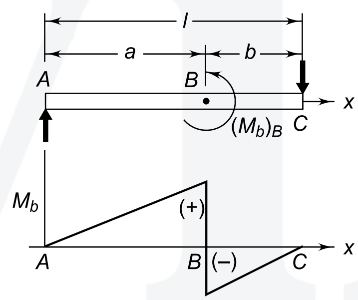
$$(M_b) \text{ at } x = \frac{(M_b)_B\, x}{l} \quad (0 < x < a) \qquad (21)$$

$$(M_b) \text{ at } x = \frac{(M_b)_B(x-l)}{l} \quad (a < x < l) \qquad (22)$$

(B) Deflections

$$\delta \text{ at } x = \frac{(M_b)_B\, x(x^2 + 3a^2 - 6al + 2l^2)}{6EIl} \quad (0 < x < \text{a}) \qquad (23)$$

$$\delta \text{ at } x = \frac{(M_b)_B[x^3 - 3l\,x^2 + x(2l^2 + 3a^2) - 3a^2 l]}{6EIl} \quad (a < x < \text{l}) \qquad (24)$$



Case 6

Case 7 Simply supported Beam—Intermediate load

(A) Bending moments

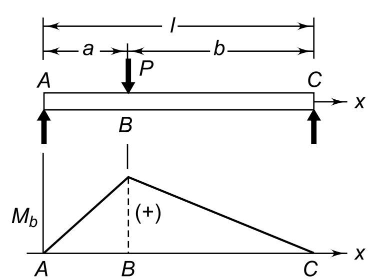
$$(M_b) \text{ at } x = \frac{Pbx}{l} \quad (0 < x < a) \qquad (25)$$

$$(M_b) \text{ at } x = \frac{Pa(l-x)}{l} \quad (\text{a} < x < l) \qquad (26)$$

(B) Deflections

$$\delta \text{ at } x = \frac{Pb\,x(x^2 + b^2 - l^2)}{6EIl} \quad (0 < x < a) \qquad (27)$$

$$\delta \text{ at } x = \frac{Pa(l-x)(x^2 + a^2 - 2lx)}{6EIl} \quad (a < x < l) \qquad (28)$$



Case 7

(Contd)

Table 9.4 *(Contd)*

Case 8 Simply supported beam—Overhang load

(A) Bending moments

$$(M_b) \text{ at } x = -\frac{Pa\,x}{l} \quad (x < l) \qquad (29)$$

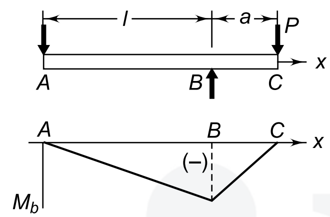
$$(M_b) \text{ at } x = P(x - l - a) \quad (x > l) \qquad (30)$$

(B) Deflections

$$\delta \text{ at } x = \frac{Pa\,x(l^2 - x^2)}{6EI\,l} \quad (x < l) \qquad (31)$$

$$\delta \text{ at } x = \frac{P(x-l)[(x-l)^2 - a(3x-l)]}{6EI} \quad (x > l) \qquad (32)$$

$$\delta \text{ at } C = -\frac{Pa^2(l+a)}{3EI} \qquad (33)$$



Case 8

Solution

Given $\delta_{max.} = 1$ mm $E = 207\,000$ N/mm^2

Step I *Deflection at Gear -B*

The deflection at gears B or C (Fig. 9.6) is calculated by the principle of superimposition. Consider the deflection at B.

Vertical plane

(i) Deflection due to a force of 1609 N [Fig. 9.47(a), Eq. (27) of Table 9.4]:

$$(\delta_B)_1 = \frac{Pbx(x^2 + b^2 - l^2)}{6EI\,l}$$

$$= \frac{(1609)(1800)(900)(900^2 + 1800^2 - 2700^2)}{6EI(2700)}$$

$$= -\frac{5.2131 \times 10^{11}}{EI} \text{ mm} \qquad \text{(i)}$$

(ii) Deflection due to a force of 6631.5 N [Fig. 9.47(b), Eq. (27) of Table 9.4]:

$$(\delta_B)_2 = \frac{Pb\,x(x^2 + b^2 - l^2)}{6EI\,l}$$

$$= \frac{(-6631.5)(900)(900)(900^2 + 900^2 - 2700^2)}{6EI(2700)}$$

$$= +\frac{18.8 \times 10^{11}}{EI} \text{ mm} \qquad \text{(ii)}$$

The vertical deflection at B denoted by $(\delta_B)_v$ is given by the principle of superimposition.

$$(\delta_B)_v = (\delta_B)_1 + (\delta_B)_2$$

$$= -\frac{5.2131 \times 10^{11}}{EI} + \frac{18.8 \times 10^{11}}{EI}$$

$$= \frac{13.5869 \times 10^{11}}{EI} \text{ mm} \qquad \text{(a)}$$

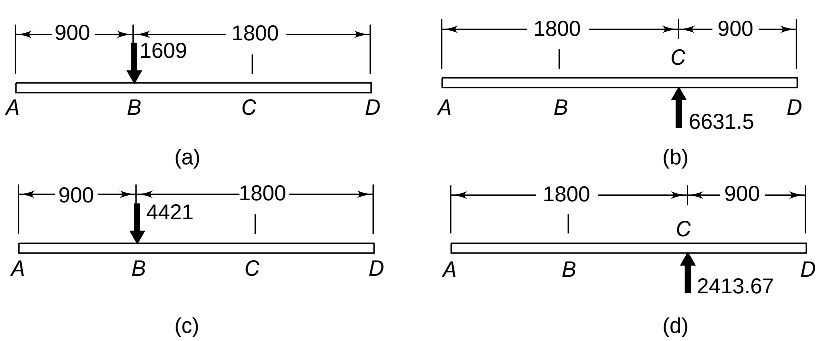


Fig. 9.47

Horizontal plane

(i) Deflection due to a force of 4421 N [Fig. 9.47(c), Eq. (27) of Table 9.4]:

$$(\delta_B)_1 = \frac{Pbx(x^2 + b^2 - l^2)}{6EI\,l}$$

$$= \frac{(4421)(1800)(900)(900^2 + 1800^2 - 2700^2)}{6E\,I(2700)}$$

$$= -\frac{14.324 \times 10^{11}}{E\,I} \text{ mm} \qquad \text{(iii)}$$

(ii) Deflection due to a force of 2413.67 N [Fig. 9.47 (d), Eq. (27) of Table 9.4]:

$$(\delta_B)_2 = \frac{Pbx(x^2 + b^2 - l^2)}{6EI\,l}$$

$$= \frac{(-2413.67)(900)(900)(900^2 + 900^2 - 2700^2)}{6E\,I(2700)}$$

$$= +\frac{6.8427 \times 10^{11}}{EI} \text{ mm} \qquad \text{(iv)}$$

By the principle of superimposition,

$$(\delta_B)_h = (\delta_B)_1 + (\delta_B)_2$$

$$= -\frac{14.324 \times 10^{11}}{EI} + \frac{6.8427 \times 10^{11}}{EI}$$

$$= -\frac{7.4813 \times 10^{11}}{EI} \text{ mm} \qquad \text{(b)}$$

The radial deflection at the gear B consists of two components—vertical deflection $(\delta_B)_v$ and horizontal component $(\delta_B)_h$. The radial deflection (δ_B) is given by,

$$\delta_B = \sqrt{[(\delta_B)_v]^2 + [(\delta_B)_h]^2}$$

$$= \sqrt{(13.5869)^2 + (7.4813)^2}\left(\frac{10^{11}}{EI}\right)$$

$$= \frac{15.51 \times 10^{11}}{EI} \text{ mm} \qquad \text{(c)}$$

Step II *Deflection at Gear-C*

A similar procedure is used to calculate the deflection at C. The final values of vertical and horizontal deflections are as follows:

$$(\delta_C)_v = +\frac{16.9245 \times 10^{11}}{EI} \text{ mm}$$

$$(\delta_C)_h = -\frac{4.7128 \times 10^{11}}{EI} \text{ mm}$$

$$\delta_C = -\frac{17.5684 \times 10^{11}}{EI} \text{ mm} \qquad \text{(d)}$$

Step III *Shaft diameter*

From (c) and (d), the radial deflection is maximum at the gear C. Equating it with permissible deflection,

$$1 = \frac{17.5684 \times 10^{11}}{(207\,000)(\pi d^4/64)}$$

$$d = 114.67 \text{ mm}$$

In Example 9.4, the shaft diameter was calculated as 68.59 mm on the basis of strength, while in this problem the same has been calculated as 114.67 mm on the basis of rigidity. Therefore, rigidity becomes the criterion of design in this case.

9.26 CASTIGLIANO'S THEOREM

One of the important techniques of determining the deflection of a complex structure is the application of Castigliano's theorem. The theorem states

'*When a body is elastically deflected by any combination of forces or moments, the deflection at any point and any direction is equal to the partial derivative of total strain energy of the body with respect to the force located at that point and acting in that direction*'.

Consider an elastic body subjected to a system of forces P_1, P_2, P_3, etc. Suppose U is the total strain energy of the body. δ_1, δ_2, δ_3,…, etc., are deflections at the point of application and in the direction of P_1, P_2, P_3, etc. Then according to this theorem,

$$\delta_1 = \frac{\partial U}{\partial P_1}$$

$$\delta_2 = \frac{\partial U}{\partial P_2}$$

$$\delta_3 = \frac{\partial U}{\partial P_3}$$

or

$$\delta_i = \frac{\partial U}{\partial P_i} \qquad (9.53)$$

In this method, it is necessary to determine the strain energy stored in a structure. The strain energy stored in tension rod shown in Fig. 4.1 (Chapter 4) is given by,

$$U = \frac{1}{2} P\delta$$

Substituting Eq. 4.6 ($\delta = Pl/AE$) in the above expression,

$$U = \frac{P^2 l}{2AE} \qquad (9.54)$$

For a transmission shaft subjected to an external torque as shown in Fig. 4.8 (Chapter 4), the strain energy is given by

$$U = \frac{1}{2} M_t \theta$$

Substituting Eq. (4.18) ($\theta = M_t\, l/JG$) in the above expression,

$$U = \frac{(M_t)^2 l}{2JG} \qquad (9.55)$$

Similarly, it can be proved that the strain energy stored in a shaft, subjected to a bending moment M_b, is given by

$$U = \int \frac{(M_b)^2 dx}{2EI} \qquad (9.56)$$

Castigliano's theorem can be used to determine the deflection at a point where no force is acting. In such a case, we assume an imaginary force Q at such a point and in the direction in which the deflection is to be determined. The partial derivative $(\partial U/\partial Q)$ gives the desired deflection when Q is equated to zero. Castigliano's theorem is applicable only in the elastic range of the materials.

Example 9.27 *A transmission shaft with a uniformly distributed load of 10 N/mm is shown in Fig. 9.48(a). The maximum permissible deflection of the shaft is (0.003L), where L is the span length. The modulus of elasticity of the shaft material is 207 000 N/mm². Determine the shaft diameter by Castigliano's theorem.*

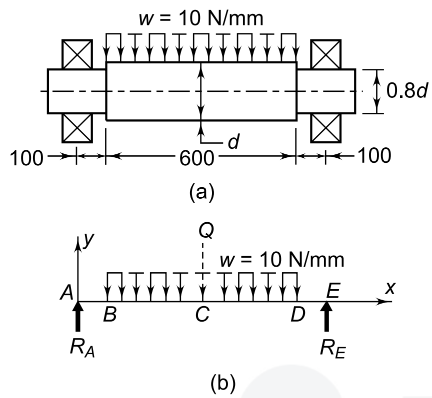


Fig. 9.48

Solution

Given $\delta_{max.} = 0.003L \quad E = 207\,000$ N/mm²

Step I *Permissible deflection*

$$\delta_{max.} = 0.003L = 0.003\,(800) = 2.4 \text{ mm} \qquad \text{(i)}$$

Step II *Deflection at centre of span*

The deflection is maximum at the centre of the span length. An imaginary force Q is assumed at the centre as shown in Fig. 9.48 (b). By symmetry, the reactions are given by

$$R_A = R_E = \left(3000 + \frac{Q}{2}\right)$$

The bending moments in the regions AB and BC are given by,

$$(M_b)_{AB} = \left(3000 + \frac{Q}{2}\right) x = 3000x + \frac{Qx}{2}$$

$$(M_b)_{BC} = \left(3000 + \frac{Q}{2}\right) x - \frac{10}{2}(x-100)(x-100)$$

$$= \frac{Qx}{2} - 5x^2 + 4000x - 50\,000$$

Differentiating partially with respect to Q,

$$\frac{\partial}{\partial Q}[(M_b)_{AB}] = \frac{x}{2} \qquad \text{(a)}$$

$$\frac{\partial}{\partial Q}[(M_b)_{BC}] = \frac{x}{2} \qquad \text{(b)}$$

The total strain energy stored in the shaft is given by,

$$U = 2U_{AB} + 2U_{BC}$$

From Eq. (9.56),

$$U = 2\int_0^{100} \frac{[(M_b)_{AB}]^2\,dx}{2EI_1} + 2\int_{100}^{400} \frac{[(M_b)_{BC}]^2\,dx}{2EI_2}$$

From Eq. (9.53),

$$\delta_{\text{max.}} = \frac{\partial U}{\partial Q}$$

$$\delta_{\text{max.}} = 2\int_0^{100} \left(\frac{2(M_b)_{AB}}{2EI_1}\right)\left(\frac{\partial}{\partial Q}[(M_b)_{AB}]\right) dx + 2\int_{100}^{400} \left(\frac{2(M_b)_{BC}}{2EI_2}\right)\left(\frac{\partial}{\partial Q}[(M_b)_{BC}]\right) dx$$

Substituting values of $[(M_b)_{AB}]$, $[(M_b)_{bc}]$, (a) and (b) in the above expression,

$$\delta_{\text{max.}} = \frac{2}{EI_1}\int_0^{100} \left(3000x + \frac{Qx}{2}\right)\left(\frac{x}{2}\right) dx + \frac{2}{EI_2}\int_{100}^{400} \left(\frac{Qx}{2} - 5x^2 + 4000x - 50\,000\right)\left(\frac{x}{2}\right) dx$$

Substituting ($Q = 0$) and integrating,

$$\delta_{\text{max.}} = \frac{10^9}{EI_1} + \frac{48.375(10^9)}{EI_2} \qquad \text{(ii)}$$

Step III *Shaft diameter*

Equating (i) and (ii) and substituting values,

$$2.4 = \frac{10^9}{(207\,000)[\pi(0.8d)^4/64]} + \frac{48.375(10^9)}{(207\,000)(\pi d^4/64)}$$

$$d = 37.99 \text{ mm}$$

9.27 AREA MOMENT METHOD

The area moment method is sometimes convenient for determining the deflection of the shaft due to bending moments. To illustrate the method, consider a simply supported beam with centre load as shown in Fig. 9.49(a). The basic principle of this method is stated as '*The vertical distance of any point C on the elastic curve of the shaft, from the tangent at any other point A on the elastic curve, is equal to the moment of the area of (M/EI) diagram between A and C with respect to the ordinate through C*'. A tangent is drawn at the point A to the elastic curve [Fig. 9.49(b)]. (δ_1) is the vertical distance of the point C from the tangent. The (M/EI) diagram is shown in Fig. 9.49(c). Taking the moment of (M/EI) diagram about the ordinate through C, we have

$$\delta_1 = \left(\frac{1}{2}\frac{Pl}{4EI}(l)\right)\left(\frac{l}{2}\right)$$

$$\delta_1 = \frac{Pl^3}{16EI} \qquad \text{(a)}$$

Similarly, (δ_2) is the vertical distance of the point B from the tangent. Taking moment,

$$\delta_2 = \left(\frac{1}{2}\frac{Pl}{4EI}\frac{l}{2}\right)\left(\frac{1}{3}\frac{l}{2}\right)$$

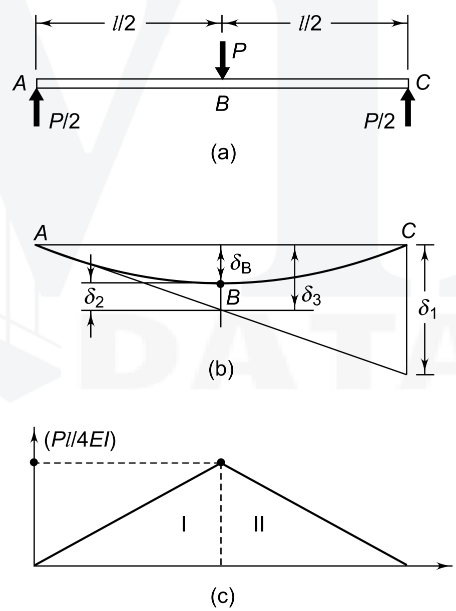


Fig. 9.49 *(a) Forces on Shaft (b) Elastic Curve (c) M/EI Diagram*

$$\therefore \quad \delta_2 = \frac{Pl^3}{96EI} \qquad \text{(b)}$$

From similar triangles,

$$\frac{\delta_3}{\delta_1} = \frac{(l/2)}{l}$$

$$\delta_3 = \frac{\delta_1}{2} = \frac{Pl^3}{32EI} \qquad (c)$$

The deflection at B is given by,

$$\delta_B = \delta_3 - \delta_2 = \frac{Pl^3}{32EI} - \frac{Pl^3}{96EI} = \frac{Pl^3}{48EI}$$

9.28 GRAPHICAL INTEGRATION METHOD

In some cases, the forces acting on the shaft and its geometry are such that it is not possible to use classical methods, such as the principle of superimposition or Castigliano's theorem, to find out the deflection. Hence, here the graphical integration method is used. The main drawback of this method is its limited accuracy. The principle of this method is illustrated in Fig. 9.50. Let us consider a function,

$$y = f(x)$$

in such a way that,

$y = 200$ units for $0 < x < 200$

$y = 350$ units for $200 < x < 400$

$y = 300$ units for $400 < x < 600$

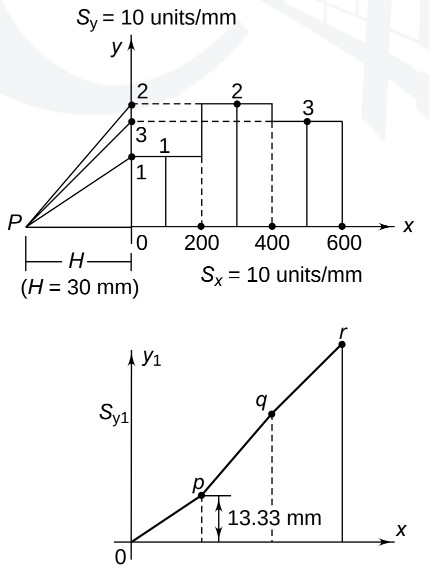


Fig. 9.50

The function is plotted to the following scales:

$S_x = 10$ units/mm $\qquad S_y = 10$ units/mm

The graphical integration method consists of the following steps:

(i) Select a pole point P. The distance OP is called the pole distance (H). In this case, $H = 30$ mm

(ii) Divide the area into suitable rectangles and project the mid-ordinates of these rectangles on the Y-axis.

Join P–1, P–2, P–3

(iii) Draw lines

Op parallel to P–1

pq parallel to P–2

qr parallel to P–3

(iv) The scale of the integral is given by

$S_{y1} = H\,S_xS_y = 30(10)(10) = 3000(\text{unit})^2/\text{mm}$

At the point p, the ordinate is 13.33 mm and the value of integration is

$$13.33 \times 3000 = 40\,000\ (\text{unit})^2/\text{mm}$$

The area of the rectangle (200 × 200) is 40000 (unit)^2, which proves the answer.

Example 9.28 *A cantilever beam carrying a load of 500 N is shown in Fig. 9.51(a). The maximum permissible deflection of the beam is 0.05 mm. The modulus of elasticity of the material is 207000 N/mm^2. Using the graphical integration method, determine the size of the cross-section of the beam.*

Solution

Given $\delta_{max.} = 0.05$ mm $\quad E = 207\,000$ N/mm^2 $\quad P = 500$ N

Step I *B-M diagram*

(M_b) at $B = 500 \times 100 = 50\,000$ N-mm

(M_b) at $A = 500 \times 200 = 100\,000$ N-mm

The bending moment diagram is shown in Fig. 9.51(b).

Step II *(M/EI) diagram*

Let us denote,

$$I = \frac{\pi d^4}{64}$$

For the portion AB,

$$I' = \frac{\pi(1.2d)^4}{64} = 2.074I$$

Considering the portion BC,

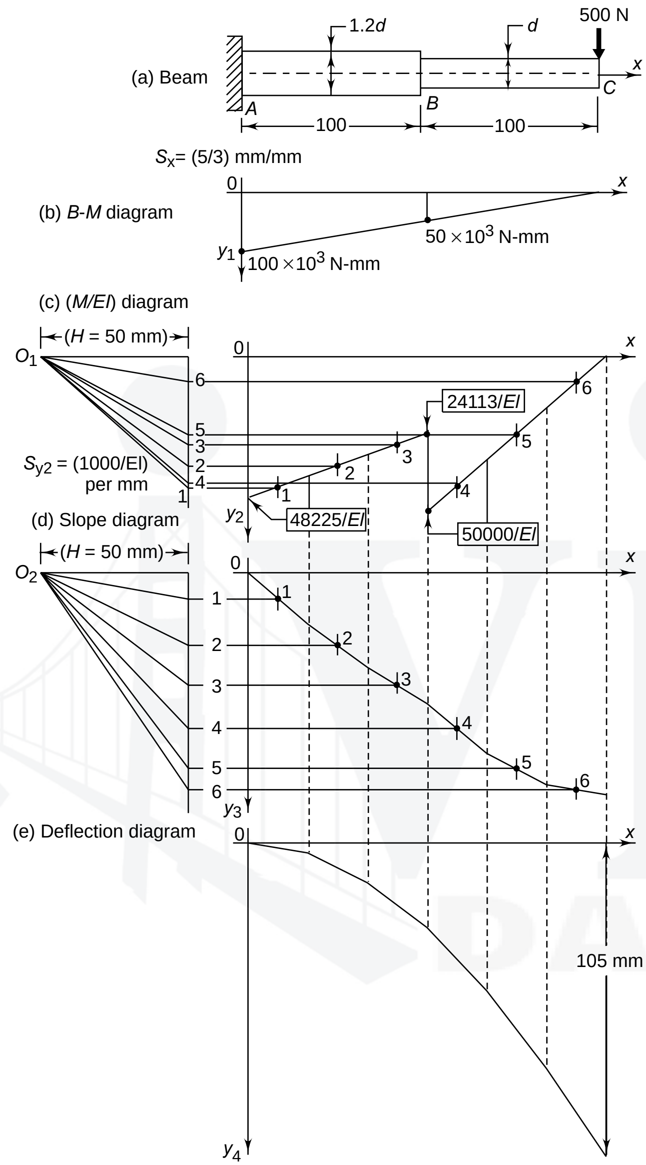


Fig. 9.51 *Graphical Integration Method*

(M/EI) at $B = 50\,000/EI$ mm^{-1}

Considering the portion AB,

(M/EI) at $B = 50\,000/2.074EI) = 24113/EI$ mm^{-1}

(M/EI) at $A = 100\,000/(2.074EI)$

$= 48\,225/EI$ mm^{-1}

The (M/EI) diagram shown in Fig. 9.51(c)

Step III *Slope and deflection Diagrams*

The (M/EI) diagram shown in Fig. 9.51(c) is drawn to the following scale:

$$S_x = (5/3) \text{ mm per mm}$$

$$S_{y2} = (1000/EI) \text{ per mm}$$

The diagram is divided into six equal parts and their mid-points are denoted as 1, 2, …, 6. The heights of these points are projected on the y_2-axis. The pole distance H is taken as 50 mm. Join lines $\overline{O_1 1}$, $\overline{O_1 2}$, ..., $\overline{O_1 6}$, The slope diagram shown in Fig. 9.51(d) is constructed by the graphical integration method. From the origin of the slope diagram, draw a line parallel to $\overline{O_1 1}$ of the (M/EI) diagram. This is the slope of part 1. Similar procedure is used for other parts. As discussed in the previous section, the scale of the slope diagram is given by,

$$S_{y3} = H\,S_x S_{y2} = 50(5/3)(1000/EI)$$
$$= (250\,000/3EI) \text{ per mm}$$

The deflection diagram shown in Fig. 9.51(e), is constructed by the graphical integration of the slope diagram. Similar procedure is used in this case. The scale of the deflection diagram is given by,

$$S_{y4} = H\,S_x S_{y3} = 50(5/3)\,(250\,000/3EI)$$
$$= (6944.44 \times 10^3/EI) \text{ per mm}$$

As shown in the figure, the actual dimension of the maximum deflection is 105 mm.
Therefore,

$$\delta_{max.} = (105)(6944.44 \times 10^3/EI)$$
$$= (729.166 \times 10^6/EI) \text{ mm}$$

Step VI *Diameter of beam*
The permissible deflection is 0.05 mm. Therefore,

$$0.05 = \frac{729.166 \times 10^6}{207\,000 \times (\pi d^4/64)}$$
$$d = 34.61 \text{ mm}$$

Example 9.29 *Consider the cantilever beam illustrated in Example 9.28. Using Castigliano's theorem, determine the size of the cross-section of the beam.*

Solution

Given $\delta_{max.} = 0.05$ mm $E = 207\,000$ N/mm^2
$P = 500$ N

Step I *Deflection by Castigliano's theorem*
Let us denote by P the force at the free end of the cantilever beam. The bending moment at a distance x from the fixed end is given by,

$$M_b = P(200 - x) \qquad \text{(i)}$$

The above relationship can be used for any point from A to C. Differentiating with respect to P,

$$\frac{\partial}{\partial P}(M_b) = (200 - x) \qquad \text{(ii)}$$

The total strain energy stored in the beam is given by

$$U = U_{AB} + U_{BC}$$

From Eq. (9.56),

$$U = \int_0^{100} \frac{(M_b)^2\,dx}{2EI'} + \int_{100}^{200} \frac{(M_b)^2\,dx}{2EI} \qquad \text{(iii)}$$

By Castigliano's theorem,

$$\delta_{max.} = \frac{\partial U}{\partial P} \qquad \text{(iv)}$$

From (iii) and (iv),

$$\delta_{max.} = \int_0^{100} \left(\frac{2(M_b)}{2EI'}\right)\left(\frac{\partial(M_b)}{\partial P}\right) dx + \int_{100}^{200} \left(\frac{2(M_b)}{2EI}\right)\left(\frac{\partial(M_b)}{\partial P}\right) dx$$

Substituting (i) and (ii) in the above expression,

$$\delta_{max.} = \frac{P}{EI'}\int_0^{100} (200 - x)^2\,dx + \frac{P}{EI}\int_{100}^{200} (200 - x)^2\,dx$$

On integration, we get,

$$\delta_{max.} = \frac{(7 \times 10^6)P}{3EI'} + \frac{(10^6)P}{3EI}$$

Substituting the values of P and I' in the above equation,

$$\delta_{max.} = \frac{(729.19 \times 10^6)}{EI} \text{ mm}$$

Step II *Diameter of beam*
The permissible deflection is 0.05 mm. Therefore,

$$0.05 = \frac{(729.19 \times 10^6)}{(207\,000)(\pi d^4/64)}$$
$$\therefore \qquad d = 34.61 \text{ mm}$$

9.29 CRITICAL SPEED OF SHAFTS

The critical speed of the shaft is the speed at which the rotating shaft becomes dynamically unstable and starts to vibrate violently in a transverse direction. It is a very dangerous condition because the amplitude of vibration is so high that the shaft may break into pieces. The critical speed of the shaft is also called the '*whirling*' *speed* or *natural frequency of vibrations*.

The centre of mass of a transmission shaft carrying gears, pulleys or sprockets never coincides with the centre of rotation. This is due to the following two reasons:

(i) It is practically impossible to machine a shaft and assemble its attachment in such a way that the mass is uniformly distributed about its geometric axis.

(ii) The shaft is subjected to forces such as gear forces, belt or chain tensions which result in lateral deflection. This moves the centre of mass away from the true axis of rotation that passes through the centreline of bearings.

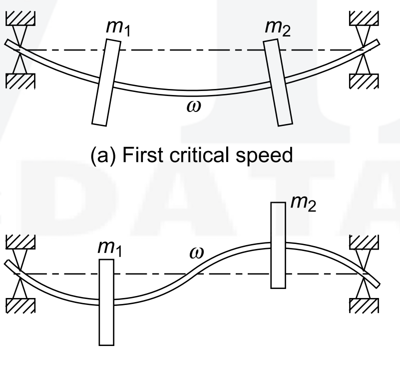
The above two factors cause the mass to rotate with small eccentricity about the geometric axis. The rotation of the shaft may start about the geometric axis. As the speed is increased, a centrifugal force acts on the mass due to eccentricity. The centrifugal force further increases the deflection of the shaft as well as its eccentricity, which in turn increases the centrifugal force itself. When the speed is further increased, a stage is reached when elastic forces within the shaft no longer balance the centrifugal force. This starts violent vibrations of the shaft and the speed at which this phenomenon occurs is called '*critical*' *speed*. This is the resonance condition, when the speed of rotation is equal to the natural frequency of vibrations. The shaft should not be run at the critical speed, because excessive deflection may cause failure of the shaft.

There is always damping in the form of internal friction and friction in bearings. Therefore, the shaft does not fail instantaneously. If the shaft passes quickly through the critical speed, no damage is done. When the speed of the shaft is further increased, i.e., more than the critical speed, a state of equilibrium is again reached and the system runs virtually about its mass centre. It has been observed that high-speed turbines that run above the critical speed give a smooth and vibration-free performance. The resonance condition can be avoided by the following two methods:

(i) In some applications, shafts are made very rigid with very high critical speed (n_{cr}), which is far away from the running speed (n). In this case, the shaft never reaches the critical speed and no resonance occurs.

(ii) There are applications where shafts are made very slender with very low critical speed (n_{cr}). The running speed (n) is two to three times of the critical speed. In this case, the shaft passes quickly through the critical speed and no damage is done. The shaft never reaches the critical speed during operating conditions and no resonance occurs.

Critical speed of shafts is a topic of the subject 'Mechanical Vibrations'. We will cover only a brief treatment of this topic. For any shaft, there are a number of critical speeds. The designer is mainly interested in the '*first*' or lowest critical speed. Occasionally, the '*second*' critical speed is considered. The other critical speeds are so high that they are outside the range of operating speeds. The bending configurations of the 'first' and 'second' critical speeds are illustrated in Fig. 9.52. In this case, a shaft supported between two end bearings and having two attached masses is considered.



(a) First critical speed

(b) Second critical speed

Fig. 9.52 *Critical Speeds of Shaft*

A transmission shaft of negligible mass and supported on two end bearings is shown in Fig. 9.53. It is carrying several concentrated masses. The first or lowest critical speed is given by,

$$\omega_n = \sqrt{\frac{g(W_1\delta_1 + W_2\delta_2 + ...)}{(W_1\delta_1^2 + W_2\delta_2^2 + ...)}} \tag{9.57}$$

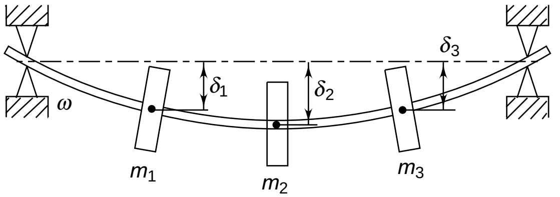


Fig. 9.53

where,

ω_n = first or lowest critical speed (rad/s)
g = gravitational constant (9.81 m/s)
$W_1 = m_1 g \quad W_2 = m_2 g$
$m_1, m_2, m_3, \ldots$ = rotating masses (kg)
$\delta_1, \delta_2, \delta_3 \ldots$ = static deflections at the respective masses (m)

The above equation is called the Rayleigh–Ritz equation.

Example 9.30 *A transmission shaft supported between two bearings and carrying two concentrated masses is shown in Fig. 9.54. It is made of steel (E = 207 000 N/mm²). Assuming that the shaft has negligible mass, calculate the critical speed.*

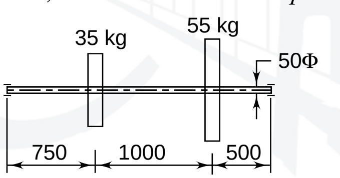


Fig. 9.54

Solution

Given $E = 207\,000$ N/mm² $\quad d = 50$ mm
$m_A = 35$ kg $\quad m_B = 55$ kg

Step I *Deflections at mass-A and mass-B*

Suppose the masses of 35 and 55 kg are attached to the shaft at points A and B respectively.

$$W_A = 35g = 35(9.81) = 343.35 \text{ N}$$
$$W_B = 55g = 55(9.81) = 539.55 \text{ N}$$

The deflections at these masses are calculated by the method of superimposition. The configuration is similar to Case 7 of Table 9.4 (simply supported beam with intermediate load). Rewriting Eqs (27) and (28) with reference to Fig. 9.55(a),

$$\delta \text{ at } x = \frac{P\,b\,x(x^2 + b^2 - l^2)}{6\,E\,I\,l} \qquad (0 < x < a) \quad \text{(a)}$$

$$\delta \text{ at } x = \frac{Pa(l-x)(x^2 + a^2 - 2lx)}{6\,E\,I\,l} \quad (a < x < l) \quad \text{(b)}$$

Deflections due to force 343.35 N [Fig. 9.55 (b)]

$a = 750$ mm $\quad b = 1500$ mm $\quad l = 2250$ mm

The deflection $(\delta_A)_A$ at the point A due to the force at A is given by Eq. (a).

$$x = 750 \text{ mm} \quad (0 < x < a)$$

$$(\delta_A)_A = \frac{Pbx(x^2 + b^2 - l^2)}{6EIl}$$

$$(\delta_A)_A = \frac{(343.55)(1500)(750)(750^2 + 1500^2 - 2250^2)}{6EI(2250)}$$

$$= \frac{-6.44(10)^{10}}{EI} \qquad \text{(i)}$$

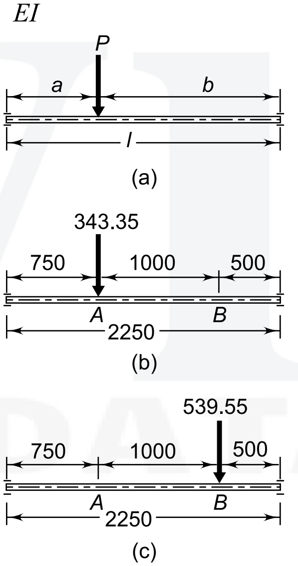


Fig. 9.55

The deflection $(\delta_B)_A$ at the point B due to the force at A is given by Eq. (b).

$$x = 1750 \text{ mm} \qquad (x > a)$$

$$(\delta_B)_A = \frac{Pa(l-x)(x^2 + a^2 - 2lx)}{6\,E\,I\,l}$$

$$(\delta_B)_A = \frac{(343.35)(750)(2250-1750)[1750^2 + 750^2 - 2(2250)(1750)]}{6EI(2250)}$$

$$= \frac{-4.05(10)^{10}}{EI} \qquad \text{(ii)}$$

Deflections due to force 539.55 N [Fig. 9.55(c)]

$a = 1750$ mm $b = 500$ mm $l = 2250$ mm

The deflection $(\delta_A)_B$ at the point A due to the force at B is given by Eq. (a).

$$x = 750 \text{ mm} \qquad (0 < x < a)$$

$$(\delta_A)_B = \frac{Pbx(x^2 + b^2 - l^2)}{6EIl}$$

$$(\delta_A)_B = \frac{(539.55)(500)(750)(750^2 + 500^2 - 2250^2)}{6EI(2250)}$$

$$= \frac{-6.37(10)^{10}}{EI} \qquad \text{(iii)}$$

The deflection $(\delta_B)_B$ at the point B due to the force at B is given by Eq. (a).

$$x = 1\,750 \text{ mm} \qquad (0 < x < a)$$

$$(\delta_B)_B = \frac{Pbx(x^2 + b^2 - l^2)}{6EIl}$$

$$(\delta_B)_B = \frac{(539.55)(500)(1750)(1750^2 + 500^2 - 2250^2)}{6EI(2250)}$$

$$= \frac{-6.12(10)^{10}}{EI} \qquad \text{(iv)}$$

Superimposing the deflections,

$$\delta_A = (\delta_A)_A + (\delta_A)_B$$

$$= \frac{-6.44(10)^{10}}{EI} + \frac{-6.37(10)^{10}}{EI} = \frac{-12.81(10)^{10}}{EI}$$

$$\delta_B = (\delta_B)_A + (\delta_B)_B$$

$$= \frac{-4.05(10)^{10}}{EI} + \frac{-6.12(10)^{10}}{EI} = \frac{-10.17(10)^{10}}{EI}$$

The negative sign indicates downward deflection and it is neglected.

$$I = \frac{\pi d^4}{64} = \frac{\pi (50)^4}{64} \text{ mm}^4$$

Substituting values of I and E,

$$\delta_A = \frac{-12.81(10)^{10}}{EI} = \frac{-12.81(10)^{10}}{(207\,000)\left(\dfrac{\pi (50)^4}{64}\right)}$$

$$= 2.017 \text{ mm} = 2.017(10^{-3}) \text{ m}$$

$$\delta_B = \frac{-10.17(10)^{10}}{EI} = \frac{-10.17(10)^{10}}{(207\,000)\left(\dfrac{\pi (50)^4}{64}\right)}$$

$$= 1.601 \text{ mm} = 1.601(10^{-3}) \text{ m}$$

Step II *Critical speed of shaft*

From Eq. (9.57),

$$\omega_n = \sqrt{\frac{g(W_1\delta_1 + W_2\delta_2)}{(W_1\delta_1^2 + W_2\delta_2^2)}}$$

$$= \sqrt{\frac{(9.81)\left[(343.35)(2.017)(10^{-3}) + (539.55)(1.601)(10^{-3})\right]}{\left[(343.35)(2.017)^2(10^{-3})^2 + (539.55)(1.601)^2(10^{-3})^2\right]}}$$

$$= \sqrt{\frac{9.81(1556.36)(10)^{-3}}{2779.82(10)^{-6}}}$$

$= 74.11$ rad/s

(1 revolution $= 2\pi$ radians)

$$= \frac{74.11(60)}{2\pi} = 707.7 \text{ rpm}$$

Short-Answer Questions

9.1 What is the function of transmission shaft?

9.2 Why is transmission shaft stepped?

9.3 What is fillet? Why is shaft provided with fillet radius?

9.4 How are commercial shafts made?

9.5 What are the disadvantages of cold drawn shafts?

9.6 What types of stresses are induced in shafts?

9.7 Which theories of failure are applicable for shafts? Why?

9.8 Define equivalent torsional moment and equivalent bending moment. State when these two terms are used in the design of shafts.

9.9 What do you understand by torsional rigidity?

9.10 What do you understand by lateral rigidity?

9.11 What is the permissible angle of twist for line shafts?

9.12 What is the permissible shear stress as per the ASME Code?

9.13 What are the advantages of hollow shaft over solid shaft?

9.14 What are the disadvantages of hollow shaft over solid shaft?

9.15 Give any two examples where hollow shafts are used.

9.16 How are hollow shafts generally manufactured?

9.17 What is flexible shaft? Where do you use it?

9.18 What are the functions of key?

9.19 How is keyway made?

9.20 What is the disadvantage of keyed joint?

9.21 What is saddle key? What are the types of saddle keys?

9.22 What are the advantages and disadvantages of saddle key over flat key?

9.23 What is sunk key? What are the types of sunk keys?

9.24 What is parallel sunk key?

9.25 What is taper sunk key?

9.26 Why is taper given to key? Why is taper given only on one side?

9.27 What is Gib-head taper sunk key? What is its advantage?

9.28 What are the advantages and disadvantages of taper key over parallel key?

9.29 What is the standard taper for sunk key?

9.30 What is feather key? Give its applications.

9.31 What are the advantages and disadvantages of feather key over flat key?

9.32 What is Woodruff key? Give its applications.

9.33 What are the advantages and disadvantages of Woodruff key over flat key?

9.34 What is Kennedy key? Give its applications.

9.35 What are the advantages and disadvantages of Kennedy key over flat key?

9.36 What are splines? Where do you use them?

9.37 What is the difference between splines and keys?

9.38 What is coupling? Where do you use it?

9.39 What is the difference between coupling and clutch?

9.40 What is the difference between rigid and flexible couplings?

9.41 Give at least three practical applications of couplings.

9.42 What is Muff coupling? Give its applications.

9.43 What are the advantages and disadvantages of Muff coupling?

9.44 What is clamp coupling? Give its applications.

9.45 What are the advantages and disadvantages of clamp coupling?

9.46 How does the working of clamp coupling differ from that of Muff coupling?

9.47 What is the purpose of spigot and recess in rigid flange coupling?

9.48 What is the difference between protected and unprotected rigid flange couplings?

9.49 What are the advantages and disadvantages of rigid flange coupling?

9.50 What is the purpose of the rubber bush in bushed-pin flexible coupling?

9.51 What are the causes of misalignment between two connecting shafts?

9.52 What are the types of misalignment between two connecting shafts?

9.53 What are the advantages and disadvantages of bushed-pin flexible coupling?

9.54 State Castigliano's theorem. Where do you use it?

9.55 What is the critical speed of shaft?

9.56 What are the first and second critical speeds of shaft?

Problems for Practice

9.1 A solid circular shaft of diameter d is subjected to a bending moment of M_b and a torsional moment of M_t. Prove that according to the maximum principal stress theory,

$$\frac{S_{yt}}{(fs)} = \frac{16}{\pi d^3}\left[M_b + \sqrt{(M_b)^2 + (M_t)^2}\right]$$

9.2 A solid circular shaft of diameter d is subjected to a bending moment of M_b and torsional moment of M_t. Prove that according to maximum shear stress theory,

$$\frac{0.5\ S_{yt}}{(fs)} = \frac{16}{\pi\ d^3}\sqrt{(M_b)^2 + (M_t)^2}$$

9.3 A solid circular shaft of diameter d is subjected to a torsional moment of M_t over a length of l. The permissible angle of twist is θ degrees. Prove that the shaft diameter is given by,

$$d = \left[\frac{584\ M_t l}{G\theta}\right]^{1/4}$$

9.4 A hollow circular shaft of outer and inner diameters of d_0 and d_i respectively is subjected to a bending moment of M_b and a torsional moment of M_t. Prove that according to the maximum principal stress theory,

$$\frac{S_{yt}}{(fs)} = \frac{16}{\pi d_0^3 (1 - C^4)} \left[M_b + \sqrt{(M_b)^2 + (M_t)^2} \right]$$

where $C = d_i / d_0$

9.5 A hollow circular shaft of outer and inner diameters of d_o and d_i respectively is subjected to a bending moment of M_b and a torsional moment of M_t. Prove that according to the maximum shear stress theory,

$$\frac{0.5\, S_{yt}}{(fs)} = \frac{16}{\pi d_0^3 (1 - C^4)} \sqrt{(M_b)^2 + (M_t)^2}$$

where, $C = d_i / d_0$

9.6 A hollow circular shaft of outer and inner diameters of d_0 and d_i respectively is subjected to a torsional moment of M_t over a length l. The permissible angle of twist is θ degrees. Prove that the shaft diameter is given by,

$$d_0 = \left[\frac{584\, M_t l}{G\theta (1 - C^4)} \right]^{1/4}$$

where, $C = d_i / d_0$

9.7 A centrifugal pump is driven by 10 kW power 1440 rpm electric motor. There is a reduction gearbox between the motor and the pump. The pump shaft rotates at 480 rpm. The design torque is 150% of the rated torque. The motor and pump shafts are made of plain carbon steel 40C8 (S_{yt} = 380 N/mm^2) and the factor of safety is 4. Assume ($S_{sy} = 0.5 S_{yt}$)
Calculate:
(i) diameter of the motor shaft ;and
(ii) diameter of the pump shaft.

[*(i) 22.01 mm, (ii) 31.75 mm*]

9.8 A transmission shaft is supported between two bearings, which are 750 mm apart. Power is supplied to the shaft through a coupling, which is located to the left of the left-hand bearing. Power is transmitted from the shaft by means of a belt pulley, 450 mm in diameter, which is located at a distance of 200 mm to the right of the left-hand bearing. The weight of the pulley is 300 N and the ratio of the belt tension of tight and slack sides is 2:1. The belt tensions act in vertically downward direction. The shaft is made of steel FeE 300 (S_{yt} = 300 N/mm^2) and the factor of safety is 3. Determine the shaft diameter, if it transmits 12.5 kW power at 300 rpm from the coupling to the pulley. Assume ($S_{sy} = 0.5 S_{yt}$)

[*45.31 mm*]

9.9 A rotating shaft, 40 mm in diameter, is made of steel FeE 580 (S_{yt} = 580 N/mm^2). It is subjected to a steady torsional moment of 250 N-m and bending moment of 1250 N-m. Calculate the factor of safety based on,
(i) maximum principal stress theory; and
(ii) maximum shear stress theory.

[*(i) 2.89, (ii) 2.86*]

9.10 A propeller shaft is required to transmit 50 kW power at 600 rpm. It is a hollow shaft, having an inside diameter 0.8 times of the outside diameter. It is made of steel (S_{yt} = 380 N/mm^2) and the factor of safety is 4. Calculate the inside and outside diameters of the shaft. Assume ($S_{sy} = 0.5 S_{yt}$)

[*41.98 and 52.48 mm*]

9.11 An intermediate shaft of a gearbox, supporting two spur gears A and B and mounted between two bearings C_1 and C_2, is shown in Fig. 9.56. The pitch circle diameters of gears A and B are 500 and 250 mm respectively. The shaft is made of alloy steel 20MnCr5. (S_{ut} = 620 and S_{yt} = 480 N/mm^2). The factors k_b and k_t of the ASME code are 2 and 1.5 respectively. The gears are keyed to the shaft. Determine the shaft diameter using the ASME code.

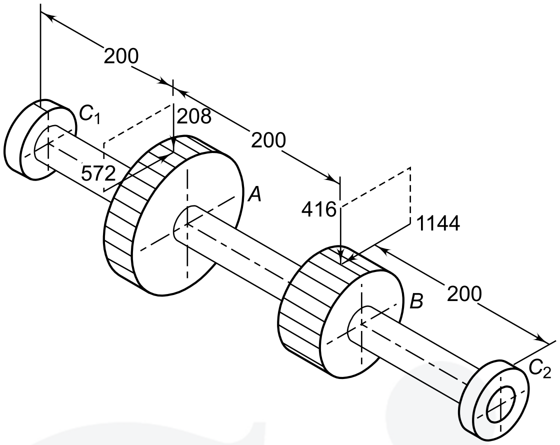


Fig. 9.56

[*27.15 mm*]

9.12 Assume the data of the intermediate shaft illustrated in Example 9.11. The permissible angle of twist for the shaft is 0.25° per metre length and the modulus of rigidity is 79 300 N/mm^2. Determine the shaft diameter on the basis of torsional rigidity.

[*45.3 mm*]

9.13 Consider the forces acting on the intermediate shaft illustrated in Example 9.11. The maximum permissible radial deflection at any gear is limited to (0.0l m), where *m* is the module. The module of the two spur gears is 10 mm and the modulus of elasticity of the shaft material is 207 000 N/mm^2. The shaft is simply supported at the bearings. Determine the radial deflections at gears *A* and *B* and find out the shaft diameter on the basis of lateral rigidity.

$$\left[\frac{25.42\times 10^8}{EI}, \frac{31.23\times 10^8}{EI} \quad \textit{and} \quad 41.87 \ mm\right]$$

9.14 A transmission shaft, supporting two pulleys *A* and *B* and mounted between two bearings C_1 and C_2 is shown in Fig. 9.57. Power is transmitted from the pulley *A* to *B*. The shaft is made of plain carbon steel 45C8 (S_{ut} = 600 and S_{yt} = 380 N/mm^2). The pulleys are keyed to the shaft. Determine the shaft diameter using the ASME code if,

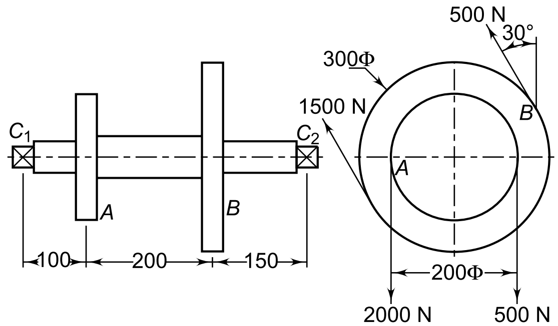


Fig. 9.57

k_b = 1.5 and k_t = 1.0

Also, determine the shaft diameter on the basis of torsional rigidity, if the permissible angle of twist between the two pulleys is 0.5° and the modulus of rigidity is 79 300 N/mm^2

[*25.35 and 25.78 mm*]

9.15 The cross-section of a flat key for a 40 mm diameter shaft is 22 × 14 mm. The power transmitted by the shaft to the hub is 25 kW at 300 rpm The key is made of steel (S_{yc} = S_{yt} = 300 N/mm^2) and the factor of safety is 2.8. Determine the length of the key.
Assume (S_{sy} = 0.577 S_{yt})

[*53.05 mm*]

9.16 It is required to design a square key for fixing a pulley on the shaft, which is 50 mm in diameter. The pulley transmits 10 kW power at 200 rpm to the shaft. The key is made of steel 45C8 (S_{yt} = S_{yc} = 380 N/mm^2) and the factor of safety is 3. Determine the dimensions of the key.
Assume (S_{sy} = 0.577S_{yt})

[*12.5 × 12.5 × 25 mm*]

9.17 A flat key is used to connect a pulley to a 45 mm diameter shaft. The standard cross section of the key is 14 × 9 mm. The key is made of commercial steel (S_{yt} = S_{yc} = 230 N/mm^2) and the factor of safety is 3. Determine the length of the key on the basis of shear and compression considerations, if 15 kW power at 360 rpm is transmitted through the keyed joint.

Assume ($S_{sy} = 0.5S_{yt}$)

[*32.95 and 51.26 mm*]

9.18 A standard splined connection 8 × 36 × 40 is used for a gear and shaft assembly rotating at 700 rpm. The dimensions of the splines are as follows:

Major diameter = 40 mm
Minor diameter = 36 mm
Number of splines = 8

The length of the gear hub is 50 mm and the normal pressure on the splines is limited to 6.5 N/mm^2. Calculate the power that can be transmitted from the gear to the shaft.

[*7.24 kW*]

9.19 A rigid coupling is used to connect a 45 kW, 1440 rpm electric motor to a centrifugal pump. The starting torque of the motor is 225% of the rated torque. There are 8 bolts and their pitch circle diameter is 150 mm. The bolts are made of steel 45C8 (S_{yt} = 380 N/mm^2) and the factor of safety is 2.5. Determine the diameter of the bolts.
Assume ($S_{sy} = 0.577S_{yt}$)
Assume that the bolts are finger tight in reamed and ground holes.

[*4.03 mm*]

9.20 The following specifications are given for a rigid coupling:

outer diameter of flanges = 160 mm
diameter of recess = 95 mm
number of bolts = 6
pre-load of each bolt = 10 kN
coefficient of friction = 0.15
speed of rotation = 100 rpm

The bolts are fitted in large clearance holes. Calculate the power transmitting capacity of the coupling.

[*6.14 kW*]

9.21 A protective flange coupling is used to connect two shafts and transmit 7.5 kW power at 720 rpm. The design torque is 150% of the rated torque. The shafts and bolts are made of plain carbon steel 30C8 (S_{yt} = 400 N/mm^2) and the factor of safety is 5. Assume,

$$S_{yc} = 1.5\ S_{yt} \quad \text{and} \quad S_{sy} = 0.5\ S_{yt}$$

The flanges are made of cast iron.
Calculate:
(i) diameter of the shafts;
(ii) number of bolts; and
(iii) diameter of the bolts.

[*(i) 30 (26.68) mm, (ii) 3 (iii) 6 (5.93) mm*]

9.22 A bushed pin type flexible coupling is used to connect two shafts and transmit 5 kW power at 720 rpm Shafts, keys and pins are made of commercial steel ($S_{yt} = S_{yc}$ = 240 N/mm^2) and the factor of safety is 3. The flanges are made of grey cast iron FG 200 (S_{ut} = 200 N/mm^2) and the factor of safety is 6. Assume,

$$S_{sy} = 0.5S_{yt} \quad \text{and} \quad S_{su} = 0.5S_{ut}$$

There are 4 pins. The pitch circle diameter of the pins is four times the shaft diameter. The permissible shear stress for the pins is 35 N/mm^2. The permissible bearing pressure for the rubber bushes is 1 N/mm^2. The keys have a square cross-section.
Calculate:
(i) diameter of the shafts;
(ii) dimensions of the key;
(iii) diameter of the pins; and
(iv) outer diameter and effective length of the bushes.

[*(i) 22 (20.36) mm, (ii) 6 × 6 × 33 mm, (iii) 9 (8.96) mm, (iv) 20 (19.41) mm each*]

9.23 A steel shaft carrying two weights of 200 and 3500 N is shown in Fig. 9.58. Neglect the weight of the shaft and estimate the first critical speed of the shaft. (E = 207 000 N/mm^2)

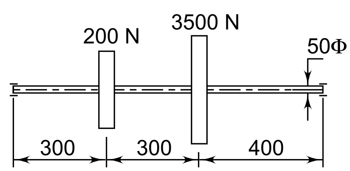


Fig. 9.58

[*903.86 rpm*]

Springs

Chapter 10

10.1 SPRINGS

A spring is defined as an elastic machine element, which deflects under the action of the load and returns to its original shape when the load is removed. It can take any shape and form depending upon the application. The important functions and applications of springs are as follows:

(i) Springs are used to absorb shocks and vibrations, e.g., vehicle suspension springs, railway buffer springs, buffer springs in elevators and vibration mounts for machinery.

(ii) Springs are used to store energy, e.g., springs used in clocks, toys, movie-cameras, circuit breakers and starters.

(iii) Springs are used to measure force, e.g., springs used in weighing balances and scales.

(iv) Springs are used to apply force and control motion. There are a number of springs used for this purpose. In the cam and follower mechanism, spring is used to maintain contact between the two elements. In engine valve mechanism, spring is used to return the rocker arm to its normal position when the disturbing force is removed. The spring used in clutch provides the required force to engage the clutch. In all these applications, the spring is used either to apply the force or to control the motion.

10.2 TYPES OF SPRINGS

Springs are classified according to their shape. The shape can be a helical coil of a wire, a piece of stamping or a flat wound-up strip. The most popular type of spring is the *helical spring*. The helical spring is made from a wire, usually of circular cross-section, which is bent in the form of a helix. There are two basic types of helical springs—*compression spring* and *extension spring* as shown in Fig. 10.1. In helical compression spring, the external force

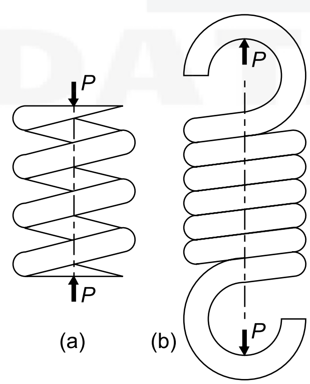


Fig. 10.1 *Helical Springs: (a) Compression Spring (b) Extension Spring*

tends to shorten the spring. In other words, the spring is compressed. In helical extension spring, the external force tends to lengthen the spring. In other words, the spring is elongated. In both the cases, the external force acts along the axis of the spring and induces torsional shear stress in the spring wire. It should be noted that although the spring is under compression, the wire of helical compression spring is not subjected to compressive stress. Also, the wire of helical extension spring is not subjected to tensile stress although the spring is under tension. In both cases, torsional shear stresses are induced in the spring wire. The words '*compression*' and '*extension*' are related to the total spring and not the stresses in spring wire.

In helical extension spring, the coils are wound tightly together, so that an initial force is required before extension begins. The helical springs are sometimes classified as closely-coiled helical spring and open-coiled helical spring. The difference between them is as follows:

(i) A helical spring is said to be closely coiled spring, when the spring wire is coiled so close that the plane containing each coil is almost at right angles to the axis of the helix. In other words, the helix angle is very small. It is usually less than 10°.

(ii) A helical spring is said to be open-coiled spring, when the spring wire is coiled in such a way, that there is large gap between adjacent coils. In other words, the helix angle is large. It is usually more than 10°.

There are few applications of open-coiled helical springs compared with closely-coiled helical springs. The analysis in this chapter is restricted to closely coiled-helical springs (CCHS). Helical springs, compression as well as extension, have the following advantages:

(i) They are easy to manufacture.
(ii) They are cheaper than other types of springs.
(iii) Their reliability is high.
(iv) The deflection of the spring is linearly proportional to the force acting on the spring.

It is due to the above advantages that helical springs are popular and extensively used in a number of applications.

A *helical torsion spring* is shown in Fig. 10.2. The construction of this spring is similar to that of compression or extension spring, except that the ends are formed in such a way that the spring is loaded by a torque about the axis of the coils. Helical torsion spring is used to transmit torque to a particular component in the machine or the mechanism. For example, the spring shown in Fig. 10.2 transmits a torque of ($P \times r$). Helical torsion spring is used in door-hinges, brush holders, automobile starters and door locks. The helical torsion spring resists the bending moment ($P \times r$), which tends to wind up the spring. The bending moment induces bending stresses in the spring wire. The term 'torsion spring' is somewhat misleading because the wire is subjected to bending stresses, unlike torsional shear stresses induced in helical compression or extension springs. It should be noted that although the spring is subjected to torsional moment, the wire of a helical torsion spring is not subjected to torsional shear stress. It is subjected to bending stresses.

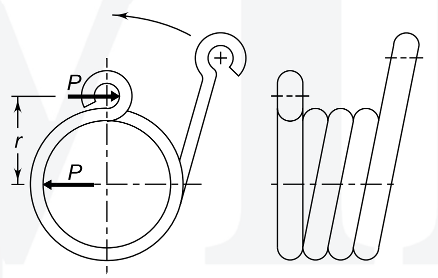


Fig. 10.2 *Helical Torsion Spring*

A *multi-leaf* or *laminated spring* consists of a series of flat plates, usually of semi-elliptical shape as shown in Fig. 10.3. The flat plates, called *leaves*,

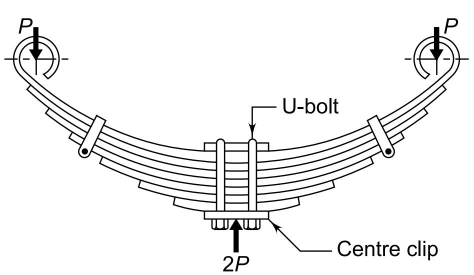


Fig. 10.3 *Semi-elliptic Leaf Spring*

have varying lengths. The leaves are held together by means of U-bolts and a centre clip. The longest leaf, called the *master leaf*, is bent at the two ends to form spring eyes. The leaves of multi-leaf spring are subjected to bending stresses. Multi-leaf springs are widely used in automobile and railroad suspensions.

In addition to the above mentioned types of springs, there are other springs such as helical springs of rectangular cross-section, spiral torsion springs, disk or belleville springs and volute springs. The discussion in this chapter is mainly restricted to helical springs and leaf springs.

10.3 TERMINOLOGY OF HELICAL SPRINGS

The main dimensions of a helical spring subjected to compressive force are shown in Fig. 10.4. They are as follows:

d = wire diameter of spring (mm)
D_i = inside diameter of spring coil (mm)
D_o = outside diameter of spring coil (mm)
D = mean coil diameter (mm)

Therefore,

$$D = \frac{D_i + D_o}{2} \tag{10.1}$$

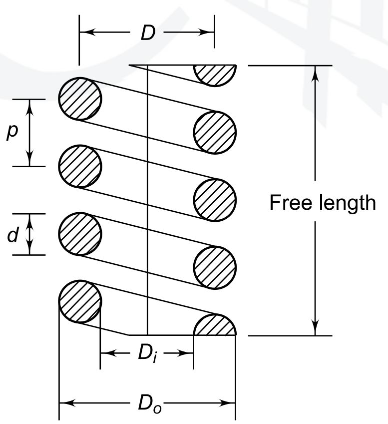


Fig. 10.4 *Dimensions of Spring*

There is an important parameter in spring design called *spring index*. It is denoted by the letter C. *The spring index is defined as the ratio of mean coil diameter to wire diameter*. Or,

$$C = \frac{D}{d} \tag{10.2}$$

In the design of helical springs, the designer should use good judgement in assuming the value of the spring index C. The spring index indicates the relative sharpness of the curvature of the coil. A low spring index means high sharpness of curvature. When the spring index is low ($C < 3$), the actual stresses in the wire are excessive due to curvature effect. Such a spring is difficult to manufacture and special care in coiling is required to avoid cracking in some wires. When the spring index is high ($C > 15$), it results in large variation in the coil diameter. Such a spring is prone to buckling and also tangles easily during handling. A spring index from 4 to 12 is considered best from manufacturing considerations. Therefore, in practical applications, the spring index usually varies from 4 to 12. However, a spring index in the range of 6 to 9 is still preferred particularly for close tolerance springs and those subjected to cyclic loading.

There are three terms—*free length, compressed length* and *solid length*, which are illustrated in Fig. 10.5. These terms are related to helical compression spring. These lengths are determined by the following way:

(i) Solid Length Solid length is defined as the axial length of the spring which is so compressed that the adjacent coils touch each other. In this case, the spring is completely compressed and no further compression is possible. The solid length is given by,

Solid length = $N_t\, d$

where,

N_t = total number of coils

(ii) Compressed Length Compressed length is defined as the axial length of the spring, which is subjected to maximum compressive force. In this case, the spring is subjected to maximum deflection δ. When the spring is subjected to maximum force, there should be some gap or clearance between the adjacent coils. The gap is essential to prevent clashing of the coils. The clashing allowance or the total axial gap is usually taken as 15% of the maximum deflection. Sometimes, an arbitrary decision is taken and it is assumed that there is a gap of 1 or 2 mm between adjacent coils under maximum load condition. In this case, the total axial gap is given by,

Total gap = $(N_t - 1) \times$ Gap between adjacent coils

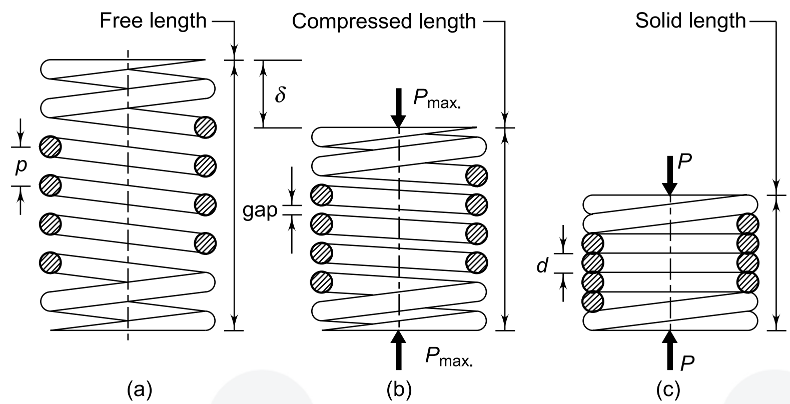


Fig. 10.5 *Spring Length Terminology*

(iii) Free Length Free length is defined as the axial length of an unloaded helical compression spring. In this case, no external force acts on the spring. Free length is an important dimension in spring design and manufacture. It is the length of the spring in free condition prior to assembly. Free length is given by,

free length = compressed length + δ

= solid length + total axial gap + δ

The *pitch of the coil* is defined as the axial distance between adjacent coils in uncompressed state of spring. It is denoted by p. It is given by,

$$p = \frac{\text{free length}}{(N_t - 1)}$$

The *stiffness of the spring* (k) is defined as the force required to produce unit deflection. Therefore,

$$k = \frac{P}{\delta} \tag{10.3}$$

where,

k = stiffness of the spring (N/mm)

P = axial spring force (N)

δ = axial deflection of the spring corresponding to the force P (mm)

There are various names for stiffness of spring such as *rate of spring*, *gradient of spring*, *scale of spring* or *simply spring constant*. The stiffness of spring represents the slope of the load-deflection line.

There are two terms related to the spring coils, viz., active coils and inactive coils. *Active coils* are the coils in the spring which contribute to spring action, support the external force and deflect under the action of force. A portion of the end coils, which is in contact with the seat, does not contribute to spring action and are called *inactive coils*. These coils do not support the load and do not deflect under the action of an external force. The number of inactive coils is given by,

inactive coils = $N_t - N$

where,

N = number of active coils.

10.4 STYLES OF END

There are four common methods which are used in forming the ends of the helical compression spring as shown in Fig. 10.6—plain ends, plain and ground ends, square ends and square and ground ends. The turns at the two ends do not affect the deflection calculated by the load-deflection equation. Therefore, while calculating the number of active turns, the end turns should be subtracted from the total number of turns. The number of active turns for different styles of end is as follows:

Type of ends	*Number of active turns (N)*
Plain ends	N_t
Plain ends (ground)	$\left(N_t - \frac{1}{2}\right)$
Square ends	$(N_t - 2)$
Square ends (ground)	$(N_t - 2)$

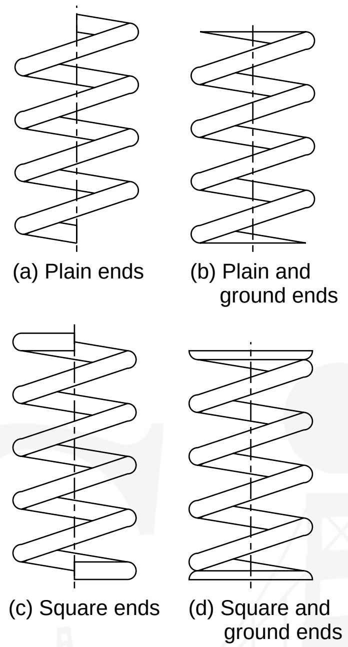


Fig. 10.6 *End Styles of Helical Compression Springs*

The different styles of end for the helical extension spring are shown in Fig. 10.7. The end should be designed in such a way that the stress concentration at the bend is minimum. Sometimes the effect of stress concentration in ends is so severe that the spring body becomes stronger than the end and failure occurs in the end coils. For helical extension ends, all coils are active coils. The number of active coils (N) is the same as the total number of coils (N_t).

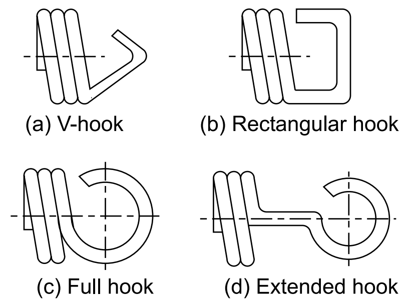


Fig. 10.7 *End Styles of Helical Extension Springs*

10.5 STRESS AND DEFLECTION EQUATIONS

There are two basic equations for the design of helical springs, viz., load-stress equation and load-deflection equation. A helical spring made from the wire of circular cross-section is shown in Fig. 10.8(a). D and d are the mean coil diameter and wire diameter respectively. The number of active coils in this spring is N. The spring is subjected to an axial force P. When the wire of the helical spring is uncoiled and straightened, it takes the shape of a bar as shown in Fig. 10.8(b). In deriving the stress equation, this bar is considered to be equivalent to the actual helical spring. The dimensions of equivalent bar are as follows:

(i) The diameter of the bar is equal to the wire diameter of the spring (d).

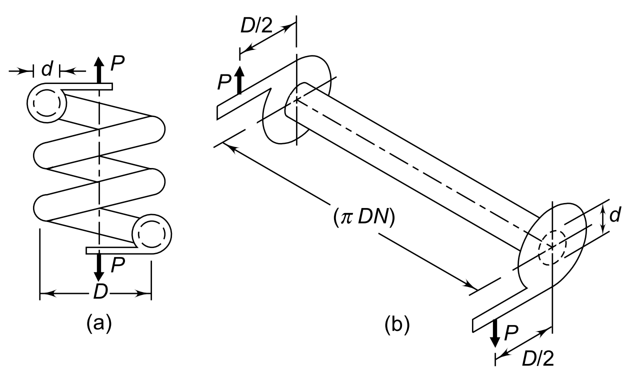


Fig. 10.8 *(a) Helical Spring (b) Helical Spring-unbent*

(ii) The length of one coil in the spring is (πD). There are N such active coils. Therefore, the length of equivalent bar is (πDN).

(iii) The bar is fitted with a bracket at each end. The length of this bracket is equal to mean coil radius of the spring ($D/2$).

The force P acting at the end of the bracket induces torsional shear stress in the bar. The torsional moment M_t is given by,

$$M_t = \frac{PD}{2}$$

The torsional shear stress in the bar is given by,

$$\tau_1 = \frac{16M_t}{\pi d^3} = \frac{16(PD/2)}{\pi d^3}$$

or
$$\tau_1 = \frac{8PD}{\pi d^3} \qquad \text{(a)}$$

When the equivalent bar is bent in the form of helical coil, there are additional stresses on account of following two factors:

(i) There is direct or transverse shear stress in the spring wire.

(ii) When the bar is bent in the form of coil, the length of the inside fibre is less than the length of the outside fibre. This results in stress concentration at the inside fibre of the coil.

The resultant stress consists of superimposition of torsional shear stress, direct shear stress and additional stresses due to the curvature of the coil. The stresses in the spring wire on account of these factors are shown in Fig. 10.9.

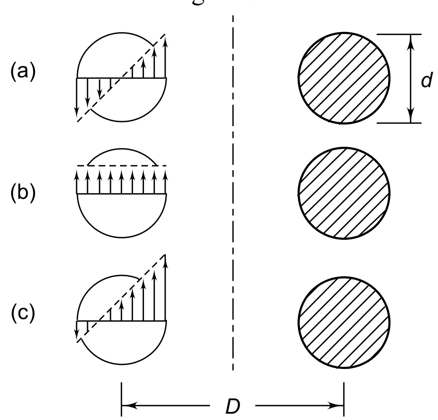


Fig. 10.9 *Stresses in Spring Wire: (a) Pure Torsional Stress (b) Direct Shear Stress (c) Combined Torsional, Direct and Curvature Shear Stresses*

Equation (a) does not take into consideration the effect of direct shear stress and stress concentration due to curvature effect. It requires modification. We will assume the following two factors to account for these effects:

K_S = factor to account for direct shear stress
K_c = factor to account for stress concentration due to curvature effect

The combined effect of these two factors is given by,

$$K = K_S K_C \qquad (10.4)$$

where K is the factor to account for the combined effect of two factors.

As shown in Fig. 10.8(b), the direct shear stress in the bar is given by,

$$\tau_2 = \frac{P}{\left(\frac{\pi}{4}d^2\right)} = \frac{4P}{\pi d^2} = \frac{8PD}{\pi d^3}\left(\frac{0.5d}{D}\right) \qquad \text{(b)}$$

Superimposing the two stresses of expressions (a) and (b), the resultant shear stress in the spring wire is given by,

$$\tau = \tau_1 + \tau_2 = \frac{8PD}{\pi d^3} + \frac{8PD}{\pi d^3}\left(\frac{0.5d}{D}\right) = \frac{8PD}{\pi d^3}\left(1 + \frac{0.5d}{D}\right) \qquad \text{(c)}$$

The shear stress correction factor (K_s) is defined as,

$$K_s = \left(1 + \frac{0.5d}{D}\right) \qquad \text{(d)}$$

or
$$K_s = \left(1 + \frac{0.5}{C}\right) \qquad (10.5)$$

Substituting the above equation in the expression (c),

$$\tau = K_s\left(\frac{8PD}{\pi d^3}\right) \qquad \text{(e)}$$

AM Wahl[1] derived the equation for resultant stress, which includes torsional shear stress, direct shear stress and stress concentration due to curvature. This equation is given by,

$$\tau = K\left(\frac{8PD}{\pi d^3}\right) \qquad (10.6)$$

where K is called the *stress factor* or *Wahl factor*.

The Wahl factor is given by,

[1] AM Wahl—*Mechanical Springs*—McGraw-Hill Inc., 1963.

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} \qquad (10.7)$$

where C is the spring index.

The Wahl factor provides a simple method to find out resultant stresses in the spring. As shown in Fig. 10.9(c), the resultant shear stress is maximum at the inside radius of the coil.

In normal applications, the spring is designed by using the Wahl factor. When the spring is subjected to fluctuating stresses, two factors K_s and K_c are separately used.

The angle of twist (θ) for the equivalent bar, illustrated in Fig. 10.8(b), is given by,

$$\theta = \frac{M_t\, l}{JG} \qquad (f)$$

where,

θ = angle of twist (radians)
M_t = torsional moment ($PD/2$)
l = length of bar (πDN)
J = polar moment of inertia of bar ($\pi d^4/32$)
G = modulus of rigidity

Substituting values in Eq. (f),

$$\theta = \frac{(PD/2)(\pi DN)}{(\pi d^4/32)\,G}$$

or
$$\theta = \frac{16PD^2N}{Gd^4} \qquad (g)$$

As shown in Fig. 10.10, the axial deflection 'δ' of the spring, for small values of θ, is given by,

$$\delta = \theta \times (\text{length of bracket})$$
$$= \theta \times (D/2) \qquad (h)$$

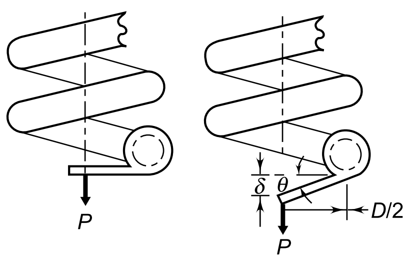


Fig. 10.10 *Deflection of Spring*

Substituting value of θ from Eq. (g) in Eq. (h),

$$\delta = \frac{8PD^3N}{Gd^4} \qquad (10.8)$$

The above equation is called the load-deflection equation.

The rate of spring (k) is given by,

$$k = \frac{P}{\delta}$$

Substituting Eq. (10.8) in the above expression,

$$k = \frac{Gd^4}{8D^3N} \qquad (10.9)$$

When a helical spring is cut into two parts, the parameters G, d and D remain same and N becomes (N/2). It is observed from Eq. (10.9), that the stiffness (k) will be double when N becomes (N/2).

It is observed from Eq. (10.8) that for a given spring,

$$\delta \propto P$$

or
$$P \propto \delta$$

The load is linearly proportional to the deflection of the spring. The load-deflection curve for helical spring is shown in Fig. 10.11. The area below the load-deflection line gives the strain energy stored in the spring. Assuming that the load is gradually applied, the energy stored in the spring is given by,

E = area below load-deflection line

$$= \text{area of triangle } OAB = \frac{1}{2}\overline{OB} \times \overline{BA} = \frac{1}{2}\delta P$$

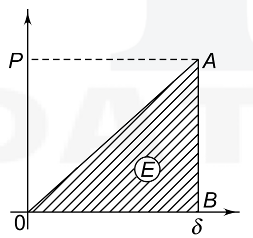


Fig. 10.11 *Load-deflection Diagram*

or,
$$E = \frac{1}{2}P\,\delta \qquad (10.10)$$

where,

E = strain energy stored in spring (N-mm)

10.6 SERIES AND PARALLEL CONNECTIONS

There are two types of spring connections—series and parallel. The objectives of series and parallel combinations are as follows:

(i) to save the space;
(ii) to change the rate of the spring at a certain deflection; and
(iii) to provide a fail-safe system.

Figure 10.12 shows two springs, with spring rates k_1 and k_2, connected in series. For series connection,

(i) The force acting on each spring is same and equal to the external force
(ii) The total deflection of the spring combination is equal to the sum of the deflections of individual springs

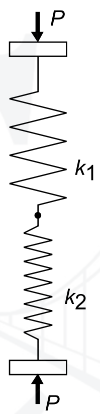


Fig. 10.12 *Springs in Series*

Therefore,

$$\delta = \delta_1 + \delta_2 \qquad \text{(a)}$$

where δ_1 and δ_2 are the deflections of the two springs.

From Eq. (10.3),

$$\delta = \frac{P}{k}$$

or,
$$\delta_1 = \frac{P}{k_1} \text{ and } \delta_2 = \frac{P}{k_2} \qquad \text{(b)}$$

Substituting Eq. (b) in Eq. (a),

$$\frac{P}{k} = \frac{P}{k_1} + \frac{P}{k_2}$$

or
$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots$$

where k is the combined stiffness of all springs in the connection.

The above expression can be written in the following form:

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{k_1 + k_2}{k_1 k_2} \text{ or } k = \frac{k_1 k_2}{k_1 + k_2}$$

$$\therefore \quad \frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \text{ or } k = \frac{k_1 k_2}{k_1 + k_2} \qquad (10.11)$$

Figure 10.13 shows two springs, with spring rates k_1 and k_2, connected in parallel. For parallel connection,

(i) The force acting on the spring combination is equal to the sum of forces acting on individual springs
(ii) The deflection of individual springs is same and equal to the deflection of the combination

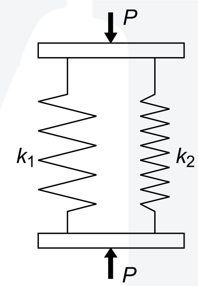


Fig. 10.13 *Springs in Parallel*

Therefore,

$$P = P_1 + P_2 \qquad \text{(c)}$$

From Eq. (10.3),

$$P = k\delta$$

or
$$P_1 = k_1\delta \quad \text{and} \quad P_2 = k_2\delta \qquad \text{(d)}$$

From Eqs (c) and (d),

$$k\delta = k_1\delta + k_2\delta$$

or
$$k = k_1 + k_2 + \cdots \qquad (10.12)$$

where k is the combined stiffness of all springs in the connection.

It is observed that deflections are additive in a series combination, while forces are additive in a parallel combination. Using this principle, the individual springs can be designed.

10.7 SPRING MATERIALS

The selection of material for the spring wire depends upon the following factors:

(i) The load acting on the spring
(ii) The range of stress through which the spring operates
(iii) The limitations on mass and volume of spring
(iv) The expected fatigue life
(v) The environmental conditions in which the spring will operate such as temperature and corrosive atmosphere
(vi) The severity of deformation encountered while making the spring.

There are four basic varieties of steel wire which are used in springs in the majority of applications[2, 3, 4]:

(i) patented and cold-drawn steel wires (unalloyed);
(ii) oil-hardened and tempered spring steel wires and valve spring wires;
(iii) oil-hardened and tempered steel wires (alloyed); and
(iv) stainless steel spring wires.

The most extensively used spring material is high-carbon hard-drawn spring steel. It is often called '*patented* and *cold-drawn*' steel wire. There are two important terms related to patented and cold-drawn steel wires, namely, '*patenting*' and '*cold drawing*'. *Patenting* is defined as heating the steel to above the critical range followed by rapid cooling to transform at an elevated temperature from 455° to 465°C. This operation produces a tough uniform structure that is suitable for severe cold drawing. After this operation, the spring wire is produced from hot rolled rods by cold drawing through carbide dies to obtain the required diameter. The patented and cold drawn steel wires are made of high carbon steel and contain 0.85–0.95% carbon. It is considered an aristocrat among springs because it has high tensile strength, high elastic limit and the ability to withstand high stresses under repeated loadings. The patented and cold-drawn steel wires are the least expensive of all spring materials. The music spring wire is the highest quality of hard-drawn steel spring. The name 'music wire' is derived from the popular '*piano*' wire that was originally used in musical instruments.

The patented and cold-drawn steel wires are mainly used in springs subjected to static forces and moderate fluctuating forces. There are four grades of this wire. *Grade-1* is used in springs subjected to static or low-load cycles. *Grade-2* is used in springs subjected to moderate-load cycles. *Grade-3* is used in highly stressed static springs or springs subjected to moderate dynamic loads. *Grade-4* is suitable for springs subjected to severe stresses. The tensile strengths of these wires are given in Table 10.1. The modulus of rigidity (G) of these wires is 81370 N/mm^2.

The second group of spring wires is unalloyed oil-hardened and tempered spring steel wires and valve spring wires. Oil-hardened and tempered spring steel wires contain 0.55–0.75% carbon. The wire is cold drawn and then hardened and tempered. A valve spring wire contains 0.60–0.75% carbon. It is the highest quality of oil-hardened and tempered steel wire. It has excellent surface finish and considered to be most reliable for applications involving fluctuating forces. It is used for applications where the stresses are severe.

There are two grades of unalloyed, oil-hardened and tempered spring steel wire and valve spring wire, viz., SW and VW. *Grade SW* is suitable for springs subjected to moderate fluctuating stresses, whereas *Grade VW* is recommended when the spring is subjected to a high magnitude of fluctuating stresses. The tensile strengths of these wires are given in Table 10.2. The modulus of rigidity (G) is 81370 N/mm^2. The limiting temperatures for SW and VW grade wires are 100° and 80°C respectively.

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[2] IS 4454–1981: Specifications for steel wires for cold-formed springs (in four parts).
[3] IS 7906–1975: Helical compression springs (in five parts).
[4] IS 7907–1976: Helical extension springs (in four parts).

Table 10.1 *Mechanical properties of patented and cold-drawn steel wires*

Wire diameter d (mm)	Minimum tensile strength (N/mm^2)			
	Gr.1	Gr.2	Gr.3	Gr.4
0.3	1720	2060	2460	2660
0.4	1700	2040	2430	2620
0.5	1670	2010	2390	2580
0.6	1650	1990	2360	2550
0.7	1630	1970	2320	2530
0.8	1610	1950	2280	2480
0.9	1590	1920	2250	2440
1.0	1570	1900	2240	2400
1.2	1540	1860	2170	2340
1.4	1500	1820	2090	2290
1.6	1470	1780	2080	2250
1.8	1440	1750	2030	2190
2.0	1420	1720	1990	2160
2.5	1370	1640	1890	2050
3.0	1320	1570	1830	1980
3.6	1270	1510	1750	1890
4.0	1250	1480	1700	1840
4.5	1230	1440	1660	1800
5.0	1190	1390	1600	1750
6.0	1130	1320	1530	1670
7.0	1090	1260	1460	1610
8.0	1050	1220	1400	1540

There are two popular varieties of alloy steel wires, namely, chromium–vanadium steel and chromium–silicon steel. Chromium–vanadium steel contains 0.48–0.53% carbon, 0.80–1.10% chromium and 0.15% vanadium. These wires are used for applications involving higher stresses and for springs subjected to impact or shock loads, such as in pneumatic hammers. Chromium–silicon spring steel is new compared with other spring steels and it was originally developed for recoil springs in aircraft guns and for control springs in torpedos in England. Chromium–silicon steel contains 0.51–0.59% carbon, 0.60–0.80% chromium and 1.2–1.6% silicon. These wires are used for applications involving highly stressed springs subjected to shock or impact loading. Alloy steel wires are superior to carbon steel wires. However, the cost is their limiting factor. Stainless steel springs, which exhibit an excellent corrosion resistance, are ideal to work in steam or other corrosive media.

Table 10.2 *Mechanical properties of oil-hardened and tempered spring steel wire and valve spring wire (unalloyed)*

Wire diameter d (mm)	Minimum tensile strength (N/mm^2)	
	SW	VW
1.0	1760	1670
1.2	1720	1620
1.5	1670	1570
2.0	1620	1520
2.5	1570	1470
3.0	1520	1430
3.6	1480	1400
4.0	1480	1400
4.5	1440	1370
5.0	1440	1370
6.0	1400	1340
7.0	1360	1300
8.0	1290	–
10.0	1250	–

In order to compare the relative cost of spring wires, the following cost indices are useful.

Spring materials	*Cost index*
Hard-drawn spring steel wire	1.0
Music wire	3.5
Oil-hardened and tempered spring steel wire and valve spring wire	1.5
Alloy steel wire	4.0
Stainless steel wire	8.5

Springs are made either by hot-working or by cold-working processes, depending upon wire diameter, spring index and desirable properties. Winding of spring wire induces residual stresses due to bending. Very often, the spring is given a mild heat treatment after winding to relieve residual stresses. Cold formed springs are wound to a smaller diameter than the desired size because of spring back or expansion that occurs after coiling. Springs made of small diameter wires are wound cold. However, helical springs made of 6 mm diameter bar or larger are usually hot wound to avoid the high residual stresses that are induced by cold forming.

There are non-ferrous materials, such as spring brass, phosphor bronze, silicon–bronze, monel and beryllium–copper, which are also used in spring wires. The discussion in this chapter is restricted to springs made of steel wires.

10.8 DESIGN OF HELICAL SPRINGS

There are three objectives for the design of the helical spring. They are as follows:

(i) It should possess sufficient strength to withstand the external load.

(ii) It should have the required load-deflection characteristic.

(iii) It should not buckle under the external load.

It is possible to design a number of springs for a given application by changing the three basic parameters, viz., wire diameter, mean coil diameter and the number of active turns. However, there are practical limitations on these parameters. In certain applications, there are space limitations, e.g., the spring is to fit in a hole of certain diameter, where the outside coil diameter (D_o) is restricted. In some applications, the spring is to fit over a rod, where the minimum inside diameter (D_i) of the coil is specified. Before proceeding to design calculations, the designer should specify the limits on these diameters.

The main dimensions to be calculated in the spring design are wire diameter, mean coil diameter and the number of active coils. The first two are calculated by the load-stress equation, while the third is calculated by the load-deflection equation. It is convenient to use the load-stress equation, which contains spring index as a parameter.

From Eq. (10.6),

$$\tau = K\left(\frac{8PD}{\pi d^3}\right)$$

Substituting $\left(\dfrac{D}{d} = C\right)$ in the above equation,

$$\tau = K\left(\frac{8PC}{\pi d^2}\right) \tag{10.13}$$

Factor of Safety The factor of safety in the design of springs is usually 1.5 or less. The use of a relatively low factor of safety is justified on the following grounds:

(i) In most of the applications, springs operate with well defined deflections. Therefore, the forces acting on the spring and corresponding stresses can be precisely calculated. It is not necessary to take higher factor of safety to account for uncertainty in external forces acting on the spring.

(ii) In case of helical compression springs, an overload will simply close up the gaps between coils without a dangerous increase in deflection and stresses.

(iii) In case of helical extension springs, usually overload stops are provided to prevent excessive deflection and stresses.

(iv) The spring material is carefully controlled at all stages of manufacturing. The thin and uniform wire cross-section permits uniform heat treatment and cold working of the entire spring.

Therefore, the factor of safety based on torsional yield strength (S_{sy}) is taken as 1.5 for the springs that are subjected to static force.

$$\tau = \frac{S_{sy}}{1.5} \tag{a}$$

Assuming, $S_{yt} = 0.75 S_{ut}$ and $S_{sy} = 0.577 S_{yt}$

Expression (a) is written as,

$$\tau = \frac{(0.577)(0.75)S_{ut}}{1.5}$$

or
$$\tau \cong 0.3\ S_{ut} \tag{10.14}$$

The permissible shear stress is, therefore, 30% of the ultimate tensile strength of the spring wire. The Indian Standard 4454–1981 has recommended a much higher value for the permissible shear stress. According to this standard,

$$\tau = 0.5\ S_{ut} \tag{10.15}$$

This is due to higher tensile yield strengths exhibited by the spring wires. In design of helical springs, the permissible shear stress (τ) is taken from 30% to 50% of the ultimate tensile strength (S_{ut}).

The basic procedure for the design of helical spring consists of the following steps:

(i) For the given application, estimate the maximum spring force (P) and the corresponding required deflection (δ) of the spring. In some cases, maximum spring force (P) and stiffness k, which is (P/δ), are specified.

(ii) Select a suitable spring material and find out ultimate tensile strength (S_{ut}) from the data. Calculate the permissible shear stress for the spring wire by following relationship:

$$\tau = 0.30\ S_{ut} \text{ or } 0.50\ S_{ut}$$

(iii) Assume a suitable value for the spring index (C). For industrial applications, the spring index varies from 8 to 10. A spring index of 8 is considered as a good value. The spring index for springs in valves and clutches is 5. The spring index should never be less than 3.

(iv) Calculate the Wahl factor by the following equation:

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

(v) Determine wire diameter (d) by Eq. (10.13).

$$\tau = K\left(\frac{8\,PC}{\pi d^2}\right)$$

(vi) Determine mean coil diameter (D) by the following relationship:

$$D = Cd$$

(vii) Determine the number of active coils (N) by Eq. (10.8).

$$\delta = \frac{8PD^3N}{Gd^4}$$

The modulus of rigidity (G) for steel wires is 81 370 N/mm^2.

(viii) Decide the style of ends for the spring depending upon the configuration of the application. Determine the number of inactive coils. Adding active and inactive coils, find out the total number of coils (N_t).

(ix) Determine the solid length of the spring by the following relationship:

$$\text{Solid length} = N_t d$$

(x) Determine the actual deflection of the spring by Eq. (10.8).

$$\delta = \frac{8PD^3N}{Gd^4}$$

(xi) Assume a gap of 0.5 to 2 mm between adjacent coils, when the spring is under the action of maximum load. The total axial gap between coils is given by,

total gap = (N_t – 1) × gap between two adjacent coils

In some cases, the total axial gap is taken as 15% of the maximum deflection:

(xii) Determine the free length of the spring by the following relationship:

free length = solid length + total gap + δ

(xiii) Determine the pitch of the coil by the following relationship:

$$p = \frac{\text{free length}}{(N_t - 1)}$$

(xiv) Determine the rate of spring by Eq. (10.9).

$$k = \frac{Gd^4}{8D^3N}$$

(xv) Prepare a list of spring specifications.

A helical compression spring that is too long compared to the mean coil diameter, acts as a flexible column and may buckle at a comparatively low axial force. The spring should be preferably designed as buckle-proof. Compression springs, which cannot be designed buckle-proof, must be guided in a sleeve or over an arbor. The thumb rules for provision of guide are as follows:

$$\frac{\text{free length}}{\text{mean coil diameter}} \leq 2.6 \qquad \text{[Guide not necessary]}$$

$$\frac{\text{free length}}{\text{mean coil diameter}} > 2.6 \qquad \text{[Guide required]}$$

However, provision of guide results in friction between the spring and the guide and this may damage the spring in the long run.

10.9 SPRING DESIGN—TRIAL AND ERROR METHOD

In practice, helical springs are designed by trial and error method. The basic procedure of spring design explained in the previous article is based on the assumption that the value of tensile strength for a given grade of spring wire is constant. However, it is observed from Table 10.1 and 10.2 that tensile strength of the material is not constant, but it varies with wire diameter. Let us consider Grade-1 of patented and cold-drawn steel wire. From Table 10.1, the values of tensile strength for various wire diameters are as follows:

Wire diameter (mm)	*Minimum Tensile Strength (N/mm^2)*
0.3	1720
0.6	1650
1.0	1570
2.0	1420
3.0	1320
4.0	1250
5.0	1190
8.0	1050

It is observed from the above values that tensile strength decreases as wire diameter increases. Therefore, tensile strength is inversely proportion to wire diameter.

$$S_{ut} \propto \frac{1}{d} \qquad \text{(a)}$$

From Eqs (10.14) and (10.15),

$$\tau = 0.30\, S_{ut} \quad \text{or} \quad \tau = 0.50\, S_{ut} \qquad \text{(b)}$$

From (a) and (b),

$$\tau \propto \frac{1}{d} \qquad \text{(c)}$$

From Eqs (10.13),

$$\tau = K\left(\frac{8\,PC}{\pi d^2}\right) \qquad \text{(d)}$$

Therefore,

$$\tau \propto \frac{1}{d^2} \qquad \text{(e)}$$

From expressions (c) and (e), it is observed that permissible stress is inversely proportional to ($1/d$), while induced stress is proportional to ($1/d^2$). Both stresses depend upon wire diameter. Such problems can be solved only by trial and error of Eqs (b) and (d). The trial and error procedure consists of the following steps:

(i) Assume some wire diameter (d).
(ii) Find out the corresponding tensile strength from Table 10.1 or 10.2 and using this value find out permissible stress by Eq. (b).
(iii) Find out induced stress by Eq. (d).
(iv) Check up whether permissible stress is more than induced stress. If not, increase the wire diameter and repeat the above steps.
(v) The above mentioned procedure is repeated till the value of induced stress comes out to be less than permissible stress.

10.10 DESIGN AGAINST FLUCTUATING LOAD

In many applications, the force acting on the spring is not constant but varies in magnitude with time. The valve spring of an automotive engine is subjected to millions of stress cycles during its lifetime. On the other hand, the springs in linkages and mechanisms are subjected to comparatively less number of stress cycles. The springs subjected to fluctuating stresses are designed on the basis of two criteria—design for infinite life and design for finite life.

Let us consider a spring subjected to an external fluctuating force, which changes its magnitude from $P_{max.}$ to $P_{min.}$ in the load cycle. The mean force P_m and the force amplitude P_a are given by,

$$P_m = \frac{1}{2}(P_{max.} + P_{min.}) \qquad (10.16)$$

$$P_a = \frac{1}{2}(P_{max.} - P_{min.}) \qquad (10.17)$$

The mean stress (τ_m) is calculated from mean force (P_m) by using shear stress correction factor (K_s). It is given by,

$$\tau_m = K_s\left(\frac{8P_m D}{\pi d^3}\right) \qquad (10.18)$$

where, $K_s = \left(1 + \frac{0.5}{C}\right)$

K_s is the correction factor for direct shear stress and it is applicable to mean stress only. For torsional stress amplitude (τ_a), it is necessary to also consider the effect of stress concentration due to curvature in addition to direct shear stress. Therefore,

$$\tau_a = K_s\ K_c \left(\frac{8P_a D}{\pi d^3} \right)$$

$$\tau_a = K \left(\frac{8P_a D}{\pi d^3} \right) \qquad (10.19)$$

where K is the Wahl factor, which takes into consideration the effect of direct shear stress as well as of stress concentration due to curvature.

There is a basic difference between the rotating-beam specimen and fatigue testing of spring wires. A spring is never subjected to a completely reversed load, changing its magnitude from tension to compression and passing through zero with respect to time. A helical compression spring is subjected to purely compressive forces. On the other hand, a helical extension spring is subjected to purely tensile forces. In general, the spring wires are subjected to pulsating shear stresses, which vary from zero to (S'_{se}) as shown in Fig. 10.14. (S'_{se}) is the endurance limit in shear for the stress variation from zero to some maximum value. The data regarding the experimental values of endurance strength of spring wires is not readily available. In absence of such values, the following relationships suggested by HJ Elmendorf[5] can be used.

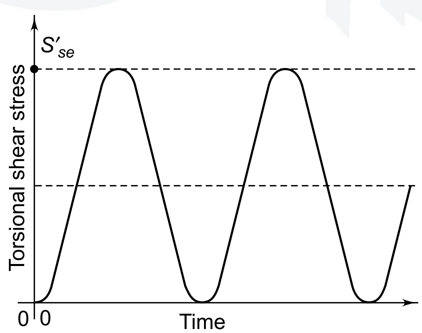


Fig. 10.14 *Pulsating Stress Cycle*

For Patented and cold-drawn steel wires (Grade-1 to 4),

$$S'_{se} = 0.21\ S_{ut}$$

$$S_{sy} = 0.42\ S_{ut} \qquad (10.20)$$

For oil-hardened and tempered steel wires (SW and VW grade),

$$S'_{se} = 0.22\ S_{ut}$$

$$S_{sy} = 0.45\ S_{ut} \qquad (10.21)$$

where S_{ut} is the ultimate tensile strength.

The fatigue diagram for the spring is shown in Fig. 10.15. The mean stress (τ_m) is plotted on the abscissa, while the stress amplitude (τ_a) on the ordinate. Point A with coordinates $\left(\frac{1}{2} S'_{se}, \frac{1}{2} S'_{se} \right)$ indicates the failure-point of the spring wire in fatigue test with pulsating stress cycle. Point B on the abscissa indicates the failure under static condition, when the mean stress (τ_m) reaches the torsional yield strength (S_{sy}). Therefore, line $\overline{AB}$ is called the line of failure.

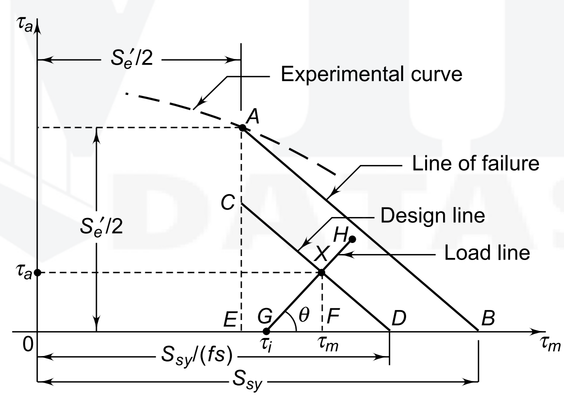


Fig. 10.15 *Fatigue Diagram for Spring Design*

To consider the effect of the factor of safety, a line $\overline{DC}$ is constructed from the point D on the abscissa in such a way that

$$\overline{OD} = \frac{S_{sy}}{(fs)}$$

[5] HJ Elmendorf—'Ratio of spring torsional elastic limit to wire tensile strength'—Metal Progress, April, 1958.

The line $\overline{DC}$ is parallel to the line $\overline{BA}$. Any point on the line $\overline{CD}$, such as X, represents a stress situation with the same factor of safety. Line $\overline{CD}$ is called the *design line* because it is used to find out permissible stresses with a particular factor of safety.

The line $\overline{GH}$ is called *load line*. It is drawn from the point G on the abscissa at a distance τ_i from the origin. The torsional shear stress due to initial pre-load on the spring (P_i) is τ_i. The line $\overline{GH}$ is constructed in such a way that its slope θ is given by,

$$\tan\theta = \frac{\tau_a}{\tau_m}$$

The point of intersection between design line $\overline{DC}$ and load line $\overline{GH}$ is X. The co-ordinates of the point X are (τ_m, τ_a).

Considering similar triangles XFD and AEB,

$$\frac{\overline{XF}}{\overline{FD}} = \frac{\overline{AE}}{\overline{EB}}$$

or
$$\frac{\tau_a}{\dfrac{S_{sy}}{(fs)} - \tau_m} = \frac{\dfrac{1}{2}S'_{se}}{S_{sy} - \dfrac{1}{2}S'_{se}} \qquad (10.22)$$

The above equation is used in the design of springs subjected to fluctuating stresses.

Examples based on Simple Analysis

Example 10.1 *It is required to design a helical compression spring subjected to a maximum force of 1250 N. The deflection of the spring corresponding to the maximum force should be approximately 30 mm. The spring index can be taken as 6. The spring is made of patented and cold-drawn steel wire. The ultimate tensile strength and modulus of rigidity of the spring material are 1090 and 81 370 N/mm² respectively. The permissible shear stress for the spring wire should be taken as 50% of the ultimate tensile strength. Design the spring and calculate:*

(i) wire diameter;
(ii) mean coil diameter;
(iii) number of active coils;
(iv) total number of coils;
(v) free length of the spring; and
(vi) pitch of the coil.

Draw a neat sketch of the spring showing various dimensions.

Solution

Given $P = 1250$ N $\quad \delta = 30$ mm $\quad C = 6$
$S_{ut} = 1090$ N/mm² $\quad G = 81\,370$ N/mm²
$\tau = 0.5\, S_{ut}$

Step I *Wire diameter*

The permissible shear stress is given by,

$\tau = 0.5\, S_{ut} = 0.5(1090) = 545$ N/mm²

From Eq. (10.7),

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4(6)-1}{4(6)-4} + \frac{0.615}{6} = 1.2525$$

From Eq. (10.13),

$$\tau = K\left(\frac{8PC}{\pi d^2}\right) \quad \text{or} \quad 545 = (1.2525)\left\{\frac{8(1250)(6)}{\pi d^2}\right\}$$

$\therefore \qquad d = 6.63$ or 7 mm $\qquad$ (i)

Step II *Mean coil diameter*

$D = Cd = 6(7) = 42$ mm $\qquad$ (ii)

Step III *Number of active coils*

From Eq. (10.8),

$$\delta = \frac{8PD^3N}{Gd^4} \quad \text{or} \quad 30 = \frac{8(1250)(42)^3 N}{(81\,370)(7)^4}$$

$\therefore \qquad N = 7.91$ or 8 coils $\qquad$ (iii)

Step IV *Total number of coils*

It is assumed that the spring has square and ground ends. The number of inactive coils is 2. Therefore,

$N_t = N + 2 = 8 + 2 = 10$ coils $\qquad$ (iv)

Step V *Free length of spring*

The actual deflection of the spring is given by,

$$\delta = \frac{8PD^3N}{Gd^4} = \frac{8(1250)(42)^3(8)}{(81\,370)(7)^4} = 30.34 \text{ mm}$$

solid length of spring $= N_t\, d = 10(7) = 70$ mm

It is assumed that there will be a gap of 1 mm between consecutive coils when the spring is subjected to the maximum force. The total number of coils is 10.

The total axial gap between the coils will be $(10–1) \times 1 = 9$ mm.

$$\text{Free length} = \text{Solid length} + \text{Total axial gap} + \delta$$
$$= 70 + 9 + 30.34$$
$$= 109.34 \text{ or } 110 \text{ mm} \qquad \text{(v)}$$

Step VI *Pitch of the coil*

$$\text{Pitch of coil} = \frac{\text{Free length}}{(N_t - 1)} = \frac{109.34}{(10-1)} = 12.15 \text{ mm} \qquad \text{(vi)}$$

The dimensions of the spring are shown in Fig. 10.16.

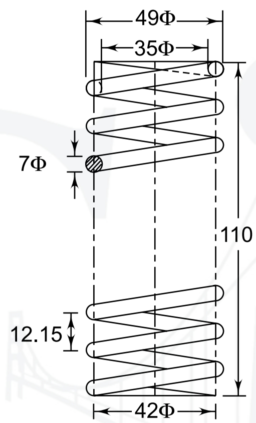


Fig. 10.16

Example 10.2 *A helical compression spring, made of circular wire, is subjected to an axial force, which varies from 2.5 kN to 3.5 kN. Over this range of force, the deflection of the spring should be approximately 5 mm. The spring index can be taken as 5. The spring has square and ground ends. The spring is made of patented and cold-drawn steel wire with ultimate tensile strength of 1050 N/mm² and modulus of rigidity of 81370 N/mm². The permissible shear stress for the spring wire should be taken as 50% of the ultimate tensile strength. Design the spring and calculate*

(i) wire diameter;
(ii) mean coil diameter;
(iii) number of active coils;
(iv) total number of coils;
(v) solid length of the spring;
(vi) free length of the spring;
(vii) required spring rate; and
(viii) actual spring rate

Solution

Given $P = 2.5$ to 3.5 kN $\quad \delta = 5$ mm $\quad C = 5$

$S_{ut} = 1050$ N/mm² $\quad G = 81\,370$ N/mm² $\quad \tau = 0.5\, S_{ut}$

Step I *Wire diameter*

The permissible shear stress for the spring is given by,

$$\tau = 0.5\, S_{ut} = 0.5\,(1050) = 525 \text{ N/mm}^2$$

From Eq. (10.7),

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4(5)-1}{4(5)-4} + \frac{0.615}{5} = 1.3105$$

From Eq. (10.13),

$$\tau = K\left(\frac{8PC}{\pi d^2}\right) \quad \text{or} \quad 525 = (1.3105)\left\{\frac{8(3500)(5)}{\pi d^2}\right\}$$

$$\therefore \qquad d = 10.55 \quad \text{or} \quad 11 \text{ mm} \qquad \text{(i)}$$

Step II *Mean coil diameter*

$$D = C\,d = 5(11) = 55 \text{ mm} \qquad \text{(ii)}$$

Step III *Number of active coils*

From Eq. (10.8),

$$\delta = \frac{8PD^3N}{Gd^4} \quad \text{or} \quad 5 = \frac{8(3500-2500)(55)^3 N}{(81\,370)(11)^4}$$

$$\therefore \qquad N = 4.48 \quad \text{or} \quad 5 \text{ coils} \qquad \text{(iii)}$$

Step IV *Total number of coils*

For square and ground ends, the number of inactive coils is 2. Therefore,

$$N_t = N + 2 = 5 + 2 = 7 \text{ coils} \qquad \text{(iv)}$$

Step V *Solid length of spring*

solid length of spring $= N_t d = 7(11) = 77$ mm $\qquad$ (v)

Step VI *Free length of spring*

The actual deflection of the spring under the maximum force of 3.5 kN is given by,

$$\delta = \frac{8PD^3N}{Gd^4} = \frac{8(3500)(55)^3(5)}{(81370)(11)^4} = 19.55 \text{ mm}$$

It is assumed that there will be a gap of 0.5 mm between the consecutive coils when the spring is subjected to the maximum force of 3.5 kN. The total number of coils is 7. Therefore, total axial gap will be $(7 - 1) \times 0.5 = 3$ mm.

Free length = Solid length + Total axial gap + δ

$$= 77 + 3 + 19.55$$
$$= 99.55 \text{ or } 100 \text{ mm} \qquad \text{(vi)}$$

Step VII *Required spring rate*

$$k = \frac{P_1 - P_2}{\delta} = \frac{3500 - 2500}{5} = 200 \text{ N/mm} \qquad \text{(vii)}$$

Step VIII *Actual spring rate*

$$k = \frac{Gd^4}{8D^3N} = \frac{(81370)(11)^4}{8(55)^3(5)} = 179.01 \text{ N/mm} \qquad \text{(viii)}$$

Example 10.3 *It is required to design a helical compression spring subjected to a maximum force of 7.5 kN. The mean coil diameter should be 150 mm from space consideration. The spring rate is 75 N/mm. The spring is made of oil-hardened and tempered steel wire with ultimate tensile strength of 1250 N/mm². The permissible shear stress for the spring wire is 30% of the ultimate tensile strength (G = 81 370 N/mm²). Calculate*

(i) wire diameter; and
(ii) number of active coils.

Solution

Given $P = 7.5$ kN $D = 150$ mm $k = 75$ N/mm
$S_{ut} = 1250$ N/mm² $G = 81\,370$ N/mm² $\tau = 0.3\ S_{ut}$

Step I *Wire diameter*

The permissible shear stress is given by,

$$\tau = 0.3\, S_{ut} = 0.3(1250) = 375 \text{ N/mm}^2$$

$$C = \frac{D}{d} = \frac{150}{d} \quad \text{or} \quad d = \frac{150}{C} \qquad \text{(a)}$$

From Eq. (10.13),

$$\tau = K\left(\frac{8PC}{\pi d^2}\right)$$

Substituting Eq. (a) in the above expression,

$$\tau = K\left\{\frac{8PC^3}{\pi(150)^2}\right\}$$

or,
$$KC^3 = \frac{\pi(150)^2\tau}{8P} = \frac{\pi(150)^2(375)}{8(7500)}$$

$$K\,C^3 = 441.79 \qquad \text{(b)}$$

where,
$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

Equation (b) is to be solved by the trial and error method. The values are tabulated in the following way:

C	K	$K\,C^3$
5	1.311	163.88
6	1.253	270.65
7	1.213	416.06
8	1.184	606.21
7.5	1.197	504.98
7.1	1.210	433.07
7.2	1.206	450.14
7.3	1.203	467.99

It is observed from the above table that the spring index should be between 7.1 and 7.2 to satisfy Eq. (b).

$$C = 7.2$$

$$d = \frac{150}{C} = \frac{150}{7.2} = 20.83 \text{ or } 21 \text{ mm} \qquad \text{(i)}$$

Step II *Number of active coils*

From Eq. (10.9),

$$k = \frac{Gd^4}{8D^3N} \qquad 75 = \frac{(81370)(21)^4}{8(150)^3 N}$$

$$\therefore \qquad N = 7.81 \text{ or } 8 \text{ coils} \qquad \text{(ii)}$$

Example 10.4 *It is required to design a helical compression spring for the valve mechanism. The axial force acting on the spring is 300 N when the valve is open and 150 N when the valve is closed. The length of the spring is 30 mm when the valve is open and 35 mm when the valve is closed. The spring is made of oil-hardened and tempered valve spring wire and the ultimate tensile strength is 1370 N/mm². The permissible shear stress for the spring wire should be taken as 30% of the ultimate tensile strength. The modulus of rigidity is 81 370 N/mm².*

The spring is to be fitted over a valve rod and the minimum inside diameter of the spring should be 20 mm. Design the spring and calculate

(i) wire diameter;
(ii) mean coil diameter;
(iii) number of active coils;

(iv) *total number of coils;*
(v) *free length of the spring; and*
(vi) *pitch of the coil.*

Assume that the clearance between adjacent coils or clash allowance is 15% of the deflection under the maximum load.

Solution

Given $P = 300$ to 150 N
Spring length = 30 to 35 mm $D_i = 20$ mm
$S_{ut} = 1370$ N/mm^2 $G = 81\ 370$ N/mm^2
$\tau = 0.3\ S_{ut}$

Step I *Wire diameter*

The spring force and spring length corresponding to closed and open positions of the valve are illustrated in Fig. 10.17. The permissible shear stress is given by,

$$\tau = 0.3\ S_{ut} = 0.3\ (1370) = 411 \text{ N/mm}^2$$
$$D_i = 20 \text{ mm}$$
$$D = D_i + d = (20 + d) \text{ mm}$$

From Eq. (10.6),

$$\tau = K\left(\frac{8PD}{\pi d^3}\right) \quad \text{or} \quad 411 = K\left\{\frac{8(300)(20+d)}{\pi d^3}\right\} \quad \text{(a)}$$

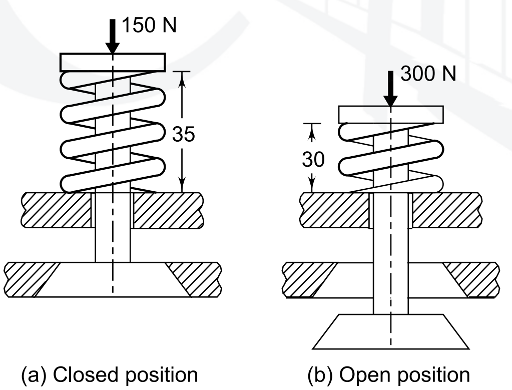


Fig. 10.17 *Valve Spring Mechanism*

It is observed from the above expression that there are two unknowns, viz., K and d and one equation. It cannot be solved. As a first trial, let us neglect the effect of the Wahl factor K or substitute ($K = 1$). At a later design stage, the wire diameter d can be increased to account for K. Substituting ($K = 1$) in Eq. (a),

$$411 = \frac{8(300)(20+d)}{\pi d^3} \quad \text{or} \quad \frac{d^3}{(20+d)} = 1.8587 \quad \text{(b)}$$

The above equation is solved by trial and error method. The values are tabulated in the following way:

d	$d^3/(20+d)$
5	5
4	2.667
3	1.174

The value of d should be between 3 to 4 mm in order to satisfy Eq. (b). The higher value of d is selected to account for the Wahl correction factor.

Therefore,

$$d = 4 \text{ mm} \quad \text{(i)}$$

Step II *Mean coil diameter*

$$D = D_i + d = 20 + 4 = 24 \text{ mm} \quad \text{(ii)}$$

$$C = \frac{D}{d} = \frac{24}{4} = 6$$

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4\times 6-1}{4\times 6-4} + \frac{0.615}{6}$$
$$= 1.2525$$

$$\tau = K\left(\frac{8PD}{\pi d^3}\right) = (1.2525)\left\{\frac{8(300)(24)}{\pi(4)^3}\right\}$$
$$= 358.81 \text{ N/mm}^2$$

Therefore,

$$\tau < 411 \text{ N/mm}^2$$

and the design is safe.

Step III *Number of active coils*

Form Eq. (10.8),

$$\delta = \frac{8PD^3 N}{Gd^4} \quad \text{or} \quad (35-30) = \frac{8(300-150)(24)^3 N}{(81\ 370)(4)^4}$$

$$\therefore \quad N = 6.28 \text{ or } 7 \text{ coils} \quad \text{(iii)}$$

Step IV *Total number of coils*

It is assumed that the spring has square and ground ends. The number of inactive coils is 2. Therefore,

$$N_t = N + 2 = 7 + 2 = 9 \text{ coils} \quad \text{(iv)}$$

Step V *Free length of spring*

The deflection of the spring for the maximum force is given by,

$$\delta = \frac{8PD^3\,N}{Gd^4} = \frac{8(300)(24)^3(7)}{(81\,370)(4)^4} = 11.15 \text{ mm}$$

The total gap between the adjacent coils is given by,

Gap = 15% of δ = 0.15 (11.15) = 1.67 mm

Solid length = $N_t\,d$ = 9(4) = 36 mm

Free length = solid length + total axial gap + δ
= 36 + 1.67 + 11.15
= 48.82 or 50 mm (v)

Step VI *Pitch of coils*

$$\text{Pitch of coil} = \frac{\text{free length}}{(N_t - 1)} = \frac{50}{(9-1)} = 6.25 \text{ mm} \quad \text{(vi)}$$

Example 10.5 *A helical tension spring is used in the spring balance to measure the weights. One end of the spring is attached to the rigid support while the other end, which is free, carries the weights to be measured. The maximum weight attached to the spring balance is 1500 N and the length of the scale should be approximately 100 mm. The spring index can be taken as 6. The spring is made of oil-hardened and tempered steel wire with ultimate tensile strength of 1360 N/mm² and modulus of rigidity of 81 370 N/mm². The permissible shear stress in the spring wire should be taken as 50% of the ultimate tensile strength. Design the spring and calculate*

(i) wire diameter;
(ii) mean coil diameter;
(iii) number of active coils;
(iv) required spring rate; and
(v) actual spring rate.

Solution

Given $P = 1500$ N $C = 6$ $S_{ut} = 1360$ N/mm²
$G = 81\,370$ N/mm² $\tau = 0.5\,S_{ut}$

Step I *Wire Diameter*

The working principle of the spring balance is illustrated in Fig. 10.18. As the load acting on the spring varies from 0 to 1500 N, the pointer attached to the free end of the spring moves over a scale between highest and lowest positions. The length of the scale between these two positions of the pointer is 100 mm. In other words, the spring deflection is 100 mm when the force is 1500 N.

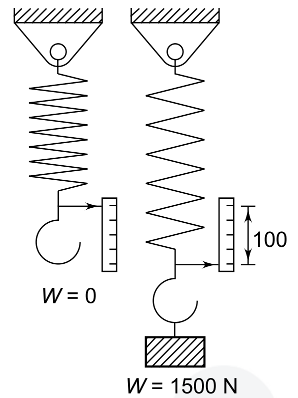


Fig. 10.18 *Mechanism of Spring Balance*

The permissible shear stress for spring wire is given by,

$$\tau = 0.5\,S_{ut} = 0.5\,(1360) = 680 \text{ N/mm}^2$$

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4\times 6-1}{4\times 6-4} + \frac{0.615}{6} = 1.2525$$

From Eq. (10.13),

$$\tau = K\left(\frac{8PC}{\pi d^2}\right) \quad \text{or} \quad 680 = 1.2525\left\{\frac{8\,(1500\,)(6)}{\pi d^2}\right\}$$

$$\therefore \qquad d = 6.5 \text{ or } 7 \text{ mm} \qquad \text{(i)}$$

Step II *Mean coil diameter*

$$D = Cd = 6\,(7) = 42 \text{ mm} \qquad \text{(ii)}$$

Step III *Number of active coils*

From Eq. (10.8),

$$\delta = \frac{8\,PD^3\,N}{Gd^4} \quad \text{or} \quad 100 = \frac{8(1500)(42)^3\,N}{(81\,370)(7)^4}$$

$$\therefore \qquad N = 21.97 \text{ or } 22 \text{ coils} \qquad \text{(iii)}$$

Step IV *Required spring rate*

$$k = \frac{P}{\delta} = \frac{1500}{100} = 15 \text{ N/mm} \qquad \text{(iv)}$$

Step V *Actual spring rate*

$$k = \frac{Gd^4}{8D^3N} = \frac{(81\,370)(7)^4}{8(42)^3(22)} = 14.98 \text{ N/mm} \qquad \text{(v)}$$

Example 10.6 *A railway wagon moving at a velocity of 1.5 m/s is brought to rest by a bumper consisting of two helical springs arranged in parallel. The mass of the wagon is 1500 kg. The springs are compressed by 150 mm in bringing the wagon to rest. The spring index can be taken as 6. The springs are made of oil-hardened and tempered steel wire with ultimate tensile strength of 1250 N/mm² and modulus of rigidity of 81 370 N/mm². The permissible shear stress for the spring wire can be taken as 50% of the ultimate tensile strength. Design the spring and calculate:*

(i) wire diameter;
(ii) mean coil diameter;
(iii) number of active coils;
(iv) total number of coils;
(v) solid length;
(vi) free length;
(vii) pitch of the coil;
(viii) required spring rate; and
(ix) actual spring rate.

Solution

Given $m = 1500$ kg $v = 1.5$ m/s $\delta = 150$ mm
$C = 6$ $S_{ut} = 1250$ N/mm² $G = 81\,370$ N/mm²
$\tau = 0.5\, S_{ut}$

Step I *Wire diameter*

The kinetic energy of the moving wagon is absorbed by the springs. The kinetic energy of the wagon is given by,

$$\text{KE} = \frac{1}{2} mv^2 = \frac{1}{2}(1500)(1.5)^2 = 1687.5 \text{ J or N-m}$$
$$= (1687.5 \times 10^3) \text{ N-mm} \qquad \text{(a)}$$

Suppose P is the maximum force acting on each spring and causing it to compress by 150 mm. The strain energy absorbed by two springs is given by,

$$E = 2\left[\frac{1}{2}\, P\,\delta\right] = 2\left[\frac{1}{2}\, P\,(150)\right] = (150P) \text{ N-mm} \qquad \text{(b)}$$

The strain energy absorbed by the two springs is equal to the kinetic energy of the wagon. Therefore,

$$(150\, P) = 1687.5 \times 10^3$$
$$P = 11\,250 \text{ N}$$

The permissible shear stress for the spring wire is given by,

$$\tau = 0.5\,(1250) = 625 \text{ N/mm}^2$$

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4(6)-1}{4(6)-4} + \frac{0.615}{6}$$
$$= 1.2525$$

From Eq. (10.13),

$$\tau = K\left(\frac{8PC}{\pi d^2}\right) \text{ or } 625 = (1.2525)\left\{\frac{8(11\,250)(6)}{\pi d^2}\right\}$$

$$\therefore \qquad d = 18.56 \text{ or } 20 \text{ mm} \qquad \text{(i)}$$

Step II *Mean coil diameter*

$$D = Cd = 6\,(20) = 120 \text{ mm} \qquad \text{(ii)}$$

Step III *Number of active coils*

From Eq. (10.8),

$$\delta = \frac{8\,PD^3\,N}{Gd^4} \text{ or } 150 = \frac{8(11\,250)(120)^3\,N}{(81\,370)(20)^4}$$

$$N = 12.56 \text{ or } 13 \text{ coils} \qquad \text{(iii)}$$

Step IV *Total number of coils*

It is assumed that the springs have square and ground ends. The number of inactive coils is 2. Therefore,

$$N_t = N + 2 = 13 + 2 = 15 \text{ coils} \qquad \text{(iv)}$$

Step V *Solid length of spring*

$$\text{Solid length} = N_t d = 15(20) = 300 \text{ mm} \qquad \text{(v)}$$

Step VI *Free length of spring*

The actual deflection of the spring is given by,

$$\delta = \frac{8PD^3N}{Gd^4} = \frac{8(11\,250)(120)^3(13)}{(81370)(20)^4} = 155.29 \text{ mm}$$

It is assumed that there will be a gap of 2 mm between adjacent coils when the spring is subjected to the maximum force of 11 250 N. Since the total number of coils is 15, the total axial gap will be $(15 - 1) \times 2 = 28$ mm.

$$\text{Free length} = \text{solid length} + \text{total axial gap} + \delta$$
$$= 300 + 28 + 155.29$$
$$= 483.29 \text{ or } 485 \text{ mm} \qquad \text{(vi)}$$

Step VII *Pitch of coils*

$$\text{Pitch of coil} = \frac{\text{free length}}{(N_t - 1)} = \frac{485}{(15-1)} = 34.64 \text{ mm} \qquad \text{(vii)}$$

Step VIII *Required spring rate*

$$k = \frac{P}{\delta} = \frac{11\,250}{150} = 75 \text{ N/mm} \qquad \text{(viii)}$$

Step IX *Actual spring rate*

$$k = \frac{Gd^4}{8D^3N} = \frac{(81370)(20)^4}{8(120)^3(13)} = 72.44 \text{ N/mm} \qquad \text{(ix)}$$

Example 10.7 *An automotive single plate clutch consists of two pairs of friction surfaces, one between the friction lining and the pressure plate and the other between the friction lining and the flywheel as shown in Fig 10.19. Eight identical helical compression springs, arranged in parallel, provide the required axial thrust on the friction surface. The total spring force exerted by all springs is 2400 N and the corresponding deflection of each spring is approximately 15 mm. The spring index can be taken as 8. The springs are made of patented and cold-drawn steel wire with ultimate tensile strength of 1390 N/mm² and modulus of rigidity of 81 370 N/mm². The permissible shear stress for the spring wire can be taken as 30% of the ultimate tensile strength. Design the springs and calculate:*

(i) wire diameter;
(ii) mean coil diameter;
(iii) number of active coils;
(iv) total number of coils;
(v) solid length;
(vi) free length;
(vii) pitch of the coil;
(viii) required stiffness of the spring; and
(ix) actual stiffness of the spring.

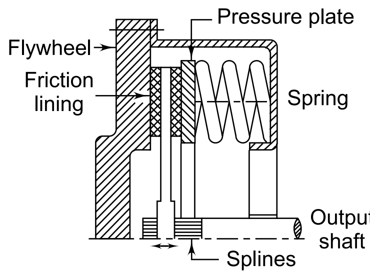


Fig. 10.19 *Clutch Mechanism*

Solution

Given $P = 2400$ N for 8 springs $\delta = 15$ mm $C = 8$
$S_{ut} = 1390$ N/mm² $G = 81\,370$ N/mm² $\tau = 0.3\, S_{ut}$

Step I *Wire diameter*

There are eight springs in parallel. The force acting on each spring is given by,

$$P = \frac{2400}{8} = 300 \text{ N}$$

The permissible shear stress for the spring wire is given by,

$$\tau = 0.3\, S_{ut} = 0.3\,(1390) = 417 \text{ N/mm}^2$$

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4\times 8-1}{4\times 8-4} + \frac{0.615}{8} = 1.184$$

From Eq. (10.13),

$$\tau = K\left(\frac{8PC}{\pi d^2}\right) \quad \text{or} \quad 417 = (1.184)\left\{\frac{8(300)(8)}{\pi d^2}\right\}$$

$$\therefore \qquad d = 4.17 \text{ or } 5 \text{ mm} \qquad \text{(i)}$$

Step II *Mean coil diameter*

$$D = Cd = 8(5) = 40 \text{ mm} \qquad \text{(ii)}$$

Step III *Number of active coils*

From Eq. (10.8),

$$\delta = \frac{8PD^3N}{Gd^4} \quad \text{or} \quad 15 = \frac{8(300)(40)^3 N}{(81370)(5)^4}$$

$$\therefore \qquad N = 4.97 \text{ or } 5 \text{ coils} \qquad \text{(iii)}$$

Step IV *Total number of coils*

It is assumed that the springs have square and ground ends. The number of inactive coils is 2. Therefore,

$$N_t = N + 2 = 5 + 2 = 7 \qquad \text{(iv)}$$

Step V *Solid length of spring*

$$\text{solid length} = N_t d = 7(5) = 35 \text{ mm} \qquad \text{(v)}$$

Step VI *Free length of spring*

The actual deflection of the spring is given by,

$$\delta = \frac{8PD^3N}{Gd^4} = \frac{8(300)(40)^3(5)}{(81\,370)(5)^4} = 15.10 \text{ mm}$$

It is assumed that there will be a gap of 1 mm between the adjacent coils when the spring is subjected to the maximum force of 300 N. The total number of coils is 7. Therefore, the total axial gap will be $(7 - 1) \times 1 = 6$ mm.

$$\begin{aligned}\text{Free length} &= \text{solid length} + \text{total axial gap} + \delta \\ &= 35 + 6 + 15.10 \\ &= 56.1 \text{ or } 57 \text{ mm} \qquad \text{(vi)}\end{aligned}$$

Step VII *Pitch of coils*

$$\text{Pitch of coil} = \frac{\text{Free length}}{(N_t - 1)} = \frac{57}{(7-1)} = 9.5 \text{ mm} \quad \text{(vii)}$$

Step VIII *Required stiffness*

$$k = \frac{P}{\delta} = \frac{300}{15} = 20 \text{ N/mm} \quad \text{(viii)}$$

Step IX *Actual stiffness*

$$k = \frac{Gd^4}{8D^3N} = \frac{(81\,370)(5)^4}{8(40)^3(5)} = 19.87 \text{ N/mm} \quad \text{(ix)}$$

Example 10.8 *A safety valve operated by a helical tension spring through the lever mechanism is schematically illustrated in Fig. 10.20. The diameter of the valve is 50 mm. In normal operating conditions, the valve is closed and the pressure inside the chamber is 0.5 N/mm². The valve is opened when the pressure inside the chamber increases to 0.6 N/mm². The maximum lift of the valve is 5 mm. The spring index can be taken as 8. The spring is made of patented and cold-drawn steel wire with ultimate tensile strength of 1200 N/mm² and modulus of rigidity of 81 370 N/mm². The permissible shear stress for the spring wire can be taken as 30% of the ultimate tensile strength. Design the spring and calculate:*

(i) wire diameter;
(ii) mean coil diameter; and
(iii) number of active coils.

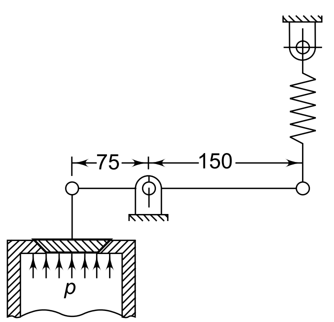


Fig. 10.20

Solution

Given $C = 8$ $S_{ut} = 1200$ N/mm²
$G = 81\,370$ N/mm² $\tau = 0.3\, S_{ut}$

Step I *Wire diameter*

Suppose, P_v and P_s are denoted as the forces acting on the valve and the spring respectively. Taking moment of these forces about the pivot,

$$P_v \times 75 = P_s \times 150 \quad \text{or} \quad P_s = \frac{1}{2} P_v \quad \text{(a)}$$

When the valve is closed,

$$P_v = \frac{\pi}{4}(50)^2(0.5) = 981.75 \text{ N}$$

From Eq. (a), the force acting on the spring is (981.75/2) or 490.87 N

When the valve is open,

$$P_v = \frac{\pi}{4}(50)^2(0.6) = 1178.10 \text{ N}$$

From Eq. (a), the force acting on the spring is (1178.10/2) or 589.05 N.

The valve is lifted through 5 mm and corresponding extension of the spring is given by,

$$\delta = 5 \times \frac{150}{75} = 10 \text{ mm}$$

Therefore, the stiffness of the spring is given by,

$$k = \frac{589.05 - 490.87}{10} = 9.82 \text{ N/mm}$$

The maximum force acting on the spring is 589.05 N.

$$P = 589.05 \text{ N}$$

The permissible shear stress for the spring wire is given by,

$$\tau = 0.3\, S_{ut} = 0.3\,(1200) = 360 \text{ N/mm}^2$$

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4(8)-1}{4(8)-4} + \frac{0.615}{8} = 1.184$$

From Eq. (10.13),

$$\tau = K\left(\frac{8PC}{\pi d^2}\right) \quad \text{or} \quad 360 = 1.184\left\{\frac{8\,(589.05)(8)}{\pi d^2}\right\}$$

$$\therefore \quad d = 6.28 \text{ or } 7 \text{ mm} \quad \text{(i)}$$

Step II *Mean coil diameter*

$$D = Cd = 8(7) = 56 \text{ mm} \quad \text{(ii)}$$

Step III *Number of active coils*

From Eq. (10.9),

$$k = \frac{Gd^4}{8D^3N} \quad \text{or} \quad 9.82 = \frac{(81\,370)(7)^4}{8(56)^3\,N}$$

$$\therefore \quad N = 14.16 \text{ or } 15 \text{ coils} \quad \text{(iii)}$$

Example 10.9 *A safety valve, 50 mm in diameter, is to blow off at a pressure of 1.5 MPa. It is held on its seat by means of a helical compression spring, with an initial compression of 25 mm. The maximum lift of the valve is 10 mm. The spring index can be taken as 6. The spring is made of patented and cold-drawn steel wire with ultimate tensile strength of 1500 N/mm² and modulus of rigidity of 81 370 N/mm². The permissible shear stress for the spring wire should be taken as 30% of the ultimate tensile strength. Design the spring and calculate:*

(i) wire diameter;
(ii) mean coil diameter;
(iii) number of active turns;
(iv) total number of turns;
(v) solid length;
(vi) free length; and
(vii) pitch of the coil.

Solution

Given $C = 6$ $S_{ut} = 1500$ N/mm²
$G = 81\,370$ N/mm² $\tau = 0.3\,S_{ut}$

Step I *Wire diameter*

Let P_1 and δ_1 denote the initial spring force and deflection respectively when the valve just begins to blow off.

$$P_1 = \frac{\pi}{4}(50)^2(1.5) = 2945.24 \text{ N}$$

$$\delta_1 = 25 \text{ mm}$$

Let P_2 and δ_2 denote the spring force and deflection respectively when the valve is open.

$$\delta_2 = \delta_1 + \text{valve lift} = 25 + 10 = 35 \text{ mm}$$

Also,

$$P \propto \delta$$

Therefore,

$$\frac{P_2}{P_1} = \frac{\delta_2}{\delta_1} \quad \text{or} \quad \frac{P_2}{(2945.24)} = \frac{35}{25}$$

$P_2 = 4123.34$ N (maximum force)

The permissible shear stress for the spring wire is given by,

$$\tau = 0.3\,S_{ut} = 0.3\,(1500) = 450 \text{ N/mm}^2$$

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4(6)-1}{4(6)-4} + \frac{0.615}{6}$$

$$= 1.2525$$

From Eq. (10.13),

$$\tau = K\left(\frac{8PC}{\pi d^2}\right) \text{ or } 450 = 1.2525\left\{\frac{8\,(4123.34)(6)}{\pi d^2}\right\}$$

$\therefore$ $d = 13.24$ or 14 mm (i)

Step II *Mean coil diameter*

$$D = Cd = 6(14) = 84 \text{ mm} \qquad \text{(ii)}$$

Step III *Number of active turns*

From Eq. (10.8),

$$\delta = \frac{8\,PD^3N}{Gd^4}$$

Substituting values of P_1 and δ_1,

$$25 = \frac{8(2945.24)(84)^3N}{(81\,370)(14)^4}$$

$\therefore$ $N = 5.6$ or 6 turns (iii)

Step IV *Total number of turns*

It is assumed that the spring has square and ground ends. The number of inactive coils is 2. Therefore,

$$N_t = N + 2 = 6 + 2 = 8 \text{ turns} \qquad \text{(iv)}$$

Step V *Solid length of spring*

$$\text{Solid length} = N_t\,d = 8(14) = 112 \text{ mm} \qquad \text{(v)}$$

Step VI *Free length of spring*

The maximum deflection of the spring under the force of 4123.34 N is given by,

$$\delta = \frac{8PD^3N}{Gd^4} = \frac{8(4123.34)(84)^3(6)}{(81\,370)(14)^4} = 37.53 \text{ mm}$$

It is assumed that there will be a gap of 2 mm between the adjacent turns, when the spring is subjected to the maximum compression. This gap is essential to avoid clashing of the coils. The total number of turns is 8. Therefore, the total axial gap will be $(8 - 1) \times 2 = 14$ mm

Free length = solid length + total axial gap + δ
= 112 + 14 + 37.53 = 163.53 mm

or free length = 165 mm (vi)

Step VII *Pitch of coils*

$$\text{Pitch of coil} = \frac{\text{free length}}{(N_t - 1)} = \frac{165}{(8-1)} = 23.57 \text{ mm}$$

(vii)

Example 10.10 *A helical compression spring is used to absorb the shock. The initial compression of the spring is 30 mm and it is further compressed by 50 mm while absorbing the shock. The spring is to absorb 250 J of energy during the process. The spring index can be taken as 6. The spring is made of patented and cold-drawn steel wire with an ultimate tensile strength of 1500 N/mm² and modulus of rigidity of 81 370 N/mm². The permissible shear stress for the spring wire should be taken as 30% of the ultimate tensile strength. Design the spring and calculate:*

(i) wire diameter;
(ii) mean coil diameter;
(iii) number of active turns;
(iv) free length; and
(v) pitch of the turns.

Solution

Given $C = 6$ $\delta_1 = 30$ mm $\delta_2 = (30 + 50)$ mm
$E = 250$ J $S_{ut} = 1500$ N/mm² $G = 81\,370$ N mm²
$\tau = 0.3\,S_{ut}$

Step I *Wire diameter*

Suppose P_1 and δ_1 denote initial spring force and deflection respectively before the shock.

$\delta_1 = 30$ mm

$P_1 = k\,\delta = (30\,k)\,N$

where k is the stiffness of the spring.

Suppose P_2 and δ_2 denote spring force and deflection respectively after absorbing shock.

$\delta_2 = 30 + 50 = 80$ mm

$P_2 = k\delta_2 = (80\,k)\,N$

$$\text{Average force during compression} = \frac{(30k + 80k)}{2} = (55\,k)\,N$$

Energy absorbed during shock = Average force × δ

$$250 \times 10^3 = (55k) \times 50$$

$$\therefore \quad k = 90.91 \text{ N/mm}$$

The maximum spring force is given by,

$P_2 = 80k = 80\,(90.91) = 7272.72\,N$

The permissible shear stress for the spring wire is given by,

$\tau = 0.3\,S_{ut} = 0.3\,(1500) = 450$ N/mm²

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4(6)-1}{4(6)-4} + \frac{0.615}{6} = 1.2525$$

From Eq. (10.13),

$$\tau = K\left(\frac{8PC}{\pi d^2}\right) \text{ or } 450 = 1.2525\left\{\frac{8\,(7272.72)(6)}{\pi d^2}\right\}$$

$$\therefore \quad d = 17.59 \text{ or } 18 \text{ mm} \qquad \text{(i)}$$

Step II *Mean coil diameter*

$$D = Cd = 6(18) = 108 \text{ mm} \qquad \text{(ii)}$$

Step III *Number of active turns*

From Eq. (10.9),

$$k = \frac{Gd^4}{8D^3N} \quad \text{or} \quad 90.91 = \frac{(81370)(18)^4}{8\,(108)^3\,N}$$

$$\therefore \quad N = 9.32 \text{ or } 10 \text{ turns} \qquad \text{(iii)}$$

Step IV *Free length*

It is assumed that the spring has square and ground ends. The number of inactive turns is 2. Therefore,

$$N_t = N + 2 = 10 + 2 = 12$$

Solid length = $N_t\,d = 12(18) = 216$ mm

It is assumed that there will be a gap of 2 mm between the adjacent turns when the spring is subjected to the maximum force of 7272.72 N. The total number of turns is 12. Therefore, the total axial gap will be $(12 - 1) \times 2 = 22$ mm. The maximum deflection is given by,

$$\delta = \frac{8\,PD^3N}{Gd^4} = \frac{8\,(7272.72)\,(108)^3\,(10)}{(81370)\,(18)^4} = 85.80 \text{ mm}$$

free length = solid length + total axial gap + δ
= 216 + 22 + 85.80 = 323.8 mm

or free length = 325 mm (iv)

Step V *Pitch of turns*

$$\text{Pitch of turns} = \frac{\text{free length}}{(N_t - 1)} = \frac{325}{(12-1)} = 29.54 \text{ mm} \qquad \text{(v)}$$

Examples Based on Trial and Error Procedure

Example 10.11 *An automotive single-plate clutch, with two pairs of friction surfaces, transmits 300 N-m torque at 1500 rpm. The inner and outer diameters of the friction disk are 170 and 270 mm respectively. The coefficient of friction is 0.35. The* *normal force on the friction surfaces is exerted by nine helical compression springs, so that the clutch is always engaged. The clutch is disengaged when the external force further compresses the springs. The spring index is 5 and the number of active coils is 6. The springs are made of patented and cold-drawn steel wires of Grade 2. (G = 81 370 N/mm^2). The permissible shear stress for the spring wire is 30% of the ultimate tensile strength. Design the springs and specify their dimensions.*

Solution

Given $C = 5$ $N = 6$ $G = 81\,370$ N/mm^2

$\tau = 0.3\, S_{ut}$ number of springs = 9

For clutch, $M_t = 300$ N-m $n = 1500$ rpm

number of friction surfaces = 2

For friction disk, $D = 270$ mm $d = 170$ mm

$\mu = 0.35$

Step I *Maximum spring force*

The construction of the clutch mechanism is illustrated in Fig. 10.19 of Example No. 10.7. There are two pairs of contacting surfaces and the torque transmitted by each pair is (300/2), or 150 N-m. Assuming uniform-wear theory (Chapter 11), the total normal force P_1 required to transmit the torque is given by Eq. (11.8), i.e.,

$$P_1 = \frac{4M_t}{\mu(D+d)} = \frac{4(150\times10^3)}{0.35(270+170)} = 3896.1 \text{ N}$$

Since there are nine springs, the force exerted by each spring is

$$P = \frac{3896.1}{9} = 432.9 \text{ N}$$

Step II *Wire diameter*

From Eq. (10.7),

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4\,(5)-1}{4\,(5)-4} + \frac{0.615}{5} = 1.3105$$

From Eq. (10.13),

$$\tau = K\left(\frac{8PC}{\pi d^2}\right) \quad \text{or} \quad \tau = (1.3105)\left[\frac{8\,(432.9)\,(5)}{\pi d^2}\right]$$

or
$$\tau = \frac{7223.28}{d^2} \text{ N/mm}^2 \qquad \text{(a)}$$

The permissible shear stress is denoted by τ_d in order to differentiate it from the induced stress τ. It is given by,

$$\tau_d = 0.3\, S_{ut} \qquad \text{(b)}$$

Equations (a) and (b) are solved by the trial and error method.

Trial 1

$d = 3$ mm

$$\tau = \frac{7223.28}{d^2} = \frac{7223.28}{(3)^2} = 802.59 \text{ N/mm}^2$$

From Table 10.1,

$S_{ut} = 1570$ N/mm^2

$\tau_d = 0.3\, S_{ut} = 0.3(1570) = 471$ N/mm^2

Therefore, $\tau > \tau_d$

The design is not safe.

Trial 2

$d = 3.6$ mm

$$\tau = \frac{7223.28}{d^2} = \frac{7223.28}{(3.6)^2} = 557.35 \text{ N/mm}^2$$

From Table 10.1,

$S_{ut} = 1510$ N/mm^2

$\tau_d = 0.3\, S_{ut} = 0.3(1510) = 453$ N/mm^2

Therefore,

$$\tau > \tau_d$$

The design is not safe.

Trial 3

$d = 4$ mm

$$\tau = \frac{7223.28}{d^2} = \frac{7223.28}{(4)^2} = 451.46 \text{ N/mm}^2$$

From Table 10.1,

$S_{ut} = 1480$ N/mm^2

$\tau_d = 0.3\, S_{ut} = 0.3(1480) = 444$ N/mm^2

Therefore,

$$\tau > \tau_d$$

The design is not safe.

Trial 4

$d = 4.5$ mm

$$\tau = \frac{7223.28}{d^2} = \frac{7223.28}{(4.5)^2} = 356.71 \text{ N/mm}^2$$

From Table 10.1,

$S_{ut} = 1440$ N/mm^2

$\tau_d = 0.3\, S_{ut} = 0.3(1440) = 432$ N/mm^2

Therefore, $\tau < \tau_d$

The design is satisfactory and the wire diameter should be 4.5 mm.

Step III *Mean coil diameter*

$$D = C\,d = 5(4.5) = 22.5 \text{ mm}$$

Step IV *Total number of coils*

It is assumed that the springs have square and ground ends.

$$N_t = N + 2 = 6 + 2 = 8$$

Step V *Free length*

From Eq. (10.8),

$$\delta = \frac{8PD^3N}{Gd^4} = \frac{8(432.9)(22.5)^3(6)}{(81370)(4.5)^4} = 7.09 \text{ mm}$$

Solid length of spring = $N_t\,d = 8(4.5) = 36$ mm

It is assumed that there will be a gap of 1 mm between consecutive coils when the spring is subjected to the maximum force. The total number of coils is 8.

The total axial gap between the coils will be $(8 - 1) \times 1 = 7$ mm.

Free length = solid length + total axial gap + δ

= 36 + 7 + 7.09

= 50.09 or 51 mm (v)

Step VI *Spring specifications*

(i) Material = patented and cold-drawn steel wire of Grade 2

(ii) Wire diameter = 4.5 mm

(iii) Mean coil diameter = 22.5 mm

(iv) Free length = 51 mm

(v) Total number of turns = 8

(vi) Style of ends = square and ground

Example 10.12 *A direct reading tension spring balance consists of a helical tension spring, which is attached to a rigid support at one end and carries masses at the other free end. The pointer attached to the free end moves on a scale and indicates the mass. The length of the scale is 100 mm, which is divided into 50 equal divisions. Each division on the scale indicates 0.5 kg. The maximum capacity of the spring balance is 25 kg. The spring index is 6. The spring is made of an oil-hardened and tempered steel wire of Grade-SW (G = 81 370 N/mm²). The permissible shear stress in the spring wire is recommended as 50% of the ultimate tensile strength. Design the spring and give its specifications.*

Solution

Given $C = 6$ $G = 81\,370$ N/mm² $\tau = 0.5\,S_{ut}$

maximum mass = 25 kg

Step I *Maximum spring force*

The working principle of spring balance is illustrated in Fig. 10.18 of Example No. 10.5. The maximum spring force is given by,

$$P = mg = 25(9.81) = 245.25 \text{ N}$$

Step II *Wire diameter*

From Eq. (10.7),

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4(6)-1}{4(6)-4} + \frac{0.615}{6} = 1.2525$$

From Eq. (10.13),

$$\tau = K\left(\frac{8PC}{\pi d^2}\right) \quad \text{or} \quad \tau = (1.2525)\left[\frac{8(245.25)(6)}{\pi d^2}\right]$$

or $$\tau = \frac{4693.3}{d^2} \text{ N/mm}^2 \tag{a}$$

The permissible shear stress is denoted by τ_d in order to differentiate it from the induced stress τ. It is given by,

$$\tau_d = 0.5\,S_{ut} \tag{b}$$

Equations (a) and (b) are solved by the trial and error method.

Trial 1

$d = 2$ mm

$$\tau = \frac{4693.3}{d^2} = \frac{4693.3}{(2)^2} = 1173.33 \text{ N/mm}^2$$

From Table 10.2,

$S_{ut} = 1620$ N/mm²

$\tau_d = 0.5S_{ut} = 0.5(1620) = 810$ N/mm²

Therefore, $\tau > \tau_d$

The design is not safe.

Trial 2

$d = 2.5$ mm

$$\tau = \frac{4693.3}{d^2} = \frac{4693.3}{(2.5)^2} = 750.93 \text{ N/mm}^2$$

From Table 10.2,

$S_{ut} = 1570$ N/mm^2

$\tau_d = 0.5\ S_{ut} = 0.5(1570) = 785$ N/mm^2

Therefore, $\tau < \tau_d$

The design is satisfactory and the wire diameter should be 2.5 mm.

Step III *Mean coil diameter*

$$D = C\,d = 6(2.5) = 15 \text{ mm}$$

Step IV *Number of active coils*

$$\text{length of each division} = \frac{\text{Length of scale}}{\text{Number of divisions}} = \frac{100}{50} = 2 \text{ mm}$$

Each division indicates 0.5 kg. Therefore,

$$k = \frac{0.5(9.81)}{2} = 2.4525 \text{ N/mm}$$

From Eq. (10.9),

$$N = \frac{Gd^4}{8D^3k} = \frac{81\,370(2.5)^4}{8(15)^3(2.4525)} = 48$$

Step V *Total number of coils*

For helical tension spring, all coils are active coils. Therefore,

$$N_t = N = 48$$

Solid length of the spring $= N_t\,d = 48(2.5)$
$= 120$ mm

Step VI *Spring specifications*

(i) material = oil-hardened and tempered steel wire of Grade-SW
(ii) wire diameter = 2.5 mm
(iii) mean coil diameter = 15 mm
(iv) total number of coils = 48
(v) solid length = 120 mm
(vi) style of ends = extended-hook

Spring Design for Fluctuating Stresses

Example 10.13 *A helical compression spring of a cam-mechanism is subjected to an initial preload of 50 N. The maximum operating force during the load cycle is 150 N. The wire diameter is 3 mm, while the mean coil diameter is 18 mm. The spring is made of oil-hardened and tempered valve spring wire of Grade-VW (S_{ut} = 1430 N/mm^2). Determine the factor of safety used in the design on the basis of fluctuating stresses.*

Solution

Given $P_{max.} = 150$ N $\quad P_{min.} = 50$ N $\quad d = 3$ mm $\quad D = 18$ mm $\quad S_{ut} = 1430$ N/mm^2

Step I *Mean and amplitude shear stresses*

$$C = \frac{D}{d} = \frac{18}{3} = 6$$

From Eq. (10.7) and (10.5),

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4(6)-1}{4(6)-4} + \frac{0.615}{6} = 1.2525$$

$$K_s = 1 + \frac{0.5}{C} = 1 + \frac{0.5}{6} = 1.0833$$

$$P_m = \frac{1}{2}(P_{max.} + P_{min.}) = \frac{1}{2}(150 + 50) = 100 \text{ N}$$

$$P_a = \frac{1}{2}(P_{max.} - P_{min.}) = \frac{1}{2}(150 - 50) = 50 \text{ N}$$

From Eq. (10.18) and (10.19),

$$\tau_m = K_s\left(\frac{8P_mD}{\pi d^3}\right) = (1.0833)\left(\frac{8\,(100)(18)}{\pi\,(3)^3}\right) = 183.91 \text{ N/mm}^2$$

$$\tau_a = K\left(\frac{8P_aD}{\pi d^3}\right) = (1.2525)\left(\frac{8(50)(18)}{\pi(3)^3}\right) = 106.32 \text{ N/mm}^2$$

Step II *Factor of safety*

From. Eq. (10.21), the relationships for oil-hardened and tempered steel wires are as follows:

$S'_{se} = 0.22\ S_{ut} = 0.22(1430) = 314.6$ N/mm^2

$S_{sy} = 0.45\ S_{ut} = 0.45(1430) = 643.5$ N/mm^2

From Eq. (10.22),

$$\frac{\tau_a}{\left(\dfrac{S_{sy}}{fs}\right) - \tau_m} = \frac{\frac{1}{2}S'_{se}}{S_{sy} - \frac{1}{2}S'_{se}}$$

or
$$\frac{106.32}{\left(\dfrac{643.5}{fs}\right) - 183.91} = \frac{\frac{1}{2}(314.6)}{643.5 - \frac{1}{2}(314.6)}$$

$$\frac{106.32}{\left(\frac{643.5}{fs}\right) - 183.91} = \frac{157.3}{486.2}$$

$$\frac{643.5}{(fs)} - 183.91 = \frac{106.32(486.2)}{157.3} = 328.63$$

$$\frac{643.5}{(fs)} = 183.91 + 328.63$$

$$(fs) = 1.26$$

Example 10.14 *An eccentric cam, 100 mm in diameter, rotates with an eccentricity of 10 mm as shown in Fig. 10.21. The roller follower is held against the cam by means of a helical compression spring. The force between the cam and the follower varies from 100 N at the lowest position to 350 N at the highest position of the follower. The permissible shear stress in the spring wire is recommended as 30% of the ultimate tensile strength. Design the spring from static considerations and determine the factor of safety against fluctuating stresses. Neglect the effect of inertia forces.*

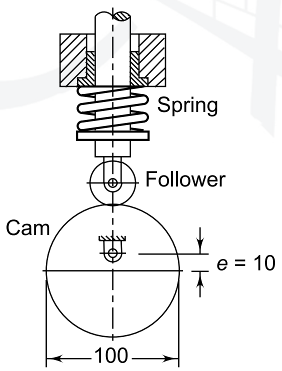


Fig. 10.21 *Cam Mechanism*

Solution

Given $P_{max.} = 350$ N $P_{min.} = 100$ N
$e = 10$ mm $\tau = 0.3\ S_{ut}$

Step I *Design against static load*

The spring is subjected to a fluctuating force. Therefore, the oil-hardened and tempered steel wire of SW grade is selected for this application. The spring index is assumed as 6.

From Eq. (10.7) and (10.5),

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4(6)-1}{4(6)-4} + \frac{0.615}{6} = 1.2525$$

$$K_s = 1 + \frac{0.5}{C} = 1 + \frac{0.5}{6} = 1.0833$$

From Eq. (10.13),

$$\tau = K\left(\frac{8PC}{\pi d^2}\right) \quad \text{or} \quad \tau = (1.2525)\left[\frac{8(350)(6)}{\pi d^2}\right]$$

or $$\tau = \frac{6697.88}{d^2}\ \text{N/mm}^2 \qquad \text{(a)}$$

The permissible shear stress is denoted by τ_d in order to differentiate it from the induced stress τ. It is given by,

$$\tau_d = 0.3\ S_{ut} \qquad \text{(b)}$$

Equations (a) and (b) are solved by the trial and error method.

Trial 1

$d = 3$ mm

$$\tau = \frac{6697.88}{d^2} = \frac{6697.88}{(3)^2} = 744.21\ \text{N/mm}^2$$

From Table 10.2,

$S_{ut} = 1520$ N/mm^2

$\tau_d = 0.3\ S_{ut} = 0.3(1520) = 456$ N/mm^2

Therefore,

$$\tau > \tau_d$$

The design is not safe.

Trial 2

$d = 3.6$ mm

$$\tau = \frac{6697.88}{d^2} = \frac{6697.88}{(3.6)^2} = 516.81\ \text{N/mm}^2$$

From Table 10.2,

$S_{ut} = 1480$ N/mm^2

$\tau_d = 0.3\ S_{ut} = 0.3(1480) = 444$ N/mm^2

Therefore,

$$\tau > \tau_d$$

The design is not safe.

Trial 3

$d = 4$ mm

$$\tau = \frac{6697.88}{d^2} = \frac{6697.88}{(4)^2} = 418.62\ \text{N/mm}^2$$

From Table 10.2,

$S_{ut} = 1480$ N/mm^2

$$\tau_d = 0.3\, S_{ut} = 0.3(1480) = 444 \text{ N/mm}^2$$

Therefore, $\tau < \tau_d$

The design is satisfactory and the wire diameter should be 4 mm. However, the spring is subjected to fluctuating stresses and to account for these stresses, the wire diameter is increased to 5 mm.

$$d = 5 \text{ mm}$$

$$D = Cd = 6(5) = 30 \text{ mm}$$

It is observed from Fig. 10.21 that the deflection of the spring between the highest and the lowest position of the follower is twice the eccentricity or 20 mm.

$$k = \frac{P_{\max.} - P_{\min.}}{\delta} = \frac{350 - 100}{20} = 12.5 \text{ N/mm}$$

From Eq. (10.9),

$$N = \frac{Gd^4}{8D^3k} = \frac{81370(5)^4}{8(30)^3(12.5)} = 18.84 \text{ or } 19$$

It is assumed that the spring has square and ground ends.

$$N_t = N + 2 = 19 + 2 = 21$$

From Eq. (10.8),

$$\delta = \frac{8PD^3N}{Gd^4} = \frac{8(350)(30)^3(19)}{(81370)(5)^4} = 28.24 \text{ mm}$$

Solid length of spring = $N_t\, d = 21(5) = 105$ mm

It is assumed that there will be a gap of 0.5 mm between consecutive coils when the spring is subjected to the maximum force. The total number of coils is 21. The total axial gap between the coils will be $(21 - 1) \times 0.5 = 10$ mm.

Free length = solid length + total axial gap + δ

Free length = 105 + 10 + 28.24

= 143.24 or 145 mm

Step II *Factor of safety against fluctuating load*

$$P_m = \frac{1}{2}(P_{\max.} + P_{\min.}) = \frac{1}{2}(350 + 100) = 225 \text{ N}$$

$$P_a = \frac{1}{2}(P_{\max.} - P_{\min.}) = \frac{1}{2}(350 - 100) = 125 \text{ N}$$

From Eq. (10.18) and (10.19),

$$\tau_m = K_s\left(\frac{8P_mD}{\pi d^3}\right) = (1.0833)\left(\frac{8(225)(30)}{\pi(5)^3}\right)$$

$$= 148.96 \text{ N/mm}^2$$

$$\tau_a = K\left(\frac{8P_aD}{\pi d^3}\right) = (1.2525)\left(\frac{8(125)(30)}{\pi(5)^3}\right)$$

$$= 95.68 \text{ N/mm}^2$$

From Table No. 10.2, ($d = 5$ mm)

$S_{ut} = 1440 \text{ N/mm}^2$

From. Eq. (10.21), the relationships for oil-hardened and tempered steel wire are as follows:

$S'_{se} = 0.22\, S_{ut} = 0.22(1440) = 316.8 \text{ N/mm}^2$

$S_{sy} = 0.45\, S_{ut} = 0.45(1440) = 648 \text{ N/mm}^2$

From Eq. (10.22),

$$\frac{\tau_a}{\left(\dfrac{S_{sy}}{fs}\right) - \tau_m} = \frac{\frac{1}{2}S'_{se}}{S_{sy} - \frac{1}{2}S'_{se}}$$

or
$$\frac{95.68}{\left(\dfrac{648}{fs}\right) - 148.96} = \frac{\frac{1}{2}(316.8)}{648 - \frac{1}{2}(316.8)}$$

$$\frac{95.68}{\left(\dfrac{648}{fs}\right) - 148.96} = \frac{158.4}{489.6}$$

$$\frac{648}{(fs)} - 148.96 = \frac{95.68\,(489.6)}{158.4} = 295.74$$

$$\frac{648}{(fs)} = 148.96 + 295.74$$

$$(fs) = 1.46$$

In Section 10.8 on 'Design of Helical Springs', it was explained that the factor of safety in spring design is usually 1.5 or less on account of four factors. Therefore, the factor of safety of 1.46 in the above application is reasonable.

Step III *Spring specifications*

(i) Material = oil-hardened and tempered steel wire of Grade-SW
(ii) Wire diameter = 5 mm
(iii) Mean coil diameter = 30 mm
(iv) Free length = 145 mm
(v) Total number of turns = 21
(vi) Style of ends = square and ground

Example 10.15 *The constructional details of an exhaust valve of a diesel engine are shown in Fig. 10.22. The diameter of the valve is 32 mm and the suction pressure in the cylinder is 0.03 N/mm². The mass of the valve is 50 g. The maximum valve lift is 10 mm. The stiffness of the spring for the valve is 10 N/mm. The spring index can be assumed as 8. The permissible shear stress in the spring wire is recommended as 30% of the ultimate tensile strength. Neglecting the effect of inertia forces, design the spring for static considerations and determine the factor of safety against fluctuating stresses.*

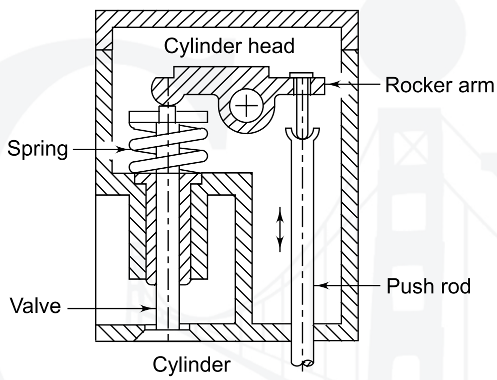


Fig. 10.22 *Valve Operating Mechanism*

Solution

Given $k = 10$ N/mm $C = 8$ $\tau = 0.3\ S_{ut}$

Step I *Maximum spring force*

The spring is subjected to fluctuating stresses. Therefore, oil-hardened and tempered valve spring wire of Grade-VW is selected for this application. Initially, the spring is fitted with a pre-load. The initial pre-load should be sufficient to hold the valve on its seat against the negative pressure inside the cylinder during the suction stroke. Since the cylinder is vertical, additional pre-load should be provided to account for the weight of the valve.

Suction force = valve area × suction pressure

$$= \frac{\pi}{4}(32)^2(0.03) = 24.13 \text{ N}$$

Weight of the valve $= mg = (0.05)(9.81) = 0.49$ N

Minimum pre-load $= 24.13 + 0.49 = 24.62$ N

To be on the safer side, the initial pre-load is taken as 30 N. During the exhaust stroke, the spring is further compressed by 10 mm (valve-lift). The maximum force acting on the spring is given by

$$P_{max.} = P_{min.} + k\delta = 30 + 10(10) = 130 \text{ N}$$

Step II *Design against static load*

From Eq. (10.7) and (10.5),

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4\,(8)-1}{4\,(8)-4} + \frac{0.615}{8} = 1.184$$

$$K_s = 1 + \frac{0.5}{C} = 1 + \frac{0.5}{8} = 1.0625$$

From Eq. (10.13),

$$\tau = K\left(\frac{8PC}{\pi d^2}\right) \quad \text{or} \quad \tau = (1.184)\left[\frac{8(130)(8)}{\pi d^2}\right]$$

$$\text{or} \quad \tau = \frac{3135.63}{d^2} \text{ N/mm}^2 \qquad \text{(a)}$$

The permissible shear stress is denoted by τ_d in order to differentiate it from the induced stress τ. It is given by,

$$\tau_d = 0.3\ S_{ut} \qquad \text{(b)}$$

Equations (a) and (b) are solved by the trial and error method.

Trial 1

$d = 2.5$ mm

$$\tau = \frac{3135.63}{d^2} = \frac{3135.63}{(2.5)^2} = 501.7 \text{ N/mm}^2$$

From Table 10.2,

$S_{ut} = 1470$ N/mm²

$\tau_d = 0.3\ S_{ut} = 0.3(1470) = 441$ N/mm²

Therefore, $\tau > \tau_d$

The design is not safe.

Trial 2

$d = 3$ mm

$$\tau = \frac{3135.63}{d^2} = \frac{3135.63}{(3)^2} = 348.4 \text{ N/mm}^2$$

From Table 10.2,

$S_{ut} = 1430$ N/mm²

$\tau_d = 0.3\ S_{ut} = 0.3(1430) = 429$ N/mm²

Therefore, $\tau < \tau_d$

The design is satisfactory and the wire diameter should be 3 mm. However, the spring is subjected to fluctuating stresses and to account for these stresses, the wire diameter is increased to 4 mm.

$$d = 4 \text{ mm}$$
$$D = Cd = 8(4) = 32 \text{ mm}$$

From Eq. (10.9),

$$N = \frac{Gd^4}{8D^3k} = \frac{81370(4)^4}{8(32)^3(10)} = 7.95 \text{ or } 8 \text{ coils}$$

It is assumed that the spring has square and ground ends.

$$N_t = N + 2 = 8 + 2 = 10 \text{ coils}$$

From Eq. (10.8),

$$\delta = \frac{8PD^3N}{Gd^4} = \frac{8(130)(32)^3(8)}{(81370)(4)^4} = 13.09 \text{ mm}$$

Solid length of spring $= N_t\, d = 10(4) = 40$ mm

It is assumed that there will be a gap of 0.5 mm between consecutive coils when the spring is subjected to the maximum force. The total number of coils is 10. The total axial gap between the coils will be $(10 - 1) \times 0.5 = 4.5$ mm.

Free length = solid length + total axial gap + δ

Free length = 40 + 4.5 + 13.09 = 57.59 or 60 mm

Step III *Factor of safety against fluctuating stresses*

$$P_m = \frac{1}{2}(P_{\text{max.}} + P_{\text{min.}}) = \frac{1}{2}(130 + 30) = 80 \text{ N}$$

$$P_a = \frac{1}{2}(P_{\text{max.}} - P_{\text{min.}}) = \frac{1}{2}(130 - 30) = 50 \text{ N}$$

From Eq. (10.18) and (10.19),

$$\tau_m = K_s\left(\frac{8P_mD}{\pi d^3}\right) = (1.0625)\left(\frac{8(80)(32)}{\pi(4)^3}\right)$$
$$= 108.23 \text{ N/mm}^2$$

$$\tau_a = K\left(\frac{8P_aD}{\pi d^3}\right) = (1.184)\left(\frac{8(50)(32)}{\pi(4)^3}\right)$$
$$= 75.38 \text{ N/mm}^2$$

From Table No. 10.2, ($d = 4$ mm)

$S_{ut} = 1400$ N/mm^2

From. Eq. (10.21), the relationships for oil-hardened and tempered steel wire are as follows:

$S'_{se} = 0.22\, S_{ut} = 0.22(1400) = 308$ N/mm^2

$S_{sy} = 0.45\, S_{ut} = 0.45(1400) = 630$ N/mm^2

From Eq. (10.22),

$$\frac{\tau_a}{\left(\frac{S_{sy}}{fs}\right) - \tau_m} = \frac{\frac{1}{2}S'_{se}}{S_{sy} - \frac{1}{2}S'_{se}}$$

or

$$\frac{75.38}{\left(\frac{630}{fs}\right) - 108.23} = \frac{\frac{1}{2}(308)}{630 - \frac{1}{2}(308)}$$

$$\frac{75.38}{\left(\frac{630}{fs}\right) - 108.23} = \frac{154}{476}$$

$$\frac{630}{(fs)} - 108.23 = \frac{75.38(476)}{154} = 232.99$$

$$\frac{630}{(fs)} = 108.23 + 232.99$$

$$(fs) = 1.85$$

The factor of safety against fluctuating stresses is reasonable.

Step IV *Spring specifications*

(i) Material = oil-hardened and tempered steel wire of Grade-VW
(ii) Wire diameter = 4 mm
(iii) Mean coil diameter = 32 mm
(iv) Free length = 60 mm
(v) Total number of coils = 10
(vi) Style of ends = square and ground

Example 10.16 *An oil-operated hydraulic compensating valve is shown in Fig. 10.23. Initially*

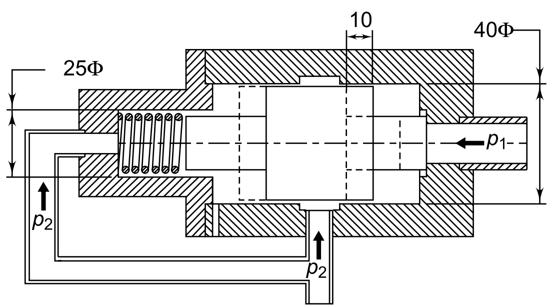


Fig. 10.23 *Compensating Valve*

when ($p_1 = p_2$) there is a pre-load of 100 N in the spring. The plunger moves through 10 mm when,

$p_1 = 0.35\ N/mm^2$
$p_2 = 0.25\ N/mm^2$

This is the normal operating condition of the valve. The spring is to be fitted in a cylindrical space of 25 mm diameter. There is no restriction on the free length of the spring. The permissible shear stress in the spring wire is recommended as 30% of the ultimate tensile strength. Design the spring from static considerations and determine the factor of safety against fluctuating stresses. Neglect the effect of inertia forces.

Solution

Given $D_{max.} = 25$ mm $\quad \tau = 0.3\ S_{ut}$

Step I *Maximum spring force*

The spring is subjected to fluctuating stresses. Therefore, an oil-hardened and tempered valve spring wire of Grade-VW is selected for this application. The spring index for such applications varies from 6 to 8. Since there is restriction on the maximum coil diameter, a low value of 5 is selected as the spring index.

The spring force is minimum when the plunger does not move through 10 mm.

$P_{min.}$ = initial pre-load = 100 N

The spring force is maximum when the plunger moves through 10 mm. The maximum spring force consists of initial pre-load and additional spring forces.

$$P_{max.} = P_{min.} + \text{additional force}$$
$$= 100 + \frac{\pi}{4}(40)^2\ (0.35 - 0.25)$$
$$= 100 + 125.66 = 225.66 \text{ N}$$

Step II *Design against static load*

When the spring force varies from 100 N to 225.66 N, the spring is compressed by 10 mm.

$$k = \frac{225.66 - 100}{10} = 12.57 \text{ N/mm}$$

From Eq. (10.7) and (10.5),

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4(5)-1}{4(5)-4} + \frac{0.615}{5}$$
$$= 1.3105$$
$$K_s = 1 + \frac{0.5}{C} = 1 + \frac{0.5}{5} = 1.1$$

From Eq. (10.13),

$$\tau = K\left(\frac{8PC}{\pi d^2}\right) \quad \text{or} \quad \tau = (1.3105)\left[\frac{8(225.66)(5)}{\pi d^2}\right]$$

or $$\tau = \frac{3765.32}{d^2} \text{ N/mm}^2 \qquad \text{(a)}$$

The permissible shear stress is denoted by τ_d in order to differentiate it from induced stress τ. It is given by,

$$\tau_d = 0.3\ S_{ut} \qquad \text{(b)}$$

Equations (a) and (b) are solved by the trial and error method.

Trial 1

$d = 2.5$ mm

$$\tau = \frac{3765.32}{d^2} = \frac{3765.32}{(2.5)^2} = 602.45 \text{ N/mm}^2$$

From Table 10.2,

$S_{ut} = 1470$ N/mm^2

$\tau_d = 0.3\ S_{ut} = 0.3(1470) = 441$ N/mm^2

Therefore, $\tau > \tau_d$

The design is not safe.

Trial 2

$d = 3$ mm

$$\tau = \frac{3765.32}{d^2} = \frac{3765.32}{(3)^2} = 418.37 \text{ N/mm}^2$$

From Table 10.2,

$S_{ut} = 1430$ N/mm^2

$\tau_d = 0.3\ S_{ut} = 0.3(1430) = 429$ N/mm^2

Therefore, $\tau < \tau_d$

The design is satisfactory and the wire diameter should be 3 mm. However, the spring is subjected to fluctuating stresses and to account for these stresses, the wire diameter is increased to 3.6 mm.

$d = 3.6$ mm

$D = Cd = 5(3.6) = 18$ mm

$D_{max.} = D + d = 18 + 3.6 = 21.6$ mm

The outer diameter of the spring is 21.6 mm, leaving sufficient margin for diametral expansion. The spring can be easily fitted in a cylindrical space of 25 mm diameter.

From Eq. (10.9),

$$N = \frac{Gd^4}{8D^3k} = \frac{81370(3.6)^4}{8(18)^3(12.57)} = 23.3 \text{ or } 24 \text{ coils}$$

It is assumed that the spring has square and ground ends.

$$N_t = N + 2 = 24 + 2 = 26 \text{ coils}$$

From Eq. (10.8),

$$\delta = \frac{8PD^3N}{Gd^4} = \frac{8(225.66)(18)^3(24)}{(81370)(3.6)^4} = 18.49 \text{ mm}$$

Solid length of spring $= N_t\, d = 26(3.6)$
$= 93.6$ mm

It is assumed that there will be a gap of 0.5 mm between consecutive coils when the spring is subjected to the maximum force. The total number of coils is 26. The total axial gap between the coils will be $(26 - 1) \times 0.5 = 12.5$ mm.

Free length = solid length + total axial gap + δ

Free length = 93.6 + 12.5 + 18.49
= 124.59 or 125 mm

Step III *Factor of safety against fluctuating stresses*

$$P_m = \frac{1}{2}(P_{\max.} + P_{\min.}) = \frac{1}{2}(225.66 + 100)$$
$$= 162.83 \text{ N}$$

$$P_a = \frac{1}{2}(P_{\max.} - P_{\min.}) = \frac{1}{2}(225.66 - 100)$$
$$= 62.83 \text{ N}$$

From Eq. (10.18) and (10.19),

$$\tau_m = K_s\left(\frac{8P_mD}{\pi d^3}\right) = (1.1)\left(\frac{8(162.83)(18)}{\pi(3.6)^3}\right)$$
$$= 175.97 \text{ N/mm}^2$$

$$\tau_a = K\left(\frac{8P_aD}{\pi d^3}\right) = (1.3105)\left(\frac{8(62.83)(18)}{\pi(3.6)^3}\right)$$
$$= 80.89 \text{ N/mm}^2$$

From Table No. 10.2, ($d = 3.6$ mm)

$$S_{ut} = 1400 \text{ N/mm}^2$$

From Eq. (10.21), the relationships for oil-hardened and tempered steel wire are as follows:

$$S'_{se} = 0.22\, S_{ut} = 0.22(1400) = 308 \text{ N/mm}^2$$
$$S_{sy} = 0.45\, S_{ut} = 0.45(1400) = 630 \text{ N/mm}^2$$

From Eq. (10.22),

$$\frac{\tau_a}{\left(\dfrac{S_{sy}}{fs}\right) - \tau_m} = \frac{\dfrac{1}{2}S'_{se}}{S_{sy} - \dfrac{1}{2}S'_{se}}$$

or
$$\frac{80.89}{\left(\dfrac{630}{fs}\right) - 175.97} = \frac{\dfrac{1}{2}(308)}{630 - \dfrac{1}{2}(308)}$$

$$\frac{80.89}{\left(\dfrac{630}{fs}\right) - 175.97} = \frac{154}{476}$$

$$\frac{630}{(fs)} - 175.97 = \frac{80.89(476)}{154} = 250.02$$

$$\frac{630}{(fs)} = 175.97 + 250.02$$

$$(fs) = 1.48$$

In Section 10.8 on 'Design of Helical Springs', it was explained that the factor of safety in spring design is usually 1.5 or less on account of four factors. Therefore, the factor of safety of 1.48 in the above application is reasonable.

Step IV *Spring specifications*

(i) Material = oil-hardened and tempered steel wire of Grade-VW
(ii) Wire diameter = 3.6 mm
(iii) Mean coil diameter = 18 mm
(iv) Free length = 125 mm
(v) Total number of turns = 26
(vi) Style of ends = square and ground

10.11 CONCENTRIC SPRINGS

A concentric spring consists of two helical compression springs, one inside the other, having the same axis. It is shown in Fig. 10.24. Concentric spring is also called a '*nested*' spring. In general, there are two springs. However, in certain applications, concentric spring consists of three coaxial springs, namely inner, middle and outer springs. Two springs shown in Fig. 10.24 have opposite hand of helices. If the outer spring has a right-hand helix, the inner spring always has a left-hand helix and vice versa.

Adjacent springs having opposite hands, prevent the locking of coils, in the event of axial misalignment or buckling of springs. Concentric spring has the following advantages:

(i) Since there are two springs, the load carrying capacity is increased and heavy load can be transmitted in a restricted space.

(ii) In concentric spring, the operation of the mechanism continues even if one of the springs breaks. This results in '*fail safe*' system.

(iii) In concentric spring, the spring vibrations called '*surge*', are eliminated.

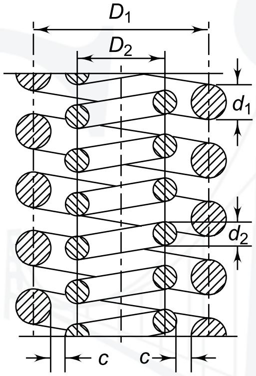


Fig. 10.24 *Concentric Springs*

Concentric springs are used as valve springs in heavy duty diesel engines, aircraft engines and railroad suspensions.

In some applications, concentric spring is used to obtain a spring force, which is not directly proportional to its deflection. Such a variable force-deflection characteristic is obtained by nesting two springs, one inside the other, having different free lengths. This type of concentric spring is shown in Fig. 10.25(a). The shorter spring begins to act only after the longer spring has been compressed to a certain amount of deflection. As shown in Fig. 10.25(b), the force-deflection characteristics of this type of concentric spring are as follows:

Force deflection characteristic of longer spring $= \overline{ab}$

Force deflection characteristic of shorter spring $= \overline{cd}$

Force deflection characteristic of composite spring $= \overline{agh}$

It is observed that $\overline{agh}$ is not a straight line. Initially, the concentric spring follows the straight line $\overline{ag}$, and at g, there is a sudden change in the load-deflection relationship and then it follows the straight line $\overline{gh}$. This results in variable force-deflection characteristic.

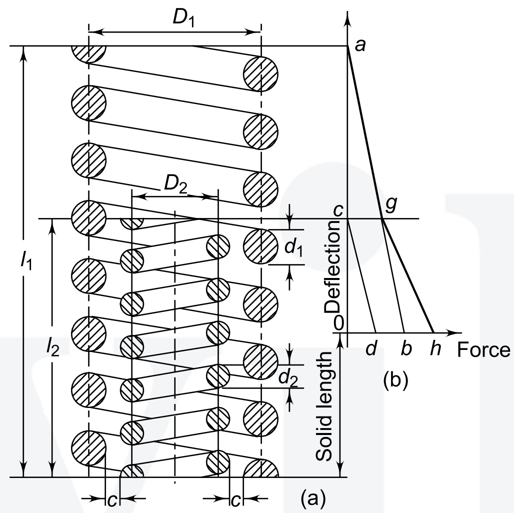


Fig. 10.25

This type of concentric spring is used in the governor of variable speed engines to take care of variable centrifugal force.

The design analysis of concentric spring shown in Fig. 10.24, is based on the following assumptions:

(i) The springs are made of the same material.

(ii) The maximum torsional shear stresses induced in outer and inner springs are equal.

(iii) They have the same free length.

(iv) Both springs are deflected by the same amount and therefore, have same solid length.

The following notations are used in the analysis:

d_1 = wire diameter of outer spring
d_2 = wire diameter of inner spring
D_1 = mean coil diameter of outer spring
D_2 = mean coil diameter of inner spring
P_1 = axial force transmitted by outer spring
P_2 = axial force transmitted by inner spring
P = total axial force
δ_1 = deflection of outer spring
δ_2 = deflection of inner spring

N_1 = number of active coils in outer spring

N_2 = number of active coils in inner spring

Since the maximum torsional shear stresses induced in both springs are equal, $\tau_1 = \tau_2$

From Eq. (10.6),

$$K_1\left(\frac{8P_1D_1}{\pi d_1^3}\right) = K_2\left(\frac{8P_2D_2}{\pi d_2^3}\right)$$

For the time being, we will neglect the effect of the Wahl factor (K) and assume, $K_1 = K_2$

Substituting in the above equation,

$$\left(\frac{P_1D_1}{d_1^3}\right) = \left(\frac{P_2D_2}{d_2^3}\right) \tag{a}$$

Since the deflections of the two springs are equal,

$$\delta_1 = \delta_2$$

From Eq. (10.8),

$$\left(\frac{8P_1D_1^3N_1}{Gd_1^4}\right) = \left(\frac{8P_2D_2^3N_2}{Gd_2^4}\right)$$

or
$$\left(\frac{P_1D_1^3N_1}{d_1^4}\right) = \left(\frac{P_2D_2^3N_2}{d_2^4}\right) \tag{b}$$

When both springs are completely compressed, their adjacent coils touch each other. The length of the spring in this case is called *solid length*. Since deflection corresponding to this condition is equal for both springs,

solid length of outer spring = solid length of inner spring

$$d_1 N_1 = d_2 N_2 \tag{c}$$

In the above expression, it is assumed that there are no inactive coils. The total number of coils is equal to the number of active coils.

Rearranging Eq. (b),

$$\left(\frac{P_1D_1^3(N_1d_1)}{d_1^5}\right) = \left(\frac{P_2D_2^3(N_2d_2)}{d_2^5}\right)$$

Substituting Eq. (c) in the above equation,

$$\left(\frac{P_1D_1^3}{d_1^5}\right) = \left(\frac{P_2D_2^3}{d_2^5}\right) \tag{d}$$

Dividing Eq. (d) by Eq. (a),

$$\frac{D_1^2}{d_1^2} = \frac{D_2^2}{d_2^2}$$

or
$$\frac{D_1}{d_1} = \frac{D_2}{d_2} = C = \text{spring index} \tag{e}$$

Therefore, two springs should have the same spring index. This also results in the same Wahl factor and confirms our earlier assumption ($K_1 = K_2$).

Substituting Eq. (e) in Eq. (a),

$$\left(\frac{P_1}{d_1^2}\right) = \left(\frac{P_2}{d_2^2}\right)$$

or
$$\frac{P_1}{P_2} = \frac{d_1^2}{d_2^2} \tag{10.23}$$

or
$$\frac{P_1}{P_2} = \frac{\pi d_1^2}{\pi d_2^2} = \frac{a_1}{a_2}$$

Therefore, the load shared by each spring is proportional to the cross-sectional area of the wire.

The clearance between the inner diameter of the outer spring and outer diameter of the inner spring is an important parameter in design of concentric springs. It should have a certain minimum value to account for diametral expansion of springs under the load. Suppose c is the radial clearance between the springs. It is illustrated in Fig. 10.26 and the following relationship can be written:

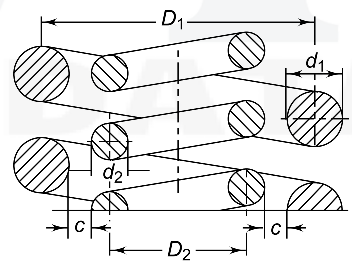


Fig. 10.26

$$D_1 = D_2 + \left[\frac{d_2}{2} + \frac{d_2}{2}\right] + 2c + \left[\frac{d_1}{2} + \frac{d_1}{2}\right]$$

or
$$2c = (D_1 - D_2) - (d_1 + d_2)$$

or
$$c = \frac{(D_1 - D_2)}{2} - \frac{(d_1 + d_2)}{2} \tag{10.24}$$

In the design of concentric springs, the diametral clearance ($2c$) is taken as the difference between wire diameters. Therefore,

$$2c = (d_1 - d_2) \tag{10.25}$$

or

$$c = \frac{(d_1 - d_2)}{2} \tag{f}$$

From Eq. (10.24) and (f),

$$\frac{(d_1 - d_2)}{2} = \frac{(D_1 - D_2)}{2} - \frac{(d_1 + d_2)}{2}$$

$$(D_1 - D_2) = (d_1 - d_2) + (d_1 + d_2) = 2d_1 \tag{g}$$

Substituting,

($D_1 = Cd_1$ and $D_2 = Cd_2$) in the above expression,

$C(d_1 - d_2) = 2d_1$ or $(C - 2)\, d_1 = Cd_2$

$$\frac{d_1}{d_2} = \frac{C}{(C-2)} \tag{10.26}$$

Equations (10.23) and (10.26) are used to find out the force transmitted by each spring.

Example 10.17 *A concentric spring is used as a valve spring in a heavy duty diesel engine. It consists of two helical compression springs having the same free length and same solid length. The composite spring is subjected to a maximum force of 6000 N and the corresponding deflection is 50 mm. The maximum torsional shear stress induced in each spring is 800 N/mm^2. The spring index of each spring is 6. Assume same material for two springs and the modulus of rigidity of spring material is 81370 N/mm^2. The diametral clearance between the coils is equal to the difference between their wire diameters. Calculate:*

(i) the axial force transmitted by each spring;

(ii) wire and mean coil diameters of each spring; and

(iii) number of active coils in each spring.

Solution

Given $P = 6000$ N $\delta = 50$ mm $\tau = 800$ N/mm^2
$C = 6$ $G = 81370$ N/mm^2

Step I *Axial force transmitted by each spring*

The diametral clearance between the coils is equal to the difference between their wire diameters. From Eq. (10.26),

$$\frac{d_1}{d_2} = \frac{C}{(C-2)} = \frac{6}{(6-2)} = 1.5$$

From Eq. (10.23),

$$\frac{P_1}{P_2} = \frac{d_1^2}{d_2^2} = \left(\frac{d_1}{d_2}\right)^2 = (1.5)^2 = 2.25 \tag{a}$$

Also, $$P_1 + P_2 = P = 6000 \text{ N} \tag{b}$$

Solving equations (a) and (b) simultaneously,

$$P_1 = 4153.85 \text{ N} \quad \text{and} \quad P_2 = 1846.15 \text{ N} \tag{i}$$

Step II *Wire and mean coil diameters*

From Eq. (10.7),

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4(6)-1}{4(6)-4} + \frac{0.615}{6} = 1.2525$$

Outer spring

From Eq. (10.13),

$$\tau = K\left(\frac{8P_1C}{\pi d_1^2}\right) \quad \text{or} \quad 800 = (1.2525)\left[\frac{8(4153.85)(6)}{\pi d_1^2}\right]$$

$$d_1 = 9.97 \text{ or } 10 \text{ mm}$$

$$D_1 = Cd_1 = 6(10) = 60 \text{ mm} \tag{ii}$$

Inner spring

From Eq. (10.13),

$$\tau = K\left(\frac{8P_2C}{\pi d_2^2}\right) \quad \text{or} \quad 800 = (1.2525)\left[\frac{8(1846.15)(6)}{\pi d_2^2}\right]$$

$$d_2 = 6.65 \text{ or } 7 \text{ mm}$$

$$D_2 = Cd_2 = 6(7) = 42 \text{ mm} \tag{ii}$$

Step III *Number of active coils*

From Eq. (10.8),

$$\delta = \frac{8P_1D_1^3N_1}{Gd_1^4} \quad \text{or} \quad 50 = \frac{8(4153.85)(60)^3 N_1}{(81370)(10)^4}$$

$$N_1 = 5.67 \text{ or } 6 \text{ coils} \tag{iii}$$

It is assumed that the springs have square and ground ends. Therefore,

$$(N_t)_1 = N_1 + 2 = 6 + 2 = 8 \text{ coils}$$

Since the springs have the same solid length,

$$(N_t)_1\, d_1 = (N_t)_2\, d_2 \quad \text{or} \quad 8(10) = (N_t)_2(7)$$

$$(N_t)_2 = 11.43 \text{ or } 12 \text{ coils}$$

$$N_2 = 12 - 2 = 10 \text{ coils} \tag{iii}$$

Example 10.18 *A concentric spring consists of two helical compression springs having the same free length. The composite spring is subjected to a maximum force of 2000 N. The wire diameter and mean coil diameter of the inner spring are 8 and 64 mm respectively. Also, the wire diameter and mean coil diameter of the outer spring are 10 and 80 mm respectively. The number of active coils in the inner and outer springs are 12 and 8 respectively. Assume same material for two springs and the modulus of rigidity of spring material is 81370 N/mm². Calculate:*

(i) the force transmitted by each spring;
(ii) the maximum deflection of the spring; and
(iii) the maximum torsional shear stress induced in each spring.

Solution

Given $P = 2000$ N $G = 81370$ N/mm^2

Step I *Force transmitted by each spring*

Suppose suffix i and o refer to inner and outer spring respectively.

$D_i = 64$ mm $d_i = 8$ mm $N_i = 12$ coils
$D_o = 80$ mm $d_o = 10$ mm $N_o = 8$ coils

Since both springs have same deflection, $\delta_i = \delta_o$

From Eq. (10.8),

$$\frac{8P_iD_i^3N_i}{Gd_i^4} = \frac{8P_oD_o^3N_o}{Gd_o^4}$$

or

$$\frac{P_iD_i^3N_i}{d_i^4} = \frac{P_oD_o^3N_o}{d_o^4}$$

$$\frac{P_i(64)^3(12)}{(8)^4} = \frac{P_o(80)^3(8)}{(10)^4}$$

$$\frac{P_o}{P_i} = 1.875 \qquad \text{(a)}$$

Also, $P_o + P_i = 2000$ N (b)

Solving equations (a) and (b) simultaneously,

$P_o = 1304.35$ N $P_i = 695.65$ N (i)

Step II *Maximum deflection of the spring*

From Eq. (10.8),

$$\delta_i = \delta_o = \frac{8P_oD_o^3N_o}{Gd_0^4} = \frac{8(1304.35)(80)^3(8)}{(81370)(10)^4}$$

$$= 52.53 \text{ mm} \qquad \text{(ii)}$$

Step III *Maximum shear stress*

$$C = \frac{D_o}{d_o} = \frac{80}{10} = 8 \qquad C = \frac{D_i}{d_i} = \frac{64}{8} = 8$$

From Eq. (10.7),

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4(8)-1}{4(8)-4} + \frac{0.615}{8} = 1.184$$

Outer spring

From Eq. (10.13),

$$\tau_o = K\left(\frac{8P_oC}{\pi d_o^2}\right) = (1.184)\left[\frac{8(1304.35)(8)}{\pi(10)^2}\right]$$

$$= 314.61 \text{ N/mm}^2 \qquad \text{(iii)}$$

Inner spring

$$\tau_i = K\left(\frac{8P_iC}{\pi d_i^2}\right) = (1.184)\left[\frac{8(695.65)(8)}{\pi(8)^2}\right]$$

$$= 262.18 \text{ N/mm}^2 \qquad \text{(iii)}$$

Example 10.19 *A concentric spring consists of two helical compression springs one inside the other. The free length of the outer spring is 15 mm greater than that of the inner spring. The wire diameter and mean coil diameter of the inner spring are 5 and 30 mm respectively. Also, the wire diameter and mean coil diameter of the outer spring are 6 and 36 mm respectively. The number of active coils in the inner and outer springs are 8 and 10 respectively. Assume same material for two springs and the modulus of rigidity of spring material is 81370 N/mm². The composite spring is subjected to a maximum axial force of 1000 N. Calculate:*

(i) the compression of each spring;
(ii) the force transmitted by each spring; and
(iii) the maximum torsional shear stress induced in each spring.

Solution

Given $P = 1000$ N $G = 81370$ N/mm^2

Step I *Compression of each spring*

Suppose suffix i and o refer to inner and outer spring respectively.

$D_i = 30$ mm $d_i = 5$ mm $N_i = 8$ coils
$D_o = 36$ mm $d_o = 6$ mm $N_o = 10$ coils

Stiffness of springs

From Eq. (10.9),

$$k_i = \frac{Gd_i^4}{8D_i^3 N_i} = \frac{(81370)(5)^4}{8(30)^3(8)} = 29.43 \text{ N/mm}$$

$$k_o = \frac{Gd_o^4}{8D_o^3 N_o} = \frac{(81370)(6)^4}{8(36)^3(10)} = 28.25 \text{ N/mm}$$

This type of spring is shown in Fig. 10.25. The free length of the outer spring is 15 mm greater than the inner spring. Therefore, the inner spring will not transmit any force till the outer spring is compressed by 15 mm. Suppose P is the axial force on the outer spring corresponding to this compression.

$$P = k_o\,\delta = 28.25\,(15) = 423.75 \text{ N}$$

After this load, both springs are active and each will transmit the force.

Remaining load shared by two springs = 1000 – 423.75 = 576.25 N

Concentric springs are parallel springs. From Eq. (10.12),

$$k = k_o + k_i = 28.25 + 29.43 = 57.68 \text{ N/mm}$$

where k is the combined stiffness of the composite spring. Suppose x is the further compression of two springs.

Remaining load = kx or $576.25 = 57.68\,x$

$$x = 9.99 \text{ mm}$$

Compression of outer spring = $\delta_o = 15 + 9.99 = 24.99$ mm (i)

Compression of inner spring = $\delta_i = 9.99$ mm (i)

Step II *Force transmitted by each spring*

Force transmitted by outer spring

$$P_o = k_o\,\delta_o = 28.25(24.99) = 705.97 \text{ N} \quad \text{(ii)}$$

Force transmitted by inner spring

$$P_i = k_i\,\delta_i = 29.43(9.99) = 294.01 \text{ N} \quad \text{(ii)}$$

Step III *Maximum shear stress*

$$C = \frac{D_o}{d_o} = \frac{36}{6} = 6 \qquad C = \frac{D_i}{d_i} = \frac{30}{5} = 6$$

From Eq. (10.7),

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4(6)-1}{4(6)-4} + \frac{0.615}{6} = 1.2525$$

Outer spring

From Eq. (10.6),

$$\tau_o = K\left(\frac{8P_oD_o}{\pi d_o^3}\right) = (1.2525)\left[\frac{8(705.97)(36)}{\pi(6)^3}\right]$$

$$= 375.28 \text{ N/mm}^2 \quad \text{(iii)}$$

Inner spring

$$\tau_i = K\left(\frac{8P_iD_I}{\pi d_i^3}\right) = (1.2525)\left[\frac{8(294.01)(30)}{\pi(5)^3}\right]$$

$$= 225.06 \text{ N/mm}^2 \quad \text{(iii)}$$

10.12 OPTIMUM DESIGN OF HELICAL SPRING

In certain applications, springs are designed with a specific objective, such as minimum weight, minimum volume or maximum energy storage capacity. In such analysis, only one objective is considered at a time for a given application. In this section, we shall discuss the optimum design of a valve spring. The operating mechanism of the exhaust valve of a diesel engine is shown in Fig. 10.22. The valve-spring is required to meet the following two conditions:

(i) The force acting on the spring at the most extended position should have a certain minimum value ($P_{\text{min.}}$). This force is required to keep the valve closed and can be considered as pre-compression.

(ii) When the spring is subjected to maximum compression and the valve is completely open, the stress in the spring wire ($\tau_{\text{max.}}$) should not exceed the permissible torsional shear stress.

For the purpose of analysis, the effect of inactive coils is neglected and the spring is designed on the basis of minimum weight.

Cross-sectional area of wire = $\dfrac{\pi}{4}d^2$

Length of one coil = πD

Length of all active coils = πDN

Volume of spring wire = $\left(\dfrac{\pi}{4}d^2\right)(\pi DN)$

Considering (ρ) as density, the weight of the spring is given by,

$$W = \rho\left(\frac{\pi^2}{4}\right)d^2DN$$

Substituting ($d = D/C$) in the above expression,

$$W = \left(\frac{\pi^2}{4}\right)\left(\frac{\rho}{C^2}\right)D^3N \quad \text{(a)}$$

In the optimization procedure, the above equation is called the *primary design equation*.

From Eq. (10.6),

$$\tau_{max.} = K\left(\frac{8P_{max.}D}{\pi d^3}\right) = K\left(\frac{8P_{max.}D}{\pi(D/C)^3}\right)$$

$$\therefore \quad P_{max.} = \frac{\pi D^2\tau_{max.}}{8KC^3} \quad \text{(b)}$$

From Eq. (10.8),

$$\delta = \frac{8PD^3N}{Gd^4} = \frac{8PD^4N}{GDd^4} = \frac{8PC^4N}{GD}$$

The deflection of the spring (Δ) for movement of the valve is given by,

$$\Delta = \delta_{max.} - \delta_{min.}$$

$$= \frac{8P_{max.}C^4N}{GD} - \frac{8P_{min.}C^4N}{GD}$$

$$= \frac{8C^4N}{GD}[P_{max.} - P_{min.}]$$

Substituting equation (b) in the above expression,

$$\Delta = \frac{8C^4N}{GD}\left[\frac{\pi D^2\tau_{max.}}{8KC^3} - P_{min.}\right]$$

$$\text{or} \quad N = \frac{\Delta GDK}{C\ \ [\pi D^2\tau_{max.} - 8KC^3P_{min.}]} \quad \text{(c)}$$

In the optimization procedure, the above equation is called the *subsidiary design equation*.

Substituting Eq. (c) in Eq. (a),

$$W = \frac{\pi^2}{4}\left(\frac{\rho\Delta GK}{C^3}\right)\frac{D^4}{[\pi D^2\tau_{max.} - 8KC^3P_{min.}]}$$

For minimum W, the right-hand factor should be minimum, or alternatively

$$\left[\frac{\pi D^2\tau_{max.} - 8KC^3P_{min.}}{D^4}\right]$$

should have maximum value. Differentiating with respect to D,

$$\frac{d}{dD}[\pi\tau_{max.}D^{-2} - 8KC^3P_{min.}D^{-4}] = 0$$

$$\left[\pi\tau_{max.}(-2D^{-3}) - 8KC^3P_{min.}(-4D^{-5})\right] = 0$$

$$P_{min.} = \frac{\pi D^2\tau_{max.}}{2(8KC^3)}$$

Substituting Eq. (b),

$$P_{min.} = \frac{P_{max.}}{2} \quad \text{(d)}$$

Therefore, if the spring is to have minimum weight, it must be designed in such a way that the force acting at the most extended position ($P_{min.}$) should be 50 per cent of the maximum force ($P_{max.}$).

Example 10.20 *A helical compression spring of the exhaust valve mechanism is initially compressed with a pre-load of 375 N. When the spring is further compressed and the valve is fully opened, the torsional shear stress in the spring wire should not exceed 750 N/mm². Due to space limitations, the outer diameter of the spring should not exceed 42 mm. The spring is to be designed for minimum weight. Calculate the wire diameter and the mean coil diameter of the spring.*

Solution

Given $P_{min.} = 375$ N $\quad \tau_{max.} = 750$ N/mm^2

$D_o = 42$ mm

Step I *Wire diameter*

For minimum weight,

$$P_{max.} = 2P_{min.} = 2(375) = 750 \text{ N}$$

Assuming the outer diameter to be 42 mm,

$$D_o = D + d = Cd + d = d\,(C + 1)$$

or
$$d = \frac{D_o}{(C+1)} = \frac{42}{(C+1)} \qquad \text{(a)}$$

From Eq. (10.6),

$$\tau_{max.} = K\left(\frac{8P_{max.}D}{\pi d^3}\right) = K\left(\frac{8P_{max.}C}{\pi d^2}\right)$$

Therefore,

$$750 = K\left(\frac{8(750)C(C+1)^2}{\pi(42)^2}\right)$$

$$C(C+1)^2 K = 692.72 \qquad \text{(b)}$$

The problem is solved by trial and error method. In practice, the spring index varies from 6 to 10. Considering values of C in this range, the results are tabulated in the following manner.

C	K Eq.(10.7)	$(C+1)^2$	$CK(C+1)^2$
5	1.311	36	235.98
6	1.253	49	368.38
7	1.213	64	543.42
8	1.184	81	767.23

Comparing Eq. (b) and the values in above table,

$$C = 8$$

$$d = \frac{42}{(C+1)} = \frac{42}{8+1} = 4.67 \text{ mm}$$

$$\therefore \quad d = 5 \text{ mm} \qquad \text{(i)}$$

Step II *Mean coil diameter*

Since $\quad D_o = D + d \qquad 42 = D + 5$

$$D = 37 \text{ mm} \qquad \text{(ii)}$$

Step III *Check for design*

$$C = \frac{D}{d} = \frac{37}{5} = 7.4$$

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4(7.4)-1}{4(7.4)-4} + \frac{0.615}{7.4} = 1.2$$

From Eq. (10.13),

$$\tau_{max.} = K\left(\frac{8PC}{\pi d^2}\right) = (1.2)\left[\frac{8(750)(7.4)}{\pi(5)^2}\right]$$

$$= 678.38 \text{ N/mm}^2$$

Therefore,

$\tau_{max.} < 750$ N/mm^2

and the dimensions are satisfactory.

10.13 SURGE IN SPRING

When the natural frequency of vibrations of the spring coincides with the frequency of external periodic force, which acts on it, resonance occurs. In this state, the spring is subjected to a wave of successive compressions of coils that travels from one end to the other and back. This type of vibratory motion is called '*surge*' of spring. Surge is found in valve springs, which are subjected to periodic force.

Let us consider a helical compression spring, one end of which is held against a flat rigid surface and the other end is subjected to a periodic external force. It always takes some 'time' to transmit this force from one end to the other. The force is transmitted by compression of coils. Initially, the end coil in contact with external force compresses and then it transmits a large portion of its compression to the adjacent second coil. The second coil compresses and in turn, transmits its major compression to the third coil. This process continues and a wave of compressed coils travels along the length of the spring till it reaches the end supported on rigid surface where it is 'reflected' back. The reflected wave travels back to the end in contact with the external force. If the time required for the wave to travel from one end to the other and back coincides with the time interval between successive load applications of periodic external force, resonance occurs and very large deflections of the coils are produced resulting in very high stresses. Many times, this stress in the coils is more than the endurance limit stress of the spring and fatigue failure occurs. Surge is the main cause of failure in valve springs.

The natural frequency of helical compression springs held between two parallel plates is given by,

$$\omega = \frac{1}{2}\sqrt{\frac{k}{m}} \qquad \text{(a)}$$

The natural frequency of helical compression springs with one end on the flat plate and the other end free, supporting the external force is given by,

$$\omega = \frac{1}{4}\sqrt{\frac{k}{m}} \qquad \text{(b)}$$

where,

k = stiffness of spring (N/m)

m = mass of spring (kg)

The mass of the spring is given by,

$$m = Al\rho \qquad \text{(c)}$$

where,

A = cross-sectional area of spring = $\left(\frac{\pi}{4}d^2\right)$

l = length of spring = $(\pi D N_t)$

ρ = mass density of spring material

Surge in springs is avoided by the following methods:

(i) The spring is designed in such a way that the natural frequency of the spring is 15 to 20 times the frequency of excitation of the external force. This prevents the resonance condition to occur.

(ii) The spring is provided with friction dampers on central coils. This prevents propagation of surge wave.

(iii) A spring made of stranded wire reduces the surge. In this case, the wire of the spring is made of three strands. The direction of winding of strands is opposite to the direction of winding of the coils while forming the spring. In case of compression of the coils, the spring tends to wind the individual wires closer together, which introduces friction. This frictional damping reduces the possibility of surge.

Surge is a serious problem in typical vibrations like valve springs and guns. It is not a problem in other applications where the external load is steady.

10.14 HELICAL TORSION SPRINGS

A helical torsion spring is a device used to transmit the torque to a particular component of a machine or mechanism. It is widely used in door hinges, brush holders, automobile starters and door locks. As shown in Fig. 10.27, the construction of the helical torsion spring is similar to that of compression or extension springs, except that the ends are formed in such a way that the spring is loaded by a torque about the axis of the coils. The helical torsion spring resists the bending moment ($P \times r$), which tends to wind up the spring. The primary stresses in this spring are flexural in contrast with torsional shear stresses in compression or extension springs. The term 'torsion spring' is somewhat misleading, because the wire of the spring is subjected to bending stresses. Each individual section of the torsion spring is, in effect, a portion of a curved beam. Using the curved beam theory, the bending stresses are given by

$$\sigma_b = K\left(\frac{M_b y}{I}\right) \qquad \text{(a)}$$

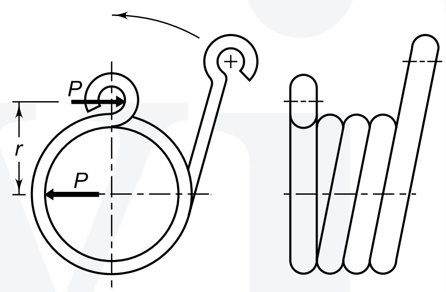


Fig. 10.27 *Helical Torsion Spring*

where K is the stress concentration factor due to curvature. For a wire of circular cross-section,

$$y = \left(\frac{d}{2}\right) \quad \text{and} \quad I = \left(\frac{\pi d^4}{64}\right)$$

Substituting the above values in Eq. (a), we get

$$\sigma_b = K\left(\frac{32 M_b}{\pi d^3}\right) \qquad (10.27)$$

AM Wahl analytically derived the expressions for the stress concentration factor K. They are given by,

$$K_i = \frac{4C^2 - C - 1}{4C(C-1)} \qquad (10.28)$$

$$K_o = \frac{4C^2 + C - 1}{4C(C+1)} \qquad (10.29)$$

where K_i and K_o are stress concentration factors at the inner and outer fibres of the coil respectively.

Referring to Fig. 10.28,

$$M_b = Pr$$

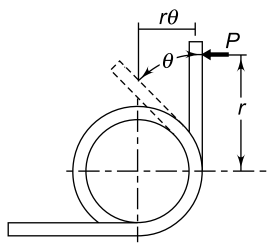


Fig. 10.28 *Angular Deflection of Spring*

The strain energy stored in the spring is given by

$$U = \int \frac{(M_b)^2\, dx}{2EI} \tag{b}$$

The integration is to be carried over the entire length of the wire, i.e., from 0 to (πDN). Therefore,

$$U = \int_0^{\pi DN} \frac{P^2 r^2 dx}{2EI}$$

$$U = \frac{P^2 r^2 (\pi DN)}{2EI} \tag{c}$$

The deflection in the direction of the force P is approximately ($r\theta$). Using Castigliano's theorem,

$$r\theta = \frac{\partial U}{\partial P} = \frac{Pr^2(\pi DN)}{EI}$$

Substituting ($I = \pi d^4/64$), we have

$$\theta = \frac{64\, PrDN}{Ed^4} \tag{10.30}$$

The stiffness of the helical torsion spring is defined as the bending moment required to produce unit angular displacement. Therefore,

$$k = \frac{Pr}{\theta}$$

or

$$k = \frac{Ed^4}{64\, DN} \tag{10.31}$$

Helical torsion springs are cold-wound and residual stresses are set up due to cold working. The direction of the external force acting on the spring should be such that it tends to wind up the spring. In this case, the resulting stresses are opposite to the residual stresses. On the other hand, when the external force tends to unwind the spring, stress-relieving treatment is required.

The design of the helical torsion spring is based on the torque-stress and the torque-deflection equations. The spring index is generally kept from 5 to 15. When it is less than 5, the strain on the coiling arbor of the torsion winder causes excessive tool breakage. When it is more than 15, the control over the spring pitch is lost.

Example 10.21 *It is required to design a helical torsion spring for a window shade. The spring is made of patented and cold-drawn steel wire of Grade-4. The yield strength of the material is 60% of the ultimate tensile strength and the factor of safety is 2. From space considerations, the mean coil diameter is kept as 18 mm. The maximum bending moment acting on the spring is 250 N-mm. The modulus of elasticity of the spring material is 207 000 N/mm². The stiffness of the spring should be 3 N-mm/rad. Determine the wire diameter and the number of active coils.*

Solution

Given $S_{yt} = 0.6\, S_{ut}$ $(fs) = 2$ $D = 18$ mm
$M_b = 250$ N-mm $E = 207\,000$ N/mm^2
$k = 3$ N-mm/rad

Step I *Wire diameter*

The wire diameter is calculated by the trial and error method.

Trial 1 $d = 1.4$ mm

From Table 10.1,

$$S_{ut} = 2290 \text{ N/mm}^2$$

The permissible stress (σ_t) is given by,

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{0.60 S_{ut}}{(fs)} = \frac{0.60\,(2290)}{(2)} = 687 \text{ N/mm}^2$$

$$C = \frac{D}{d} = \frac{18}{1.4} = 12.857$$

$$K_i = \frac{4C^2 - C - 1}{4C(C-1)} = \frac{4\,(12.857)^2 - (12.857) - 1}{4(12.857)\,(12.857 - 1)}$$
$$= 1.0616$$

$$\sigma_b = K_i \left(\frac{32 M_b}{\pi d^3}\right) = (1.0616)\left(\frac{32(250)}{\pi (1.4)^3}\right)$$
$$= 985.18 \text{ N/mm}^2$$

Therefore, $(\sigma_b) > (\sigma_t)$

The design is not safe.

Trial 2 $d = 1.6$ mm

From Table 10.1,

$$S_{ut} = 2250 \text{ N/mm}^2$$

The permissible stress (σ_t) is given by,

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{0.60\ S_{ut}}{(fs)} = \frac{0.60\ (2250)}{(2)}$$
$$= 675 \text{ N/mm}^2$$

$$C = \frac{D}{d} = \frac{18}{1.6} = 11.25$$

$$K_i = \frac{4C^2 - C - 1}{4C(C-1)} = \frac{4(11.25)^2 - (11.25) - 1}{4(11.25)(11.25 - 1)}$$
$$= 1.071$$

$$\sigma_b = K_i\left(\frac{32M_b}{\pi d^3}\right) = (1.071)\left(\frac{32(250)}{\pi(1.6)^3}\right)$$
$$= 665.84 \text{ N/mm}^2$$

Therefore, $(\sigma_b) < (\sigma_t)$

The design is satisfactory and the wire diameter should be 1.6 mm.

Step II *Number of active coils*

From. (10.31),

$$N = \frac{Ed^4}{64Dk} = \frac{(207\,000)(1.6)^4}{64(18)(3)} = 392.53$$

or $N = 393$ coils

10.15 SPIRAL SPRINGS

A spiral spring consists of a thin strip of rectangular cross-section, which is wound in the form of spiral as shown in Fig. 10.29. It is also called '*power*' spring or '*flat*' spiral spring. The inner end of this strip is fixed in the arbor at the centre. The outer end is clamped to a drum called 'retaining' drum. The spring is wound by rotating the arbor and during the winding process, energy is stored in the spring. This energy is released as mechanical torque through the drum when it gradually rotates and unwinds the spring. The strip is made of high quality steel. It is very thin which results in high flexibility and enables the arbor to have large angular deflection. In many applications, the angular deflection of the arbor is several dozens of revolutions. This results in accumulation of considerable amount of energy. Very often, the drum is integral with a gear and energy is released by rotation of this gear to another gear that drives a mechanism. Spiral springs are widely used in watches, cameras, instruments and automatic weapons. All types of toys are powered by this type of spring. They are also used as starters for small engines.

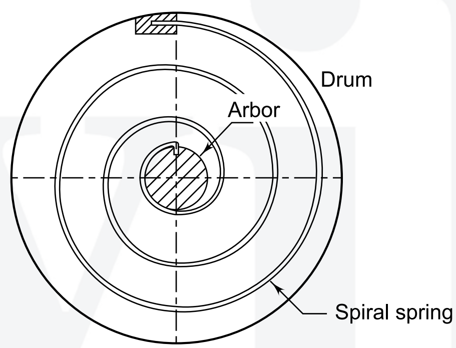


Fig. 10.29 *Spiral Spring*

As shown in Fig. 10.30, the strip of spiral spring is subjected to pure bending moment. The following

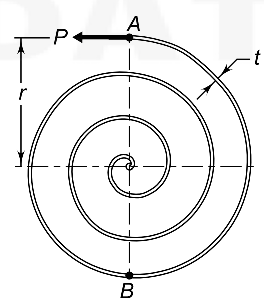


Fig. 10.30

notations are used in the analysis of spiral spring:

P = force induced at the outer end A due to winding of the arbor (N)

r = distance of centre of gravity of spiral from outer end (mm)

t = thickness of strip (mm)

b = width of strip perpendicular to plane of paper (mm)

l = length of strip from outer end to inner end (mm)

The outer end A of the spring is pulled by the force P. The bending moment M due to the force P acting at a distance r is given by,

$$M = Pr \tag{a}$$

Point B is at a farthest distance from the line of action of the force P. Therefore, bending moment is maximum at the point B. The maximum bending moment (M_b) is given by,

$$M_b = P\,(2r) = 2\,(Pr) = 2M$$

The maximum bending stress induced at the point B is given by,

$$\sigma_b = \frac{M_b y}{I}$$

where,

$$M_b = 2M \qquad y = \frac{t}{2} \qquad I = \frac{bt^3}{12}$$

Substituting,

$$\sigma_b = \frac{12M}{bt^2} \tag{10.32}$$

When both ends are clamped, the angle of rotation of the arbor (θ) with respect to the drum or the point A is given by,

$$\theta = \frac{Ml}{EI} = \frac{Ml}{E\left(\dfrac{bt^3}{12}\right)}$$

or

$$\theta = \frac{12Ml}{Ebt^3} \tag{10.33}$$

The deflection (δ) of one end of the spring with respect to the other is given by,

$$\delta = r\theta$$

or

$$\delta = \frac{12Mlr}{Ebt^3} \tag{10.34}$$

The strain energy (U) stored in the spring is given by,

$$U = \frac{1}{2}M\theta = \frac{1}{2}M\left(\frac{12Ml}{Ebt^3}\right)$$

or

$$U = \frac{6M^2 l}{Ebt^3} \tag{10.35}$$

Example 10.22 *A flat spiral spring is required to provide a maximum torque of 1200 N-mm. The spring is made of a steel strip and the maximum bending stress should not exceed 800 N/mm^2. When the stress in the spring decreases from 800 to 0 N/mm^2, the arbor turns through three complete revolutions with respect to the retaining drum. The thickness of the steel strip is 1.25 mm and the modulus of elasticity is 207 000 N/mm^2. Calculate the width and length of the steel strip.*

Solution

Given $M = 1200$ N-mm $\quad \sigma_b = 800$ N/mm^2
$E = 207\,000$ N/mm^2 $\quad t = 1.25$ mm

Step I *Width of strip*

From Eq. (10.32),

$$\sigma_b = \frac{12M}{bt^2} \quad \text{or} \quad 800 = \frac{12\,(1200)}{b\,(1.25)^2}$$

$\therefore \quad b = 11.52$ or 12 mm

Step II *Length of strip*

$\theta = 3$ revolutions $= 3\,(2\pi) = 6\pi$ radians

From Eq. (10.33),

$$\theta = \frac{12Ml}{Ebt^3}$$

or

$$6\pi = \frac{12(1200)l}{(207\,000)(12)(1.25)^3}$$

$\therefore \quad l = 6350.68$ mm or 6.35 m

Example 10.23 *A flat spiral spring, used in an electrical instrument, is required to exert a maximum force of 5 N at the free end against the retaining drum. The line of action of this force is 75 mm from the centre of gravity of the spiral. The spring is made of brass strip (E = 106 000 N/mm^2) and the maximum bending stress should not exceed 100 N/mm^2. The width and length of the strip are 12.5 and 750 mm. Calculate*

(i) the thickness of strip; and

(ii) the number of degrees of rotation through which the arbor should be turned to produce the required force.

Solution

Given $P = 5$ N $\quad r = 75$ mm $\quad b = 12.5$ mm

$l = 750$ mm $\quad \sigma_b = 100$ N/mm^2
$E = 106\,000$ N/mm^2

Step I *Thickness of strip*

$$M = Pr = 5\,(75) = 375 \text{ N-mm}$$

From Eq. (10.32),

$$\sigma_b = \frac{12M}{bt^2} \quad \text{or} \quad 100 = \frac{12(375)}{12.5t^2}$$

$$\therefore \qquad t = 1.897 \text{ or } 2 \text{ mm} \qquad \text{(i)}$$

Step II *Degrees of rotation*

From Eq. (10.33),

$$\theta = \frac{12Ml}{Ebt^3} = \frac{12(375)(750)}{(106\,000)(12.5)(2)^3} \text{ radians}$$

or $$\theta = \frac{12(375)(750)}{(106\,000)(12.5)(2)^3}\left(\frac{180}{\pi}\right) \text{ degrees}$$

$$\theta = 18.24° \qquad \text{(ii)}$$

10.16 MULTI-LEAF SPRING

Multi-leaf springs are widely used for the suspension of cars, trucks and railway wagons. A multi-leaf spring consists of a series of flat plates, usually of semi-elliptical shape, as shown in Fig. 10.31. The flat plates are called *leaves* of the spring. The leaves have graduated lengths. The leaf at the top has maximum length. The length gradually decreases from the top leaf to the bottom leaf. The longest leaf at the top is called *master leaf*. It is bent at both ends to from the spring eyes. Two bolts are inserted through these eyes to fix the leaf spring to the automobile body. The leaves are held together by means of two U-bolts and a centre clip. Rebound clips are provided to keep the leaves in alignment and prevent lateral shifting of the leaves during operation. At the centre, the leaf spring is supported on the axle. Multi-leaf springs are provided with one or two extra full length leaves in addition to master leaf. The extra full-length leaves are stacked between the master leaf and the graduated length leaves. The extra full-length leaves are provided to support the transverse shear force.

For the purpose of analysis, the leaves are divided into two groups namely, master leaf along with graduated-length leaves forming one group and extra full-length leaves forming the other. The following notations are used in the analysis:

n_f = number of extra full-length leaves
n_g = number of graduated-length leaves including master leaf
n = total number of leaves
b = width of each leaf (mm)
t = thickness of each leaf (mm)

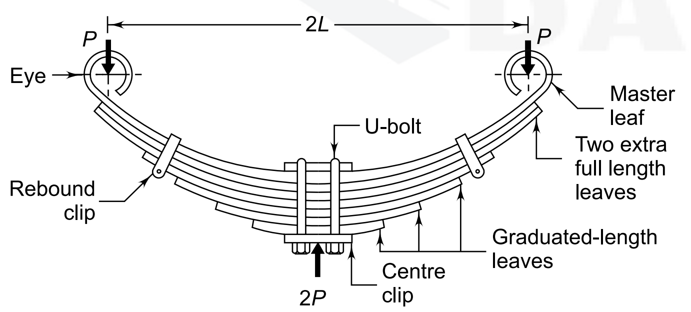


Fig. 10.31 *Semi-elliptic Leaf Spring*

L = length of the cantilever or half the length of semi-elliptic spring (mm)
P = force applied at the end of the spring (N)
P_f = portion of P taken by the extra full-length leaves (N)
P_g = portion of P taken by the graduated-length leaves (N)

The group of graduated-length leaves along with the master leaf can be treated as a triangular plate, as shown in Fig. 10.32. In this case, it is assumed that the individual leaves are separated and the master leaf placed at the centre. Then the second leaf is cut longitudinally into two halves, each of width (b/2) and placed on each side of the master leaf. A similar procedure is repeated for the other leaves. The resultant shape is approximately a triangular plate of thickness t and a maximum width at the support as ($n_g\, b$). The bending stress in the plate at the support is given by,

$$(\sigma_b)_g = \frac{M_b y}{I} = \frac{(P_g L)\,(t/2)}{\left[\dfrac{1}{12}\,(n_g b)\,(t^3)\right]}$$

or, $$(\sigma_b)_g = \frac{6P_g L}{n_g bt^2} \qquad \text{(a)}$$

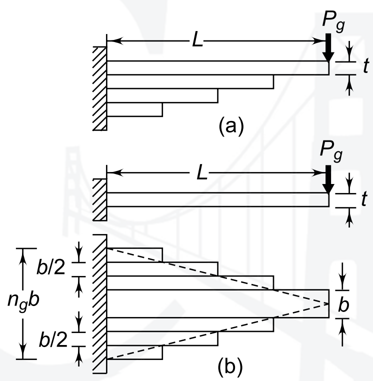


Fig. 10.32 *Graduated-length Leaves as Triangular Plate*

It can be proved that the deflection (δ_g) at the load point of the triangular plate is given by,

$$\delta_g = \frac{P_g L^3}{2EI_{\text{max.}}} = \frac{P_g L^3}{2E\left[\dfrac{1}{12}\,(n_g b)\,(t^3)\right]}$$

or, $$\delta_g = \frac{6P_g L^3}{En_g bt^3} \qquad \text{(b)}$$

Similarly, the extra full-length leaves can be treated as a rectangular plate of thickness t and uniform width ($n_f\, b$), as shown in Fig. 10.33. The bending stress at the support is given by,

$$(\sigma_b)_f = \frac{M_b\, y}{I} = \frac{(P_f L)(t/2)}{\left[\dfrac{1}{12}\,(n_f b)(t^3)\right]}$$

or $$(\sigma_b)_f = \frac{6P_f L}{n_f bt^2} \qquad \text{(c)}$$

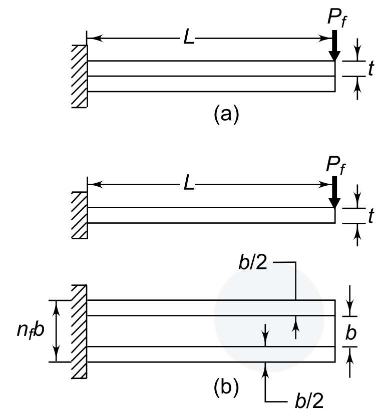


Fig. 10.33 *Extra full-length leaves as rectangular plate*

The deflection at the load point is given by,

$$\delta_f\ \frac{P_f L^3}{3EI} = \frac{P_f L^3}{3E\left[\dfrac{1}{12}\,(n_f b)(t^3)\right]}$$

or $$\delta_f = \frac{4P_f L^3}{En_f bt^3} \qquad \text{(d)}$$

Since the deflection of full-length leaves is equal to the deflection of graduated- length leaves,

$$\delta_g = \delta_f$$

$$\frac{6P_g L^3}{En_g bt^3} = \frac{4P_f L^3}{En_f bt^3}$$

or $$\frac{P_g}{P_f} = \frac{2n_g}{3n_f} \qquad \text{(e)}$$

also, $$P_g + P_f = P \qquad \text{(f)}$$

From Eqs (e) and (f),

$$P_f = \frac{3n_f P}{(3n_f + 2n_g)} \qquad \text{(g)}$$

$$P_g = \frac{2n_g P}{(3\,n_f + 2n_g)} \qquad \text{(h)}$$

Substituting the above values in Eqs. (a) and (c),

$$(\sigma_b)_g = \frac{12PL}{(3n_f + 2n_g)\,bt^2} \tag{10.36}$$

$$(\sigma_b)_f = \frac{18PL}{(3n_f + 2n_g)\,bt^2} \tag{10.37}$$

It is seen from the above equations that bending stresses in full-length leaves are 50% more than those in graduated-length leaves. The deflection at the end of the spring is determined from Eqs. (b) and (h). It is given by,

$$\delta = \frac{12PL^3}{Ebt^3(3n_f + 2n_g)} \tag{10.38}$$

Multi-leaf springs are designed using load-stress and load-deflection equations. The standard dimensions for the width and thickness of the leaf section are as follows:[6]

Nominal thickness (mm): 3.2, 4.5, 5, 6, 6.5, 7, 7.5, 8, 9, 10, 11, 12, 14, and 16

Nominal width (mm): 32, 40, 45, 50, 55, 60, 65, 70, 75, 80, 90, 100 and 125

The leaves are usually made of steels, 55Si2Mn90, 50Cr1 or 50CrlV23. The plates are hardened and tempered. The factor of safety based on the yield strength is from 2 to 2.5 for the automobile suspension.

10.17 NIPPING OF LEAF SPRINGS

As discussed in the previous section, the stresses in extra full-length leaves are 50% more than the stresses in graduated-length leaves. One of the methods of equalising the stresses in different leaves is to pre-stress the spring. The pre-stressing is achieved by bending the leaves to different radii of curvature, before they are assembled with the centre clip. As shown in Fig. 10.34, the full-length leaf is given a greater radius of curvature than the adjacent leaf. The radius of curvature decreases with shorter leaves. The initial gap C between the extra full-length leaf and the graduated-length leaf before the assembly, is called a '*nip*'. Such pre-stressing, achieved by a difference in radii of curvature, is known as '*nipping*'. Nipping is common in automobile suspension springs.

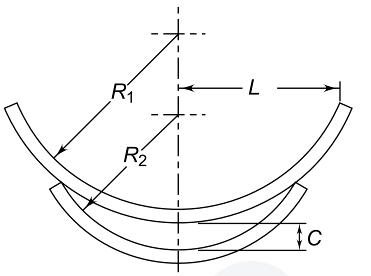


Fig. 10.34 *Nipping of Leaf Spring*

Rewriting Eqs (a) and (c) of the previous section,

$$(\sigma_b)_g = \frac{6P_gL}{n_gbt^2} \tag{a}$$

$$(\sigma_b)_f = \frac{6P_fL}{n_fbt^2} \tag{b}$$

Assuming that pre-stressing results in stress-equalisation,

$$(\sigma_b)_g = (\sigma_b)_f$$

From (a) and (c),

$$\frac{P_g}{P_f} = \frac{n_g}{n_f} \tag{i}$$

Also, $$P_g + P_f = P \tag{ii}$$

Solving Eqs (i) and (ii),

$$P_g = \frac{n_gP}{n} \tag{iii}$$

$$P_f = \frac{n_fP}{n} \tag{iv}$$

where $$n = n_g + n_f$$

Rewriting Eqs (b) and (d) of the previous section,

$$\delta_g = \frac{6P_gL^3}{En_gbt^3} \tag{c}$$

$$\delta_f = \frac{4P_fL^3}{En_fbt^3} \tag{d}$$

[6] IS 1135–1966: Specifications for leaf-springs for automobile suspension.

Under the maximum force P, the deflection of graduated-length leaves will exceed the deflection of extra full-length leaves by an amount equal to the initial nip C.

Therefore,

$$C = \frac{6P_g L^3}{E n_g b t^3} - \frac{4P_f L^3}{E n_f b t^3}$$

Substituting Eqs (iii) and (iv) in the above expression,

$$C = \frac{2PL^3}{Enbt^3} \tag{10.39}$$

The initial pre-load P_i required to close the gap C between the extra full-length leaves and graduated-length leaves is determined by considering the initial deflection of leaves.

Under the action of pre-load P_i,

$$C = (\delta_g)_i + (\delta_f)_i$$

$$\frac{2PL^3}{Enbt^3} = \frac{6(P_i/2)L^3}{E n_g b t^3} + \frac{4(P_i/2)L^3}{E n_f b t^3}$$

or

$$P_i = \frac{2 n_g n_f P}{n(3n_f + 2n_g)} \tag{10.40}$$

The resultant stress in the extra full-length leaves is obtained by superimposing the stresses due to initial pre-load (P_i) and the external force (P). From Eq. (b),

$$(\sigma_b)_f = \frac{6L(P_f - 0.5P_i)}{n_f b t^2}$$

Substituting Eq. (g) of the previous section and Eq. (10.40) in the above expression,

$$(\sigma_b)_f = \frac{6PL}{nbt^2}$$

Since the stresses are equal in all leaves, the above expression is written as

$$\sigma_b = \frac{6PL}{nbt^2} \tag{10.41}$$

The deflection of the multi-leaf spring due to the external force P is the same as given by Eq. (10.38).

Example 10.24 *A semi-elliptic leaf spring used for automobile suspension consists of three extra full-length leaves and 15 graduated-length leaves, including the master leaf. The centre-to-centre distance between two eyes of the spring is 1 m. The maximum force that can act on the spring is 75 kN. For each leaf, the ratio of width to thickness is 9:1. The modulus of elasticity of the leaf material is 207 000 N/mm². The leaves are pre-stressed in such a way that when the force is maximum, the stresses induced in all leaves are same and equal to 450 N/mm². Determine*

(i) the width and thickness of the leaves;
(ii) the initial nip; and
(iii) the initial pre-load required to close the gap C between extra full-length leaves and graduated-length leaves.

Solution

Given $2P = 75$ kN $2L = 1$ m $b = 9t$ $n_f = 3$ $n_g = 15$ $E = 207\,000$ N/mm² $\sigma_b = 450$ N/mm²

Step I *Width and thickness of the leaves*

$2P = 75$ kN or $P = 37\,500$ N
$2L = 1$ m or $L = 500$ mm
$b = 9t$ $n = n_f + n_g = 3 + 15 = 18$

From Eq. (10.41),

$$\sigma_b = \frac{6PL}{nbt^2} \quad \text{or} \quad (450) = \frac{6(37\,500)(500)}{(15+3)(9t)t^2}$$

$\therefore$ $t = 11.56$ or 12 mm

$$b = 9\,t = 9\,(12) = 108 \text{ mm} \tag{i}$$

Step II *Initial nip*

From Eq. (10.39),

$$C = \frac{2PL^3}{Enbt^3} = \frac{2(37\,500)(500)^3}{(207\,000)(18)(108)(12)^3}$$
$$= 13.48 \text{ mm} \tag{ii}$$

Step III *Initial pre-load*

From Eq. (10.40),

$$P_i = \frac{2n_g n_f P}{n(3n_f + 2n_g)} = \frac{2(15)(3)(37\,500)}{18(3\times 3 + 2\times 15)}$$
$$= 4807.69 \text{ N} \tag{iii}$$

Example 10.25 *A semi-elliptic multi-leaf spring is used for the suspension of the rear axle of a truck. It consists of two extra full-length leaves and ten graduated-length leaves including the master leaf. The centre-to-centre distance between the spring eyes is 1.2 m. The leaves are made of steel 55Si2Mo90 (S_{yt} = 1500 N/mm² and E = 207 000 N/mm²) and the factor of safety is 2.5. The spring is to be designed for a maximum force of 30 kN. The leaves are pre-stressed so as to equalize stresses in all leaves. Determine*

(i) the cross-section of leaves; and

(ii) the deflection at the end of the spring.

Solution

Given $2P = 30$ kN $2L = 1.2$ m $n_f = 2$ $n_g = 10$ $E = 207\,000$ N/mm² $S_{yt} = 1500$ N/mm² $(fs) = 2.5$

Step I *Cross-section of the leaves*

$2P = 30$ kN or $P = 15\,000$ N

$2L = 1.2$ m or $L = 600$ mm

$$\sigma_b = \frac{S_{yt}}{(fs)} = \frac{1500}{2.5} = 600 \text{ N/mm}^2$$

From Eq. (10.41),

$$\sigma_b = \frac{6PL}{nbt^2} \quad \text{or} \quad (600) = \frac{6(15\,000)(600)}{(2+10)bt^2}$$

$$bt^2 = 7500 \text{ mm}^3$$

Assuming a standard width of 60 mm,

$$t^2 = \frac{7500}{60} \quad \text{or} \quad t = 11.18 \text{ or } 12 \text{ mm}$$

Cross-section of the leaves = 60 × 12 mm (i)

Step II *Deflection at the end of the spring*

From Eq. (10.38),

$$\delta = \frac{12PL^3}{Ebt^3(3n_f + 2n_g)}$$

$$= \frac{12(15\,000)(600)^3}{(207\,000)(60)(12)^3(3\times 2 + 2\times 10)}$$

$$= 69.68 \text{ mm} \quad \text{(ii)}$$

10.18 BELLEVILLE SPRING

A Belleville spring consists of a coned disk as shown in Fig. 10.35. It resembles a dinner plate without a bottom. This type of spring is also called '*coned disk*' spring. It is called Belleville spring because it was invented by Julian Belleville, who patented its design in France in 1867. The Belleville

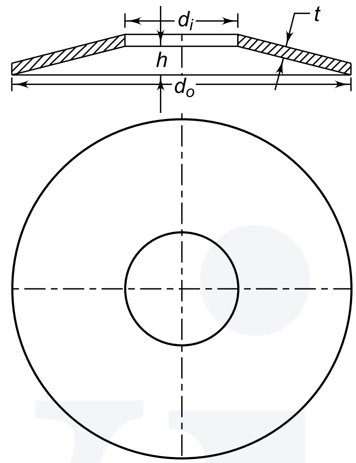


Fig. 10.35 *Belleville Spring*

spring has typical load-deflection characteristics as shown in Fig. 10.36. The variation of (h/t) ratio produces a wide variety of load deflection curves. For example, when (h/t) ratio is 3.5, an S-curve is obtained which is useful in applications involving snap acting mechanism. When (h/t) is reduced to 2.1, the central portion of the curve becomes horizontal, which means that the load is constant for

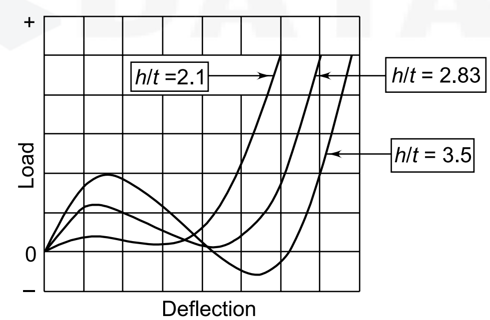


Fig. 10.36 *Load Deflection Curves for Belleville Springs*

this range of deflection. This portion of the curve is useful for engaging or disengaging the clutch, when the Belleville spring is used as a clutch spring. The Belleville spring offers the following advantages:

(i) It is simple in construction and easy to manufacture.

(ii) The Belleville spring is a compact spring unit.

(iii) It is especially useful where very large force is desired for small deflection of the spring.

(iv) It provides a wide range of spring constants making it versatile.

(v) It can provide any linear or non-linear load-deflection characteristic.

(vi) The individual coned disks of a particular size and thickness can be stacked in series, parallel or series–parallel combinations as shown in Fig. 10.37. These combinations provide a variety of spring constants without changing the design. When two Belleville springs are arranged in series, double deflection is obtained for the same force. On the other hand, when two Belleville springs are in parallel, almost double force is obtained for a given deflection.

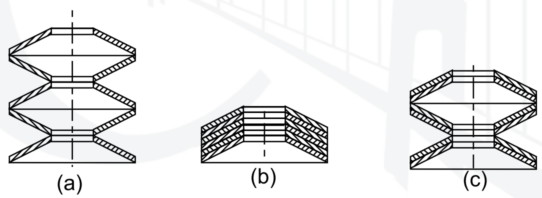


Fig. 10.37 *Nesting of Belleville Springs: (a) Series Combination (b) Parallel Combination (c) Parallel Series Combination*

Belleville springs are used in plate clutches and brakes, gun recoil mechanisms, relief valves and a wide variety of bolted connections.

The analysis of the Belleville spring is exceedingly complex, and mathematical treatment is beyond the scope of this book. The load-deflection and load-stress equations of the Belleville spring are as follows:

$$P = \frac{E\delta}{(1-\mu^2)M(d_o/2)^2}[(h-\delta/2)(h-\delta)t+t^3] \tag{10.42}$$

$$\sigma = \frac{E\delta}{(1-\mu^2)M(d_o/2)^2}[C_1(h-\delta/2)+C_2 t] \tag{10.43}$$

where,

P = axial force (N)
d = deflection of spring (m)
t = thickness of washer (m)
h = free height minus thickness (m)
E = modulus of elasticity (N/m^2)
σ = stress at the inside circumference (N/m^2)
d_0 = outer diameter of washer (m)
d_i = inner diameter of washer (m)
m = Poisson's ratio (0.3 for steel)

$$M = \frac{6}{\pi \log_e(d_o/d_i)}\left[\frac{(d_o/d_i)-1}{(d_o/d_i)}\right]^2 \tag{10.44}$$

$$C_1 = \frac{6}{\pi \log_e(d_o/d_i)}\left[\frac{(d_o/d_i)-1}{\log_e(d_o/d_i)}-1\right] \tag{10.45}$$

$$C_2 = \frac{6}{\pi \log_e(d_o/d_i)}\left[\frac{(d_o/d_i)-1}{2}\right] \tag{10.46}$$

Example 10.26 *A Belleville spring is made of silicon steel. The spring is compressed completely flat when it is subjected to an axial force of 4500 N. The corresponding maximum stress is (1375 × 10^6) N/m^2. Assume,*

$$\frac{d_o}{d_i} = 1.75 \quad \text{and} \quad \frac{h}{t} = 1.5$$

Calculate

(i) thickness of the washer;

(ii) free height of the washer minus thickness (h);

(iii) outer diameter of the washer; and

(iv) inner diameter of the washer.

Solution

Given $P = 4500$ N $\sigma_{max.} = 1375 \times 10^5$ N/m^2

Step I *Thickness of washer*

When the spring is compressed completely flat,

$$\delta = h$$

From Eqs (10.44), (10.45) and (10.46),

$$M = \frac{6}{\pi \log_e(d_o/d_i)}\left[\frac{(d_o/d_i)-1}{(d_o/d_i)}\right]^2$$

$$= \frac{6}{\pi \log_e(1.75)}\left[\frac{1.75-1}{1.75}\right]^2 = 0.6268$$

$$C_1 = \frac{6}{\pi \log_e(d_o/d_i)}\left[\frac{(d_o/d_i)-1}{\log_e(d_o/d_i)} - 1\right]$$

$$= \frac{6}{\pi \log_e(1.75)}\left[\frac{1.75-1}{\log_e(1.75)} - 1\right] = 1.161$$

$$C_2 = \frac{6}{\pi \log_e(d_o/d_i)}\left[\frac{(d_o/d_i)-1}{2}\right]$$

$$= \frac{6}{\pi \log_e(1.75)}\left[\frac{1.75-1}{2}\right] = 1.28$$

Dividing Eq. (10.42) by Eq. (10.43)

$$\frac{P}{\sigma} = \frac{(h-\delta/2)(h-\delta)t + t^3}{C_1(h-\delta/2) + C_2 t} = \frac{t^3}{C_1(h-\delta/2) + C_2 t}$$

because $(h = \delta)$

Substituting values,

$$\frac{4500}{1375(10^6)} = \frac{t^3}{(1.161)(1.5t - 1.5t/2) + (1.28)t} = \frac{t^2}{2.15}$$

$$t = 2.653(10^{-3}) \text{ m} = 2.653 \text{ mm} = 2.65 \text{ mm} \qquad \text{(i)}$$

Step II *Free height of washer minus thickness (h)*

$$h = 1.5\,t = 1.5\,(2.65) = 3.98 \text{ mm} = 4 \text{ mm} \qquad \text{(ii)}$$

Step III *Outer diameter of washer*

From Eq. (10.42),

$$P = \frac{E\delta}{(1-\mu^2)M(d_o/2)^2}[(h-\delta/2)(h-\delta)t + t^3]$$

or $$P = \frac{E\delta}{(1-\mu^2)M(d_o/2)^2}[t^3] \qquad \text{because } (h = \delta)$$

$E = 207\,000$ N/mm^2 = $(207\,000 \times 10^6)$ N/m^2

$\mu = 0.3$

$t = (2.65 \times 10^{-3})$ m $\qquad h = \delta = (4 \times 10^{-3}) = 0.004$ m

Substituting these values,

or $$4500 = \frac{(207\,000 \times 10^6)(0.004)}{(1-0.3^2)\,(0.6268)\,(d_o/2)^2}[(2.65 \times 10^{-3})^3$$

$$d_o = 154.96 \times 10^{-3} \text{ m} = 155 \text{ mm} \qquad \text{(iii)}$$

Step IV *Inner diameter of washer*

$$d_i = \frac{d_o}{1.75} = \frac{155}{1.75} = 88.57 \text{ mm} \qquad \text{(iv)}$$

10.19 SHOT PEENING

In a large number of applications, the external force acting on the spring fluctuates with respect to time resulting in fatigue failure. Due to poor surface finish of the spring wire, the fatigue crack usually begins with some surface irregularity and propagates due to tensile stresses. It has been observed that propagation of fatigue crack is always due to tensile stresses. In order to reduce the chances of crack propagation, a layer of residual compressive stress is induced in the surface of the spring wire. One of the methods of creating such a layer is shot peening. In this process, small steel balls are impinged on the wire surface with high velocities either by an air blast or by centrifugal action. The balls strike against the wire surface and induce residual compressive stresses. The depth of the layer of the residual compressive stresses depends upon a number of factors, such as size of the balls, velocity of striking, original hardness and ductility of the spring wire. These parameters are adjusted in such a manner as to produce the required depth of layer of compressive stress. Shot peening is effective for springs loaded only in one direction, such as helical compression, helical extension or torsion bar springs.

Short-Answer Questions

10.1 What are the functions of spring?

10.2 What are the applications of spring?

10.3 What type of stress is induced in helical compression spring?

10.4 What type of stress is induced in helical extension spring?

10.5 Distinguish between closely coiled and open-coiled helical springs.

10.6 What is helical torsion spring? How does it differ from helical compression spring?

10.7 What type of stress is induced in helical torsion spring?

10.8 What are the applications of multi-leaf spring?

10.9 What is the spring index?

10.10 What is stiffness of spring?

10.11 What are active coils of spring?

10.12 What are inactive coils of spring?

10.13 What is the Wahl factor? Why is it used?

10.14 What are the objectives of series and parallel connections of springs?

10.15 What is pulsating shear stress? Why are springs subjected to pulsating shear stress?

10.16 What is surge in spring?

10.17 What is concentric spring?

10.18 What are the advantages of concentric spring?

10.19 What are the applications of concentric spring?

10.20 What is spiral spring?

10.21 What are the advantages of spiral spring?

10.22 What are the applications of spiral spring?

10.23 What are graduated-length and full-length leaves in multi-leaf spring?

10.24 What is nip of leaf spring?

10.25 What is the objective of nipping of leaf spring?

Problems for Practice

10.1 It is required to design a helical compression spring subjected to a force of 500 N. The deflection of the spring corresponding to this force is approximately 20 mm. The spring index should be 6. The spring is made of cold-drawn steel wire with ultimate tensile strength of 1000 N/mm^2. The permissible shear stress for the spring wire can be taken as 50% of the ultimate tensile strength (G = 81 370 N/mm^2). Design the spring and calculate:

(i) wire diameter;
(ii) mean coil diameter;
(iii) number of active coils;
(iv) total number of coils;
(v) free length of the spring; and
(vi) pitch of the coils.

Assume a gap of 1 mm between adjacent coils under maximum load condition. The spring has square and ground ends.

[*(i) 4.37 or 5 mm (ii) 30 mm (iii) 10 (iv) 12 (v) 92.24 mm (vi) 8.39 mm*]

10.2 In an automotive plate clutch, six helical compression springs arranged in parallel provide the axial thrust of 1500 N. The springs are compressed by 10 mm to provide this thrust force. The springs are identical and the spring index is 6. The springs are made of cold-drawn steel wires with ultimate tensile strength of 1200 N/mm^2. The permissible shear stress for the spring wire can be taken as 50% of the ultimate tensile strength (G = 81 370 N/mm^2). The springs have square and ground ends. There should be a gap of 1 mm between adjacent coils when the springs are subjected to the maximum force. Design the springs and calculate:

(i) wire diameter;
(ii) mean coil diameter;
(iii) number of active coils;
(iv) total number of coils;
(v) solid length;
(vi) free length;
(vii) required spring rate; and
(viii) actual spring rate.

[*(i) 2.82 or 3 mm (ii) 18 mm (iii) 6 (iv) 8 (v) 24 mm (vi) 41.62 mm (vii) 25 N/mm (8) 23.54 N/mm*]

10.3 A direct reading spring balance consists of a helical tension spring, which is attached to a rigid support at one end and carries weights at the other free end. The pointer attached to the free end moves on a scale and indicates the weight. The length of the scale is 75 mm. The maximum capacity of the balance is to measure the weight of 500 N. The spring index is 6. The spring is made of oil-hardened and tempered steel wire with ultimate tensile strength of 1400 N/mm^2. The permissible shear stress for spring wire can be taken as 50% of the ultimate tensile strength (G = 81 370 N/mm^2). Design the spring and calculate:

(i) wire diameter;
(ii) mean coil diameter;
(iii) number of active coils;
(iv) required spring rate; and
(v) actual spring rate.

[*(i) 3.7 or 4 mm (ii) 24 mm (iii) 29 (iv) 6.67 N/mm (v) 6.5 N/mm*]

10.4 A railway wagon moving at a velocity of 2 m/s is brought to rest by a bumper consisting of two helical compression springs arranged in parallel. The springs are compressed by 150 mm in bringing the wagon to rest. The mass of the wagon is 1000 kg. The spring index can be taken as 6. The springs are made of oil-hardened and tempered steel wire with ultimate tensile strength of 1500 N/mm^2 and modulus of rigidity of 81 370 N/mm^2. The permissible shear stress for the spring wire can be taken as 50% of the ultimate tensile strength. Design the springs and calculate:

(i) maximum force on each spring;
(ii) wire diameter;
(iii) mean coil diameter; and
(iv) number of active coils.

[*(i) 13333.33N (ii) 18.44 or 19 mm (iii) 114 mm (iv) 11*]

10.5 A helical compression spring is required to deflect through approximately 25 mm when the external force acting on it varies from 500 to 1000 N. The spring index is 8. The spring has square and ground ends. There should be a gap of 2 mm between adjacent coils when the spring is subjected to the maximum force of 1000 N. The spring is made of cold-drawn steel wire with ultimate tensile strength of 1000 N/mm^2 and permissible shear stress in the spring wire should be 50% of the ultimate tensile strength (G = 81 370 N/mm^2). Design the spring and calculate:

(i) wire diameter;
(ii) mean coil diameter;
(iii) number of active coils;
(iv) total number of coils;
(v) solid length;
(vi) free length;
(vii) required spring rate; and
(viii) actual spring rate.

[*(i) 6.95 or 7 mm (ii) 56 mm (iii) 7 (iv) 9 (v) 63 mm (vi) 129.34 mm (vii) 20 N/mm (viii) 19.87 N/mm*]

10.6 A safety valve, 40 mm in diameter, is to blow off at a pressure of 1.2 MPa. It is held on its seat by means of a helical compression spring, with initial compression of 20 mm. The maximum lift of the valve is 12 mm. The spring index is 6. The spring is made of cold-drawn steel wire with ultimate tensile strength of 1400 N /mm^2. The permissible shear stress can be taken as 50% of this strength. (G = 81 370 N/mm^2). Calculate:

(i) wire diameter;
(ii) mean coil diameter; and
(iii) number of active coils.

[*(i) 8.12 or 9 mm (ii) 54 mm (iii) 6*]

10.7 An automotive engine develops maximum torque at a speed of 1000 rpm At this speed, the power developed by the engine is 25 kW. The engine is equipped with a single plate clutch having two pairs of friction surfaces. The mean diameter of the friction disk is 190 mm and the coefficient of friction is 0.35. Six springs, with a spring index of 6, provide the necessary axial force. The springs are made of patented and cold-drawn steel wires of Grade 2 (G = 81 370 N/mm^2). The permissible shear stress can be taken as 50% of the ultimate tensile strength. Determine the wire diameter of the spring.

[*4 mm*]

10.8 A spring-loaded relief valve is shown in Fig. 10.38. It consists of a plunger, which is mounted in the main body covering the outlet opening. It is held against the inlet opening by means of a helical compression spring, with adjustable compression. When the force due to hydraulic pressure acting on the plunger exceeds the initial setting of the spring, the plunger is lifted up, permitting the oil to relieve into the outlet opening. Since the valve is occasionally used, the spring is to be designed for static load.

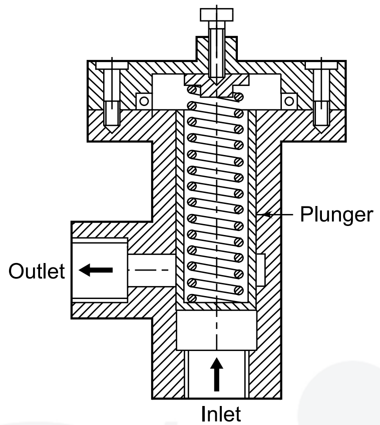


Fig. 10.38 *Spring Loaded Relief Valve*

The diameter of the plunger is 25 mm and the outer diameter of the spring should not exceed 20 mm because of space limitations. The normal working pressure is 0.25 N/mm^2 and the valve should open at a pressure of 1 N/mm^2, with a valve-lift of 6 mm from the normal position. The spring is made of oil-hardened and tempered valve spring wire of Grade-VW (G = 81 370 N/mm^2). The permissible shear stress in the wire can be taken as 50% of the ultimate tensile strength. Design the spring and determine:

(i) the wire diameter;
(ii) the mean coil diameter;
(iii) the stiffness of the spring; and
(iv) the number of active coils.

[*(i) 3.6 mm (ii) 15 mm (iii) 61.36 N/mm (iv) 9*]

10.9 A helical compression spring of a mechanism is subjected to an initial pre-load of 50 N and the maximum force during the load cycle is 300 N. The wire diameter is 5 mm, while the spring index is 5. The spring is made of oil-hardened and tempered steel wire of Grade-SW (S_{ut} = 1440 N/mm^2). Determine the factor of safety against fluctuating stresses.

[*1.82*]

10.10 A concentric spring consists of two helical compression springs, one inside the other. The free length of the outer spring is 25 mm greater than the inner spring. The wire diameter and mean coil diameter of the inner spring are 8 and 64 mm respectively. Also, the wire diameter and mean coil diameter of the outer spring are 10 and 80 mm respectively. The number of active coils in inner and outer springs are 10 and 15 respectively. Assume same material for two springs and the modulus of rigidity of spring material is 81370 N/mm^2. Calculate:

(i) the stiffness of spring when the deflection is from 0 to 25 mm
(ii) the stiffness of spring when the deflection is more than 25 mm.

[*(i) 13.24 N/mm (ii) 29.13 N/mm*]

10.11 A flat spiral spring is required to provide a maximum torque of 1000 N-mm. It is made of steel strip (E = 207 000 N/mm^2) and the maximum bending stress in the strip should not exceed 750 N/mm^2. The ratio of width to thickness of the strip is 10. The arbor turns through 2.5 revolutions with respect to the retaining drum to provide the required torque.
Calculate the thickness, width and length of strip.

[*1.2 mm, 12 mm and 5.62 m*]

10.12 A semi-elliptic leaf spring consists of two extra full-length leaves and six graduated-length leaves, including the master leaf. Each leaf is 7.5 mm thick and 50 mm wide. The centre-to-centre distance between the two eyes is 1 m. The leaves are pre-stressed in such a way that when the load is maximum, stresses induced in all the leaves are equal to 350 N/mm^2. Determine the maximum force that the spring can withstand.

[*5250 N*]

10.13 A semi-elliptic leaf spring consists of two extra full-length leaves and eight graduated-length leaves, including the master leaf. The centre-to-centre distance between the two eyes of the spring is 1 m. The maximum force acting on the spring is 10 kN and the width of each leaf is 50 mm. The spring is initially pre-loaded in such a way that when the load is maximum, the stresses induced in all the leaves are equal to 350 N/mm^2. The modulus of elasticity of the leaf material is 207 000 N/mm^2. Determine

(i) the thickness of leaves; and
(ii) the deflection of the spring at maximum load.

[*(i) 10 (9.26) mm (ii) 32.94 mm*]

10.14 A semi-elliptic spring used for automobile suspension, consists of two extra full-length leaves and eight graduated-length leaves, including the master leaf. The centre-to-centre distance between the two eyes is 1 m. The leaves are made of steel 55Si2Mo90 (S_{yt} = 1500 N/mm^2 and E = 207 000 N/mm^2) and the factor of safety is 2. The maximum spring load is 30 kN. The leaves are pre-stressed so as to equalize stresses in all leaves under maximum load. Determine the dimensions of the cross-section of the leaves and the deflection at the end of the spring.

[*32 × 14 (13.69) mm and 56.27 mm*]

Friction Clutches

Chapter 11

11.1 CLUTCHES

The clutch is a mechanical device, which is used to connect or disconnect the source of power from the remaining parts of the power transmission system at the will of the operator. An automotive clutch can permit the engine to run without driving the car. This is desirable when the engine is to be started or stopped, or when the gears are to be shifted.

Very often, three terms are used together, namely, *couplings, clutches and brakes*. There is a basic difference between the coupling and the clutch. A coupling, such as a flange coupling, is a permanent connection. The driving and driven shafts are permanently attached by means of coupling and it is not possible to disconnect the shafts, unless the coupling is dismantled. On the other hand, the clutch can connect or disconnect the driving and driven shafts, as and when required by the operator. Similarly, there is a basic difference between initial and final conditions in clutch and brake operations. In the operation of clutch, the conditions are as follows:

(i) Initial Condition The driving member is rotating and the driven member is at rest.

(ii) Final Condition Both members rotate at the same speed and have no relative motion.

In the operation of brake, the conditions are as follows:

(i) Initial Condition One member such as the brake drum is rotating and the braking member such as the brake shoe is at rest.

(ii) Final Condition Both members are at rest and have no relative motion.

Clutches are classified into the following four groups:

(i) Positive contact Clutches They include square jaw clutches; spiral jaw clutches and toothed clutches. In these clutches, power transmission is achieved by means of interlocking of jaws or teeth. Their main advantage is positive engagement and once coupled, they can transmit large torque with no slip.

(ii) Friction Clutches They include single and multi-plate clutches, cone clutches and centrifugal clutches. In these clutches, power transmission is achieved by means of friction between contacting surfaces. This chapter is restricted to friction clutches.

(iii) Electromagnetic Clutches They include magnetic particle clutches, magnetic hysteresis clutches and eddy current clutches. In these clutches, power transmission is achieved by means of the magnetic field. These clutches have many advantages, such as rapid response time, ease of control, and smooth starts and stops.

(iv) Fluid Clutches and Couplings In these clutches, power transmission is achieved by means of hydraulic pressure. A fluid coupling provides extremely smooth starts and absorbs shock.

The simplest form of positive contact clutches is the square jaw clutch as shown in Fig. 11.1. It consists of two halves carrying projections or jaws. One clutch half is fixed and the other can move along the axis of the shaft over either a feather key or splines by means of shift lever. During the engagement, the jaws of the moving half enter into the sockets of the mating half. The engaging surfaces of jaw and socket form a rigid mechanical junction. Jaw clutches can be used to connect shafts, when the driving shaft is stationary or rotating at very low velocity. There are two types of jaws, namely, square and spiral. The spiral jaws can be engaged at slightly higher speed without clashing. Frequent engagement results in wear of jaws. The jaw clutches have the following advantages:

(a) They do not slip and engagement is positive.

(b) No heat is generated during engagement or disengagement.

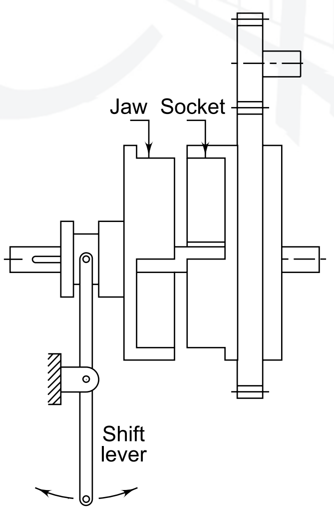


Fig. 11.1 *Square Jaw Clutch*

The jaw clutches have the following drawbacks:

(a) Jaw clutches can be engaged only when both shafts are stationary or rotate with very small speed difference.

(b) They cannot be engaged at high speeds because engagement of jaws and sockets results in shock.

In general, positive contact clutches are rarely used as compared with friction clutches. However, they have some important applications where synchronous operation is required like power presses and rolling mills.

A single plate friction clutch consisting of two flanges is shown in Fig. 11.2. One flange is rigidly keyed to the driving shaft, while the other is connected to the driven shaft by means of splines. The splines permit free axial movement of the driven flange with respect to the driven shaft. This axial movement is essential for engagement and disengagement of the clutch. The actuating force is provided by a helical compression spring, which forces the driven flange to move towards the driving flange. Power is transmitted from the driving shaft to the driving flange by means of the key. Power is then transmitted from the driving flange to the driven flange by means of frictional force. Finally, power is transmitted from the driven flange to the driven shaft by means of the splines. Since the power is transmitted by means of frictional force between the driving and driven

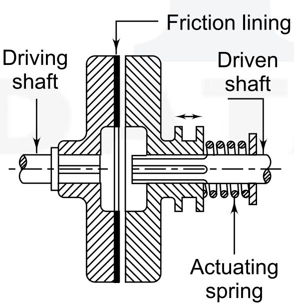


Fig. 11.2 *Single Plate Clutch*

flanges, the clutch is called '*friction*' clutch. In order to disengage the clutch, a fork is inserted in the collar on the driven flange to shift it axially to the right side. This relieves the pressure between the driving and the driven flanges and no torque can be transmitted. In the working condition, the clutch is in an engaged position under the action of spring force. Levers or forks are operated to '*disengage*' the clutch.

The main advantages of friction clutch are as follows:

(i) The engagement is smooth.

(ii) Slip occurs only during engaging operation and once the clutch is engaged, there is no slip between the contacting surfaces. Therefore, power loss and consequent heat generation do not create problems, unless the operation requires frequent starts and stops.

(iii) In certain cases, the friction clutch serves as a safety device. It slips when the torque transmitted through it exceeds a safe value. This prevents the breakage of parts in the transmission chain.

Depending upon the number of friction surfaces, the friction clutches are classified as single-plate or multi-plate clutches. Depending upon the shape of the friction material, the clutches are classified as disk clutches, cone clutches or expanding shoe clutches.

The following factors should be considered while designing friction clutches:

(i) Selection of a proper type of clutch that is suitable for the given application

(ii) Selection of suitable friction material at the contacting surfaces

(iii) Designing the clutch for sufficient torque capacity

(iv) Engagement and disengagement should be without shock or jerk

(v) Provision for holding the contacting surfaces together by the clutch itself and without any external assistance

(vi) Low weight for rotating parts to reduce inertia forces, particularly in high-speed applications

(vii) Provision for taking or compensating wear of rubbing surfaces

(viii) Provision for carrying away the heat generated at the rubbing surfaces

11.2 TORQUE TRANSMITTING CAPACITY

A friction disk of a single plate clutch is shown in Fig. 11.3. The following notations are used in the derivation:

D = outer diameter of friction disk (mm)
d = inner diameter of friction disk (mm)
p = intensity of pressure at radius r (N/mm^2)
P = total operating force (N)
M_t = torque transmitted by the clutch (N-mm)

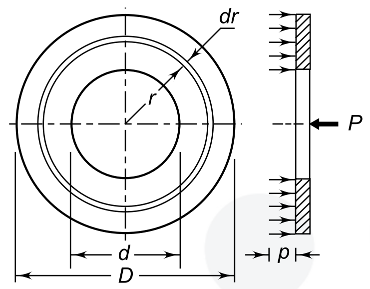


Fig. 11.3 *Friction Disk*

The intensity of pressure p at radius r may be constant or may be variable.

Consider an elemental ring of radius r and radial thickness dr as shown in Fig. 11.4. For this ring,

$$\text{elemental area} = (2\pi r\, dr)$$

$$\text{elemental axial force} = p(2\pi r\, dr) = 2\pi(pr\, dr) \qquad \text{(a)}$$

$$\text{elemental friction force} = \mu p(2\pi r\, dr)$$

$$\text{elemental friction torque} = \mu p(2\pi r\, dr)\, r = 2\pi\mu\,(pr^2\, dr) \qquad \text{(b)}$$

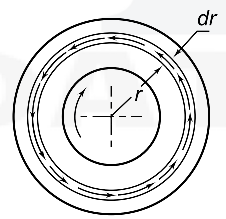


Fig. 11.4 *Friction Force on Elemental Ring*

Integrating the expression (a),

$$P = \int 2\pi\,(pr\, dr)$$

or

$$P = 2\pi \int_{d/2}^{D/2} pr\, dr \qquad (11.1)$$

Integrating the expression (b),

$$M_t = \int 2\pi\mu\,(pr^2\, dr)$$

or

$$M_t = 2\pi\mu \int_{d/2}^{D/2} pr^2\, dr \qquad (11.2)$$

Two theories are used to obtain the torque capacity of the clutch. They are called uniform pressure theory and uniform wear theory.

(i) Uniform Pressure Theory In case of new clutches employing a number of springs, the pressure remains constant over the entire surface area of the friction disk. With this assumption, p is assumed to be constant. This constant pressure distribution is illustrated in Fig. 11.5(a). From Eq. (11.1)

$$P = 2\pi \int_{d/2}^{D/2} pr\, dr = 2\pi p \int_{d/2}^{D/2} r\, dr = 2\pi p \left(\frac{r^2}{2}\right)_{d/2}^{D/2}$$

or
$$P = \frac{\pi p}{4}(D^2 - d^2) \tag{11.3}$$

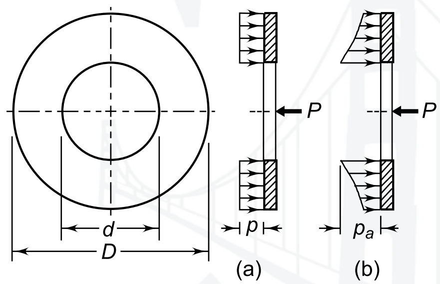


Fig. 11.5 *Pressure Distribution*

From Eq. (11.2),

$$M_t = 2\pi\mu \int_{d/2}^{D/2} pr^2\, dr = 2\pi\mu\, p \int_{d/2}^{D/2} r^2\, dr$$

$$= 2\pi\mu\, p \left(\frac{r^3}{3}\right)_{d/2}^{D/2}$$

or
$$M_t = \frac{\pi\mu p}{12}(D^3 - d^3) \tag{11.4}$$

The following formulae are used in deriving Eqs (11.3) and (11.4),

$$\int x^n dx = \frac{x^{n+1}}{n+1} \qquad \int x dx = \frac{x^2}{2} \qquad \int x^2\, dx = \frac{x^3}{3}$$

Dividing Eq. (11.4) by Eq. (11.3),

$$M_t = \frac{\mu P}{3}\frac{(D^3 - d^3)}{(D^2 - d^2)} \tag{11.5}$$

The above equations have been derived for a single pair of contacting surfaces. When there are a number of friction surfaces in contact, as in the case of the multi-disk clutch, Eq. (11.5) should be multiplied by the number of pairs of contacting surfaces to obtain the resultant torque transmitting capacity.

(ii) Uniform wear Theory According to the second theory, it is assumed that the wear is uniformly distributed over the entire surface area of the friction disk. This assumption is used for worn-out clutches. The axial wear of the friction disk is proportional to the frictional work. The work done by the friction force at radius r is proportional to the frictional force (μp) and rubbing velocity ($2\pi r\, n$) where n is speed in rev/min.

$\therefore$ wear $\propto (\mu\, p)\,(2\pi\, rn)$

Assuming speed n and the coefficient of friction μ as constant for a given configuration,

wear $\propto pr$

When the wear is uniform,

pr = constant

The pressure distribution according to uniform wear theory is illustrated in Fig. 11.5(b). In this case, p is inversely proportional to r. Therefore, pressure is maximum at the inner radius and minimum at the outer periphery. The maximum pressure intensity at the inner diameter ($d/2$) is denoted by p_a. It is also the permissible intensity of pressure. Since p_r is constant,

$pr = p_a\,(d/2)$

From Eq. (11.1),

$$P = 2\pi \int_{d/2}^{D/2} pr\, dr = 2\pi \left(p_a \times \frac{d}{2}\right) \int_{d/2}^{D/2} dr$$

$$P = \frac{\pi p_a d}{2}(D - d) \tag{11.6}$$

From Eq. (11.2),

$$M_t = 2\pi\mu \int_{d/2}^{D/2} pr^2 dr = 2\pi\mu \left(p_a \times \frac{d}{2}\right) \int_{d/2}^{D/2} rdr$$

$$M_t = \frac{\pi\mu p_a d}{8}(D^2 - d^2) \tag{11.7}$$

Dividing Eq. (11.7) by Eq. (11.6),

$$M_t = \frac{\mu P}{4}(D+d) \tag{11.8}$$

The above equation gives the torque transmitting capacity for a single pair of contacting surfaces.

There are two phases of wear mechanism in friction lining. They are as follows:

(i) When the friction lining is new, uniform-pressure theory is applicable. Therefore,

p = constant

Since, wear $\propto pr$ $\therefore$ wear $\propto r$

The wear at the outer radius will be more. Since the pressure plate is rigid, this wear will release the pressure at the outer edge.

(ii) Since the pressure is released, there will be no further wear at the outer edge. The wear will now take place at the inner edge due to contact of the pressure plate. This will release pressure and stop further wear at the inner edge. This process of wear mechanism alternatively at the outer and inner radii will continue, till the pressure is adjusted in such a manner that the product pr is constant resulting in uniform wear at any radius.

Based on above discussion, the following observations are made:

(i) The uniform-pressure theory is applicable only when the friction lining is new. When the lining is put into service, wear occurs. Therefore, a major portion of the life of friction lining comes under the uniform-wear criterion and it is more logical to use uniform wear theory in design of clutches.

Equation (11.8) can be written in the following way:

$$M_t = \mu P R_f$$

where R_f is called the *friction radius*.

For uniform wear theory or worn-out clutches,

$$R_f = \frac{1}{4}(D+d) \tag{a}$$

Similarly, Eq. (11.5) can be written in the following way:

$$M_t = \mu P R_f$$

For uniform pressure theory or new clutches,

$$R_f = \frac{1}{3}\frac{(D^3-d^3)}{(D^2-d^2)} \tag{b}$$

We will consider some numerical values for D and d and calculate the friction radii. The results are tabulated in the following way:

D	d	$\frac{1}{3}\frac{(D^3-d^3)}{(D^2-d^2)}$	$\frac{1}{4}(D+d)$
140	80	56.36	55
200	100	77.78	75
200	180	95.088	95

(ii) A comparison of Eqs. (a) and (b) in the above table shows that the friction radius for new clutches is slightly greater than for worn-out clutches. However, the difference between the two values is very small.

(iii) Since the friction radius of new clutches is more, the torque transmitting capacity ($\mu P R_f$) is also more. On the other hand, the torque transmitting capacity of worn-out clutches is low due to lower friction radius. When we use uniform wear theory, we are on the safe side and assume lower torque carrying capacity for given dimensions. Such a clutch will have little more torque carrying capacity when it is new.

Conclusions

(i) The uniform-pressure theory is applicable only when the friction lining is new.

(ii) The uniform-wear theory is applicable when the friction lining gets worn out.

(iii) The friction radius for new clutches is slightly greater than that of worn-out clutches.

(iv) The torque transmitting capacity of new clutches is slightly more than that of worn-out clutches.

(v) A major portion of the life of friction lining comes under the uniform wear criterion.

(vi) It is more logical and safer to use uniform-wear theory in the design of clutches.

From Eq. (11.8),

$$M_t = \frac{\mu P}{4}(D+d) = \mu P \frac{1}{2}\left(\frac{D}{2}+\frac{d}{2}\right) = \mu P r_m$$

Therefore, the torque transmitting capacity can be increased by three methods:

(i) Use friction material with a higher coefficient of friction (μ)

(ii) Increase the plate pressure (P)

(iii) Increase the mean radius of the friction disk (r_m)

In design of clutches, the following factors should be considered:

(i) Service Factor In order to start the machine from rest and accelerate it to the rated speed, the clutch should have torque capacity substantially higher than the nominal torque rating. In most of the cases, the accelerating or starting torque is much more than the running torque. If the clutch is not designed for this increased torque, it will slip under the load and no power can be transmitted. There is another factor to account for additional torque. In many applications, the torque developed by the prime mover fluctuates and also, the torque requirement by driven machinery fluctuates as in the case of presses. These two factors are accounted by means of service factor. For design purpose,

$$(M_t)_{des} = K_s (M_t)$$

where,

K_s = service factor

$(M_t)_{des}$ = torque capacity of clutch for design purpose

(M_t) = rated torque

There is no standardisation of service factors. Various manufacturers recommend different service factors for the applications.

(ii) Location of Clutch Let us consider a mill driven by a diesel engine. The optimum operating speed of the engine is too high for direct connection to the mill shaft. Therefore, a gearbox is provided to reduce the speed. In this set-up, a clutch is also required so that the engine can be started and brought up to the full speed before connecting to the mill shaft. In such applications, the question arises about the location of clutch—whether the clutch should be located between the engine and the gearbox or between the gearbox and the mill?

The clutch is required to transmit a given power. The power transmitted by the clutch is the product of torque and speed. Therefore, greater the speed, lower is the torque to be transmitted. It is, therefore, logical to place the clutch at the high-speed side, that is, between the engine and the gearbox. Since the torque capacity is low, the cost of the clutch is also low. On the other hand, the speed is low between the gearbox and the mill and the clutch will have to transmit high torque, increasing the cost.

(iii) The coefficient of friction for automotive clutches, which use asbestos lining in contact with a cast iron surface, is taken from 0.3 to 0.4. The allowable pressure on the friction lining varies from 0.1 N/mm^2 for large heavy-duty double-plate clutches to 0.25 N/mm^2 for an average passenger-car clutch. The allowable pressure for clutches with metal plates is from 0.7 to 1.05 N/mm^2.

Example 11.1 *A plate clutch consists of one pair of contacting surfaces. The inner and outer diameters of the friction disk are 100 and 200 mm respectively. The coefficient of friction is 0.2 and the permissible intensity of pressure is 1 N/mm^2. Assuming uniform-wear theory, calculate the power-transmitting capacity of the clutch at 750 rpm.*

Solution

Given $D = 200$ mm $d = 100$ mm $\mu = 0.2$ $p_a = 1$ N/mm^2 $n = 750$ rpm

Step I *Operating force*

From Eq. (11.6),

$$P = \frac{\pi p_a d}{2}(D-d) = \frac{\pi(1)(100)}{2}(200-100)$$

$$= 15\,707.96\ N$$

Step II *Power transmitting capacity*

From Eq. (11.8),

$$M_t = \frac{\mu P}{4}(D+d) = \frac{(0.2)(15\,707.96)}{4}(200+100)$$

$$= 235\,619.4 \text{ N-mm}$$

$$\text{kW} = \frac{2\pi n M_t}{60\times10^6} = \frac{2\pi(750)\,(235\,619.4)}{60\times10^6} = 18.51$$

Example 11.2 *Assume the data given in Example 11.1 and calculate the power transmitting capacity of the clutch using uniform pressure theory.*

Solution

Given $D = 200$ mm $d = 100$ mm $\mu = 0.2$
$p_a = 1$ N/mm^2 $n = 750$ rpm

Step I *Operating force*

From Eq. (11.3),

$$P = \frac{\pi p}{4}(D^2 - d^2) = \frac{\pi(1)}{4}(200^2 - 100^2)$$
$$= 23\,561.95 \text{ N}$$

Step II *Power transmitting capacity*

From Eq. (11.5),

$$M_t = \frac{\mu P}{3}\frac{(D^3 - d^3)}{(D^2 - d^2)}$$
$$= \frac{(0.2)(23\,561.95)}{3}\frac{(200^3 - 100^3)}{(200^2 - 100^2)}$$
$$= 366\,519.22 \text{ N-mm}$$
$$\text{kW} = \frac{2\pi n M_t}{60 \times 10} = \frac{2\pi(750)(366\,519.22)}{60 \times 10^6} = 28.79$$

Example 11.3 *An automotive plate clutch consists of two pairs of contacting surfaces with an asbestos friction lining. The torque transmitting capacity of the clutch is 550 N-m. The coefficient of friction is 0.25 and the permissible intensity of pressure is 0.5 N/mm^2. Due to space limitations, the outer diameter of the friction disk is fixed as 250 mm. Using uniform wear theory, calculate*

(i) the inner diameter of the friction disk; and
(ii) the spring force required to keep the clutch in an engaged position.

Solution

Given $M_t = 550$ N-m $D = 250$ mm $\mu = 0.25$
$p_a = 0.5$ N/mm^2 number of pairs of contacting surfaces = 2

Step I *Inner diameter of friction disk*

The friction disk of the automotive clutch is fixed between the flywheel on one side and the pressure plate on the other. The friction lining is provided on both sides of the friction disk. There are two pairs of contacting surfaces—one between the flywheel and the friction disk and the other between the friction disk and the pressure plate. Therefore, the torque transmitted by one pair of contacting surfaces is (550/2) or 275 N-m.

From Eq. (11.7),

$$M_t = \frac{\pi \mu P_a d}{8}(D^2 - d^2)$$
$$(275 \times 10^3) = \frac{\pi(0.25)(0.5)d}{8}(250^2 - d^2)$$

Rearranging the terms, we have

$$d(250^2 - d^2) = 5\,602\,254$$

The above equation is solved by trial and error method. It is a cubic equation, with the following three roots:

(i) $d = 174.16$ mm
(ii) $d = 112.29$ mm
(iii) $d = -286.46$ mm

Mathematically, all the three answers are correct. The inner diameter cannot be negative. As a design engineer, one should select the inner diameter as 174.16 mm, which results in a minimum area of friction lining compared with 112.29 mm. For minimum cost of friction lining,

$$d = 174 \text{ mm} \qquad \text{(i)}$$

Step II *Required spring force*

From Eq. (11.6),

$$P = \frac{\pi P_a d}{2}(D - d) = \frac{\pi(0.5)(174)}{2}(250 - 174)$$
$$= 10\,386.11 \text{ N} \qquad \text{(ii)}$$

Example 11.4 *A single plate clutch consists of one pair of contacting surfaces. Because of space limitations, the outer diameter of the friction disk is fixed as D. The permissible intensity of pressure is p_a and the coefficient of friction, μ. Assuming uniform wear theory, plot the variation of the torque transmitting capacity against the ratio of diameters (d/D). Show that the torque transmitting capacity of the clutch is maximum, when (d/D) is equal to 0.577.*

Solution

Step I *Variation of torque capacity against the ratio (d/D)*

$$\frac{d}{D} = x \qquad \text{(a)}$$

From Eq. (11.7),

$$M_t = \frac{\pi \mu P_a D}{8} (D^2 - d^2) \qquad \text{(b)}$$

Substituting Eq. (a) in Eq. (b), we have

$$M_t = \frac{\pi \mu P_a D^3}{8} [x(1 - x^2)] \qquad \text{(c)}$$

For maximum torque capacity,

$$\frac{\partial}{\partial x}(M_t) = 0 \qquad \text{or} \qquad \frac{\partial}{\partial x}[x(1 - x^2)] = 0$$

$\therefore \; 1 - 3x^2 = 0$

$$\therefore \; x = \sqrt{\frac{1}{3}} = 0.577 \qquad \text{(d)}$$

Equation (c) is rearranged in the following form:

$$\frac{8M_t}{\pi \mu p_a D^3} = x(1 - x^2)$$

The left hand side is called the torque characteristic. The variation of the torque characteristic against x is shown in Fig. 11.6.

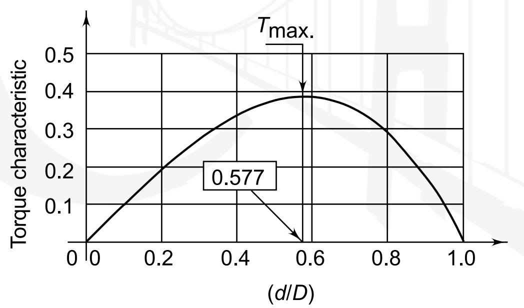


Fig. 11.6 *Variation of Torque against d/D*

Step II *Optimum ratio*

From (d), the condition for maximum torque capacity is given by,

$$\frac{d}{D} = \sqrt{\frac{1}{3}} = 0.577$$

$$\frac{D}{d} = \sqrt{3} = 1.732$$

Therefore, it is a good practice to use (D/d) ratio between 1.5 and 2.

Example 11.5 *A single plate clutch consists of only one pair of contacting surfaces. It is used for an engine, which develops a maximum torque of 120 N-m. Assume a factor of safety of 1.5 to account for slippage at full-engine torque. The permissible intensity of pressure is 350 kPa and the coefficient of friction is 0.35. Assuming uniform wear theory, calculate the inner and outer diameters of the friction lining.*

Solution

Given $M_t = 120$ N-m $\quad (fs) = 1.5 \quad \mu = 0.35$
$p_a = 350$ kPa

Step I *Inner diameter of friction lining*

$M_t = 1.5(120) = 180$ N-m or 180 000 N-mm

$p_a = 350$ kPa $= 350 \times 10^{-3}$ MPa or 0.35 N/mm^2

From Eq. (11.7),

$$M_t = \frac{\pi \mu p_a d}{8} (D^2 - d^2)$$

or

$$180\,000 = \frac{\pi (0.35)(0.35) d}{8} (D^2 - d^2)$$

$$\therefore \; d(D^2 - d^2) = 3\,741\,765.19 \qquad \text{(a)}$$

The above expression indicates two unknowns, D and d, and one equation. It cannot be solved, unless we make some assumption. We will assume that the clutch is transmitting maximum torque. As explained in the previous example, the condition for this objective is written as,

$$\frac{d}{D} = 0.577 \qquad \text{or} \qquad D = \frac{d}{0.577}$$

$$\therefore \; D^2 = \frac{d^2}{0.577^2} = 3d^2 \qquad \text{(b)}$$

Substituting Eq. (b) in Eq. (a),

$d(3d^2 - d^2) = 3\,741\,765.19$

or $2d^3 = 3\,741\,765.19$

$\therefore \; d = 123.22$ mm

Step II *Outer diameter of friction lining*

$$D = \frac{d}{0.577} = \frac{123.22}{0.577} = 213.55 \text{ mm}$$

Example 11.6 *An automotive plate clutch consists of two pairs of contacting surfaces with asbestos friction lining. The maximum engine torque is 250 N-m. The coefficient of friction is 0.35. The inner and outer diameters of friction lining are 175 and 250 mm respectively. The clamping force is provided by nine springs, each* *compressed by 5 mm to give a force of 800 N, when the clutch is new.*

(i) *What is the factor of safety with respect to slippage when the clutch is brand new?*

(ii) *What is the factor of safety with respect to slippage after initial wear has occurred?*

(iii) *How much wear of friction lining can take place before the clutch will slip?*

Solution

Given $M_t = 250$ N-m $D = 250$ mm
$d = 175$ mm $\mu = 0.35$ number of pairs = 2
For springs, $P = 800$ N $\delta = 5$ mm
number of springs = 9

Step I *Factor of safety against slippage for new clutch*

There are two pairs of contacting surfaces. The torque transmitted by one pair should be (250/2) or 125 N-m.

$$M_t = 125 \times 10^3 = 125\ 000 \text{ N-mm} \quad \text{(a)}$$

$$P = 9\,(800) = 7200 \text{ N}$$

When the clutch is brand new, the uniform pressure theory is applicable.

From Eq. (11.5),

$$M_t = \frac{\mu P}{3}\frac{(D^3 - d^3)}{(D^2 - d^2)} = \frac{0.35(7200)}{3}\frac{(250^3 - 175^3)}{(250^2 - 175^2)}$$

$$= 270\ 529.41 \text{ N-mm} \quad \text{(b)}$$

$$(fs) = \frac{270\ 529.41}{125\ 000} = 2.16 \quad \text{(i)}$$

Step II *Factor of safety against slippage after initial wear*

When initial wear has occurred, the uniform wear theory is applicable. We will assume that there is negligible change in the spring force after initial wear has occurred.

From Eq. (11.8),

$$M_t = \frac{\mu P}{4}(D + d) = \frac{0.35(7200)}{4}(250 + 175)$$

$$= 267\ 750 \text{ N-mm}$$

$$(fs) = \frac{267\ 750}{125\ 000} = 2.14 \quad \text{(ii)}$$

Step III *Wear of friction lining before slippage*

The clutch will slip when the spring force is no longer in a position to transmit the engine torque. The spring force required to transmit the engine torque of 125 000 N-m is given by,

$$M_t = \frac{\mu P}{4}(D + d)$$

or

$$125\ 000 = \frac{0.35P}{4}(250 + 175) \quad \text{(c)}$$

$$\therefore \quad P = 3361.34 \text{ N}$$

Since there are nine springs, the force per spring is (3361.34/9) or 373.48 N. When the force per spring will be less than 373.48 N, the clutch will slip. Suppose x is the wear of friction lining for this condition.

Compression of spring = initial compression $- x$
$= (5 - x)$ mm

$$\text{Stiffness of spring} = \left(\frac{373.48}{5 - x}\right) \text{ N/mm} \quad \text{(d)}$$

Initially, each spring is compressed by 5 mm to give a force of 800 N.

$$\text{Stiffness of spring} = \left(\frac{800}{5}\right) \text{ N/mm} \quad \text{(e)}$$

From (d) and (e),

$$\left(\frac{373.48}{5 - x}\right) = \left(\frac{800}{5}\right) \qquad \therefore \; x = 2.67 \text{ mm} \quad \text{(iii)}$$

When the wear of friction lining is more than 2.67 mm, the spring force will be less than 373.48 N and the clutch will slip.

11.3 MULTI-DISK CLUTCHES

A multi-disk clutch, as shown in Fig. 11.7, consists of two sets of disks—A and B. Disks of Set A are usually made of hardened steel, while those of Set B are made of bronze. Disks of Set A are connected to the driven shaft by means of splines. Because of splines, they are free to move in an axial direction on the splined sleeve. There are four through bolts, which pass through the holes in the disks of Set B. A clearance fit between the bolt and the holes in the plates allows disks of Set B to move in an axial direction. The bolts are rigidly fixed to a rotating drum, which is keyed to the driving shaft. The axial force P, which is required to hold the disks together, is provided by means of levers or springs. When the driving shaft rotates, the drum, along with bolts and disks of Set *B*, also rotate. Power is transmitted from the disks of Set *B* to those of *A* by means of friction. When the disks of Set *A* rotate, they transmit the power to the driven shaft through splined sleeve.

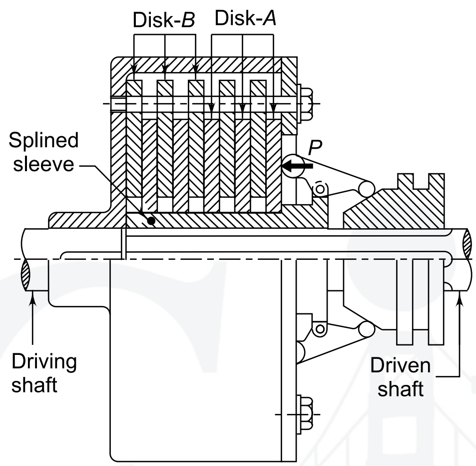


Fig. 11.7 *Multi-disk Clutch*

Equations derived for torque transmitting capacity of the single plate clutch are modified to account for the number of pairs of contacting surfaces in the following way:

For the uniform-pressure criterion,

$$M_t = \frac{\mu P z}{3}\frac{(D^3 - d^3)}{(D^2 - d^{3)}} \qquad (11.9)$$

For the uniform-wear criterion,

$$M_t = \frac{\mu P z}{4}(D + d) \qquad (11.10)$$

where z is the number of pairs of contacting surfaces.

In the design of multi-disk clutches, very often it is required to determine the number of disks rather than the number of pairs of contacting surfaces. In multi-disk clutch, illustrated in Fig.11.7, there are two types of disks called disks of Set *A* and disks of Set *B*. We can use steel disks for Set *A* and bronze disks for Set *B*. Or, we can use plane steel disks for Set *A* and Set *B* consisting of steel disks with asbestos lining. Let us consider 5 disks—3 disks of Set *A* and 2 disks of Set *B*. As shown in Fig.11.8, the number of pairs of contacting surfaces is 4. Therefore,

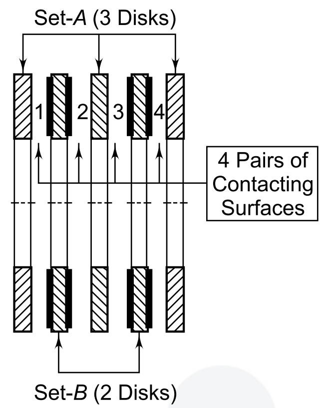


Fig. 11.8 *Number of Disks*

Number of disks = number of pairs of contacting surfaces + 1 = z +1 (a)

Suppose,

z_1 = number of disks on driving shaft

z_2 = number of disks on driven shaft

Substituting in (a),

$z_1 + z_2$ = number of pairs of contacting surfaces + 1

or number of pairs of contacting surfaces $= (z_1 + z_2 - 1)$

It should be noted that the two outer disks have contacting surface on one side only.

The difference between single and multi-plate clutches is as follows:

(i) The number of pairs of contacting surfaces in the single plate clutch is one or at the most, two. There are more number of contacting surfaces in the multi-disk clutch.

(ii) As the number of contacting surfaces is increased, the torque transmitting capacity is also increased, other conditions being equal. In other words, for a given torque capacity, the size of the multi-plate clutch is smaller than that of the single plate clutch, resulting in compact construction.

(iii) The work done by friction force during engagement is converted into heat. More heat is generated in the multi-plate clutch due to increased number of contacting surfaces. Heat dissipation is a serious problem in the multi-plate clutch. Therefore, multi-plate clutches are wet clutches, while single plate clutches are dry.

(iv) The coefficient of friction decreases due to cooling oil, thereby reducing the torque transmitting capacity of the multi-plate clutch. The coefficient of friction is high in dry single plate clutches.

(v) Single plate clutches are used in applications where large radial space is available, such as trucks and cars. Multi-disk clutches are used in applications where compact construction is desirable, e.g., scooter and motorcycle.

The difference between dry and wet clutches is as follows:

(i) A dry clutch has higher coefficient of friction. In wet clutches, the coefficient of friction is reduced due to oil. The coefficient of friction for dry operation is 0.3 or more, while it is 0.1 or less for wet operation.

(ii) The torque capacity of dry clutch is high compared with the torque capacity of wet clutch of the same dimensions.

(iii) For dry clutch, it is necessary to prevent contamination due to moisture or nearby lubricated machinery, by providing seals. Such a problem is not serious in wet clutches.

(iv) Heat dissipation is more difficult in dry clutches. In wet clutches, the lubricating oil carries away the frictional heat.

(v) Rate of wear is far less in wet clutches compared to dry clutches. The wear rate in wet clutches is about 1% of the rate expected in dry clutches.

(vi) The engagement in wet clutch is smoother than in the case of dry clutch.

(vii) In wet clutches, the clutch facings are grooved to provide for passage of lubricant. This reduces the net face area for transmitting torque.

Example 11.7 *An oil immersed multi-disk clutch with cork sheet as the friction material is used on a scooter engine. The friction disk of such a clutch is shown in Fig. 11.9. The torque transmitted by the clutch is 10 N-m. The coefficient of friction between the cork sheet and the steel plate in the wet condition is 0.2. The permissible pressure on the cork sheet is 0.1 N/mm². The inner and outer diameters of the friction lining are 65 and 95 mm respectively. There are radial slots, on the friction surface for the circulation of the coolant, which reduces the effective friction area. To account for these slots, the number of contacting surfaces can be increased by 5%. Assuming uniform-wear theory, calculate the required number of contacting surfaces.*

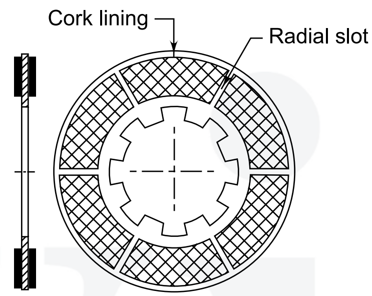


Fig. 11.9

Solution

Given $M_t = 10$ N-m $D = 95$ mm $d = 65$ mm $\mu = 0.2$ $p_a = 0.1$ N/mm²

Step I *Operating force*

For Eq. (11.6),

$$P = \frac{\pi\, p_a\, d}{2}(D - d) = \frac{\pi(0.1)(65)}{2}(95 - 65)$$
$$= 306.31 \text{ N}$$

Step II *Number of contacting surfaces*

From Eq. (11.10),

$$z = \frac{4M_t}{\mu P(D+d)} = \frac{4(10 \times 10^3)}{(0.2)(306.31)(95+65)}$$
$$= 4.08$$

Considering the effect of radial slots,

$$z = 1.05\,(4.08) = 4.28 \text{ or } 5 \text{ surfaces}$$

Example 11.8 *A multi-disk clutch consists of five steel plates and four bronze plates. The inner and outer diameters of the friction disks are 75 and 150 mm respectively. The coefficient of friction is 0.1 and the intensity of pressure on friction lining is limited to 0.3 N/mm². Assuming uniform wear theory, calculate:*

(i) the required force to engage the clutch; and
(ii) power transmitting capacity at 750 rpm.

Solution

Given $D = 150$ mm $d = 75$ mm $\mu = 0.1$
$n = 750$ rpm $p_a = 0.3$ N/mm^2
Number of steel plates = 5
Number of bronze plates = 4

Step I *Required operating force*
From Eq. (11.6),

$$P = \frac{\pi p_a d}{2}(D - d) = \frac{\pi(0.3)(75)}{2}(150 - 75)$$
$$= 2650.72 \text{ N} \qquad \text{(i)}$$

Step II *Power transmitting capacity*
There are five steel plates and four bronze plates. The total number of plates is 9.

Number of disks = $z + 1 = 9$
or $z = 8$

From Eq. (11.10),

$$M_t = \frac{\mu Pz}{4}(D + d) = \frac{(0.1)(2650.72)(8)}{4}(150 + 75)$$
$$= 119\ 282.4 \text{ N-mm}$$

Therefore,

$$\text{kW} = \frac{2\pi n M_t}{60 \times 10^6} = \frac{2\pi(750)(119\ 282.4)}{60 \times 10^6} = 9.37 \qquad \text{(ii)}$$

Example 11.9 *An oil immersed multi-disk clutch, with moulded asbestos on one side and steel disks on the other, is used in an application. The torque transmitted by the clutch is 75 N-m. The coefficient of friction between the asbestos lining and the steel plate in the wet condition is 0.1. The permissible intensity of pressure on the asbestos lining is 500 kPa. The outer diameter of the friction lining is kept as 100 mm due to space limitation. Assuming uniform wear theory, calculate the inside diameter of the disks, the required number of disks and the clamping force.*

Solution

Given $M_t = 75$ N-m $D = 100$ mm $\mu = 0.1$
$p_a = 500$ kPa

Step I *Inside diameter of disks*

$M_t = 75$ N-m = 75 000 N-mm
$p_a = 500$ kPa = 500×10^{-3} MPa = 0.5 N/mm^2

From Eq. (11.10),

$$M_t = \frac{\mu Pz}{4}(D + d)$$

It is observed from the above expression that there are two unknowns d and z, and one equation. It cannot be solved unless we make some assumption. We will assume that the clutch is transmitting maximum torque. As explained in Example 11.4, the condition for this objective is written as,

$$\frac{d}{D} = 0.577 \quad \text{or} \quad d = 0.577\,D = 0.577(100)$$
$$= 57.7 \text{ or } 58 \text{ mm}$$

Step II *Clamping force*
For Eq. (11.6),

$$P = \frac{\pi P_a d}{2}(D - d) = \frac{\pi(0.5)(58)}{2}(100 - 58)$$
$$= 1913.23 \text{ N}$$

Step III *Required number of disks*
From Eq. (11.10)

$$z = \frac{4M_t}{\mu P(D + d)} = \frac{4(75\ 000)}{(0.1)(1913.23)(100 + 58)}$$
$$= 9.92 \text{ or } 10$$

Number of disks = $z + 1 = 10 + 1 = 11$

We will use 6 plane steel disks and 5 steel disks with attached asbestos lining.

11.4 FRICTION MATERIALS

For light loads and low speeds, wood, cork and leather are used as friction materials. The present trend for high speeds and heavy loads has given a stimulus to the development of new friction materials, which are capable of withstanding severe service conditions.

The desirable properties of a good friction material are as follows:

(i) It should have high coefficient of friction.
(ii) The coefficient of friction should remain constant over the entire range of temperatures encountered in applications.
(iii) It should have good thermal conductivity.
(iv) It should remain unaffected by environmental conditions like moisture, or dirt particles.

(v) It should have high resistance to abrasive and adhesive wear.
(vi) It should have good resilience to provide good distribution of pressure at the contacting surfaces.

The coefficient of friction depends upon a number of factors. They include materials of contacting surfaces, surface finish, surface temperature, rubbing speed, foreign particles on rubbing surfaces and atmospheric conditions like moisture.

There are two types of friction materials in common use—asbestos-base and sintered metals.

There are two types of asbestos friction materials—woven and moulded. A woven asbestos friction disk consists of asbestos fibre woven around brass, copper or zinc wires and impregnated with rubber or asphalt. They have an endless circular weave, which increases the centrifugal bursting strength. Moulded asbestos friction disks are prepared from the wet mixture of brass chips and asbestos, which is poured into the mould and given the shape of the disk. The mixture is then heated and pressed for a specific curing time. The difference between woven and moulded asbestos materials is as follows:

(i) Woven material is flexible, while moulded asbestos is rigid.
(ii) Woven material has higher coefficient of friction.
(iii) Woven material conforms more readily to clutch surface while moulded materials take longer time to wear in the seat.
(iv) Woven materials are not only costly, but also wear at a faster rate, resulting in high cost in the long run.

Asbestos material, whether woven or moulded, is an organic material and is subject to destruction by heat at comparatively low temperature. Sintered-metal friction materials solve this difficulty.

There are two varieties of friction disks made from sintered metals—bronze-base and iron-base, depending upon the major constituent. The advantages of sintered-metal friction disks are as follows:

(i) They have higher wear resistance.
(ii) They can be used at high temperatures.
(iii) The coefficient of friction is constant over a wide range of temperature and pressure.
(iv) They are unaffected by environmental conditions, such as dampness, salt water or fungi.

Sintered-metal friction materials offer an excellent design with lighter, cheaper and compact construction. The maximum permissible intensity of pressure for woven and moulded asbestos materials is 0.3 N/mm^2 and 1.0 N/mm^2 respectively, while for sintered metals it can be taken between 1 and 2 N/mm^2. The values of coefficient of friction for different combinations are given in Table 11.1.

Table 11.1 *Coefficient of friction*

Contacting surfaces	*Coefficient of friction*	
	Wet	*Dry*
Woven asbestos—cast iron	0.1–0.2	0.3–0.6
Moulded asbestos—cast iron	0.08–0.12	0.2–0.5
Bronze-base sintered metal—cast iron	0.05–0.1	0.1–0.4
Bronze-base sintered metal—steel	0.05–0.1	0.1–0.3

It has been found that if asbestos dust is inhaled, it may lead to cancer. The body cells which come in contact with asbestos particles are agitated and develop into cancer cells. Lung cancer is common among operators working in atmospheres of asbestos dust. There are federal regulations in a number of countries, which prohibit the use of asbestos in clutch or brake linings. Nowadays, metallic or semi-metallic fibres or powder is used in place of asbestos fibres.

Modern friction lining consists of four basic ingredients, namely, fibres, filler, binder and friction modifiers. Fibres provide rigidity and strength for the friction lining. Nowadays, steel wool or aramid is used as fibre material instead of asbestos. A filler fills the space between the fibres and extend the lining life. Filler materials are barytes, clay and calcium carbonate. In case of metallic lining, fine metal power is used as filler material. Binder is a glue that holds the lining ingredients together. Phenolformaldehyde is extensively used as binder material. Friction modifier improves frictional and wear properties. Brass and zinc particles are added as friction modifiers to control the abrasive properties of the lining.

11.5 CONE CLUTCHES

A cone clutch, as shown in Fig. 11.10, consists of inner and outer conical surfaces. The outer cone is keyed to the driving shaft, while the inner cone is free to slide axially on the driven

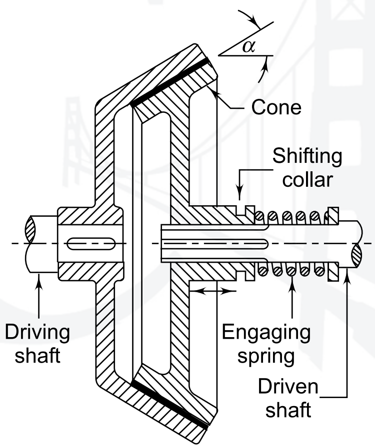


Fig. 11.10 *Cone Clutch*

shaft due to splines. The axial force required to engage the clutch is provided by means of helical compression spring. In engaged position, power is transmitted from the driving shaft to the outer cone by means of the key. Power is then transmitted from the outer cone to the inner cone by means of friction. Finally, power is transmitted from the inner cone to the driven shaft by means of the splines. In order to disengage the clutch, a fork is inserted in the shifting collar to shift it axially towards right side. This releases pressure between inner and outer cones and no torque can be transmitted. Leather, cork or asbestos are used for the friction lining on the inner cone. The conical surface results in considerable friction force even with a small engaging force due to the wedge action. The recommended semi-cone angle (α) is 12.5°. The cone clutches are simple in construction and easy to disengage. Their main drawback is the strict requirement for the coaxiality of two shafts. The equations for the torque transmitting capacity of the cone clutch are derived in a manner similar to that of a single plate clutch. The dimensions of the friction cone are shown in Fig. 11.11. An elemental frustum of the cone bounded by circles of radii r and $(r + dr)$ is considered for the purpose of analysis. Figure 11.12(a) shows this elemental frustum of cone. For this frustum,

$$\text{area} = \delta A = 2\pi r\left(\frac{dr}{\sin\alpha}\right)$$

$$\text{normal force} = p\ (\text{area}) = p\ \delta A = 2\pi pr\left(\frac{dr}{\sin\alpha}\right)$$

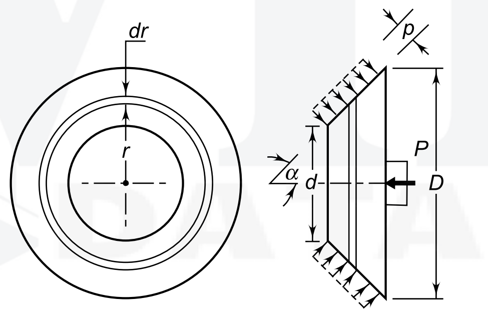


Fig. 11.11

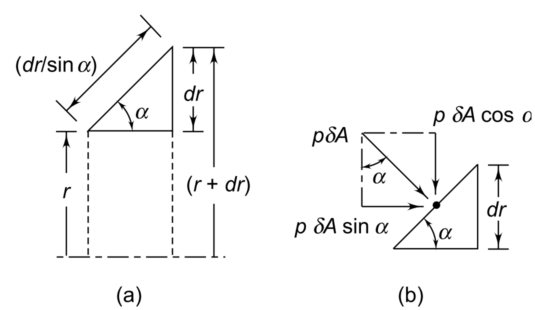


Fig. 11.12

friction force = μ (normal force)

$$= 2\pi\mu pr\left(\frac{dr}{\sin\alpha}\right)$$

friction torque = r (friction force)

$$= 2\pi\mu pr^2\left(\frac{dr}{\sin\alpha}\right) \qquad \text{(a)}$$

From Fig. 11.12(b),

axial force = $\delta P = p\ \delta A \sin\alpha$

$$= p\left[2\pi r\left(\frac{dr}{\sin\alpha}\right)\right]\sin\alpha = 2\pi\, prdr \qquad \text{(b)}$$

Integrating expressions (a) and (b),

$$M_t = \frac{2\pi\mu}{\sin\alpha}\int_{d/2}^{D/2} pr^2 dr \qquad (11.11)$$

$$P = 2\pi\int_{d/2}^{D/2} prdr \qquad (11.12)$$

Two theories are used to obtain the torque capacity of the cone clutch.

(i) Uniform pressure Theory According to uniform pressure theory, pressure p at radius r is constant. This constant pressure distribution is illustrated in Fig. 11.13(a). From Eq. (11.12),

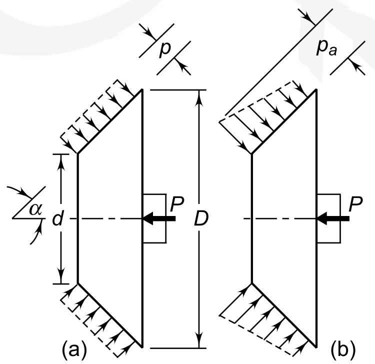


Fig. 11.13 *Pressure Distribution*

$$P = 2\pi\int_{d/2}^{D/2} prdr = 2\pi p\int_{d/2}^{D/2} rdr$$

$$P = 2\pi p\left[\frac{r^2}{2}\right]_{d/2}^{D/2} = 2\pi p\left[\frac{D^2 - d^2}{8}\right]$$

$$\therefore \quad P = \frac{\pi p}{4}(D^2 - d^2) \qquad (11.13)$$

From Eq. (11.11),

$$M_t = \frac{2\pi\mu}{\sin\alpha}\int_{d/2}^{D/2} pr^2 dr = \frac{2\pi\mu p}{\sin\alpha}\int_{d/2}^{D/2} r^2\, dr$$

$$= \frac{2\pi\mu p}{\sin\alpha}\left[\frac{r^3}{3}\right]_{d/2}^{D/2} = \frac{2\pi\mu p}{\sin\alpha}\left[\frac{(D^3 - d^3)}{24}\right]$$

$$\therefore \quad M_t = \frac{\pi\mu p}{12\sin\alpha}(D^3 - d^3) \qquad (11.14)$$

Dividing Eq. (11.14) by Eq. (11.13),

$$M_t = \frac{\mu P}{3\sin\alpha}\frac{(D^3 - d^3)}{(D^2 - d^2)} \qquad (11.15)$$

(ii) Uniform Wear Theory According to uniform wear theory, the product (pr) is constant. The pressure distribution as per this theory is illustrated in Fig.11.13(b). Since pr is constant,

$$pr = p_a\,(d/2)$$

In the above expression, p_a is the intensity of pressure at the inner diameter. It is also the permissible intensity of pressure. From Eq. (11.12),

$$P = 2\pi\int_{d/2}^{D/2} prdr = 2\pi\left(p_a\frac{d}{2}\right)\int_{d/2}^{D/2} dr$$

$$= 2\pi\left(p_a\frac{d}{2}\right)[r]_{d/2}^{D/2} = 2\pi\left(p_a\frac{d}{2}\right)\left[\frac{(D-d)}{2}\right]$$

$$\therefore \quad P = \frac{\pi p_a d}{2}(D - d) \qquad (11.16)$$

From Eq. (11.11),

$$M_t = \frac{2\pi\mu}{\sin\alpha}\int_{d/2}^{D/2} pr^2\, dr = \frac{2\pi\mu}{\sin\alpha}\left(p_a\frac{d}{2}\right)\int_{d/2}^{D/2} rdr$$

$$= \frac{2\pi\mu}{\sin\alpha}\left(p_a\frac{d}{2}\right)\left[\frac{r^2}{2}\right]_{d/2}^{D/2}$$

or $M_t = \dfrac{2\pi\mu}{\sin\alpha}\left(p_a \dfrac{d}{2}\right)\left[\dfrac{(D^2 - d^2)}{8}\right]$

$$\therefore M_t = \frac{\pi\mu p_a d}{8\sin\alpha}(D^2 - d^2) \qquad (11.17)$$

Dividing Eq. (11.17) by (11.16),

$$M_t = \frac{\mu P}{4\sin\alpha}(D + d) \qquad (11.18)$$

The following observations are made:

(i) It is observed from the above equation, that the torque capacity is inversely proportional to $\sin\alpha$. The value of α should be as small as possible so that $\sin\alpha$ will be less and M_t will be more. Therefore, the torque capacity of a cone clutch increases as the semi-cone angle decreases.

However, when α is less than the angle of static friction (ϕ), the clutch has a tendency to grab, resulting in self-engagement. This is not desirable because the clutch should engage or disengage at the will of the operator. To avoid self-engagement and to facilitate disengagement,

$$\alpha > \text{angle of static friction}$$

Taking the coefficient of friction as 0.2,

$$\alpha > \tan^{-1}\mu \quad \text{or} \quad \alpha > \tan^{-1}(0.2) \text{ or } 11.3°$$

Therefore, the semi-cone angle (α) is taken as 12.5°.

(ii) For a given torque,

$$M_t = \text{constant}$$

From Eq. (11.18)

$$M_t = \frac{\mu P}{4\sin\alpha}(D + d) \quad \text{or} \quad P \propto \sin\alpha$$

Therefore, a relatively small axial force can transmit a given torque if the semi-cone angle is decreased. However, as the semi-cone angle decreases, there is more wedging action and the force required to disengage increases. Thus, a clutch with a small semi-cone angle requires a relatively small force to engage the clutch but a relatively large force to disengage the clutch.

From Eq. (11.8), the torque transmitting capacity of single plate clutch is,

$$M_t = \frac{\pi P}{4}(D + d)$$

Dividing Eq. (11.18) by Eq. (11.8),

$$\frac{(M_t)_{\text{cone}}}{(M_t)_{\text{plate}}} = \frac{1}{\sin\alpha} = \frac{1}{\sin(12.5)} = 4.62$$

Thus, for given dimensions, the torque transmitting capacity of cone clutch is higher than that of single plate clutch.

Refer to Fig. 11.14. The face width of a friction lining (b) is given by,

$$b = \frac{D - d}{2\sin\alpha} \qquad (11.19)$$

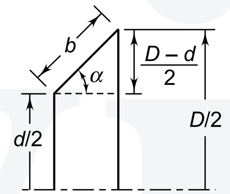


Fig. 11.14

Example 11.10 *A cone clutch with asbestos friction lining transmits 30 kW power at 500 rpm. The coefficient of friction is 0.2 and the permissible intensity of pressure is 0.35 N/mm². The semi-cone angle α is 12.5°. The outer diameter is fixed as 300 mm from space limitations. Assuming uniform wear theory, calculate:*

(i) the inner diameter;

(ii) the face width of the friction lining; and

(iii) the force required to engage the clutch.

Solution

Given kW = 30 $\quad n = 500$ rpm $\quad \mu = 0.2$

$p_a = 0.35$ N/mm² $\quad \alpha = 12.5°$ $\quad D = 300$ mm

Step I *Inner diameter*

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n} = \frac{60 \times 10^6 (30)}{2\pi(500)}$$

$$= 572\,957.8 \text{ N-mm}$$

From Eq. (11.17),

$$M_t = \frac{\pi\mu p_a d}{8\sin\alpha}(D^2 - d^2)$$

$$\therefore \qquad 572\,957.8 = \frac{\pi(0.2)(0.35)d}{8\sin(12.5°)}(300^2 - d^2)$$

Rearranging the terms, we have

$$d(300^2 - d^2) = 4\,511\,297.43$$

The above equation is solved by trial and error method. The nearest value of d is 270.8 mm. Therefore, the inner diameter is taken as 270 mm.

$$d = 270 \text{ mm} \qquad \text{(i)}$$

Step II *Face width of friction lining*

From Eq. (11.19),

$$b = \frac{D-d}{2\sin\alpha} = \frac{300-270}{2\sin(12.5°)} = 69.3 \text{ mm} \qquad \text{(ii)}$$

Step III *Force required to engage clutch*

From Eq. (11.18),

$$P = \frac{4M_t\sin\alpha}{\mu(D+d)} = \frac{4(572\,957.8)\sin(12.5°)}{0.2(300+270)}$$

$$= 4351.25 \text{ N} \qquad \text{(iii)}$$

Example 11.11 *A cone clutch is used to connect an electric motor running at 1440 rpm with a machine which is stationary. The machine is equivalent to a rotor of 150 kg mass and radius of gyration as 250 mm. The machine has to be brought to the full speed of 1440 rpm from stationary condition in 40s. The semi-cone angle α is 12.5°. The mean radius of the clutch is twice the face width. The coefficient of friction is 0.2 and the normal intensity of pressure between contacting surfaces should not exceed 0.1 N/mm². Assuming uniform wear criterion, calculate:*

(i) the inner and outer diameters;
(ii) the face width of friction lining;
(iii) the force required to engage the clutch; and
(iv) the amount of heat generated during each engagement of clutch.

Solution

Given $n = 1440$ rpm $\mu = 0.2$ $r_m = 2b$
$p_a = 0.1$ N/mm² $\alpha = 12.5°$
For machine, $m = 150$ kg $k = 250$ mm $t = 40$ s

Step I *Inner and outer diameters*

$$\omega_1 = 0$$

$$\omega_2 = \frac{2\pi n}{60} = \frac{2\pi(1440)}{60} = 150.80 \text{ rad/s}$$

$$\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{150.80 - 0}{40} = 3.77 \text{ rad/s}^2$$

$$M_t = I\alpha = mk^2\alpha = 150(0.25)^2(3.77)$$

$$= 35.34292 \text{ N-m} = 35\,342.92 \text{ N-mm}$$

From Eq. (11.17),

$$M_t = \frac{\pi\mu p_a d}{8\sin\alpha}(D^2 - d^2)$$

$$\therefore \quad 35\,342.92 = \frac{\pi(0.2)(0.1)d}{8\sin(12.5°)}(D^2 - d^2)$$

Rearranging the terms, we have

$$d(D^2 - d^2) = 973\,978.34 \qquad \text{(a)}$$

From Eq. (11.19),

$$D - d = 2b\sin\alpha \qquad \text{(b)}$$

Since the mean radius of the clutch is twice the face width,

$$\frac{D+d}{4} = 2b \quad \text{or} \quad D + d = 8b \qquad \text{(c)}$$

Dividing Eq. (c) by (b),

$$\frac{D+d}{D-d} = \frac{8b}{2b\sin\alpha} = \frac{4}{\sin\alpha}$$

Therefore,

$$\frac{D}{d} = \frac{4+\sin\alpha}{4-\sin\alpha} = \frac{4+\sin(12.5°)}{4-\sin(12.5°)} = 1.1144$$

$$\frac{D^2}{d^2} = 1.2419 \quad \text{or} \quad D^2 = 1.2419\,d^2$$

Substituting this value in Eq. (a), we get

$$d(1.2419\,d^2 - d^2) = 973\,978.34$$

$$\therefore \quad d = 159.09 \text{ mm}$$

$$D = 1.1144\,d = 1.1144\,(159.09)$$

$$= 177.29 \text{ mm} \qquad \text{(i)}$$

Step II *Face width of friction lining*

From Eq. (c),

$$b = \frac{D+d}{8} = \frac{177.29+159.09}{8} = 42.05 \text{ N} \qquad \text{(ii)}$$

Step III *Force required to engage clutch*

From Eq. (11.18),

$$P = \frac{4M_t\sin\alpha}{\mu(D+d)} = \frac{4(35\,342.92)\sin(12.5°)}{0.2(177.29+159.09)}$$

$$= 454.82 \text{ N} \qquad \text{(iii)}$$

Step IV *Heat generated during each engagement*

$$\omega_{ave.} = \frac{\omega_1+\omega_2}{2} = \frac{0+150.8}{2} = 75.4 \text{ rad/s}$$

$\theta = \omega_{ave}$ (time) = 75.4(40) = 3016 radians

Heat generated during engagement = work done by frictional torque

$H_g = M_t \theta$ = 35.34 (3016) = 106 585.44 N-m or J =106.59 kJ (iv)

11.6 CENTRIFUGAL CLUTCHES

Whenever it is required to engage the load after the driving member has attained a particular speed, a centrifugal clutch is used. The centrifugal clutches permit the drive-motor or engine to start, warm up and accelerate to the operating speed without load. Then the clutch is automatically engaged and the driven equipment is smoothly brought up to the operating speed. These clutches are particularly useful with internal combustion engines, which cannot start under load.

The centrifugal clutch works on the principle of centrifugal force. The centrifugal force increases with speed. The construction of centrifugal clutch is shown in Fig. 11.15. It consists of a spider, which is mounted on the input shaft, and which is provided with four equally spaced radial guides. A sliding shoe is retained in each guide by means of a spring. The outer surface of the sliding shoe is provided with a lining of friction material like asbestos. The complete assembly of spider, shoes and springs is enclosed in a coaxial drum, which is mounted on the output shaft. As the angular speed of the input shaft increases, the centrifugal force acting on the sliding shoes increases, causing the shoes to move in a radially outward direction. The shoes continue to move with increasing speed until they contact the inner surface of the drum. Power is transmitted due to frictional force between the shoe lining and the inner surface of the drum. The clutch is disengaged automatically. When the angular velocity of the shoes decreases, the centrifugal force decreases. This reduces the normal force between the friction lining and the drum. The friction force, which is proportional to normal force, also reduces. The lining slips with respect to the drum and no torque can be transmitted.

The forces acting on the shoe are shown in Fig. 11.16, where following notations are used:

r_d = radius of the drum (mm)

r_g = radius of the centre of gravity of the shoe in engaged position (mm)

m = mass of each shoe (kg)

P_{cf} = centrifugal force (N)

P_s = spring force (N)

z = number of shoes

ω_2 = running speed (rad/s)

ω_1 = speed at which engagement starts (rad/s)

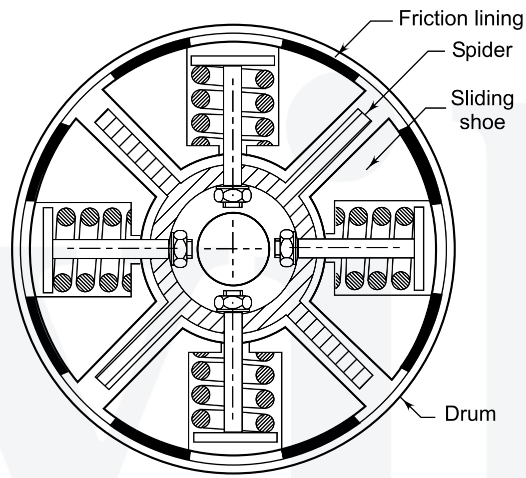


Fig. 11.15 *Centrifugal Clutch*

The centrifugal forces corresponding to speed ω_1 and ω_2 are given by,

$$(P_{cf})_1 = \frac{m\omega_1^2 r_g}{1000} \text{ and } (P_{cf})_2 = \frac{m\omega_2^2 r_g}{1000}$$

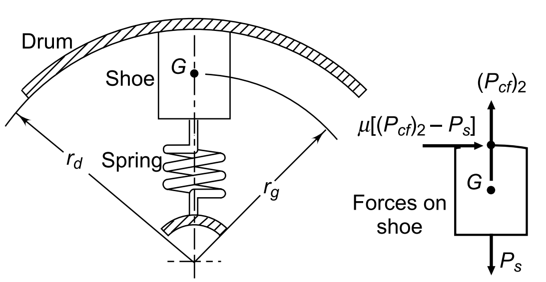


Fig. 11.16 *Forces on Shoe*

The term (1000) in the denominator is taken because r_g is taken in mm. The centrifugal force $(P_{cf})_1$ is balanced by an equal and opposite spring force at the beginning of the engagement.

Therefore,

$$P_s = (P_{cf})_1 = \frac{m\omega_1^2 r_g}{1000}$$

or
$$P_s = \frac{m\omega_1^2 r_g}{1000} \tag{11.20}$$

When the centrifugal force is slightly more than the spring force, the shoe begins to move in a radially outward direction. In the running condition, the net force acting on the drum is given by

$$\text{Net force on drum} = (P_{cf})_2 - P_s$$

$$= (P_{cf})_2 - (P_{cf})_1 = \frac{mr_g(\omega_2^2 - \omega_1^2)}{1000}$$

$$\therefore \quad \text{Net force on drum} = \frac{mr_g(\omega_2^2 - \omega_1^2)}{1000} \tag{11.21}$$

$$\text{Friction force} = \frac{\mu mr_g(\omega_2^2 - \omega_1^2)}{1000} \tag{11.22}$$

$$\text{Friction torque} = \frac{\mu mr_g r_d(\omega_2^2 - \omega_1^2)}{1000}$$

The above expression gives frictional torque in N-mm because r_d is in mm. Since the number of shoes is z, the torque transmitting capacity of the clutch is given by

$$M_t = \frac{\mu mr_g r_d z(\omega_2^2 - \omega_1^2)}{1000} \tag{11.23}$$

There are two distinct applications of centrifugal clutch, namely, light-duty applications and heavy-duty applications. They are as follows:

(i) In centrifugal clutch, the engagement is very smooth because the electric motor has a chance to accelerate and reach the operating speed, before it has to take up the load. Chain saws, lawnmowers, golf carts and small recreational vehicles use centrifugal clutch on this account.

(ii) The centrifugal clutch is also useful in heavy-duty applications, where a high inertia load is to be brought up to the operating speed. By providing a '*time delay*' that is sufficient to permit the prime mover to gain momentum before taking over the load, centrifugal clutches provide smooth engagement. Heavy mobile equipment such as cranes, cement mills, and ball mills use centrifugal clutches on this account.

The centrifugal clutches are used in small two wheelers like mopeds as well as large army vehicles like battle tanks.

Example 11.12 *A centrifugal clutch, transmitting 20 kW at 750 rpm consists of four shoes. The clutch is to be engaged at 500 rpm The inner radius of the drum is 165 mm. The radius of the centre of gravity of the shoes is 140 mm, when the clutch is engaged. The coefficient of friction is 0.3, while the permissible pressure on friction lining is 0.1 N/mm² Calculate:*

(i) the mass of each shoe; and

(ii) the dimensions of friction lining.

Solution

Given kW = 20 $n_2 = 750$ rpm $n_1 = 500$ rpm $z = 4$ $r_d = 165$ mm $r_g = 140$ mm $\mu = 0.3$ $p_a = 0.1$ N/mm^2

Step I *Mass of each shoe*

$$\omega_2 = \frac{2\pi n_2}{60} = \frac{2\pi(750)}{60} = 78.54 \text{ rad/s}$$

$$\omega_1 = \frac{2\pi n_1}{60} = \frac{2\pi(500)}{60} = 52.36 \text{ rad/s}$$

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n_2} = \frac{60 \times 10^6 (20)}{2\pi(750)}$$

$$= 254\,647.9 \text{ N-mm}$$

From Eq. (11.23),

$$m = \frac{1000 M_t}{\mu r_g r_d z(\omega_2^2 - \omega_1^2)}$$

$$= \frac{1000(254\,647.9)}{0.3(140)(165)(4)(78.54^2 - 52.36^2)}$$

$$= 2.68 \text{ kg} \tag{i}$$

Step II *Dimensions of friction lining*

Refer to Fig. 11.15, which illustrates four shoes with spider. It is assumed that the arc of contact of friction lining or shoe subtends an angle (θ) of 70° at the centre of the spider.

Let the dimensions of friction lining be designated as l and b as shown in Fig. 11.17.

l = contact length of friction lining with drum (mm)
b = width of friction lining (mm)

$$\therefore \quad l = r_d\,\theta = 165\left(\frac{70}{180}\pi\right) = 201.59 \text{ or } 200 \text{ mm}$$

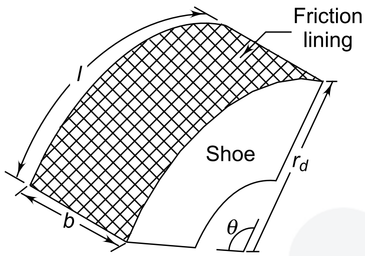


Fig. 11.17

The area of friction lining is (lb). The net force acting on the drum will be

$$(p_a\, lb). \tag{a}$$

From Eq. (11.21), the net force acting on the drum is given by,

$$\text{Net force on drum} = \frac{mr_g(\omega_2^2 - \omega_1^2)}{1000} \tag{b}$$

From (a) and (b),

$$(p_a\, lb) = \frac{mr_g(\omega_2^2 - \omega_1^2)}{1000}$$

$$(0.1)\,(200)\,b = \frac{2.68(140)(78.54^2 - 52.36^2)}{1000}$$

$$\therefore \quad b = 64.29 \text{ or } 65 \text{ mm}$$

Example 11.13 *A centrifugal clutch consists of four shoes, each having a mass of 1.5 kg. In the engaged position, the radius to the centre of gravity of each shoe is 110 mm, while the inner radius of the drum is 140 mm. The coefficient of friction is 0.3. The pre-load in the spring is adjusted in such a way that the spring force at the beginning of engagement is 700 N. The running speed is 1440 rpm. Calculate*

(i) the speed at which the engagement begins; and

(ii) the power transmitted by the clutch at 1440 rpm

Solution

Given $n_2 = 1440$ rpm $\quad m = 1.5$ kg $\quad z = 4$
$r_d = 140$ mm $\quad r_g = 110$ mm $\quad \mu = 0.3 \quad P_s = 700$ N

Step I *Speed at which the engagement begins*

From Eq. (11.20), the spring force is given by,

$$P_s = \frac{m\omega_1^2\, r_g}{1000} \quad \text{or} \quad 700 = \frac{1.5\omega_1^2(110)}{1000}$$

$$\therefore \quad \omega_1 = 65.13 \text{ rad/s}$$

$$n_1 = \frac{60\omega_1}{2\pi} = \frac{60(65.13)}{2\pi} = 621.98 \text{ rpm} \tag{i}$$

Step II *Power transmitted by the clutch*

$$\omega_2 = \frac{2\pi n_2}{60} = \frac{2\pi(1440)}{60} = 150.80 \text{ rad/s}$$

From Eq. (11.23), $M_t = \dfrac{\mu m r_g r_d z(\omega_2^2 - \omega_1^2)}{1000}$

$$\therefore \quad M_t = \frac{0.3(1.5)(110)(140)(4)(150.80^2 - 65.13^2)}{1000}$$

$$= 512\,784.60 \text{ N-mm}$$

$$\text{kW} = \frac{2\pi n_2 M_t}{60 \times 10^6} = \frac{2\pi(1440)(512\,784.60)}{60 \times 10^6}$$

$$= 77.33 \tag{ii}$$

11.7 ENERGY EQUATION

A dynamic system consisting of driving and driven shafts and the clutch is illustrated in Fig. 11.18. The driving shaft is connected to the prime mover while the driven shaft to the machine. In many applications, the initial angular velocity of the driven shaft is zero. It is brought to full speed from rest by the clutching operation. When the clutch is fully engaged, the driving and driven shafts rotate at the same speed. However, during the clutching operation, slippage occurs because the two sides rotate at different speeds. Energy is dissipated during slippage, which is converted into frictional heat. When the clutch is fully engaged, there is no slip between the two surfaces and there is no friction or frictional heat. When the rate of heat generation is more than the rate at which the heat is dissipated to the surrounding, the temperature of the clutch assembly increases. Therefore, the permissible temperature rise is a limiting factor in the design of the clutch.

In dynamic analysis of the clutch, the following assumptions are made:

(i) The driving and driven shafts are rigid.

(ii) Torque remains constant during clutching operation.

The following notations are used for the dynamic analysis of the system shown in Fig. 11.18,

I_1 = moment of inertia of driving shaft (kg-m^2)
α_1 = angular acceleration of driving shaft (rad/s^2)
ω_1 = angular velocity of driving shaft (rad/s)
θ_1 = angular displacement of driving shaft (rad)
I_2 = moment of inertia of driven shaft (kg-m^2)
α_2 = angular acceleration of driven shaft (rad/s^2)
ω_2 = angular velocity of driven shaft (rad/s)
θ_2 = angular displacement of driven shaft (rad)
M_t = clutch torque (N-m)

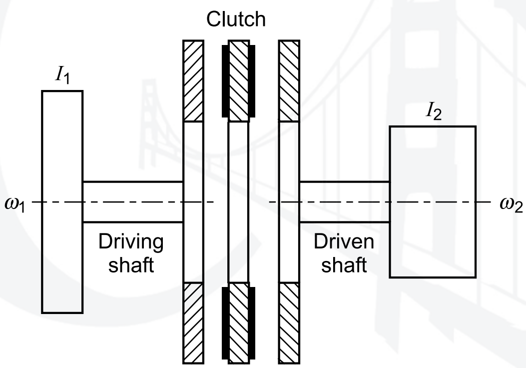


Fig. 11.18 *Dynamic System*

The equation of motion for the driving shaft is written as,

$$I_1 \alpha_1 = -M_t \tag{a}$$

The equation of motion for the driven shaft is written as,

$$I_2 \alpha_2 = M_t \tag{b}$$

From (a) and (b),

$$\alpha_1 = \frac{d^2\theta_1}{dt^2} = -\frac{M_t}{I_1} \tag{c}$$

$$\alpha_1 = \frac{d^2\theta_2}{dt^2} = \frac{M_t}{I_2} \tag{d}$$

Integrating Eqs (c) and (d) with respect to time t,

$$\frac{d\theta_1}{dt} = \left(-\frac{M_t}{I_1}\right)t + C_1 \tag{e}$$

$$\frac{d\theta_2}{dt} = \left(\frac{M_t}{I_2}\right)t + C_2 \tag{f}$$

where C_1 and C_2 are the constants of integration. The initial conditions are as follows:

$$t = 0 \qquad \frac{d\theta_1}{dt} = \omega_1 \qquad \frac{d\theta_2}{dt} = \omega_2$$

Substituting the above values in equations (e) and (f),

$$C_1 = \omega_1 \quad \text{and} \quad C_2 = \omega_2$$

Substituting the above values and rewriting equations (e) and (f),

$$\frac{d\theta_1}{dt} = \left(-\frac{M_t}{I_1}\right)t + \omega_1 \tag{g}$$

$$\frac{d\theta_2}{dt} = \left(\frac{M_t}{I_2}\right)t + \omega_2 \tag{h}$$

The difference in velocities of driving and driven shafts is called the relative velocity at time t. It is given by,

$$\omega = \frac{d\theta}{dt} = \frac{d\theta_1}{dt} - \frac{d\theta_2}{dt}$$

$$= \left[\left(-\frac{M_t}{I_1}\right)t + \omega_1\right] - \left[\left(\frac{M_t}{I_2}\right)t + \omega_2\right]$$

$$\therefore \quad \omega = (\omega_1 - \omega_2) - M_t\left(\frac{I_1 + I_2}{I_1 I_2}\right)t \tag{11.24}$$

The above equation gives the relative velocity at any time t. The clutching action is complete at the instance when the angular velocities of two shafts become equal or the relative velocity becomes zero. Suppose at this instance, time is t_1. Equating relative velocity to zero and substituting ($t = t_1$) in Eq. (11.24),

$$0 = (\omega_1 - \omega_2) - M_t\left(\frac{I_1 + I_2}{I_1 I_2}\right)t_1$$

$$\therefore \qquad t_1 = \frac{(\omega_1 - \omega_2) I_1 I_2}{(I_1 + I_2) M_t} \tag{11.25}$$

It is observed from the above equation that the time required for engaging the clutch is directly proportional to the difference in the angular velocities of driving and driven shafts and inversely proportional to the torque. During the clutching operation, the energy is dissipated in the form of frictional heat. The rate of energy dissipation, denoted by u, is given by,

$$u = M_t \frac{d\theta}{dt}$$

Substituting Eq. (11.24),

$$u = M_t \left[(\omega_1 - \omega_2) - M_t \left(\frac{I_1 + I_2}{I_1 I_2} \right) t \right] \tag{11.26}$$

The total energy dissipated during the clutching operation is obtained by integrating u from ($t = 0$) to ($t = t_1$). E denotes the total energy. It is given by,

$$E = \int_0^{t_1} u \; dt = M_t \int_0^{t_1} \left[(\omega_1 - \omega_2) - M_t \left(\frac{I_1 + I_2}{I_1 I_2} \right) t \right] dt$$

$$\therefore \; E = M_t (\omega_1 - \omega_2) \int_0^{t_1} dt - M_t^2 \left(\frac{I_1 + I_2}{I_1 I_2} \right) \int_0^{t_1} t \; dt$$

$$= M_t (\omega_1 - \omega_2) t_1 - M_t^2 \left(\frac{I_1 + I_2}{I_1 I_2} \right) \frac{t_1^2}{2}$$

Substituting the value of t_1 from Eq. (11.25) in the above expression,

$$E = \frac{1}{2} \frac{(\omega_1 - \omega_2)^2 I_1 I_2}{(I_1 + I_2)} \tag{11.27}$$

It is observed from the above expression that energy dissipated during clutching operation is independent of the torque and directly proportional to the square of relative velocity of the driving and driven shafts.

11.8 THERMAL CONSIDERATIONS

The energy dissipated during the clutching operation is converted into frictional heat, which increases the temperature of the clutch assembly. The temperature rise is given by,

$$\Delta_t = \frac{E}{mc} \tag{11.28}$$

where,

m = mass of the clutch assembly (kg)

c = specific heat of the clutch assembly (J/kg °C)

Δ_t = temperature rise (°C)

The specific heat (c) of the clutch assembly made of either steel or cast iron is taken as 500 J/kg °C.

The actual temperature rise will be less than Δ_t because some heat will be radiated to the surrounding atmosphere and some will be carried away by air flow in the vicinity of the clutch assembly. Equation (11.28) gives approximate value of the temperature rise.

When the clutch assembly is allowed to cool in the air after the clutching operation, the thermal analysis is carried by '*lumped-heat-capacity*' method. It is based on the assumption that there is no temperature gradient within the clutch assembly. This is an idealised state because a temperature gradient must exist in the material if the heat is to be conducted into or out of the material. The assumption of uniform temperature throughout the clutch assembly is realistic provided that the size of the clutch assembly is small. The equation of convective heat transfer from the clutch assembly to the atmosphere is given by,

$$T - T_a = (T_i - T_a) e^{-(Ah/mc)t} \tag{11.29}$$

where,

T = instantaneous temperature at time t (°C)

T_a = ambient temperature (°C)

T_i = initial temperature (°C)

A = area for heat transfer (m^2)

h = surface heat transfer coefficient (W/m^2 °C)

t = time (s)

The graph of temperature of the clutch assembly against time is shown in Fig. 11.19. It consists of the following parts:

(i) Initially, the clutch is disengaged and rotates in the atmosphere from time ($t = t_o$) to time ($t = t_1$). The temperature of the clutch assembly is equal to the atmospheric temperature T_a. The line AB on the temperature–time curve shows this stage.

(ii) At time ($t = t_1$) or the point B, there is first clutching operation. This results in instantaneous temperature rise Δ_t, which will increase the temperature of the clutch assembly from T_a to T_1. The line *BC* on temperature–time curve shows this stage.

(iii) The clutch assembly is allowed to cool in the air from time ($t = t_1$) to time ($t = t_2$). The temperature decreases according to Eq. (11.29). It will continue to fall along the curve *CDG* and reach atmospheric temperature unless there is another clutching operation.

(iv) At the point *D* on the curve *CDG*, there is second clutching operation, which will result in instantaneous temperature rise Δ_t and the temperature will increase from *D* to *E*.

(v) The temperature will fall along the curve *EF* according to Eq. (11.29) and reach atmospheric temperature in due course.

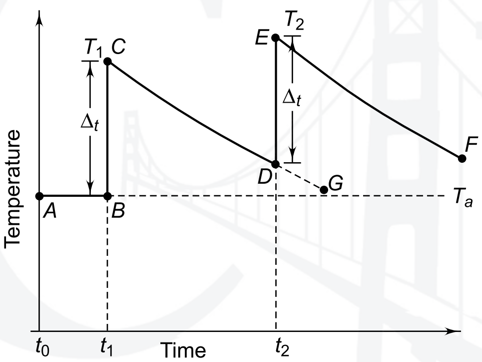


Fig. 11.19

It is observed that frequency of clutching operation is an important parameter in thermal analysis of clutches.

Example 11.14 *A single plate clutch is designed to transmit 10 kW power at 2000 rpm. The equivalent mass and radius of gyration of the input shaft are 20 kg and 75 mm respectively. The equivalent mass and radius of gyration of the output shaft are 35 kg and 125 mm respectively. Calculate:*

(i) the time required to bring the output shaft to the rated speed from rest; and

(ii) the heat generated during the clutching operation.

Solution

Given kW = 10 $n_1 = 2000$ rpm

For input shaft, $m_1 = 20$ kg $k_1 = 75$ mm

For output shaft, $m_2 = 35$ kg $k_2 = 125$ mm

Step I *Time required to bring output shaft to the rated speed*

$$I_1 = m_1k_1^2 = 20(0.075)^2 = 0.1125 \text{ kg-m}^2$$

$$I_1 = m_2k_2^2 = 35(0.125)^2 = 0.5469 \text{ kg-m}^2$$

$$\omega_1 = \frac{2\pi n_1}{60} = \frac{2\pi(2000)}{60} = 209.44 \text{ rad/s and } \omega_2 = 0$$

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n_1} = \frac{60 \times 10^6 (10)}{2\pi(2000)}$$

$$= 47\,746.48 \text{ N-mm} \quad \text{or} \quad 47.746 \text{ N-m}$$

From Eq. (11.25),

$$t_1 = \frac{(\omega_1 - \omega_2)I_1I_2}{(I_1 + I_2)M_t}$$

$$= \frac{(209.44 - 0)(0.1125)(0.5469)}{(0.1125 + 0.5469)(47.746)} = 0.41 \text{ s} \qquad \text{(i)}$$

Step II *Heat generated during clutching operation*

From Eq. (11.27),

$$E = \frac{1}{2}\frac{(\omega_1 - \omega_2)^2 I_1I_2}{(I_1 + I_2)}$$

$$= \frac{1}{2}\frac{(209.44 - 0)^2(0.1125)(0.5469)}{(0.1125 + 0.5469)}$$

$$= 2046.45 \text{ J} \qquad \text{(ii)}$$

Short-Answer Questions

11.1 What is the difference between clutch and flange coupling?

11.2 What is the difference between the clutch and the brake?

11.3 Where do you use clutch?

11.4 What is the function of an automotive clutch?

11.5 Name the different types of clutches. Give at least one practical application of each.

11.6 What are the advantages of single plate clutch over multi-plate clutch?

11.7 What are the two theories applied to friction plates?

11.8 Why are clutches usually designed on the basis of uniform wear?

11.9 Where do you use single plate clutch?

11.10 Where do you use multi-plate clutch?

11.11 Why is heat-dissipation necessary in clutches?

11.12 Name the friction materials used in clutches and brakes.

11.13 What are the drawbacks of asbestos friction materials?

11.14 What are the advantages of sintered-metal friction materials over asbestos friction materials?

11.15 Why is the semi-cone angle of a cone clutch made 12.5°?

11.16 What are the advantages of cone clutch?

11.17 What are the drawbacks of cone clutch?

11.18 Give practical applications of cone clutch.

11.19 What are the advantages of centrifugal clutch?

11.20 What are the drawbacks of centrifugal clutch?

11.21 Give examples of centrifugal clutch.

Problems for Practice

11.1 A single plate clutch consists of one pair of contacting surfaces. The inner and outer diameters of the friction disk are 125 and 250 mm respectively. The coefficient of friction is 0.25 and the total axial force is 15 kN. Calculate the power transmitting capacity of the clutch at 500 rpm using :

(i) uniform wear theory; and

(ii) uniform pressure theory.

[*(i) 18.41 kW (ii) 19.09 kW*]

11.2 An automotive single plate clutch consists of two pairs of contacting surfaces. The outer diameter of the friction disk is 270 mm. The coefficient of friction is 0.3 and the maximum intensity of pressure is 0.3 N/mm^2. The clutch is transmitting a torque of 531 N-m. Assuming uniform wear theory, calculate:

(i) the inner diameter of the friction disk; and

(ii) spring force required to keep the clutch engaged.

[*(i) 167 mm (ii) 8100.69 N*]

11.3 A multi-disk clutch consists of two steel disks with one bronze disk. The inner and outer diameters of the contacting surfaces are 200 and 250 mm respectively. The coefficient of friction is 0.1 and the maximum pressure between the contacting surfaces is limited to 0.4 N/mm^2. Assuming uniform wear theory, calculate the required force to engage the clutch and the power transmitting capacity at 720 rpm

[*(i) 6283.19 N (ii) 10.66 kW*]

11.4 A multi-disk clutch consists of steel and bronze plates. It transmits 15 kW power at 1400 rpm The inner and outer diameters of the contacting surfaces are 100 and 200 mm respectively. The coefficient of friction is 0.15 and the permissible intensity of pressure is 0.5 N/mm^2. Assuming uniform-wear theory, calculate the number of steel and bronze disks.

[*2 steel plates with 1 bronze plate (z = 1.1258)*]

11.5 A leather faced cone clutch transmits power at 500 rpm. The semi-cone angle α is 12.5°. The mean diameter of the clutch is 300 mm, while the face width of the contacting surface of the friction lining is 100 mm. The coefficient of friction is 0.2 and the maximum intensity of pressure is limited to 0.07 N/mm^2. Calculate the force to engage the clutch and the power transmitting capacity.

[*1324.68 N and 9.61 kW*]

11.6 A centrifugal clutch, transmitting 18.5 kW at 720 rpm, consists of four shoes. The clutch is to be engaged at 75% of the running speed. The inner radius of the drum is 165 mm, while the radius of the centre of gravity of each shoe, during engaged position, is 140 mm. The coefficient of friction is 0.25. Calculate the mass of each shoe.

[*4.27 kg*]

Brakes

Chapter 12

12.1 BRAKES

A brake is defined as a mechanical device, which is used to absorb the energy possessed by a moving system or mechanism by means of friction. The primary purpose of the brake is to slow down or completely stop the motion of a moving system, such as a rotating drum, machine or vehicle. It is also used to hold the parts of the system in position at rest. An automobile brake is used either to reduce the speed of the car or to bring it to rest. It is also used to keep the car stationary on the downhill road. The energy absorbed by the brake can be either kinetic or potential or both. In automobile application, the brake absorbs the kinetic energy of the moving vehicle. In hoists and elevators, the brake absorbs the potential energy released by the objects during the braking period. The energy absorbed by the brake is converted into heat energy and dissipated to the surroundings. Heat dissipation is a serious problem in brake applications.

Brakes are classified into the following three groups:

(i) *Mechanical brakes*, which are operated by mechanical means such as levers, springs and pedals. Depending upon the shape of the friction material, the mechanical brakes are classified as block brakes, internal or external shoes brakes, disk brakes and band brakes. Brakes are also classified into two groups according to the direction of the actuating force, namely, radial brakes and axial brakes. Internal and external shoe brakes are radial brakes, while disk brakes are axial brakes. The discussion in this chapter is restricted to mechanical brakes.

(ii) *Hydraulic and pneumatic brakes*, which are operated by fluid pressure such as oil pressure or air pressure.

(iii) *Electrical brakes*, which are operated by magnetic forces and which include magnetic particle brakes, hysteresis brakes and eddy-current brakes.

Brake capacity depends upon the following three factors:

(i) The unit pressure between braking surfaces
(ii) The contacting area of braking surface
(iii) The radius of the brake drum
(iv) The coefficient of friction
(v) The ability of the brake to dissipate heat that is equivalent to the energy being absorbed

12.2 ENERGY EQUATIONS

The first step in the design of a mechanical brake is to determine the braking-torque capacity for the given application. The braking-torque depends upon the amount of energy absorbed by the brake. When a mechanical system of mass m moving with a velocity v_1 is slowed down to the velocity

v_2 during the period of braking, the kinetic energy absorbed by the brake is given by

$$KE = \frac{1}{2} m(v_1^2 - v_2^2) \tag{12.1}$$

where

KE = kinetic energy absorbed by the brake (J)

m = mass of the system (kg)

v_1, v_2 = initial and final velocities of the system (m/s)

Similarly, the kinetic energy of the rotating body is given by

$$KE = \frac{1}{2} I(\omega_1^2 - \omega_2^2) \tag{12.2}$$

or

$$KE = \frac{1}{2} mk^2(\omega_1^2 - \omega_2^2) \tag{12.3}$$

where,

I = mass moment of inertia of the rotating body (kg-m^2)

k = radius of gyration of the body (m)

ω_1, ω_2 = initial and final angular velocities of the body (rad/s)

In certain applications like hoists, the brake absorbs the potential energy released by the moving weight during the braking period. When a body of mass m falls through a distance h, the potential energy absorbed by the brake during the braking period is given by

$$PE = mgh \tag{12.4}$$

where,

g = gravitational constant (9.81 m/s^2)

Depending upon the type of application, the total energy absorbed by the brake is determined by adding the respective quantities of Eqs (12.1) to (12.4). This energy is equated to the work done by the brake.

Therefore,

$$E = M_t\theta \tag{12.5}$$

where

E = total energy absorbed by the brake (J)

M_t = braking-torque (N-m)

θ = angle through which the brake drum rotates during the braking period (rad)

Example 12.1 *A solid cast iron disk, 1 m in diameter and 0.2 m thick, is used as a flywheel. It is rotating at 350 rpm. It is brought to rest in 1.5 s by means of a brake. Calculate*

(i) the energy absorbed by the brake; and

(ii) the torque capacity of the brake.

Solution

Given $D = 1$ m $\quad t = 0.2$ m $\quad n_1 = 350$ rpm $\quad t = 1.5$ s

Step I *Energy absorbed by brake*

The brake absorbs the kinetic energy of the rotating flywheel. The mass density of cast iron is taken as 7200 kg/m^3. The radius of gyration of a solid disk about its axis of rotation is $(d/\sqrt{8})$. Therefore,

$$m = \frac{\pi}{4}(1)^2(0.2)(7200) = 1130.97 \text{ kg}$$

$$k^2 = \frac{D^2}{8} = \frac{1}{8} \text{ m}^2$$

$$\omega_1 = \frac{2\pi n_1}{60} = \frac{2\pi(350)}{60} = 36.65 \text{ rad/s}$$

and $\omega_2 = 0$

$$E = \frac{1}{2} mk^2(\omega_1^2 - \omega_2^2)$$

$$= \frac{1}{2}(1130.97)\left(\frac{1}{8}\right)(36.65)^2 = 94\,946.52 \text{ J} \quad \text{(i)}$$

Step II *Torque capacity of brake*

The average velocity during the braking period is $(\omega_1 + \omega_2)/2$ or $(\omega_1/2)$. Therefore,

$$\theta = \left(\frac{\omega_1}{2}\right)t = \left(\frac{36.65}{2}\right)(1.5) = 27.49 \text{ rad}$$

From Eq. (12.5),

$$M_t = \frac{E}{\theta} = \frac{94\,946.52}{27.49} = 3453.86 \text{ N-m} \quad \text{(ii)}$$

Example 12.2 *A four-wheeled automobile car has a total mass of 1000 kg. The moment of inertia of each wheel about a transverse axis through its centre of gravity is 0.5 kg-m^2. The rolling radius of the wheel is 0.35 m. The rotating and reciprocating parts of the engine and the transmission system are equivalent to a moment of inertia of 2.5 kg-m^2, which rotates at five times the road-wheel speed. The car is traveling at a speed of 100 km/h on a* *plane road. When the brakes are applied, the car decelerates at 0.5 g. There are brakes on all four wheels. Calculate:*

(i) the energy absorbed by each brake; and

(ii) the torque capacity of each brake.

Solution

Given For car, $m = 1000$ kg $v_1 = 100$ km/h

Deceleration = $(0.5g)$

For wheels, $I = 0.5$ kg-m^2 $R = 0.35$ m

For engine and the transmission system,

$$I = 2.5 \text{ kg-m}^2$$

speed = 5 (wheel speed)

Step I *Energy absorbed by each brake*

(i) KE of the car

$$v_1 = 100 \text{ km/h} = \frac{100 \times 10^3}{60 \times 60} \text{ m/s}$$

$$= 27.78 \text{ m/s} \quad \text{and} \quad v_2 = 0$$

$$\text{KE} = \frac{1}{2}m(v_1^2 - v_2^2) = \frac{1}{2}(1000)(27.78)^2$$

$$= 385\,802.44 \text{ J}$$

(ii) KE of the wheels

$$\omega_1 = \frac{v_1}{R} = \frac{27.78}{0.35} = 79.37 \text{rad/s} \quad \text{and} \quad \omega_2 = 0$$

$$\text{KE of four wheels} = 4\left[\frac{1}{2}I(\omega_1^2 - \omega_2^2)\right]$$

$$= 4\left[\frac{1}{2}(0.5)(79.37)^2\right] = 6298.81 \text{ J}$$

(iii) KE of the engine and transmission system

$$\omega_1 = 5(79.37) = 396.83 \text{ rad/s} \quad \text{and} \quad \omega_2 = 0$$

$$\text{KE} = \frac{1}{2}I(\omega_1^2 - \omega_2^2) = \frac{1}{2}(2.5)(396.83)^2$$

$$= 196\,837.97 \text{ J}$$

The energy absorbed by the four brakes consists of the kinetic energy of the car, the kinetic energy of the wheel and the kinetic energy of the engine and the transmission system.

$$E = \frac{1}{4}(385\,802.44 + 6298.81 + 196\,837.97)$$

$$= 147\,234.8 \text{ J} \qquad \text{(i)}$$

Step II *Torque capacity of brake*

The braking time t is given by

$$\frac{v_1 - v_2}{t} = 0.5g \qquad \therefore \frac{27.78 - 0}{t} = 0.5(9.81)$$

$$\therefore \qquad t = 5.66 \text{ s}$$

The average velocity during the braking time is $(\omega_1 + \omega_2)/2$ or $(\omega_1/2)$. Therefore,

$$\theta = \left(\frac{\omega_1}{2}\right)t = \left(\frac{79.37}{2}\right)(5.66) = 224.6 \text{ rad}$$

$$M_t = \frac{E}{\theta} = \frac{147\,234.8}{224.6} = 665.54 \text{ N-m} \qquad \text{(ii)}$$

Example 12.3 *A mass of 2500 kg is lowered at a velocity of 1.5 m/s from the drum as shown in Fig. 12.1. The mass of the drum is 50 kg and its radius of gyration can be taken as 0.7 m. On applying the brake, the mass is brought to rest in a distance of 0.5 m. Calculate*

(i) the energy absorbed by the brake; and

(ii) the torque capacity of the brake.

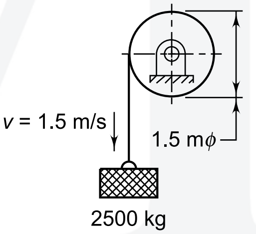


Fig. 12.1

Solution

Given $m = 2500$ kg $v = 1.5$ m/s

For drum, $m = 50$ kg $k = 0.7$ m $h = 0.5$ m

$R = 0.75$ m

Step I *Energy absorbed by brake*

KE of the mass

$$\text{KE} = \frac{1}{2}m(v_1^2 - v_2^2) = \frac{1}{2}(2500)(1.5)^2 = 2812.5 \text{ J}$$

KE of the drum

$$\omega_1 = \frac{v_1}{R} = \frac{1.5}{0.75} = 2 \text{ rad/s}$$

$$\text{KE} = \frac{1}{2}mk^2(\omega_1^2 - \omega_2^2) = \frac{1}{2}(50)(0.7)^2(2)^2 = 49 \text{ J}$$

PE of the mass

$$\text{PE} = mgh = (2500)(9.81)(0.5) = 12\,262.5 \text{ J}$$

$$E = 2812.5 + 49 + 12\,262.5 = 15\,124 \text{ J} \qquad \text{(i)}$$

Step II *Torque capacity of brake*

During the braking action, the mass moves through a distance of 0.5 m. If θ is the angle through which the drum rotates during the braking period,

$$\theta \times \text{drum radius} = 0.5$$

or $$\theta = \frac{0.5}{0.75} = 0.667 \text{ rad}$$

$$\therefore \quad M_t = \frac{E}{\theta} = \frac{15124}{0.667} = 22\,686 \text{ N-m} \qquad \text{(ii)}$$

12.3 BLOCK BRAKE WITH SHORT SHOE

A block brake consists of a simple block, which is pressed against the rotating drum by means of a lever as shown in Fig. 12.2. The friction between the block and the brake drum causes the retardation of the drum. This type of brake is commonly employed in railway trains. The block is either rigidly attached to the lever or, in some applications, pivoted to the lever. The angle of contact between the block and the brake drum is usually small. When it is less than 45°, the intensity of pressure between the block and brake drum is uniform. The free-body diagram of forces acting on the drum and the lever is shown in Fig. 12.3. The analysis is based on the following assumptions:

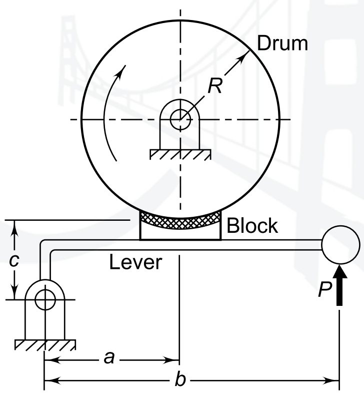


Fig. 12.2 *Block Brake*

(i) The block is rigidly attached to the lever.

(ii) The angle of contact between the block and brake drum is small, resulting in uniform pressure distribution.

(iii) The brake drum is rotating in clockwise direction.

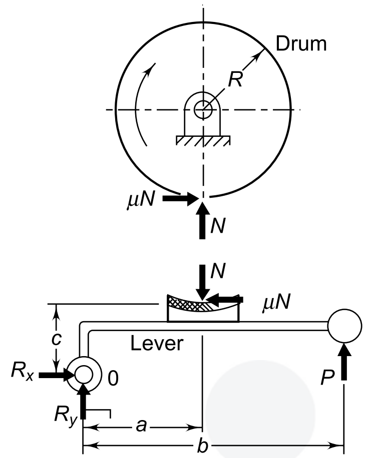


Fig. 12.3 *Free-body Diagram (Clockwise Rotation)*

Considering the forces acting on the brake drum,

$$M_t = \mu NR \qquad (12.6)$$

where

M_t = braking torque (N-mm)

R = radius of the brake drum (mm)

μ = coefficient of friction

N = normal reaction (N)

The dimensions of the block are determined by the following expression,

$$N = plw \qquad (12.7)$$

where

p = permissible pressure between the block and the brake drum (N/mm^2)

l = length of the block (mm)

w = width of the block (mm)

The width of the block w is usually between one-fourth and one-half of the drum diameter. When the width is small, the length of the block increases and the size of brake increases. Therefore, a narrower block results in an unnecessarily large size of brake. If the width is more, it becomes difficult to maintain uniform pressure distribution all over the surface of friction lining on the block. Therefore, a wider block results in variation in normal pressure distribution. The width of the block should be optimum between these two limits. The optimum value is given by,

$$\frac{1}{4}\text{(drum diameter)} < w < \frac{1}{2}\text{(drum diameter)}$$

Considering the equilibrium of forces in vertical and horizontal directions,

$$R_x = \mu N$$
$$R_y = (N - P) \tag{12.8}$$

Taking moment of forces acting on the lever about the hinge point O,

$$P \times b - N \times a + \mu N \times c = 0$$

or $$P = \frac{(a - \mu c)}{b} \times N \tag{12.9}$$

where P is the actuating force on the lever. Depending upon the magnitude of coefficient of friction (μ) and location of hinge pin (c), there are three different cases.

Case I $a > \mu c$

In this case, the friction force (μN) helps to reduce the magnitude of the actuating force P. It is seen from the free-body diagram, that the moment due to braking effort ($P \times b$), and moment due to friction force ($\mu N \times c$) are both anticlockwise. Such a brake is called a partially '*self-energizing*' brake. However, the brake is not self-locking, because a small magnitude of the positive force P is required for the braking action. This is a very desirable condition.

Case II $a = \mu c$

In this case, the actuating force P is zero, as seen from Eq. (12.9). This indicates that no external force is required for the braking action. Such a brake is called a '*self-locking*' brake. This is not a desirable condition in normal applications. Some positive braking effort (P) should be required to apply the brake, otherwise the brake will be out of control of the operator.

Case III $a < \mu c$

Under this condition, the actuating force P becomes negative, as seen from Eq. (12.9). This is a dangerous operating condition, resulting in uncontrolled braking and grabbing. The brake is out of control of the operator because he cannot apply it.

In designing block brakes, care should be taken to see that the brake is not self-locking and, at the same time, full advantage of the partial self-energizing effect should be taken to reduce the magnitude of the braking effort P. The condition to avoid self-locking is given by

$$a > \mu c \tag{12.10}$$

Conclusion

In order to prevent the brake arm from grabbing, the moment of friction force about the brake arm pivot (μNc) should be less than the moment of brake effort about the pivot (Pb).

The above analysis was based on three assumptions and the third assumption was that the brake drum is rotating in a clockwise direction. Let us consider the free-body diagram of forces when the brake drum rotates in an anti-clockwise direction. It is illustrated in Fig. 12.4. Taking moment of forces acting on the lever about the hinge point O,

$$P \times b - N \times a - \mu N \times c = 0$$

or $$P = \frac{(a + \mu c)}{b} \times N$$

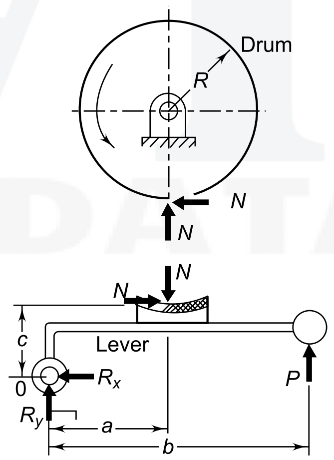


Fig. 12.4 *Free-body Diagram (Anti-clockwise Rotation)*

Therefore, the braking effort (P) also depends upon the direction of rotation of the brake drum. Obviously, for anti-clockwise rotation, the actuating force or braking effort (P) is more than that of clockwise rotation, which is given by Eq. (12.9). In design, the objective will be smaller values of braking effort.

It is observed from the above analysis that the percentage of total brake effort that results from self-energizing action depends upon the following three factors:

(i) The location of pivot for the brake lever or brake arm, namely, dimensions *a* and *c*
(ii) The coefficient of friction (μ)
(iii) The direction of rotation of brake drum

The main disadvantage of the block brake is the tendency of the brake drum shaft to bend under the action of normal reaction. The remedy is to use two symmetrical blocks at the opposite sides of the brake drum.

Example 12.4 *A single block brake with a torque capacity of 250 N-m is shown in Fig. 12.5(a). The brake drum rotates at 100 rpm and the coefficient of friction is 0.35. Calculate*

(i) the actuating force and the hinge-pin reaction for clockwise rotation of the drum;
(ii) the actuating force and hinge-pin reaction for anticlockwise rotation of the drum;
(iii) the rate of heat generated during the braking action; and
(iv) the dimensions of the block, if the intensity of pressure between the block and brake drum is 1 N/mm². The length of the block is twice its width.

State whether the brake is self-locking.

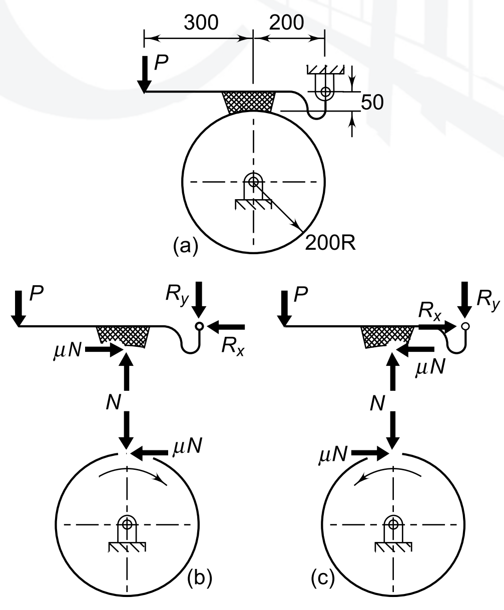


Fig. 12.5

Solution

Given $M_t = 250$ N-m $\quad n = 100$ rpm $\quad \mu = 0.35$
$p = 1$ N/mm^2 $\quad l = 2w$

Step I *Actuating force and hinge-pin reaction for clockwise rotation*

From Eq. (12.6),

$$N = \frac{M_t}{\mu R} = \frac{(250\times10^3)}{0.35(200)} = 3571.43 \text{ N}$$

The free-body diagram of forces for clockwise rotation of the drum is shown in Fig. 12.5(b). Taking moments about the hinge-pin,

$\mu N(50) + P(500) - N(200) = 0$

or $0.35(3571.43)(50) + P(500) - 3571.43(200) = 0$

$\therefore \quad P = 1303.57$ N

$R_x = \mu N = 0.35(3571.43) = 1250$ N

$R_y = N - P = 3571.43 - 1303.57 = 2267.86$ N

$$R = \sqrt{(1250)^2 + (2267.86)^2} = 2589.53 \text{ N} \qquad \text{(i)}$$

Step II *Actuating force and hinge-pin reaction for anticlockwise rotation*

The free-body diagram of forces for anti-clockwise rotation of the drum is shown in Fig. 12.5(c). Taking moments about the hinge-pin,

$-\mu N(50) + P(500) - N(200) = 0$

or $-0.35(3571.43)(50) + P(500) - 3571.43(200) = 0$

$\therefore \quad P = 1553.57$ N

$R_x = \mu N = 0.35(3571.43) = 1250$N

$R_y =$ N– P=3571.43 – 1553.57 = 2017.86N

$$R = \sqrt{(1250)^2 + (2017.86)^2} = 2373.66\text{N} \qquad \text{(ii)}$$

Step III *Rate of heat generated during braking action*

Initial velocity of the drum

$$= \omega r = \frac{2\pi(100)}{60}(0.2) = 2.094 \text{ m/s}$$

Final velocity of the drum = 0

Average velocity of the drum $= \dfrac{2.094 + 0}{2}$
$= 1.047$ m/s

The rate of heat generated during the braking period is equal to the rate of work done by the frictional force.

Rate of heat generated = frictional force × average velocity

$$= \mu N(1.047)$$
$$= 0.35\ (3571.43)\ (1.047)$$
$$= 1308.75 \text{ N-m/s or W} \qquad \text{(iii)}$$

Step IV *Dimensions of the block*

From Eq. (12.7),

$$N = plw$$
$$3571.43 = (1)(2w)(w)$$
$$\therefore \quad w = 42.26 \text{ mm or } 45 \text{ mm}$$
$$l = 2w = 90 \text{ mm} \qquad \text{(iv)}$$

Step V *Self-locking property*

Referring to Fig. 12.5 (a),

$a = 200$ mm $\quad c = 50$ mm $\quad \mu = 0.35$

$$\left(\frac{a}{c}\right) = 4 \qquad \therefore \left(\frac{a}{c}\right) > \mu$$

The brake is not self-locking.

Example 12.5 *A double block brake is shown in Fig. 12.6. The brake drum rotates in a clockwise direction and the actuating force is 500 N. The coefficient of friction between the blocks and the drum is 0.35. Calculate the torque absorbing capacity of the brake.*

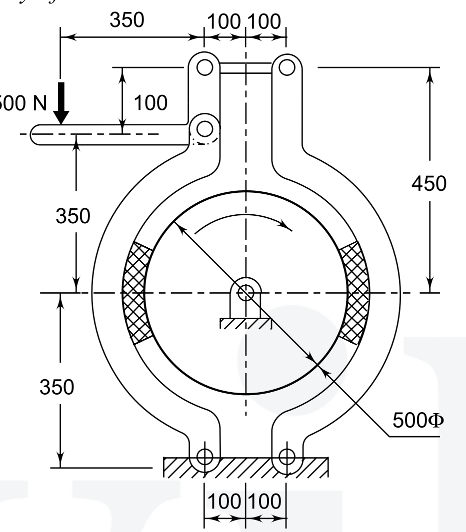


Fig. 12.6 *Double Block Brake*

Solution

Given $P = 500$ N $\quad \mu = 0.35 \quad R = 250$ mm

Step I *Force analysis*

The free-body diagram of forces acting on various parts is shown in Fig. 12.7. Considering the forces

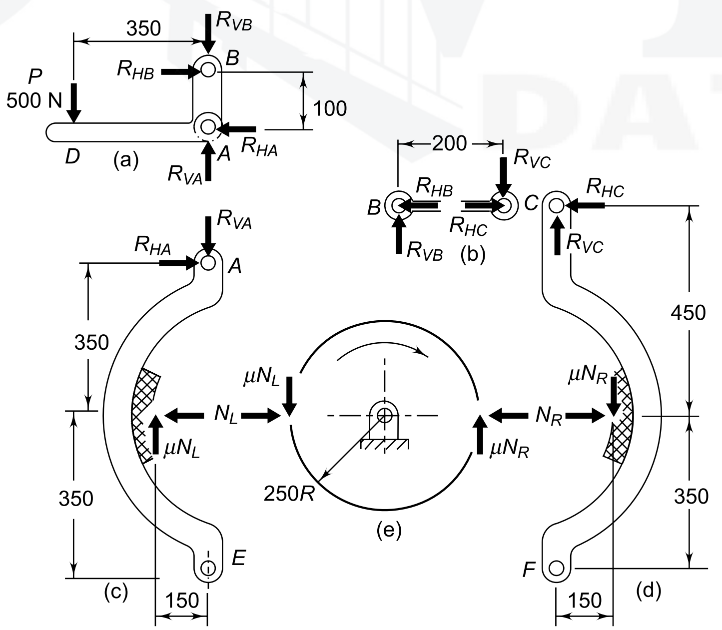


Fig. 12.7 *Free-body Diagram of Forces*

acting on the link DAB and taking moments about the pin A,

$$R_{HB}(100) = 500(350) \quad \text{or} \quad R_{HB} = 1750 \text{ N} \qquad \text{(a)}$$

Considering equilibrium of horizontal forces,

$$R_{HA} = R_{HB} = 1750 \text{ N} \qquad \text{(b)}$$

Considering equilibrium of vertical forces,

$$R_{VA} = 500 + R_{VB} \qquad \text{(c)}$$

Considering forces acting on the link BC,

$$R_{HC} = R_{HB} = 1750 \text{ N} \qquad \text{(d)}$$

Taking moments about the pin C,

$$R_{VB}(200) = 0 \qquad \therefore R_{VB} = 0 \qquad \text{(e)}$$

Considering equilibrium of vertical forces on the link BC,

$$R_{VC} = R_{VB} = 0 \qquad \text{(f)}$$

From Eqs (c) and (f),

$$R_{VA} = 500 \text{ N} \qquad \text{(g)}$$

Considering the free-body diagram of forces acting on the right-side link CF and taking moment of forces about the pin F,

$$R_{HC}(350 + 450) - N_R(350) - \mu N_R(150) = 0$$

or $1750(350 + 450) - N_R(350) - 0.35\, N_R(150) = 0$

$$\therefore \quad N_R = 3478.26\, N \qquad \text{(h)}$$

Similarly, taking moment of forces about the pin E of the left-side-link AE,

$$R_{HA}(350 + 350) + \mu\, N_L(150) - N_L(350) = 0$$

$$1750(350 + 350) + 0.35\, N_L(150) - N_L(350) = 0$$

$$\therefore \quad N_L = 4117.65 \text{ N} \qquad \text{(j)}$$

Step II *Torque absorbing capacity of the brake*

Referring to Fig. 12.7(e), the torque-absorbing capacity of the brake is given by

$$M_t = \mu(N_R + N_L)R$$
$$= 0.35(3478.26 + 4117.65)(0.25)$$
$$= 664.64 \text{ N-m}$$

Example 12.6 *A block brake with a short shoe is shown in Fig. 12.8. It is to be designed so that the product pv is limited to 2.*

where,

p = normal pressure between friction lining and the brake drum (N/mm²)

v = peripheral velocity of brake drum (m/s)

The coefficient of friction between the brake drum and the friction lining is 0.2. The cable drum is connected to the brake drum by means of a pair of spur gears. The brake drum rotates four times as fast as the cable drum. The permissible intensity of pressure on friction lining is 1 N/mm². Calculate:

(i) The magnitude of the brake shoe force (P)

(ii) The area of friction lining

(iii) The uniform velocity at which the mass can be lowered. What happens at higher speeds?

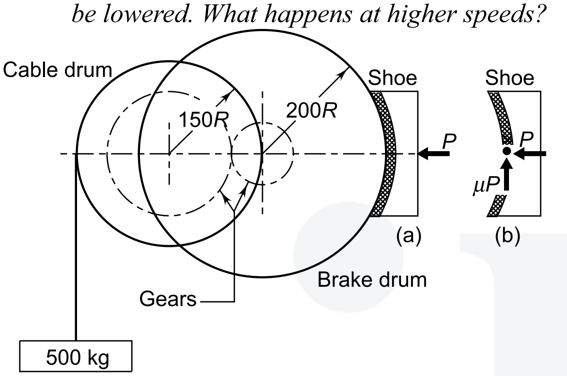


Fig. 12.8

Solution

Given $(pv) = 2 \quad \mu = 0.2 \quad p_{max.} = 1 \text{ N/mm}^2$

Step I *Brake shoe force*

The following notations are used in this example:

n_1 = speed of rotation of cable drum (rpm)

n_2 = speed of rotation of brake drum (rpm)

$(M_t)_1$ = torque on cable drum (N-mm)

$(M_t)_2$ = torque on brake drum (N-mm)

v_1 = peripheral velocity of cable drum (m/s). It is also the velocity with which the mass is lowered

v_2 = peripheral velocity of brake drum (m/s)

The brake drum rotates four times as fast as the cable drum.

$$n_2 = 4n_1 \qquad \text{(a)}$$

Referring to Fig. 12.8(a),

$$(M_t)_1 = (mg) \times 150 = (500 \times 9.81) \times 150 \text{ N-mm} \qquad \text{(b)}$$

Referring to Fig.12.8(b),

$$(M_t)_2 = \mu P \times 200 = (0.2\, P) \times 200 \text{ N-mm} \qquad \text{(c)}$$

Since the same power is transmitted,

$$\frac{2\pi n_1 (M_t)_1}{60 \times 10^6} = \frac{2\pi n_2 (M_t)_2}{60 \times 10^6}$$

$$\therefore \quad n_1(M_t)_1 = n_2(M_t)_2$$

$\therefore \quad n_1(500 \times 9.81) \times 150 = 4n_1(0.2\,P) \times 200$

$\therefore \quad P = 4598.44$ N (i)

Step II *Area of friction lining*

$P = p_{max.} \times$ Area of lining

$$\therefore \text{ Area of friction lining} = \frac{P}{p_{max.}} = \frac{4598.44}{1}$$

$$= 4598.44 \text{ mm}^2 \qquad \text{(ii)}$$

Step III *Uniform velocity at which mass can be lowered*

$$v_2 p_2 = 2 \quad \therefore \; v_2 = \frac{2}{p_2} = \frac{2}{1} = 2 \text{ m/s} \qquad \text{(d)}$$

$$\omega_2 = \frac{v_2}{R_2} = \frac{2}{0.2} = 10 \text{ rad/s}$$

and
$$\omega_1 = \frac{\omega_2}{4} = \frac{10}{4} = 2.5 \text{ rad/s}$$

$$v_1 = \omega_1 R_1 = 2.5(0.15) = 0.375 \text{ m/s}$$

or
$$v_1 = 0.375 \times 60 = 22.5 \text{ m/min} \qquad \text{(iii)}$$

When the coefficient of friction is constant, the rate of heat generated is proportional to the product pv. Therefore, at higher speeds the brake drum will be overheated.

12.4 BLOCK BRAKE WITH LONG SHOE

In the previous section, a block brake with short shoe was discussed. The angle of contact between the block and brake drum in such cases is usually small and less than 45°. It is, therefore, reasonable to assume that the normal reaction (N) and frictional force (μN) are concentrated at the midpoint of the shoe. This assumption is not applicable for the brake with the long shoe. A block brake with long shoe is shown in Fig. 12.9. The semi-block angle is denoted by θ. It is one half of the total angle of the contacting surface of block with the drum.

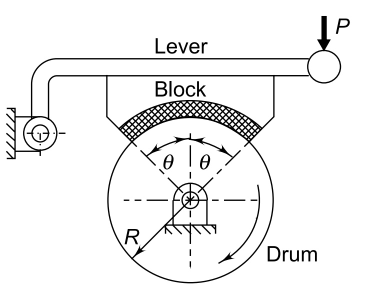
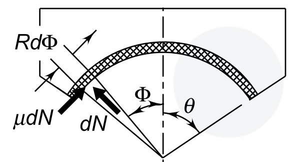


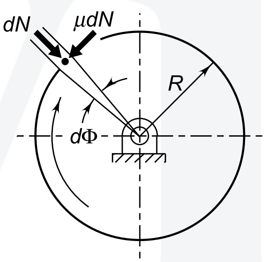
Fig. 12.9 *Block Brake with Long Shoe*

An element of friction lining, located at an angle (ϕ) and subtending an angle ($d\phi$) is shown in Fig. 12.10. The area of the element is ($Rd\phi w$), where w is the width of the friction lining parallel to the axis of the brake drum. If the intensity of pressure at the element is p, the normal reaction dN on the element is given by

$$dN = (Rd\phi w)p \qquad \text{(a)}$$



(a) Forces on lining



(b) Forces on drum

Fig. 12.10

The force of friction on the elementary area is given by,

$$\mu dN = \mu(Rd\phi w)p \qquad \text{(b)}$$

The torque transmitted by the force of friction on the elementary area is given by,

$$\delta M_t = \mu dNR = \mu(R^2 d\phi w)p \qquad \text{(c)}$$

$$M_t = \mu R^2 w \int p\, d\phi \qquad \text{(d)}$$

Since the brake drum is made of a hard material like cast iron or steel, the wear occurs on the friction lining, which is attached to the block. As shown in Fig. 12.11, the block or the lining will retain the cylindrical shape of the brake drum when wear occurs. After the radial wear takes place, a point such as Y' moves to Y due to the force P on the actuating lever and maintains contact of the lining on block with the brake drum. In

Fig. 12.11(b), (δy) is the wear in the Y direction and (δr) is the wear in the radial direction. If it is assumed that the block is constrained to move in the Y direction towards the brake drum to compensate for wear, (δy) should be constant because it is same for all points. Therefore,

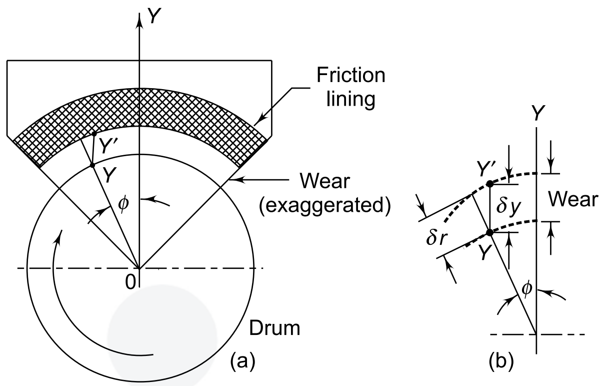


Fig. 12.11 *Wear on Friction Lining*

$$\delta y = \frac{\delta r}{\cos\phi} = \text{constant} \tag{e}$$

The radial wear (δr) is proportional to the work done by the frictional force. The work done by the frictional force depends upon the frictional force (μN) and the rubbing velocity. Since the rubbing velocity is constant for all points on the friction lining,

$$\delta r \propto \mu dN$$

Substituting in the expression (a),

$$\delta r \propto \mu R d\phi\, wp$$

$$\therefore \qquad \delta r \propto p \tag{f}$$

From the expressions (e) and (f),

$$\frac{p}{\cos\phi} = \text{constant}$$

or $$p = C_1 \cos\phi \tag{g}$$

where C_1 is the constant of proportionality. The pressure is maximum when $\phi = 0$. Therefore,

$$p = p_{\text{max.}} \quad \text{when} \quad \phi = 0°$$

Substituting this condition,

$$p_{\text{max.}} = C_1 \tag{h}$$

From Eqs (g) and (h),

$$p = p_{\text{max.}} \cos\phi \tag{i}$$

Substituting this value in Eq. (a),

$$dN = (R d\phi w) p_{\text{max.}} \cos\phi \tag{j}$$

Refer to forces acting on the element of the friction lining as shown in Fig. 12.10(a). The elemental force (dN) is resolved into the following two components:

(i) Vertical component ($dN \cos\phi$)

(ii) Horizontal component ($dN \sin\phi$)

The horizontal components on two sides of the lining will cancel each other. Therefore, the net force (N) is vertical and it is given by,

$$N = \text{Vertical force } = \int dN \cos\phi$$

$$= \int (R d\phi w) p_{\text{max.}} \cos\phi \cos\phi$$

or $$N = R\, w\, p_{\text{max.}} \int_{-\theta}^{+\theta} \cos^2\phi\, d\phi \tag{k}$$

$$\int \cos^2\phi = \int \frac{(1+\cos 2\phi)}{2} d\phi$$

since $$\left[\cos^2 A = \left(\frac{1+\cos 2A}{2}\right)\right]$$

$$\int \cos^2\phi = \frac{1}{2}\int (1+\cos 2\phi) d\phi$$

$$= \frac{1}{2}\left[\phi + \frac{\sin 2\phi}{2}\right] = \frac{1}{4}(2\phi + \sin 2\phi) \tag{l}$$

since $$\left[\int \cos ax\, dx = \frac{\sin ax}{a}\right]$$

From (k) and (l),

$$N = \frac{1}{4} Rwp_{\max.} [2\phi + \sin 2\phi]_{-\theta}^{+\theta}$$

$$= \frac{1}{4} Rwp_{\max.} \{2[\theta - (-\theta)] + [(\sin 2\theta - \sin(-2\theta)]\}$$

Therefore,

$$N = \frac{1}{4} Rwp_{\max.} (4\theta + 2\sin 2\theta)$$

$$= \frac{1}{2} Rwp_{\max.} (2\theta + \sin 2\theta) \qquad \text{(m)}$$

Substituting [$p = p_{\max.} \cos\phi$] in (d)

$$M_t = \mu R^2 wp_{\max.} \int_{-\theta}^{+\theta} \cos\phi \, d\phi = \mu R^2 wp_{\max.} [\sin\phi]_{-\theta}^{+\theta}$$

$$= \mu R^2 wp_{\max.} [\sin\theta - \sin(-\theta)]$$

$$M_t = \mu R^2 wp_{\max.} (2\sin\theta) \qquad \text{(n)}$$

Dividing Eq. (n) by (m),

$$M_t = \mu NR \left[\frac{4\sin\theta}{2\theta + \sin 2\theta}\right]$$

The tangential frictional force on the block can be found by dividing the torque by the radius, or

(friction force = M_t/R)

$$\text{Friction force} = \mu N \left[\frac{4\sin\theta}{2\theta + \sin 2\theta}\right]$$

In the above expression, the term,

$$\mu \left[\frac{4\sin\theta}{2\theta + \sin 2\theta}\right]$$

is called '*equivalent*' coefficient of friction and denoted by (μ'). Therefore,

$$\text{Friction force} = \mu' N \qquad \text{(o)}$$

and $$M_t = \mu' N R \qquad \text{(p)}$$

where,

$$\mu' = \mu \left[\frac{4\sin\theta}{2\theta + \sin 2\theta}\right] \qquad (12.11)$$

It is observed that the equation (*p*) is similar to Eq. (12.6) of block brake with short shoe except that (μ) is replaced by (μ').

Therefore, examples of block brakes with 'long' shoe, can be solved by using the same equations, which are derived for block brake with 'short' shoe, by replacing the coefficient of friction (μ) by an equivalent coefficient of friction (μ').

12.5 PIVOTED BLOCK BRAKE WITH LONG SHOE

When the block is rigidly fixed to the lever, the tendency of the frictional force (μN) is to unseat the block with respect to the lever as shown in Fig. 12.3. In case of the pivoted shoe brake, the location of the pivot can be selected in such a way that the moment of frictional force about the pivot is zero. This is the main advantage of the pivoted shoe brake. A double block brake with two symmetrical and pivoted shoes is shown in Fig. 12.12.

An element of friction lining, located at an angle (ϕ) and subtending an angle ($d\phi$) is shown in Fig. 12.13. The area of the element is ($Rd\phi w$), where w is the width of the friction lining parallel to the axis of the brake drum. If the intensity of pressure at the element is p, the normal reaction dN on the element is given by

$$dN = (Rd\phi w)p \qquad \text{(a)}$$

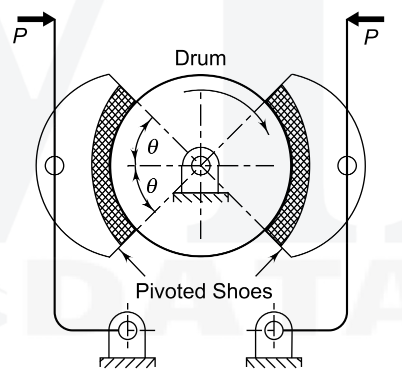


Fig. 12.12 *Pivoted Double Block Brake*

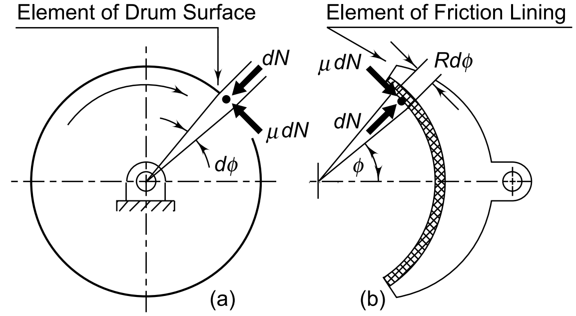


Fig. 12.13

Since the brake drum is made of a hard material like cast iron or steel, the wear occurs on the friction lining, which is attached to the shoe. As shown in Fig. 12.14, the lining will retain the cylindrical shape of the brake drum when wear occurs. After the radial wear has taken place, a point such as X' moves to X in order to maintain contact of the lining with the brake drum. In Fig. 12.14(b), (δx) is the wear in the X direction and (δr) is the wear in the radial direction. If it is assumed that the shoe is constrained to move towards the brake drum to compensate for wear, (δx) should be constant because it is same for all points. Therefore,

$$\delta x = \frac{\delta r}{\cos\phi} = \text{constant} \tag{b}$$

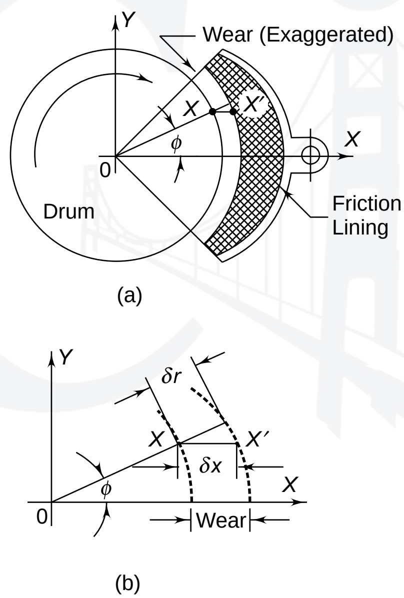


Fig. 12.14 *Wear on Friction Lining*

The radial wear (δr) is proportional to the work done by the frictional force. The work done by the frictional force depends upon the frictional force (μN) and the rubbing velocity. Since the rubbing velocity is constant for all points on friction lining,

$$\delta r \propto \mu \, dN$$

$$\therefore \quad \delta r \propto \mu R \, d\phi \, wp$$

$$\therefore \quad \delta r \propto p \tag{c}$$

From the expressions (b) and (c),

$$\frac{p}{\cos\phi} = \text{constant}$$

or $\quad p = C_1 \cos\phi \quad$ (d)

where C_1 is the constant of proportionality. The pressure is maximum when $\phi = 0$. Substituting,

$$p_{\text{max.}} = C_1 \tag{e}$$

From Eqs (d) and (e),

$$p = p_{\text{max.}} \cos\phi$$

Substituting this value in Eq. (a),

$$dN = (R d\phi w) p_{\text{max.}} \cos\phi \tag{f}$$

The forces acting on the element of the friction lining are shown in Fig. 12.15(a). The distance h of the pivot is selected in such a manner that the moment of frictional force about it is zero. Therefore,

$$M_f = 2\int_0^{\theta} \mu dN (h\cos\phi - R) = 0$$

Substituting dN from Eq. (f),

$$\int_0^{\theta} (h\cos^2\phi - R\cos\phi) d\phi = 0$$

$$h\int_0^{\theta} \left[\frac{1+\cos 2\phi}{2}\right] d\phi - R\int_0^{\theta} \cos\phi \, d\phi = 0$$

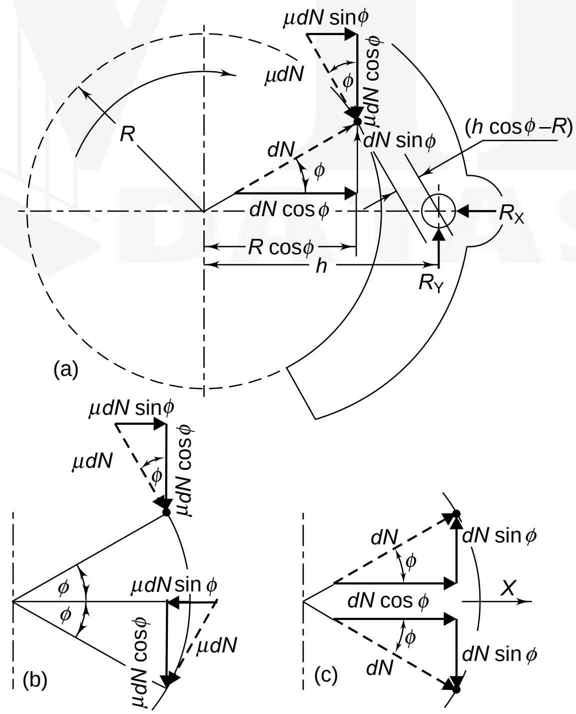


Fig. 12.15 *Forces Acting on Shoe*

or
$$h\left[\frac{\phi+\frac{1}{2}\sin 2\phi}{2}\right]_0^{\theta}-R(\sin\phi)_0^{\theta}=0$$

$$\therefore \quad h=\frac{4R\sin\theta}{2\theta+\sin 2\theta} \qquad (12.12)$$

The elemental torque of frictional force (μdN) about the axis of the brake drum is (μdNR). Therefore,

$$M_t=2\int_0^{\theta}\mu dNR$$

Substituting the value of dN from Eq. (f),

$$M_t=2\mu R^2 wp_{\max.}\int_0^{\theta}\cos\phi\, d\phi$$

$$\therefore \quad M_t=2\mu R^2 wp_{\max.}\sin\theta \qquad (12.13)$$

The reaction R_X can be determined by considering two components ($dN\cos\phi$) and ($\mu dN\sin\phi$) as shown in Fig.12.15(a). Due to symmetry,

$$\int \mu dN\sin\phi=0$$

This is illustrated in Fig. 12.15(b), where two symmetrical elements in mirror image position with respect to the X-axis are considered.

Therefore,

$$R_X=2\int_0^{\theta}dN\cos\phi=2Rwp_{\max.}\int_0^{\theta}\cos^2\phi\, d\phi$$

$$=2Rwp_{\max.}\left[\frac{2\theta+\sin 2\theta}{4}\right]$$

or
$$R_X=\frac{1}{2}Rwp_{\max.}(2\theta+\sin 2\theta) \qquad (12.14)$$

The reaction R_Y can be determined by considering two components ($dN\sin\phi$) and ($\mu dN\cos\phi$) as shown in Fig. 12.15(a). Due to symmetry,

$$\int dN\sin\phi=0$$

This is illustrated in Fig. 12.15(c), where two symmetrical elements in mirror image position with respect to the X-axis are considered.

Therefore,

$$R_Y=2\int_0^{\theta}\mu dN\cos\phi=2\mu Rwp_{\max.}\int_0^{\theta}\cos^2\phi\, d\phi$$

or
$$R_Y=\frac{1}{2}\mu Rwp_{\max.}(2\theta+\sin 2\theta) \qquad (12.15)$$

Pivoted shoe brakes are mainly used in hoists and cranes. The applications of these brakes are limited because of the physical problem in locating a pivot so close to the drum surface.

Example 12.7 *A pivoted double-block brake, similar to that in Fig. 12.12, has two shoes, which subtend an angle (2θ) of 100°. The diameter of the brake drum is 500 mm and the width of the friction lining is 100 mm. The coefficient of friction is 0.2 and the maximum intensity of pressure between the lining and the brake drum is 0.5 N/mm². The pivot of each shoe is located in such a manner that the moment of the frictional force on the shoe is zero. Calculate:*

(i) the distance of the pivot from the axis of the brake drum;
(ii) the torque capacity of each shoe; and
(iii) the reactions at the pivot.

Solution

Given $D = 500$ mm $\quad w = 100$ mm $\quad \mu = 0.2$
$p_{\max.} = 0.5$ N/mm² $\quad 2\theta = 100°$

Step I *Distance of pivot from axis of drum*

From Eq. (12.12),

$$h=\frac{4R\sin\theta}{2\theta+\sin 2\theta}=\frac{4(250)\sin(50°)}{\left(\frac{100\pi}{180}\right)+\sin(100°)}$$

$$=280.59 \text{ mm} \qquad (i)$$

Step II *Torque capacity of each shoe*

From Eq. (12.13),

$$M_t=2\mu R^2 wp_{\max.}\sin\theta$$
$$=2(0.2)(250)^2(100)(0.5)\sin(50°)$$
$$=957\,555 \text{ N-mm} \qquad (ii)$$

Step III *Reactions at pivot*

From Eqs (12.14) and (12.15),

$$R_X=\frac{1}{2}Rwp_{\max.}(2\theta+\sin 2\theta)$$

$$=\frac{1}{2}(250)(100)(0.5)\left[\left(\frac{100\pi}{180}\right)+\sin(100°)\right]$$

$$=17063 \text{ N}$$

$$R_Y=\frac{1}{2}\mu Rwp_{\max.}(2\theta+\sin 2\theta)$$

$$=\mu R_x=0.2\,(17\,063)=3412.6 \text{ N} \qquad (iii)$$

12.6 INTERNAL EXPANDING BRAKE

The construction of an internal expanding brake is shown in Fig. 12.16. It consists of a shoe, which is pivoted at one end and subjected to an actuating force P at the other end. A friction lining is fixed on

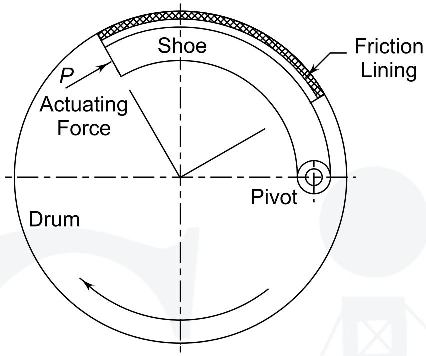


Fig. 12.16 *Internal Expanding Brake*

the shoe and the complete assembly of shoe, lining and pivot is placed inside the brake drum. Internal shoe brakes, with two symmetrical shoes, are used on all automobile vehicles. The actuating force is usually provided by means of a hydraulic cylinder or a cam mechanism. The analysis of the internal shoe brake is based on the following assumptions:

(i) The intensity of normal pressure between the friction lining and the brake drum at any point is proportional to its vertical distance from the pivot.

(ii) The brake drum and the shoe are rigid.

(iii) The centrifugal force acting on the shoe is negligible.

(iv) The coefficient of friction is constant.

The free-body diagram of forces acting on an element on the surface of the drum and the surface of friction lining is shown in Fig.12.17. It should be noted that angles ϕ, θ_1, θ_2 begin with a line drawn from the centre of the drum to the centre of the pivot of the shoe. The friction material begins at an angle θ_1 from this line and ends at an angle θ_2.

Consider an elemental area on the friction lining located at an angle ϕ and subtending an angle $d\phi$. The elemental area will be $(Rd\phi w)$ where w is the width of the friction lining parallel to the axis of the brake drum. If p is the intensity of normal pressure on this elemental area, the normal reaction dN is given by,

$$dN = pR\,wd\phi \qquad \text{(a)}$$

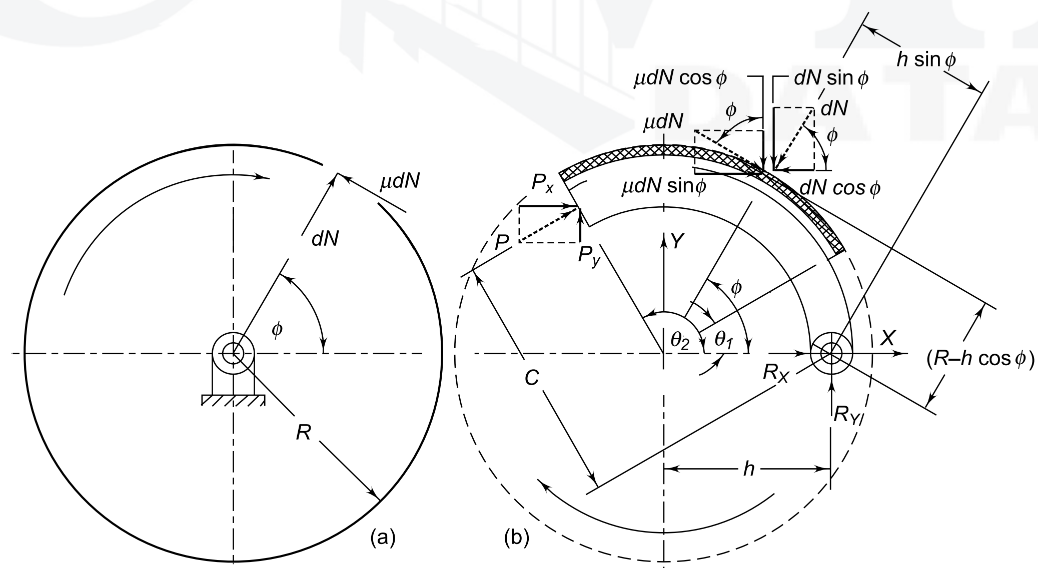


Fig. 12.17 *Free-body Diagram of Forces*

As mentioned in the first assumption, the normal pressure p is proportional to the vertical distance ($R \sin\phi$) of the element from the pivot. Therefore,

$$p \propto \sin\phi \quad \text{or} \quad p = C_1 \sin\phi \tag{b}$$

Assuming $p = p_{max.}$ when $\phi = \phi_{max.}$ we have,

$$p_{max.} = C_1 \sin\phi_{max.} \tag{c}$$

From Eqs (b) and (c),

$$p = \frac{p_{max.}\sin\phi}{\sin\phi_{max.}} \tag{12.16}$$

It can be seen from Eq. (b) that

$\phi_{max.} = 90°$ when $\theta_2 > 90°$

$\phi_{max.} = \theta_2$ when $\theta_2 < 90°$

Substituting Eq. (12.16) in Eq. (a),

$$dN = \frac{p_{max.}Rw}{\sin\phi_{max.}}\sin\phi\, d\phi \tag{d}$$

The moment M_f of the frictional force (μdN) about the pivot point is given by

$$M_f = \int \mu dN(R - h\cos\phi)$$

Substituting Eq. (d),

$$M_f = \frac{\mu p_{max.}Rw}{\sin\phi_{max.}}\int_{\theta_1}^{\theta_2} \sin\phi(R - h\cos\phi)d\phi \tag{12.17}$$

The moment M_n of the normal force (dN) about the pivot point is given by

$$M_n = \int dN(h\sin\phi)$$

Substituting Eq. (d),

$$M_n = \frac{p_{max.}Rwh}{\sin\phi_{max.}}\int_{\theta_1}^{\theta_2} \sin^2\phi\, d\phi \tag{12.18}$$

$$\int_{\theta_1}^{\theta_2} \sin\phi(R - h\cos\phi)d\phi$$

$$= R\int_{\theta_1}^{\theta_2} \sin\phi\, d\phi - h\int_{\theta_1}^{\theta_2}\left(\frac{\sin 2\phi}{2}\right)d\phi$$

$$= R[-\cos\phi]_{\theta_1}^{\theta_2} - h\left[-\frac{\cos 2\phi}{4}\right]_{\theta_1}^{\theta_2}$$

$$= \frac{1}{4}[4R(\cos\theta_1 - \cos\theta_2) - h(\cos 2\theta_1 - \cos 2\theta_2)]$$

From Eq. (12.17)

$$M_f = \frac{\mu p_{max.}Rw[4R(\cos\theta_1 - \cos\theta_2) - h(\cos 2\theta_1 - \cos 2\theta_2)]}{4\sin\phi_{max.}} \tag{12.19}$$

Similarly,

$$\int_{\theta_1}^{\theta_2} \sin^2\phi\, d\phi = \int_{\theta_1}^{\theta_2}\left(\frac{1-\cos 2\phi}{2}\right)d\phi = \left[\frac{\phi}{2} - \frac{\sin 2\phi}{4}\right]_{\theta_1}^{\theta_2}$$

$$= \frac{1}{4}[2(\theta_2 - \theta_1) - (\sin 2\theta_2 - \sin 2\theta_1)]$$

From Eq. (12.18),

$$M_n = \frac{p_{max.}Rwh[2(\theta_2 - \theta_1) - (\sin 2\theta_2 - \sin 2\theta_1)]}{4\sin\phi_{max.}} \tag{12.20}$$

Referring to Fig. 12.17, the elemental torque due to frictional force (μdN) is (μdNR). Therefore,

$$M_t = \int \mu\, dN\, R$$

Substituting Eq. (d),

$$M_t = \frac{\mu R^2 p_{max.}w}{\sin\phi_{max.}}\int_{\theta_1}^{\theta_2}\sin\phi\, d\phi$$

or

$$M_t = \frac{\mu R^2 p_{max.}w(\cos\theta_1 - \cos\theta_2)}{\sin\phi_{max.}} \tag{12.21}$$

Considering the forces acting on the shoe and taking moments about the pivot,

$$P \times C + M_f - M_n = 0$$

The couple ($P \times C$) is clockwise. The couple due to (dN), i.e., M_n is anticlockwise, while the couple due to ($\mu\, dN$), i.e., M_f is clockwise. Therefore,

$$P = \frac{M_n - M_f}{C} \tag{12.22}$$

The above equation is derived for the clockwise rotation of the brake drum. The direction of the frictional force (μdN) is reversed for the anticlockwise rotation of the brake drum. In that case, the couple due to (μdN), i.e. M_f will be anticlockwise and Eq. (12.22) will be written as

$$P = \frac{M_n + M_f}{C} \tag{12.23}$$

Let us assume that the vehicle is moving in the forward direction for clockwise rotation of the brake drum. The vehicle will be travelling in 'reverse' for anti-clockwise rotation of brake drum. The following observations are made:

(i) When the vehicle is moving forward (clockwise rotation of the brake drum), the couple due to actuating force ($P \times C$) and the couple due to friction force ($\mu\ dN$), i.e. M_f are both clockwise. Therefore, friction force helps to reduce the actuating force and consequently the force on the brake pedal is reduced. This is self-energizing effect [$P = (M_n - M_f)/C$].

(ii) When the vehicle is moving in 'reverse' (anti-clockwise rotation of the brake drum), the couple due to actuating force ($P \times C$) and the couple due to friction force ($\mu\ dN$), i.e., M_f are opposite. Therefore, friction force tends to increase the actuating force and consequently, the force on the brake pedal is increased [$P = (M_n + M_f)/C$].

(iii) Therefore, braking action when traveling in 'reverse' is not as effective as when traveling 'forward'.

Internal expanding brake offers the following advantages:

(i) It has simple construction with small number of parts. It is cheaper compared with other types of brakes.

(ii) It is more reliable due to small number of parts.

(iii) It requires little maintenance.

(iv) In an internal expanding brake, a small actuating force can produce a large braking torque.

(v) It offers protection against entry of foreign particles.

The disadvantages of internal expanding brake are as follows:

(i) It has relatively poor heat dissipating capacity.

(ii) It becomes self-locking due to wear, if not properly designed.

An internal expanding brake is mainly used in vehicle, conveyor and hoist. A hydraulic brake system is usually used in an automotive brake for cars, trucks and buses. It uses hydraulic fluid as a medium for transmitting force from the brake pedal to the wheel brakes. Some of the guidelines for the design of automotive brake are as follows[1]:

(i) The maximum force exerted with the right foot for the fifth percentile female is 22 N and for the male, approximately 42 N. Therefore, from ergonomic considerations, the brake system should be designed for a maximum pedal force of 22 to 25 N. With booster, the pedal force can be as low as 11 to 17 N.

(ii) From ergonomic considerations, the pedal travel should not exceed 150 mm.

(iii) The brake system should be designed so as to obtain a deceleration of 1 g (i.e., 9.81 m/s^2), when the vehicle is loaded full.

Example 12.8 *An automotive type internal-expanding double-shoe brake is shown in Fig. 12.18. The face width of the friction lining is 40 mm and the maximum intensity of normal pressure is limited to 1 N/mm^2. The coefficient of friction is 0.32. The angle θ_1 can be assumed to be zero. Calculate:*

(i) the actuating force P; and

(ii) the torque-absorbing capacity of the brake.

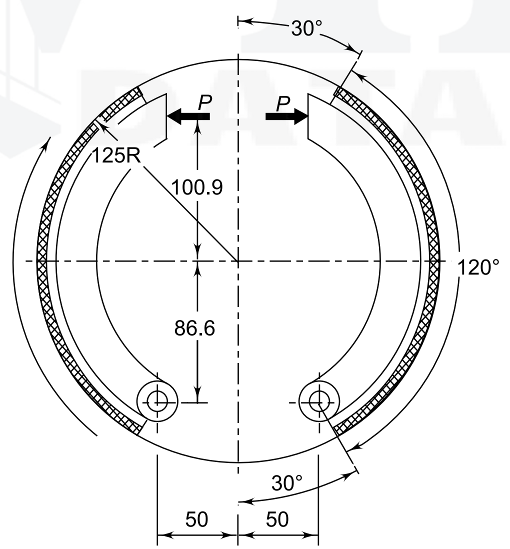


Fig. 12.18 *Automotive Double Shoe Brake*

[1] Limpert Rudolf – *Brake Design and Safety* – Society of Automotive Engineers (SAE), USA—1992.

Solution

Given $w = 40$ mm $\mu = 0.32$ $p_{max.} = 1$ N/mm^2 $R = 125$ mm

Step I *Actuating force*

It is assumed that the maximum normal pressure will occur between the lining on the right-hand shoe and the brake drum. For the right-hand shoe,

$\theta_1 = 0$ $\theta_2 = 120°$ $\phi_{max.} = 90°$ $\sin \phi_{max.} = 1$

The distance h of the pivot from the axis of the brake drum is given by

$$h = \sqrt{86.6^2 + 50^2} = 100 \text{ mm}$$

From Eq. (12.19),

$$M_f = \frac{\mu p_{max.} Rw[4R(\cos\theta_1 - \cos\theta_2) - h(\cos 2\theta_1 - \cos 2\theta_2)]}{4 \sin \phi_{max.}}$$

$$= \frac{0.32(1)(125)(40)[4(125)(1 - \cos 120°) - 100(1 - \cos 240°)]}{4(1)}$$

$= 240\,000$ N-mm

From Eq. (12.20),

$$M_n = \frac{p_{max.} Rwh[2(\theta_2 - \theta_1) - (\sin 2\theta_2 - \sin 2\theta_1)]}{4 \sin \phi_{max.}}$$

$$= \frac{1(125)(40)(100)\left[2\left(\frac{120\pi}{180}\right) - \sin(240°)\right]}{4(1)}$$

$= 631\,851.95$ N-mm

From Eq. (12.22),

$$P = \frac{M_n - M_f}{C}$$

$$= \frac{631\,851.95 - 240\,000}{100.9 + 86.6} = 2089.88 \text{ N} \qquad \text{(i)}$$

Step II *Torque-absorbing capacity*

From Eq. (12.21), the torque $(M_t)_R$ for the right-hand shoe is given by

$$(M_t)_R = \frac{\mu R^2 p_{max.} w(\cos\theta_1 - \cos\theta_2)}{\sin \phi_{max.}}$$

$$= \frac{0.32(125)^2(1)(40)(1 - \cos 120°)}{1}$$

$= 300\,000$ N-mm

The maximum intensity of pressure for the left-hand shoe is unknown. For identical shoes, it can be seen from the expressions of M_n and M_f, that both are proportional to ($p_{max.}$). For left-hand shoe, the maximum intensity of pressure is taken as ($p'_{max.}$). Therefore, for the left-hand shoe,

$$M'_f = \frac{240\,000\, p'_{max.}}{p_{max.}} = \frac{(240\,000)\, p'_{max.}}{(1)}$$

$= 240\,000\, p'_{max.}$

Similarly,

$$M'_n = \frac{631\,851.95\, p'_{max.}}{p_{max.}} = \frac{(631\,851.95)\, p'_{max.}}{(1)}$$

$= 631\,851.95\, p'_{max.}$

For the left-hand shoe,

$$P = \frac{M'_n + M'_f}{C}$$

or $$2089.88 = \frac{(240\,000 + 631\,851.95)\, p'_{max.}}{(100.9 + 86.6)}$$

$\therefore$ $p_{max.} = 0.45$ N/mm^2

Since the shoes are identical,

$$(M_t)_L = 300\,000\left(\frac{p'_{max.}}{p_{max.}}\right) = 300\,000\left(\frac{0.45}{1}\right)$$

$= 135\,000$ N-mm

The total torque-absorbing capacity of the brake is given by,

$M_t = 300\,000 + 135\,000 = 435\,000$ N-mm

or 435 N-m (ii)

Example 12.9 *An internal-expanding brake with four identical shoes is shown in Fig. 12.19. Each hinge pin supports a pair of shoes. The actuating mechanism is designed in such a way that it produces the same force P on each of the four shoes. The face width of the friction lining is 50 mm and the maximum intensity of normal pressure is limited to 1 N/mm². The coefficient of friction is 0.30. Calculate:*

(i) the actuating force P; and

(ii) the torque-absorbing capacity of the brake.

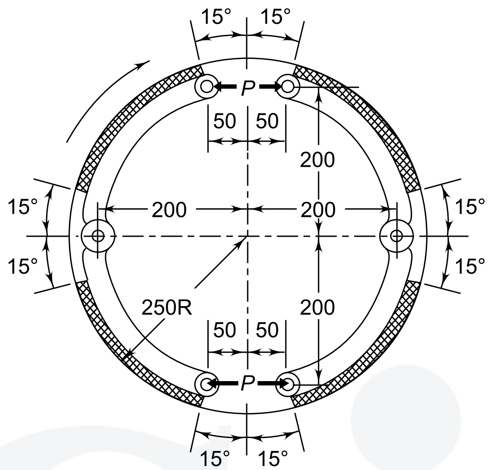


Fig. 12.19

Solution

Given $w = 50$ mm $\mu = 0.3$ $p_{max.} = 1$ N/mm^2
$R = 250$ mm $h = 200$ mm

Step I *Actuating force*

It is assumed that the maximum normal pressure will occur between the drum and the lining on right-hand shoe of the upper half. For the right-hand shoe,

$\theta_1 = 15°$ $\theta_2 = 75°$ $\phi_{max.} = 75°$

$\sin \phi_{max.} = 0.9659$ $h = 200$ mm $c = 200$ mm

where h is the distance of the pivot axis from the axis of the brake drum.

From Eq. (12.19),

$$M_f = \frac{\mu p_{max.} Rw[4R(\cos\theta_1 - \cos\theta_2) - h(\cos 2\theta_1 - \cos 2\theta_2)]}{4\sin\phi_{max.}}$$

$$= \frac{0.3(1)(250)(50)[4(250)(\cos 15° - \cos 75°) - 200(\cos 30° - \cos 150°)]}{4(0.9659)}$$

$= 350\,091.19$ N-mm

From Eq. (12.20),

$$M_n = \frac{p_{max.} Rwh[2(\theta_2 - \theta_1) - (\sin 2\theta_2 - \sin 2\theta_1)]}{4\sin\phi_{max.}}$$

$$= \frac{1(250)(50)(200)\left[2\left(\frac{60\pi}{180}\right) - (\sin 150° - \sin 30°)\right]}{4(0.9659)}$$

$= 1\,355\,209.59$ N-mm

From Eq. (12.22),

$$P = \frac{M_n - M_f}{C}$$

$$= \frac{1\,355\,209.59 - 350\,091.19}{200} = 5025.59 \text{ N} \qquad \text{(i)}$$

Step II *Torque-absorbing capacity*

From Eq. (12.21), the torque $(M_t)_R$ for the right hand shoe in the upper half is given by

$$(M_t)_R = \frac{\mu R^2 p_{max.} w(\cos\theta_1 - \cos\theta_2)}{\sin\phi_{max.}}$$

$$= \frac{0.3(250)^2(1)(50)(\cos 15° - \cos 75°)}{0.9659}$$

$= 686\,315.98$ N-mm

The maximum intensity of pressure for the left-hand shoe in the upper half is unknown. For identical shoes, it can be seen from the expressions of M_n and M_f that both are proportional to ($p_{max.}$). For the left-hand shoe, the maximum intensity of pressure is taken as ($p'_{max.}$). Therefore, for the left-hand shoe,

$$M'_f = \frac{350\,091.19\, p'_{max.}}{p_{max.}} = \frac{(350\,091.19)\, p'_{max.}}{(1)}$$

$= 350\,091.19\, p'_{max.}$

Similarly,

$$M'_n = \frac{1355\,209.59\, p'_{max.}}{p_{max.}} = \frac{(1355\,209.59)\, p'_{max.}}{(1)}$$

$= 1355\,209.59\, p'_{max.}$

Using Eq. (12.23) for the left-hand shoe in the upper half,

$$P = \frac{M'_n + M'_f}{C}$$

or $$5025.59 = \frac{(350\,091.19 + 1355\,209.59)\, p'_{max.}}{200}$$

$\therefore$ $p'_{max.} = 0.59$ N/mm^2

$$(M_t)_L = 686\,315.98\left(\frac{p'_{max.}}{p_{max.}}\right) = 686\,315.98\left(\frac{0.59}{1}\right)$$

$= 404\,926.43$ N-mm

The total torque-absorbing capacity of the pair of shoes in the upper half of the brake drum is given by,

$$(M_t)_{uh} = (M_t)_R + (M_t)_L = 686\,315.98 + 404\,926.43$$
$$= 1\,091\,242.41 \text{ N-mm}$$

Since all four shoes are identical, the two pairs are also identical. Therefore,

$$M_t = (M_t)_{uh} + (M_t)_{lf} = 2(M_t)_{uh} = 2(1\,091\,242.41)$$
$$= 2\,182\,484.82 \text{ N-mm}$$

or, $$M_t = 2182.48 \text{ N-m} \qquad \text{(ii)}$$

12.7 BAND BRAKES

The construction of a simple band brake is shown in Fig.12.20. It consists of a flexible steel strip lined with friction material, which is pressed against the rotating brake drum. When one end of the steel band passes through the fulcrum of the actuating lever, the brake is called the simple band brake.

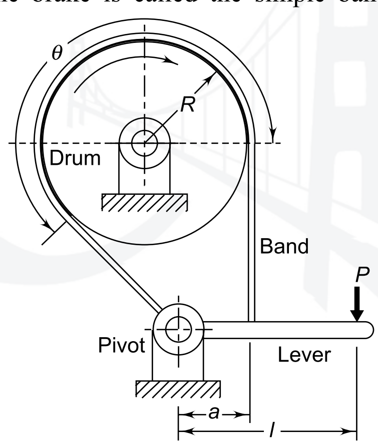


Fig. 12.20 *Simple Band Brake*

The working of the steel band is similar to that of a flat belt operating at zero velocity. The free-body diagram of forces acting on the band and the lever is shown in Fig. 12.21. The ratio of band tensions is given by,

$$\frac{P_1}{P_2} = e^{\mu\theta} \qquad (12.24)$$

where,

P_1 = tension on the tight side of the band (N)
P_2 = tension on the loose side of the band (N)
μ = coefficient of friction between the friction lining and the brake drum
θ = angle of wrap (rad)

The torque M_t absorbed by the brake is given by,

$$M_t = (P_1 - P_2)R \qquad (12.25)$$

where,

M_t = torque capacity of the brake (N-mm)
R = radius of the brake drum (mm)

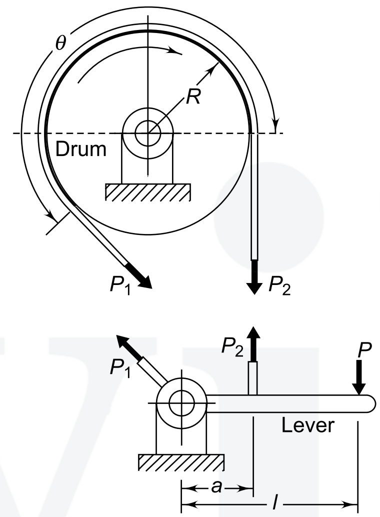


Fig. 12.21 *Free-body Diagram of Forces*

Considering the forces acting on the lever and taking moments about the pivot,

$$P_2 \times a = P \times l$$

or $$P = \frac{P_2 a}{l} \qquad (12.26)$$

An element of the band subtending an angle ($d\phi$) is shown in Fig. 12.22. The elemental area of the friction lining is ($Rd\phi w$), where w is the width of the lining parallel to the axis of the brake drum. In the figure, (P) and ($P + dP$) are tensions in the band in the loose and tight sides respectively. If p is the intensity of pressure, the normal reaction (dN) is given by

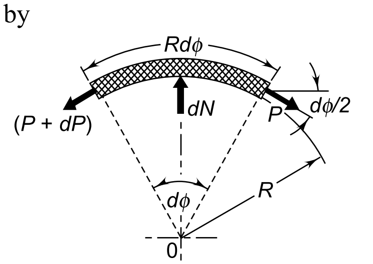


Fig. 12.22

$$dN = pRwd\phi \qquad \text{(a)}$$

Considering equilibrium of vertical forces on the element,

$$dN = P\sin\left(\frac{d\phi}{2}\right) + (P + dP)\sin\left(\frac{d\phi}{2}\right) \qquad \text{(b)}$$

For small angles,

$$\sin\left(\frac{d\phi}{2}\right) = \left(\frac{d\phi}{2}\right)$$

Neglecting higher order differentials,

$$dN = Pd\phi \qquad \text{(c)}$$

From Eqs (a) and (c),

$$p = \frac{P}{Rw}$$

The intensity of pressure is maximum, when the band tension P is equal to P_1,

or $$p_{\text{max.}} = \frac{P_1}{Rw} \qquad (12.27)$$

A differential band brake is shown in Fig. 12.23(a). In this case, neither end of the band passes through the fulcrum of the actuating lever. Such brakes can be designed for the condition of self-locking. The free-body diagram of forces acting on the band and the lever is shown in Fig.12.23(b). Considering forces acting on the lever and taking moments about the pivot,

$$P \times l + P_1 \times b - P_2 \times a = 0$$

or $$P = \frac{P_2 a - P_1 b}{l}$$

Substituting Eq. (12.24) in the above expression,

$$P = \frac{P_2(a - b \times e^{\mu\theta})}{l}$$

For the self-locking condition,

$$P = 0 \text{ or negative}$$

or $$a \le b \times e^{\mu\theta}$$

Therefore, the condition of self-locking is given by

$$\left(\frac{a}{b}\right) \le e^{\mu\theta} \qquad (12.28)$$

The self-locking property is undesirable in most of the applications of the brake, since the brake is out of the operator's control. In such a situation, no external force is required to apply the brake. A band in contact with the drum automatically grabs

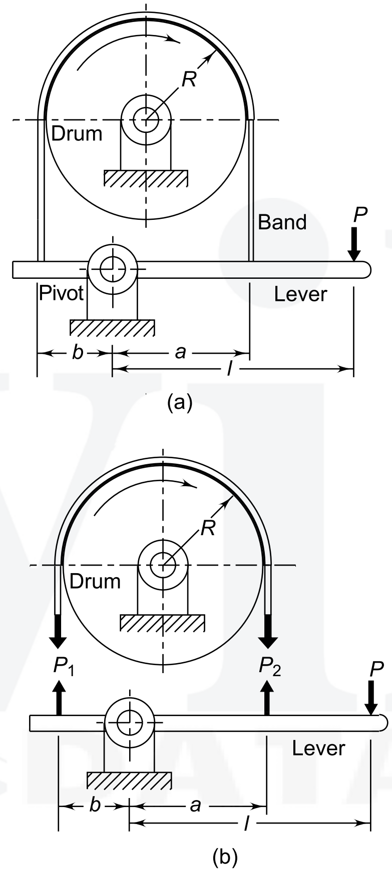


Fig. 12.23 *Differential Band Brake: (a) Construction (b) Free-body Diagram*

and the operator cannot prevent it. In general, for the satisfactory operation of braking, the operator should exert a small but positive force. The brake should be applied as and when the operator requires.

Although the self-locking feature is undesirable in speed control brakes, it is used to advantage in back stop-mechanism. A back-stop brake is a device, which is used to prevent the reverse rotation of the drum when such a rotation would have harmful effects. The back-stop brake is a self-locking differential band brake illustrated in Fig. 12.23(a) which satisfies the condition of self-locking. With this condition, the band will lock up the drum for clockwise rotation. On the other hand, the friction tends to loosen the band and the drum revolves freely for anti-clockwise rotation. Should the rotation reverse and become clockwise, the rotation will stop. The back-stop brakes are popular in hoisting and materials handling equipment. It is particularly used in bucket elevator. It prevents the loaded buckets from reversing their direction in case of power failure or power shut off at the end of a shift.

Band brake offers the following advantages:

(i) Band brake has simple construction. It has small number of parts. These features reduce the cost of band brake.

(ii) Most equipment manufacturers can easily produce band brake without requiring specialized facilities like foundry or forging shop. The friction lining is the only part which must be purchased from outside agencies.

(iii) Band brake is more reliable due to small number of parts.

(iv) Band brake requires little maintenance.

The disadvantages of band brake are as follows:

(i) The heat dissipation capacity of a band brake is poor.

(ii) The wear of friction lining is uneven from one end to the other.

Band brakes are used in applications like bucket conveyors, hoists and chain saws. They are more popular as back-stop devices.

Example 12.10 *A differential band brake is shown in Fig. 12.24(a). The width and the thickness of the steel band are 100 mm and 3 mm respectively and the maximum tensile stress in the band is 50 N/mm². The coefficient of friction between the friction lining and the brake drum is 0.25. Calculate:*

(i) the tensions in the band;

(ii) the actuating force; and

(iii) the torque capacity of the brake.

Find out whether the brake is self-locking.

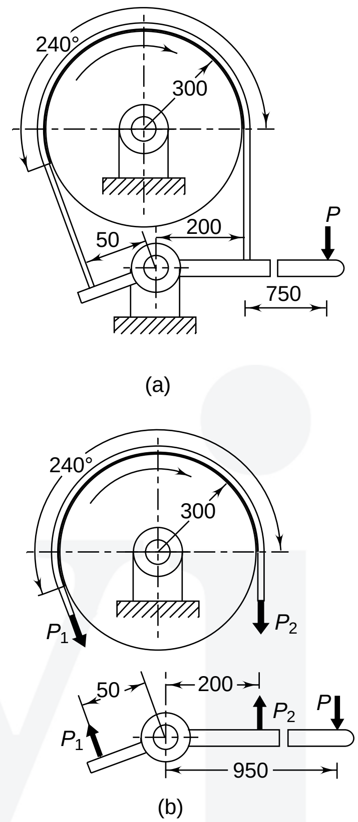


Fig. 12.24

Solution

Given $w = 100$ mm $t = 3$ mm $\mu = 0.25$
$\sigma_t = 50$ N/mm² $R = 300$ mm $= 0.3$ m

Step I *Tensions in band*

The maximum tension in the band is P_1.

$$P_1 = \sigma_t wt = 50(100)3 = 15\,000 \text{ N}$$

$$\frac{P_1}{P_2} = e^{\mu\theta} = e^{\left\{\frac{(0.25 \times 240)\pi}{180}\right\}} = 2.85$$

$$\therefore \qquad P_2 = \frac{P_1}{2.85} = \frac{15\,000}{2.85} = 5263 \text{ N} \qquad \text{(i)}$$

Step II *Actuating force*

The free-body diagram of forces acting on the band and the actuating lever is shown in Fig. 12.24(b). Taking moment of forces about the fulcrum,

$$P_1(50) + P(950) - P_2(200) = 0$$
$$(15\,000)(50) + P(950) - (5263)(200) = 0$$
or $$P = 318.5 \text{ N} \qquad \text{(ii)}$$

Step III *Torque capacity of brake*

$$M_t = (P_1 - P_2)R = (15\,000 - 5263)(0.3)$$
$$= 2921.1 \text{ N-m} \qquad \text{(iii)}$$

Step IV *Self-locking property*

Since $\left(\dfrac{a}{b}\right) = \dfrac{200}{50} = 4$ and $e^{\mu\theta} = 2.85$

$$\therefore \qquad \left(\frac{a}{b}\right) > e^{\mu\theta}$$

The brake is not self-locking.

Example 12.11 *A band brake working on the back-stop principle is shown in Fig. 12.25(a). The width of friction lining perpendicular to the axis of the drum is 75 mm. In normal operating condition, the drum rotates in clockwise direction. Calculate:*

(i) the minimum value of coefficient of friction between the lining and the drum so that the brake acts as a back-stop brake.

(ii) If the normal pressure between the drum and the lining is to be limited to 0.3 N/mm², what could be the maximum braking torque?

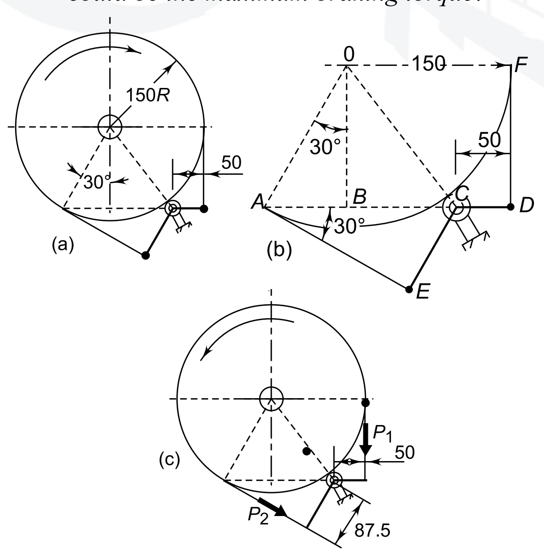


Fig. 12.25

Solution

Given $w = 75$ mm $\quad p_{max.} = 0.3$ N/mm^2
$R = 150$ mm

Step I *Minimum coefficient of friction for back-stop brake*

Refer to Fig. 12.25(b). Construct a line OB perpendicular to AC.

$$\therefore \qquad OB \perp AC \quad \text{and} \quad AE \perp OA$$

It is a pair of two perpendicular lines.

Since $\angle AOB = 30°$ $\quad \therefore \angle EAC = 30°$

$AB = OA \sin(30°) = 150 \sin(30°) = 75$ mm

$DA = DB + BA = FO + AB$
$= 150 + 75 = 225$ mm

$AC = AD - CD = 225 - 50 = 175$ mm

$EC = AC \sin(30°) = 175 \sin(30°) = 87.5$ mm

The direction of rotation for back-stop action is anti-clockwise as shown in Fig. 12.25(c). In order to prevent rotation in anti-clockwise direction,

$$P_2 \times 87.5 < P_1 \times 50$$

or $$\frac{P_1}{P_2} > \frac{87.5}{50} \qquad \therefore \frac{P_1}{P_2} > 1.75$$

$$\frac{P_1}{P_2} = e^{\mu\theta} = e^{\left\{\frac{(\mu \times 240)\pi}{180}\right\}} > 1.75$$

$$\therefore \qquad \frac{(\mu \times 240)\pi}{180} > \log_e(1.75)$$

$$\therefore \qquad \mu > 0.1336 \qquad \text{(i)}$$

Step II *Maximum braking torque*

From Eq. (12.27),

$$P_1 = Rw\,p_{max.} = 150(75)(0.3) = 3375 \text{ N}$$

and $$\frac{P_1}{P_2} = 1.75$$

$$\therefore \qquad P_2 = 1928.57 \text{ N}$$

$\therefore$ braking torque $= (3375 - 1928.57) \times 150$
$= 216\,964.5$ N-mm

$$\therefore \qquad = 216.96 \text{ N-m} \qquad \text{(ii)}$$

12.8 DISK BRAKES

A disk brake is similar to a plate clutch, except that one of the shafts is replaced by a fixed member.

Disk brakes can be observed on the front wheel of most motorcycles. A bicycle brake is another example of a disk brake. In this case, the wheel rim constitutes the disk. The friction lining on the caliper contacts only a small portion of the rim, leaving the remaining portion to dissipate the heat to the surrounding. Figure 12.26 shows the principle of disk brake. There are two pads, on either side of the disk, in the form of annular

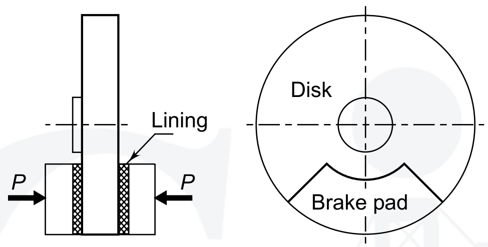


Fig. 12.26 *Principle of Disk Brake*

sector. The friction lining is attached to each pad. A caliper attached to non-rotating member exerts a force P on each pad. When the pads are pressed against the rotating disk, the friction force between the surfaces of friction lining and the disk retards the speed and finally stops the disk. It is seen that the brake pad occupies only a small portion of the disk, where the heat is generated due to friction. On the contrary, the complete surface area of the disk is available for dissipation of heat. Since a caliper is used to exert force on the pads, this type of brake is called '*caliper*' disk brake. Since the caliper contains two pads, the axial forces on two sides of the disk balance each other and leave no thrust load on the bearings. Disk brakes have the following advantages:

(i) Disk brake is simple to install and service.

(ii) Disk brake has high torque transmitting capacity in small volume.

(iii) In drum brakes, as the temperature increases, the coefficient of friction decreases. Due to inherent ability of the disk brake to dissipate heat, it is insensitive to changes in the coefficient of friction. There is no mechanical 'fade' of friction lining. Fading is the loss of torque-transmitting capacity of friction lining at higher temperatures.

(iv) The disk brake is easy to control.

(v) The brake can never become self-locking.

(vi) The brake is equally effective for both directions of rotation of the disk.

(vii) The disk brake has 'linearity', that is, the braking torque is linearly proportional to the actuating force.

Caliper disk brakes are used in many applications like front wheel of motorcycle, lift trucks, farm machinery and light mobile equipment.

The equations derived for single-plate clutch are also applicable to disk brake. However, disk brakes are never made with friction lining covering the entire circumference of the plate, because it would result in overheating. The braking pad presses only a small fraction of the circumference.

There are two types of shapes for the pads of caliper disk brakes, namely, annular and circular. The disk brake with an annular pad is illustrated in Fig. 12.27. The dimensions of the annular pad are as follows:

R_o = outer radius of pad (mm)

R_i = inner radius of pad (mm)

θ = angular dimension of pad (radians)

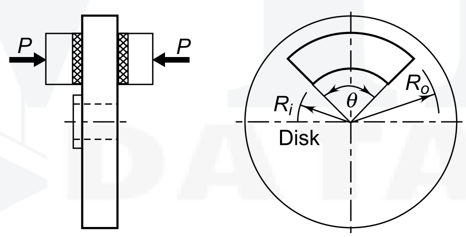


Fig. 12.27 *Disk Brake with Annular Pad*

Since the area of the pad is comparatively small, it is assumed that pressure on friction lining is uniform. The uniform pressure theory is discussed in Sectuon 11.2 of Chapter 11 on 'Friction clutches'. The friction radius (R_f) for uniform pressure theory is given by,

$$R_f = \frac{1}{3}\frac{(D^3 - d^3)}{(D^2 - d^2)}$$

Substituting,

($D = 2R_o$) and ($d = 2R_i$) in the above expression,

$$R_f = \frac{2}{3}\frac{(R_o^3 - R_i^3)}{(R_o^2 - R_i^2)} \tag{12.29}$$

The torque capacity of the disk brake is given by,

$$M_t = \mu P R_f \tag{12.30}$$

where,

M_t = torque capacity of pad (N-mm)
μ = coefficient of friction
P = actuating force (N)
R_f = friction radius (mm)

The area of the pad is given by,

$$A = \left(\frac{\theta}{2\pi}\right)\pi(R_o^2 - R_i^2)$$

or

$$A = \frac{1}{2}\theta(R_o^2 - R_i^2) \tag{12.31}$$

where (θ) is in radians.

The actuating force P is given by,

$$P = \text{average pressure} \times \text{area of pad}$$

or

$$P = p_{\text{ave.}} A \tag{12.32}$$

where A is the area of the pad (mm^2)

The disk brake with a circular pad is illustrated in Fig. 12.28. The dimensions of a circular pad are as follows:

R = radius of pad (mm)
e = distance of pad centre from the axis of disk (mm)

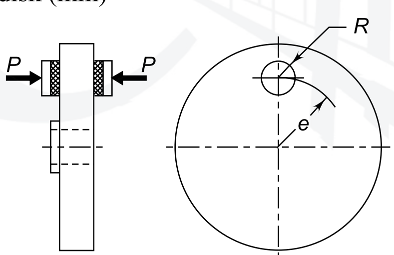


Fig. 12.28 *Disk Brake with Circular Pad*

The friction radius (R_f) of circular pad is given by,

$$R_f = \delta e \tag{12.33}$$

The values of (δ) are given in Table 12.1.

Table 12.1 *Values of δ for circular-pad caliper disk brakes*

R/e	δ
0.0	1.0000
0.1	0.9833
0.2	0.9693
0.3	0.9572
0.4	0.9467
0.5	0.9375

Example 12.12 *Following data is given for a caliper disk brake with annular pad, for the front wheel of the motorcycle:*

torque capacity = 1500 N-m
outer radius of pad = 150 mm
inner radius of pad = 100 mm
coefficient of friction = 0.35
average pressure on pad = 2 MPa
number of pads = 2

Calculate the angular dimension of the pad.

Solution

Given $M_t = 1500$ N-m $R_o = 150$ mm $R_i = 100$ mm $p_a = 2$ MPa $\mu = 0.35$
Number of pads = 2

Step I *Actuating force*

Since there are two pads, the torque capacity of one pad is (1500/2) or 750 N-m.

From Eq. (12.29),

$$R_f = \frac{2}{3}\frac{(R_o^3 - R_i^3)}{(R_o^2 - R_i^2)} = \frac{2}{3}\frac{(150^3 - 100^3)}{(150^2 - 100^2)} = 126.67 \text{ mm}$$

From Eq. (12.30),

$$M_t = \mu P R_f$$

$$750(10)^3 = 0.35P(126.67) \quad \text{or} \quad P = 16\,916.85 \text{ N}$$

Step II *Angular dimension of pad*

P = average pressure × area of pad
[2 MPa = 2 N/mm^2]

$$16\,916.85 = 2A \quad \text{or} \quad A = 8458.42 \text{ mm}^2$$

From Eq. (12.31),

$$A = \frac{1}{2}\theta(R_o^2 - R_i^2)$$

or

$$8458.42 = \frac{1}{2}\theta(150^2 - 100^2)$$

$$\theta = 1.3533 \text{ radians}$$

or

$$\theta = 1.3533\left(\frac{180}{\pi}\right) = 77.54°$$

The angular dimension of the pad can be taken as 80°.

Example 12.13 *The following data is given for a caliper disk brake, with circular pad, for the lightweight two-wheeler,*

torque capacity = 1500 N-m
number of caliper brakes on the wheel = 3
number of pads on each caliper brake = 2
coefficient of friction = 0.35
average pressure on pad = 2 MPa

The ratio of pad radius to the distance of the pad center from axis of disk is 0.2. Calculate the radius of the pad.

Solution

Given $M_t = 1500$ N-m $p_a = 2$ MPa $\mu = 0.35$
$(R/e) = 0.2$ Number of brakes = 3
Number of pads on each brake = 2

Step I *Radius of pad*

Since there are three caliper brakes, each with two pads, the torque capacity of one pad is (1500/6) or 250 N-m.

$$\left(\frac{R}{e}\right) = 0.2 \quad \text{from Table 12.1A, } \delta = 0.9693$$

From Eq. (12.33),

$$R_f = \delta e = 0.9693e = 0.9693\left(\frac{R}{0.2}\right)$$
$$= (4.8465)R \text{ mm}$$

P = average pressure × area of pad = $2(\pi R^2)$
[2 MPa = 2 N/mm^2]

From Eq. (12.30),

$$M_t = \mu P R_f$$
$$250(10^3) = 0.35(2\pi R^2)(4.8465\, R)$$
$$R = 28.63 \text{ mm}$$

The radius of the pad can be taken as 30 mm.

12.9 THERMAL CONSIDERATIONS

The energy absorbed by the brake is converted into heat, which increases the temperature at the rubbing surfaces. When the temperature increases, the coefficient of friction decreases, adversely affecting the torque capacity of the brake. At high temperature, there is rapid wear of the friction lining, which reduces the life of the lining. Therefore, the temperature rise should be kept within permissible range. The permissible temperature for different friction materials is given in Table 12.2.

Table 12.2 *Properties of friction materials for brakes*

Material	*Coefficient of friction*	*Permissible temperature (°C)*	*Intensity of pressure (N/mm^2)*
Cast iron on cast iron	0.15–0.20	300	1.00
Wood on cast iron	0.25–0.30	60	0.35
Leather on cast iron	0.30–0.50	60	0.25
Woven-asbestos on metal	0.35–0.40	250	0.65
Moulded-asbestos on metal	0.40–0.45	250	1.00
Sintered metal on metal	0.20–0.40	300	2.75

It is very difficult to precisely calculate temperature rise. In preliminary design analysis, very often the product (pv) is considered in place of temperature rise. When the coefficient of friction is constant, the rate of heat generated is proportional to the product (pv) where p is the intensity of normal pressure (N/mm^2) and v is the rubbing speed (m/min).

The recommended values of product (pv) are given in Table 12.3.

Table 12.3

Application	*(pv)*
(i) Intermittent applications, comparatively long period of rest and poor dissipation of heat	115
(ii) Continuous application and poor dissipation of heat	58
(iii) Continuous application and good dissipation of heat	175
(iv) Vehicle-brakes	125

The above values are based on past experience.

The temperature rise depends upon the mass of the brake drum assembly, the ratio of the braking period to the rest period and the specific heat of the material. For peak short-time requirements, it is assumed that all the heat generated during the braking period is absorbed by the brake drum assembly. In that case, the temperature rise is given by

$$\Delta_t = \frac{E}{mc} \tag{12.34}$$

where

Δ_t = temperature rise of the brake drum assembly (°C)
E = total energy absorbed by the brake (J)
m = mass of the brake drum assembly (kg)
c = specific heat of the brake drum material (J/kg°C)

The actual temperature rise will be less than that calculated from Eq. (12.34). Some heat will be radiated to the atmosphere and some carried away by the air flow. The equation gives approximate value and the actual temperature rise is obtained by experiments.

Example 12.14 *A flywheel of 100 kg mass and 350 mm radius of gyration is rotating at 500 rpm. It is brought to rest by means of a brake. The mass of the brake drum assembly is 5 kg. The brake drum is made of cast-iron FG260(c = 460 J/kg°C). Assuming that the total heat generated is absorbed by the brake drum only, calculate the temperature rise.*

Solution

Given For brake assembly, $m = 5$ kg
$c = 460$ J/kg°C For flywheel, $m = 100$ kg
$k = 350$ mm $n = 500$ rpm

Step I *KE of flywheel*

$$\omega_1 = \frac{2\pi n_1}{60} = \frac{2\pi(500)}{60} = 52.36 \text{ rad/s} \quad \text{and} \quad w_2 = 0$$

$$\text{KE} = \frac{1}{2} mk^2(\omega_1^2 - \omega_2^2) = \frac{1}{2}(100)(0.35)^2(52.36)^2$$
$$= 167\,921 \text{ J}$$

Step II *Temperature rise*

$$\therefore \quad \Delta_t = \frac{E}{mc} = \frac{167\,921}{5(460)} = 73°\text{C}$$

Short-Answer Questions

12.1 What is the function of brake?
12.2 State different types of brakes and give at least one practical application of each.
12.3 What is block brake with short shoe? Where do you use it?
12.4 What is the disadvantage of block brake with one short shoe? What is the remedy?
12.5 What is self-energizing block brake?
12.6 What is self-locking block brake?
12.7 What is the condition for self-locking block brake?
12.8 What is partially self-energizing block brake?
12.9 What is the advantage of pivoted shoe brake over fixed shoe brake?
12.10 What is internal expanding shoe brake? Where do you use it?
12.11 What are the advantages of internal expanding shoe brake?
12.12 What are the disadvantages of internal expanding shoe brake?
12.13 What is differential band brake?
12.14 What is the condition of self-locking in differential band brake? Why should it be avoided in speed-control brakes?
12.15 In which applications are self-locking differential band brakes used?
12.16 What are the advantages of band brake?
12.17 What are the disadvantages of band brake?
12.18 What are the applications of band brakes?
12.19 What is 'back-stop' band brake?
12.20 What are the advantages of disk brakes over drum brakes?

Problems for Practice

12.1 An automobile vehicle weighing 13.5 kN is moving on a level road at a speed of 95 km/h. When the brakes are applied, it is subjected to a uniform deceleration of 6 m/s^2. There are brakes on all four wheels. The tyre diameter is 750 mm. The kinetic energy of the rotating parts is 10% of the kinetic energy of the moving vehicle. The mass of each brake drum assembly is 10 kg and the specific heat capacity is 460 J/kg°C. Calculate

(i) the braking time;
(ii) the braking distance;
(iii) the total energy absorbed by each brake;
(iv) the torque capacity of each brake; and
(v) the temperature rise of brake drum assembly.

[*(i) 4.4 s (ii) 58.06 m (iii) 131 779 J (iv) 851.18 N-m (v) 28.65°C*]

12.2 A double block brake consists of two symmetrical pivoted shoes. The diameter of the brake drum is 300 mm and the angle of wrap (2θ) for each shoe is 90°. The pivot of the shoe is located in such a way as to avoid the couple due to frictional force. Determine the distance of pivot from the axis of the brake drum.

[*165 mm*]

12.3 A single block brake with a torque capacity of 15 N-m is shown in Fig. 12.29. The coefficient of friction is 0.3 and the

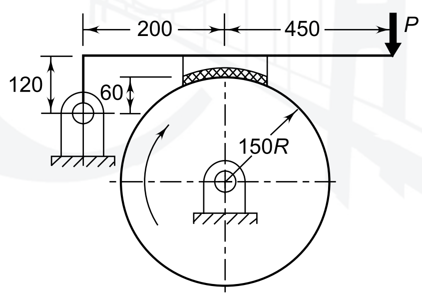


Fig. 12.29

maximum pressure on the brake lining is 1 N/mm^2. The width of the block is equal to its length. Calculate

(i) the actuating force;
(ii) the dimensions of the block;
(iii) the resultant hinge-pin reaction; and
(iv) the rate of heat generated, if the brake drum rotates at 50 rpm.

[*(i) 93.33 N (ii) 18.26 × 18.26 mm (iii) 260 N (iv) 39.27 W*]

12.4 An automotive-type internal-expanding brake is shown in Fig. 12.30. The face width of the friction lining is 50 mm and the coefficient of friction is 0.4. The maximum intensity of pressure on the lining is 0.8 N/mm^2. The angle θ_1 can be assumed to be zero. Calculate:

(i) the actuating force;
(ii) the torque capacity of the brake.

[*(i) 2141.52 N (ii) 530.78 N-m*]

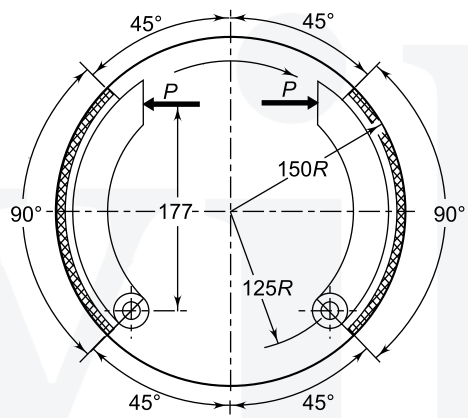


Fig. 12.30

12.5 Refer to the simple band-brake shown in Fig. 12.20 and assume the following data:

$a = 250$ mm $l = 750$ mm $\theta = 225°$

$R = 250$ mm

The width of the friction lining is 60 mm and the coefficient of friction is 0.4. The maximum intensity of pressure is 0.25 N/mm^2. Calculate

(i) the band tension on tight and loose sides;
(ii) the actuating force; and
(iii) the torque capacity of the brake.

[*(i) 3750 and 779.63 N (ii) 259.88 N (iii) 742.59 N-m*]

Belt Drives

Chapter 13

13.1 BELT DRIVES

Belt, chain and rope drives are called '*flexible*' drives. There are two types of drives—rigid and flexible. Gear drives are called rigid or non-flexible drives. In gear drives, there is direct contact between the driving and driven shafts through the gears. In flexible drives, there is an intermediate link such as belt, rope or chain between the driving and driven shafts. Since this link is flexible, the drives are called '*flexible*' drives. In gear drives, rotary motion of the driving shaft is directly converted into rotary motion of the driven shaft by means of pinion and gear. In flexible drives, the rotary motion of the driving shaft is first converted into translatory motion of the belt or chain and then again converted into rotary motion of the driven shaft. Thus, a flexible element is superimposed between the driving and driven elements. The advantages of flexible drives over rigid drives are as follows:

(i) Flexible drives transmit power over a comparatively long distance due to an intermediate link between driving and driven shafts.

(ii) Since the intermediate link is long and flexible, it absorbs shock loads and damps vibrations.

(iii) Flexible drives provide considerable flexibility in the location of the driving and driven shafts. The tolerances on the centre distance are not critical as compared with a gear drive.

(iv) Flexible drives are cheap compared to gear drives. Their initial and maintenance costs are low.

The disadvantages of flexible drives are as follows:

(i) They occupy more space.

(ii) The velocity ratio is relatively small.

(iii) In general, the velocity ratio is not constant.

Belts are used to transmit power between two shafts by means of friction. A belt drive consists of three elements—driving and driven pulleys and an endless belt, which envelopes them. Belt drives offer the following advantages compared with other types of drives:

(i) Belt drives can transmit power over considerable distance between the axes of driving and driven shafts.

(ii) The operation of belt drive is smooth and silent.

(iii) They can transmit only a definite load, which if exceeded, will cause the belt to slip over the pulley, thus protecting the parts of the drive against overload.

(iv) They have the ability to absorb the shocks and damp vibration.

(v) They are simple to design.

(vi) They have low initial cost.

The disadvantages of belt drives compared to other types of drives are as follows:

(i) Belt drives have large dimensions and occupy more space.
(ii) The velocity ratio is not constant due to belt slip.
(iii) They impose heavy loads on shafts and bearings.
(iv) There is considerable loss of power resulting in low efficiency.
(v) Belt drives have comparatively short service life.

Belt drives are mainly used in electric motors, automobiles, machine tools and conveyors.

Depending upon the shape of the cross-section, belts are classified as flat belts and V-belts. *Flat belts* have a narrow rectangular cross-section, while *V-belts* have a trapezoidal cross-section. Flat belts offer the following advantages over V-belts:

(i) They are relatively cheap and easy to maintain. Their maintenance consists of periodic adjustment in the centre distance between two shafts in order to compensate for stretching and wear. They do not require precise alignment of shafts and pulleys. When worn out, they are easy to replace.
(ii) A flat belt drive can be used as a clutch by making a simple provision of shifting the belt from tight to loose pulley and vice versa.
(iii) Different velocity ratios can be obtained by using a stepped pulley, where the belt is shifted from one step to another, having different diameters.
(iv) They can be used in dusty and abrasive atmosphere and require no closed casing.
(v) The design of flat-belt drive is simple and inexpensive.
(vi) They can be used for long centre distances, even up to 15 m.
(vii) The efficiency of flat belt drive is more than V-belt drive.

The major disadvantage of flat belt drives over V-belt drives are as follows:

(i) The power transmitting capacity of flat-belt drive is low.
(ii) The velocity ratio of flat belt-drive is lower than V-belt drive.
(iii) Flat-belt drives have large dimensions and occupy more space compared to V-belt drives.
(iv) Flat belts generate more noise than V-belts.
(v) In general, flat-belt drives are horizontal and not vertical.

It is due to these reasons that flat belts are becoming less popular on the shop floor. Flat belts are used in belt conveyors, baking machinery, brick and clay machinery, crushers, saw mills, textile machinery, line shafts and bucket elevators.

Compared with flat belts, V-belts offer the following advantages:

(i) The force of friction between the surfaces of the belt and V-grooved pulley is high due to wedge action. This wedging action permits a smaller arc of contact, increases the pulling capacity of the belt and consequently results in increase in the power transmitting capacity.
(ii) V-belts have short centre distance, which results in compact construction.
(iii) They permit high speed-reduction even up to seven to one.
(iv) Flat belts are hinged, while V-belts are endless which results in smooth and quiet operation, even at high operating speeds.
(v) The drive is positive because the slip is negligible due to wedge action.
(vi) V-belt drive can operate in any position, even when the belt is vertical.

The disadvantages of a V-belt drive over a flat-belt drive are as follows:

(i) The ratio of the cross-sectional height to the pulley diameter is large in case of the V-belt. This increases bending stresses in the belt cross-section and adversely affects the durability.
(ii) The efficiency of the V-belt is lower than that of the flat-belt and the creep is also higher.
(iii) The construction of V-grooved pulleys is complicated and costlier compared with pulleys of the flat-belt drive.

V-belts are very popular where an electric motor is used as the prime mover to drive compressors, pumps, fans, positive displacement pumps, blowers and machine tools. They are also popular in automobiles to drive accessories on petrol or diesel engines.

The velocity ratio for flat belt is up to 4:1. For V-belts the velocity ratio is up to 7:1. For chain drives it can be up to 15:1.

Flat and V-belts are widely used. However there are certain applications, where '*round*' belts are used. The round belt is illustrated in Fig. 13.1. Round belts are made of leather, canvas or rubber. The diameter of round belts is usually from 3 mm to 12 mm. The minimum diameter of the smaller pulley of round belt is 30 times the diameter of the belt. There are

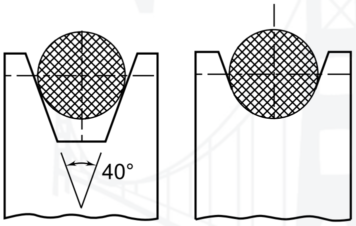


Fig. 13.1 *Round Belt*

two types of grooves for pulley—trapezoidal with an angle of 40° between the sides and half round with a radius equal to that of the belt. The advantages of round belts are as follows:

(i) Round belts can operate satisfactorily over pulleys in several different planes. They are suitable for 90° twist, reverse bends or serpentine drives.

(ii) They can be stretched over the pulley and snapped into the groove very easily. This makes the assembly and replacement simple.

Round belts are limited to light duties. They are used in dishwasher drives, sewing machines, vacuum cleaners and light textile machinery.

13.2 BELT CONSTRUCTIONS

The desirable properties of belt materials are as follows:

(i) The belt material should have high coefficient of friction with the pulleys.

(ii) The belt material should have high tensile strength to withstand belt tensions.

(iii) The belt material should have high wear resistance.

(iv) The belt material should have high flexibility and low rigidity in bending in order to avoid bending stresses while passing over the pulley.

Belts are made of leather, canvas, rubber or rubberized fabric and synthetic materials.

There are two types of flat belts—*leather belt* and *fabric rubber belt*. The leather belt is made of the best quality leather obtained from either sides of the backbone of a steer. There are two varieties of leather—oak-tanned and mineral or chrome-tanned. The main advantage of leather belt is the high coefficient of friction and consequently, high power transmitting capacity.

There is a specific term '*ply*' of the belt. In order to make a practical thick belt, the layers of belt material are cemented together as shown in Fig. 13.2(a). These layers are called '*plies*' of belt. Belts are specified according to the number of layers or plies, e.g., single-ply, double-ply or triple-ply belts. The power rating of the belt is also specified per ply of belt, e.g., the power rating of the Dunlop 'high speed' belt is 0.0118 kW per ply per mm width of belt.

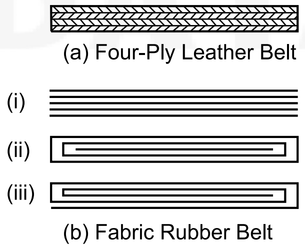


Fig. 13.2 *Flat Belts*

The fabric rubber belts are made from several layers of canvas or cotton-duck impregnated with rubber as shown in Fig.13.2(b). The fabric transmits major portion of the load. The rubber protects the fabric against damage and increases the coefficient of friction. It is due to the presence of rubber that different plies of fabric work together as one belt.

Fabric rubber belts are widely used in engineering industries. They offer the following advantages:

(i) They have high load carrying capacity.
(ii) They have long service life.
(iii) They can operate at high operating speeds of up to 300 m/s.

The disadvantages of fabric rubber belts are as follows:

(i) They cannot operate on small diameter pulleys.
(ii) They are subjected to destruction in an environment of mineral oil, gasoline and alkalis.

Three types of construction of fabric rubber belts are shown in Fig.13.2(b). Type (i) is called a *raw-edge belt*. It consists of plies of fabric cut to the width of the belt. There are layers of rubber between the plies of fabric. The edges of fabric are protected by waterproof compound. The raw-edge belts are flexible. Therefore, they are widely used for high-speed applications and with small diameter pulleys. Type (ii) is called *layer by layer folded edge belt*. It consists of a central ply wrapped around with rectangular plies. Such belts may or may not have rubber layer between the plies. Type (iii) is called *spirally wrapped folded edge belts*. They are made of a single piece of fabric without any rubber layer in between. Compared with raw-edge belts, layer by layer and spirally wrapped folded edge belts have edges of semicircular shape which is advantageous for operation on pulleys having flanges, for cross-belt drives and where belts are to be shifted.

Flat belts are produced in the form of long bands and stored in the form of coils. The ends of these belts are joined by methods as illustrated in Fig. 13.3. There are three methods—cementing, lacing and by using metal fasteners.

(i) Cemented Joints Leather belts are cemented with a tapered lap joint of length of 20 to 25 times the belt thickness. Multiple ply belts are cemented in the form of stepped joint. Cementing is widely used for rubber and leather belts. The strength of cemented joint is 80 to 85% of the strength of the belt.

(ii) Laced Joint Lacing is done with catgut or with rawhide strips. The strength of a laced joint is 50% of the strength of the belt.

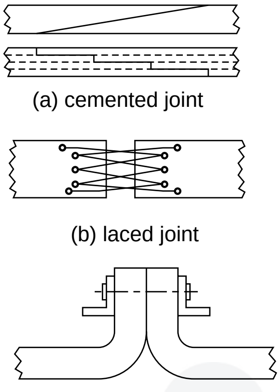


Fig. 13.3 *Flat Belt Joints*

(iii) Joints with Metal Fasteners Metal fasteners require least time to make the joint. However, they are not suitable for high-speed operations. The strength of screwed or hinged joint is 25% of the strength of belt.

V-belts are made of polyester fabric and cords, with rubber reinforcement. The cross-section of the V-belt is shown in Fig. 13.4. It consists of the following three parts:

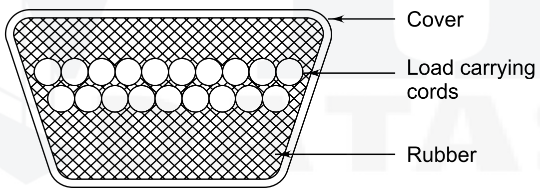


Fig. 13.4 *Cross-section of V-belt*

(i) the central load carrying layers of polyester cords or polyester fabric, which are located on horizontal lines near the centre of gravity of the belt cross-section,
(ii) the surrounding layer of rubber to transmit force from cords to side walls, and
(iii) outer polychloroprene impregnated elastic cover.

The polyester cords or fabric transmit force from the driving pulley to the driven pulley, which in turn transmit power. Since they are located near the neutral axis of the cross-section of the belt, the stresses due to bending of the belt around the pulley are almost negligible. The layer of rubber located above the load carrying cords is subjected to tension and called *tension layer*. Similarly, the layer of rubber below the central cords is subjected to compression and called *compression layer*.

13.3 GEOMETRICAL RELATIONSHIPS

There are two types of belt construction—open and crossed as illustrated in Fig. 13.5 and 13.6 respectively. The difference between these two constructions is as follows:

(i) An open belt drive is a belt drive in which the belt proceeds from the top of one pulley to the top of another without crossing. A crossed belt drive is a belt drive in which the belt proceeds from the top of one pulley to the bottom of another and crosses over itself. In both cases, the driving and driven shafts are parallel.

(ii) In an open belt drive, both driving and driven pulleys rotate in the same direction. In a crossed belt drive, driving and driven pulleys rotate in the opposite direction.

(iii) In crossed belt drive, the angle of wrap is more. Therefore, power transmitting capacity of a crossed belt drive is more than that of an open belt drive.

(iv) In crossed belt drive, the belt rubs against itself while crossing. Also, the belt has to bend in two different planes. These two factors increase the wear and reduce the life of the belt.

(v) In open belt drives, when the centre distance is more, the belt whips, i.e., vibrates in a direction perpendicular to the direction of motion. When the centre distance is small, the belt slip increases. Both these factors limit the use of an open belt drive. Crossed belt drives do not have these limitations.

(vi) Open belt drives are more popular than crossed belt drives.

An open belt drive is shown in Fig. 13.5. The dimensions of the open belt drive are as follows:

α_s = wrap angle for small pulley (degrees)
α_b = wrap angle for big pulley (degrees)
D = diameter of big pulley (mm)
d = diameter of small pulley (mm)
C = centre distance (mm)

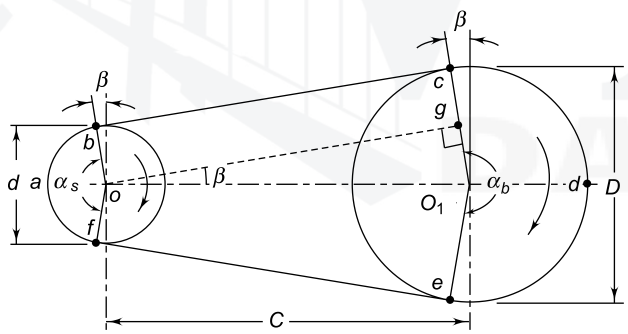


Fig. 13.5 *Open belt drive*

Construction Draw a line $\overline{og}$ perpendicular to the line $\overline{o_1c}$. The area *ogcb* is a rectangle.

$$\therefore \qquad ob = gc$$

From triangle ogo_1,

$$\sin\beta = \frac{o_1g}{oo_1} = \frac{o_1c - gc}{oo_1} = \frac{o_1c - ob}{oo_1}$$

$$= \frac{D/2 - d/2}{C} = \frac{D-d}{2C}$$

$$\therefore \qquad \sin\beta = \frac{D-d}{2C}$$

also $\alpha_s = (180 - 2\beta)$ and $\alpha_b = (180 + 2\beta)$

$$\text{Therefore, } \alpha_s = 180 - 2\sin^{-1}\left(\frac{D-d}{2C}\right) \qquad (13.1)$$

$$\alpha_b = 180 + 2\sin^{-1}\left(\frac{D-d}{2C}\right) \qquad (13.2)$$

Refer to Fig.13.5 again. The length of the belt (L) is given by,

$$L = \text{arc }(fab) + \overline{bc} + \text{arc }(cde) + \overline{ef}$$

$$= \frac{d}{2}(\alpha_s) + \overline{og} + \frac{D}{2}(\alpha_b) + \overline{og}$$

$$= \frac{d}{2}(\pi - 2\beta) + C\cos\beta + \frac{D}{2}(\pi + 2\beta) + C\cos\beta$$

or
$$L = \frac{\pi(D+d)}{2} + \beta(D-d) + 2C\cos\beta \qquad \text{(a)}$$

For small values of β,

$$\beta = \sin\beta = \left(\frac{D-d}{2C}\right)$$

and
$$\cos\beta = 1 - 2\sin^2\left(\frac{\beta}{2}\right) = 1 - \frac{\beta^2}{2}$$

$$= 1 - \frac{(D-d)^2}{8C^2}$$

Substituting these values of β and $\cos\beta$ in Eq. (a), we get,

$$L = \frac{\pi(D+d)}{2} + \left(\frac{D-d}{2C}\right)(D-d) + 2C\left[1 - \frac{(D-d)^2}{8C^2}\right]$$

$$= \frac{\pi(D+d)}{2} + \frac{(D-d)^2}{2C} + 2C - \frac{(D-d)^2}{4C}$$

$$\therefore \quad L = 2C + \frac{\pi(D+d)}{2} + \frac{(D-d)^2}{4C} \qquad (13.3)$$

A crossed belt drive is shown in Fig. 13.6.

Construction Draw a line $\overline{og}$ perpendicular to the line $\overline{o_1c}$ The area $ofcg$ is a rectangle.

$$\therefore \quad cg = of$$

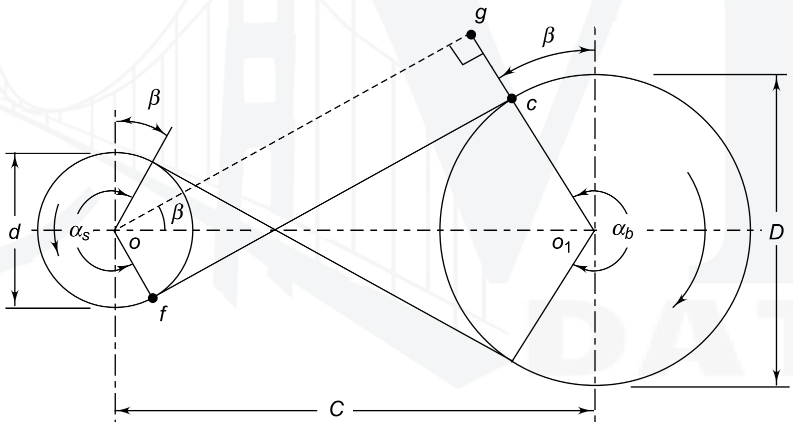


Fig. 13.6 *Crossed Belt Drive*

From triangle ogo_1,

$$\sin\beta = \frac{o_1g}{oo_1} = \frac{o_1c + cg}{oo_1} = \frac{o_1c + of}{oo_1}$$

$$= \frac{D/2 + d/2}{C} = \frac{D+d}{2C}$$

$$\therefore \quad \sin\beta = \frac{D+d}{2C}$$

$$\alpha_s = \alpha_b = (180° + 2\beta)$$

$$\alpha_s = \alpha_b = 180 + 2\sin^{-1}\left(\frac{D+d}{2C}\right) \qquad (13.4)$$

The procedure used for open belt drive can be used for crossed belt drive and it can be proved that the belt length L for a crossed belt drive is given by,

$$L = 2C + \frac{\pi(D+d)}{2} + \frac{(D+d)^2}{4C} \qquad (13.5)$$

It should be noted that in the above expressions, D and d are pitch diameters of pulleys while L is the pitch length of the belt.

13.4 ANALYSIS OF BELT TENSIONS

The forces acting on the element of a flat belt are shown in Fig. 13.7. The following notations are used in the derivations:

P_1 = belt tension in the tight side (N)
P_2 = belt tension in the loose side (N)
m = mass of the one meter length of belt (kg/m)
v = belt velocity (m/s)
f = coefficient of friction
α = angle of wrap for belt (radians)

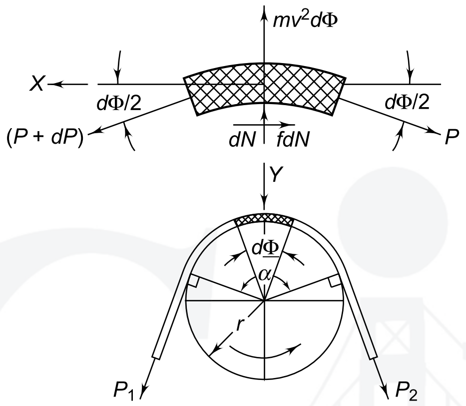


Fig. 13.7 *Forces on Flat Belt*

An element of the belt subtending an angle ($d\phi$) is in equilibrium under the action of the following forces:

(i) tensions (P) and ($P + dP$) on the loose and tight sides respectively;
(ii) the normal reaction between the surfaces of the belt and pulley (dN) and the frictional force ($f \times dN$) at the interface; and
(iii) centrifugal force in radially outward direction.

The expression for centrifugal force is given by,

Centrifugal force = mass × acceleration (i)

The length of element is ($rd\phi$) and the mass per unit length is m. Therefore,

Mass of element = $mrd\phi$ (ii)

In the subject of Theory of Machines, the motion of a particle along the circular path is analysed. If a particle revolves with a linear velocity v at a radius r from the axis of rotation, the normal or centripetal acceleration is given by,

$$f = \left(\frac{v^2}{r}\right) \qquad \text{(iii)}$$

From (i), (ii) and (iii),

$$\text{Centrifugal force} = (mrd\phi)\left(\frac{v^2}{r}\right) = (mv^2 d\phi)$$

Considering equilibrium of forces in X and Y directions,

$$(P + dP)\cos\left(\frac{d\phi}{2}\right) - P\cos\left(\frac{d\phi}{2}\right) - fdN = 0 \qquad \text{(a)}$$

$$(P + dP)\sin\left(\frac{d\phi}{2}\right) + P\sin\left(\frac{d\phi}{2}\right) - mv^2 d\phi - dN = 0 \qquad \text{(b)}$$

For small values of $\left(\frac{d\phi}{2}\right)$,

$\cos\left(\frac{d\phi}{2}\right)$ is approximately 1

$\sin\left(\frac{d\phi}{2}\right)$ is approximately $\left(\frac{d\phi}{2}\right)$

Substituting these values in Eq. (a),

$$(P + dP) - P - f\,dN = 0$$

or

$$dP - f\,dN = 0$$

$$\therefore \qquad dN = \frac{dP}{f} \qquad \text{(c)}$$

Similarly, substituting $\sin\left(\frac{d\phi}{2}\right)$ as $\left(\frac{d\phi}{2}\right)$ in Eq. (b),

$$(P + dP)\left(\frac{d\phi}{2}\right) + P\left(\frac{d\phi}{2}\right) - mv^2 d\phi - dN = 0$$

Neglecting the differential of the second order ($dP \times d\phi$ ($dP \times d\phi$),

$$Pd\phi - mv^2\, d\phi - dN = 0$$

Substituting Eq. (c) in the above expression,

$$(P - mv^2)d\phi - \frac{dP}{f} = 0$$

or

$$\frac{dP}{(P - mv^2)} = fd\phi$$

Integrating the above expression,

$$\int_{P_2}^{P_1} \frac{dP}{(P - mv^2)} = f\int_0^{\alpha} d\phi$$

$$[\log_e (P - mv^2)]_{P_2}^{P_1} = f[\phi]_0^{\alpha}$$

$$\log_e (P_1 - mv^2) - \log_e (P_2 - mv^2) = f(\alpha - 0)$$

$$\log_e \left[\frac{P_1 - mv^2}{P_2 - mv^2}\right] = f\alpha$$

Therefore, the relationship is given by

$$\frac{P_1 - mv^2}{P_2 - mv^2} = e^{f\alpha} \qquad (13.6)$$

In the above derivation, the following mathematical equations are used:

$$\int \frac{dx}{x} = \log_e x \qquad \int dx = x$$

$$(\log_e a - \log_e b) = \log_e \left(\frac{a}{b}\right)$$

The forces acting on the element of V-belt are shown in Fig. 13.8. The force components P, $(P + dP)$ and $(mv^2 d\phi)$ are the same as those of the flat belt. The difference lies in the normal reaction dN. The normal reaction, which acts on two sides of the V-belt, is assumed as $\left(\frac{1}{2}dN\right)$ on each side. The resultant reaction in the X–Y plane is $\left[dN \times \sin\left(\frac{\theta}{2}\right)\right]$, where θ is the belt angle. The frictional force $2\left(f \times \frac{1}{2}dN\right)$ or (fdN) remains unchanged. Considering the equilibrium of forces in X and Y directions, we have

$$(P + dP)\cos\left(\frac{d\phi}{2}\right) - P\cos\left(\frac{d\phi}{2}\right) - f\,dN = 0 \qquad \text{(d)}$$

$$(P + dP)\sin\left(\frac{d\phi}{2}\right) + P\sin\left(\frac{d\phi}{2}\right)$$

$$-(m\,v^2 d\phi) - dN\sin\left(\frac{\theta}{2}\right) = 0 \qquad \text{(e)}$$

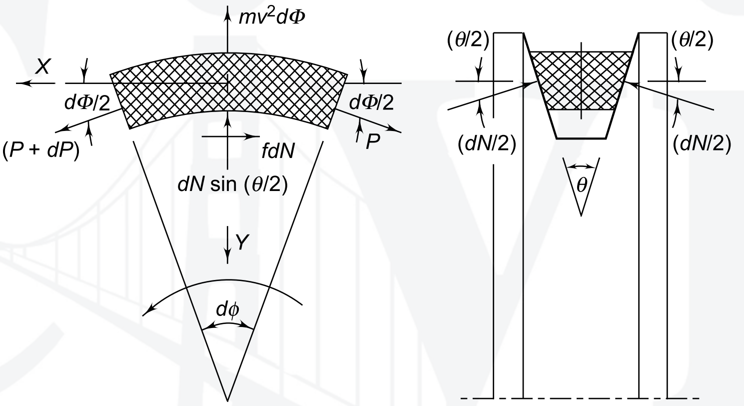


Fig. 13.8 *Forces on V-belt*

For small values of $\left(\frac{d\phi}{2}\right)$,

$\cos\left(\frac{d\phi}{2}\right)$ is approximately 1

$\sin\left(\frac{d\phi}{2}\right)$ is approximately $\left(\frac{d\phi}{2}\right)$

Substituting these values in Eq. (d),

$$(P + dP) - P - f\,dN = 0$$

or

$$dP - fdN = 0$$

$\therefore$

$$dN = \frac{dP}{f} \qquad \text{(f)}$$

Similarly, substituting $\sin\left(\frac{d\phi}{2}\right)$ as $\left(\frac{\theta}{2}\right)$ in Eq. (e),

$$(P + dP)\left(\frac{d\phi}{2}\right) + P\left(\frac{d\phi}{2}\right) - mv^2 d\phi$$

$$-dN\sin\left(\frac{\theta}{2}\right) = 0$$

Neglecting the term of the second order differential $(dP \times d\phi)$,

$$Pd\phi - mv^2 d\phi - dN\sin\left(\frac{\theta}{2}\right) = 0$$

Substituting Eq. (f) in the above expression,

$$(P - mv^2)\,d\phi - \frac{dP}{f}\sin\left(\frac{\theta}{2}\right) = 0$$

or

$$\frac{dP}{(P - mv^2)} = \frac{fd\phi}{\sin\left(\frac{\theta}{2}\right)}$$

Integrating the above expression,

$$\int_{P_2}^{P_1} \frac{dP}{(P - mv^2)} = \frac{f}{\sin\left(\frac{\theta}{2}\right)} \int_0^{\alpha} d\phi$$

$$[\log_e (P - mv^2)]_{P_2}^{P_1} = \frac{f}{\sin\left(\frac{\theta}{2}\right)} \times [\phi]_0^{\alpha}$$

Substituting limits,

$$\log_e(P_1 - mv^2) - \log_e(P_2 - mv^2) = \frac{f}{\sin\left(\frac{\theta}{2}\right)} (\alpha - 0)$$

$$\log_e \left[\frac{P_1 - mv^2}{P_2 - mv^2}\right] = \frac{f\alpha}{\sin\left(\frac{\theta}{2}\right)}$$

or
$$\frac{P_1 - mv^2}{P_2 - mv^2} = e^{f\alpha/\sin(\theta/2)} \qquad (13.7)$$

The superiority of V-belt over flat belt can be explained by comparing Eqs (13.6) and (13.7). The following observations are made:

(i) The equations of flat and V-belt are identical except that the coefficient of friction f in flat belt drive is replaced by $f/\sin(\theta/2)$ in case of V-belt. In other words, the effective coefficient of friction in V-belt is $[f/\sin(\theta/2)]$ as compared to $[f]$ of flat belt.

(ii) For a V-belt, $\theta = 40°$ or $[f/\sin(\theta/2)] = 2.92 f$
Therefore, for identical materials of belt and pulleys, the coefficient of friction of V-belt is 2.92 times that of flat belt. Consequently, the power-transmitting capacity of V-belt is much more than that of flat belt. Therefore, V-belts are more powerful.

(iii) Due to increased frictional force, the slip is less in V-belt compared with flat belt.

The effective pull in the belt is $(P_1 - P_2)$. Therefore,

Power transmitted = $(P_1 - P_2)\, v$ N-m/s or W

or
$$\text{kW} = \frac{(P_1 - P_2)v}{1000} \qquad (13.8)$$

13.5 CONDITION FOR MAXIMUM POWER

The belt is given an initial tension P_i in order to transmit power. The initial tension depends upon the length of the belt, the elasticity of the belt material, the geometry of pulleys and the centre distance. In order to derive an expression for initial tension, the following assumptions are made:

(i) The length of the belt is constant.

(ii) The belt has linear elasticity.

When the driving pulley begins to rotate, the elongation on the tight side is proportional to $(P_1 - P_i)$ while the contraction on the loose side is proportional to $(P_i - P_2)$. For constant belt length, the elongation on the tight side is equal to the contraction on the loose side. Therefore,

$$(P_1 - P_i) = (P_i - P_2)$$

$$\therefore \quad P_i = \frac{1}{2}(P_1 + P_2) \qquad (13.9)$$

From Eq. (13.6),

$$\frac{P_1 - mv^2}{P_2 - mv^2} = e^{f\alpha}$$

$$\therefore \quad \frac{P_2 - mv^2}{P_1 - mv^2} = \frac{1}{e^{f\alpha}} = \frac{e^{-f\alpha}}{1}$$

or
$$\frac{(P_2 - mv^2) + (P_1 - mv^2)}{(P_2 - mv^2) - (P_1 - mv^2)} = \frac{e^{-f\alpha} + 1}{e^{-f\alpha} - 1}$$

$$\frac{(P_1 + P_2) - 2mv^2}{(P_2 - P_1)} = \frac{e^{-f\alpha} + 1}{e^{-f\alpha} - 1}$$

Substituting Eq. (13.9) in the above expression,

$$\frac{2P_i - 2mv^2}{-(P_1 - P_2)} = \frac{e^{-f\alpha} + 1}{e^{-f\alpha} - 1}$$

or
$$(P_1 - P_2) = 2(P_i - mv^2) \times \frac{(1 - e^{-f\alpha})}{(1 + e^{-f\alpha})}$$

Therefore,

$$\text{Power} = (P_1 - P_2)v = 2(P_i v - mv^3) \times \frac{(1 - e^{-f\alpha})}{(1 + e^{-f\alpha})}$$

Differentiating power with respect to v and equating the result to zero,

$$\frac{\partial}{\partial v}(\text{power}) = 0 \quad \text{or} \quad \frac{\partial}{\partial v}(P_i v - mv^3) = 0$$

or
$$P_i - 3mv^2 = 0$$

The optimum velocity of the belt for maximum power transmission is given by,

$$v = \sqrt{\frac{P_i}{3m}} \qquad (13.10)$$

13.6 CONDITION FOR MAXIMUM POWER (ALTERNATIVE APPROACH)

When the belt passes over the pulley, the centrifugal force due to its own weight tends to lift the belt from the surface of the pulley. An element of belt subtending an angle ($d\phi$) at the centre of the pulley is shown in Fig. 13.9.

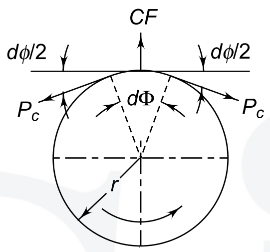


Fig. 13.9

Length of belt element = $r\,d\phi$

Mass of element = $mr\,d\phi$

where m is the mass per unit length of the belt.

The acceleration of the belt element rotating about the axis of the pulley is $\left(\dfrac{v^2}{r}\right)$.

Centrifugal force on belt element = CF = mass × acceleration

$$\text{CF} = mr\,d\phi\left(\frac{v^2}{r}\right) = mv^2 d\phi \qquad \text{(a)}$$

The centrifugal force CF induces belt tension P_c. By symmetry, the centrifugal force induces equal tension on two sides of belt. Resolving the forces acting on the belt element in vertical direction,

$$\text{CF} = 2P_c \sin\left(\frac{d\phi}{2}\right) \qquad \text{(b)}$$

From (a) and (b),

$$mv^2\,d\phi = 2P_c \sin\left(\frac{d\phi}{2}\right) \qquad \text{(c)}$$

For small values,

$$\sin\left(\frac{d\phi}{2}\right) = \left(\frac{d\phi}{2}\right) \qquad \text{(d)}$$

From (c) and (d),

$$mv^2\,d\phi = 2P_c\left(\frac{d\phi}{2}\right)$$

$$P_c = mv^2 \qquad (13.11)$$

A belt can transmit maximum power when the following two conditions are simultaneously satisfied:

(i) The tension on the belt reaches the maximum permissible value for the belt cross-section.

(ii) The belt is on the point of slipping, i.e., maximum frictional force is developed in the belt.

Suppose,

b = width of belt (mm)

t = thickness of belt (mm)

σ = maximum permissible tensile stress for the belt material (N/mm^2)

The maximum belt tension ($P_{max.}$) is given by,

$$P_{max.} = bt\sigma \qquad \text{(e)}$$

Since there is tension due to centrifugal forece,

$$P_1 = P_{max.} - P_c \qquad \text{(f)}$$

Also,

$$\frac{P_1}{P_2} = e^{f\alpha}$$

or

$$P_2 = \frac{P_1}{e^{f\alpha}} \qquad \text{(g)}$$

The power transmitted by the belt is given by,

Power = $(P_1 - P_2)\,v$

$$= \left[P_1 - \frac{P_1}{e^{f\alpha}}\right]v = P_1 v\left[1 - \frac{1}{e^{f\alpha}}\right]$$

Suppose,

$$\left[1 - \frac{1}{e^{f\alpha}}\right] = k$$

Power = $P_1\,v\,k = (P_{max.} - P_c)vk = (P_{max.} - mv^2)vk$

Power = $(P_{max.}\,v - mv^3)k$

The power transmitted will be maximum when,

$$\frac{\partial}{\partial v}(\text{Power}) = 0 \quad \text{or} \quad \frac{\partial}{\partial v}(P_{max.}\,v - mv^3) = 0$$

or

$$P_{max.} - 3mv^2 = 0 \qquad (13.12)$$

The optimum velocity of the belt for maximum power transmission is given by,

$$v = \sqrt{\frac{P_{max.}}{3m}} \qquad (13.13)$$

From Eqs (13.11) and (13.12),

$$P_{max.} - 3P_c = 0 \quad \text{or} \quad P_{max.} = 3P_c \qquad (13.14)$$

$$P_c = \frac{P_{max.}}{3} \qquad (13.15)$$

From the expression (f),

$$P_1 = P_{max.} - P_c = 3P_c - P_c = 2P_c$$

$$P_1 = 2P_c \tag{13.16}$$

Conclusions

The condition for maximum power transmission is,

(i) The maximum permissible tension in the belt should be three times the tension due to centrifugal force ($P_{max.} = 3P_c$).

(ii) Alternatively, the tension in the tight side of the belt should be twice the tension due to centrifugal force ($P_1 = 2P_c$).

(iii) Alternatively, the belt velocity should be,

$$v = \sqrt{\frac{P_{max.}}{3m}}$$

13.7 CHARACTERISTICS OF BELT DRIVES

There is a peculiar phenomenon in the belt drive, which is called '*creep*'. Creep is a slight relative motion of the belt as it passes over the pulley. Figure 13.10 shows exaggerated mechanics of creep. While moving from tight to loose side over the pulley, the belt element is transferred from the zone of higher tension to the zone of lower tension. As the tension in the belt is reduced, the belt becomes shortened and creeps along the surface of the pulley. This causes relative motion between the belt and pulley surface. Creep results in a decrease in the angular velocity of the driven pulley from that calculated by considering the ratio of diameters of pulleys. The efficiency of the belt drive is reduced by 1 to 2 per cent as a result of creep.

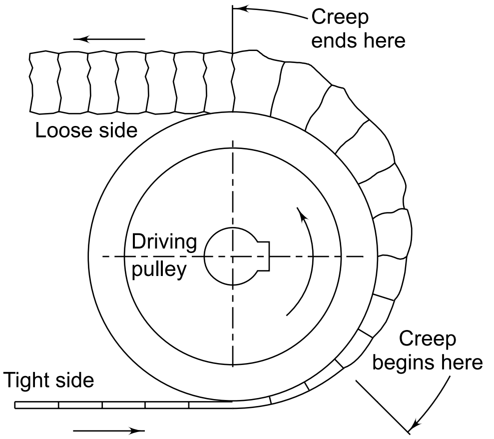


Fig. 13.10 *Belt Creep*

Both creep and slip lower the expected velocity of the driven member. However, there is a basic difference between the creep and slip. Slip is caused by overloads and in this case, the belt slides over the entire arc of contact on the pulley.

The power losses in the belt drive are made up of the following factors:

(i) Power loss due to belt creep on the pulley.

(ii) Power loss due to internal friction between the particles of the belt in alternate bending and unbending over the pulley.

(iii) Power loss due aerodynamic resistance to the motion of pulleys and belt.

(iv) Power loss due to friction in bearings of pulleys.

The losses in V-belt drives are comparatively more than those of in flat belt drives, because of increased internal friction and creep on the pulleys. *For medium service conditions, the efficiency of flat-belt drives is 96% while the efficiency of V-belt is 95%.*

The flat-belt drives can be horizontal or vertical as illustrated in Fig. 13.11. The belt drive is said to be horizontal if the centres of driving and driven pulleys are in a horizontal line. In horizontal belt drives, the loose side is usually kept on the top. On the upper side, the sag of the belt due to its own weight slightly increases the arc of contact with the pulleys and increases the efficiency of drive. On the contrary, when the lower side is loose, sag will reduce the angle of wrap with the pulleys.

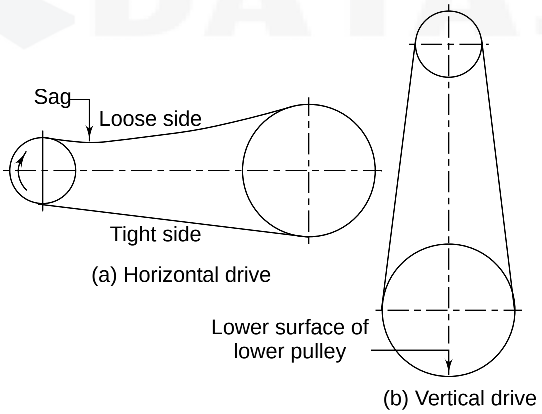


Fig. 13.11 *Horizontal and Vertical Drives*

The belt drive is said to vertical if the centres of driving and driven pulleys are in a vertical line. As far as possible, vertical drives should be avoided. Due to gravitational force on belt, the natural tendency of the belt is to fall away from the lower surface of the lower pulley. This causes slip and reduces the efficiency of the drive. To run such a drive, the belt has to run with excessive tension with consequent increase in bearing reactions and reduced belt life.

Conclusions

(i) Vertical belt drive should be avoided.

(ii) In horizontal belt drive, the upper side should be loose side.

Very often, flat belts are used between two non-parallel shafts. Figure 13.12 shows one such drive called '*quarter-turn*' drive. The belt in this case should satisfy the law of belting. *The law of belting states —'The centreline of the belt when it approaches a pulley must lie in the midplane of that pulley*'. However, a belt leaving a pulley may be drawn out of the midplane of that pulley. As seen in Fig. 13.12, the centreline of belt approaching the lower pulley lies in its midplane. This is also true for the upper pulley. It is also observed that it is not possible to operate the belt in the reverse direction without violating the law of belting. Therefore, for non-parallel shafts, motion is possible only in one direction. Otherwise, the belt is thrown off the pulley.

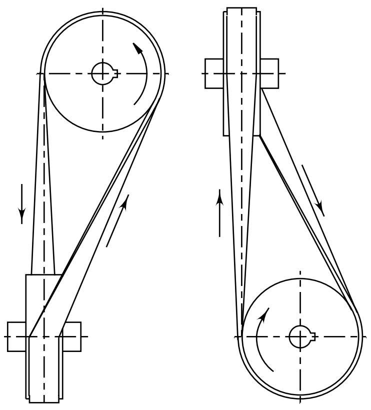


Fig. 13.12 *Quarter-turn Belt Drive*

Many times, it is required to decide optimum centre distance for the belt drive. A shorter centre distance is always preferred due to the following reasons:

(i) It results in compact construction.

(ii) It is more economical.

(iii) The operation is more stable.

However, there are two limitations for decreasing the centre distance, viz., the physical dimensions of pulleys and minimum angle of wrap to transmit a given power. When the centre distance is unknown, it is estimated by the following guidelines:

For velocity ratio less than 3,

$$C = D + 1.5\,d$$

For velocity ratio more than 3,

$$C = D$$

In general, horizontal belt drives are installed on the shop floor. The conditions which should be satisfied, when a flat-belt drive is installed are as follows:

(i) The axes of driving and driven shafts should be parallel.

(ii) The centre distance between driving and driven shafts should be optimum.

(iii) The upper side of the belt should be loose side.

Various portions of the belt are subjected to different stresses. The components of these stresses are as follows:

(i) Tensile stress in the belt due to initial tension (P_i)

(ii) Tensile stress in the belt due to transmitted power (P_1 or P_2)

(iii) Tensile stress in the belt due to centrifugal force (P_c)

(iv) Bending stresses in the belt as it passes over the pulley

It is observed that maximum stress in the belt is induced in the tight side of belt when it passes over a small pulley.

Example 13.1 *The layout of a leather belt drive transmitting 15 kW of power is shown in Fig. 13.13. The centre distance between the pulleys is twice the diameter of the bigger pulley. The belt should operate at a velocity of 20 m/s approximately and the stresses in the belt should not exceed*

2.25 N/mm². The density of leather is 0.95 g/cc and the coefficient of friction is 0.35. The thickness of the belt is 5 mm. Calculate:

(i) the diameter of pulleys;
(ii) the length and width of the belt; and
(iii) the belt tensions.

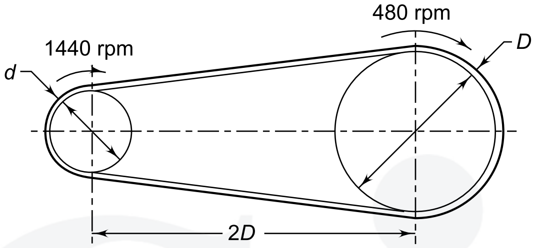


Fig. 13.13

Solution

Given kW = 15 $v = 20$ m/s $C = 2D$
$t = 5$ mm $\rho = 0.95$ g/cc $\sigma = 2.25$ N/mm^2
$f = 0.35$

Step I *Diameter of pulleys*

$$v = \frac{\pi d n}{60(1000)}$$

or $$d = \frac{60(1000)v}{\pi n} = \frac{60(1000)(20)}{\pi(1440)}$$

$$= 265.26 \text{ mm (or 270 mm)}$$

The diameters of pulleys are

$$d = 270 \text{ mm}$$

and $$D = \frac{270(1440)}{(480)} = 810 \text{ mm} \qquad \text{(a)}$$

Step II *Belt length*

$$C = 2D = 2(810) = 1620 \text{ mm}$$

From Eq. (13.3),

$$L = 2C + \frac{\pi(D+d)}{2} + \frac{(D-d)^2}{4C}$$

$$= 2(1620) + \frac{\pi(810+270)}{2} + \frac{(810-270)^2}{4(1620)}$$

$$= 4981.46 \text{ mm} \qquad \text{(b1)}$$

Step III *Belt width and belt tensions*

The correct belt velocity is given by,

$$v = \frac{\pi d n}{60(1000)} = \frac{\pi(270)(1440)}{60(1000)} = 20.36 \text{ m/s}$$

$$\alpha_s = 180 - 2\sin^{-1}\left(\frac{D-d}{2C}\right)$$

$$= 180 - 2\sin^{-1}\left(\frac{810-270}{2\times 1620}\right) = 160.8°$$

or $$\alpha_s = \left(\frac{160.8}{180}\right)\pi = 2.81 \text{ rad}$$

The density of leather is given as 0.95 g per cubic cm. The volume of 1 metre belt in cubic cm is given by,

$$\text{volume} = (\text{length}) \times (\text{width}) \times (\text{thickness})$$

$$= (100)\left(\frac{b}{10}\right)\left(\frac{5}{10}\right)$$

where b is the width in mm.

$\therefore$ $$m = (0.95)(100)\left(\frac{b}{10}\right)\left(\frac{5}{10}\right) \text{ g/m}$$

$$= (0.95)(100)\left(\frac{b}{10}\right)\left(\frac{5}{10}\right)(10^{-3}) \text{ kg/m}$$

$$= (4.75 \times 10^{-3})b \text{ kg/m}$$

or $$mv^2 = (4.75 \times 10^{-3})b\,(20.36)^2 = 1.97b$$

also $$e^{f\alpha} = e^{(0.35)(2.81)} = 2.67$$

From Eq. (13.6),

$$\frac{P_1 - mv^2}{P_2 - mv^2} = e^{f\alpha} \quad \text{or} \quad \frac{P_1 - 1.97b}{P_2 - 1.97b} = 2.67$$

or $$P_1 - 2.67P_2 + 3.29b = 0 \qquad \text{(i)}$$

The maximum stress in the belt is given as 2.25 N/mm^2.

$$\sigma = \frac{\text{maximum tension}}{\text{area of cross-section of belt}} = \frac{P_1}{A}$$

$\therefore$ $$P_1 = \sigma A = 2.25(5b) = (11.25\,b) \text{ N} \qquad \text{(ii)}$$

From Eq. (13.8),

$$P_1 - P_2 = \frac{1000(\text{kW})}{v} = \frac{1000(15)}{20.36} = 736.74 \text{ N} \qquad \text{(iii)}$$

Solving Eqs (i), (ii) and (iii) simultaneously, we have

$$b = 127.02 \text{ mm or } 130 \text{ mm} \qquad \text{(b2)}$$

$$P_1 = 1428.98 \text{ N} \quad \text{and} \quad P_2 = 692.26 \text{ N} \qquad \text{(c)}$$

Example 13.2 *The following data is given for a V-belt drive connecting a 20 kW motor to a compressor.*

	Motor-pulley	*Compressor-pulley*
Pitch diameter (mm)	*300*	*900*
Speed (rpm)	*1440*	*480*
Coefficient of friction	*0.2*	*0.2*

The centre distance between pulleys is 1 m and the dimensions of the cross-section of the belt are given in Fig. 13.14(a). The density of the composite belt is 0.97 g/cc and the allowable tension per belt is 850 N.

How many belts are required for this application?

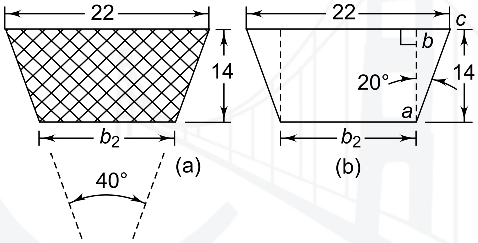


Fig. 13.14

Solution

Given kW = 20 $D = 900$ mm $d = 300$ mm $C = 1$ m $\rho = 0.97$ g/cc $P_1 = 850$ N $f = 0.2$ $n = 1440$ rpm $\theta = 40°$

Step I *Ratio of belt tensions*

From Eq. (13.1),

$$\alpha_s = 180 - 2\sin^{-1}\left(\frac{D-d}{2C}\right)$$

$$= 180 - 2\sin^{-1}\left(\frac{900-300}{2\times 1000}\right) = 145.07°$$

or $$\alpha_s = \left(\frac{145.07}{180}\right)\pi = 2.53 \text{ rad}$$

$$e^{f\alpha/\sin(\theta/2)} = e^{(0/20)(2.53)/\sin(20°)} = 4.4$$

Step II *Mass of belt per metre length*

Refer to Fig. 13.14(b). Draw $ab \perp bc$ From $\Delta\, abc$,

$$\frac{\overline{bc}}{\overline{ba}} = \tan 20° \qquad \overline{bc} = \overline{ba}\tan 20° = 14\tan 20°$$

The width b_2 at the base is given by,

$$b_2 = 22 - 2\,\overline{bc} = 22 - 2(14\tan 20°) = 11.81 \text{ mm}$$

$$\text{Area of cross-section} = \frac{1}{2}(11.81 + 22)(14)$$

$$= 236.67 \text{ mm}^2 = (236.67 \times 10^{-2}) \text{ cm}^2$$

The length of a one metre belt is 100 cm. Therefore, the mass of the belt (m) per metre length is given by,

mass = density × volume of belt
= density × (area of cross-section × length)
= $(0.97) \times (236.67 \times 10^{-2})(100)$ gm/m
= $(0.97)(10^{-3})(236.67 \times 10^{-2})(100)$ kg/m.
= 0.23 kg/m

Step III *kW rating per belt*

$$v = \frac{\pi d n}{60(1000)} = \frac{\pi(300)(1440)}{60(1000)} = 22.62 \text{ m/s}$$

$$\therefore \quad mv^2 = (0.23)(22.62)^2 = 117.68$$

From Eq. (13.7),

$$\frac{P_1 - mv^2}{P_2 - mv^2} = e^{f\alpha/\sin(\theta/2)}$$

$$\frac{P_1 - 117.68}{P_2 - 117.68} = 4.4 \qquad \text{(a)}$$

The allowable tension in the belt is given as 850 N.

$$\therefore \quad P_1 = 850 \text{ N} \qquad \text{(b)}$$

From Eqs (a) and (b),

$$P_2 = 284.11 \text{ N}$$

$$\text{kW per belt} = \frac{(P_1 - P_2)v}{1000} = \frac{(850 - 284.11)(22.62)}{1000}$$

$$= 12.8$$

Step IV *Number of belts*

$$\text{Number of belts} = \frac{20}{12.8} = 1.56 \text{ or } 2 \text{ belts}$$

Example 13.3 *The following data is given for an open-type V-belt drive:*

diameter of driving pulley = 150 mm
diameter of driven pulley = 300 mm

centre distance = 1 m
groove angle = 40°
mass of belt = 0.25 kg/m
maximum permissible tension = 750 N
coefficient of friction = 0.2

Plot a graph of maximum tension and power transmitted against the belt velocity. Calculate the maximum power transmitted by the belt and the corresponding belt velocity. Neglect power losses.

Solution

$D = 300$ mm $\quad d = 150$ mm $\quad C = 1$ m $\quad P_1 = 750$ N
$f = 0.2 \quad m = 0.25$ kg/m $\quad \theta = 40°$

Step I *Belt velocity at maximum power*

From Eq. (13.1),

$$\alpha_s = 180 - 2\sin^{-1}\left(\frac{D-d}{2C}\right)$$

$$= 180 - 2\sin^{-1}\left(\frac{300-150}{2\times 1000}\right) = 171.4°$$

or $\quad \alpha_s = \left(\frac{171.4}{180}\right)\pi = 2.99$ rad

$$e^{f\alpha/\sin(\theta/2)} = e^{(0.2)(2.99)/\sin(20°)} = 5.75$$

From Eq. (13.7),

$$\frac{P_1 - mv^2}{P_2 - mv^2} = e^{f\alpha/\sin(\theta/2)}$$

$$\frac{P_1 - 0.25v^2}{P_2 - 0.25v^2} = 5.75 \qquad \text{(a)}$$

The belt tension is maximum when $v = 0$. For this condition,

$$\frac{P_1}{P_2} = 5.75$$

The maximum permissible tension is 750 N.

$\therefore \quad P_1 = 750$ N

$$P_2 = \frac{P_1}{5.75} = \frac{750}{5.75} = 130.43 \text{ N}$$

From Eq. (13.9),

$$\therefore \quad P_i = \frac{1}{2}(P_1 + P_2) = \frac{1}{2}(750 + 130.43)$$

$$= 440.22 \text{ N}$$

For maximum power transmission,

$$v = \sqrt{\frac{P_i}{3m}} = \sqrt{\frac{(440.22)}{3(0.25)}} = 24.23 \text{ m/s}$$

Step II *Maximum power transmission*

$$P_1 + P_2 = 2P_i$$

$$P_1 + P_2 - 2(440.22) = 0$$

or $\quad P_1 + P_2 - 880.44 = 0 \qquad$ (b)

Substituting ($v = 24.23$ m/s) in Eq. (a),

$$\frac{P_1 - 0.25(24.23)^2}{P_2 - 0.25(24.23)^2} = 5.75 \qquad \text{(c)}$$

Solving (b) and (c),

$$P_1 = 646.73 \text{ N} \quad \text{and} \quad P_2 = 233.70 \text{ N}$$

$$\therefore \quad \text{kW} = \frac{(P_1 - P_2)v}{1000} = \frac{(646.73 - 233.70)(24.23)}{1000}$$

$$= 10.01$$

Step III *Variation of tension and power against belt velocity*

For an intermediate velocity of $v = 10$ m/s,

$$\frac{P_1 - 0.25(10)^2}{P_2 - 0.25(10)^2} = 5.75 \qquad \text{(d)}$$

Solving (b) and (d),

$$P_1 = 732.4 \text{ N} \quad \text{and} \quad P_2 = 148.03 \text{ N}$$

$$\therefore \quad \text{kW} = \frac{(P_1 - P_2)v}{1000} = \frac{(732.4 - 148.03)(10)}{1000}$$

$$= 5.84$$

Similar calculations can be made for other values of the belt velocity. The variations of belt tension and transmitted power against the belt velocity are shown in Fig. 13.15. The transmitted power will be zero when,

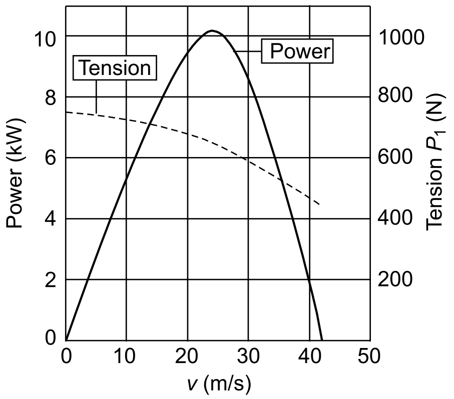


Fig. 13.15

$$P_1 = P_2$$

From Eq. (b),

$$P_i = P_1 = P_2 = 440.22 \text{ N}$$

Now $$\frac{P_1 - 0.25v^2}{P_2 - 0.25v^2} = 5.75$$

or $$P_1 = 0.25\, v^2$$

or $$v = \sqrt{\frac{P_1}{0.25}} = \sqrt{\frac{440.22}{0.25}} = 41.96 \text{ m/s}$$

The power transmitted by the belt at 41.96 m/s is zero.

13.8 SELECTION OF FLAT-BELTS FROM MANUFACTURER'S CATALOGUE

In practice, the designer has to select a belt from the manufacturer's catalogue. For the selection of a proper belt for a given application, the following information is required:

(i) power to be transmitted;
(ii) the input and the output speeds;
(iii) the centre distance depending upon the availability of space; and
(iv) type of load.

The basic procedure for belt selection from the catalogue of The Dunlop Rubber Co. (India) Ltd.[1] is discussed in this section. The maximum power transmitted by the belt is obtained by multiplying the rated power by a load correction factor. The relationship is given by,

$$(\text{kW})_{\text{max.}} = F_a\,(\text{kW}) \tag{13.17}$$

where,

$(\text{kW})_{\text{max.}}$ = power transmitted by the belt for design purpose
kW = actual power transmitted by the belt in a given application
F_a = load correction factor

The values of the load correction factor are given in Table 13.1.

Table 13.1 *Load correction factor (F_a)*

Type of load	F_a
(i) Normal load	1.0
(ii) Steady load, e.g., centrifugal pumps, fans, light machine tools, conveyors	1.2
(iii) Intermittent load, e.g., heavy duty fans, blowers, compressors, reciprocating pumps, line shafts, heavy-duty machines	1.3
(iv) Shock load, e.g., vacuum pumps, rolling mills, hammers, grinders	1.5

The power transmitting capacities of the belt are developed for 180° of arc of contact. The actual arc of contact will be different in different applications. When the arc of contact is less than 180°, there will be additional tension in the belt, to account for which a factor called '*arc of contact factor*' (F_d) is used in calculations. The values of the arc of contact factor are given in Table 13.2. It is not advisable to use an arc of contact less than 150° for a flat belt drive. Therefore, the minimum arc of contact should be 150°.

Table 13.2 *Arc of contact factor (F_d)*

α_s (Deg.)	120	130	140	150	160	170	180	190	200
F_d	1.33	1.26	1.19	1.13	1.08	1.04	1.00	0.97	0.94

There are two varieties of Dunlop transmission belts—HI-SPEED duck belting and FORT duck belting. HI-SPEED duck belting is used in general-purpose applications. FORT duck belting is recommended for heavy duty applications. The range of optimum belt velocity for these two belts is 17.8 to 22.9 m/s. The power transmitting capacities of the belts are as follows:

HI-SPEED	0.0118 kW per mm width per ply
FORT	0.0147 kW per mm width per ply

The above values are based on the following two assumptions:

[1] 'Dunlop Transmission Belting, Technical Data' published by Dunlop Industrial Products Service, Kolkata.

(i) The arc of contact is 180°.

(ii) The belt velocity is 5.08 m/s.

There is a specific term '*power rating*' or '*load rating*' of flat belts. *Power rating* of flat belt is defined as the power transmitting capacity of the belt per mm width per ply at 180° arc of contact. For example, the power rating of FORT belt is 0.0147 kW. It indicates that a single-ply Fort belt of 1 mm width will transmit 0.0147 kW power for 180° arc of contact with the pulley.

The standard widths of these belts (in mm) are as follows:

3-Ply	25	40	50	63	76					
4-Ply	40	44	50	63	76	90	100	112	125	152
5-Ply	76	100	112	125	152					
6-Ply	112	125	152	180	200					

The preferred diameters (in mm) of cast iron and mild steel pulleys are as follows:

100, 112, 125, 140, 160, 180, 200, 224, 250, 280, 315, 355, 400, 450, 500, 560, 630, 710, 800, 900 and 1000

The basic procedure for selection of flat belt consists of the following steps:

(i) The optimum belt velocity of Dunlop belts is from 17.8 to 22.9 m/s. Assume some belt velocity such as 18 m/s in this range and calculate the diameter of the smaller pulley by the following relationship:

$$d = \frac{60(1000)v}{\pi n}$$

where n is the input speed or rpm of the smaller pulley.

The diameter of the bigger pulley is obtained by the following relationship:

$$D = d\left[\frac{\text{speed of smaller pulley}}{\text{speed of bigger pulley}}\right]$$

$$= d\left[\frac{\text{input speed}}{\text{output speed}}\right]$$

Modify the values of d and D to the nearest preferred diameters. Determine the correct belt velocity for these preferred pulley diameters and check whether the actual velocity is in the range of optimum belt velocity.

(ii) Determine the load correction factor F_a from Table 13.1. It depends upon the type of load. Find out the maximum power for the purpose of belt selection by following relationship,

$$(\text{kW})_{\text{max.}} = F_a\,(\text{kW})$$

(iii) Calculate the angle of wrap for the smaller pulley by the following relationship:

$$\alpha_s = 180 - 2\sin^{-1}\left(\frac{D-d}{2C}\right)$$

Find out the arc of contact factor F_d from Table 13.2.

(iv) Calculate the corrected power by the following relationship:

$$(\text{kW})_{\text{corrected}} = (\text{kW})_{\text{max.}} \times F_d$$

(v) Calculate the corrected power rating for the belt by the following relationships:

For HI-SPEED belt,

$$\text{Corrected kW rating} = \frac{0.0118v}{(5.08)}$$

For FORT belt,

$$\text{Corrected kW rating} = \frac{0.0147v}{(5.08)}$$

where v is the correct belt velocity in m/s.

(vi) Calculate the product of (width × number of plies) by dividing the corrected power by the corrected kW rating. Or,

$$(\text{Width} \times \text{No. of plies}) = \frac{\text{corrected power}}{\text{corrected belt rating}}$$

$$= \frac{(\text{kW})_{\text{corrected}}}{\text{Corrected kW rating of belt}}$$

Calculate the belt width by assuming a suitable number of plies. In this step, there are a number alternative solutions. A belt whose width is near the value of the standard width is the optimum solution. For this belt, select the standard belt width.

(vii) Calculate the belt length by using the following relationship:

$$L = 2C + \frac{\pi(D+d)}{2} + \frac{(D-d)^2}{4C}$$

The above procedure of belt selection is explained in Example 13.4.

Example 13.4 *It is required to select a flat belt drive for a compressor running at 720 rpm, which is driven by a 25 kW, 1440 rpm motor. Space is available for a centre distance of 3 m. The belt is open-type.*

Solution

Given kW = 25 $n_1 = 1440$ rpm $n_2 = 720$ rpm $C = 3$ m

Step I *Diameter of smaller and bigger pulleys*

(Refer Fig. 13.16)

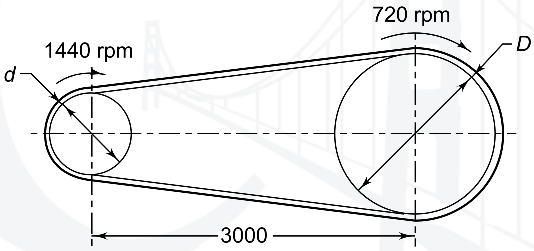


Fig. 13.16

$$v = \frac{\pi d n_1}{60(1000)}$$

Using a belt velocity of 18 m/s,

$$d = \frac{60(1000)v}{\pi n_1} = \frac{60(1000)(18)}{\pi(1440)} = 238.73 \text{ mm}$$

Selecting the preferred pulley diameter of 250 mm,

$$d = 250 \text{ mm} \quad \text{and} \quad D = 2 \times 250 = 500 \text{ mm}$$

The belt velocity for these dimensions is given by,

$$v = \frac{\pi d n_1}{60(1000)} = \frac{\pi(250)(1440)}{60\,(1000)} = 18.85 \text{ m/s}$$

Step II *Maximum power for selection*

From Table 13.1, the load correction factor for the compressor is 1.3. Therefore,

Maximum power = 1.3(25) = 32.5 kW

Step III *Arc of contact factor (F_d)*

From Eq. (13.1),

$$\alpha_s = 180 - 2\sin^{-1}\left(\frac{D-d}{2C}\right)$$

$$= 180 - 2\sin^{-1}\left(\frac{500-250}{2\times 3000}\right) = 175.23°$$

Assuming linear interpolation, the arc of contact factor is given by,

$$F_d = 1 + \frac{(1.04-1)(180-175.23)}{(180-170)} = 1.019$$

Step IV *Corrected power*

Corrected power = 1.019(32.5) = 33.12 kW

Step V *Corrected power rating of belt*

Selecting a HI-SPEED belt, the corrected rating at a belt speed of 18.85 m/s is given by,

$$\text{Corrected belt rating} = \frac{0.0118(18.85)}{5.08}$$
$$= 0.0438 \text{ kW}$$

Step VI *Selection of belt*

$$(\text{Width} \times \text{No. of plies}) = \frac{\text{corrected power}}{\text{corrected belt rating}}$$
$$= \frac{33.12}{0.0438} = 756.17$$

Belt widths,

4 plies $\quad w = \dfrac{756.17}{4} = 189.04$ mm

5 plies $\quad w = \dfrac{756.17}{5} = 151.23$ mm

6 plies $\quad w = \dfrac{756.17}{6} = 126.03$ mm

We shall select a HI-SPEED belt of 152 mm preferred width and 5 plies.

Step VII *Belt length*

The belt length is given by

$$L = 2C + \frac{\pi(D+d)}{2} + \frac{(D-d)^2}{4C}$$

$$= 2(3000) + \frac{\pi(500+250)}{2} + \frac{(500-250)^2}{4(3000)}$$

$$= 7183.31 \text{ mm}$$

Belt Specification 7.2 m length of 152 mm width and 5 plies HI-SPEED belting.

13.9 PULLEYS FOR FLAT BELTS

The pulleys for flat belts consist of three parts, viz., rim, hub and arms or web. The rim carries the belt. The hub connects the pulley to the shaft. The arms or web join the hub with the rim. There are two types of pulleys that are used for flat belts, viz., cast iron pulleys and mild steel pulleys[2]. The pulley diameters are calculated in belt drive design. They should comply with standard values. The recommended values of minimum pulley diameters are given in Table 13.3. The minimum pulley diameter depends upon the following two factors:

(i) The number of plies in the belt
(ii) The belt speed

Preferred values of pulley diameters are given in Table 13.4. There is a relationship between the width of the belt and the width of the pulley, or to be more specific, width of the rim of the pulley. The difference between the width of the rim and the width of the belt is given in Table 13.5. Preferred values for width of cast iron and mild steel pulleys are given in Table 13.6.

Table 13.3 *Minimum pulley diameters for given belt speeds and belt plies (mm)*

No. of plies	*Maximum belt speed (m/s)*				
	10	*15*	*20*	*25*	*30*
3	90	100	112	140	180
4	140	160	180	200	250
5	200	224	250	315	355
6	250	315	355	400	450
7	355	400	450	500	560
8	450	500	560	630	710
9	560	630	710	800	900
10	630	710	800	900	1000

Table 13.4 *Recommended diameters of cast iron and mild steel flat pulleys*

Nominal diameter (mm)	*Tolerance on diameter (mm)*
40	± 0.5
45, 50	± 0.6
56, 63	± 0.8
71, 80	± 1.0
90, 100, 112	± 1.2
125, 140	± 1.6
160, 180, 200	± 2.0
224, 250	± 2.5
280, 315, 355	± 3.2
400, 450, 500	± 4.0
560, 630, 710	± 5.0
800, 900, 1000	± 6.3
1120, 1250, 1400	± 8.0
1600, 1800, 2000	± 10.0

Table 13.5 *Relationship between belt and pulley widths*

Belt width (mm)	*Pulley to be wider than the belt width by (mm)*
Up to and including 125	13
From 125 up to and including 250	25
From 250 up to and including 375	38
From 375 up to and including 500	50

Table 13.6 *Recommended widths of cast iron and mild steel flat pulleys*

Width (mm)	*Tolerance (mm)*
20, 25, 32, 40, 50, 63, 71	± 1.0
80, 90, 100, 112, 125, 140	± 1.5
160, 180, 200, 224, 250, 280, 315	± 2.0
355, 400, 450, 500, 560, 630	± 3.0

There is a specific term, '*crowning*' of pulley in flat belt drive. The thickness of the rim is slightly

[2] IS 1691–1980: Specifications for cast iron and mild steel flat pulleys.

increased in the centre to give it a convex or conical shape as shown in Fig. 13.17. This is called '*crown*' of the pulley. The crown is provided only on one of the two pulleys. The objectives of providing crown are as follows:

(i) The crown on the pulley helps to hold the belt on the pulley in running condition.
(ii) The crown on the pulley prevents the belt from running off the pulley.
(iii) The crown on the pulley brings the belt to running equilibrium position near the midplane of the pulley.

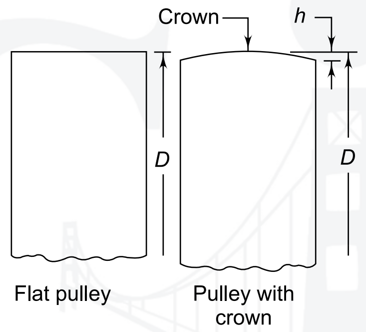


Fig. 13.17 *Crown for Pulley*

The crown on pulley is essential, particularly when the pulleys are mounted inaccurately or there is a possibility of slip due to non-parallelism between connected shafts. Values of crown for cast iron and mild steel pulleys are given in Table 13.7 and 13.8.

Table 13.7 *Crown of cast iron and mild steel flat pulleys (pulley diameter from 40 to 355 mm)*

Pulley diameter D (mm)	*Crown h (mm)*
40–112	0.3
125–140	0.4
160–180	0.5
200–224	0.6
250–280	0.8
315–355	1.0

Table 13.8 *Crown of cast iron and mild steel flat pulleys (pulley diameter from 400 to 2000 mm)*

Pulley diameter D (mm)	*Crown (in mm) of pulleys of width (in mm)*						
	125 and smaller	*140 and 160*	*180 and 200*	*224 and 250*	*280 and 315*	*355*	*400 and larger*
400	1	1.2	1.2	1.2	1.2	1.2	1.2
450	1	1.2	1.2	1.2	1.2	1.2	1.2
500	1	1.5	1.5	1.5	1.5	1.5	1.5
560	1	1.5	1.5	1.5	1.5	1.5	1.5
630	1	1.5	2.0	2.0	2.0	2.0	2.0
710	1	1.5	2.0	2.0	2.0	2.0	2.0
800	1	1.5	2.0	2.5	2.5	2.5	2.5
900	1	1.5	2.0	2.5	2.5	2.5	2.5
1000	1	1.5	2.0	2.5	3.0	3.0	3.0
1120	1.2	1.5	2.0	2.5	3.0	3.0	3.5
1250	1.2	1.5	2.0	2.5	3.0	3.0	3.5
1400	1.5	2.0	2.5	3.0	3.5	4.0	4.0
1600	1.5	2.0	2.5	3.0	3.5	4.0	5.0
1800	2.0	2.5	3.0	3.5	4.0	5.0	5.0
2000	2.0	2.5	3.0	3.5	4.0	5.0	6.0

Cast iron pulleys are made of grey cast iron of Grade FG 200. The proportions of these pulleys are given in Fig. 13.18 and Table No. 13.9.

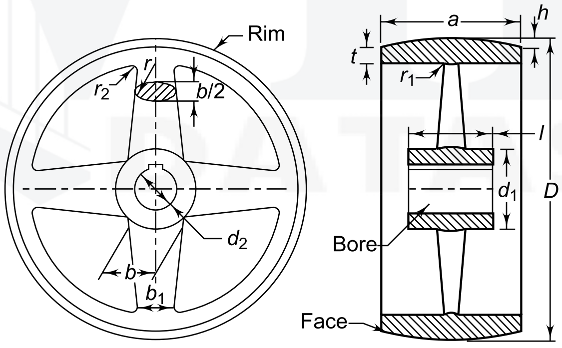


Fig. 13.18 *Cast Iron Pulley*

Table 13.9 *Proportions of cast iron pulleys (Refer to Fig. 13.18)*

(a)	*Number of arms*
	(1) For pulleys up to 200 mm diameter, use webs.
	(2) For pulleys above 200 mm diameter and up to 450 mm diameter, use 4 arms.
	(3) For pulleys above 450 mm diameter, use 6 arms.

(Contd)

Table 13.9 *(Contd)*

(b) *Cross-section of arms-elliptical*

(c) *Thickness of arm b near boss*

$$b = 2.94\sqrt[3]{\frac{aD}{4n}} \text{ for single belt}$$

$$b = 2.94\sqrt[3]{\frac{aD}{2n}} \text{ for double belt}$$

where, a = width of pulley

D = diameter of pulley

n = number of arms in the pulley

(d) *Thickness of arm b_1 near rim* = use taper 4 mm per 100 mm (from boss to rim)

(e) *Radius of the cross-section of arms*

$$r = \frac{3}{4}b$$

(f) *Minimum length l of the bore*

$$l = \frac{2}{3}a$$

It may be more for loose pulleys, but in no case it should exceed a.

(g) $$\frac{d_1 - d_2}{2} = 0.412 \times \sqrt[3]{aD} + 6 \text{ mm for single belt}$$

$$\frac{d_1 - d_2}{2} = 0.529 \times \sqrt[3]{aD} + 6 \text{ mm for double belt}$$

(h) *Radius r_1 and r_2*

$$r_1 = \frac{b}{2} \text{(near rim)}$$

$$r_2 = \frac{b}{2} \text{(near rim)}$$

(i) Thickness of rim

$$\text{Rim thickness} = \left(\frac{D}{200} + 3\right) \text{mm (for single belt)}$$

$$\text{Rim thickness} = \left(\frac{D}{200} + 6\right) \text{mm (for double belt)}$$

Mild steel pulleys are made of structural steels. The rim and the arms or web are made of low carbon steel, while the hub which is subjected to crushing stress at the keyway, is made of medium carbon steel. The rim is roll-formed from a steel plate and joined either by bolts or welded. The arms are made of round steel bar and welded to the rim. The arms are welded to the hub, if it is made of steel. The arms are screwed to the hub, if it is made of cast iron. Mild steel pulleys have lightweight construction compared with cast iron pulleys. Their proportions are shown in Fig. 13.19 and given Table 13.10 and Table 13.11.

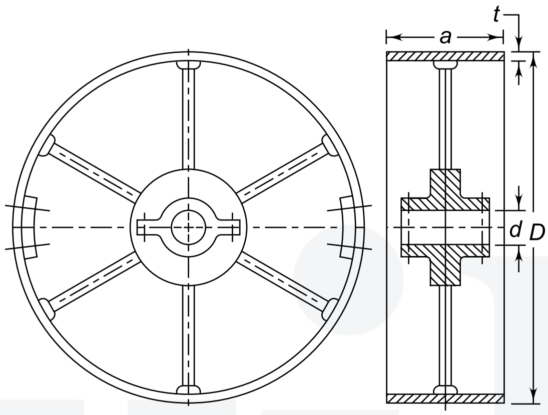


Fig. 13.19 *Mild Steel Pulley*

Table 13.10 *Proportions of mild steel pulleys (Refer to Fig. 13.19)*

(a) *Arrangement of arms*

Pulleys up to 300 mm width are normally supplied with a single row of spokes. Wider pulleys requiring double row of spokes sometimes used.

(b) *Minimum length of boss*

The length of boss is equal to half width of face, subject to a minimum of 100 mm in case of pulleys with 19 mm diameter spokes and minimum of 138 mm for pulleys with 22 mm diameter spokes.

(c) *Thickness of rims* 5 mm (for all pulleys in above table)

Table 13.11 *Number of arms in pulleys*

Diameter of pulley (mm)	*Details of spokes*	
	Number of spokes	*Diameter of spokes (mm)*
280 to 500	6	19
560 to 710	8	19
800 to 1 000	10	22
1 120	12	22
1 250	14	22
1 400	16	22
1 600	18	22
1 800	18	22
2 000	22	22

13.10 ARMS OF CAST IRON PULLEY

There are three important things about the arms of the pulley. They are as follows:

(i) The arms of pulley have elliptical cross-section.

(ii) The major axis of elliptical cross-section is in the plane of rotation.

(iii) The major axis of elliptical cross-section is usually twice the minor axis.

Elliptical cross-section reduces aerodynamic losses during the rotation of pulley as compared with rectangular cross-section. Therefore invariably, the arms have elliptical cross-section.

The design of these arms illustrates the application of simple formula for bending stresses. It is assumed that the belt wraps around the rim of the pulley through approximately 180° and one-half of the arms carry the load at any moment. This is illustrated in Fig. 13.20. The torque transmitted by the pulley is given by,

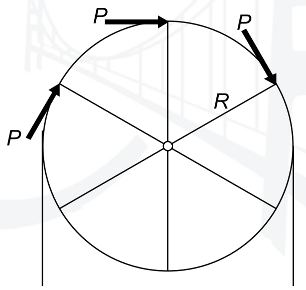


Fig. 13.20

$$M_t = PR\left(\frac{N}{2}\right) \quad \text{or,} \quad P = \frac{2M_t}{RN} \qquad \text{(a)}$$

where,

M_t = torque transmitted by the pulley (N-mm)

P = tangential force at the end of each arm (N)

R = radius of the rim (mm)

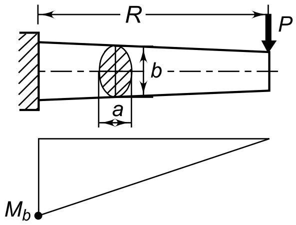
N = number of arms

As shown in Fig. 13.21(a), the bending moment acting on the arm is given by,

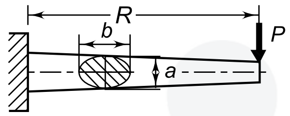
$$M_b = PR \qquad \text{(b)}$$

From (a) and (b),

$$M_b = \frac{2M_t}{N} \qquad \text{(c)}$$



(a) Correct (Major axis in plane of rotation)



(b) Incorrect (Minor axis in plane of rotation)

Fig. 13.21

The major axis of the elliptical cross-section is in the plane of rotation. For this elliptical cross-section with a and b as minor and major axes respectively,

$$I = \frac{\pi a b^3}{64} \qquad \text{(d)}$$

Since the major axis is twice of the minor axis,

$$b = 2a \qquad \text{(e)}$$

Substituting (e) in (d),

$$I = \frac{\pi a^4}{8} \quad \text{and} \quad y = \frac{b}{2} = a$$

The bending stress in the arm is given by,

$$\sigma_b = \frac{M_b y}{I} = \frac{(2M_t/N)(a)}{(\pi a^4/8)}$$

or,

$$a^3 = \frac{16\,M_t}{\pi\,N\,\sigma_b}$$

$$\therefore \qquad a = 1.72\sqrt[3]{\frac{M_t}{N\sigma_b}} \qquad \text{(f)}$$

where σ_b is the permissible bending stress.

If we consider the minor axis in the plane of rotation as illustrated in Fig. 13.21(b),

$$I = \frac{\pi\, b a^3}{64}$$

Since the major axis is twice of the minor axis,

$$b = 2a$$

$$I = \frac{\pi a^4}{32} \quad \text{and} \quad y = \frac{a}{2}$$

The bending stresses in the arm is given by,

$$\sigma_b = \frac{M_b y}{I} = \frac{(2M_t/N)(a/2)}{(\pi a^4/32)} \quad \text{or,} \quad a^3 = \frac{32M_t}{\pi N \sigma_b}$$

$$\therefore \quad a = 2.17\sqrt[3]{\frac{M_t}{N\sigma_b}} \tag{g}$$

It is observed from Eqs (f) and (g) that keeping the minor axis in the plane of rotation increases the cross-sectional area. It is therefore 'economical' to keep major axis of elliptical cross-section in the plane of rotation.

Example 13.5 *A pulley, made of grey cast iron FG 150, transmits 10 kW of power at 720 rpm. The diameter of the pulley is 500 mm. The pulley has four arms of elliptical cross-section, in which the major axis is twice of the minor axis. Determine the dimensions of the cross-section of the arm, if the factor of safety is 5.*

Solution

Given kW = 10 $n = 720$ rpm $D = 500$ mm
For pulley, $S_{ut} = 150$ N/mm^2 $(fs) = 5$ $N = 4$
$b = 2a$

Step I *Permissible stress*

$$\sigma_t = \frac{S_{ut}}{(fs)} = \frac{150}{5} = 30 \text{ N/mm}^2$$

Step II *Bending moment on arm*

The torque transmitted by the pulley is given by,

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n} = \frac{60 \times 10^6 (10)}{2\pi (720)}$$

$$= 132\ 629.12 \text{ N-mm}$$

Assuming that one-half of the arms carry the load at any instant, the tangential force at the end of each arm is given by

$$P = \frac{M_t}{R\left(\dfrac{N}{2}\right)} = \frac{132\ 629.12}{(250)\left(\dfrac{4}{2}\right)} = 265.26 \text{ N}$$

$$\therefore M_b = P \times R = 265.26 \times 250 = 66\ 314.56 \text{ N-mm}$$

Step III *Cross-section of arm*

For an elliptical cross-section with a and b as minor and major axes respectively,

$$I = \frac{\pi a b^3}{64} \quad \text{and} \quad b = 2a$$

$$I = \frac{\pi a (2a)^3}{64} = \frac{\pi a^4}{8} \text{ mm}^2 \quad \text{and} \quad y = \frac{b}{2} = a \text{ mm}$$

The bending stresses in the arm are given by,

$$\sigma_b = \frac{M_b\, y}{I} \quad \text{or} \quad 30 = \frac{(66\ 314.56)(a)}{\left(\dfrac{\pi a^4}{8}\right)}$$

$$\therefore \quad a = 17.78 \text{ or } 20 \text{ mm}$$

$$b = 2a = 40 \text{ mm}$$

Alternatively, using Eq. (f),

$$a = 1.72\sqrt[3]{\frac{M_t}{N\sigma_b}} = 1.72\sqrt[3]{\frac{132\ 629.12}{4(30)}}$$

$$= 17.78 \text{ or } 20 \text{ mm}$$

Example 13.6 *A belt pulley, made of grey cast iron FG150, has four arms of elliptical cross-section, in which the major axis is twice of the minor axis. The tensions on tight and slack sides of the belt are 750 and 250 N respectively. The mean diameter of the pulley is 300 mm, while the hub diameter 60 is mm. Assume that half the number of arms transmit torque at any time. The factor of safety is 5. Determine the dimensions of the cross-section of the pulley arm near the hub.*

Solution

Given $P_1 = 750$ N $P_2 = 250$ N $D = 300$ mm
hub diameter = 60 mm $S_{ut} = 150$ N/mm^2 $(fs) = 5$
$N = 4$ $b = 2a$

Step I *Permissible stress*

$$\sigma_t = \frac{S_{ut}}{(fs)} = \frac{150}{5} = 30 \text{ N/mm}^2$$

Step II *Bending moment on arm*

As shown in Fig. 13.22,

$$M_t = (P_1 - P_2)(D/2) = (750 - 250)(300/2)$$

$$= 75000 \text{ N-mm}$$

Assuming that one-half of the arms carry the load at any time, the tangential force P at the end of the arm is given by,

$$M_t = PR\left(\frac{N}{2}\right) \quad \text{or,}$$

$$P = \frac{2M_t}{RN} = \frac{2(75000)}{(150)(4)} = 250 \text{ N}$$

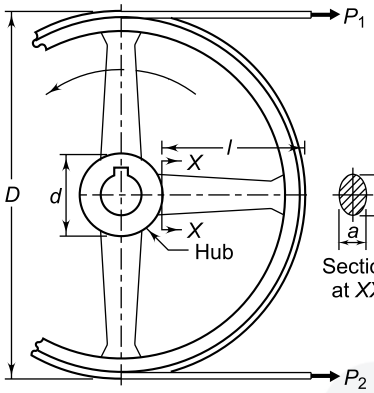


Fig. 13.22

The arm l for the bending moment at the section XX near the hub is given by,

$$l = \frac{D}{2} - \frac{d}{2} = \frac{300}{2} - \frac{60}{2} = 120 \text{ mm}$$

At the section XX,

$$M_b = P \times l = 250 \times 120 = 30000 \text{ N-mm}$$

Step III *Cross-section of arm*

$$b = 2a$$

$$I = \frac{\pi ab^3}{64} = \frac{\pi a(2a)^3}{64} = \frac{\pi a^4}{8} \text{ mm}^4$$

$$y = \frac{b}{2} = \frac{(2a)}{2} = a \text{ mm}$$

$$\sigma_b = \frac{M_b y}{I} \quad \text{or} \quad 30 = \frac{(30000)(a)}{\left(\frac{\pi a^4}{8}\right)}$$

$\therefore$ $a = 13.66$ or 15 mm

$b = 2a = 30$ mm

13.11 V-BELTS

The dimensions for the cross-section of V-belt are shown in Fig. 13.23. The following notations are used for the dimensions of the cross-section:

(i) Pitch Width (W_p) It is the width of the belt at its pitch zone. This is the basic dimension for standardisation of belt and corresponding pulley groove.

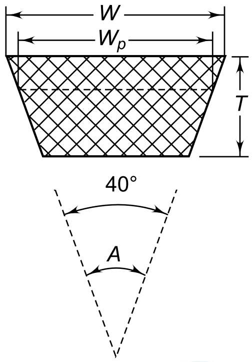


Fig. 13.23 *Dimensions of V belt*

(ii) Nominal Top Width (W) It is the top width of the trapezium outlined on the cross-section of the belt.

(iii) Nominal Height (T) It is the height of the trapezium outlined on the cross-section of the belt.

(iv) Angle of Belt (A) It is the included angle obtained by extending the sides of the belt. The standard value of the belt angle is 40°.

(v) Pitch Length (L_p) It is the length of the pitch line of the belt. This is the circumferential length of the belt at the pitch width.

The manufacturers and the Bureau of Indian Standards have standardised the dimensions of the cross-section[3]. The cross-sectional dimensions are given in Table 13.12. There are six basic symbols—*Z*, *A*, *B*, *C*, *D* and *E*—for the cross-section of V-belts. Z-section belts are occasionally used for low power transmission and small pulley diameters, while *A*, *B*, *C*, *D* and *E* section belts are widely used as general purpose belts. V-belts are designated by the symbol of cross-section followed by nominal pitch length along with symbol L_p., e.g., a V-belt of cross-section *B* and with pitch length 4430 mm is designated as *B* 4430L_p. The recommended values of standard pitch lengths (L_p) are given in Table 13.14. The groove angle for the belt is 40°. The groove angle for the pulley is from 34° to 38°. This results in wedging action between the belt and the groove, thereby increasing the frictional force, and consequently the transmitted power.

[3] IS 2494–1994: Endless V-belts for industrial purposes (Part 1: General Purpose-Specification).

Table 13.12 *Dimensions of standard cross-sections*

Belt section	*Pitch width W_p (mm)*	*Nominal top width W(mm)*	*Nominal Height T (mm)*	*Recommended Minimum pitch diameter of pulley (mm)*	*Permissible Minimum pitch diameter of pulley (mm)*
Z	8.5	10	6	85	50
A	11	13	8	125	75
B	14	17	11	200	125
C	19	22	14	315	200
D	27	32	19	500	355
E	32	38	23	630	500

The selection of the cross-section depends upon two factors, namely, the power to be transmitted and speed of the faster shaft. Figure 13.24 shows the range of speed and power for various cross-sections of the belt. Depending upon the power and speed of the faster pulley, a point can be plotted on this diagram and the corresponding cross-section selected. In borderline cases, alternative design calculations are made to determine the best solution.

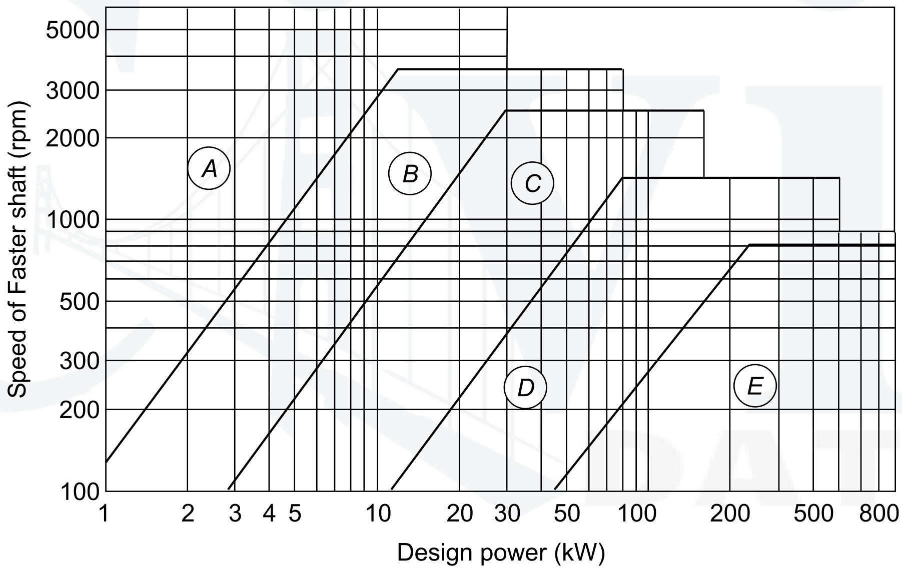


Fig. 13.24 *Selection of Cross-section of V belt*

The calculations of V-belts are based on preferred pitch diameters of pulleys and pitch lengths. The series of preferred values for pitch diameters and pitch lengths (in mm) are given in Tables 13.13 and 13.14 respectively.

The number of belts required for a given application is calculated by the following relationship:

Number of belts

$$= \frac{(\text{transmitted power in kW}) \times (F_a)}{(\text{kW rating of single V-belt}) \times (F_c) \times (F_d)}$$

$$= \frac{P \times F_a}{P_r \times F_c \times F_d} \tag{13.18}$$

where,

P = drive power to be transmitted (kW)

F_a = correction factor for industrial service (Table 13.15)

P_r = power rating of single V-belt (from Table 13.16 to Table 13.20)

F_c = correction factor for belt length (Table 13.21)

F_d = correction factor for arc of contact (Table 13.22)

An extensive data regarding the correction factors, kW ratings of belts and other details is given in the standard as well as in manufacturer's catalogues. However, the discussion in this article is restricted to the selection procedure of general purpose belts.

Table 13.13 *Preferred pitch diameters of pulleys (mm)*

Belt section					
Z	*A*	*B*	*C*	*D*	*E*
50	75	125	200	355	500
53	80	132	212	375	530
56	85	140	224	400	560
60	90	150	236	425	600
63	95	160	250	450	630
67	100	170	265	475	670
71	106	180	280	500	710
75	112	190	300	530	750
80	118	200	315	560	800
85	125	224	355	600	900
90	132	250	375	630	1000
95	140	280	400	710	1120
100	150	300	450	750	1250
112	160	315	500	800	1400
125	170	355	530	900	1500
140	180	375	560	1000	1600
160	190	400	600	1060	1800
180	200	450	630	1120	1900
200	224	500	710	1250	2000
250	250	530	750	1400	2240
315	280	560	800	1500	2500
400	300	600	900	1600	—
500	315	630	1000	1800	—
630	350	710	1200	2000	—
800	400	750	1250	—	—
—	450	800	1400	—	—
—	500	900	1600	—	—
—	560	1000	—	—	—
—	630	1120	—	—	—
—	710	—	—	—	—
—	800	—	—	—	—

Tolerance on the pitch diameter is ± 0.8 per cent

Table 13.14 *Nominal pitch lengths for standard sizes of V-belts*

Pitch lengths of belts L_p (mm)					
Z	*A*	*B*	*C*	*D*	*E*
405	630	930	1560	2740	4660
475	700	1000	1760	3130	5040
530	790	1100	1950	3330	5420
625	890	1210	2190	3730	6100
700	990	1370	2420	4080	6850
780	1100	1560	2720	4620	7650
920	1250	1690	2880	5400	9150
1080	1430	1760	3080	6100	12230
1330	1550	1950	3310	6840	13750
1420	1640	2180	3520	7620	15280
1540	1750	2300	4060	8410	16800
	1940	2500	4600	9140	
	2050	2700	5380	10700	
	2200	2870	6100	12200	
	2300	3200	6820	13700	
	2480	3600	7600	15200	
	2570	4060	9100		
	2700	4430	10700		
	2910	4820			
	3080	5370			
	3290	6070			
	3540				

Table 13.15 *Correction factors according to service (F_a)*

Service	*Type of driven Machine*	*Type of driving units*					
		AC Motor: normal torque, squirrel cage, synchronous and split phase *DC Motor*: shunt wound—multi cylinder IC engine over 600 rpm			*AC Motor*: high torque, induction, single phase *DC Motor*: series and compound wound—*single cylinder IC engine*, Multi cylinder IC engine under 600 rpm—line shaft, *clutches and brakes*		
		Operational hours per day (h)			Operational hours per day (h)		
		0–10	10–16	16–24	0–10	10–16	16–24
Light duty	Agitator, blower, exhauster, centrifugal pumps, compressor and fans up to 7.5 kW and light duty conveyor	1.0	1.1	1.2	1.1	1.2	1.3
Medium duty	Belt conveyor, fans over 7.5 kW, generator, line shaft, machine tools, presses, positive displacement pumps and vibrating screen	1.1	1.2	1.3	1.2	1.3	1.4
Heavy duty	Bucket elevator, hammer mill, piston pump, saw mill, exciter and wood working machinery	1.2	1.3	1.4	1.4	1.5	1.6
Extra-heavy duty	Crusher, mill and hoist	1.3	1.4	1.5	1.5	1.6	1.8

Table 13.16 *Power ratings in kW (P_r) for A-Section V- Belts, 13 mm wide with 180° Arc of contact on smaller pulley*

Speed of faster shaft	Power rating for smaller pulley pitch diameter of									Additional power increment per belt for speed ratio of									
	75 mm	80 mm	85 mm	90 mm	100 mm	106 mm	112 mm	118 mm	125 mm	1.00 to 1.01	1.02 to 1.04	1.05 to 1.08	1.09 to 1.12	1.13 to 1.18	1.19 to 1.24	1.25 to 1.34	1.35 to 1.51	1.52 to 1.99	2.00 and over
rpm	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW
720	0.53	0.60	0.68	0.75	0.90	0.99	1.07	1.16	1.26	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
960	0.66	0.76	0.86	0.95	1.14	1.25	1.37	1.49	1.61	0.00	0.01	0.03	0.04	0.05	0.06	0.08	0.09	0.10	0.12
1440	0.91	1.04	1.17	1.31	1.58	1.73	1.90	2.07	2.24	0.00	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.17
2880	1.42	1.67	1.91	2.14	2.59	2.76	3.11	3.36	3.63	0.00	0.04	0.08	0.12	0.16	0.20	0.23	0.27	0.31	0.35
100	0.11	0.13	0.12	0.14	0.17	0.18	0.20	0.21	0.23	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01
200	0.19	0.22	0.24	0.26	0.31	0.33	0.36	0.39	0.42	0.00	0.00	0.01	0.01	0.01	0.01	0.02	0.02	0.02	0.03
300	0.26	0.29	0.33	0.37	0.43	0.46	0.51	0.55	0.60	0.00	0.00	0.01	0.01	0.02	0.02	0.02	0.03	0.03	0.04
400	0.33	0.37	0.42	0.46	0.55	0.60	0.66	0.71	0.77	0.00	0.01	0.01	0.02	0.02	0.03	0.03	0.04	0.04	0.05
500	0.39	0.45	0.51	0.56	0.67	0.72	0.79	0.86	0.93	0.00	0.01	0.01	0.02	0.03	0.03	0.04	0.05	0.05	0.06
600	0.46	0.52	0.59	0.65	0.78	0.85	0.93	1.00	1.08	0.00	0.01	0.02	0.02	0.03	0.04	0.05	0.06	0.07	0.07
700	0.52	0.59	0.66	0.74	0.88	0.96	1.05	1.14	1.23	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
800	0.57	0.66	0.74	0.82	0.98	1.08	1.18	1.27	1.38	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.09	0.10
900	0.63	0.72	0.81	0.90	1.08	1.18	1.30	1.41	1.52	0.00	0.01	0.02	0.04	0.05	0.06	0.07	0.08	0.10	0.11
1000	0.68	0.78	0.88	0.98	1.18	1.29	1.42	1.54	1.66	0.00	0.01	0.03	0.04	0.05	0.07	0.08	0.09	0.11	0.12
1100	0.73	0.84	0.95	1.06	1.28	1.40	1.53	1.66	1.80	0.00	0.02	0.03	0.04	0.06	0.07	0.09	0.10	0.12	0.13
1200	0.78	0.90	1.02	1.13	1.37	1.50	1.64	1.78	1.93	0.00	0.02	0.03	0.05	0.07	0.08	0.10	0.11	0.13	0.15
1300	0.83	0.95	1.10	1.21	1.45	1.60	1.75	1.90	2.06	0.00	0.02	0.04	0.05	0.07	0.09	0.11	0.12	0.14	0.16
1400	0.88	1.01	1.15	1.28	1.54	1.69	1.86	2.02	2.19	0.00	0.02	0.04	0.06	0.08	0.09	0.11	0.13	0.15	0.17
1500	0.92	1.07	1.21	1.35	1.63	1.79	1.96	2.13	2.31	0.00	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18
1600	0.97	1.12	1.27	1.42	1.72	1.89	2.06	2.24	2.43	0.00	0.02	0.04	0.06	0.09	0.11	0.13	0.15	0.17	0.19
1700	1.01	1.17	1.33	1.48	1.79	1.97	2.16	2.35	2.54	0.00	0.02	0.05	0.07	0.09	0.11	0.14	0.16	0.18	0.21
1800	1.05	1.22	1.39	1.55	1.88	2.07	2.26	2.46	2.66	0.00	0.02	0.05	0.07	0.10	0.12	0.15	0.17	0.19	0.22
1900	1.09	1.27	1.44	1.61	1.95	2.14	2.35	2.56	2.77	0.00	0.03	0.05	0.08	0.10	0.13	0.15	0.18	0.20	0.23
2000	1.13	1.31	1.50	1.68	2.03	2.23	2.44	2.65	2.87	0.00	0.03	0.05	0.08	0.11	0.13	0.16	0.19	0.22	0.24
2100	1.17	1.36	1.55	1.74	2.10	2.31	2.53	2.75	2.98	0.00	0.03	0.06	0.09	0.11	0.14	0.17	0.20	0.23	0.25
2200	1.21	1.40	1.60	1.79	2.17	2.38	2.61	2.84	3.07	0.00	0.03	0.06	0.09	0.12	0.15	0.18	0.21	0.24	0.27
2300	1.24	1.45	1.65	1.85	2.24	2.46	2.69	2.93	3.16	0.00	0.03	0.06	0.09	0.12	0.16	0.18	0.22	0.25	0.28
2400	1.28	1.48	1.70	1.90	2.31	2.54	2.78	3.02	3.26	0.00	0.03	0.07	0.10	0.13	0.16	0.19	0.23	0.26	0.29
2500	1.31	1.53	1.75	1.95	2.37	2.61	2.85	3.09	3.34	0.00	0.03	0.07	0.10	0.14	0.17	0.20	0.24	0.27	0.30

(Contd)

Table 13.16 *(Contd)*

Speed of faster shaft	*Power rating for smaller pulley pitch diameter of*									*Additional power increment per belt for speed ratio of*									
	75 mm	*80 mm*	*85 mm*	*90 mm*	*100 mm*	*106 mm*	*112 mm*	*118 mm*	*125 mm*	*1.00 to 1.01*	*1.02 to 1.04*	*1.05 to 1.08*	*1.09 to 1.12*	*1.13 to 1.18*	*1.19 to 1.24*	*1.25 to 1.34*	*1.35 to 1.51*	*1.52 to 1.99*	*2.00 and over*
rpm	*kW*	*kW*	*kW*	*kW*	*kW*	*kW*	*kW*	*kW*	*kW*	*kW*	*kW*	*kW*	*kW*	*kW*	*kW*	*kW*	*kW*	*kW*	*kW*
2600	1.34	1.57	1.80	2.01	2.43	2.67	2.92	3.17	3.42	0.00	0.04	0.07	0.11	0.14	0.17	0.21	0.24	0.28	0.31
2700	1.37	1.60	1.84	2.06	2.49	2.74	2.99	3.25	3.51	0.00	0.04	0.07	0.11	0.15	0.18	0.22	0.25	0.29	0.33
2800	1.40	1.64	1.88	2.10	2.55	2.80	3.06	3.31	3.57	0.00	0.04	0.08	0.11	0.15	0.19	0.23	0.26	0.30	0.34
2900	1.43	1.67	1.91	2.15	2.60	2.86	3.12	3.38	3.65	0.00	0.04	0.08	0.12	0.16	0.19	0.23	0.27	0.30	0.35
3000	1.46	1.71	1.95	2.19	2.66	2.92	3.18	3.45	3.72	0.00	0.04	0.08	0.12	0.16	0.20	0.24	0.28	0.32	0.36
3100	1.48	1.74	1.98	2.23	2.71	2.98	3.24	3.50	3.77	0.00	0.04	0.08	0.13	0.17	0.21	0.25	0.29	0.33	0.37
3200	1.51	1.77	2.02	2.28	2.75	3.03	3.29	3.56	3.83	0.00	0.04	0.09	0.13	0.17	0.22	0.26	0.30	0.34	0.39
3300	1.53	1.80	2.06	2.31	2.80	3.07	3.34	3.61	3.88	0.00	0.04	0.09	0.13	0.18	0.22	0.27	0.31	0.36	0.40
3400	1.55	1.82	2.10	2.34	2.83	3.11	3.30	3.65	3.92	0.00	0.05	0.09	0.14	0.18	0.23	0.27	0.32	0.37	0.41
3500	1.57	1.85	2.12	2.38	2.87	3.10	3.43	3.70	3.97	0.00	0.05	0.09	0.14	0.19	0.24	0.28	0.33	0.38	0.42
3600	1.59	1.87	2.14	2.41	2.91	3.19	3.47	3.73	4.00	0.00	0.05	0.10	0.15	0.19	0.24	0.29	0.34	0.39	0.44
3700	1.61	1.89	2.17	2.44	2.95	3.22	3.50	3.77	4.04	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45
3800	1.62	1.92	2.19	2.46	2.98	3.25	3.53	3.80	4.06	0.00	0.05	0.10	0.15	0.20	0.26	0.31	0.36	0.41	0.46
3900	1.64	1.93	2.22	2.49	2.99	3.28	3.56	3.82	4.08	0.00	0.05	0.11	0.16	0.21	0.26	0.31	0.37	0.42	0.47
4000	1.65	1.95	2.24	2.51	3.03	3.30	3.58	3.84	4.10	0.00	0.05	0.11	0.16	0.22	0.27	0.32	0.38	0.43	0.48
4100	1.67	1.96	2.25	2.53	3.05	3.32	3.60	3.85	4.10	0.00	0.06	0.11	0.17	0.22	0.28	0.33	0.39	0.44	0.50
4200	1.68	1.98	2.27	2.55	3.07	3.35	3.61	3.85	4.10	0.00	0.06	0.11	0.17	0.23	0.28	0.34	0.40	0.45	0.51
4300	1.69	1.99	2.28	2.57	3.08	3.35	3.62	3.85	4.10	0.00	0.06	0.12	0.17	0.23	0.29	0.35	0.40	0.46	0.52
4400	1.69	2.00	2.30	2.57	3.09	3.35	3.63	3.85	4.10	0.00	0.06	0.12	0.18	0.24	0.30	0.35	0.41	0.47	0.53
4500	1.70	2.01	2.31	2.59	3.10	3.35	3.63	3.85	4.07	0.00	0.06	0.12	0.18	0.24	0.30	0.36	0.42	0.48	0.54
4600	1.71	2.01	2.31	2.60	3.10	3.34	3.63	3.84	4.06	0.00	0.06	0.12	0.19	0.25	0.31	0.37	0.43	0.49	0.56
4700	1.71	2.02	2.32	2.60	3.10	3.34	3.62	3.82		0.00	0.06	0.13	0.19	0.25	0.32	0.38	0.44	0.51	0.57
4800	1.71	2.02	2.32	2.60	3.10	3.33	3.60	3.81		0.00	0.06	0.13	0.19	0.26	0.32	0.39	0.45	0.52	0.58
4900	1.71	2.02	2.32	2.60	3.10	3.32	3.58	3.79		0.00	0.07	0.13	0.20	0.26	0.33	0.39	0.46	0.53	0.59
5000	1.71	2.02	2.32	2.60	3.09	3.32	3.56	3.77		0.00	0.07	0.14	0.20	0.27	0.34	0.40	0.47	0.54	0.60

Table 13.17 *Power ratings in kW (Pr) for B-Section V-Belts, 17 mm wide with 180° arc of contact on smaller pulley*

Speed of faster shaft	Power rating for smaller pulley pitch diameter of									Additional power increment per belt for speed ratio of									
	125 mm	132 mm	140 mm	150 mm	160 mm	170 mm	180 mm	190 mm	200 mm	1.00 to 1.01	1.02 to 1.04	1.05 to 1.08	1.09 to 1.12	1.13 to 1.18	1.19 to 1.24	1.25 to 1.34	1.35 to 1.51	1.52 to 1.99	2.00 and over
rpm	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW
720	1.61	1.79	1.99	2.24	2.48	2.73	2.97	3.21	3.45	0.00	0.03	0.05	0.08	0.10	0.13	0.15	0.18	0.20	0.23
960	2.02	2.24	2.50	2.82	3.13	3.44	3.75	4.05	4.35	0.00	0.03	0.07	0.10	0.14	0.17	0.20	0.24	0.27	0.30
1440	2.72	3.03	3.39	3.83	4.26	4.68	5.09	5.50	5.90	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.36	0.41	0.46
2880	3.96	4.44	4.95	5.55	6.11	6.62	7.08	7.48	–	0.00	0.10	0.20	0.30	0.41	0.50	0.61	0.71	0.81	0.91
100	0.32	0.35	0.38	0.43	0.47	0.51	0.55	0.59	0.63	0.00	0.00	0.01	0.01	0.01	0.02	0.02	0.02	0.03	0.03
200	0.57	0.63	0.69	0.77	0.85	0.92	1.00	1.08	1.15	0.00	0.01	0.01	0.02	0.03	0.04	0.04	0.05	0.06	0.06
300	0.80	0.88	0.97	1.08	1.19	1.31	1.42	1.53	1.64	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
400	1.01	1.11	1.23	1.38	1.52	1.67	1.81	1.96	2.10	0.00	0.01	0.03	0.04	0.06	0.07	0.08	0.10	0.11	0.13
500	1.21	1.33	1.48	1.66	1.84	2.01	2.19	2.36	2.54	0.00	0.02	0.04	0.05	0.07	0.09	0.11	0.12	0.14	0.16
600	1.40	1.55	1.72	1.93	2.14	2.35	2.55	2.76	2.96	0.00	0.02	0.04	0.06	0.08	0.11	0.13	0.15	0.17	0.19
700	1.58	1.75	1.94	2.19	2.43	2.66	2.90	3.13	3.37	0.00	0.02	0.05	0.07	0.10	0.12	0.15	0.17	0.20	0.22
800	1.75	1.94	2.16	2.44	2.70	2.97	3.24	3.50	3.78	0.00	0.03	0.06	0.08	0.11	0.14	0.17	0.20	0.23	0.25
900	1.92	2.13	2.37	2.68	2.97	3.27	3.56	3.85	4.13	0.00	0.03	0.06	0.10	0.13	0.16	0.19	0.22	0.25	0.29
1000	2.08	2.31	2.58	2.91	3.23	3.55	3.87	4.18	4.49	0.00	0.04	0.07	0.10	0.14	0.18	0.21	0.25	0.28	0.32
1100	2.23	2.49	2.78	3.13	3.48	3.83	4.17	4.51	4.84	0.00	0.04	0.08	0.12	0.16	0.19	0.23	0.27	0.31	0.35
1200	2.38	2.66	2.96	3.35	3.72	4.09	4.46	4.81	5.17	0.00	0.04	0.08	0.13	0.17	0.21	0.25	0.30	0.34	0.38
1300	2.53	2.82	3.15	3.55	3.95	4.34	4.73	5.11	5.48	0.00	0.05	0.09	0.14	0.18	0.23	0.27	0.32	0.37	0.41
1400	2.66	2.97	3.32	3.75	4.17	4.59	4.98	5.39	5.78	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.39	0.44
1500	2.79	3.12	3.49	3.94	4.38	4.82	5.24	5.66	6.06	0.00	0.05	0.10	0.16	0.21	0.26	0.32	0.37	0.42	0.48
1600	2.92	3.26	3.65	4.12	4.58	5.04	5.48	5.91	6.33	0.00	0.06	0.11	0.17	0.23	0.28	0.34	0.39	0.45	0.51
1700	3.04	3.40	3.80	4.29	4.77	5.24	5.70	6.14	6.58	0.00	0.06	0.12	0.18	0.24	0.30	0.36	0.42	0.48	0.54
1800	3.15	3.52	3.94	4.45	4.95	5.44	5.90	6.36	6.80	0.00	0.06	0.13	0.19	0.25	0.32	0.38	0.44	0.51	0.57
1900	3.26	3.65	4.08	4.61	5.12	5.62	6.10	6.56	7.00	0.00	0.07	0.13	0.20	0.27	0.33	0.40	0.47	0.54	0.60
2000	3.36	3.76	4.21	4.75	5.28	5.78	6.27	6.74	7.19	0.00	0.07	0.14	0.21	0.28	0.35	0.42	0.49	0.56	0.63
2100	3.45	3.87	4.33	4.88	5.42	5.94	6.43	6.91	7.36	0.00	0.07	0.15	0.22	0.30	0.37	0.44	0.52	0.59	0.67
2200	3.54	4.00	4.44	5.01	5.55	6.07	6.58	7.05	7.50	0.00	0.08	0.16	0.23	0.31	0.39	0.46	0.54	0.62	0.70
2300	3.62	4.06	4.54	5.12	5.68	6.20	6.70	7.18	7.62	0.00	0.08	0.16	0.24	0.32	0.41	0.49	0.57	0.65	0.73

(Contd)

Table 13.17 *(Cond)*

Speed of faster shaft	Power rating for smaller pulley pitch diameter of									Additional power increment per belt for speed ratio of									
	125 mm	132 mm	140 mm	150 mm	160 mm	170 mm	180 mm	190 mm	200 mm	1.00 to 1.01	1.02 to 1.04	1.05 to 1.08	1.09 to 1.12	1.13 to 1.18	1.19 to 1.24	1.25 to 1.34	1.35 to 1.51	1.52 to 1.99	2.00 and over
rpm	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW
2400	3.70	4.14	4.63	5.22	5.78	6.31	6.81	7.28	7.72	0.00	0.08	0.17	0.25	0.34	0.42	0.51	0.59	0.68	0.76
2500	3.77	4.22	4.64	5.23	5.80	6.33	6.83	7.29	7.79	0.00	0.09	0.18	0.26	0.35	0.44	0.53	0.62	0.70	0.79
2600	3.82	4.28	4.79	5.39	5.95	6.48	6.97	7.42	7.83	0.00	0.09	0.18	0.27	0.37	0.46	0.55	0.64	0.73	0.82
2700	3.88	4.35	4.86	5.46	6.03	6.55	7.03	7.47	7.86	0.00	0.10	0.19	0.29	0.38	0.48	0.57	0.67	0.76	0.86
2800	3.93	4.40	4.91	5.52	6.08	6.60	7.06	7.48	7.85	0.00	0.10	0.20	0.29	0.39	0.49	0.59	0.69	0.79	0.89
2900	3.97	4.44	4.96	5.56	6.12	6.62	7.08	7.48		0.00	0.10	0.20	0.31	0.41	0.51	0.61	0.72	0.82	0.92
3000	4.00	4.48	4.99	5.59	6.14	6.63	7.07	7.44		0.00	0.11	0.21	0.32	0.42	0.53	0.63	0.74	0.85	0.95
3100	4.02	4.50	5.02	5.61	6.15	6.63	7.04			0.00	0.11	0.22	0.33	0.44	0.55	0.65	0.76	0.87	0.98
3200	4.04	4.52	5.03	5.62	6.14	6.60	6.99			0.00	0.11	0.23	0.34	0.45	0.56	0.68	0.79	0.90	1.01
3300	4.05	4.52	5.03	5.61	6.11	6.55				0.00	0.12	0.23	0.35	0.47	0.58	0.70	0.81	0.93	1.05
3400	4.05	4.52	5.02	5.58	6.07	6.48				0.00	0.12	0.24	0.36	0.48	0.60	0.72	0.84	0.96	1.08
3500	4.04	4.50	5.00	5.55	6.01					0.00	0.12	0.25	0.37	0.49	0.62	0.74	0.86	0.99	1.11
3600	4.02	4.48	4.96	5.49						0.00	0.13	0.25	0.38	0.51	0.63	0.76	0.89	1.01	1.14
3700	3.99	4.45	4.92	5.43						0.00	0.13	0.26	0.39	0.52	0.65	0.78	0.91	1.04	1.17
3800	3.95	4.40	4.86	5.34						0.00	0.13	0.27	0.40	0.54	0.67	0.80	0.94	1.07	1.20
3900	3.91	4.34	4.76							0.00	0.14	0.28	0.41	0.55	0.69	0.82	0.96	1.10	1.24
4000	3.85	4.28	4.70							0.00	0.14	0.28	0.42	0.56	0.70	0.84	0.99	1.13	1.27
4100	3.78	4.20								0.00	0.14	0.29	0.43	0.58	0.72	0.87	1.01	1.16	1.30
4200	3.71	4.11								0.00	0.15	0.30	0.44	0.59	0.74	0.89	1.04	1.18	1.33
4300	3.62	4.00								0.00	0.15	0.30	0.45	0.61	0.76	0.91	1.06	1.21	1.36
4400	3.53									0.00	0.15	0.31	0.46	0.62	0.78	0.93	1.08	1.24	1.39
4500	3.42									0.00	0.16	0.32	0.48	0.63	0.79	0.95	1.11	1.27	1.43

Table 13.18 *Power ratings in kW (P_r) for C-section V-Belts, 22 mm wide with 180° arc of contact on smaller pulley*

Speed of faster shaft	Power rating for smaller pulley pitch diameter of										Additional power increment per belt for speed ratio of									
	200 mm	212 mm	224 mm	236 mm	250 mm	265 mm	280 mm	315 mm	355 mm	400 mm	1.00 to 1.01	1.02 to 1.04	1.05 to 1.08	1.09 to 1.12	1.13 to 1.18	1.19 to 1.24	1.25 to 1.34	1.35 to 1.51	1.52 to 1.99	2.00 and over
rpm	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW
720	4.65	5.18	5.70	6.22	6.81	7.44	8.06	9.49	11.05	12.75	0.00	0.07	0.14	0.21	0.28	0.35	0.42	0.49	0.56	0.63
960	5.76	6.42	7.08	7.72	8.46	9.24	10.00	11.72	13.58	15.51	0.00	0.09	0.19	0.28	0.38	0.47	0.56	0.66	0.75	0.85
1440	7.49	8.36	9.21	10.03	10.95	11.91	12.82	14.76	16.67	–	0.00	0.14	0.28	0.42	0.56	0.71	0.85	0.99	1.13	1.27
100	0.93	1.02	1.11	1.20	1.30	1.42	1.53	1.78	2.07	2.39	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
200	1.66	1.83	2.00	2.16	2.36	2.57	2.77	3.25	3.79	4.39	0.00	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18
300	2.32	2.56	2.80	3.05	3.33	3.62	3.92	4.61	5.38	6.23	0.00	0.03	0.06	0.09	0.12	0.15	0.18	0.21	0.24	0.26
400	2.93	3.24	3.56	3.87	4.23	4.62	5.00	5.88	6.87	7.96	0.00	0.04	0.08	0.12	0.16	0.20	0.23	0.27	0.31	0.35
500	3.50	3.89	4.27	4.65	5.09	5.55	6.02	7.08	8.27	9.58	0.00	0.05	0.10	0.15	0.20	0.24	0.29	0.34	0.39	0.44
600	4.04	4.49	4.94	5.38	5.90	6.44	6.98	8.21	9.59	11.09	0.00	0.06	0.12	0.18	0.24	0.29	0.35	0.41	0.47	0.53
700	4.56	5.07	5.58	6.08	6.66	7.28	7.89	9.28	10.82	12.48	0.00	0.07	0.14	0.21	0.27	0.34	0.41	0.48	0.55	0.62
800	5.04	5.61	6.18	6.74	7.39	8.07	8.75	10.28	11.96	13.75	0.00	0.08	0.16	0.23	0.31	0.39	0.47	0.55	0.63	0.71
900	5.50	6.13	6.75	7.36	8.07	8.82	9.55	11.20	13.00	14.90	0.00	0.09	0.18	0.26	0.35	0.44	0.53	0.62	0.71	0.79
1000	5.93	6.61	7.29	7.95	8.71	9.51	10.29	12.05	13.94	15.90	0.00	0.10	0.20	0.29	0.39	0.49	0.59	0.69	0.78	0.88
1100	6.33	7.06	7.79	8.49	9.30	10.15	10.98	12.82	14.77	16.75	0.00	0.11	0.22	0.32	0.43	0.54	0.65	0.75	0.86	0.97
1200	6.71	7.49	8.25	9.00	9.85	10.74	11.60	13.50	15.48	17.43	0.00	0.12	0.24	0.35	0.47	0.59	0.70	0.82	0.94	1.06
1300	7.05	7.87	8.67	9.46	10.34	11.27	12.15	14.09	16.07	17.95	0.00	0.13	0.26	0.38	0.51	0.64	0.76	0.89	1.02	1.15
1400	7.37	8.23	9.06	9.87	10.79	11.74	12.64	14.59	16.52	18.27	0.00	0.14	0.27	0.41	0.55	0.69	0.82	0.96	1.10	1.23
1500	7.66	8.55	9.41	10.24	11.18	12.14	13.05	14.99	16.84		0.00	0.15	0.29	0.44	0.59	0.73	0.88	1.03	1.18	1.32
1600	7.91	8.83	9.71	10.56	11.52	12.48	13.39	15.28	17.00		0.00	0.16	0.31	0.47	0.63	0.78	0.94	1.10	1.25	1.41
1700	8.14	9.07	9.97	10.83	11.79	12.75	13.65	15.46			0.00	0.17	0.33	0.50	0.67	0.83	1.00	1.17	1.33	1.50
1800	8.32	9.28	10.19	11.05	12.00	12.95	13.82	15.52			0.00	0.18	0.35	0.53	0.71	0.88	1.06	1.23	1.41	1.59
1900	8.48	9.44	10.35	11.22	12.15	13.07	13.90				0.00	0.19	0.37	0.56	0.75	0.93	1.12	1.30	1.49	1.67
2000	8.59	9.56	10.47	11.32	12.24	13.12	13.89				0.00	0.20	0.39	0.59	0.78	0.98	1.17	1.37	1.57	1.76

(Contd)

Table 13.18 *(Contd)*

Speed of faster shaft	Power rating for smaller pulley pitch diameter of										Additional power increment per belt for speed ratio of									
	200 mm	212 mm	224 mm	236 mm	250 mm	265 mm	280 mm	315 mm	355 mm	400 mm	1.00 to 1.01	1.02 to 1.04	1.05 to 1.08	1.09 to 1.12	1.13 to 1.18	1.19 to 1.24	1.25 to 1.34	1.35 to 1.51	1.52 to 1.99	2.00 and over
rpm	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW
2100	8.67	9.64	10.54	11.37	12.25	13.08					0.00	0.21	0.41	0.62	0.82	1.03	1.23	1.44	1.65	1.85
2200	8.71	9.67	10.55	11.35	12.19						0.00	0.22	0.43	0.65	0.86	1.08	1.29	1.51	1.72	1.94
2300	8.71	9.65	10.50	11.27							0.00	0.23	0.45	0.68	0.90	1.13	1.35	1.58	1.80	2.03
2400	8.67	9.58	10.40	11.12							0.00	0.23	0.47	0.70	0.94	1.18	1.41	1.65	1.88	2.12
2500	8.58	9.47	10.24								0.00	0.24	0.49	0.74	0.98	1.22	1.47	1.71	1.96	2.20
2600	8.45	9.30									0.00	0.25	0.51	0.76	1.02	1.27	1.53	1.78	2.04	2.29
2700	8.28	9.07									0.00	0.26	0.53	0.79	1.06	1.32	1.59	1.85	2.12	2.38
2800	8.05										0.00	0.27	0.55	0.82	1.10	1.37	1.64	1.92	2.19	2.47

Table 13.19 *Power ratings in kW (P_r) for D-Section V- Belts, 32 mm wide with 180° arc of contact on smaller pulley*

Speed of faster shaft	Power rating for smaller pulley pitch diameter of										Additional power increment per belt for speed ratio of									
	355 mm	375 mm	400 mm	425 mm	450 mm	475 mm	500 mm	530 mm	560 mm	600 mm	1.00 to 1.01	1.02 to 1.04	1.05 to 1.08	1.09 to 1.12	1.13 to 1.18	1.19 to 1.24	1.25 to 1.34	1.35 to 1.51	1.52 to 1.99	2.00 and over
rpm	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW
720	16.26	17.90	19.90	21.85	23.75	23.59	27.38	29.44	31.42	33.91	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25
960	19.26	21.16	23.45	25.63	27.70	29.65	31.47	33.50	35.32		0.00	0.33	0.67	1.00	1.34	1.67	2.00	2.33	2.67	3.00
1440	21.22	23.03	–	–	–	–	–	–	–	–	0.00	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50
100	3.39	3.70	4.08	4.45	4.83	5.20	5.57	6.02	6.46	7.04	0.00	0.03	0.07	0.10	0.14	0.17	0.21	0.24	0.28	0.31
200	6.04	6.61	7.32	8.02	8.72	9.42	10.11	10.93	11.75	12.83	0.00	0.07	0.14	0.21	0.28	0.35	0.42	0.49	0.56	0.63
300	8.41	9.22	10.24	11.24	12.24	13.22	14.20	15.37	16.52	18.04	0.00	0.10	0.21	0.31	0.42	0.52	0.62	0.73	0.83	0.94
400	10.57	11.61	12.91	14.19	15.45	16.70	17.94	19.40	20.85	22.74	0.00	0.14	0.28	0.42	0.56	0.70	0.83	0.97	1.11	1.25
500	12.55	13.80	15.35	16.87	18.38	19.86	21.32	23.03	24.72	26.91	0.00	0.17	0.35	0.52	0.70	0.87	1.04	1.22	1.39	1.56
600	14.34	15.79	17.56	19.30	21.01	22.68	24.32	26.23	28.09	30.49	0.00	0.21	0.42	0.62	0.83	1.04	1.25	1.46	1.67	1.88
700	15.96	17.57	19.54	21.46	23.33	25.15	26.91	28.96	30.92	33.41	0.00	0.24	0.49	0.73	0.97	1.22	1.46	1.70	1.95	2.19
800	17.39	19.14	21.26	23.32	25.31	27.22	29.06	31.17	33.16	35.62	0.00	0.28	0.56	0.83	1.11	1.39	1.67	1.95	2.22	2.50
900	18.62	20.48	22.72	24.87	26.92	28.87	30.73	32.81	34.73	37.02	0.00	0.31	0.63	0.94	1.25	1.56	1.87	2.19	2.50	2.81
1000	19.64	21.57	23.88	26.07	28.14	30.07	31.86	33.82	35.57		0.00	0.35	0.70	1.04	1.39	1.74	2.08	2.43	2.78	3.13
1100	20.43	22.40	24.74	26.91	28.92	30.76	32.42				0.00	0.38	0.77	1.15	1.53	1.91	2.29	2.63	3.06	3.44
1200	20.98	22.96	25.26	27.36	29.25						0.00	0.42	0.84	1.25	1.67	2.09	2.50	2.92	3.34	3.75
1300	21.27	23.21	25.42	27.38							0.00	0.45	0.91	1.35	1.81	2.26	2.71	3.16	3.61	4.06
1400	21.29	23.15	25.21								0.00	0.49	0.98	1.46	1.95	2.43	2.92	3.40	3.89	4.38
1500	21.03	22.76									0.00	0.52	1.05	1.56	2.09	2.61	3.12	3.65	4.17	4.69
1600	20.46										0.00	0.56	1.11	1.67	2.23	2.78	3.33	3.89	4.45	5.00

Table 13.20 *Power ratings in kW (P_r) for E-Section V-Belts, 32 mm wide with 180° arc of contact on smaller pulley*

Speed of faster shaft	Power rating for smaller pulley pitch diameter of										Additional power increment per belt for speed ratio of									
	500 mm	530 mm	560 mm	600 mm	630 mm	650 mm	710 mm	750 mm	800 mm	900 mm	1.00 to 1.01	1.02 to 1.04	1.05 to 1.08	1.09 to 1.12	1.13 to 1.18	1.19 to 1.24	1.25 to 1.34	1.35 to 1.51	1.52 to 1.99	2.00 and over
rpm	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW	kW
585	26.6	28.2	32.1	35.6	37.7	40.1	43.7	47.4	50.8	56.0	0.00	0.44	0.89	1.33	1.77	2.21	3.07	3.11	3.54	3.98
720	30.8	32.0	35.3	39.0	41.1	44.2	47.2	50.1			0.00	0.55	1.09	1.64	2.19	2.73	3.28	3.85	4.37	4.92
960	31.1	33.5	36.5	-	-						0.00	0.73	1.46	2.18	2.91	3.64	4.37	5.08	5.82	6.54
50	3.83	4.13	4.92	5.03	5.33	5.77	6.20	6.80	7.38	8.45	0.00	0.04	0.08	0.08	0.14	0.19	0.23	0.26	0.31	0.34
100	6.85	7.41	8.19	9.11	9.68	10.49	11.3	12.4	13.4	15.3	0.00	0.08	0.14	0.23	0.31	0.37	0.45	0.53	0.61	0.68
150	9.59	10.4	11.4	12.7	13.6	14.7	15.9	17.6	19.1	21.8	0.00	0.11	0.23	0.34	0.45	0.56	0.68	0.79	0.91	1.02
200	12.1	13.1	14.6	16.3	17.3	18.0	20.4	22.3	24.3	27.7	0.00	0.14	0.31	0.45	0.61	0.76	0.91	1.06	1.20	1.36
250	14.4	13.7	17.5	19.5	20.8	22.6	24.3	26.7	29.1	32.9	0.00	0.19	0.37	0.56	0.76	0.95	1.14	1.33	1.52	1.70
300	16.6	18.2	20.1	22.4	24.0	26.1	28.2	30.9	33.7	38.2	0.00	0.23	0.45	0.68	0.91	1.14	1.36	1.59	1.83	2.05
350	18.7	20.5	22.8	25.3	27.0	29.4	31.7	34.8	37.7	42.8	0.00	0.26	0.53	0.79	1.06	1.33	1.59	1.86	2.12	2.43
400	20.7	22.4	25.1	27.9	29.7	31.5	34.9	38.2	41.4	46.6	0.00	0.31	0.61	0.91	1.21	1.52	1.83	2.12	2.43	2.73
450	22.4	24.4	27.3	30.4	32.2	35.0	37.7	41.3	44.7	50.2	0.00	0.34	0.68	1.02	1.36	1.71	2.05	2.39	2.73	3.07
500	24.0	26.3	29.2	32.6	34.5	37.5	40.4	44.0	47.4	50.5	0.00	0.37	0.76	1.13	1.52	1.89	2.28	2.65	3.03	3.41
550	25.6	27.9	31.0	34.4	36.6	39.6	42.5	45.2	49.6	55.0	0.00	0.42	0.84	1.24	1.66	2.08	2.50	2.91	3.33	3.75
600	27.0	29.3	32.6	36.2	38.3	41.5	44.3	48.0	51.4	56.4	0.00	0.45	0.91	1.38	1.83	2.28	2.73	3.18	3.63	4.09
650	28.1	30.5	33.9	37.5	39.7	42.8	45.8	49.3	52.5		0.00	0.50	0.98	1.47	1.97	2.45	2.95	3.44	3.94	4.43
700	29.0	31.6	35.0	38.7	40.8	43.9	46.8	50.2	53.0		0.00	0.53	1.06	1.60	2.12	2.65	3.18	3.70	4.25	4.77
750	29.8	32.5	35.9	39.5	41.7	44.7	47.3	50.4			0.00	0.56	1.14	1.70	2.28	2.84	3.41	3.98	4.54	5.11
800	30.5	33.1	36.4	40.0	42.1	44.9	47.3				0.00	0.61	1.21	1.83	2.43	3.04	3.63	4.25	4.85	5.46
850	30.8	33.4	36.7	40.3	42.1	44.9					0.00	0.65	1.29	1.93	2.57	3.22	3.86	4.50	5.15	5.80
900	31.1	33.7	36.9	40.2	41.8						0.00	0.68	1.36	2.05	2.73	3.41	4.09	4.77	5.46	6.15
950	31.0	33.4	36.6	39.5							0.00	0.73	1.44	2.16	2.88	3.60	4.31	5.04	5.75	6.48
1000	30.8	33.1	36.0								0.00	0.76	1.52	2.28	3.04	3.80	4.54	5.05	6.06	6.82
1050	30.4	32.6									0.00	0.79	1.60	2.39	3.18	3.98	4.77	5.57	6.37	7.16
1100	29.6										0.00	0.84	1.66	2.50	3.33	4.17	4.99	5.83	6.67	7.50

Table 13.21 *Correction factors for belt pitch length (F_c)*

Correction Factor	Belt pitch length (mm)					
	Belt cross section					
	Z	*A*	*B*	*C*	*D*	*E*
0.80		630				
0.81			930			
0.82		700		1560	2740	
0.83			1000			
0.84		790		1760		
0.85			1100			
0.86	405	890			3130	
0.87			1210	1950	3330	
0.88		990				
0.89						
0.90	475	1100	1370	2190	3730	4660
0.91				2340		
0.92	530		1560	2490	4080	5040
0.93		1250				
0.94				2720	4620	5420
0.95	625		1760	2800		
0.96		1430		3080		6100
0.97			1950		5400	
0.98	700	1550		3310		
0.99		1640	2180	3520		6850
1.00	780	1750	2300		6100	
1.02		1940	2500	4060		7650
1.03					6840	
1.04	920	2050	2700			
1.05		2200	2850	4600	7620	9150
1.06		2300				
1.07	1080				8410	9950
1.08		2480	3200	5380		
1.09		2570			9140	10710
1.10		2700	3600			
1.11				6100		
1.12		2910			10700	12230
1.13		3080	4060			
1.14		3290		6860		13750
1.15			4430			
1.16		3540	4820	7600	12200	
1.17			5000		13700	15280
1.18			5370			
1.19			6070		15200	16800
1.20				9100		
1.21				10700		

Table 13.22 *Correction factor for arc of contact (F_d)*

$\dfrac{D-d}{C}$	*Arc of contact on smaller pulley (in degrees)*	*Correction Factor F_d*
0.00	180	1.00
0.05	177	0.99
0.10	174	0.99
0.15	171	0.98
0.20	169	0.97
0.25	166	0.97
0.30	163	0.96
0.35	160	0.95
0.40	157	0.94
0.45	154	0.93
0.50	151	0.93
0.55	148	0.92
0.60	145	0.91
0.65	142	0.90
0.70	139	0.89
0.75	136	0.88
0.80	133	0.87
0.85	130	0.86
0.90	127	0.85
0.95	123	0.83
1.00	120	0.82

13.12 SELECTION OF V-BELTS

In practice, the designer has to select a V-belt from the catalogue of the manufacturer. The following information is required for the selection:

(i) Type of driving unit
(ii) Type of driven machine
(iii) Operational hours per day
(iv) Power to be transmitted
(v) Input and output speeds
(vi) Approximate centre distance depending upon the availability of space

The basic procedure for the selection of V-belts consists of the following steps:

(i) Determine the correction factor according to service (F_a) from Table 13.15. It depends upon the type of driving unit, the type of driven machine and the operational hours per day.
(ii) Calculate the design power by the following relationship:

$$\text{Design power} = F_a \text{ (transmitted power)}$$

(iii) Plot a point with design power as X co-ordinate and input speed as Y co-ordinate in Fig. 13.24. The location of this point decides the type of cross-section of the belt. In a borderline case, such as the point located on the borderline of cross-sections B and C, alternative calculations are made to find out the best cross-section.

(iv) Determine the recommended pitch diameter of the smaller pulley from Table 13.12. It depends upon the cross-section of the belt. Calculate the pitch diameter of the bigger pulley by the following relationship:

$$D = d\left[\frac{\text{speed of smaller pulley}}{\text{speed of bigger pulley}}\right]$$

$$= d\left[\frac{\text{input speed}}{\text{output speed}}\right]$$

The above values of D and d are compared with the preferred pitch diameters given in Table 13.13. In case of non-standard value, the nearest values of d and D should be taken from Table 13.13.

(v) Determine the pitch length of belt L by the following relationship,

$$L = 2C + \frac{\pi(D+d)}{2} + \frac{(D-d)^2}{4C}$$

(vi) Compare the above value of L with the preferred pitch length L in Table 13.14. In case of a non-standard value, the nearest value of pitch length from Table 13.14 should be taken.

(vii) Find out the correct centre distance C by substituting the above value of L in the following equation:

$$L = 2C + \frac{\pi(D+d)}{2} + \frac{(D-d)^2}{4C}$$

It is a quadratic equation in C.

(viii) Determine the correction factor (F_c) for belt pitch length from Table 13.21. It depends upon the type of cross-section and the pitch length of the belt.

(ix) Calculate the arc of contact for the smaller pulley by the following relationship:

$$\alpha_s = 180 - 2\sin^{-1}\left(\frac{D-d}{2C}\right)$$

Determine the correction factor (F_d) for the arc of contact from Table 13.22. It is not advisable to use an arc of contact less than 120° for V-belt drive. Therefore, the minimum arc of contact should be 120°.

(x) Depending upon the type of cross-section, refer to the respective table from Table 13.16 to Table 13.20 and determine the power rating (P_r) of single V-belt. It depends upon three factors—speed of faster shaft, pitch diameter of smaller pulley and the speed ratio.

(xi) The last step in the selection procedure is to find out the number of belts. It depends upon the design power and the power transmitting capacity of one belt. The number of belts is obtained by the following relationship:

$$\text{Number of belts} = \frac{P \times F_a}{P_r \times F_c \times F_d}$$

13.13 V-GROOVED PULLEY

The dimensions of V-grooved pulley[4] for V-belts are given in Table 13.23 and shown in Fig. 13.25. Such pulleys are usually made of grey cast iron of Grade-FG 250. In some applications, the pulleys are made of carbon steel casting. The notations used in the table are as follows:

l_p = Pitch width of pulley groove or pitch width of belt. It is the width of the belt at its neutral axis. The line in the V-belt, whose length remains unchanged when the belt is deformed under tension, is called its neutral axis.

b = Minimum height of groove above the pitch line

h = Minimum depth of groove below the pitch line

e = Centre to centre distance of adjacent grooves.

f = Distance of the edge of pulley to first groove center

α = Groove angle

d_p = Pitch diameter of pulley. It is diameter of the pulley measured at the pitch width of the groove

g = Minimum top width of the groove

[4] IS 3142–1993: Pulleys-V-Grooved Pulleys for endless V-belts sections Z, A, B, C, D and E and endless wedge belts sections SPZ, SPA, SPB and SPC-Specifications.

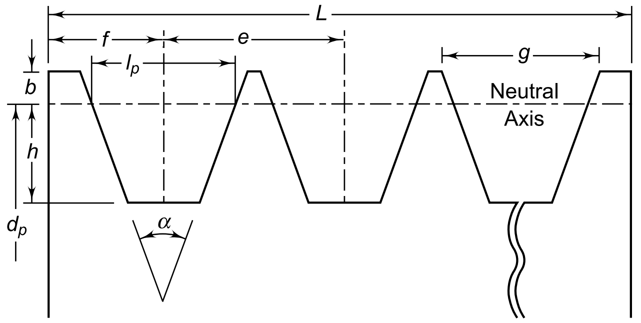


Fig. 13.25 *Dimensions of V-grooved pulley*

Table 13.23 *Dimensions of V-grooved pulleys*

Groove section	l_p *mm*	*b mm*	*h mm*	*e mm*	*f mm*	$\alpha°$	d_p *mm*	*g mm*	*Outside diameter*
Z	8.5	2.00	9	12 ± 0.3	7–9	34	Up to 80	9.7	$d_p + 4.0$
A	11	2.75	11	15 ± 0.3	9–12	34	Up to 118	12.7	$d_p + 5.5$
B	14	3.50	14	19 ± 0.4	11.5–14.5	34	Up to 190	16.1	$d_p + 7.0$
C	19	4.80	19	25.5 ± 0.5	16–19	34	Up to 315	21.9	$d_p + 9.6$
D	27	8.10	19.9	37 ± 0.6	23–27	36	Up to 499	21.9	$d_p + 16.2$
E	32	9.60	23.4	44.5 ± 0.7	28–33	36	Up to 629	38.2	$d_p + 19.2$
$L = (x - 1)\, e + 2f$ where x is the number of grooves									

Example 13.7 *The following data is given for an open-type V-belt drive:*

diameter of driving pulley = 200 mm
diameter of driven pulley = 600 mm
groove angle for sheaves = 34°
mass of belt = 0.5 kg/m
maximum permissible tension in belt = 500 N
coefficient of friction = 0.2
contact angle for smaller pulley = 157°
speed of smaller pulley = 1440 rpm
power to be transmitted = 10 kW

How many V-belts should be used, assuming each belt takes its proportional part of the load?

Solution

Given $kW = 10$ $n = 1440$ rpm $D = 600$ mm $d = 200$ mm $\theta = 34°$ $m = 0.5$ kg/m $f = 0.2$ $\alpha_s = 157°$ allowable belt tension = 500 N

Step I *Belt tensions*

$$\frac{f\alpha}{\sin\left(\frac{\theta}{2}\right)} = \frac{0.2\left(\frac{157}{180}\right)\pi}{\sin\left(\frac{34}{2}\right)} = 1.874$$

$$e^{f\alpha/\sin(\theta/2)} = e^{1.874} = 6.52$$

$$v = \frac{\pi d n}{60 \times 10^3} = \frac{\pi(200)(1440)}{60 \times 10^3} = 15.08 \text{ m/s}$$

$$mv^2 = 0.5(15.08)^2 = 113.70$$

From Eq. (13.7),

$$\frac{P_1 - mv^2}{P_2 - mv^2} = e^{f\alpha/\sin(\theta/2)}$$

$$\therefore \quad \frac{P_1 - 113.70}{P_2 - 113.70} = 6.52$$

$$P_1 - 6.52P_2 + 627.61 = 0 \qquad \text{(i)}$$

From Eq. (13.8),

$$\text{kW} = \frac{(P_1 - P_2)v}{1000} \quad \text{or} \quad 10 = \frac{(P_1 - P_2)(15.08)}{1000}$$

$$P_1 - P_2 - 663.13 = 0 \qquad \text{(ii)}$$

Solving Eq. (i) and (ii),

$$P_1 = 896.96 \text{ N} \quad \text{and} \quad P_2 = 233.83 \text{ N}$$

Step II *Number of belts*

$$\text{Number of belts} = \frac{\text{Maximum tension in belt}}{\text{Allowable belt load}}$$

$$= \frac{896.96}{500} = 1.79 \text{ or } 2 \text{ belts}$$

Example 13.8 *It is required to design a V-belt drive to connect a 7.5 kW, 1440 rpm induction motor to a fan, running at approximately 480 rpm, for a service of 24 h per day. Space is available for a centre distance of about 1 m.*

Solution

Given kW = 7.5 $n_1 = 1440$ rpm $n_2 = 480$ rpm
$C = 1$ m service = 24 h per day

Step I *Correction factor according to service (F_a)*

In this application, an induction motor is driving a fan of 7.5 kW capacity for 24 hours per day. From Table 13.15, the correction factor according to service (F_a) is 1.3.

Step II *Design power*

$$\text{Design power} = F_a \text{ (transmitted power)}$$
$$= 1.3(7.5) = 9.75 \text{ kW}$$

Step III *Type of cross-section for belt*

Plot a point with co-ordinates of 9.75 kW and 1440 rpm speed in Fig. 13.24. It is observed that the point is located in the region of the B-section belt. Therefore, for this application, the cross-section of V-belt is *B*.

Step IV *Pitch diameter of smaller and bigger pulleys*

From Table 13.12, the minimum pitch diameter for the smaller pulley is 200 mm.

$$\text{Speed ratio} = \frac{1440}{480} = 3$$

$$d = 200 \text{ mm} \quad \text{and} \quad D = 3(200) = 600 \text{ mm}$$

It is observed from Table 13.13 that both diameters have preferred values.

Step V *Pitch length of belt*

From Eq. (13.3),

$$L = 2C + \frac{\pi(D+d)}{2} + \frac{(D-d)^2}{4C}$$

$$= 2(1000) + \frac{\pi(600+200)}{2} + \frac{(600-200)^2}{4(1000)}$$

$$= 3296.64 \text{ mm}$$

Step VI *Preferred pitch length*

From Table 13.14, the preferred pitch length for B-section belt is 3200 or 3600 mm. It is assumed that the pitch length of the belt is 3200 mm.

Step VII *Correct centre distance*

Substituting this value of pitch length in Eq. (13.3),

$$L = 2C + \frac{\pi(D+d)}{2} + \frac{(D-d)^2}{4C}$$

$$3200 = 2C + \frac{\pi(600+200)}{2} + \frac{(600-200)^2}{4C}$$

Simplifying the above expression,

$$C^2 - 971.68C + 20000 = 0$$

$$C = \frac{971.68 \pm \sqrt{971.68^2 - 4(20000)}}{2}$$

$$= 950.64 \text{ mm}$$

The correct centre distance is 950.64 mm.

Step VIII *Correction factor for belt pitch length (F_c)*

From Table 13.21 (B-section and 3200 mm pitch length),

$$F_c = 1.08$$

Step IX *Correction factor for arc of contact (F_d)*

From Eq. (13.1),

$$\alpha_s = 180 - 2\sin^{-1}\left(\frac{D-d}{2C}\right)$$

$$= 180 - 2\sin^{-1}\left(\frac{600-200}{2 \times 950.64}\right)$$

$$= 155.71° \text{ or } 156°$$

From Table 13.22, F_d is approximately 0.94.

Step X *Power rating of single V-belt*

From Table 13.17, (1440 rpm, 200 mm pulley, B-section) (speed ratio = 3)

$$P_r = 5.90 + 0.46 = 6.36 \text{ kW}$$

Step XI *Number of belts*

From Eq.(13.12),

$$\text{Number of belts} = \frac{P \times F_a}{P_r \times F_c \times F_d}$$

$$= \frac{7.5(1.30)}{6.36(1.08)(0.94)}$$

$= 1.51$ or 2 belts

Example 13.9 *It is required to select a V-belt drive to connect a 15 kW, 2880 rpm normal torque A.C. motor to a centrifugal pump, running at approximately 2400 rpm, for a service of 18 hours per day. The centre distance should be approximately 400 mm. Assume that the pitch diameter of the driving pulley is 125 mm.*

Solution

Given kW = 15 $n_1 = 2880$ rpm $n_2 = 2400$ rpm $C = 400$ mm service = 18 h per day $d = 125$ mm

Step I *Correction factor according to service (F_a)*
In this application, a normal torque A.C. motor is driving the centrifugal pump of 15 kW capacity.

From Table 13.15 (normal torque A.C. motor and 18 h per day),

$$F_a = 1.2$$

Step II *Design power*

Design power = F_a (transmitted power)
= 1.2(15) = 18 kW

Step III *Type of cross-section for belt*
Plot a point with co-ordinates of 18 kW and 2880 rpm speed in Fig. 13.24. It is observed that the point is located in the region of the B-section belt. Therefore, for this application, the cross-section of the V-belt is *B*.

Step IV *Pitch diameter of smaller and bigger pulleys*
The pitch diameter for the smaller pulley is given as 125 mm.

$$\text{Speed ratio} = \frac{2880}{2400} = 1.2$$

$$d = 125 \text{ mm} \quad \text{and} \quad D = 1.2(125) = 150 \text{ mm}$$

It is observed from Table 13.13 that both diameters have preferred values.

Step V *Pitch length of belt*
From Eq. (13.3),

$$L = 2C + \frac{\pi(D+d)}{2} + \frac{(D-d)^2}{4C}$$

$$= 2(400) + \frac{\pi(150+125)}{2} + \frac{(150-125)^2}{4(400)}$$

$= 1232.36$ mm

Step VI *Preferred pitch length*
From Table 13.14, the preferred pitch length for B-section belt is 1210 or 1370 mm. It is assumed that the pitch length of the belt is 1210 mm.

Step VII *Correct centre distance*
Substituting this value of pitch length in Eq. (13.3),

$$L = 2C + \frac{\pi(D+d)}{2} + \frac{(D-d)^2}{4C}$$

$$1210 = 2C + \frac{\pi(150+125)}{2} + \frac{(150-125)^2}{4C}$$

Simplifying the above expression,

$$C^2 - 389\,C + 78.12 = 0$$

$$C = \frac{389 \pm \sqrt{389^2 - 4(78.12)}}{2} = 388.80 \text{ mm}$$

The correct centre distance is 388.80 mm.

Step VIII *Correction factor for belt pitch length (F_c)*
From Table 13.21 (B-section and 1210 mm pitch length),

$$F_c = 0.87$$

Step IX *Correction factor for arc of contact (F_d)*
From Eq. (13.1),

$$\alpha_s = 180 - 2\sin^{-1}\left(\frac{D-d}{2C}\right)$$

$$= 180 - 2\sin^{-1}\left(\frac{150-125}{2 \times 388.80}\right)$$

$= 176.32°$ or $177°$

From Table 13.22, $F_d = 0.99$

Step X *Power rating of single V-belt*
From Table 13.17, (2880 rpm, 125 mm pulley, B-section) (speed ratio = 1.2)

$$P_r = 3.96 + 0.50 = 4.46 \text{ kW}$$

Step XI *Number of belts*
From Eq.(13.12),

$$\text{Number of belts} = \frac{P \times F_a}{P_r \times F_c \times F_d} = \frac{15(1.20)}{4.46(0.87)(0.99)}$$

$= 4.69$ or 5 belts

Example 13.10 *It is required to select a V-belt drive from 5 kW normal torque motor, which runs at 1440 rpm to a light duty compressor running at 970 rpm. The compressor runs for 24 hours per day. Space is available for a centre distance of about 500 mm. Assume that the pitch diameter of the driving pulley is 150 mm.*

Solution

Given kW = 5 $n_1 = 1440$ rpm $n_2 = 970$ rpm
$C = 500$ mm service = 24 hours per day
$d = 150$ mm

Step I *Correction factor according to service (F_a)*
In this application, a normal torque motor is driving a compressor of 5-kW capacity. From Table 13.15, the correction factor according to service (F_a) is 1.2.

Step II *Design power*

$$\text{Design power} = F_a \text{ (transmitted power)} = 1.2(5) = 6 \text{ kW}$$

Step III *Type of cross-section for belt*
Plot a point with co-ordinates of 6 kW and 1440 rpm speed in Fig. 13.24. It is observed that the point is located in the region of the B-section belt. Therefore, for this application the cross-section of the V-belt is *B*.

Step IV *Pitch diameter of smaller and bigger pulleys*
The pitch diameter for the smaller pulley is given as 150 mm.

$$\text{Speed ratio} = \frac{1440}{970} = 1.485$$

$$d = 150 \text{ mm and } D = 1.485(150) = 222.68 \text{ mm}$$

It is observed from Table 13.13 that standard diameter for bigger pulley is 224 mm.

$$D = 224 \text{ mm}$$

Step V *Pitch length of belt*
From Eq. (13.3),

$$L = 2C + \frac{\pi(D+d)}{2} + \frac{(D-d)^2}{4C}$$

$$= 2(500) + \frac{\pi(224+150)}{2} + \frac{(224-150)^2}{4(500)}$$

$$= 1590.22 \text{ mm}$$

Step VI *Preferred pitch length*
From Table 13.14, the preferred pitch length for B-section belt is 1560 or 1690 mm. It is assumed that the pitch length of the belt is 1560 mm.

Step VII *Correct centre distance*
Substituting this value of pitch length in Eq. (13.3),

$$L = 2C + \frac{\pi(D+d)}{2} + \frac{(D-d)^2}{4C}$$

$$1560 = 2C + \frac{\pi(224+150)}{2} + \frac{(224-150)^2}{4C}$$

Simplifying the above expression,

$$C^2 - 486.26C + 684.5 = 0$$

$$C = \frac{486.26 \pm \sqrt{486.26^2 - 4(684.5)}}{2} = 484.85 \text{ mm}$$

The correct centre distance is 484.85 mm.

Step VIII *Correction factor for belt pitch length (F_c)*
From Table 13.21 (B-section and 1560 mm pitch length),

$$F_c = 0.92$$

Step IX *Correction factor for arc of contact (F_d)*
From Eq. (13.1),

$$\alpha_s = 180 - 2\sin^{-1}\left(\frac{D-d}{2C}\right)$$

$$= 180 - 2\sin^{-1}\left(\frac{224-150}{2 \times 484.85}\right) = 171.25° \text{ or } 171°$$

From Table 13.22, $F_d = 0.98$

Step X *Power rating of single V-belt*
From Table 13.17, (1440 rpm, 150 mm pulley, B-section) (speed ratio = 1.485)

$$P_r = 3.83 + 0.36 = 4.19 \text{ kW}$$

Step XI *Number of belts*
From Eq. (13.18),

$$\text{Number of belts} = \frac{P \times F_a}{P_r \times F_c \times F_d}$$

$$= \frac{5(1.20)}{4.19(0.92)(0.98)}$$

$$= 1.59 \text{ or } 2 \text{ belts}$$

13.14 BELT TENSIONING METHODS

A loose belt mounted on the pulleys does not transmit any load. Therefore, belts are provided with initial tension in order to transmit power. When a new belt is mounted on pulleys under tension, it loses its initial tension due to elongation during its service life. Therefore, a provision should be made to adjust the belt tension from time to time. There are number of methods to adjust the belt tensions. They are as follows:

(i) When the belts are hinged or laced, a short length of belt is cut periodically to remove the slack. The belt is joined again and mounted on the pulleys. This is not possible with endless belts.

(ii) In the second method, the centre distance between the pulleys is slightly increased by means of an adjusting screw. The electric motor carrying the driving pulley moves with respect to the bedplate by the adjusting screw.

(iii) A device for adjusting belt tension by means of an idler pulley is shown in Fig. 13.26. The idler pulley is held against the belt by its own weight, in addition to an adjustable weight. The idler pulley should be located next to the small pulley on the loose side of the belt. The belt is flat-faced without any crown and it can be removed from the belt when the machine is not working. The idler pulley increases the arc of contact and consequently the power transmitting capacity is also increased.

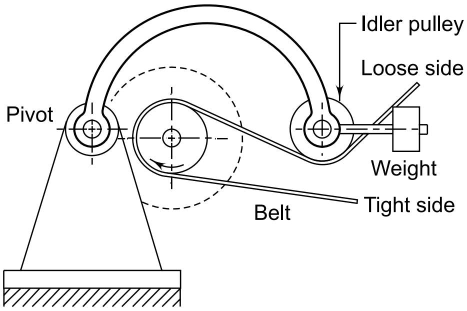


Fig. 13.26 *Idler Pulley Mechanism*

(iv) The most popular method is the Rockwood belt drive or the pivoted motor mounting as shown in Fig. 13.27. In this case, the belt tension is adjusted by changing the eccentricity e of the centre of gravity of the motor from the pivot. For the given configuration,

$$e = \frac{(P_1 a + P_2 b)}{W}$$

where W is the weight of the motor.

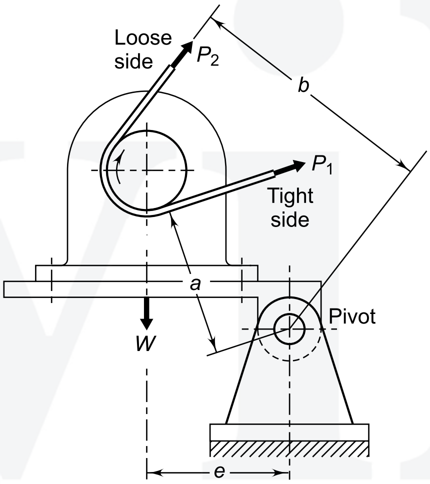


Fig. 13.27 *Pivoted Motor Construction*

13.15 RIBBED V-BELTS

Ribbed V-belts are flat belts with a series of evenly spaced teeth on the inside of the circumference, which mesh with the teeth in the pulley or sprocket as shown in Fig. 13.28(a). Therefore, they can maintain exactly the same angular position of the driven shaft with respect to driving shaft. A ribbed V-belt is a positive drive. These belts combine the high velocity characteristic of the belt drive with positive power transmission of the chain drive. There are different names for ribbed V-belt, such *as synchronous belt, timing belt* or *toothed belt*.

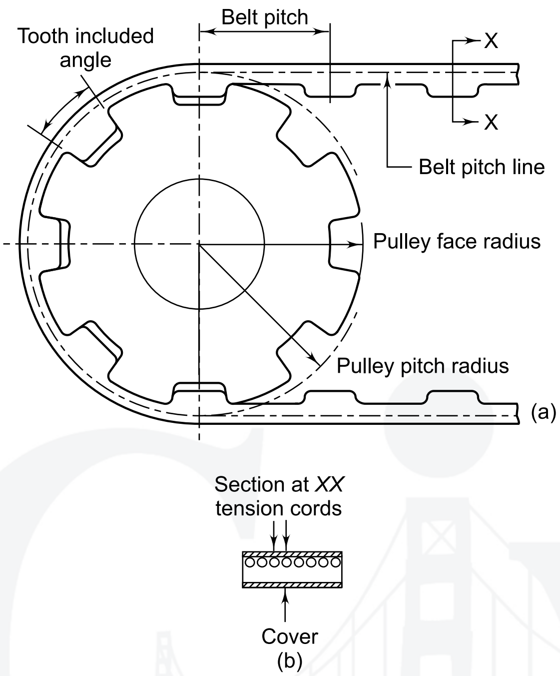


Fig. 13.28 *Ribbed V-belt*

The cross-section of ribbed V-belt is shown in Fig. 13.28(b). It consists of the following four components:

(i) Tension members consisting of steel or fibre cords, for transmitting the force or load

(ii) Rubber backing for protecting the load carrying tension members

(iii) Rubber teeth for engagement with the sprocket

(iv) Facing or cover, which protects the belt and prevents wear

There are three principle dimensions of ribbed V-belt, viz., pitch, pitch length and width. *Pitch* is the distance between two adjacent tooth centres measured along the pitch line of the belt. It is shown in Fig. 13.28(a). On the sprocket, pitch is the distance between two adjacent groove centres measured along the pitch circle of the sprocket. The pitch circle of the sprocket coincides with the pitch line of the belt. The pitch circle diameter of the sprocket is always more than the pulley face diameter. The *pitch length* is the total length of the belt measured along the belt pitch line. Ribbed V-belts are standardised on the basis of belt pitch. The standard pitches are as follows[5]:

Pitch code	*Belt pitch (mm)*	*Standard widths (mm)*
XL	5.080	6.4, 7.9, 9.5
L	9.525	12.7, 19.1, 25.4
H	12.700	19.1, 25.4, 38.1, 50.8, 76.2
XH	22.225	50.8, 76.2, 101.6
XXH	31.750	50.8, 76.2, 101.6, 127.0

Ribbed V-belts are selected from manufacturer's catalogue like flat or V-belts.

Ribbed V-belts offer the following advantages:

(i) It is a positive drive; there is no slip and no variation in output speed.

(ii) It has high strength to weight ratio, which allows for high pitch line velocities of up to 80 m/s.

(iii) The belt is thin and flexible, which permits the use of small diameter pulleys as small as 15 mm in diameter.

(iv) The length of the belt does not increase appreciably during service due to steel cords. Therefore, no tensioning device is required like flat belt drive.

(v) The ribbed V-belt drive does not require any lubrication like chain drive.

(vi) The ribbed V-belt does not require initial tension like flat belt. This reduces bearing reactions.

The disadvantages of ribbed V-belt drives are as follows:

(i) It is costly compared with flat or V-belts.

(ii) The construction of sprocket is difficult compared with the pulleys for flat or V-belts.

(iii) It is more sensitive to misalignment than V or flat belts.

Ribbed V-belts are used in automobiles for driving camshaft from an engine crankshaft. They are also used in business machines, sewing machines, portable wood working machines, timing devices and power transmission units.

[5] IS 9804–1981—Specification for synchronous belt drive belts.

Short-Answer Questions

13.1 Why are belt drives called 'flexible' drives?
13.2 What are the advantages of flat belt drive?
13.3 What are the disadvantages of flat belt drive?
13.4 What are the applications of flat belt drives?
13.5 What are the advantages of V-belts over flat belts?
13.6 What are the disadvantages of V-belts over flat belts?
13.7 What are the applications of V-belt drive?
13.8 What are the desirable properties of belt material?
13.9 What are the advantages of leather belts over fabric rubber belts?
13.10 What are the advantages of fabric rubber belts?
13.11 What do you understand by single-ply and double-ply belts?
13.12 Distinguish between open and cross belt drives.
13.13 State the law of belting.
13.14 What is belt rating?
13.15 State the different types of pulleys used in belt drives.
13.16 Why is the cross-section of the pulley an elliptical arm? Why is the major axis of the cross-section in the plane of rotation?
13.17 What is creep in belts?
13.18 What is the standard value of the belt angle for V-belt?
13.19 How will you designate V-belt?
13.20 What is the standard value for the groove angle of V-belt pulley?
13.21 What are the advantages of ribbed V-belts?
13.22 What are the disadvantages of ribbed V-belts?
13.23 What are the applications of ribbed V-belts?

Problems for Practice

13.1 The layout of a double-ply leather belt drive is shown in Fig. 13.29. The mass of the belt is 2 kg per metre length and the coefficient of friction is 0.35. Calculate (i) the tensions on the tight and loose sides, and (ii) the length of the belt. The belt is transmitting 35 kW of power.

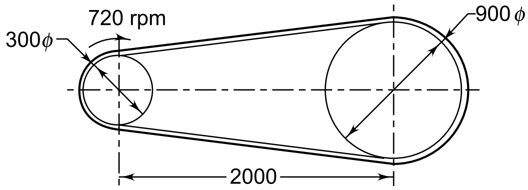


Fig. 13.29

[*(i) 5170.80 and 2076.19 N (ii) 5929.96 mm*]

13.2 The layout of a crossed leather belt drive transmitting 7.5 kW is shown in Fig. 13.30. The mass of the belt is 0.55 kg per metre length and the coefficient of friction is 0.30. Calculate (i) the belt tensions on the tight and loose sides, and (ii) the length of the belt.

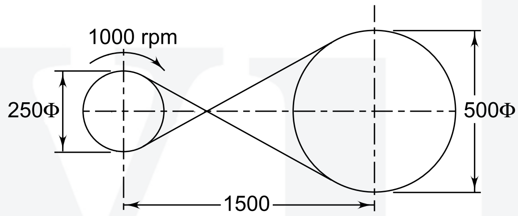


Fig. 13.30

[*(i) 955.12 and 382.16 N (ii) 4271.85 mm*]

13.3 The layout of a crossed leather belt drive is shown in Fig. 13.31. The belt, 6 mm thick, transmits 7.5 kW and operates at a velocity of 13 m/s approximately. The coefficient of friction is 0.3 and the permissible tensile stress for the belt material is 1.75 N/mm^2. The density of leather is 0.95 g/cc. Calculate (i) the diameters of pulleys; (ii) the length and width of the belt and (iii) belt tensions on the tight and loose sides.

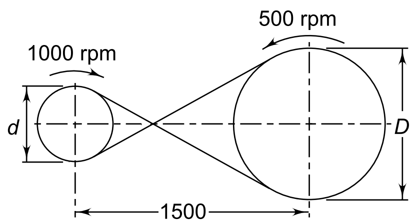


Fig. 13.31

[*(i) 250 and 500 mm (ii) 4271.85 and 90.4 mm (iii) 949.2 and 376.24 N*]

13.4 It is required to select a flat-belt drive for a fan running at 360 rpm which is driven by a 10 kW, 1440 rpm motor. The belt drive is open-type and space is available for a centre distance of 2 m approximately. The belt velocity should be between 17.8 to 22.9 m/s. The power transmitting capacity of the belt per mm width per ply at 180° arc of contact and at a belt velocity of 5.08 m/s is 0.0118 kW. The load correction factor can be taken as 1.2. Suggest preferred diameters for motor and fan pulleys and give complete specifications of belting.

[*250 and 1000 mm; 6.035 m length of 76 mm wide × 4 plies belting (L = 6033.81 mm)*] *(width × number of plies = 298.15)*

13.5 It is required to select a flat-belt drive to connect two transmission shafts rotating at 800 and 400 rpm respectively. The centre to centre distance between the shafts is approximately 3 m and the belt drive is open-type. The power transmitted by the belt is 30 kW and the load correction factor is 1.3. The belt should operate at a velocity between 17.8 to 22.9 m/s. The power transmitting capacity of the belt per mm width per ply at 180º arc of contact and at a belt velocity of 5.08 m/s is 0.0147 kW. Select preferred pulley diameters and specify the belt.

[*(i) 450 and 900 mm (ii) 8.2 m length of 152 mm wide × 5 plies belting (L = 8137.45 mm)*] *(width × number of plies = 740)*

13.6 It is required to select a V-belt drive to connect a 20-kW, 1440 rpm motor to a compressor running at 480 rpm for 15 hours per day. Space is available for a centre distance of approximately 1.2 m. Determine

(i) the specifications of the belt;
(ii) diameters of motor and compressor pulleys;
(iii) the correct centre distance; and
(iv) the number of belts.

[*(i) C 4600 L_p: V-belt (ii) 315 and 945 mm (iii) 1271.38 mm (iv) 2 (1.53) belts*]

13.7 A V-belt drive is required for a 15-kW, 1440 rpm electric motor, which drives a centrifugal pump running at 360 rpm for a service of 24 hours per day. From space considerations, the centre distance should be approximately 1 m. Determine

(i) belt specifications;
(ii) number of belts;
(iii) correct centre distance; and
(iv) pulley diameters.

[*(i) B 3600 L_p: V-belt (ii) 3 (3.097) belts (iii) 968.12 mm (iv) 200 and 800 mm*]

13.8 A Rockwood belt drive is shown in Fig. 13.27. The tensions on the tight and slack sides of the belt are 1500 N and 500 N respectively and the mass of the motor is 100 kg. The distances *a* and *b* are 75 mm and 250 mm respectively. Determine the required moment arm *e* for motor mounting.

[*242.1 mm*]

Chain Drives

Chapter 14

14.1 CHAIN DRIVES

A chain drive consists of an endless chain wrapped around two sprockets as shown in Fig. 14.1. *A chain can be defined as a series of links connected by pin joints*. The sprocket is a toothed wheel with a special profile for the teeth. The chain drive is intermediate between belt and gear drives. It has some features of belt

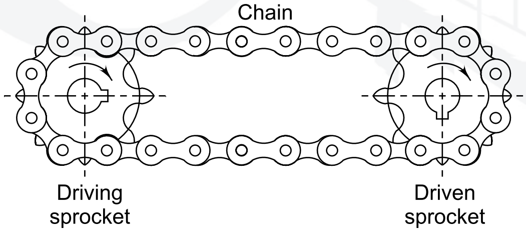


Fig. 14.1 *Chain Drives*

drives and some of gear drives. The advantages of chain drives compared with belt and gear drives are as follows:

(i) Chain drives can be used for long as well as short centre distances. They are particularly suitable for medium centre distance, where gear drives will require additional idler gears. Thus, chain drives can be used over a wide range of centre distances.

(ii) As shown in Fig. 14.2, a number of shafts can be driven in the same or opposite direction by means of the chain from a single driving sprocket.

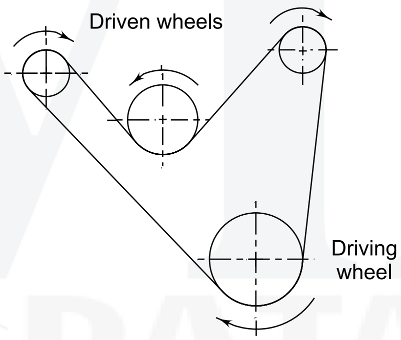


Fig. 14.2

(iii) Chain drives have small overall dimensions than belt drives, resulting in compact unit.

(iv) A chain does not slip and to that extent, chain drive is a positive drive.

(v) The efficiency of chain drives is high. For properly lubricated chain, the efficiency of chain drive is from 96% to 98%.

(vi) Chain does not require initial tension. Therefore, the forces acting on shafts are reduced.

(vii) Chains are easy to replace.

(viii) Atmospheric conditions and temperatures do not affect the performance of chain drives. They do not present any fire hazard.

The disadvantages of chain drives are as follows:

(i) Chain drives operate without full lubricant film between the joints unlike gears. This results in more wear at the joints. The wear increases the pitch of the chain. The chain is stretched out and may leave the sprocket, if tension is not adjusted from time to time.

(ii) Chain drives are not suitable for non-parallel shafts. Bevel and worm gears and quarter-turn belt drives can be used for non-parallel shafts.

(iii) Chain drive is unsuitable where precise motion is required due to polygonal effect. The velocity of the chain is not constant resulting in non-uniform speed of the driven shaft.

(iv) Chain drives require housing.

(v) Compared with belt drives, chain drives require precise alignment of shafts. However, the centre distance is not as critical as in the case of gear drive.

(vi) Chain drives require adjustment for slack, such as a tensioning device. Compared with the belt drive, chain drives require proper maintenance, particularly lubrication.

(vii) Chain drives generate noise.

Chain drives are popular in the transportation industry, such as bicycle, motorcycle and automobile vehicle. They are used in metal and wood working machinery for the transmission of power. They are widely used in agricultural machinery, oil-well drilling rigs, building construction and materials handling equipment. Chain drives are used for velocity ratios less than 10 : 1 and chain velocities of up to 25 m/s. In general, they are recommended to transmit power up to 100 kW.

There are different types of chains. With respect to their purpose, chains are classified into the following three groups:

(i) Load lifting chains
(ii) Hauling chains
(iii) Power transmission chains

Load lifting chains are used for suspending, raising or lowering loads in materials handling equipment. The popular example of this category is a '*link*' chain as illustrated in Fig. 14.3. Link chains are used in low capacity hoists, winches

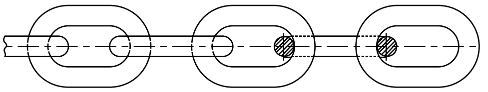


Fig. 14.3 *Link Chain*

and hand operated cranes. They offer the following advantages:

(i) They have good flexibility in all directions.
(ii) Link chains can operate with small diameter pulleys and drums.
(iii) They are simple to design and easy to manufacture.
(iv) They produce low noise and are practically noiseless at low speeds of less than 0.1 m/s.

The disadvantages of link chains are as follows:

(i) Link chain is heavy in weight.
(ii) It is susceptible to jerks and overloads.
(iii) The failure of link chain is sudden and total.
(iv) Link chains operate at low speed.

Hauling chains are used for carrying materials continuously by sliding, pulling or carrying in conveyors. The popular example of this category is a '*block*' chain as illustrated in Fig. 14.4. It consists of side plates of simple shapes and pins. It operates at medium velocities of up to 2 to 4 m/s. In general, hauling chains have long pitches because they have considerable length and mesh with sprockets whose size is not strictly limited. These chains are relatively noisy and wear rapidly because of the impact between the blocks and the sprocket. These chains are used only for conveyor applications.

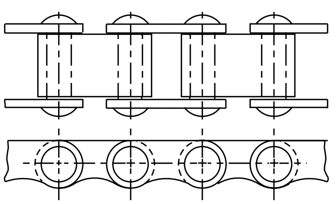


Fig. 14.4 *Block Chain*

Power transmission chains are used for transmitting power from one shaft to another. The discussion in this chapter is restricted to power transmission chains.

14.2 ROLLER CHAINS

The construction of a roller chain is shown in Fig. 14.5. It consists of alternate links made of inner and outer link plates. A roller chain consists of following five parts:

(i) Pin
(ii) Bushing
(iii) Roller
(iv) Inner link plate
(v) Outer link plate

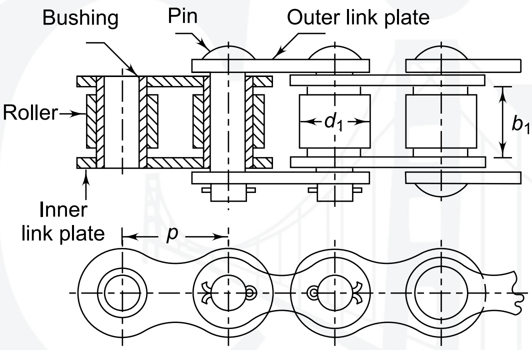


Fig. 14.5 *Construction of Roller Chain*

The pin is press fitted to two outer link plates, while the bush is press fitted to inner link plates. The bush and the pin form a swivel joint and the outer link is free to swivel with respect to the inner link. The rollers are freely fitted on bushes and, during engagement, turn with the teeth of the sprocket wheels. This results in rolling friction instead of sliding friction between roller and sprocket teeth. The rolling friction reduces wear and frictional power loss and improves the efficiency of the chain drive.

The inner and outer link plates are made of medium carbon steels. These link plates are blanked from cold-rolled sheets and hardened to 50 HRC. The pins, bushes and rollers are made of case carburising alloy steels and hardened to 50 HRC.

The pitch (p) of the chain is the linear distance between the axes of adjacent rollers. Roller chains are standardized and manufactured on the basis of the pitch. These chains are available in single-strand or multi-strand constructions such as simple, duplex or triplex chains as shown in Fig. 14.6. The dimensions and breaking load of standard chains[1] are given in Table 14.1. The roller chains are designated on the basis of 'pitch'. It is designated in the following way.

(i) In Table 14.1, the chain number is given in the first column, e.g., 08B or 16A. It consists of two parts—a number followed by a letter. The number in two digits expresses the 'pitch' in sixteenths of an '*inch*'. The letter A means American Standard ANSI series and the letter B means British

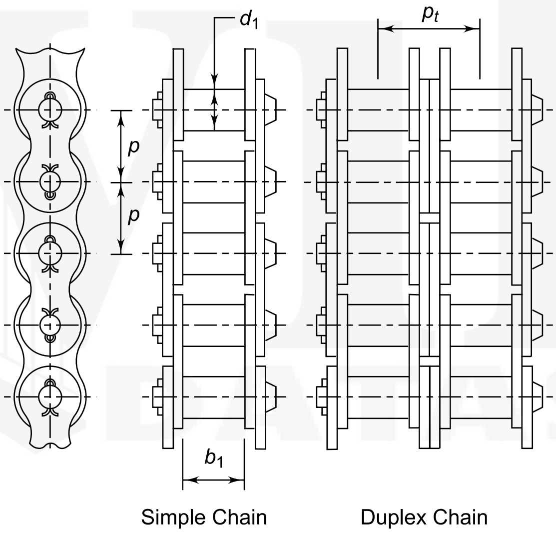


Fig. 14.6 *Simple and Duplex Chains*

Standard series. Most of the chain manufacturers are American and their ANSI series is popular in engineering industries.

(a) Let us consider the designation '08B'. The pitch of this chain is (08/16) inch or (08/16) × (25.4) mm, i.e., 12.7 mm. The letter B indicates British standard series.

(b) Let us consider the designation '16A'.

[1] IS 2403–1991: Short pitch transmission precision Chains and Chain Wheels.

The pitch of this chain is (16/16) inch or 1 inch, i.e., 25.4 mm. The letter A indicates American Standard ANSI series.

(ii) The chain number given in the first column is supplemented by a hyphenated suffix 1 for simple chain, 2 for duplex chain, 3 for triplex chain, and so on. For example,

08B-2 or 16A-1

These designations indicate

(a) 08B chain with duplex construction (double strand)

(b) 16A chain with simple construction (single strand)

There is a specific term, breaking load, in Table 14.1. *Breaking load* is defined as the maximum tensile load, which if applied will result in chain failure. In this definition, the failure is considered to have occurred at the first point, where increasing extension is no longer accompanied by increasing load, i.e., the limit of load-extension diagram.

The numbers given in brackets in Table 14.1 are equivalent and approximate chain numbers.

Table 14.1 *Dimensions and breaking loads of roller chains*

ISO chain number	*Pitch p (mm)*	*Roller diameter d_1 (mm) (max.)*	*Width b_1 (mm) (min.)*	*Transverse pitch p_t (mm)*	*Breaking load (min) N*		
					Simple	*Duplex*	*Triplex*
05B	8.00	5.00	3.00	5.64	4 400	7 800	11 100
06B	9.525	6.35	5.72	10.24	8 900	16 900	24 900
08A (ANSI-40)	12.70	7.95	7.85	14.38	13 800	27 600	41 400
08B	12.70	8.51	7.75	13.92	17 800	31 100	44 500
10A (ANSI-50)	15.875	10.16	9.4	18.11	21 800	43 600	65 400
10B	15.875	10.16	9.65	16.59	22 200	44 500	66 700
12A (ANSI-60)	19.05	11.91	12.57	22.78	31 100	62 300	93 400
12B	19.05	12.07	11.68	19.46	28 900	57 800	86 700
16A (ANSI-80)	25.40	15.88	15.75	29.29	55 600	111 200	166 800
16B	25.40	15.88	17.02	31.88	42 300	84 500	126 800
20A(ANSI-100)	31.75	19.05	18.90	35.76	86 700	173 500	260 200
20B	31.75	19.05	19.56	36.45	64 500	129 000	193 500
24A (ANSI-120)	38.10	22.23	25.22	45.44	124 600	249 100	373 700
24B	38.10	25.40	25.40	48.36	97 900	195 700	293 600
28A(ANSI-140)	44.45	25.40	25.22	48.87	169 000	338 100	507 100
28B	44.45	27.94	30.99	59.56	129 000	258 000	387 000
32A(ANSI-160)	50.80	28.58	31.55	58.55	222 400	444 800	667 200
32B	50.80	29.21	30.99	58.55	169 000	338 100	507 100
40A(ANSI-200)	63.50	39.68	37.85	71.55	347 000	693 900	1040 900
40B	63.50	39.37	38.10	72.29	262 400	524 900	787 300
48A	76.20	47.63	47.35	87.83	500 400	1000 800	1501 300
48B	76.20	48.26	45.72	91.21	400 300	800 700	1201 000
64B	101.60	63.50	60.96	119.89	711 700	1423 400	—

14.3 GEOMETRIC RELATIONSHIPS

The engagement of chain on sprocket wheel is shown in Fig. 14.7. D is the pitch circle diameter of the sprocket and α is called the *pitch angle*. The pitch circle diameter of the sprocket is defined as the diameter of an imaginary circle that passes through the centres of link pins as the chain is wrapped on the sprocket.

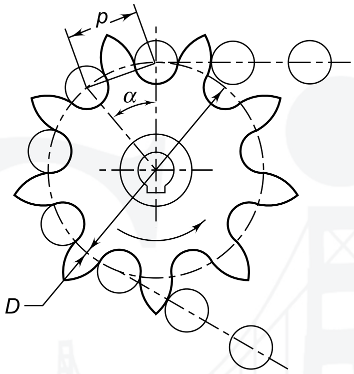


Fig. 14.7

$$\alpha = \frac{360}{z} \tag{14.1}$$

where z is the number of teeth on the sprocket. From the figure, it can be proved that

$$\sin\left(\frac{\alpha}{2}\right) = \frac{(p/2)}{(D/2)} \quad \text{or} \quad D = \frac{p}{\sin\left(\frac{\alpha}{2}\right)}$$

$$\therefore \quad D = \frac{p}{\sin\left(\frac{180}{z}\right)} \tag{14.2}$$

The velocity ratio i of the chain drives is given by,

$$i = \frac{n_1}{n_2} = \frac{z_2}{z_1} \tag{14.3}$$

where

n_1, n_2 = speeds of rotation of driving and driven shafts (rpm)

z_1, z_2 = number of teeth on driving and driven sprockets.

The average velocity of the chain is given by,

$$v = \frac{\pi D n}{60 \times 10^3}$$

$$\therefore \quad v = \frac{zpn}{60 \times 10^3} \tag{14.4}$$

where v is the average velocity in m/s.

The length of the chain is always expressed in terms of the number of links, or

$$L = L_n \times p \tag{14.5}$$

where

L = length of the chain (mm)

L_n = number of links in the chain

The number of links in the chain is determined by the following approximate relationships:

$$L_n = 2\left(\frac{a}{p}\right) + \left(\frac{z_1 + z_2}{2}\right) + \left(\frac{z_2 - z_1}{2\pi}\right)^2 \times \left(\frac{p}{a}\right) \tag{14.6}$$

where,

a = centre distance between axes of driving and driven sprockets (mm)

z_1 = number of teeth on the smaller sprocket

z_2 = number of teeth on the larger sprocket

The above formula is derived by analogy with the length of the belt. The first two terms represent the number of links when ($z_1 = z_2$) and the sides of the chain are parallel. The third term takes into consideration the inclination of the sides. It is obvious that the chain should contain a whole number of links. Therefore, the number of links (L_n) is adjusted to the previous or next digit so as to get an even number. It is always preferred to have an '*even*' number of links, since the chain consists of alternate pairs of inner and outer link plates.

When the chain has an odd number of links, an additional link, called '*offset*' link, is provided. The offset link is, however, weaker than the main links. After selecting the exact number of links, the centre to centre distance between the axes of the two sprockets is calculated by the following formula:

$$a = \frac{p}{4}\left\{\left[L_n - \left(\frac{z_1 + z_2}{2}\right)\right] + \sqrt{\left[L_n - \left(\frac{z_1 + z_2}{2}\right)\right]^2 - 8\left[\frac{z_2 - z_1}{2\pi}\right]^2}\right\} \tag{14.7}$$

The above equation can be easily derived from Eq. (14.6). The centre distance calculated by the formula does not provide any sag. In practice, a small amount of sag is essential for the links to take the best position on the sprocket wheel. The centre distance is, therefore, reduced by a margin of (0.002a to 0.004a) to account for the sag.

14.4 POLYGONAL EFFECT

The chain passes around the sprocket as a series of chordal links. This action is similar to that of a non-slipping belt wrapped around a rotating polygon. The chordal action is illustrated in Fig. 14.8, where the sprocket has only four teeth. It is assumed that the sprocket is rotating at a constant speed of n rpm. In Fig. 14.8(a), the chain link AB is at a distance of $\left(\frac{D}{2}\right)$ from the centre of the sprocket wheel and its linear velocity is given by,

$$v_{max.} = \frac{\pi D n}{60 \times 10^3} \text{ m/s} \qquad \text{(a)}$$

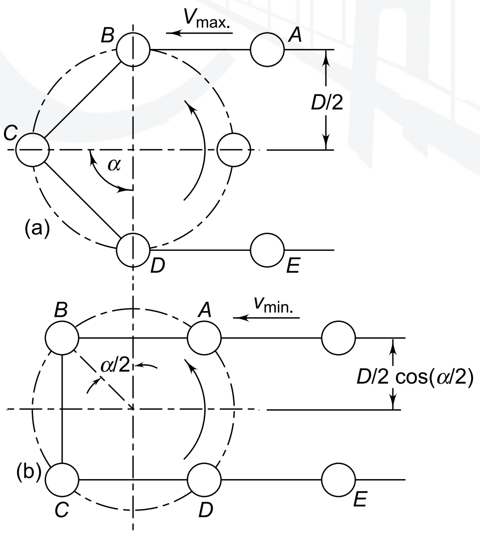


Fig. 14.8 *Polygonal action of Chain*

As the sprocket rotates through an angle $\left(\frac{\alpha}{2}\right)$, the position of the chain link AB is shown in Fig. 14.8(b). In this case, the link is at a distance of $\frac{D}{2} \times \cos\left(\frac{\alpha}{2}\right)$ from the centre of the sprocket and its linear velocity is given by,

$$v_{min.} = \frac{\pi D n \cos\left(\frac{\alpha}{2}\right)}{60 \times 10^3} \text{ m/s} \qquad \text{(b)}$$

It is evident that the linear speed of the chain is not uniform but varies from $v_{max.}$ to $v_{min.}$ during every cycle of tooth engagement. This results in a pulsating and jerky motion. The variation in velocity is given by

$$(v_{max.} - v_{min.}) \propto \left[1 - \cos\left(\frac{\alpha}{2}\right)\right]$$

or
$$(v_{max.} - v_{min.}) \propto \left[1 - \cos\left(\frac{180}{z}\right)\right]$$

As the number of teeth (z) increases to ∞, cos (180/z) or cos (180/∞), i.e., cos (0°) will approach unity and ($v_{max.} - v_{min.}$) will become zero. Therefore, the variation will be zero. In order to reduce the variation in chain speed, the number of teeth on the sprocket should be increased. It has been observed that the speed variation is 4% for a sprocket with 11 teeth, 1.6% for a sprocket with 17 teeth, and less than 1% for a sprocket with 24 teeth.

For smooth operation at moderate and high speeds, it is considered a good practice to use a driving sprocket with at least 17 teeth. From durability and noise considerations, the minimum number of teeth on the driving sprocket should be 19 or 21.

14.5 POWER RATING OF ROLLER CHAINS

The power transmitted by the roller chain can be expressed by the elementary equation

$$\text{kW} = \frac{P_1 v}{1000}$$

where

P_1 = allowable tension in the chain (N)

v = average velocity of chain (m/s)

However, it is not easy to determine the allowable tension in the chain. It depends upon a number of factors, such as the type of chain, pitch of the chain link, number of teeth on the smaller sprocket, chain velocity, the type of power source and driven machinery and the system of lubrication. In practice, the power rating of the roller chain is obtained on the basis of four failure criteria, viz., wear, fatigue, impact and galling.

(i) Wear The wear of the chain is caused by the articulation of pins in the bushings. The wear results in elongation of the chain, or in other words, the chain pitch is increased. This makes the chain '*ride out*' on the sprocket teeth, resulting in a faulty engagement. When the elongation is excessive, it becomes necessary to replace the chain. The permissible elongation for the chain is 1.5 to 2.5%. When the chain is properly lubricated, a layer of oil film between the contacting surfaces of the pin and the bushing reduces wear.

(ii) Fatigue As the chain passes around the sprocket wheel, it is subjected to a tensile force, which varies from a maximum on the tight side to a minimum on the loose side. The chain link is, therefore, subjected to one complete cycle of fluctuating stresses during every revolution of the sprocket wheel. This results in a fatigue failure of side link plates. For infinite life, the tensile stress should be lower than the endurance limit of the link plates.

(iii) Impact The engagement of rollers with the teeth of the sprocket results in impact. When excessive, this may lead to the breakage of roller or bushing. Increasing the number of teeth on the sprocket or reducing chain tension and speed reduces the magnitude of the impact force.

(iv) Galling Galling is a stick-slip phenomenon between the pin and the bushing. When the chain tension is high, welds are formed at the high spots of the contacting area. Such microscopic welds are immediately broken due to relative motion of contacting surfaces and leads to excessive wear, even in the presence of the lubricant.

The manufacturer's catalogues give extensive details, like charts and tables, for the power rating of chains. They are based on the above criteria of chain failure. Since the chain manufacturing is a specialized industry, like ball and roller bearings, it is necessary for the designer to select a proper chain from these catalogues. Table 14.2 gives values of power rating for simple (single-strand) roller chains. These values are based on the assumption that there are 17 teeth on the driving sprocket wheel.

Table 14.2 *Power rating of simple roller chain*

Pinion speed (rpm)	*Power (kW)*								
	06 B	*08A*	*08 B*	*10A*	*10 B*	*12A*	*12 B*	*16A*	*16 B*
50	0.14	0.28	0.34	0.53	0.64	0.94	1.07	2.06	2.59
100	0.25	0.53	0.64	0.98	1.18	1.74	2.01	4.03	4.83
200	0.47	0.98	1.18	1.83	2.19	3.40	3.75	7.34	8.94
300	0.61	1.34	1.70	2.68	3.15	4.56	5.43	11.63	13.06
500	1.09	2.24	2.72	4.34	5.01	7.69	8.53	16.99	20.57
700	1.48	2.95	3.66	5.91	6.71	10.73	11.63	23.26	27.73
1000	2.03	3.94	5.09	8.05	8.97	14.32	15.65	28.63	34.89
1400	2.73	5.28	6.81	11.18	11.67	14.32	18.15	18.49	38.47
1800	3.44	6.98	8.10	8.05	13.03	10.44	19.85	—	—
2000	3.80	6.26	8.67	7.16	13.49	8.50	20.57	—	—

For a given application, the kW rating of the chain is determined by the following relationship:

kW rating of chain

$$= \frac{(\text{kW to be transmitted}) \times K_s}{K_1 \times K_2} \tag{14.8}$$

where

K_s = service factor

K_1 = multiple strand factor

K_2 = tooth correction factor

The service factor takes into consideration the effect of shocks and vibrations on the power to be transmitted. The power rating in Table 14.2 is based on 17 teeth on the driving wheel. In a given application, the number of teeth on the driving wheel can be less than or more than 17. The tooth correction factor K_2 accounts for this variation. Similarly, the values of the power rating given in Table 14.2 are based on the assumption that the chain has a single strand. There are duplex and triplex chains too. This variation in the number of strands is taken into account by the multiple strand factor K_1. The values of K_s, K_1 and K_2 are given in Tables 14.3, 14.4 and 14.5 respectively[2].

Table 14.3 *Service factor (K_s)*

Type of driven load	*Type of input power*		
	IC engine with hydraulic drive	*Electric motor*	*IC engine with mechanical drive*
(i) *Smooth:* agitator, fan, light conveyor	1.0	1.0	1.2
(ii) *Moderate shock*: machine tools, crane, heavy conveyor, food mixer, grinder	1.2	1.3	1.4
(iii) *Heavy shock:* punch press, hammer mill, reciprocating conveyor, rolling mill drive	1.4	1.4	1.7

Table 14.4 *Multiple strand factor (K_1)*

Number of strands	K_1
1	1.0
2	1.7
3	2.5
4	3.3
5	3.9
6	4.6

Table 14.5 *Tooth correction factor (K_2)*

Number of teeth on the driving sprocket	K_2
15	0.85
16	0.92
17	1.00
18	1.05
19	1.11
20	1.18
21	1.26
22	1.29
23	1.35
24	1.41
25	1.46
30	1.73

For a satisfactory performance of roller chains, the centre distance between the sprockets should provide at least a 120° wrap angle on the smaller sprocket. In practice, the recommended centre distance is between 30 to 50 chain pitches.

Therefore, $$30p < a < 50p \tag{14.9}$$

The expected service life of these chains is 15,000 hours. The velocity ratio should be kept below 6 : 1 to get a satisfactory performance.

14.6 SPROCKET WHEELS

There are different constructions for sprocket wheels as shown in Fig. 14.9. Small sprockets up to 100 mm in diameter are usually made of a disk or a solid disk with a hub on one side (Fig. 14.9a and b). They are machined from low carbon steel bars. Large sprockets with more than 100 mm diameter are either welded to a steel hub or bolted to a cast iron hub (Fig. 14.9c and d). In general, sprockets are made of low carbon or medium carbon steels. In certain applications, stainless steel is used for sprockets. When the chain velocity is less than 180 m/min, the teeth of the sprocket wheel are heat-treated to obtain a hardness of 180 BHN. For

[2] *Courtesy*: Diamond Chain Company, Indianapolis, Indiana, USA.

high speed applications, the recommended surface hardness is 300 to 500 BHN. The teeth are hardened either by carburising in case of low carbon steel or by quenching and tempering in case of high carbon steel.

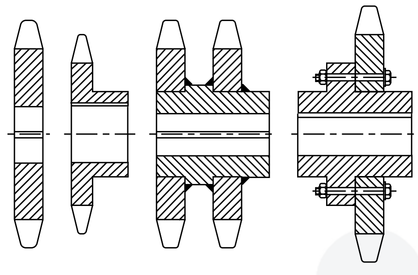


Fig. 14.9 *Construction of Sprocket Wheels*

The difference between the gear and the sprocket is as follows:

(i) A gear meshes with another gear. A sprocket meshes with an '*intermediate*' link, namely chain, which in turn meshes with another sprocket.

(ii) The face width of gear is usually more with respect to its diameter. The sprockets are comparatively thin so as to fit between inner link plates of the chain.

(iii) The teeth of gears have involute profile, while circular arcs are used for the profile of sprocket teeth.

There are standard profiles for the teeth of a sprocket wheel, as illustrated in Figs 14.10 and 14.11. The principal dimensions of the tooth profile are given in Table 14.6.

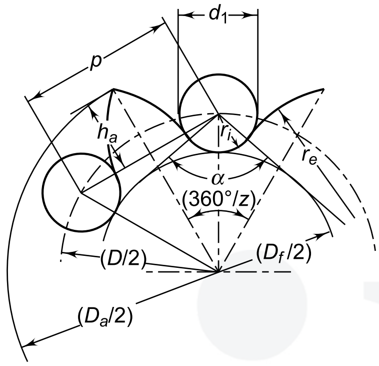


Fig. 14.10 *Tooth Profile of Sprocket*

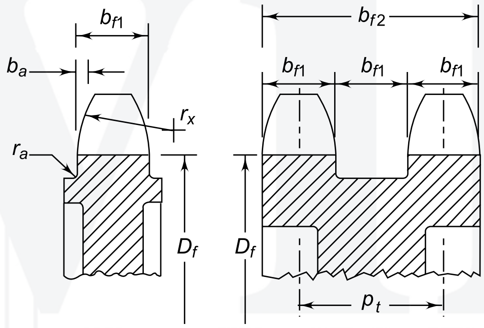


Fig. 14.11 *Rim Profile of Sprocket*

Table 14.6 *Proportions of the sprocket wheel (Figs 14.10 and 14.11)*

Dimension	*Notation*	*Equation*
1. Chain pitch	p	(Table 14.1)
2. Pitch circle diameter	D	$D = \dfrac{p}{\sin\left(\dfrac{180}{z}\right)}$
3. Roller diameter	d_1	(Table 14.1)
4. Width between inner plates	b_1	(Table 14.1)
5. Transverse pitch	p_t	(Table 14.1)
6. Top diameter	D_a	$(D_a)_{max.} = D + 1.25p - d_1$ $(D_a)_{min.} = D + p\left(1 - \dfrac{1.6}{z}\right) - d_1$
7. Root diameter	D_f	$D_f = D - 2r_i$

(Contd)

Table 14.6 *(Contd)*

8. Roller seating radius	r_i	$(r_i)_{max.} = (0.505d_1 + 0.069\sqrt[3]{d_1})$ $(r_i)_{min.} = 0.505d_1$
9. Tooth flank radius	r_e	$(r_e)_{max.} = 0.008d_1\,(z^2 + 180)$ $(r_e)_{min.} = 0.12d_1\,(z + 2)$
10. Roller seating angle	α	$\alpha_{max.} = \left[120 - \dfrac{90}{z}\right]$ $\alpha_{min.} = \left[140 - \dfrac{90}{z}\right]$
11. Tooth height above the pitch polygon	h_a	$(h_a)_{max.} = 0.625p - 0.5d_1 + \dfrac{0.8p}{z}$ $(h_a)_{min.} = 0.5(p - d_1)$
12. Tooth side radius	r_x	$(r_x)_{min.} = p$
13. Tooth width	b_{f1}	$b_{f1} = 0.93\,b_1$ if $p \le 12.7$ mm $b_{f1} = 0.95\,b_1$ if $p > 12.7$ mm
14. Tooth side relief	b_a	$b_a = 0.1p$ to $0.15p$

14.7 DESIGN OF CHAIN DRIVE

There are two important rules in the design of a chain drive. They are as follows:

(i) The number of pitches or links of the chain should be always 'even'.

(ii) The number of teeth on the driving sprocket should be always 'odd', such as 17, 19 or 21.

The odd number of teeth of the sprocket, in combination of even number of chain links, facilitates uniform wear. In this combination, every time a new link comes in contact with a particular tooth on the sprocket and the wear is distributed. In a sprocket having even number of teeth, alternate teeth shows greater wear than the intermediate ones. Uniform wear of all teeth is particularly desirable when an old chain is to be replaced by a new one.

As the chain is elongated due to wear, it has a tendency to shift 'outward' upon the profiles of sprocket teeth. Smaller the angular pitch of the sprocket (angle between adjacent teeth), greater is the outward shift. As the number of teeth on a sprocket increases, the angular pitch decrease and a small elongation of the chain leads to a large outward shift of the chain. In such cases, the chain leaves the sprocket. This outward shift limits the number of teeth on the sprocket. The maximum number of teeth on the driven sprocket is 100 to120.

Chain drives can be arranged as vertical or horizontal. As far as possible, vertical chain drives should be avoided. In case of a vertical drive, due to sag, the tendency of the chain is to leave the profile of teeth at the lower side of the lower sprocket. Therefore, vertical arrangement requires more careful adjustment of chain tension in order to prevent the outward shift of chain. Horizontal chain drives are always preferred. As shown in Fig. 14.12, the lower strand should be the slack side. This is

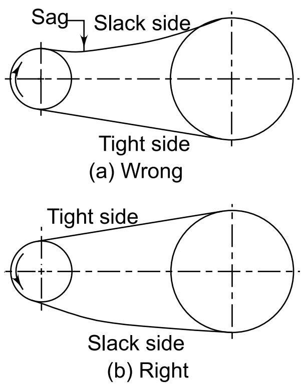


Fig. 14.12 *Drive Arrangement*

opposite to that of a belt drive. Keeping the driving or tight side on the top has the following advantages:

(i) The tendency of sagging portion of the chain to engage additional teeth on the sprocket is prevented.

(ii) In case of chain drives with very long centre distances, the contact between the upper and lower strands is avoided. This is because the tendency of the lower strand is to move away from the upper strand. On the contrary, when the slack side is on the top, the tendency of the upper strand is to move towards the lower strands.

Conclusions

(i) The chain drive should be horizontal.

(ii) The driving or tight side should be on the top.

Adequate tension in the chain is the most important requirement of chain drive. Chain life is reduced if the chain is too tight or too loose. Too tight a chain results in an unnecessary additional load in the chain and increased bearing reactions. Too loose a chain causes vibrations called '*whipping*' in the slack strand. These vibrations cause extra wear at the joints and induce fluctuating stresses in the parts of the chain.

There are two methods for adjustment of chain tension. They are as follows:

(i) Change the centre distance by moving the axis of one of the sprockets.

(ii) Provide an adjustable idler sprocket.

In the first method, the centre distance between the sprockets is periodically increased by means of an adjusting screw. The electric motor carrying the driving sprocket moves with respect to the bedplate by the adjusting screw. This device is capable to compensate for chain elongation up to the length of two links. After this, two links of the chain are removed and the chain is reassembled and reused. The second method of using the idler sprocket is illustrated in Fig. 14.13. The idler sprocket is installed on the driven (slack) side of the chain at a place where the sag is maximum.

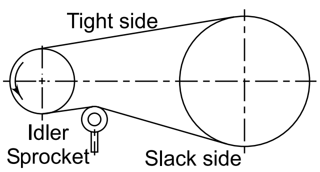


Fig. 14.13 *Chain Tightener*

The general recommendations for the design of chain drive are as follows:

(i) The minimum number of teeth on the driving sprocket is 17. From durability and noise considerations, the minimum number of teeth should be 21.

(ii) When the drive operates at low speed such as 100 rpm, the number of teeth on the driving sprocket can be less than 17. In such cases, the number of teeth is taken as 13 or 15.

(iii) The minimum number of teeth and consequently minimum sprocket diameter is sometimes restricted by the size of the shaft on which the driving sprocket is mounted. The diameter of the driving sprocket should be more than the shaft diameter.

(iv) The centre distance between the axes of the driving and driven shafts should be between 30 to 50 times of the pitch of the chain.

(v) The arc of contact of the chain on the smaller sprocket should not be less than 120°.

(vi) The preferred arrangement for chain drive is with the centrelines of sprockets horizontal. Also, the tight side should be on the top of the drive.

(vii) The length of the chain should be in multiples of pitch. The exact centre distance should be adjusted to account for integer number of pitches for chain length.

(viii) The speed reduction of a single-stage chain drive should not be more than 7 : 1.

(ix) Multi-strand chains are recommended when high power is to be transmitted. Overhanging shafts should be avoided when multi-strand chains are used.

(x) Idler sprockets can be used to reduce the slack in the chain. When they are used, the idler sprocket should be kept outside the closed span of the chain. Also, the idler sprocket should be kept near to the smaller sprocket on the slack or loose side of the chain.

(xi) The chain drive should be provided with a wire mesh or sheet metal guard. This is necessary to protect the operator as well as the chain drive.

(xii) The chain should be properly lubricated as per the recommendations of the manufacturer or the standard.

(xiii) The expected service life of these chains is 15 000 hours.

14.8 CHAIN LUBRICATION

The frictional losses in chain drive consists of the following factors:

(i) Loss due to friction between the rollers and the bushes
(ii) Loss due to friction between bushes and pins
(iii) Loss due to friction between the sprocket teeth and the rollers

The efficiency of a well lubricated chain drive is from 96% to 98%.

The objectives of chain lubrication are as follows:

(i) To reduce the wear of chain components
(ii) To protect the chain against rust and corrosion
(iii) To carry away the frictional heat
(iv) To prevent seizure of pins and bushes
(v) To cushion shock loads and protect the chain

There are three basic methods for the lubrication of chains. They are designated as Type-A, Type-B and Type-C. The type depends upon the power rating and the velocity of chain. The ANSI standard as well as the chain manufacturers recommend a particular type of lubrication depending upon these two factors.

(i) Type-A (Manual or Drip Lubrication) In manual lubrication, the lubricating oil is applied to the chain links with a brush or an oil can after every eight hours of operation. The frequency of applying the oil should be adjusted so as to prevent overheating of the chain or discoloration of the chain joints. In drip lubrication, a drip lubricator continuously supplies the oil to a wick-packed horizontal pipe. This pipe is provided with fine holes through which the oil comes out in the form of drops on the link plates. It is necessary to take precautions to avoid wind circulation in the vicinity of oil drops; otherwise it may result in carrying the oil drops in the wrong direction.

(ii) Type-B (Bath or Disk Lubrication) In bath lubrication, the lower side of the chain is made to pass through a sump containing the lubricating oil. The oil level should be maintained up to the pitch line of the chain. In disk lubrication, the entire chain is kept above the level of lubricating oil in the sump. There is a disk, which is attached to one of the shafts and which picks up the oil from the sump and deposits it onto the chain by means of a trough.

(iii) Type-C (Oil Stream Lubrication) In this method, there is a separate oil pump, which supplies a continuous stream of lubricating oil to the chain. The oil is applied on the inside of the chain loop. It is directed at the slack side of the chain.

Example 14.1 *A single-strand chain No. 12A is used in a mechanical drive. The driving sprocket has 17 teeth and rotates at 1000 rpm. What is the factor of safety used for standard power rating? Neglect centrifugal force acting on the chain.*

Solution

Given $z_1 = 17 \quad n_1 = 1000$ rpm Chain-12A

Step I *Chain tension*

The pitch of the chain is given as 19.05 mm in Table 14.1. From Eq. (14.4),

$$v = \frac{z_1 p n_1}{60 \times 10^3} = \frac{17(19.05)(1000)}{60 \times 10^3} = 5.4 \text{ m/s}$$

The kW rating of chain 12A at 1000 rpm is given as 14.32 kW in Table 14.2. Therefore, the chain tension P_1 at the rated power is given by,

$$P_1 = \frac{1000(\text{kW})}{v} = \frac{1000(14.32)}{5.4} = 2651.85 \text{ N}$$

Step II *Factor of safety*

In Table 14.1, the breaking load for the above chain is given as 31 100 N.

$$\therefore \qquad (fs) = \frac{31\,100}{2651.81} = 11.73$$

Example 14.2 *A chain drive is used in a special purpose vehicle. The vehicle is run by a 30 kW rotary engine. There is a separate mechanical drive from the engine shaft to the intermediate shaft. The* *driving sprocket is fixed to this intermediate shaft. The efficiency of the drive between the engine and the intermediate shafts is 90%. The driving sprocket has 17 teeth and it rotates at 300 rpm. The driven sprocket rotates at 100 rpm. Assume moderate shock conditions and select a suitable four-strand chain for this drive.*

Solution

Given Engine power = 30 kW $\eta = 0.9$
$z_1 = 17$ $n_1 = 300$ rpm $n_2 = 100$ rpm

Step I *kW rating of chain*

The power transmitted to the driving sprocket is given by,

$$\text{kW} = \eta\,(30) = 0.9\,(30) = 27 \text{ kW}$$

For moderate shock condition the service factor is given as 1.4 in Table 14.3.

$$K_s = 1.4$$

From Table 14.4,

$$K_1 = 3.3$$

From Table 14.5,

$$K_2 = 1.0$$

From Eq. (14.8),

$$\text{kW rating of chain} = \frac{(\text{kW to be transmitted}) \times K_s}{K_1 \times K_2}$$

$$= \frac{27 \times 1.4}{3.3 \times 1} = 11.45 \text{ kW}$$

Step II *Selection of chain*

Refer to Table 14.2. The required kW rating is 11.45 kW at 300 rpm speed of driving sprocket. Chain No.16A (kW rating = 11.63) is suitable for the above application.

Example 14.3 *It is required to design a chain drive to connect 5 kW, 1400 rpm electric motor to a drilling machine. The speed reduction is 3 : 1. The centre distance should be approximately 500 mm.*

(i) Select a proper roller chain for the drive.
(ii) Determine the number of chain links.
(iii) Specify the correct centre distance between the axes of sprockets.

Solution

Given kW = 5 $a = 500$ mm $n_1 = 1400$ rpm
$i = 3$

Step I *kW rating of chain*

The number of teeth on the driving sprocket is selected as 21. It is further assumed that the chain is a simple roller chain with only one strand. From Table 14.3, the service factor is taken as 1.3 assuming moderate shock conditions. For single strand chain,

$$K_1 = 1$$

For 21 teeth,

$$K_2 = 1.26$$

From Eq. (14.8),

$$\text{kW rating of chain} = \frac{(\text{kW to be transmitted}) \times K_s}{K_1 \times K_2}$$

$$= \frac{5 \times 1.3}{1 \times 1.26} = 5.16 \text{ kW}$$

Step II *Selection of chain*

Referring to Table 14.2, the power rating of the chain 8A at 1400 rpm is 5.28 kW. Therefore, the chain number 8A is selected.

Step III *Number of chain links*

The pitch dimension (p) of this chain (Table 14.1) is 12.70 mm. (i)

$z_1 = 21$ teeth $z_2 = iz_1 = 3(21) = 63$ teeth
$p = 12.70$ mm $a = 500$ mm

From Eq. (14.6),

$$L_n = 2\left(\frac{a}{p}\right) + \left(\frac{z_1 + z_2}{2}\right) + \left(\frac{z_2 - z_1}{2\pi}\right)^2 \times \left(\frac{p}{a}\right)$$

$$= 2\left(\frac{500}{12.70}\right) + \left(\frac{21 + 63}{2}\right) + \left(\frac{63 - 21}{2\pi}\right)^2 \times \left(\frac{12.70}{500}\right)$$

$$= 121.87 \text{ or } 122 \text{ links} \qquad \text{(ii)}$$

Step IV *Correct centre distance*

$$\left[L_n - \left(\frac{z_1 + z_2}{2}\right)\right] = \left[122 - \left(\frac{21 + 63}{2}\right)\right] = 80$$

From Eq. (14.7),

$$a = \frac{p}{4}\left\{\left[L_n - \left(\frac{z_1+z_2}{2}\right)\right] + \sqrt{\left[L_n - \left(\frac{z_1+z_2}{2}\right)\right]^2 - 8\left[\frac{z_2-z_1}{2\pi}\right]^2}\right\}$$

$$= \frac{12.70}{4}\left\{80 + \sqrt{(80)^2 - 8\left[\frac{63-21}{2\pi}\right]^2}\right\}$$

$$= 500.81 \text{ mm} \qquad \text{(iii)}$$

Example 14.4 *It is required to design a chain drive to connect a 12 kW, 1400 rpm electric motor to a centrifugal pump running at 700 rpm. The service conditions involve moderate shocks.*

(i) Select a proper roller chain and give a list of its dimensions.

(ii) Determine the pitch circle diameters of driving and driven sprockets.

(iii) Determine the number of chain links.

(iv) Specify the correct centre distance between the axes of sprockets.

Solution

Given kW = 12 $n_1 = 1400$ rpm $n_2 = 700$ rpm

Step I *kW rating of chain*

In order to reduce the polygonal effect, the number of teeth on the driving sprocket is selected as 17 (K_2 = 1). It is further assumed that the chain is simple roller chain with only one strand (K_1 = 1). The service factor from Table 14.3 is 1.3. From Eq. (14.8),

$$\text{kW rating of chain} = \frac{(\text{kW to be transmitted}) \times K_s}{K_1 \times K_2}$$

$$= \frac{12 \times 1.3}{1 \times 1} = 15.6 \text{ kW}$$

Step II *Selection of chain*

Referring to Table 14.2, the power rating of the chain 12B at 1400 rpm is 18.15 kW. Therefore the chain number 12B is selected.

The dimensions of this chain (Table 14.1) are as follows:

$$p = 19.05 \text{ mm} \quad d_1 = 12.07 \text{ mm} \quad b_1 = 11.68 \text{ mm} \qquad \text{(i)}$$

Step III *Pitch circle diameter of driving and driven pulleys*

From Eq. 14.2,

$$D = \frac{p}{\sin\left(\frac{180}{z}\right)} = \frac{19.05}{\sin\left(\frac{180}{17}\right)} = 103.67 \text{ mm} \qquad \text{(iia)}$$

For the driven sprocket,

$$z_2 = z_1\left(\frac{n_1}{n_2}\right) = 17\left(\frac{1400}{700}\right) = 34$$

$$D_2 = \frac{p}{\sin\left(\frac{180}{z}\right)} = \frac{19.05}{\sin\left(\frac{180}{34}\right)} = 206.46 \text{ mm} \qquad \text{(iib)}$$

Step IV *Number of chain links*

The centre distance between the sprocket wheels should be between (30p) to (50p). Taking a mean value of (40p), the approximate centre distance is calculated.

$$a = 40\,p = 40(19.05) = 762 \text{ mm}$$

From Eq. (14.6),

$$L_n = 2\left(\frac{a}{p}\right) + \left(\frac{z_1+z_2}{2}\right) + \left(\frac{z_2-z_1}{2\pi}\right)^2 \times \left(\frac{p}{a}\right)$$

$$= 2\left(\frac{762}{19.05}\right) + \left(\frac{17+34}{2}\right) + \left(\frac{34-17}{2\pi}\right)^2 \times \left(\frac{19.05}{762}\right)$$

$$= 105.68 \text{ or } 106 \text{ links} \qquad \text{(iii)}$$

Step V *Correct centre distance*

$$\left[L_n - \left(\frac{z_1+z_2}{2}\right)\right] = \left[106 - \left(\frac{17+34}{2}\right)\right] = 80.5$$

From Eq. (14.7),

$$a = \frac{p}{4}\left\{\left[L_n - \left(\frac{z_1+z_2}{2}\right)\right] + \sqrt{\left[L_n - \left(\frac{z_1+z_2}{2}\right)\right]^2 - 8\left[\frac{z_2-z_1}{2\pi}\right]^2}\right\}$$

$$= \frac{19.05}{4}\left\{80.5 + \sqrt{(80.5)^2 - 8\left[\frac{34-17}{2\pi}\right]^2}\right\}$$

$$= 765.03 \text{ mm}$$

To provide small sag, for allowing the chain links to take the best position on the sprocket teeth, the centre distance is reduced by $(0.002a)$. Therefore, the correct centre distance is given by,

$$a = 0.998 \times 765.03 = 763.5 \text{ mm} \qquad \text{(iv)}$$

Example 14.5 *Refer to the data of the previous problem and calculate the following dimensions of the driving sprocket wheel:*

(i) outer diameter;
(ii) root diameter;
(iii) roller seating radius;
(iv) tooth flank radius;
(v) tooth side radius;
(vi) tooth width; and
(vii) tooth side relief.

Solution

Given (From the previous problem),
$p = 19.05$ mm $\quad d_1 = 12.07$ mm $\quad b_1 = 11.68$ mm
$z = 17$ teeth $\quad D = 103.67$ mm

Step I *Outer diameter*
Referring to Table 14.6,

$$(D_a)_{\text{max.}} = D + 1.25\,p - d_1$$
$$= 103.67 + 1.25(19.05) - 12.07$$
$$= 115.41 \text{ mm}$$

$$(D_a)_{\text{min.}} = D + p\left(1 - \frac{1.6}{17}\right) - d_1$$
$$= 103.67 + 19.05\left(1 - \frac{1.6}{17}\right) - 12.07$$
$$= 108.86 \text{ mm}$$

$\therefore$ outer diameter $D_a = 112$ mm (i)

Step II *Root diameter and roller seating radius*

$$(r_i)_{\text{max.}} = 0.505 d_1 + 0.069\sqrt[3]{d_1}$$
$$= 0.505(12.07) + 0.069\sqrt[3]{12.07}$$
$$= 6.25 \text{ mm}$$

$$(r_i)_{\text{min.}} = 0.505 d_i = 0.505(12.07) = 6.10 \text{ mm}$$

$\therefore$ roller seating radius $r_i = 6.15$ mm (iii)

Root diameter $D_f = D - 2r_i = 103.67 - 2(6.15)$
$= 91.37$ mm (ii)

Step III *Tooth flank radius*

$$(r_e)_{\text{max.}} = 0.008 d_1 (z^2 + 180)$$
$$= 0.008(12.07)(17^2 + 180) = 45.29 \text{ mm}$$

$$(r_e)_{\text{min.}} = 0.12 d_1 (z + 2) = 0.12\ (12.07)\ (17+2)$$
$$= 27.52$$

Therefore,

Tooth flank radius $r_e = 35$ mm (iv)

Step IV *Tooth side radius*
Tooth side radius $= p = 19.05$ mm (v)

Step V *Tooth width*
Tooth width $b_{f1} = 0.95\ b_1 = 0.95\ (11.68)$
$= 11.10$ mm (vi)

Step VI *Tooth side relief*

$b_a = (0.10p)$ to $(0.15p)$
$= 1.905$ to 2.858 mm or $b_a = 2.4$ mm (vii)

Example 14.6 *It is required to design a chain drive to connect a 10 kW, 900 rpm petrol engine to a conveyor. The driving sprocket is mounted on engine shaft. The driven sprocket is mounted on conveyor shaft. The conveyor shaft should run between 225 to 245 rpm. The service conditions involve moderate shocks.*

(i) Select a proper roller chain and give a list of its dimensions.
(ii) Determine the pitch circle diameters of the driving and driven sprockets.
(iii) Determine the number of chain links.
(iv) Specify the correct centre distance between the axes of sprockets.

Create an alternative design for the above application, which will result in compact construction using multi-strand chain. For this design,

(v) Select the roller chain with multi-strand construction,
(vi) Determine the number of chain links,
(vii) Specify the correct centre distance between the axes of sprockets.

Solution

Given kW = 10 $\quad n_1 = 900$ rpm
$n_2 = 225$ to 245 rpm

Part I *Initial design*

Step I *kW rating of chain*

The number of teeth on the driving sprocket is selected as 21. It is further assumed that the chain is a simple roller chain with only one strand ($K_1 = 1$). The service factor K_s from Table 14.3 is 1.4. The tooth correction factor (K_2) is 1.26 for 21 teeth. From Eq. (14.8),

kW rating of chain

$$= \frac{(\text{kW to be transmitted}) \times K_s}{K_1 \times K_2}$$

$$= \frac{10 \times 1.4}{1 \times 1.26} = 11.11 \text{ kW}$$

Step II *Selection of chain*

Referring to Table 14.2, the power rating of the chain 12A is 10.73 and 14.32 kW at 700 and 1000 rpm respectively. Therefore, the chain number 12A is selected. The power rating of the chain 12A at the operating speed of 900 rpm is calculated by linear interpolation.

kW rating at 900 rpm

$$= 10.73 + \frac{(14.32 - 10.73)}{(1000 - 700)} \times (900 - 700) = 13.12 \text{ kW}$$

The required power rating is 11.13 kW, while the chain has a power rating of 13.12 kW. Therefore, chain 12A is suitable for the above application. (i)

Step III *Pitch circle diameter of driving and driven pulleys*

The pitch of the chain 12A is given as 19.05 mm in Table 14.1.

From Eq. 14.2,

$$D_1 = \frac{p}{\sin\left(\frac{180}{z_1}\right)} = \frac{19.05}{\sin\left(\frac{180}{21}\right)} = 127.82 \text{ mm} \quad \text{(iia)}$$

The speed of the conveyor shaft should be approximately between 225 to 245 rpm. Taking a mean speed of 235 rpm,

$$z_2 = z_1\left(\frac{n_1}{n_2}\right) = 21\left(\frac{900}{235}\right) = 80.43 \text{ or } 80 \text{ teeth}$$

$$D_2 = \frac{p}{\sin\left(\frac{180}{z_2}\right)} = \frac{19.05}{\sin\left(\frac{180}{80}\right)} = 485.23 \text{ mm} \quad \text{(iib)}$$

Step IV *Number of chain links*

The centre distance between the sprocket wheels should be between ($30p$) to ($50p$). Taking a mean value of ($40p$), the approximate centre distance is given by,

$$a = 40p = 40(19.05) = 762 \text{ mm}$$

From Eq. (14.6),

$$L_n = 2\left(\frac{a}{p}\right) + \left(\frac{z_1 + z_2}{2}\right) + \left(\frac{z_2 - z_1}{2\pi}\right)^2 \times \left(\frac{p}{a}\right)$$

$$= 2\left(\frac{762}{19.05}\right) + \left(\frac{21 + 80}{2}\right) + \left(\frac{80 - 21}{2\pi}\right)^2 \times \left(\frac{19.05}{762}\right)$$

$$= 132.7 \text{ or } 132 \text{ links} \quad \text{(iii)}$$

Step V *Correct centre distance*

$$\left[L_n - \left(\frac{z_1 + z_2}{2}\right)\right] = \left[132 - \left(\frac{21 + 80}{2}\right)\right] = 81.5$$

From Eq. (14.7),

$$a = \frac{p}{4}\left\{\left[L_n - \left(\frac{z_1 + z_2}{2}\right)\right] + \sqrt{\left[L_n - \left(\frac{z_1 + z_2}{2}\right)\right]^2 - 8\left[\frac{z_2 - z_1}{2\pi}\right]^2}\right\}$$

$$= \frac{19.05}{4}\left\{81.5 + \sqrt{(81.5)^2 - 8\left[\frac{80 - 21}{2\pi}\right]^2}\right\}$$

$$= 755.1 \text{ mm} \quad \text{(iv)}$$

Part II *Alternative design*

Step I *kW rating of chain*

Four-strand chain is used to permit a smaller pitch chain. This results in smaller centre distance and compact construction. The multi-strand factor (K_1) is given as 3.3 for four strands in Table 14.4.

From Eq. (14.8),

$$\text{kW rationg of chain} = \frac{(\text{kW to be transmitted}) \times K_s}{K_1 \times K_2}$$

$$= \frac{10 \times 1.4}{3.3 \times 1.26} = 3.37 \text{ kW}$$

Step II *Selection of chain*

Referring to Table 14.2, the power rating of the chain 8A is 2.95 and 3.94 kW at 700 and 1000 rpm respectively. Therefore, the chain number 8A is selected. The power rating of the chain 8A at the operating speed of 900 rpm is calculated by linear interpolation.

kW rating at 900 rpm

$$= 2.95 + \frac{(3.94 - 2.95)}{(1000 - 700)} \times (900 - 700) = 3.61 \text{ kW}$$

The required power rating is 3.37 kW, while the chain has a power rating of 3.61 kW. Therefore the chain 8A is suitable for the application. (v)

Step III *Number of chain links*

The pitch of chain 8A is given as 12.7 mm in Table 14.1. It is assumed that the driving and the driven sprockets have 25 and 96 teeth respectively. The output speed is given by,

$$n_2 = \left(\frac{z_1}{z_2}\right) n_1 = \left(\frac{25}{96}\right) 900 = 234.37 \text{ rpm}$$

The output speed is within the range of 225 to 245 rpm. To make compact construction, the centre distance is taken as $30p$.

$$a = 30p = 30 \times 12.7 = 381 \text{ mm}$$

$$z_1 = 25 \quad z_2 = 96 \quad p = 12.7 \text{ mm}$$

From Eq. (14.6),

$$L_n = 2\left(\frac{a}{p}\right) + \left(\frac{z_1 + z_2}{2}\right) + \left(\frac{z_2 - z_1}{2\pi}\right)^2 \times \left(\frac{p}{a}\right)$$

$$= 2\left(\frac{381}{12.7}\right) + \left(\frac{25 + 96}{2}\right) + \left(\frac{96 - 25}{2\pi}\right)^2 \times \left(\frac{12.7}{381}\right)$$

$$= 124.76 \text{ or } 124 \text{ links} \qquad \text{(vi)}$$

Step IV *Correct centre distance*

$$\left[L_n - \left(\frac{z_1 + z_2}{2}\right)\right] = \left[124 - \left(\frac{25 + 96}{2}\right)\right] = 63.5$$

From Eq. (14.7),

$$a = \frac{p}{4}\left\{\left[L_n - \left(\frac{z_1 + z_2}{2}\right)\right] + \sqrt{\left[L_n - \left(\frac{z_1 + z_2}{2}\right)\right]^2 - 8\left[\frac{z_2 - z_1}{2\pi}\right]^2}\right\}$$

$$= \frac{12.7}{4}\left\{63.5 + \sqrt{(63.5)^2 - 8\left[\frac{96 - 25}{2\pi}\right]^2}\right\}$$

$$= 375.83 \text{ mm} \qquad \text{(vii)}$$

Example 14.7 A simple chain No. 10B is used to transmit power from a 1400 rpm electric motor to a line shaft running at 350 rpm. The number of teeth on the driving sprocket wheel is 19. The operation is smooth without any shocks. Calculate

(i) the rated power for which the chain drive can be recommended;

(ii) the tension in the chain for this rated power; and

(iii) the factor of safety for the chain based on the breaking load.

Solution

Given Chain 10B $z_1 = 19$ $n_1 = 1400$ rpm $n_2 = 350$ rpm

Step I *Rated power of chain drive*

From Table 14.3,

$$K_s = 1.0$$

From Table 14.4 (single strand),

$$K_1 = 1.0$$

From Table 14.5 (19 teeth),

$$K_2 = 1.11$$

From Table 14.2, the power rating for the chain 10B is 11.67 kW at 1400 rpm. From Eq. (14.8),

kW to be transmitted =

$$= \frac{(\text{kW rating of the chain}) K_1 K_2}{K_S}$$

$$= \frac{11.67(1)(1.11)}{1}$$

$$= 12.95 \text{ kW} \qquad \text{(i)}$$

Step II *Tension in the chain*

From Table 14.1, $p = 15.875$ mm

From Eq. (14.4),

$$v = \frac{z_1 p n_1}{60 \times 10^3} = \frac{19(15.875)(1400)}{60 \times 10^3} = 7.04 \text{ m/s}$$

The chain tension is given by

$$P_1 = \frac{1000\,\text{kW}}{v} = \frac{1000(12.95)}{7.04} = 1839.49\text{ N} \qquad \text{(ii)}$$

Step III Factor of safety

From Table 14.1, the breaking load for the chain 10B is 22 200 N. Thus,

$$(fs) = \frac{22\,200}{1839.49} = 12.07 \qquad \text{(iii)}$$

Example 14.8 *A chain drive consisting of a driver sprocket A and two driven sprockets B and C is shown in Fig. 14.14. The sprocket A rotates at 100 rpm and transmits 0.2 kW of power each to both sprockets B and C. The chain used is simple roller chain 08B (pitch = 12.7 mm). The centre distances given in the figure are in terms of pitches. Calculate the tensions in various parts of the chain and determine the shaft reactions for the sprocket B.*

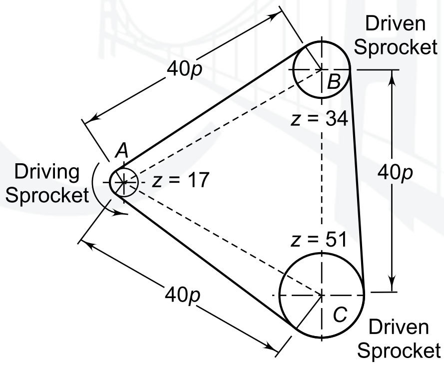


Fig. 14.14

Solution

Given $z_A = 17 \quad z_B = 34 \quad z_C = 51 \quad n_A = 100$ rpm

$p = 12.7$ mm

power to each sprocket B and $C = 0.2$ kW

Chain-08B

Step I Chain tensions

The chain velocity is given by

$$v = \frac{z_A p n_A}{60 \times 10^3} = \frac{17(12.7)(100)}{60 \times 10^3} = 0.36\text{ m/s}$$

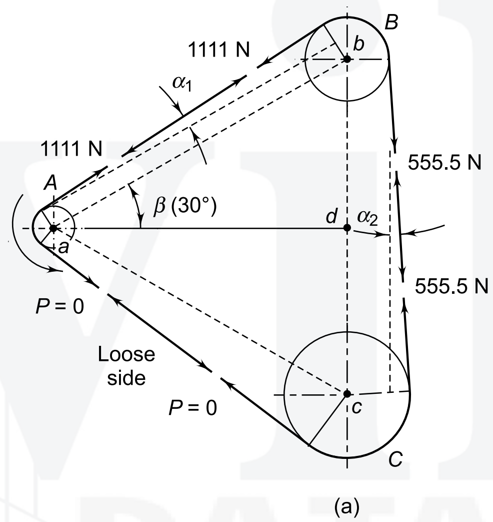
The total power transmitted by the sprocket A is (0.2 + 0.2) or 0.4 kW and the chain tension is given by

$$P_A = \frac{1000\,\text{kW}}{v} = \frac{1000(0.4)}{0.36} = 1111\text{ N}$$

The power transmitted to each of the sprockets B and C is 0.2 kW. Therefore, the net tension is given by

$$P_B = P_C = \frac{1000(0.2)}{0.36} = 555.5\text{ N}$$

The free-body diagram showing the tensions in various parts of the chain is shown in Fig. 14.15(a).



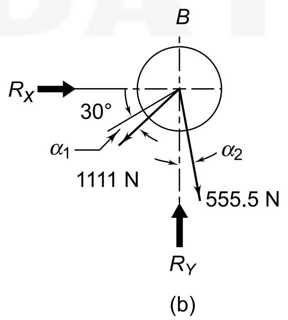


Fig. 14.15 *(a) Free body diagram of forces (b) Reactions at pin B*

Step II Shaft reactions at the sprocket B

The pitch circle diameters of the sprockets are as follows:

$$D_A = \frac{p}{\sin\left(\frac{180}{z_A}\right)} = \frac{12.7}{\sin\left(\frac{180}{17}\right)} = 69.12 \text{ mm}$$

$$D_B = \frac{p}{\sin\left(\frac{180}{z_B}\right)} = \frac{12.7}{\sin\left(\frac{180}{34}\right)} = 137.64 \text{ mm}$$

$$D_C = \frac{p}{\sin\left(\frac{180}{z_C}\right)} = \frac{12.7}{\sin\left(\frac{180}{51}\right)} = 206.30 \text{ mm}$$

and $\quad 40p = 40(12.7) = 508$ mm

Refer to Fig.14.15(a). a, b and c are the centres of circles A, B and C respectively. Construct a line ad perpendicular to the line bc.

$$\sin\beta = \frac{bd}{ab} = \frac{20p}{40p} = 0.5 \quad \therefore \beta = 30°$$

Referring to Fig. 14.15(a),

$$\sin\alpha_1 = \frac{D_B - D_A}{2a} = \left[\frac{137.64 - 69.12}{2(508)}\right]$$

or $\quad \alpha_1 = 3.87°$

$$\sin\alpha_2 = \frac{D_C - D_B}{2a} = \left[\frac{206.3 - 137.64}{2(508)}\right]$$

or $\quad \alpha_2 = 3.87°$

The forces acting on the shaft of the sprocket B are shown in Fig.14.15(b). Considering equilibrium of vertical and horizontal forces,

$$R_Y = 1111 \sin(30 + \alpha_1) + 555.5 \cos\alpha_2$$
$$= 1111 \sin(30 + 3.87) + 555.5 \cos(3.87)$$
$$= 1173.41 \text{ N}$$
$$R_X = 1111 \cos(30 + \alpha_1) - 555.5 \sin\alpha_2$$
$$= 1111 \cos(30 + 3.87) - 555.5 \sin(3.87)$$
$$= 884.98 \text{ N}$$

The resultant reaction R_B is given by

$$R_B = \sqrt{(1173.41)^2 + (884.98)^2} = 1469.72 \text{ N}$$

14.9 SILENT CHAIN

The silent or inverted-tooth chain as shown in Fig. 14.16 consists of a series of links formed from laminated steel plates. Each plate has two teeth with a space between them to accommodate the mating tooth of the sprocket. The sprocket teeth have a trapezoidal profile. Depending upon the type of joint between links, the silent chains are divided into two groups—Reynold chain and Morse chain. In the *Reynold chain*, the links are connected by pins resulting in sliding friction. Rocker joints are used in Morse chain.

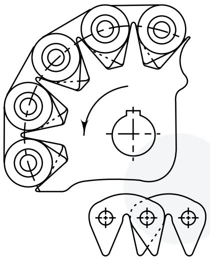


Fig. 14.16 *Silent chain*

Compared with roller chains, the silent chains can be used for high speed applications. They operate more smoothly and almost noiselessly. Their reliability is more due to laminated construction. They are, however, more heavier, more difficult to manufacture and more expensive than roller chains. Their applications are limited due to these reasons.

Short-Answer Questions

14.1 What are the advantages of chain drives?
14.2 What are the disadvantages of chain drives?
14.3 What are the applications of chain drives?
14.4 What are the advantages of link chains?
14.5 What are the disadvantages of link chains?
14.6 What are the applications of link chains?
14.7 What are the five parts of roller chain?
14.8 How will you designate roller chain?
14.9 What are simple and duplex roller chains?
14.10 What is the offset link of roller chain?
14.11 What is the polygonal action in roller chain? How will you reduce it?
14.12 In chain drives, the sprocket has odd number of teeth and the chain has even number of links. Why?
14.13 What are the types of failure in roller chains?
14.14 What are the advantages of silent chains?
14.15 What are the disadvantages of silent chains?
14.16 What are the applications of silent chains?

Problems for Practice

14.1 A simple roller chain 10B is used to drive the camshaft of an internal combustion engine. Both shafts rotate at 350 rpm and the centre distance between their axes should be approximately 550 mm. The number of teeth on each sprocket wheel is 19. Calculate

(i) the number of chain links; and

(ii) the correct centre distance.

[*(i) 88 (88.29) links (ii) 547.69 mm*]

14.2 A simple chain 08B is used to transmit power from a transmission shaft running at 200 rpm to another shaft running at 100 rpm. There are 19 teeth on the driving sprocket wheel and the operation is smooth without any shocks. Calculate

(i) the power transmitting capacity of the chain drive;

(ii) the chain velocity;

(iii) the chain tension; and

(iv) the factor of safety based on the breaking load.

[*(i) 1.31 kW (ii) 0.8 m/s (iii) 1637.5 N (iv) 10.87*]

14.3 A chain drive consists of a 21 teeth driving sprocket, running at 500 rpm and another 35 teeth driven sprocket. The sprockets are connected by a simple roller chain 06B, which transmits 1 kW of power. Calculate

(i) the pitch circle diameters of the driving and driven sprocket wheels;

(ii) the chain velocity;

(iii) the chain tension; and

(iv) the torque on the driven shaft.

[*(i) 63.91 and 106.26 mm (ii) 1.667 m/s (iii) 599.88 N (iv) 31.87 N-m*]

14.4 It is required to design a chain drive with a duplex chain to connect a 15 kW, 1400 rpm electric motor to a transmission shaft running at 350 rpm. The operation involves moderate shocks.

(i) Specify the number of teeth on the driving and driven sprockets.

(ii) Select a proper roller chain.

(iii) Calculate the pitch circle diameters of the driving and driven sprockets.

(iv) Determine the number of chain links.

(v) Specify the correct centre distance.

During preliminary stages, the centre distance can be assumed to be 40 times the pitch of the chain.

[*(i) 17 and 68 teeth (ii) Chain 10B (iii) 86.39 and 343.74 mm (iv) 124 (124.15) links (v) 633.81 mm*]

Rolling Contact Bearings

Chapter 15

15.1 BEARINGS

Bearing is a mechanical element that permits relative motion between two parts, such as the shaft and the housing, with minimum friction. The functions of the bearing are as follows:

(i) The bearing ensures free rotation of the shaft or the axle with minimum friction.

(ii) The bearing supports the shaft or the axle and holds it in the correct position.

(iii) The bearing takes up the forces that act on the shaft or the axle and transmits them to the frame or the foundation.

Bearings are classified in different ways. Depending upon the direction of force that acts on them, bearings are classified into two categories—radial and thrust bearings, as shown in Fig. 15.1. A *radial bearing* supports the load, which is perpendicular to the axis of the shaft. A *thrust bearing* supports the load, which acts along the axis of the shaft.

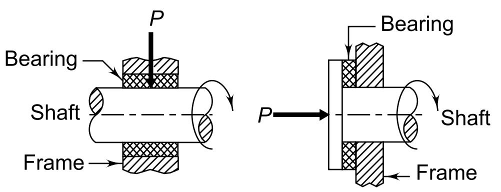


Fig. 15.1 *Radial and Thrust Bearings*

The most important criterion to classify the bearings is the type of friction between the shaft and the bearing surface. Depending upon the type of friction, bearings are classified into two main groups—sliding contact bearings and rolling contact bearings as shown in Fig. 15.2. *Sliding contact bearings* are also called plain bearings, journal bearings or sleeve bearings. In this case, the surface of the shaft slides over the surface of the bush resulting in friction and wear. In order to reduce the friction, these two surfaces are separated by a film of lubricating oil. The bush is made of special bearing material like white metal or bronze. *Rolling contact bearings* are also called antifriction bearings or simply ball bearings. Rolling elements, such as balls or rollers, are introduced between the surfaces that are in relative motion. In this type of bearing, sliding friction is replaced by rolling friction.

Sliding contact bearings are used in the following applications:

(i) crankshaft bearings in petrol and diesel engines;

(ii) centrifugal pumps;

(iii) large size electric motors;

(iv) steam and gas turbines; and

(v) concrete mixers, rope conveyors and marine installations.

Rolling contact bearings are used in the following applications:

(i) machine tool spindles;
(ii) automobile front and rear axles;
(iii) gear boxes;
(iv) small size electric motors; and
(v) rope sheaves, crane hooks and hoisting drums.

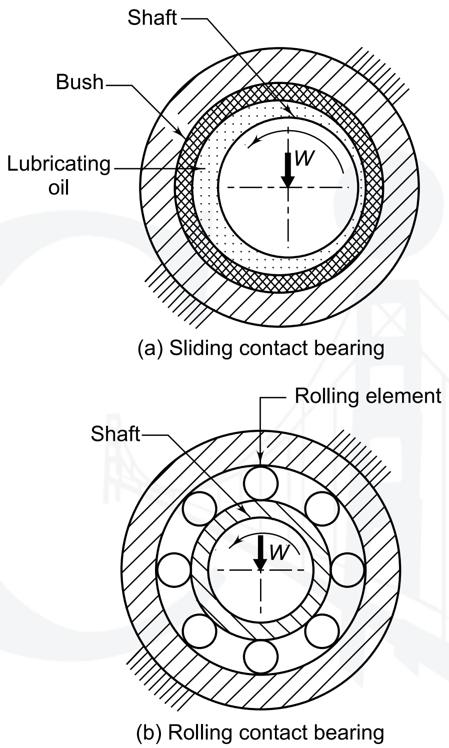


Fig. 15.2 *Sliding and Rolling Contact Bearing*

15.2 TYPES OF ROLLING CONTACT BEARINGS

For starting conditions and at moderate speeds, the frictional losses in rolling contact bearing are lower than that of equivalent hydrodynamic journal bearing. This is because the sliding contact is replaced by rolling contact resulting in low coefficient of friction. Therefore, rolling contact bearings are called '*antifriction*' bearings. However, this is a misnomer. There is always friction at the contacting surfaces between the rolling element and the inner and outer cages.

A rolling contact bearing consists of four parts—inner and outer races, a rolling element like ball, roller or needle and a cage which holds the rolling elements together and spaces them evenly around the periphery of the shaft. Depending upon the type of rolling element, the bearings are classified as ball bearing, cylindrical roller bearing, taper roller bearing and needle bearing. Depending upon the direction of load, the bearings are also classified as radial bearing and thrust bearing. There is, however, no clear distinction between these two groups. Certain types of radial bearings can also take thrust load, while some thrust bearings are capable of taking radial loads.

The types of rolling contact bearings, which are frequently used, are shown in Fig. 15.3. The characteristics of these bearings are as follows:

(i) Deep Groove Ball Bearing The most frequently used bearing is the deep groove ball bearing. It is

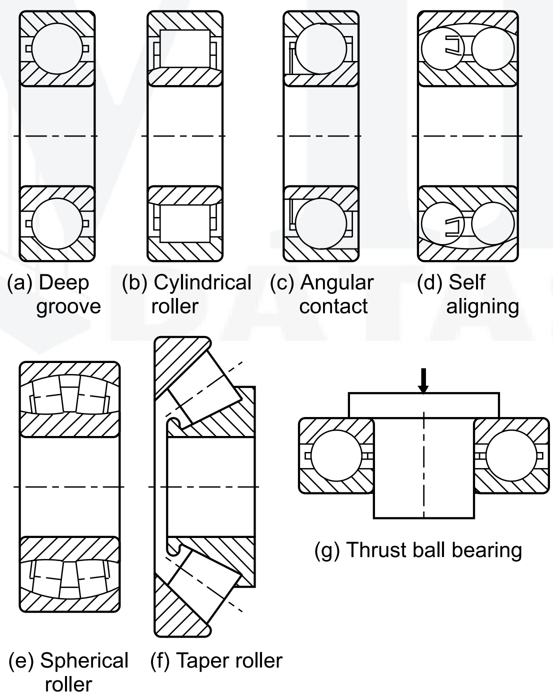


Fig. 15.3 *Types of Rolling Contact Bearing*

found in almost all kinds of products in general mechanical engineering. In this type of bearing, the radius of the ball is slightly less than the radii of curvature of the grooves in the races. Kinematically, this gives a point contact between the balls and the races. Therefore, the balls and the races may roll freely without any sliding. Deep groove ball bearing has the following advantages:

(a) Due to relatively large size of the balls, deep groove ball bearing has high load carrying capacity.
(b) Deep groove ball bearing takes loads in the radial as well as axial direction.
(c) Due to point contact between the balls and races, frictional loss and the resultant temperature rise is less in this bearing. The maximum permissible speed of the shaft depends upon the temperature rise of the bearing. Therefore, deep groove ball bearing gives excellent performance, especially in high speed applications.
(d) Deep groove ball bearing generates less noise due to point contact.
(e) Deep groove ball bearings are available with bore diameters from a few millimetres to 400 millimetres.

The disadvantages of deep groove ball bearings are as follows:

(a) Deep groove ball bearing is not self-aligning. Accurate alignment between axes of the shaft and the housing bore is required.
(b) Deep groove ball bearing has poor rigidity compared with roller bearing. This is due to the *point contact* compared with the *line contact* in case of roller bearing. It is unsuitable for machine tool spindles where rigidity is important consideration.

(ii) Cylindrical Roller Bearing When maximum load carrying capacity is required in a given space, the point contact in ball bearing is replaced by the line contact of roller bearing. A cylindrical roller bearing consists of relatively short rollers that are positioned and guided by the cage. Cylindrical roller bearing offers the following advantages:

(a) Due to line contact between rollers and races, the radial load carrying capacity of the cylindrical roller bearing is very high.
(b) Cylindrical roller bearing is more rigid than ball bearing.
(c) The coefficient of friction is low and frictional loss is less in high-speed applications.

The disadvantages of cylindrical roller bearing are as follows:

(a) In general, cylindrical roller bearing cannot take thrust load.
(b) Cylindrical roller bearing is not self-aligning. It cannot tolerate misalignment. It needs precise alignment between axes of the shaft and the bore of the housing.
(c) Cylindrical roller bearing generates more noise.

(iii) Angular Contact Bearing In angular contact bearing, the grooves in inner and outer races are so shaped that the line of reaction at the contact between balls and races makes an angle with the axis of the bearing. This reaction has two components—radial and axial. Therefore, angular contact bearing can take radial and thrust loads. Angular contact bearings are often used in pairs, either side by side or at the opposite ends of the shaft, in order to take the thrust load in both directions. These bearings are assembled with a specific magnitude of pre-load. Angular contact bearings offer the following advantages:

(a) Angular contact bearing can take both radial and thrust loads.
(b) In angular contact bearing, one side of the groove in the outer race is cut away to permit the insertion of larger number of balls than that of deep groove ball bearing. This permits the bearing to carry relatively large axial and radial loads. Therefore, the load carrying capacity of angular contact bearing is more than that of deep groove ball bearing.

The disadvantages of angular contact bearings are as follows:

(a) Two bearings are required to take thrust load in both directions.
(b) The angular contact bearing must be mounted without axial play.
(c) The angular contact bearing requires initial pre-loading.

(iv) Self-aligning Bearings There are two types of self-aligning rolling contact bearings, viz., self-aligning ball bearing and spherical roller bearing. The principle of self-aligning bearing is explained in the next section. The self-aligning ball bearing consists of two rows of balls, which roll on a common spherical surface in the outer race. In this case, the assembly of the shaft, the inner race and the balls with cage can freely roll and adjust itself to the angular misalignment of the shaft. There is similar arrangement in the spherical roller bearing, where balls are replaced by two rows of spherical rollers, which run on a common spherical surface in the outer race. Compared with the self-aligning ball bearing, the spherical roller bearing can carry relatively high radial and thrust loads. Both types of self-aligning bearing permit minor angular misalignment of the shaft relative to the housing. They are therefore particularly suitable for applications where misalignment can arise due to errors in mounting or due to deflection of the shaft. They are used in agricultural machinery, ventilators, and railway axle-boxes.

(v) Taper Roller Bearing The taper roller bearing consists of rolling elements in the form of a frustum of cone. They are arranged in such a way that the axes of individual rolling elements intersect in a common apex point on the axis of the bearing. In kinematics' analysis, this is the essential requirement for pure rolling motion between conical surfaces. In taper roller bearing, the line of resultant reaction through the rolling elements makes an angle with the axis of the bearing. Therefore, taper roller bearing can carry both radial and axial loads. In fact, the presence of either component results in the other, acting on the bearing. In other words, a taper roller bearing subjected to pure radial load induces a thrust component and vice versa. Therefore, taper roller bearings are always used in pairs to balance the thrust component. Taper roller bearing has separable construction. The outer ring is called '*cup*' and the inner ring is called '*cone*'. The cup is separable from the remainder assembly of the bearing elements including the rollers, cage and the cone. Taper roller bearings offer the following advantages:

(a) Taper roller bearing can take heavy radial and thrust loads.
(b) Taper roller bearing has more rigidity.
(c) Taper roller bearing can be easily assembled and disassembled due to separable parts.

The disadvantages of taper roller bearing are as follows:

(a) It is necessary to use two taper roller bearings on the shaft to balance the axial force.
(b) It is necessary to adjust the axial position of the bearing with pre-load. This is essential to coincide the apex of the cone with the common apex of the rolling elements.
(c) Taper roller bearing cannot tolerate misalignment between the axes of the shaft and the housing bore.
(d) Taper roller bearings are costly.

Taper roller bearings are used for cars and trucks, propeller shafts and differentials, railroad axle-boxes and as large size bearings in rolling mills.

(vi) Thrust Ball Bearing A thrust ball bearing consists of a row of balls running between two rings—the shaft ring and the housing ring. Thrust ball bearing carries thrust load in only one direction and cannot carry any radial load. The use of a large number of balls results in high thrust load carrying capacity in smaller space. This is the major advantage of thrust bearing. The disadvantages of thrust bearings are as follows:

(a) Thrust ball bearing cannot take radial load.
(b) It is not self-aligning and cannot tolerate misalignment.
(c) Their performance is satisfactory at low and medium speeds. At high speeds, such bearings give poor service because the balls are subjected to centrifugal forces and gyroscopic couple.
(d) Thrust ball bearings do not operate as well on horizontal shafts as they do on vertical shafts.
(e) Thrust ball bearing requires continuous pressure applied by springs to hold the rings together.

Thrust ball bearings are used where heavy thrust loads are to be carried, for example, worm gear boxes and crane hooks.

There are specific materials for the parts of rolling contact bearings. They are as follows:

(a) The balls and the inner and outer races are made of high carbon chromium steel (SAE 52100 or AISI 5210). It contains 1 per cent carbon and 1.5 per cent chromium. The balls and races are through-hardened to obtain a minimum hardness of 58 Rockwell C.

(b) The cages are made from stampings of low carbon steel.

(c) The rollers are made of case hardened steels (AISI 3310, 4620 or 8620). They are case carburized to obtain a surface hardness of 58 Rockwell C.

It should be noted that balls are through hardened, while the rollers are case hardened.

15.3 PRINCIPLE OF SELF-ALIGNING BEARING

In the previous section, two types of self-aligning bearings, namely, self-aligning ball bearing and spherical roller bearing, were explained. In many applications, the bearing is required to tolerate a small amount of misalignment between the axes of the shaft and the bearing. The misalignment may be due to deflection of the shaft under load or due to tolerances of individual components. Self-aligning bearings are used in these applications. The principle of self-aligning bearing is illustrated in Fig. 15.4. A shaft perfectly aligned with the bearing is shown in Fig. 15.4(a). When the shaft is deflected under the load, it exerts pressure at the edges of the bearing as shown in Fig. 15.4(b). The edge pressure is dangerous and may result in undue wear and breakdown of the oil film. In self-aligning bearing, the external surface of the bearing bush is made spherical as shown in Fig. 15.4(c). The centre of this spherical surface is at the centre of the bearing. Therefore, the bush is free to roll in its seat and align itself with the journal. Arrangement is made to provide lubricant between the spherical surfaces of the bush and its seat in order to reduce the friction. This principle is used in self-aligning ball bearings and spherical roller bearings. It is illustrated in Fig. 15.5. The angular misalignment α is exaggerated in the figure. Self-aligning bearings are commonly employed when accurate alignment is impossible or unfeasible.

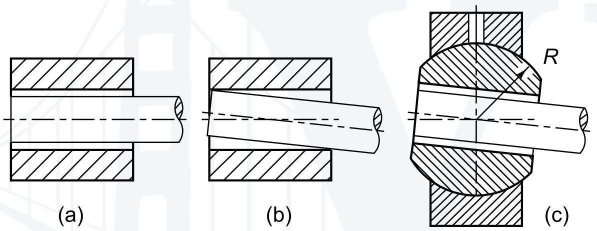


Fig. 15.4 *Self aligning Bearing: (a) Shaft aligned with Bearing (b) Shaft misaligned with Bearing (c) Self aligning Bearing*

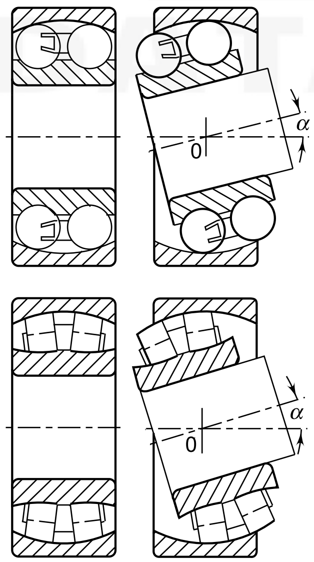


Fig. 15.5 *Self aligning Principle*

15.4 SELECTION OF BEARING-TYPE

The selection of the type of bearing in a particular application depends upon the requirement of the application and the characteristics of different types of bearings. The guidelines for selecting a proper type of bearing are as follows:

(i) For low and medium radial loads, ball bearings are used, whereas for heavy loads and large shaft diameters, roller bearings are selected.

(ii) Self-aligning ball bearings and spherical roller bearings are used in applications where a misalignment between the axes of the shaft and housing is likely to exist.

(iii) Thrust ball bearings are used for medium thrust loads whereas for heavy thrust loads, cylindrical roller thrust bearings are recommended. Double acting thrust bearings can carry the thrust load in either direction.

(iv) Deep groove ball bearings, angular contact bearings and spherical roller bearings are suitable in applications where the load acting on the bearing consists of two components—radial and thrust.

(v) The maximum permissible speed of the shaft depends upon the temperature rise in the bearing. For high speed applications, deep groove ball bearings, angular contact bearings and cylindrical roller bearings are recommended.

(vi) Rigidity controls the selection of bearings in certain applications like machine tool spindles. Double row cylindrical roller bearings or taper roller bearings are used under these conditions. The line of contact in these bearings, as compared with the point of contact in ball bearings, improves the rigidity of the system.

(vii) Noise becomes the criterion of selection in applications like household appliances. For such applications, deep groove ball bearings are recommended.

Knowledge of the design characteristics of different types of bearings and proper appreciation of the needs of an application enables the designer to select a proper type of bearing. The characteristics of the bearing should match with the requirements of the application.

[1] IS 3823 – 1988: Rolling Bearings—Static load ratings

15.5 STATIC LOAD CARRYING CAPACITY

Static load is defined as the load acting on the bearing when the shaft is stationary. It produces permanent deformation in balls and races, which increases with increasing load. The permissible static load, therefore, depends upon the permissible magnitude of permanent deformation. From past experience, it has been found that a total permanent deformation of 0.0001 of the ball or roller diameter occurring at the most heavily stressed ball and race contact, can be tolerated in practice, without any disturbance like noise or vibrations. *The static load carrying capacity of a bearing is defined as the static load which corresponds to a total permanent deformation of balls and races, at the most heavily stressed point of contact, equal to 0.0001 of the ball diameter*.

Formulae are given in standards[1] for calculating the static load carrying capacity of different types of bearings. However, while selecting the bearings, it is not necessary to use these formulae. The values of static load carrying capacities are directly given in the manufacturer's catalogues, which are based on the above formulae. Where conditions of friction, noise and smoothness are not critical, a much higher permanent deformation can be tolerated and consequently static loads up to four times the static load carrying capacity may be permissible. On the other hand, where extreme smoothness of operation is desired, a smaller permanent deformation is permitted.

15.6 STRIBECK'S EQUATION

Stribeck's equation gives the static load capacity of bearing. It is based on the following assumptions:

(i) The races are rigid and retain their circular shape.

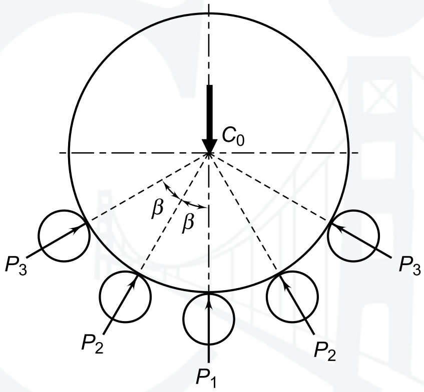
(ii) The balls are equally spaced.

(iii) The balls in the upper half do not support any load.

Figure 15.6 (a) shows the forces acting on the inner race through the rolling elements, which support the static load C_0. It is assumed that there is a single row of balls. Considering the equilibrium of forces in the vertical direction,

$$C_o = P_1 + 2P_2 \cos \beta + 2P_3 \cos (2\beta) + \ldots\ldots \qquad \text{(a)}$$

As the races are rigid, only balls are deformed. Suppose δ_1 is the deformation at the most heavily stressed Ball No.1. Due to this deformation, the inner race is deflected with respect to the outer race through δ_1. As shown in Fig. 15.6(b), the centre



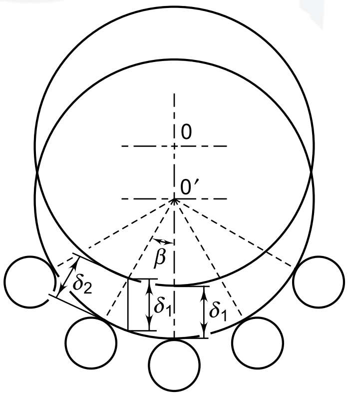


Fig. 15.6 *(a) Forces acting on Inner Race (b) Deflection of Inner Race*

of the inner ring moves from O to O' through the distance δ_1 without changing its shape. Suppose δ_1, δ_2 … are radial deflections at the respective balls.

Also, $\quad \delta_2 = \delta_1 \cos \beta \quad$ or $\quad \dfrac{\delta_2}{\delta_1} = \cos \beta \qquad$ (b)

According to Hertz's equation, the relationship between the load and deflection at each ball is given by,

$$\delta \propto (P)^{2/3}$$

Therefore,

$$\delta_1 = C_1 P_1^{2/3} \quad \text{and} \quad \delta_2 = C_1 P_2^{2/3}$$

$$\therefore \qquad \frac{\delta_2}{\delta_1} = \left(\frac{P_2}{P_1}\right)^{2/3} \qquad \text{(c)}$$

From Eq. (b) and (c),

$$\left(\frac{P_2}{P_1}\right)^{2/3} = \cos \beta$$

or $$P_2 = P_1 (\cos\beta)^{3/2}$$

In a similar way,

$$P_3 = P_1 (\cos 2\beta)^{3/2}$$

Substituting these values in Eq. (a),

$$C_0 = P_1 + 2[P_1(\cos \beta)^{3/2}]\cos \beta + 2[P_1(\cos 2\beta)^{3/2}]\cos 2\beta + \ldots\ldots$$

$$= P_1[1 + 2(\cos \beta)^{5/2} + 2(\cos 2\beta)^{5/2} + \ldots\ldots]$$

or $\quad C_o = P_1 M \qquad$ (d)

where

$$M = [1 + 2(\cos \beta)^{5/2} + 2(\cos 2\beta)^{5/2} + \ldots\ldots] \qquad \text{(e)}$$

If z is the number of balls,

$$\beta = \frac{360}{z}$$

The values of M for different values of z are tabulated as follows:

Z	8	10	12	15
M	1.84	2.28	2.75	3.47
(z/M)	4.35	4.38	4.36	4.37

It is seen from the above table that (z/M) is practically constant. Stribeck suggested the value for (z/M) as 5.

or $$M = \left(\frac{1}{5}\right) z$$

Substituting this value in Eq. (d),

$$C_o = \left(\frac{1}{5}\right) zP_1 \qquad \text{(f)}$$

From experimental evidence, it is found that the force P_1 required to produce a given permanent deformation of the ball is given by,

$$P_1 = kd^2 \qquad \text{(g)}$$

where d is the ball diameter and the factor k depends upon the radii of curvature at the point of contact, and on the modulii of elasticity of materials. From Eqs (f) and (g),

$$C_o = \frac{kd^2 z}{5} \qquad (15.1)$$

The above equation is known as Stribeck's equation.

15.7 DYNAMIC LOAD CARRYING CAPACITY

The life of a ball bearing is limited by the fatigue failure at the surfaces of balls and races. The dynamic load carrying capacity of the bearing is, therefore, based on the fatigue life of the bearing. *The life of an individual ball bearing is defined as the number of revolutions (or hours of service at some given constant speed), which the bearing runs before the first evidence of fatigue crack in balls or races*. Since the life of a single bearing is difficult to predict, it is necessary to define the life in terms of the statistical average performance of a group of bearings. Bearings are rated on one of the two criteria—the average life of a group of bearings or the life, which 90% of the bearings will reach or exceed. The second criterion is widely used in bearing industry. *The rating life of a group of apparently identical ball bearings is defined as the number of revolutions that 90% of the bearings will complete or exceed before the first evidence of fatigue crack*. There are a number of terms used for this rating life. They are minimum life, catalogue life, L_{10} life or B_{10} life. These terms are synonyms for rating life. In this chapter, we will use the term L_{10} life.

The life of an individual ball bearing may be different from rating life. Statistically, it can be proved that the life, which 50% of a group of bearings will complete or exceed, is approximately five times the rating or L_{10} life. This means that for the majority of bearings, the actual life is considerably more than the rated life.

The dynamic load carrying capacity of a bearing is defined as the radial load in radial bearings (or thrust load in thrust bearings) that can be carried for a minimum life of one million revolutions. The minimum life in this definition is the L_{10} life, which 90% of the bearings will reach or exceed before fatigue failure. The dynamic load carrying capacity is based on the assumption that the inner race is rotating while the outer race is stationary. The formulae for calculating the dynamic load capacity for different types of bearings are given in standards[2]. However, the manufacturer's catalogues give ready-made values of dynamic load capacities of bearings.

15.8 EQUIVALENT BEARING LOAD

In actual applications, the force acting on the bearing has two components—radial and thrust. It is therefore necessary to convert the two components acting on the bearing into a single hypothetical load, fulfilling the conditions applied to the dynamic load carrying capacity. Then the hypothetical load can be compared with the dynamic load capacity. *The equivalent dynamic load is defined as the constant radial load in radial bearings (or thrust load in thrust bearings), which if applied to the bearing would give same life as that which the bearing will attain under actual condition of forces*. The expression for the equivalent dynamic load is written as,

$$P = XVF_r + YF_a \qquad (15.2)$$

where,

P = equivalent dynamic load (N)
F_r = radial load (N)
F_a = axial or thrust load (N)
V = race-rotation factor

X and Y are radial and thrust factors respectively and their values are given in the manufacturer's catalogues.

The race-rotation factor depends upon whether the inner race is rotating or the outer race. The value

[2] IS 3824–1983: Method for evaluating dynamic load ratings of rolling bearings (in four parts).

of V is 1 when the inner race rotates while the outer race is held stationary in the housing. The value of V is 1.2 when the outer race rotates with respect to the load, while the inner race remains stationary. In most of the applications, the inner race rotates and the outer race is fixed in the housing. Assuming V as unity, the general equation for equivalent dynamic load is given by,

$$P = XF_r + YF_a \tag{15.3}$$

In this chapter, we will use the above equation for calculating equivalent dynamic load. The effect of V should be considered in special cases, where the outer race rotates and the inner race is stationary.

When the bearing is subjected to pure radial load F_r,

$$P = F_r \tag{15.4}$$

When the bearing is subjected to pure thrust load F_a,

$$P = F_a \tag{15.5}$$

15.9 LOAD-LIFE RELATIONSHIP

The relationship between the dynamic load carrying capacity, the equivalent dynamic load, and the bearing life is given by,

$$L_{10} = \left(\frac{C}{P}\right)^p \tag{15.6}$$

where,

L_{10} = rated bearing life (in million revolutions)
C = dynamic load capacity (N), and
p = 3 (for ball bearings)
p = 10/3 (for roller bearings)

Rearranging Eq. (15.6),

$$C = P\,(L_{10})^{1/p}$$

For all types of ball bearings,

$$C = P\,(L_{10})^{1/3} \tag{15.7}$$

For all types of roller bearings,

$$C = P\,(L_{10})^{0.3} \tag{15.8}$$

The relationship between life in million revolutions and life in working hours is given by

$$L_{10} = \frac{60nL_{10h}}{10^6} \tag{15.9}$$

where,

L_{10h} = rated bearing life (hours)
n = speed of rotation (rpm)

Example 15.1 *In a particular application, the radial load acting on a ball bearing is 5 kN and the expected life for 90% of the bearings is 8000 h. Calculate the dynamic load carrying capacity of the bearing, when the shaft rotates at 1450 rpm.*

Solution

Given $F_r = 5$ kN $\quad L_{10h} = 8000$ h $\quad n = 1450$ rpm

Step I *Bearing life (L_{10})*

$$L_{10} = \frac{60nL_{10h}}{10^6} = \frac{60(1450)(8000)}{10^6}$$
$$= 696 \text{ million rev.}$$

Step II *Dynamic load capacity*

Since the bearing is subjected to purely radial load,

$$P = F_r = 5000 \text{ N}$$

From Eq. (15.7),

$$C = P(L_{10})^{1/3} = (5000)(696)^{1/3} = 44\,310.48 \text{ N}$$

Example 15.2 *A taper roller bearing has a dynamic load capacity of 26 kN. The desired life for 90% of the bearings is 8000 h and the speed is 300 rpm. Calculate the equivalent radial load that the bearing can carry.*

Solution

Given $C = 26$ kN $\quad L_{10h} = 8000$ h $\quad n = 300$ rpm

Step I *Bearing life (L_{10})*

$$L_{10} = \frac{60nL_{10h}}{10^6} = \frac{60(300)(8000)}{10^6}$$
$$= 144 \text{ million rev.}$$

Step II *Equivalent radial load*

From Eq. (15.8), $C = P\,(L_{10})^{0.3}$

$$\therefore \qquad P = \frac{C}{(L_{10})^{0.3}} = \frac{26\,000}{(144)^{0.3}} = 5854.16 \text{ N}$$

Since the bearing is subjected to purely radial load,

$$F_r = P = 5854.16 \text{ N}$$

15.10 SELECTION OF BEARING LIFE

While selecting the proper size of a bearing, it is necessary to specify the expected life of the bearing for the given application. The information regarding the life expectancy is generally vague and values based on past experience are used. For all kinds of vehicles, the speed of rotation is not constant and the desired life is expressed in terms of millions of revolutions. The recommended bearing life for wheel applications is given in Table15.1.

Table 15.1 *Bearing life for wheel applications*

Wheel application	*Life (million rev.)*
Automobile cars	50
Trucks	100
Trolley cars	500
Rail-road cars	1000

In the other applications, the speed of rotation is relatively constant and the desired life is expressed in terms of hours of service. The recommended bearing life for some of the applications is given in Table 15.2.

Table 15.2 *Bearing life for industrial applications*

(i) Machines used intermittently such as lifting tackle, hand tools and household appliances	4000–8000 h
(ii) Machines used for eight hours of service per day, such as electric motors and gear drives	12 000–20 000 h
(iii) Machines used for continuous operation (24 h per day) such as pumps, compressors and conveyors	40 000–60 000 h

The values given in the above tables are only for general guidance. For a particular application, the designer should consider the past experience, the difficulties faced by the customer in replacing the bearing and the economics of breakdown costs.

15.11 LOAD FACTOR

The forces acting on the bearing are calculated by considering the equilibrium of forces in vertical and horizontal planes. These elementary equations do not take into consideration the effect of dynamic load. The forces determined by these equations are multiplied by a *load factor* to determine the dynamic load carrying capacity of the bearing. Load factors are used in applications involving gear, chain and belt drives. In gear drives, there is an additional dynamic load due to inaccuracies of the tooth profile and the elastic deformation of teeth. In chain and belt drives, the dynamic load is due to vibrations. The values of load factor are given in Table 15.3.

Table 15.3 *Values of load factor*

Types of drive	*Load factor*
(A) *Gear drives*	
(i) Rotating machines free from impact like electric motors and turbo-compressors	1.2–1.4
(ii) Reciprocating machines like internal combustion engines and compressors	1.4–1.7
(iii) Impact machines like hammer mills	2.5–3.5
(B) *Belt drives*	
(i) V-belts	2.0
(ii) Single-ply leather belt	3.0
(iii) Double-ply leather belt	3.5
(C) *Chain drives*	1.5

The values of the load factor given in the above table are used in the absence of precise analysis of dynamic forces.

15.12 SELECTION OF BEARING FROM MANUFACTURER'S CATALOGUE

The basic procedure for the selection of a bearing from the manufacturer's catalogue consists of the following steps:

(i) Calculate the radial and axial forces acting on the bearing and determine the diameter of the shaft where the bearing is to be fitted.

(ii) Select the type of bearing for the given application.

(iii) Determine the values of X and Y, the radial and thrust factors, from the catalogue. The values of X and Y factors for single-row deep groove ball bearings are given in Table 15.4. The values depend upon two ratios, $\left(\frac{F_a}{F_r}\right)$ and $\left(\frac{F_a}{C_0}\right)$, where C_0 is the static load capacity. The selection of the bearing is, therefore, done by trial and error. The static and dynamic load capacities of single-row deep groove ball bearings of different series are given in Table 15.5. To begin with, a bearing of light series, such as 60, is selected for the given diameter of the shaft and the value of C_0 is found from Table 15.5. Knowing the ratios $\left(\dfrac{F_a}{C_0}\right)$ and $\left(\dfrac{F_a}{F_r}\right)$ the values of X and Y factors are found from Table 15.4.

(iv) Calculate the equivalent dynamic load from the equation.

$$P = XF_r + YF_a$$

(v) Make a decision about the expected bearing life and express the life L_{10} in million revolutions.

(vi) Calculate the dynamic load capacity from the equation

$$C = P\,(L_{10})^{1/3}$$

(vii) Check whether the selected bearing of series 60 has the required dynamic capacity. If not, select the bearing of the next series and go back to Step (iii) and continue.

Ball bearings are thus selected by the trial and error procedure. The above procedure is also applicable to other types of bearings.

Table 15.4 *X and Y factors for single-row deep groove ball bearings*[3]

$\left(\dfrac{F_a}{C_0}\right)$	$\left(\dfrac{F_a}{F_r}\right) \le e$		$\left(\dfrac{F_a}{F_r}\right) > e$		e
	X	Y	X	Y	
0.025	1	0	0.56	2.0	0.22
0.040	1	0	0.56	1.8	0.24
0.070	1	0	0.56	1.6	0.27
0.130	1	0	0.56	1.4	0.31
0.250	1	0	0.56	1.2	0.37
0.500	1	0	0.56	1.0	0.44

In order to explain the selection procedure, let us consider a numerical example. Suppose it is required to select a single-row deep groove ball bearing, for a shaft that is 75 mm in diameter and which rotates at 125 rpm. The bearing is subjected to a radial load of 21 kN and there is no thrust load. The expected life of the bearing is 10 000 hours.

Step (i)

$F_r = 21\,000$ N $\quad F_a = 0 \quad d = 75$ mm

Step (ii) Type: single-row deep groove ball bearing

Step (iii) Since there is no axial load,

$$P = F_r = 21\,000 \text{ N}$$

Step (iv) $$L_{10} = \frac{60n\,L_{10h}}{10^6} = \frac{60\,(125)(10\,000)}{10^6}$$

$$= 75 \text{ million rev.}$$

Step (v) $$C = P\,(L_{10})^{1/3} = 21\,000\,(75)^{1/3}$$

$$= 88\,560.43 \text{ N}$$

Step (vi) It is observed from Table 15.5, that following bearings are available with 75 mm bore diameter,

No. 6015 (C = 39 700 N)
No. 6215 (C = 66 300 N)
No. 6315 (C = 112 000 N)
No. 6415 (C = 153 000 N)

Therefore, bearing No. 6315 is selected for the above application.

Table 15.5 *Dimensions and static and dynamic load capacities of single-row deep groove ball bearings*[4]

Principal dimensions (mm)			*Basic load ratings (N)*		*Designation*
d	*D*	*B*	*C*	*C_0*	
10	19	5	1480	630	61800
	26	8	4620	1960	6000
	30	9	5070	2240	6200
	35	11	8060	3750	6300

(*Contd*)

[3] The values of X and Y factors are based on normal clearance. For other clearances, refer to SKF Catalogue. (Courtesy of Associated Bearing Company Limited, Pune)

[4] In Table No. 15.5 and 15.6, the following notations are used:

d = inner diameter of the bearing
D = outer diameter of the bearing
B = axial width of the bearing

Table 15.5 (*Contd*)

Principal dimensions (mm)			Basic load ratings (N)		Designation
d	*D*	*B*	*C*	C_0	
12	21	5	1430	695	61801
	28	8	5070	2240	6001
	32	10	6890	3100	6201
	37	12	9750	4650	6301
15	24	5	1560	815	61802
	32	9	5590	2500	6002
	35	11	7800	3550	6202
	42	13	11400	5400	6302
17	26	5	1680	930	61803
	35	10	6050	2800	6003
	40	12	9560	4500	6202
	47	14	13500	6550	6303
	62	17	22900	11800	6403
20	32	7	2700	1500	61804
	42	8	7020	3400	16404
	42	12	9360	4500	6004
	47	14	12700	6200	6204
	52	15	15900	7800	6304
	72	19	30700	16600	6404
25	37	7	3120	1960	61805
	47	8	7610	4000	16005
	47	12	11200	5600	6005
	52	15	14000	6950	6205
	62	17	22500	11400	6305
	80	21	35800	19600	6405
30	42	7	3120	2080	61806
	55	9	11200	5850	16006
	55	13	13300	6800	6006
	62	16	19500	10000	6206
	72	19	28100	14600	6306
	90	23	43600	24000	6406
35	47	7	4030	3000	61807
	62	9	12400	6950	16007
	62	14	15900	8500	6007
	72	17	25500	13700	6207
	80	21	33200	18000	6307
	100	25	55300	31000	6407

(*Contd*)

Table 15.5 (*Contd*)

Principal dimensions (mm)			Basic load ratings (N)		Designation
d	*D*	*B*	*C*	C_0	
40	52	7	4160	3350	61808
	68	9	13300	7800	16008
	68	15	16800	9300	6008
	80	18	30700	16600	6208
	90	23	41000	22400	6308
	110	27	63700	36500	6408
45	58	7	6050	3800	61809
	75	10	15600	9300	16009
	75	16	21200	12200	6009
	85	19	33200	18600	6209
	100	25	52700	30000	6309
	120	29	76100	45500	6409
50	65	7	6240	4250	61810
	80	10	16300	10000	16010
	80	16	21600	13200	6010
	90	20	35100	19600	6210
	110	27	61800	36000	6310
	130	31	87100	52000	6410
55	72	9	8320	5600	61811
	90	11	19500	12200	16011
	90	18	28100	17000	6011
	100	21	43600	25000	6211
	120	29	71500	41500	6311
	140	33	99500	63000	6411
60	78	10	8710	6100	61812
	95	11	19900	13200	16012
	95	18	29600	18300	6012
	110	22	47500	28000	6212
	130	31	81900	48000	6312
	150	35	108000	69500	6412
65	85	10	11700	8300	61813
	100	11	21200	14600	16013
	100	18	30700	19600	6013
	120	23	55900	34000	6213
	140	33	92300	56000	6313
	160	37	119000	78000	6413

(*Contd*)

Table 15.5 (*Contd*)

Principal dimensions (mm)			*Basic load ratings (N)*		*Designation*
d	*D*	*B*	*C*	C_0	
70	90	10	12100	9150	61814
	110	13	28100	19000	16014
	110	20	37700	24500	6014
	125	24	61800	37500	6214
	150	35	104000	63000	6314
	180	42	143000	104000	6414
75	95	10	12500	9800	61815
	115	13	28600	20000	10615
	115	20	39700	26000	6015
	130	25	66300	40500	6215
	160	37	112000	72000	6315
	190	45	153000	114000	6415

While selecting a ball bearing from the manufacturer's catalogue, very often a term '*series*' of the bearing is used. Some manufacturers use terms such as light series, medium series and heavy series. For a given diameter of the shaft, the dimensions of balls and races are smaller in the light series. The load capacities are less and cost is also less. For the same shaft diameter, the dynamic load carrying capacity of the bearing belonging to the heavy series is more and the cost is also more. The trial and error method for the selection of a ball bearing begins with the light series. If that is not suitable, a medium series bearing is selected. The last choice is the costliest heavy series bearing.

A rolling contact bearing is usually designated by three or four digits. The meaning of these digits is as follows:

(i) The last two digits indicate the bore diameter of the bearing in mm (bore diameter divided by 5). For example, XX07 indicates a bearing of 35 mm bore diameter.

(ii) The third digit from the right indicates the series of the bearing. The numbers used to indicate the series are as follows:
Extra light series –1
Light series – 2
Medium series – 3
Heavy series – 4
For example, X307 indicates a medium series bearing with a bore diameter of 35 mm.

(iii) The fourth digit and sometimes fifth digit from the right specifies the type of rolling contact bearing. For example, the digit 6 indicates deep groove ball bearings

Light series bearings permit smallest bearing width and outer diameter for a given bore diameter. They have lowest load carrying capacities. Medium series bearings have 30 to 40 per cent higher dynamic load carrying capacities compared with light series bearings of the same bore diameter. However, they occupy more radial and axial space. Heavy series bearings have 20 to 30 per cent higher dynamic load carrying capacities compared with medium series bearings of the same bore diameter.

The ISO plan for the dimension series of the bearing is illustrated in Fig. 15.7(a). It consists of two digit numbers. The first number indicates the width series 8, 0, 1, 2, 3, 4, 5, and 6 in order of increasing width. The second number indicates the diameter series 7, 8, 9, 0, 1, 2, 3, and 4 in order of ascending outer diameter of the bearing. Figure 15.7(b) shows the proportionate dimensions of SKF bearings belonging to different series with a 50 mm bore diameter (Bearing No. 6010, 6210, 6310 and 6410).

Example 15.3 *A single-row deep groove ball bearing is subjected to a pure radial force of 3 kN from a shaft that rotates at 600 rpm. The expected life L_{10h} of the bearing is 30 000 h. The minimum acceptable diameter of the shaft is 40 mm. Select a suitable ball bearing for this application.*

Solution

Given $F_r = 3$ kN $L_{10h} = 30\,000$ h $n = 600$ rpm $d = 40$ mm

Step I *Dynamic load capacity*

The bearing is subjected to pure radial load.

$$\therefore \quad P = F_r = 3000 \text{ N}$$

From Eq. (15.9),

$$L_{10} = \frac{60n\,L_{10h}}{10^6} = \frac{60(600)(30\,000)}{10^6}$$

$$=1080 \text{ million rev.}$$

From Eq. (15.7),

$$C = P\,(L_{10})^{1/3} = (3000)(1080)^{1/3} = 30\,779.57 \text{ N}$$

Step II *Selection of bearing*

In Table 15.5, six different bearings are available for the shaft diameter of 40 mm. They are 61808, 16008, 6008, 6208, 6308 and 6408. Out of these, Bearing No. 6208

(C = 30 700 N) is suitable for the above application. The small difference between the expected and actual dynamic capacities, viz., (30 779.57 – 30 700 = 79.57 N) is considered to be negligible.

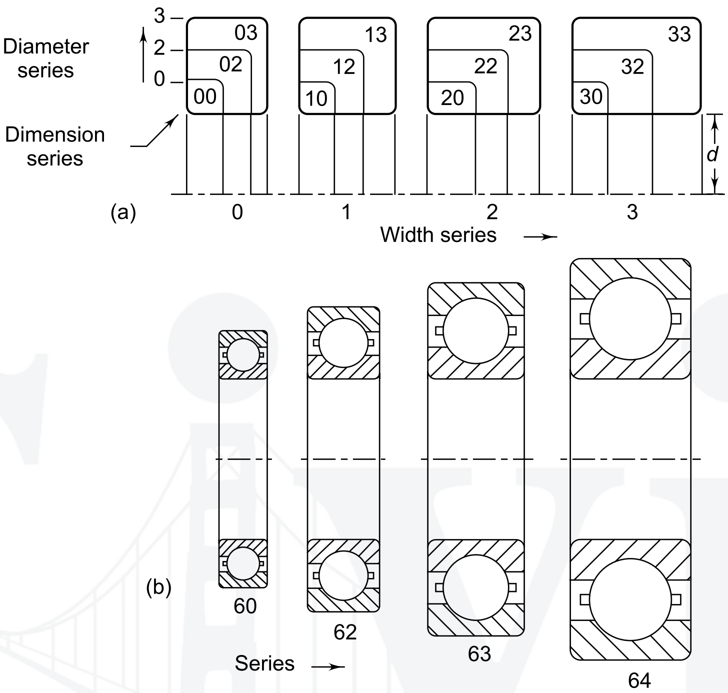


Fig. 15.7 *Dimension Series of Bearing*

Example 15.4 *A single-row deep groove ball bearing is subjected to a radial force of 8 kN and a thrust force of 3 kN. The shaft rotates at 1200 rpm. The expected life L_{10h} of the bearing is 20 000 h. The minimum acceptable diameter of the shaft is 75 mm. Select a suitable ball bearing for this application.*

Solution

Given $F_r = 8$ kN $F_a = 3$ kN $L_{10h} = 20\,000$ hr
$n = 1200$ rpm $d = 75$ mm

Step I *X and Y factors*

When the bearing is subjected to radial as well as axial load, the values of X and Y factors are obtained from Table 15.4 by trial and error procedure. It is observed from Table 15.4, that values of X are constant and the values of Y vary only in case when,

$$\left(\frac{F_a}{F_r}\right) > e$$

In this case, the value of Y varies from 1.0 to 2.0. We will assume the average value 1.5 as the first trial value for the factor Y. Therefore,

$X = 0.56$ $Y = 1.5$ $F_r = 8000$ N $F_a = 3000$ N

From Eq. (15.3),

$P = XF_r + YF_a = 0.56(8000) + 1.5(3000) = 8980$ N

From Eq. (15.9),

$$L_{10} = \frac{60n\,L_{10h}}{10^6} = \frac{60(1200)(20\,000)}{10^6}$$

$= 1440$ million rev.

From Eq. (15.7),

$C = P\,(L_{10})^{1/3} = (8980)(1440)^{1/3} = 101\,406.04$ N

From Table 15.5, it is observed that for the shaft of 75 mm diameter, Bearing No. 6315 ($C = 112\,000$) is suitable for the above data. For this bearing,

$$C_o = 72\,000 \text{ N}$$

Therefore,

$$\left(\frac{F_a}{F_r}\right) = \left(\frac{3000}{8000}\right) = 0.375$$

and $$\left(\frac{F_a}{C_o}\right) = \left(\frac{3000}{72000}\right) = 0.04167$$

Referring to Table 15.4,

$$e = 0.24 \text{ (approximately)} \quad \text{and} \quad \left(\frac{F_a}{F_r}\right) > e$$

The value of Y is obtained by linear interpolation.

$$Y = 1.8 - \frac{(1.8 - 1.6)}{(0.07 - 0.04)} \times (0.04167 - 0.04) = 1.79$$

and $\quad X = 0.56$

Step II *Dynamic load capacity*

$$P = XF_r + YF_a = 0.56(8000) + 1.79(3000) = 9850 \text{ N}$$

$$C = P\,(L_{10})^{1/3} = 9850\,(1440)^{1/3} = 111\,230.46 \text{ N}$$

Step III *Selection of bearing*

From Table 15.5, Bearing No. 6315 (C = 112 000) is suitable for the above application.

Example 15.5 *A transmission shaft rotating at 720 rpm and transmitting power from the pulley P to the spur gear G is shown in Fig. 15.8(a). The belt tensions and the gear tooth forces are as follows:*

$P_1 = 498$ N $\quad P_2 = 166$ N $\quad P_t = 497$ N $\quad P_r = 181$ N

The weight of the pulley is 100 N. The diameter of the shaft at bearings B_1 and B_2 is 10 mm and 20 mm respectively. The load factor is 2.5 and the expected life for 90% of the bearings is 8000 h. Select single-row deep groove ball bearings at B_1 and B_2.

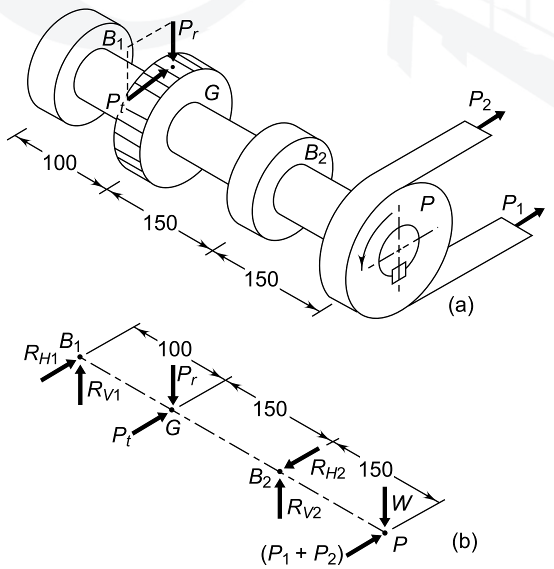


Fig. 15.8

Solution

Given $\quad n = 720$ rpm $\quad d_1 = 10$ mm $\quad d_2 = 20$ mm

$L_{10h} = 8000$ h $\quad$ load factor = 2.5

Step I *Radial and axial forces*

The forces acting on the shaft are shown in Fig. 15.8(b).

Considering forces in the vertical plane and taking

also $\quad R_{V1} + R_{V2} = P_r + W$

or $\quad R_{V1} + 232.4 = 181 + 100$

$\therefore \quad R_{V1} = 48.6$ N $\quad$ (ii)

Considering forces in the horizontal plane and taking moments of forces about the bearing B_1,

$$P_t\,(100) + (P_1 + P_2)(400) - R_{H2}(250) = 0$$

or $\quad 497(100) + (498 + 166)(400) - R_{H2}(250) = 0$

$\therefore \quad R_{H2} = 1261.2$ N $\quad$ (iii)

$$R_{H2} = R_{H1} + P_t + (P_1 + P_2)$$

or $\quad 1261.2 = R_{H1} + 497 + (498 + 166)$

$\therefore \quad R_{H1} = 100.2$ N $\quad$ (iv)

The reactions at the two bearings are given by

$$R_1 = \sqrt{(R_{V1})^2 + (R_{H1})^2} = \sqrt{(48.6)^2 + (100.2)^2}$$

$$= 111.36 \text{ N}$$

$$R_2 = \sqrt{(R_{V2})^2 + (R_{H2})^2} = \sqrt{(232.4)^2 + (1261.2)^2}$$

$$= 1282.43 \text{ N}$$

The bearing reactions are in the radial direction. Therefore,

$$F_{r1} = R_1 = 111.36 \text{ N}$$

$$F_{r2} = R_2 = 1282.43 \text{ N}$$

There is no axial thrust on these bearings; hence,

$$F_{a1} = F_{a2} = 0$$

Step II *Dynamic load capacities*

From Eq. (15.4),

$$P_1 = F_{r1} = 111.36 \text{ N}$$

$$P_2 = F_{r2} = 1282.43 \text{ N}$$

From Eq. (15.9),

$$L_{10} = \frac{60nL_{10h}}{10^6} = \frac{60(720)(8000)}{10^6}$$

$$= 345.6 \text{ million rev.}$$

Considering the load factor and using Eq. (15.7),

$$C_1 = P_1(L_{10})^{1/3} \text{ (Load factor)}$$

$$= (111.36)(345.6)^{1/3}(2.5) = 1953.71 \text{ N}$$

$$C_2 = P_2(L_{10})^{1/3} \text{ (Load factor)}$$

$$= (1282.43)(345.6)^{1/3}(2.5) = 22499.09 \text{ N}$$

Step III *Selection of bearings*

Referring to Table 15.5, the following bearings are available for 10 mm and 20 mm shaft diameter;

Bearing Nos. 6000 and 6404 are suitable at B_1 and B_2 respectively.

Example 15.6 *A shaft transmitting 50 kW at 125 rpm from the gear G_1 to the gear G_2 and mounted on* *two single-row deep groove ball bearings B_1 and B_2 is shown in Fig. 15.9(a). The gear tooth forces are*

$$P_{t1} = 15915\ N \qquad P_{r1} = 5793\ N$$

$$P_{t2} = 9549\ N \qquad P_{r2} = 3476\ N$$

The diameter of the shaft at bearings B_1 and B_2 is 75 mm. The load factor is 1.4 and the expected life for 90% of the bearings is 10000 h. Select suitable

10 mm	*20 mm*
(i) No. 61800 (C = 1480 N)	(i) No. 61804 (C = 2700 N)
(ii) No. 6000 (C = 4620 N)	(ii) No. 16404 (C = 7020 N)
(iii) No. 6200 (C = 5070 N)	(iii) No. 6004 (C = 9360 N)
(iv) No. 6300 (C = 8060 N)	(iv) No. 6204 (C = 12700 N)
	(v) No. 6304 (C = 15900 N)
	(vi) No. 6404 (C = 30700 N)

ball bearings.

Solution

Given $kW = 50 \quad n = 125$ rpm $\quad d = 75$ mm

$L_{10h} = 10\ 000$ h $\quad$ load factor = 1.4

Step I *Radial and axial forces*

The forces acting on the shaft are shown in Fig. 15.9(b). Considering forces in the vertical plane and taking moments about bearing B_1,

$$P_{r1}(125) + P_{t2}(775) - R_{V2}(625) = 0$$

or $\quad 5793(125) + 9549(775) - R_{V2}(625) = 0$

$\therefore \quad R_{V2} = 12\,999$ N

Considering equilibrium of vertical forces,

$$P_{t2} + P_{r1} = R_{v2} + R_{v1}$$

or $\quad 9549 + 5793 = 12\,999 + R_{V1}$

$\therefore \quad R_{V1} = 2343$ N

A similar procedure is repeated for forces in the horizontal plane and the reactions calculated as follows:

$$R_{H1} = 11\,898\ \text{N} \qquad R_{H2} = 7493\ \text{N}$$

The radial forces at the two bearings are given by

$$F_{r1} = \sqrt{(R_{V1})^2 + (R_{H1})^2} = \sqrt{(2343)^2 + (11898)^2}$$

$$= 12\ 127\ \text{N}$$

$$F_{r2} = \sqrt{(R_{V2})^2 + (R_{H2})^2} = \sqrt{(12999)^2 + (7493)^2}$$

$$= 15\ 004\ \text{N}$$

Since there is no axial thrust,

$$F_{a1} = F_{a2} = 0$$

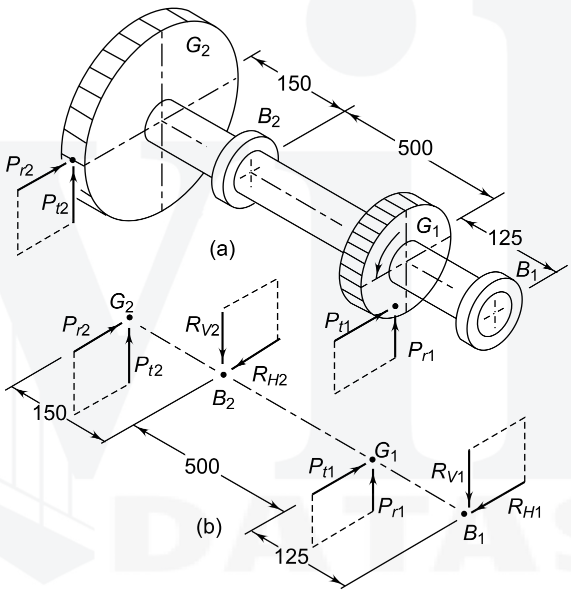


Fig. 15.9

Step II *Dynamic load capacities*

$$P_1 = F_{r1} = 12\ 127\ \text{N}$$

$$P_2 = F_{r2} = 15\ 004\ \text{N}$$

From Eq. (15.9),

$$L_{10} = \frac{60nL_{10h}}{10^6} = \frac{60(125)(10000)}{10^6}$$

$$= 75 \text{ million rev.}$$

Considering the load factor and using Eq. (15.7), the dynamic load capacities are given by,

$$C_1 = P_1(L_{10})^{1/3} \text{ (Load factor)}$$

$$= (12127)(75)^{1/3}(1.4) = 71\ 598\ \text{N}$$

$$C_2 = P_2(L_{10})^{1/3} \text{ (Load factor)}$$

$$= (15004)(75)^{1/3}(1.4) = 88\,584 \text{ N}$$

Step III *Selection of bearings*

From Table 15.5, the available bearings at B_1 and B_2 are as follows:

B_1 and B_2 (d = 75 mm)
No. 6015 (C = 39 700 N)
No. 6215 (C = 66 300 N)
No. 6315 (C = 112 000 N)
No. 6415 (C = 153 000 N)

Therefore, Bearing No. 6315 is suitable at B_1 as well as B_2.

Example 15.7 *A single-row deep groove ball bearing No. 6002 is subjected to an axial thrust of 1000 N and a radial load of 2200 N. Find the expected life that 50% of the bearings will complete under this condition.*

Solution

Given F_a = 1000 N F_r = 2200 N
Bearing = No. 6002

Step I *X and Y factors*

Referring to Table 15.5, the capacities of bearing No. 6002 are,

$$C_o = 2500 \text{ N} \quad \text{and} \quad C = 5590 \text{ N}$$

Also, $F_a = 1000$ N $F_r = 2200$ N

$$\left(\frac{F_a}{F_r}\right) = \left(\frac{1000}{2200}\right) = 0.455$$

and

$$\left(\frac{F_a}{C_o}\right) = \left(\frac{1000}{2500}\right) = 0.4$$

Referring to Table 15.4,

$$\left(\frac{F_a}{F_r}\right) > e$$

The value of Y is obtained by linear interpolation.

$$Y = 1.2 - \frac{(1.2 - 1.0)}{(0.5 - 0.25)} \times (0.4 - 0.25) = 1.08 \text{ and}$$

$$X = 0.56$$

Step II *Bearing life (L_{10})*

$$P = XF_r + YF_a = 0.56(2200) + 1.08(1000)$$
$$= 2312 \text{ N}$$

From Eq. (15.7),

$$C = P\,(L_{10})^{1/3} \qquad 5590 = 2312\,(L_{10})^{1/3}$$

$$\therefore \qquad L_{10} = 14.13 \text{ million rev.}$$

Step III *Bearing life (L_{50})*

It can be proved that the life (L_{50}), which 50% of the bearings will complete or exceed, is approximately five times the life L_{10} which 90% of the bearings will complete or exceed.

Therefore,

$$L_{50} = 5L_{10} = 5\,(14.13) = 70.65 \text{ million rev.}$$

15.13 SELECTION OF TAPER ROLLER BEARINGS

The terminology related to taper roller bearings is slightly different from that of ball or cylindrical roller bearings. As mentioned in the earlier discussion, in taper roller bearing, the inner race is called a cone, and the outer race, the cup. The cup is separable from the remaining assembly of the bearing, consisting of the cone, cage and rollers. These two parts can be separately mounted to the housing and the journal. In this type of bearing, it is possible to make adjustment for radial clearance. There are two varieties of taper roller bearings involving single-row and double-row constructions. The discussion in this chapter is restricted to single-row taper roller bearings.

In taper roller bearings, the line of action of the resultant reaction makes an angle with the axis of the bearing. This reaction can be resolved into radial and axial components. Therefore, taper roller bearings are suitable for carrying combined axial and radial loads. The conical surface of each roller is subjected to pressure, which acts normal to the surface. Therefore, even if the external force acting on the bearing is purely radial, it induces a thrust reaction within the bearing. To avoid separation of the cup from the cone, this thrust reaction must be balanced by an equal and opposite force. One of the methods of creating this force is to use at least two taper roller bearings on the same shaft. It such a case, the thrust reactions of two bearings balance each other. There are two types of popular construction, with two bearings on the same shaft. When two bearings are mounted on the shaft, with their backs facing each other, the mounting is said to be 'back-to-back' or indirect mounting. The construction, which involves two bearings with their fronts facing each other, is called 'face-to-face' or direct mounting. These constructions are illustrated in Figs 15.10 and 15.11.

The thrust component F_a created due to radial load F_r is approximately given by

$$F_a = \frac{0.5F_r}{Y} \tag{15.10}$$

where Y is the thrust factor. In the preliminary stages of bearing selection, the value of Y is taken as 1.5. The equivalent dynamic load for single-row taper roller bearings is given by

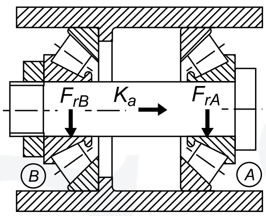
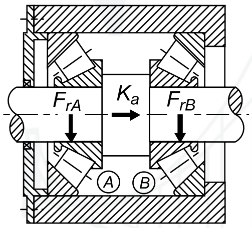
Arrangements	Load case	Axial loads
 Back to back	Case 1(a): $F_{rA} \geq F_{rB}$ $K_a \geq 0$	$F_{aA} = \dfrac{0.5F_{rA}}{Y}$ $F_{aB} = F_{aA} + K_a$
	Case 1(b): $F_{rA} < F_{rB}$ $K_a \geq 0.5(F_{rB} - F_{rA})$	$F_{aA} = \dfrac{0.5F_{rA}}{Y}$ $F_{aB} = F_{aA} + K_a$
 Face to face	Case 1(c): $F_{rA} < F_{rB}$ $K_a < 0.5(F_{rB} - F_{rA})$	$F_{aA} = F_{aB} - K_a$ $F_{aB} = \dfrac{0.5F_{rB}}{Y}$

Fig. 15.10 *Axial Loading of Taper Roller Bearings*

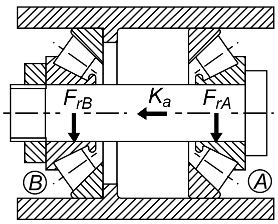
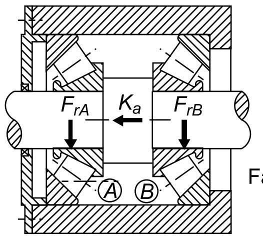
Arrangements	Load case	Axial loads
 Back to back	Case 2 (a): $F_{rA} < F_{rB}$ $K_a \geq 0$	$F_{aA} = F_{aB} + K_a$ $F_{aB} = \dfrac{0.5F_{rB}}{Y}$
	Case 2 (b): $F_{rA} > F_{rB}$ $K_a \geq 0.5(F_{rA} - F_{rB})$	$F_{aA} = F_{aB} + K_a$ $F_{aB} = \dfrac{0.5F_{rB}}{Y}$
 Face to face	Case 2 (c): $F_{rA} > F_{rB}$ $K_a < 0.5(F_{rA} - F_{rB})$	$F_{aA} = \dfrac{0.5F_{rA}}{Y}$ $F_{aB} = F_{rA} - K_a$

Fig. 15.11 *Axial Loading of Taper Roller Bearings*

$P = F_r$ when $(F_a/F_r) \leq e$ (15.11)

$P = 0.4F_r + YF_a$ when $(F_a/F_r) > e$ (15.12)

The dimensions, dynamic load carrying capacity, values of factor Y, value of e and designation of single-row taper roller bearing are given in Table 15.6. The equations for calculating thrust load for various bearing arrangements and load cases are given in Figs 15.10 and 15.11. The equations given in the figures are based on the following assumptions:

(i) The bearings are adjusted against each other to give zero clearance in operation but are without pre-load.

(ii) Bearings A and B are exactly identical ($Y_A = Y_B = Y$).

In the bearing arrangements shown in the figures, the bearing A is subjected to the radial load F_{rA} while bearing B to radial load F_{rB}. K_a is the external axial force acting on the shaft. The radial loads F_{rA} and F_{rB} are always considered positive, even in cases when both act in the direction opposite to that shown in figures.

Table 15.6 *Dimensions (mm), dynamic capacities (N) and calculation factors for single-row taper roller bearings*

d	*D*	*B*	*C*	*Designation*	*e*	*Y*
20	42	15	22 900	32004 *X*	0.37	1.6
	47	15.25	26 000	30204	0.35	1.7
	52	16.25	31 900	30304	0.30	2.0
	52	22.25	41 300	32304	0.30	2.0
25	47	15	25 500	32005 *X*	0.43	1.4
	52	16.25	29 200	30205	0.37	1.6
	52	19.25	34 100	32205 *B*	0.57	1.05
	52	22	44 000	33205	0.35	1.7
	62	18.25	41 800	30305	0.30	2
	62	18.25	35 800	31305	0.83	0.72
	62	25.25	56 100	32305	0.30	2
30	55	17	33 600	32006 *X*	0.43	1.4
	62	17.25	38 000	30206	0.37	1.6
	62	21.25	47 300	32206	0.37	1.6
	62	21.25	45 700	32206 *B*	0.57	1.05
	62	25	60 500	33206	0.35	1.7
	72	20.75	52 800	30306	0.31	1.9
	72	20.75	44 600	31306	0.83	0.72
	72	28.75	72 100	32306	0.31	1.9
35	62	18	40 200	32007 *X*	0.46	1.3
	72	18.25	48 400	30207	0.37	1.6
	72	24.25	61 600	32207	0.37	1.6
	72	24.25	57 200	32207 *B*	0.57	1.05
d	*D*	*B*	*C*	*Designation*	*e*	*Y*

(*Contd*)

Table 15.6 (*Contd*)

35	72	28	79 200	33207	0.35	1.7
	80	22.75	68 200	30307	0.31	1.9
	80	22.75	57 200	31307	0.83	0.72
	80	32.75	89 700	32307	0.31	1.9
	80	32.75	88 000	32307 *B*	0.54	1.1
40	68	19	49 500	32008 *X*	0.37	1.6
	75	26	74 800	33108	0.35	1.7
	80	19.75	58 300	30208	0.37	1.6
	80	24.75	70 400	32208	0.37	1.6
	80	32	96 800	33208	0.35	1.7
	85	33	114 000	T2EE040	0.35	1.7
	90	25.25	80 900	30308	0.35	1.7
	90	25.25	69 300	31308	0.83	0.72
	90	35.25	110 000	32308	0.35	1.7
45	75	20	55 000	32009 *X*	0.40	1.5
	80	26	79 200	33109	0.37	1.6
	85	20.75	62 700	30209	0.40	1.5
	85	24.75	74 800	32209	0.40	1.5
	85	32	101 000	33209	0.40	1.5
	95	29	84 200	T7FC045	0.88	0.68
	95	36	140 000	T2ED045	0.33	1.8
	100	27.25	101 000	30309	0.35	1.7
	100	27.25	85 800	31309	0.83	0.72
	100	38.25	132 000	32309	0.35	1.7
	100	38.25	128 000	32309 *B*	0.54	1.1
50	80	20	57 200	32010 *X*	0.43	1.4
	80	24	64 400	33010	0.31	1.9
	85	26	80 900	33110	0.40	1.5
	90	21.75	70 400	30210	0.43	1.4
	90	24.75	76 500	32210	0.43	1.4
	90	32	108 000	33210	0.40	1.5
	100	36	145 000	T2ED050	0.35	1.7
	105	32	102 000	T7FC050	0.88	0.68
	110	29.25	117 000	30310	0.35	1.7
	110	29.25	99 000	31310	0.83	0.72
	110	42.25	161 000	32310	0.35	1.7
	110	42.25	151 000	32310 *B*	0.54	1.1
60	95	23	76 500	32012 *X*	0.43	1.4
	95	27	85 800	33012	0.33	1.8

(*Contd*)

Table 15.6 (*Contd*)

d	*D*	*B*	*C*	*Designation*	*e*	*Y*
60	100	30	110 000	33112	0.40	1.5
	110	23.75	91 300	30212	0.40	1.5
	110	29.75	119 000	32212	0.40	1.5
	110	38	157 000	33212	0.40	1.5
	115	39	157 000	T5ED060	0.54	1.1
	115	40	183 000	T2EE060	0.33	1.8
60	125	37	145 000	T7FC060	0.83	0.72
	130	33.5	161 000	30312	0.35	1.7
	130	33.5	134 000	31312	0.83	0.72
	130	48.5	216 000	32312	0.35	1.7
	130	48.5	205 000	32312 *B*	0.54	1.1
70	110	25	95 200	32014 *X*	0.43	1.4
	110	31	121 000	33014	0.28	2.1
	120	37	161 000	33114	0.37	1.6
	125	26.25	119 000	30214	0.43	1.4
	125	33.25	147 000	32214	0.43	1.4
	125	41	190 000	33214	0.40	1.5
	130	43	220 000	T2ED070	0.33	1.8
	140	39	168 000	T7FC070	0.88	0.68
	140	52	264 000	T4FE070	0.44	1.35
	150	38	209 000	30314	0.35	1.7
	150	38	176 000	31314	0.83	0.72
	150	54	275 000	32314	0.35	1.7
	150	54	264 000	32314 *B*	0.54	1.1
80	125	29	128 000	32016 *X*	0.43	1.4
	125	36	157 000	33016	0.28	2.1
	130	37	168 000	33116	0.43	1.4
	140	28.25	140 000	30216	0.43	1.4
	140	35.25	176 000	32216	0.43	1.4
	140	46	233 000	33216	0.43	1.4
	145	46	264 000	T2ED080	0.31	1.9
	170	42.5	255 000	30316	0.35	1.7
	170	42.5	212 000	31316	0.83	0.72
	170	61.5	358 000	32316	0.35	1.7
	170	61.5	336 000	32316 *B*	0.54	1.1
90	140	32	157 000	32018 *X*	0.43	1.4
	140	39	205 000	33018	0.27	2.2
	150	45	238 000	33118	0.40	1.5
	155	46	270 000	T2ED090	0.33	1.8

(*Contd*)

Table 15.6 (*Contd*)

d	D	B	C	*Designation*	e	Y
90	160	32.5	183 000	30218	0.43	1.4
	160	42.5	238 000	32218	0.43	1.4
	190	46.5	308 000	30318	0.35	1.7
	190	46.5	251 000	31318	0.83	0.72
	190	67.5	429 000	32318	0.35	1.7
100	145	24	119 000	T4CB100	0.48	1.25
	150	32	161 000	32020 *X*	0.46	1.3
	150	39	212 000	33020	0.28	2.1
	165	47	292 000	T2EE100	0.31	1.9
	180	37	233 000	30220	0.43	1.4
	180	49	297 000	32220	0.43	1.4
	180	63	402 000	33220	0.40	1.5
	215	51.5	380 000	30320	0.35	1.7
	215	56.5	352 000	31320 *X*	0.83	0.72
	215	77.5	539 000	32320	0.35	1.7
150	225	48	347 000	32030 *X*	0.46	1.3
	270	49	402 000	30230	0.43	1.4
	270	77	682 000	32230	0.43	1.4
	320	72	765 000	30330	0.35	1.7
	320	82	837 000	31330 *X*	0.83	0.72
200	280	51	446 000	32940	0.40	1.5
	310	70	704 000	32040 *X*	0.43	1.4
	360	64	737 000	30240	0.43	1.4
	360	104	1 140 000	32240	0.40	1.5
300	420	76	990 000	32960	0.40	1.5

Example 15.8 *A transmission shaft, transmitting 8 kW of power at 400 rpm from a bevel gear G_1 to a helical gear G_2 and mounted on two taper roller bearings B_1 and B_2 is shown in Fig. 15.12(a). The gear tooth forces on the helical gear act at a pitch circle radius of 55 mm, while those on the bevel gear can be assumed to act at the large end of the tooth at a radius of 50 mm. The diameter of the journal at the bearings B_1 and B_2 is 40 mm. The load factor is 1.2 and the expected life for 90% of bearings is 10 000 h. Bearings B_1 and B_2 are identical. The thrust force due to bevel and helical gears is taken by the bearing B_2. Select suitable taper roller bearings for this application.*

Solution

Given $kW = 8$ $n = 400$ rpm $d = 40$ mm
load factor = 1.2 $L_{10h} = 10\,000$ h

Step I *Radial and axial forces on bearings*
The forces acting on the shaft in the vertical and horizontal planes are shown in Fig. 15.12(b). Considering forces in the vertical plane and taking moments about the bearing B_1,

$$3473(150) + 439(100) - 1319(50) - R_{V2}(300) = 0$$

$$\therefore \quad R_{V2} = 1663 \text{ N}$$

Considering equilibrium of vertical forces,

$$3473 - R_{V1} - 1663 - 439 = 0$$

$$\therefore \quad R_{V1} = 1371 \text{ N}$$

Considering forces in the horizontal plane and taking moments about the bearing B_1,

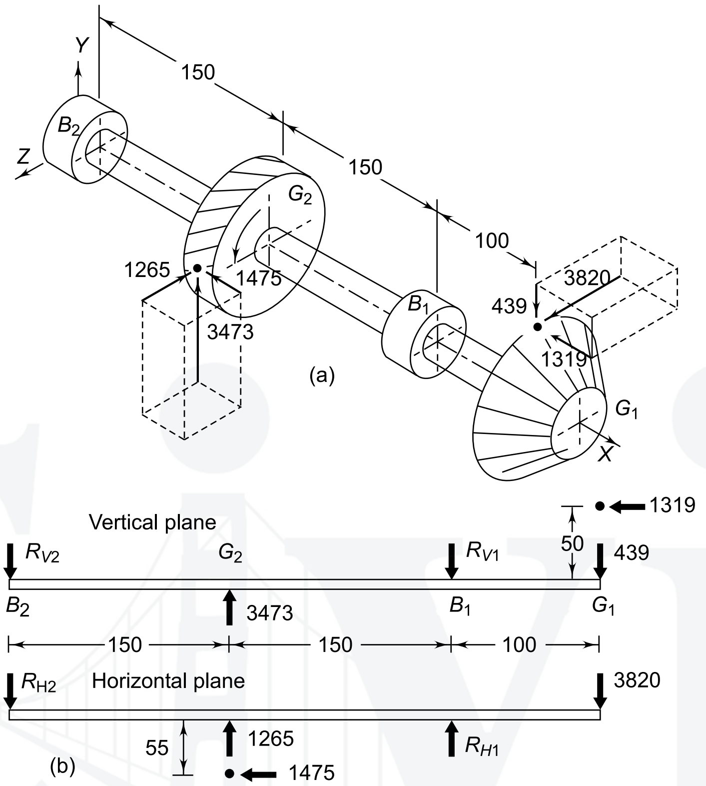


Fig. 15.12

$$3820(100) + 1265(150) + 1475(55) - R_{H2}(300) = 0$$

$$\therefore \quad R_{H2} = 2176.25 \text{ N}$$

Considering equilibrium of horizontal forces,

$$2176.25 + 3820 - 1265 - R_{H1} = 0$$

$$\therefore \quad R_{H1} = 4731.25 \text{ N}$$

The radial forces acting on the bearing are as follows:

$$F_{r1} = \sqrt{(R_{V1})^2 + (R_{H1})^2}$$
$$= \sqrt{(1371)^2 + (4731.25)^2} = 4926 \text{ N}$$

$$F_{r2} = \sqrt{(R_{V2})^2 + (R_{H2})^2}$$
$$= \sqrt{(1663)^2 + (2176.25)^2} = 2739 \text{ N}$$

and $K_a = 1319 + 1475 = 2794$ N

Step II *Tentative selection of bearing*

From Eq. (15.9),

$$L_{10} = \frac{60nL_{10h}}{10^6} = \frac{60(400)(10\,000)}{10^6}$$
$$= 240 \text{ million rev.}$$

For the purpose of referring to Figs 15.10 and 15.11, the bearing B_2 is called Bearing A, the bearing B_1 is called B, and face-to-face construction is selected. With these notations,

$$F_{rA} = 2739 \text{ N}$$
$$F_{rB} = 4926 \text{ N}$$
$$K_a = 2794 \text{ N}$$

Trial 1 Tentatively, we select Bearing 30208. Referring to Table 15.6,

$$e = 0.37 \quad \text{and} \quad Y = 1.6$$

Since $F_{rA} < F_{rB}$ and $K_a > 0$

The load case is similar to Case 2(a) of Fig. 15.11.

$$F_{aB} = \frac{0.5F_{rB}}{Y} = \frac{0.5(4926)}{1.6} = 1539.38 \text{ N}$$

$$F_{aA} = F_{aB} + K_a = 1539.38 + 2794 = 4333.38 \text{ N}$$

Step III *Final check for selection*

Bearing A

$$F_{rA} = 2739 \text{ N} \quad \text{and} \quad F_{aA} = 4333.38 \text{ N}$$

$$F_{aA}/F_{rA} = (4333.38)/(2739) = 1.58 > e$$

$$P = 0.4\, F_r + YF_a$$

$$= 0.4(2739) + 1.6\,(4333.38) = 8029 \text{ N}$$

$$C = P(L_{10})^{0.3} \text{ (Load factor)}$$

$$= 8029(240)^{0.3}\,(1.2) = 49877.66 \text{ N}$$

Bearing 30208 (C = 58 300 N) is suitable.

Bearing B

$$F_{rB} = 4926 \text{ N} \quad \text{and} \quad F_{aB} = 1539.38 \text{ N}$$

$$F_{aB}/F_{rB} = (1539.38)/(4926) = 0.31 < e$$

$$P = F_r = 4926 \text{ N}$$

$$C = P(L_{10})^{0.3} \text{ (Load factor)}$$

$$= 4926\,(240)^{0.3}\,(1.2) = 30\,601.24 \text{ N}$$

Bearing 30208 (C = 58 300 N) is suitable.

Example 15.9 *A machine shaft, supported on two identical taper roller bearings A and B, is shown in Fig.15.13. It is subjected to a radial force of 30 kN and a thrust force of 10 kN. The thrust is taken by Bearing A alone. The shaft rotates at 300 rpm. The machine is intermittently used and the expected life* L_{10h} *of the bearings is 4000 h. The minimum acceptable diameter of the shaft, where the bearings are mounted, is 60 mm. Select suitable taper roller bearings for the shaft.*

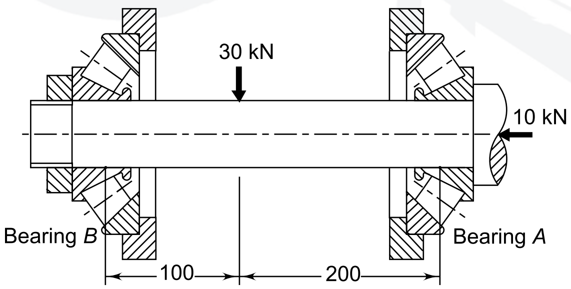


Fig. 15.13

Solution

Given $K_a = 10$ kN $\quad n = 300$ rpm $\quad d = 60$ mm

$L_{10h} = 4000$ h

Step I *Radial and axial forces on bearings*

Refer to forces acting on the shaft in the vertical plane as shown in Fig. 15.13. Taking moments about the bearing B,

$$F_{rA} \times 300 = 30\,000 \times 100 \quad \text{or} \quad F_{rA} = 10\,000 \text{ N}$$

Taking moments about the bearing A,

$$F_{rB} \times 300 = 30\,000 \times 200 \text{ or } F_{rB} = 20\,000 \text{ N}$$

Also,

$$K_a = 10\,000 \text{ N}$$

Step II *Tentative selection of bearing*

$$F_{rA} < F_{rB} \quad \text{and} \quad K_a > 0$$

The above conditions are similar to Case 2(a) illustrated in Fig. 15.11. For this case,

$$F_{aA} = F_{aB} + K_a \tag{a}$$

$$F_{aB} = \frac{0.5F_{rB}}{Y} \tag{b}$$

Tentatively, we will consider the following taper roller bearings from Table 15.6, which are available for the shaft of 60 mm diameter.

d	*D*	*B*	*C*	*Designation*	*e*	*Y*
60	95	23	76 500	32012 *X*	0.43	1.4
	95	27	85 800	33012	0.33	1.8
	100	30	110 000	33112	0.40	1.5
	110	23.75	91 300	30212	0.40	1.5
	110	29.75	119 000	32212	0.40	1.5
	110	38	157 000	33212	0.40	1.5
	115	39	157 000	T5ED060	0.54	1.1
	115	40	183 000	T2EE060	0.33	1.8
	125	37	145 000	T7FC060	0.83	0.72
	130	33.5	161 000	30312	0.35	1.7
	130	33.5	134 000	31312	0.83	0.72
	130	48.5	216 000	32312	0.35	1.7
	130	48.5	205 000	323 12 *B*	0.54	1.1

It is also observed that the values of Y vary from 0.72 to 1.8. Taking an average value of 1.3 as the first trial value,

$$F_{aB} = \frac{0.5F_{rB}}{Y} = \frac{0.5\,(20\,000)}{1.3} = 7692.31 \text{ N}$$

$$F_{aA} = F_{aB} + K_a = 7692.31 + 10\,000 = 17\,692.31 \text{ N}$$

Bearing A is more critical because it is subjected to maximum load. For the bearing A,

$$F_{rA} = 10\,000 \text{ N} \quad F_{aA} = 17\,692.31 \text{ N}$$

$$K_a = 10\,000 \text{ N}$$

Therefore,

$$\frac{F_{aA}}{F_{rA}} = \frac{17\,692.31}{10000} = 1.769$$

It is observed from the above table that for all values of e,

$$\frac{F_{aA}}{F_{rA}} > e$$

From Eq. (15.12),

$$P = 0.4F_r + YF_a = 0.4(10\,000) + 1.3(17\,692.31)$$
$$= 27\,000 \text{ N}$$

From Eq. (15.9),

$$L_{10} = \frac{60nL_{10h}}{10^6} = \frac{60(300)(4000)}{10^6}$$
$$= 72 \text{ million rev.}$$

The dynamic load carrying capacity of the bearing A is given by,

$$C = P\,(L_{10})^{0.3} = 27\,000\,(72)^{0.3} = 97\,401.02 \text{ N}$$

It is observed that Bearing No. 33112 (C = 111 000 N) may be suitable.

Step III *Final check for selection*

For Bearing No. 33112,

$$e = 0.4 \quad Y = 1.5 \quad C = 111\,000 \text{ N}$$

$$F_{aB} = \frac{0.5F_{rB}}{Y} = \frac{0.5(20\,000)}{1.5} \quad 6666.67 \text{ N}$$

$$F_{aA} = F_{aB} + K_a = 6666.67 + 10\,000$$
$$= 16\,666.67 \text{ N}$$

Since,

$$\frac{F_{aA}}{F_{rA}} = \frac{16666.67}{10\,000} = 1.67 \quad \therefore \frac{F_{aA}}{F_{rA}} > e$$

$$\therefore \quad P = 0.4\,F_r + YF_a$$
$$= 0.4(10\,000) + 1.5\,(16\,666.67)$$
$$= 29\,000 \text{ N}$$
$$C = P\,(L_{10})^{0.3} = 29\,000\,(72)^{0.3}$$
$$= 104\,615.91 \text{ N} < 110\,000 \text{ N}$$

Therefore, Bearing No. 33112 (C = 111 000 N) is suitable for the application. We will check whether Bearing No. 33112 is also suitable at B. For the bearing B,

$$F_{rB} = 20\,000 \text{ N}$$

$$F_{aB} = \frac{0.5F_{rB}}{Y} = \frac{0.5\,(20\,000)}{1.5} = 6666.67 \text{ N}$$

Therefore,

$$\frac{F_{aB}}{F_{rB}} = \frac{6666.67}{20\,000} = 0.33 \quad \text{and} \quad e = 0.4$$

$$\frac{F_{aB}}{F_{rB}} < e$$

From Eq. (15.11),

$$P = F_{rB} = 20\,000 \text{ N}$$
$$C = P\,(L_{10})^{0.3} = 20\,000\,(72)^{0.3}$$
$$= 72\,148.91 \text{ N} < 110\,000 \text{ N}$$

Therefore, Bearing No. 33112 is selected at A as well as at B.

15.14 DESIGN FOR CYCLIC LOADS AND SPEEDS

In certain applications, ball bearings are subjected to cyclic loads and speeds. As an example, consider a ball bearing operating under the following conditions:

(i) radial load 2500 N at 700 rpm for 25% of the time,

(ii) radial load 5000 N at 900 rpm for 50% of the time, and

(iii) radial load 1000 N at 750 rpm for the remaining 25% of the time.

Under these circumstances, it is necessary to consider the complete work cycle while finding out the dynamic load capacity of the bearing. The procedure consists of dividing the work cycle into a number of elements, during which the operating conditions of load and speed are constant.

Suppose that the work cycle is divided into x elements. Let $P_1, P_2, \ldots P_x$ be the loads and $n_1, n_2, \ldots, n_x$ be the speeds during these elements. During the first element, the life L_1 corresponding to load P_1, is given by

$$L_1 = \left(\frac{C}{P_1}\right)^3 \times 10^6 \text{ rev.}$$

In one revolution, the life consumed is $\left(\frac{1}{L_1}\right)$ or $\left(\frac{P_1^3}{C^3} \times \frac{1}{10^6}\right)$

Let us assume that the first element consists of N_1 revolutions. Therefore, the life consumed by the first element is given by,

$$\frac{N_1 P_1^3}{10^6\,C^3}$$

Similarly, the life consumed by the second element is given by

$$\frac{N_2 P_2^3}{10^6 C^3}$$

Adding these expressions, the life consumed by the complete work cycle is given by

$$\frac{N_1 P_1^3}{10^6 C^3} + \frac{N_2 P_2^3}{10^6 C^3} + \cdots + \frac{N_x P_x^3}{10^6 C^3} \qquad \text{(a)}$$

If P_e is the equivalent load for the complete work cycle, the life consumed by the work cycle is given by,

$$\frac{N P_e^3}{10^6 C^3} \qquad \text{(b)}$$

where, $N = N_1 + N_2 + \ldots + N_x$

Equating expressions (a) and (b),

$$N_1 P_1^3 + N_2 P_2^3 + \ldots + N_x P_x^3 = N P_e^3$$

or
$$P_e = \sqrt[3]{\left[\frac{N_1 P_1^3 + N_2 P_2^3 + \cdots}{N_1 + N_2 + \cdots}\right]} \qquad (15.13)$$

or
$$P_e = \sqrt[3]{\left[\frac{\Sigma N P^3}{\Sigma N}\right]} \qquad (15.14)$$

The above equation is used for calculating the dynamic load capacity of a bearing.

When the load does not vary in steps of constant magnitude, but varies continuously with time, the above equation is modified and written as

$$P_e = \left[\frac{\int_0^N P^3 dN}{\int_0^N dN}\right]^{1/3}$$

or
$$P_e = \left[\frac{1}{N} \int P^3 dN\right]^{1/3} \qquad (15.15)$$

In case of bearings, where there is a combined radial and axial load, it should be first converted into equivalent dynamic load before the above computations are carried out.

Example 15.10 *A single-row deep groove ball bearing has a dynamic load capacity of 40500 N and operates on the following work cycle:*

(i) radial load of 5000 N at 500 rpm for 25% of the time;

(ii) radial load of 10000 N at 700 rpm for 50% of the time; and

(iii) radial load of 7000 N at 400 rpm for the remaining 25% of the time.

Calculate the expected life of the bearing in hours.

Solution

Given $C = 40500$ N

Step I *Equivalent load for complete work cycle*

Consider the work cycle of one minute duration. The values of load P and revolutions N are tabulated as follows:

Element No.	*P (N)*	*Element time (minute)*	*Speed (rpm)*	*Revolutions N in element time*
1	5000	0.25	500	125
2	10 000	0.5	700	350
3	7000	0.25	400	100
Total		1.00		575

From Eq. (15.13),

$$P_e = \sqrt[3]{\left[\frac{N_1 P_1^3 + N_2 P_2^3 + N_3 P_3^3}{N_1 + N_2 + N_3}\right]}$$

$$= \sqrt[3]{\left[\frac{125(5000)^3 + 350(10\,000)^3 + 100(7000)^3}{575}\right]}$$

$$= 8860.06 \text{ N}$$

Step II *Bearing life (L_{10h})*

According to the load life relationship,

$$L_{10} = \left(\frac{C}{P_e}\right)^3 = \left(\frac{40\,500}{8860.06}\right)^3 = 95.51 \text{ million rev.}$$

$$L_{10h} = \frac{L_{10} \times 10^6}{60n} = \frac{95.51 \times 10^6}{60(575)} = 2768.45 \text{ h}$$

Example 15.11 *A ball bearing is operating on a work cycle consisting of three parts—a radial load of 3000 N at 1440 rpm for one quarter cycle, a radial load of 5000 N at 720 rpm for one half cycle, and radial load of 2500 N at 1440 rpm for the remaining cycle. The expected life of the bearing is 10 000 h. Calculate the dynamic load carrying capacity of the bearing.*

Solution

Given $L_{10h} = 10\,000$ h

Step I *Equivalent load for complete work cycle*

Considering the work cycle of one minute duration,

$$N_1 = \frac{1}{4}(1440) = 360 \text{ rev.}$$

$$N_2 = \frac{1}{2}(720) = 360 \text{ rev.}$$

$$N_3 = \frac{1}{4}(1440) = 360 \text{ rev.}$$

The average speed of rotation is given by,

$$n = N_1 + N_2 + N_3 = 1080 \text{ rpm}$$

From Eq. (15.13),

$$P_e = \sqrt[3]{\left[\frac{N_1P_1^3 + N_2P_2^3 + N_3P_3^3}{N_1 + N_2 + N_3}\right]}$$

$$= \sqrt[3]{\left[\frac{360(3000)^3 + 360(5000)^3 + 360(2500)^3}{1080}\right]}$$

$$= 3823 \text{ N}$$

Step II *Dynamic load carrying capacity of bearing*

$$L_{10} = \frac{60nL_{10h}}{10^6} = \frac{60(1080)(10\,000)}{10^6}$$

$$= 648 \text{ million rev.}$$

From Eq. (15.7),

$$C = P(L_{10})^{1/3} = 3823(648)^{1/3} = 33\,082 \text{ N}$$

Example 15.12 *A single-row deep groove ball bearing is subjected to a 30 second work cycle that consists of the following two parts:*

	Part I	*Part II*
duration (s)	*10*	*20*
radial load (kN)	*45*	*15*
axial load (kN)	*12.5*	*6.25*
speed (rpm)	*720*	*1440*

The static and dynamic load capacities of the ball bearing are 50 and 68 kN respectively. Calculate the expected life of the bearing in hours.

Solution

Given $C_o = 50$ kN $\quad C = 68$ kN

Step I *Equivalent load for complete work cycle*

For Part I,

$$\left(\frac{F_a}{F_r}\right) = \left(\frac{12.5}{45}\right) = 0.278$$

and

$$\left(\frac{F_a}{C_o}\right) = \left(\frac{12.5}{50}\right) = 0.25$$

From Table 15.4, $e = 0.37$

$$\therefore \quad \left(\frac{F_a}{F_r}\right) < e$$

$$\therefore \quad X = 1 \qquad Y = 0$$

$$P_1 = F_r = 45\,000 \text{ N}$$

also

$$N_1 = \frac{10}{60} \times (720) = 120 \text{ N}$$

For Part II,

$$\left(\frac{F_a}{F_r}\right) = \left(\frac{6.25}{15}\right) = 0.417$$

and

$$\left(\frac{F_a}{C_o}\right) = \left(\frac{6.25}{50}\right) = 0.125$$

From Table 15.4, $e = 0.31$ (approximately)

$$\therefore \quad \left(\frac{F_a}{F_r}\right) > e$$

Assuming linear interpolation,

$$Y = 1.6 - \frac{(1.6 - 1.4)}{(0.130 - 0.07)} \times (0.125 - 0.07)$$

$$= 1.42 \text{ and } X = 0.56$$

$$P_2 = XF_r + YF_a = 0.56(15\,000) + 1.42(6250)$$

$$= 17\,275 \text{ N}$$

$$N_2 = \frac{20}{60} \times (1440) = 480 \text{ rev.}$$

$$N_1 + N_2 = 120 + 480 = 600 \text{ rev.}$$

The number of revolutions completed in one work cycle of 30 second duration is 600. Therefore, 1200 revolutions will be completed in one minute. Or,

$$n = 1200 \text{ rpm}$$

From Eq. (15.13),

$$P_e = \sqrt[3]{\left[\frac{N_1P_1^3 + N_2P_2^3}{N_1 + N_2}\right]}$$

$$= \sqrt[3]{\left[\frac{120(45\,000)^3 + 480(17\,275)^3}{600}\right]}$$

$$= 28\,167.89 \text{ N}$$

Step II *Bearing life* (L_{10h})

$$L_{10} = \left(\frac{C}{P_e}\right)^3 = \left(\frac{68000}{28167.89}\right)^3 = 14.069 \text{ million rev.}$$

$$L_{10h} = \frac{L_{10} \times 10^6}{60\,n} = \frac{14.069 \times 10^6}{60(1200)} = 195.4 \text{ h}$$

Example 15.13 *The magnitude of radial force acting on a ball bearing varies in a sinusoidal manner as shown in Fig. 15.14, while the direction remains fixed. Determine the equation for the variation of the force P against the angle of rotation θ.*

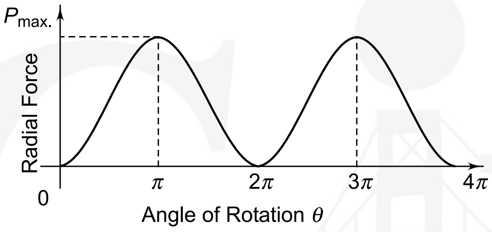


Fig. 15.14 *Variation of force*

Solution

In order to determine the equation of the sinusoidal curve shown in Fig.15.14, we will use the basic

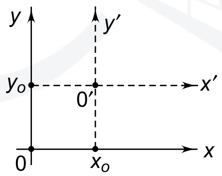


Fig. 15.15

relationships of plane analytical geometry. Figure 15.15 shows the following two co-ordinate systems:

(i) Co-ordinate system (x, y) with origin at O.

(ii) Co-ordinate system (x', y') with origin at O'.

The co-ordinates of the origin O' with respect to the origin O are (x_o, y_o). The relationships for transformation of co-ordinates with pure translation[5] are as follows:

$$x' = x - x_o \quad \text{(a)}$$

$$y' = y - y_o \quad \text{(b)}$$

Refer to the sine curve shown in Fig. 15.16. The equation of this curve with respect to the (x', y') co-ordinate system is given by,

$$y' = \left(\frac{1}{2} P_{max.}\right) \sin x' \quad \text{(c)}$$

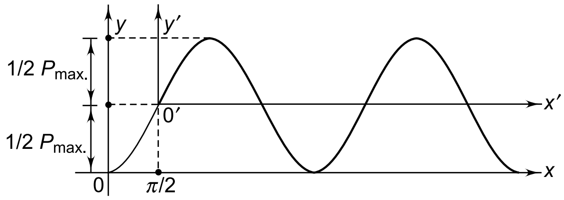


Fig. 15.16

Also,

$$x_o = \frac{\pi}{2} \quad \text{and} \quad y_o = \frac{P_{max.}}{2} \quad \text{(d)}$$

From expressions (a), (b) and (d),

$$x' = x - \frac{\pi}{2} \quad \text{and} \quad y' = y - \frac{P_{max.}}{2} \quad \text{(e)}$$

Substituting expressions (e) in Eq. (c),

$$\left(y - \frac{P_{max.}}{2}\right) = \left(\frac{P_{max.}}{2}\right) \sin\left(x - \frac{\pi}{2}\right)$$

Using the relationship $\sin(-\theta) = -\sin\theta$,

$$\left(y - \frac{P_{max.}}{2}\right) = -\left(\frac{P_{max.}}{2}\right) \sin\left(\frac{\pi}{2} - x\right)$$

$$= -\left(\frac{P_{max.}}{2}\right) \cos x$$

Therefore,

$$y = \frac{P_{max.}}{2} - \frac{P_{max.}}{2} \cos x = \frac{P_{max.}}{2} (1 - \cos x)$$

Replacing y by P and x by θ, (Fig.15.14)

$$P = \frac{P_{max.}}{2} (1 - \cos\theta)$$

Example 15.14 *A ball bearing is subjected to a radial force which varies in sinusoidal way as shown in Fig.15.14 and discussed in Ex. 15.13. The direction of force remains fixed. The amplitude of the force is 1500 N and the speed of rotation is 720 rpm. Determine the dynamic load capacity of the bearing for the expected life of 8000 h.*

Solution

Given $P_{max.} = 1500$ N $\quad n = 720$ rpm

$L_{10h} = 8000$ h

[5] Spiegel MR – 'Mathematical Handbook of Formulas and Tables' – McGraw Hill Book Co. – 1968.

Step I *Equivalent load for complete work cycle*

As derived in the previous example, the equation for force P at angle of rotation θ is given by,

$$P = \frac{1}{2} P_{\max.}(1 - \cos\theta)$$

Considering the work cycle from $\theta = 0$ to $\theta = 2\pi$ and applying Eq. (15.15),

$$P_e = \left[\frac{1}{N}\int P^3 dN\right]^{1/3}$$

$$= \left[\frac{1}{2\pi}\int \frac{P^3_{\max.}}{8}(1-\cos\theta)^3 d\theta\right]^{1/3}$$

$$= \frac{P_{\max.}}{2}\left[\frac{1}{2\pi}\int (1-\cos\theta)^3 d\theta\right]^{1/3} \quad \text{(a)}$$

Also, $\int (1-\cos\theta)^3 d\theta = \int (1 - 3\cos\theta + 3\cos^2\theta - \cos^3\theta) d\theta$

$$= \int \left[1 - 3\cos\theta + \frac{3(1+\cos 2\theta)}{2} - \cos\theta(1-\sin^2\theta)\right] d\theta$$

$$= \int (2.5 - 4\cos\theta + 1.5\cos 2\theta + \cos\theta \sin^2\theta) d\theta$$

$$= \left[2.5\theta - 4\sin\theta + 0.75\sin 2\theta + \frac{\sin^3\theta}{3}\right] \quad \text{(b)}$$

The following formulae are used in above derivation:

$$\int \cos\theta \, d\theta = \sin\theta \qquad \int \cos 2\theta \, d\theta = \frac{\sin 2\theta}{2}$$

$$\int \cos\theta \sin^2\theta = \frac{\sin^3\theta}{3}$$

Since

$$\int_0^{2\pi} (1-\cos\theta)^3 d\theta = \int_0^{\pi} (1-\cos\theta)^3 d\theta + \int_{\pi}^{2\pi} (1-\cos\theta)^3 d\theta$$

$$= \left[2.5\theta - 4\sin\theta + 0.75\sin 2\theta + \frac{\sin^3\theta}{3}\right]_0^{\pi} + \left[2.5\theta - 4\sin\theta + 0.75\sin 2\theta + \frac{\sin^3\theta}{3}\right]_{\pi}^{2\pi}$$

$$= [2.5\pi - 0] + [2.5(2\pi - \pi)] = 5\pi$$

$$\therefore \quad \int_0^{2\pi} (1-\cos\theta)^3 d\theta = 5\pi \quad \text{(c)}$$

from (a) and (c),

$$P_e = \frac{P_{\max.}(2.5)^{1/3}}{2} = \frac{1500(2.5)^{1/3}}{2} = 1017.9 \text{ N}$$

Step II *Dynamic load carrying capacity of bearing*

$$L_{10} = \frac{60nL_{10h}}{10^6} = \frac{60(720)(8000)}{10^6}$$

$$= 345.6 \text{ million rev.}$$

From Eq. (15.7),

$$C = P_e (L_{10})^{1/3} = 1017.9(345.6)^{1/3}$$
$$= 7143.26 \text{ N}$$

15.15 BEARING WITH A PROBABILITY OF SURVIVAL OTHER THAN 90 PER CENT

In the definition of rating life, it is mentioned that the rating life is the life that 90% of a group of identical bearings will complete or exceed before fatigue failure. The reliability R is defined as,

$$R = \frac{\text{No. of bearings which have successfully completed } L \text{ million revolutions}}{\text{Total number of bearings under test}}$$

Therefore, the reliability of bearings selected from the manufacturer's catalogue is 0.9 or 90%.

In certain applications, where there is risk to human life, it becomes necessary to select a bearing having a reliability of more than 90%. Figure 15.17 shows the distribution of bearing failures. The relationship between bearing life and reliability is given by a statistical curve known as *Wiebull distribution*.

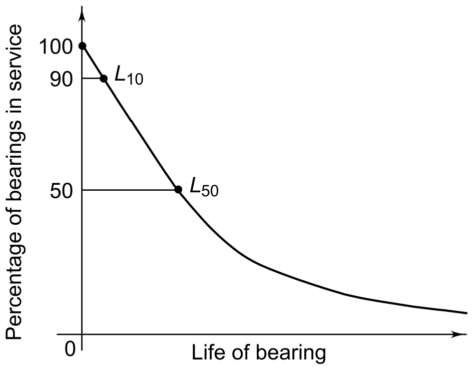


Fig. 15.17

For Wiebull distribution,

$$R = e^{-(L/a)^b} \tag{15.16}$$

where R is the reliability (in fraction), L is the corresponding life and a and b are constants. Rearranging the above equation, we have

$$\left(\frac{1}{R}\right) = e^{(L/a)^b}$$

or
$$\log_e\left(\frac{1}{R}\right) = \left(\frac{L}{a}\right)^b \tag{a}$$

If L_{10} is the life corresponding to a reliability of 90% or R_{90}, then,

$$\log_e\left(\frac{1}{R_{90}}\right) = \left(\frac{L_{10}}{a}\right)^b \tag{b}$$

Dividing Eq. (a) by Eq. (b), we have

$$\left(\frac{L}{L_{10}}\right) = \left[\frac{\log_e\left(\frac{1}{R}\right)}{\log_e\left(\frac{1}{R_{90}}\right)}\right]^{1/b} \tag{15.17}$$

where $R_{90} = 0.9$

The values of a and b are

$a = 6.84$ and $b = 1.17$

These values are obtained from the condition,

$$L_{50} = 5L_{10} \tag{15.18}$$

where L_{50} is the median life or life which 50% of the bearings will complete or exceed before fatigue failure. Equation (15.17) is used for selecting the bearing when the reliability is other than 90%.

In a system, if there are a number of bearings, the individual reliability of each bearing should be fairly high. If there are N bearings in the system, each having the same reliability R then the reliability of the complete system is given by,

$$R_s = (R)^N \tag{15.19}$$

where R_s indicates the probability of one out of N bearings failing during its lifetime.

Example 15.15 *A single-row deep groove ball bearing is subjected to a radial force of 8 kN and a thrust force of 3 kN. The values of X and Y factors are 0.56 and 1.5 respectively. The shaft rotates at 1200 rpm. The diameter of the shaft is 75 mm and Bearing No. 6315 (C =112 000 N) is selected for this application.*

(i) Estimate the life of this bearing, with 90% reliability.

(ii) Estimate the reliability for 20 000 h life.

Solution

Given $F_r = 8$ kN $F_a = 3$ kN $X = 0.56$ $Y = 1.5$
$n = 1200$ rpm $d = 75$ mm $C = 112\,000$ N

Step I *Bearing life with 90% reliability*

From Eq. (15.3),

$$P = XF_r + YF_a = 0.56\,(8000) + 1.5\,(3000) = 8980 \text{ N}$$

From Eq. (15.6),

$$L_{10} = \left(\frac{C}{P}\right)^3 = \left(\frac{112\,000}{8980}\right)^3 = 1940.10 \text{ million rev.}$$

$$L_{10h} = \frac{L_{10}(10^6)}{60n} = \frac{1940.10(10^6)}{60(1200)} = 26\,945.83 \text{ h} \tag{i}$$

Step II *Reliability for 20 000 hr life*

From Eq. (15.17),

$$\left(\frac{L}{L_{10}}\right) = \left[\frac{\log_e\left(\frac{1}{R}\right)}{\log_e\left(\frac{1}{R_{90}}\right)}\right]^{1/b}$$

or
$$\left(\frac{L}{L_{10}}\right)^b = \left[\frac{\log_e\left(\frac{1}{R}\right)}{\log_e\left(\frac{1}{R_{90}}\right)}\right]$$

Substituting the following values,

$L = 20\,000$ h $L_{10} = 26\,945.83$ h $R_{90} = 0.90$ $b = 1.17$ we get,

$$\left(\frac{20\,000}{26\,945.83}\right)^{1.17} = \left[\frac{\log_e\left(\frac{1}{R}\right)}{\log_e\left(\frac{1}{0.90}\right)}\right]$$

$$\therefore \quad R = 0.9283 \text{ or } 92.83\% \tag{ii}$$

Example 15.16 *A ball bearing, subjected to a radial load of 5 kN, is expected to have a life of 8000 h at 1450 rpm with a reliability of 99%. Calculate the dynamic load capacity of the bearing, so that it can be selected from the manufacturer's catalogue based on a reliability of 90%.*

Solution

Given $F_r = 5$ kN $n = 1450$ rpm $L_{99h} = 8000$ h

Step I *Bearing life with 99% reliability*

$$L_{99} = \frac{60nL_{99h}}{10^6} = \frac{60(1450)(8000)}{10^6}$$

$$= 696 \text{ million rev.}$$

Step II *Bearing life with 90% reliability*

From Eq. (15.17),

$$\left(\frac{L_{99}}{L_{10}}\right) = \left[\frac{\log_e\left(\frac{1}{R_{99}}\right)}{\log_e\left(\frac{1}{R_{90}}\right)}\right]^{1/1.17} = \left[\frac{\log_e\left(\frac{1}{0.99}\right)}{\log_e\left(\frac{1}{0.90}\right)}\right]^{1/1.17}$$

$$= 0.1342$$

Therefore,

$$L_{10} = \frac{L_{99}}{0.1342} = \frac{696}{0.1342} = 5186.29 \text{ million rev.}$$

Step III *Dynamic load carrying capacity of bearing*

$$C = P\,(L_{10})^{1/3} = 5000\,(5186.29)^{1/3} = 86\,547.7 \text{ N}$$

Example 15.17 *A single-row deep groove ball bearing is used to support the lay shaft of a four speed automobile gear box. It is subjected to the following loads in respective speed ratios:*

Gear	*Axial load (N)*	*Radial load (N)*	*% time engaged*
First gear	3250	4000	1%
Second gear	500	2750	3%
Third gear	50	2750	21%
Fourth gear	Nil	Nil	75%

The lay shaft is fixed to the engine shaft and rotates at 1750 rpm. The static and dynamic load carrying capacities of the bearing are 11 600 and 17 600 N respectively. The bearing is expected to be in use for 4000 hours of operation. Find out the reliability with which the life could be expected.

Solution

Given $n = 1750$ rpm $C_o = 11\,600$ N
$C = 17\,600$ N $L_h = 4000$ h

Step I *Equivalent load for complete work cycle*

Considering the work cycle of one minute duration,

$$N_1 = \frac{1}{100}(1750) = 17.50 \text{ rev.}$$

$$N_2 = \frac{3}{100}(1750) = 52.50 \text{ rev.}$$

$$N_3 = \frac{21}{100}(1750) = 367.50 \text{ rev.}$$

$$N_4 = \frac{75}{100}(1750) = 1312.50 \text{ rev.}$$

$$(N_1 + N_2 + N_3 + N_4) = 1750 \text{ rev.}$$

First gear

$$\left(\frac{F_a}{F_r}\right) = \left(\frac{3250}{4000}\right) = 0.8125$$

and
$$\left(\frac{F_a}{C_o}\right) = \left(\frac{3250}{11\,600}\right) = 0.28$$

From Table 15.4, it is observed that the value of e will be from 0.37 to 0.44.

$$\therefore \quad \left(\frac{F_a}{F_r}\right) > e$$

The value of factor Y is obtained by linear interpolation.

$$Y = 1.2 - \frac{(1.2 - 1.0)}{(0.5 - 0.25)} \times (0.28 - 0.25) = 1.176$$

and $X = 0.56$

From Eq. (15.3),

$$P_1 = XF_r + YF_a = 0.56(4000) + 1.176(3250)$$
$$= 6062 \text{ N}$$

Second gear

$$\left(\frac{F_a}{F_r}\right) = \left(\frac{500}{2750}\right) = 0.182$$

and
$$\left(\frac{F_a}{C_o}\right) = \left(\frac{500}{11\,600}\right) = 0.0431$$

From Table 15.4, it is observed that the value of e will be from 0.24 to 0.27.

$$\therefore \quad \left(\frac{F_a}{F_r}\right) < e$$

From Eq. (15.4),

$$P_2 = F_r = 2750 \text{ N}$$

Third gear

$$\left(\frac{F_a}{F_r}\right) = \left(\frac{50}{2750}\right) = 0.0182$$

and $$\left(\frac{F_a}{C_o}\right) = \left(\frac{50}{11600}\right) = 0.00431$$

From Table 15.4, it is observed that the value of e will be 0.22 or less. Assuming,

$$\therefore \qquad \left(\frac{F_a}{F_r}\right) < e$$

From Eq. (15.4),

$$P_3 = F_r = 2750 \text{ N}$$

Fourth gear

$$P_4 = 0$$

From Eq. (15.13),

$$P_e = \sqrt[3]{\left[\frac{N_1P_1^3 + N_2P_2^3 + N_3P_3^3 + N_4P_4^3}{N_1 + N_2 + N_3 + N_4}\right]}$$

$$= \sqrt[3]{\left[\frac{17.5(6062)^3 + 52.5(2750)^3 + 367.5(2750)^3 + 1312.5(0)}{1750}\right]}$$

$= 1932.67$ N

Step II *Bearing life L_{10} and L*

From Eq. (15.6),

$$L_{10} = \left(\frac{C}{P}\right)^3 = \left(\frac{17\,600}{1932.67}\right)^3 = 755.2 \text{ million rev.}$$

From Eq. (15.9),

$$L = \frac{60nL_h}{10^6} = \frac{60\,(1750)(4000)}{10^6} = 420 \text{ million rev.}$$

Step III *Reliability of bearing*

From Eq. (15.17),

$$\left(\frac{L}{L_{10}}\right) = \left[\frac{\log_e\left(\frac{1}{R}\right)}{\log_e\left(\frac{1}{R_{90}}\right)}\right]^{1/b}$$

or $$\left(\frac{L}{L_{10}}\right)^b = \left[\frac{\log_e\left(\frac{1}{R}\right)}{\log_e\left(\frac{1}{R_{90}}\right)}\right]$$

Substituting the following values,

$L = 420$ million rev. $\quad L_{10} = 755.2$ million rev.

$R_{90} = 0.90 \quad b = 1.17$

we get,

$$\left(\frac{420}{755.2}\right)^{1.17} = \left[\frac{\log_e\left(\frac{1}{R}\right)}{\log_e\left(\frac{1}{0.90}\right)}\right]$$

$$\therefore \qquad R = 0.9483 \text{ or } 94.83\%$$

15.16 NEEDLE BEARINGS

Needle bearings are characterised by cylindrical rollers of very small diameter and relatively long length. They are also called '*quill*' bearings. The length to diameter ratio of needles is more than four. Needle bearings are used with or without inner and outer races as shown in Fig. 15.18. Very often, needle bearings are used without the races as shown in Fig. 15.18(a). In this case, the needles run directly on the surface of the shaft. The shaft is hardened and ground with a surface hardness of 50 HRC. This type of construction is suitable where limited radial space is available. Needle bearings offer following advantages:

(i) They have a small outer diameter. It is due to this reason that they are often used to replace sleeve bearings. This allows replacement with little or no changes in design.

(ii) They are compact and lightweight compared with other types of bearings.

(iii) They have large load carrying capacity compared to their size.

(iv) They have large load carrying capacity particularly at low peripheral speeds.

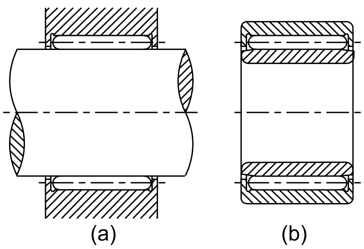


Fig. 15.18 *Needle Bearings*

Needle bearings are ideally suited for applications involving oscillatory motion such as piston pin bearings, rocker arms and universal joints. They are also suitable for continuous rotation where the load is variable or intermittent.

Although needle bearings are considered as a variety of cylindrical roller bearings, they have altogether different characteristics. Short roller bearings can be manufactured with a high degree of accuracy. The needles, which are considerably longer than their diameter, cannot be manufactured with the same degree of accuracy. Short rollers are accurately guided in their cage and races. Needles are not guided to that extent. This results in high friction in needle bearings. The coefficient of friction in cylindrical roller bearings is 0.0011. On the other hand, the coefficient of friction in needle bearings is 0.0045 or almost four times.

15.17 BEARING FAILURE—CAUSES AND REMEDIES

There are two basic types of bearing failure—breakage of parts like races or cage and the surface destruction. The fracture in the outer race of the ball bearing occurs due to overload. When the bearing is misaligned, the load acting on some balls or rollers sharply increases and may even crush them. The failure of the cage is caused due to the centrifugal force acting on the balls. The complete breakage of the parts of the ball bearing can be avoided by selecting the correct ball bearing, adjusting the alignment between the axes of the shaft and the housing and operating within permissible speeds.

In general, the failure of antifriction bearing occurs not due to breakage of parts but due to damage of working surfaces of their parts. The principal types of surface wear are as follows:

(i) Abrasive Wear Abrasive wear occurs when the bearing is made to operate in an environment contaminated with dust, foreign particles, rust or spatter. Remedies against this type of wear are provision of oil seals, increasing surface hardness and use of high viscosity oils. The thick lubricating film developed by these oils allows fine particles to pass without scratching.

(ii) Corrosive Wear The corrosion of the surfaces of bearing parts is caused by the entry of water or moisture in the bearing. It is also caused due to corrosive elements present in the Extreme Pressure (EP) additives that are added in the lubricating oils. These elements attack the surfaces of the bearing, resulting in fine wear uniformly distributed over the entire surface. Remedies against this type of wear are, providing complete enclosure for the bearing free from external contamination, selecting proper additives and replacing the lubricating oil at regular intervals.

(iii) Pitting Pitting is the main cause of the failure of antifriction bearings. Pitting is a surface fatigue failure which occurs when the load on the bearing part exceeds the surface endurance strength of the material. This type of failure is characterised by pits, which continue to grow resulting in complete destruction of the bearing surfaces. Pitting depends upon the magnitude of Hertz' contact stress and the number of stress cycles. The surface endurance strength can be improved by increasing the surface hardness.

(iv) Scoring Excessive surface pressure, high surface speed and inadequate supply of lubricant result in breakdown of the lubricant film. This results in excessive frictional heat and overheating at the contacting surfaces. Scoring is a stick-slip phenomenon, in which alternate welding and shearing takes place rapidly at high spots. Here, the rate of wear is faster. Scoring can be avoided by selecting the parameters, such as surface speed, surface pressure and the flow of lubricant in such a way that the resulting temperature at the contacting surfaces is within permissible limits.

15.18 LUBRICATION OF ROLLING CONTACT BEARINGS

The purpose of lubrication in antifriction bearings is to reduce the friction between balls and races. The other objectives are dissipation of frictional heat, prevention of corrosion and protection of the bearing from dirt and other foreign particles. There are two types of lubricants—oil and grease. Compared with grease, oil offers the following advantages:

(i) It is more effective in carrying frictional heat.

(ii) It feeds more easily into contact areas of the bearing under load.

(iii) It is more effective in flushing out dirt, corrosion and foreign particles from the bearing.

The advantages offered by grease lubricated bearings are simple housing design, less maintenance cost, better sealing against rust and less possibility of leakage. The guidelines for selecting the lubricant are as follows:

(i) When the temperature is less than 100°C, grease is suitable, while lubricating oils are preferred for applications where the temperature exceeds 100°C.

(ii) When the product of bore (in mm) × speed (in rpm) is below 200 000, grease is suitable. For higher values, lubricating oils are recommended.

(iii) Grease is suitable for low and moderate loads, while lubricating oils are used for heavy duty applications.

(iv) If there is a central lubricating system, which is required for the lubrication of other parts, the same lubricating oil is used for bearings, e.g., gearboxes.

The choice of lubricating oil is necessary for high speed, heavy load applications, while in the remaining majority of applications, grease offers the simplest and cheapest mode of lubrication.

15.19 MOUNTING OF BEARING

The inner race of the bearing is fitted on the shaft by means of an interference fit. It prevents the relative rotation and the corresponding wear between the inner race and the shaft. Tolerances for shaft diameter, corresponding to this type of interference fit, are given in the manufacturer's catalogue. Care should be taken to select the fit in such a way that it provides sufficient tightness to give a firm mounting and at the same time, it is not too tight a fit to cause deformation of the inner race and destroying clearance between the rolling elements and the races. The outer race is also mounted in the housing with interference fit, but to a lesser degree of tightness than that of the inner race. Insufficient tightness of the outer race in the housing seat may cause '*creep*'. In bearing terminology, creep is slow rotation of the outer race relative to its seating. It is caused when the shaft is subjected to external force that rotates and changes its direction. When two bearings are mounted on the same shaft, the outer race of one of them should be permitted to shift axially to take care of axial deflection of the shaft caused either by thrust load or by the temperature variation.

It is necessary to position inner and outer races axially by positive means. There are several methods such as providing shoulders for the shaft or the housing, lock nut, snap ring or cover plates as shown in Fig. 15.19. The basic principle is to restrict the displacement of inner as well as outer race in axial direction by positive means. Figure 15.19(a) shows the mounting suitable for a long and continuous shaft. It consists of an adapter sleeve, which is provided with a small taper. The bearing is press fitted on this adapter sleeve. Because of the taper, the displacement of the inner race to the left side is restricted. A washer and lock nut is provided to restrict the displacement of the inner race to the right side. Two methods of restricting the displacement of the inner race are illustrated in Figs. 15.19(b) and (c). In both the cases, the shaft is provided with a shoulder to restrict the displacement of the inner race to the left side. In Fig. 15.19(b), the displacement of the race to the right side is restricted by a plate, which is bolted to the shaft. In Fig. 15.19(c), a snap-ring is used in place of the plate. In Fig. 15.19(d), the housing is provided with a shoulder to restrict the displacement of the outer race to the left side. A circular ring of the cover plate restricts the displacement to the right side. The cover plate is bolted to the housing. Commercial oil seal unit is used to prevent the leakage of lubricating oil. The shoulders for the shaft and housing bore have standards dimension, which can be obtained from the manufacturer's catalogue.

Shafts and spindles in machine tools and precision equipment should rotate without any play or clearance either in axial or radial direction. This is achieved by preloading the ball bearings. The objective of preloading is to remove the internal clearance usually found in the bearing. Preloading of cylindrical roller bearing is obtained by the following methods:

(i) The roller bearing is mounted on a taper shaft or sleeve, which causes the inner race to expand and remove the radial clearance.

(ii) The outer race is fitted in the housing bore by an interference fit. It causes the outer race to contract and remove the radial clearance.

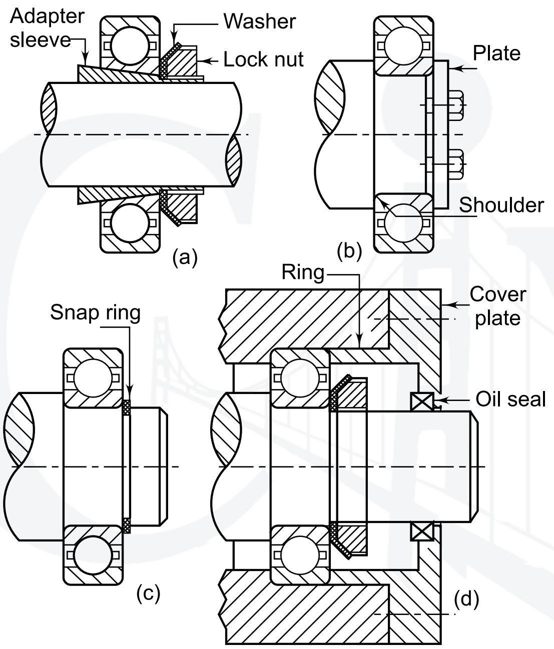


Fig. 15.19 *Mountings of Bearing*

Ball bearings, such as angular contact bearing, are preloaded by axial force by tightening the lock nut during the assembly.

It is essential to use the correct method of mounting and to observe cleanliness if the bearing is to function with satisfaction and achieve the required life. The precautions to be taken during the mounting operation are as follows:

(i) Mounting should be carried out in a dust-free and dry environment. Machines which produce metal particles, chips or sawdust should not be located in the vicinity of the mounting operation.

(ii) Before assembly, the shaft and the housing bore should be inspected. The burrs on the shaft and the shoulders should be removed. The accuracy of the form and dimensions of the shaft and bearing seat in the housing should be inspected.

(iii) The bearing should not be taken out from its package until before it is assembled. The rust-inhibiting compound on the bearing should not be wiped except on the outer diameter and bore surface. These inner and outer surfaces are cleaned with white spirit and wiped with clean cloth.

(iv) Small bearings are mounted on the shaft with the help of a small piece of tube or ring. Blows are applied by means of a hammer on this tube or ring. Direct blows should never be applied to the bearing surface, otherwise the race or the cage may get damaged. The tube or the metallic ring is placed against the inner race and the blows are applied with an ordinary hammer all around the periphery of the ring.

(v) Medium size bearings are mounted on the shaft by pressing the tube or the metallic ring by means of a hydraulic or mechanical press. Large size bearing is mounted by heating it to 80° to 90°C above the ambient temperature by induction heating and then shrinking it on the shaft. The bearing should never be heated by direct flame.

The interference fit between the outer race and the housing is obtained by similar methods, viz., by applying hammer blows on a metallic ring or tube which is in contact with the outer race or by using hydraulic or mechanical press or by heating the housing.

Short-Answer Questions

15.1 What are the functions of bearing?
15.2 What is radial bearing?
15.3 What is thrust bearing?
15.4 What is sliding-contact bearing?
15.5 What is rolling-contact bearing?
15.6 What are the applications of sliding-contact bearing?

15.7 What are the applications of rolling-contact bearing?

15.8 Why are ball and roller bearings called 'antifriction' bearings?

15.9 Name the various types of ball bearings.

15.10 Name the various types of roller bearings.

15.11 State any two advantages and two disadvantages of deep groove ball bearing.

15.12 State any two advantages and two disadvantages of cylindrical roller bearing.

15.13 State any two advantages and two disadvantages of angular contact bearing.

15.14 Where do you use self-aligning ball bearings and spherical roller bearings?

15.15 Why are taper roller bearings used in pairs?

15.16 State any two advantages and two disadvantages of taper roller bearings.

15.17 Enumerate any two advantages and disadvantages of rolling-contact bearings over sliding-contact bearings.

15.18 What is the criterion for static load carrying capacity of ball bearing?

15.19 Define static load carrying capacity of ball bearing.

15.20 Define rating life of bearing.

15.21 What is the criterion for dynamic load carrying capacity of ball bearing?

15.22 Define dynamic load carrying capacity of rolling-contact bearing.

15.23 What is L_{10} life?

15.24 What is L_{50} life?

15.25 What is the reliability of rolling-contact bearing selected from the manufacturer's catalogue?

15.26 Enumerate any three advantages of needle roller bearings.

15.27 Where do you use needle roller bearings?

15.28 What is the objective of preloading of rolling-contact bearings?

15.29 Where do you use preloaded rolling-contact bearings?

Problems for Practice

15.1 A ball bearing with a dynamic load capacity of 22.8 kN is subjected to a radial load of 10 kN. Calculate

(i) the expected life in million revolutions that 90% of the bearings will reach;

(ii) the corresponding life in hours, if the shaft is rotating at 1450 rpm; and

(iii) the life that 50% of the bearings will complete or exceed before fatigue failure.

[*(i) 11.85 (ii) 136.23 (iii) 681.17*]

15.2 A cylindrical roller bearing with bore diameter of 40 mm is subjected to a radial force of 25 kN. The coefficient of friction is 0.0012 and the speed of rotation is 1440 rpm. Calculate the power lost in friction.

[*0.09 kW*]

15.3 A ball bearing is subjected to a radial force of 2500 N and an axial force of 1000 N. The dynamic load carrying capacity of the bearing is 7350 N. The values of X and Y factors are 0.56 and 1.6 respectively. The shaft is rotating at 720 rpm. Calculate the life of the bearing.

[*340.42 h*]

15.4 A ball bearing operates on the following work cycle:

Element No.	*Radial load (N)*	*Speed (rpm)*	*Element time (%)*
1	3000	720	30
2	7000	1440	50
3	5000	900	20

The dynamic load capacity of the bearing is 16.6 kN. Calculate

(i) the average speed of rotation;

(ii) the equivalent radial load; and

(iii) the bearing life.

[*(i) 1116 rpm (ii) 6271.57 N (iii) 276.94 h*]

15.5 The radial load acting on a ball bearing is 2500 N for the first five revolutions and reduces to 1500 N for the next ten revolutions. The load variation then repeats itself. The expected life of the bearing is 20 million revolutions. Determine the dynamic load carrying capacity of the bearing.

[*5303.43 N*]

15.6 A ball bearing subjected to a radial load of 3000 N is expected to have a satisfactory life of 10000 h at 720 rpm with a reliability of 95%. Calculate the dynamic load carrying capacity of the bearing, so that it can be selected from a manufacturer's catalogue based on 90% reliability. If there are four such bearings, each with a reliability of 95% in a system, what is the reliability of the complete system?

[*27840.94 N and 81.45%*]

15.7 A system involves four identical ball bearings, each subjected to a radial load of 2500 N. The reliability of the system, i.e., one out of four bearings failing during the lifetime of five million revolutions, is 82%. Determine the dynamic load carrying capacity of the bearing, so as to select it from the manufacturer's catalogue based on 90% reliability.

[*5247.92 N*]

Sliding Contact Bearings

Chapter 16

16.1 BASIC MODES OF LUBRICATION

Lubrication is the science of reducing friction by application of a suitable substance called lubricant, between the rubbing surfaces of bodies having relative motion. The lubricants are classified into following three groups:

(i) Liquid lubricants like mineral or vegetable oils
(ii) Semi-solid lubricants like grease
(iii) Solid lubricants like graphite or molybdenum disulphide

The objectives of lubrication are as follows:

(i) to reduce friction
(ii) to reduce or prevent wear
(iii) to carry away heat generated due to friction
(iv) to protect the journal and the bearing from corrosion

The basic modes of lubrication are thick- and thin film lubrication. In addition, sometimes a term *'zero film'* bearing is used. Zero film bearing is a bearing which operates without any lubricant, i.e., without any film of lubricating oil.

Thick film lubrication describes a condition of lubrication, where two surfaces of the bearing in relative motion are completely separated by a film of fluid. Since there is no contact between the surfaces, the properties of surface, like surface finish, have little or no influence on the performance of the bearing. The resistance to relative motion arises from the viscous resistance of the fluid. Therefore, the viscosity of the lubricant affects the performance of the bearing.

Thick film lubrication is further divided into two groups: hydrodynamic and hydrostatic lubrication. *Hydrodynamic lubrication is defined as a system of lubrication in which the load-supporting fluid film is created by the shape and relative motion of the sliding surfaces.* The principle of hydrodynamic lubrication in journal bearings is shown in Fig. 16.1. Initially, the shaft is at rest (a) and it

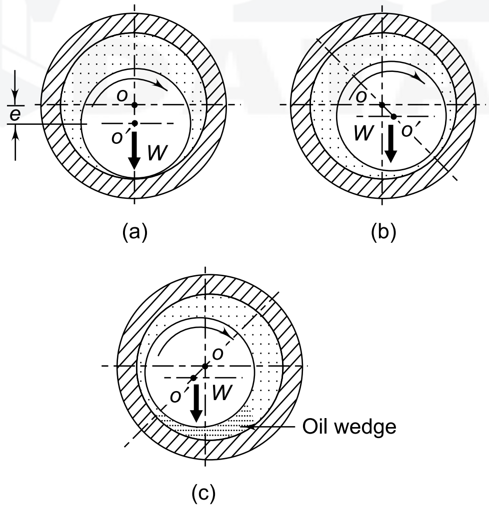


Fig. 16.1 *Hydrodynamic Lubrication (a) Journal at Rest (b) Journal Starts to Rotate (c) Journal at Full Speed*

sinks to the bottom of the clearance space under the action of load *W*. The surfaces of the journal and bearing touch during *'rest'*. As the journal starts to rotate, it climbs the bearing surface (b) and as the speed is further increased, it forces the fluid into the wedge-shaped region (c). Since more and more fluid is forced into the wedge-shaped clearance space, pressure is generated within the system. The pressure distribution around the periphery of the journal is shown in Fig. 16.2. Since the pressure is created within the system due to rotation of the shaft, this type of bearing is known as *self-acting bearing*. The pressure generated in the clearance space supports the external load (*W*). In this case, it is not necessary

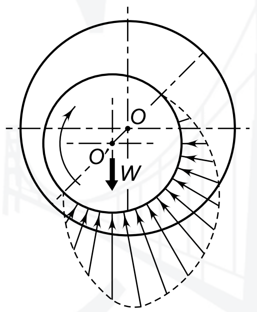


Fig. 16.2 *Pressure Distribution in Hydrodynamic Bearing*

to supply the lubricant under pressure and the only requirement is sufficient and continuous supply of the lubricant. This mode of lubrication is seen in bearings mounted on engines and centrifugal pumps. Frequently, a term *'journal'* bearing is used. *A journal bearing is a sliding contact bearing working on hydrodynamic lubrication and which supports the load in radial direction.* The portion of the shaft inside the bearing is called journal and hence the name 'journal' bearing.

There are two types of hydrodynamic journal bearings, namely, full journal bearing and partial bearing. The construction of full and partial bearings is illustrated in Fig. 16.3. In *full journal bearing*, the angle of contact of the bushing with the journal is 360°. Full journal bearing can take loads in any radial direction. Most of the bearings used in industrial applications are full journal bearings. In *partial bearings*, the angle of contact between the bush and the journal is always less than 180°. Most of the partial bearings in practice have 120° angle of contact. Partial bearing can take loads in only one radial direction. Partial bearings are used in railroad-cars. The advantages of partial bearings compared to full journal bearing are as follows:

(i) Partial bearing is simple in construction.
(ii) It is easy to supply lubricating oil to the partial bearing.
(iii) The frictional loss in partial bearing is less. Therefore, temperature rise is low.

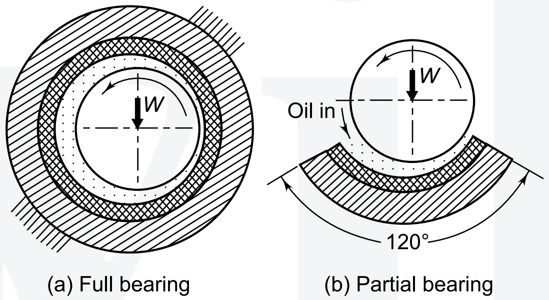


Fig. 16.3 *Full and Partial Bearings*

There are two terms with reference to full and partial bearings, namely, 'clearance' bearing and 'fitted' bearing. *A clearance bearing is a bearing in which the radius of the journal is less than the radius of the bearing.* Therefore, there is a clearance space between the journal and the bearing. Most of the journal bearings are of this type. *A fitted bearing is a bearing in which the radius of the journal and the bearing are equal.* Obviously, fitted bearing must be partial bearing and the journal must run eccentric with respect to the bearing in order to provide space for lubricating oil.

There are two types of thrust bearings which take axial load, namely *'footstep'* bearing and '*collar*' bearing as shown in Fig. 16.4. *The footstep bearing or simply 'step' bearing is a thrust bearing in which the end of the shaft is in contact with the bearing surface. The collar bearing is a thrust* *bearing in which a collar integral with the shaft is in contact with the bearing surface.* In this case, the shaft continues through the bearing. The shaft can be with single collar or can be with multiple collars.

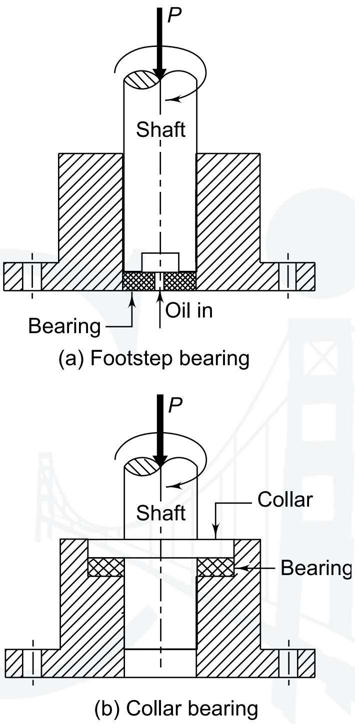


Fig. 16.4 *Types of Thrust Bearing*

Hydrostatic lubrication is defined as a system of lubrication in which the load supporting fluid film, separating the two surfaces is created by an external source, like a pump, supplying sufficient fluid under pressure. Since the lubricant is supplied under pressure, this type of bearing is called *externally pressurised bearing*. The principle of hydrostatic lubrication in journal bearing is illustrated in Fig. 16.5. Initially, the shaft rests on the bearing surface [Fig.16.5(a)]. As the pump starts, high pressure fluid is admitted in the clearance space, forcing the surfaces of the bearing and journal to separate out [Fig.16.5(b)]. Hydrostatic bearings are used on vertical turbo generators, centrifuges and ball mills.

Compared with hydrostatic bearings, hydrodynamic bearings are simple in construction, easy to maintain and lower in initial as well as maintenance cost. Hydrostatic bearings, although costly, offer the following advantages:

(i) high load carrying capacity even at low speeds;
(ii) no starting friction; and
(iii) no rubbing action at any operating speed or load.

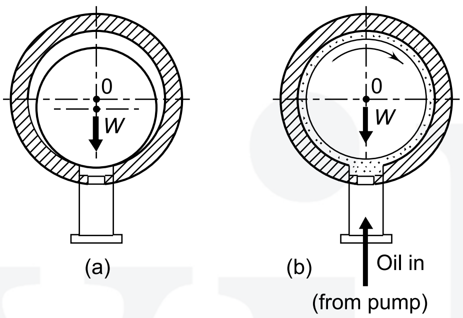


Fig. 16.5 *Hydrostatic Lubrication: (a) Journal at Rest (b) Journal at Full Speed*

Thin film lubrication, which is also called boundary lubrication, is defined as a condition of lubrication where the lubricant film is relatively thin and there is partial metal to metal contact. This mode of lubrication is seen in door hinges and machine tool slides. The conditions resulting in boundary lubrication are excessive load, insufficient surface area or oil supply, low speed and misalignment. The mechanism of boundary lubrication is shown in Fig. 16.6. There are certain fatty acids which contain polar molecules.

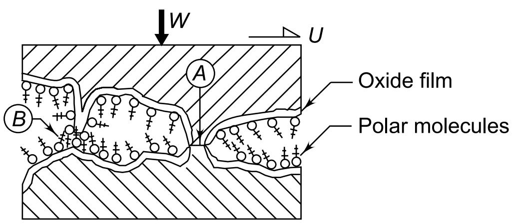


Fig. 16.6 *Boundary Lubrication: (a) Metal to Metal Contact (b) Cluster of Molecules*

Molecules in which there is a permanent separation of positive and negative charges are called *polar molecules*. Their polarity has a tendency to orient and stick to the surface in a particular fashion. The clusters of polar molecules, cohering to one another and adhering to the surface, form a compact film which prevents metal to metal contact as is seen in the region B. This results in partial lubrication. There is also a zone (region A) where metal to metal contact takes place, junctions are formed at high spots and shearing takes place due to relative motion. The performance of bearing under boundary lubrication depends upon two factors, namely, the chemical composition of the lubricating oil, such as polar molecules (at the region B), and surface roughness (at region A). The hydrodynamic bearing also operates under the boundary lubrication when the speed is very low or when the load is excessive.

There is a particular mode of lubrication known as *elastohydrodynamic lubrication*. When the fluid film pressure is high and the surfaces to be separated are not sufficiently rigid, there is elastic deformation of the contacting surfaces. This elastic deflection is useful in the formation of the fluid film in certain cases. Since the hydrodynamic film is developed due to elastic deflection of the parts, this mode of lubrication is called elastohydrodynamic lubrication. This type of lubrication occurs in gears, cams and rolling contact bearings.

16.2 VISCOSITY

Viscosity is defined as the internal frictional resistance offered by a fluid to change its shape or relative motion of its parts. An oil film placed between two parallel plates is shown in Fig. 16.7. The lower plate is stationary while the upper plate is moved with a velocity U by means of a force P.

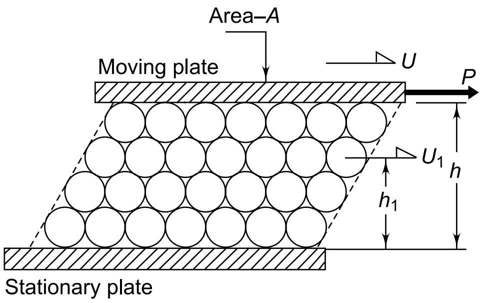


Fig. 16.7 *Newton's Law of Viscosity*

The molecules of oil are visualised as small balls, which roll in layers between two plates. The oil will stick to both the surfaces, and therefore the layer of molecules in contact with the stationary plate has zero velocity. Similarly, the layer of molecules in contact with the upper plate will move with a velocity U. The intermediate layers will move with velocities which are proportional to their distance from the stationary plate. Therefore,

$$\frac{U}{h} = \frac{U_1}{h_1} = \frac{U_2}{h_2}$$

This type of orderly movement is called *streamline, laminar* or *viscous* flow.

The tangential force per unit area, i.e., (P/A), is shear stress, while the ratio (U/h) is the rate of shear. According to Newton's law of viscosity, the shear stress is proportional to the rate of shear at any point in the fluid. Therefore,

$$\left(\frac{P}{A}\right) \propto \left(\frac{U}{h}\right)$$

or

$$P = \mu A\left(\frac{U}{h}\right) \tag{16.1}$$

When the velocity distribution is non-linear with respect to h, the term (U/h) in the above equation is replaced by (dU/dh) and the equation is rewritten as

$$P = \mu A\left(\frac{dU}{dh}\right) \tag{16.2}$$

The constant of proportionality μ in the above equations is called the absolute viscosity. The unit of absolute viscosity is given by

$$\mu = \frac{Ph}{AU} = \frac{(\text{N})(\text{mm})}{(\text{mm}^2)(\text{mm/s})}$$

$$= \text{N-s/mm}^2 \quad \text{or} \quad \text{MPa-s}$$

The popular unit of viscosity is the *Poise*, which gives absolute viscosity in dyne-s/cm^2. Poise is a large unit and viscosities of most of the lubricating oils are given in terms of centi-Poise (cP), which is one-hundredth of a Poise. Therefore, following two separate notations are used for viscosity:

μ = viscosity in units of (N-s/mm^2) or (MPa-s)

z = viscosity in units of (cP)

The relationship between z and μ is as follows:

$$1\,\text{cP} = \frac{1}{10^2}\text{ Poise}$$

$$= \frac{1}{10^2}\,\frac{\text{dyne-s}}{\text{cm}^2}$$

$$= \frac{1}{10^2}\times\left(\frac{\text{N}}{10^5}\right)\left[\frac{\text{s}}{10^2\,\text{mm}^2}\right]$$

$$= (10^{-9})\text{ N-s/mm}^2$$

$$\therefore \quad 1\text{ N-s/mm}^2 = 1\text{ MPa s} = (10^9)\text{ cP}$$

Therefore,

$$\mu = \frac{z}{10^9} \tag{16.3}$$

16.3 MEASUREMENT OF VISCOSITY

In practice, it is difficult to carry out an experiment with two parallel plates for the measurement of viscosity. The popular method of determining viscosity is to measure the time required for a given volume of oil to pass through a capillary tube of standard dimensions. The oil is kept in a reservoir, which is immersed in the constant temperature bath. Based on this principle, there are three commercial viscometers named after Saybolt, Redwood and Engler. The Saybolt universal viscometer is widely used in USA, the Redwood viscometer in UK and the Engler viscometer in the Indian subcontinent. In the Saybolt universal viscometer, 60 cm^3 of lubricating oil is passed through a capillary tube of standard dimensions and the time is measured in seconds. The unit of viscosity is called Saybolt Universal Seconds (SUS), which is related to kinematic viscosity by the following relationship:

$$z_k = \left[0.22t - \frac{180}{t}\right] \tag{16.4}$$

where t is viscosity in Saybolt Universal Seconds (SUS) and z_k is kinematic viscosity in centiStokes (cSt). The kinematic viscosity is defined as the ratio of absolute viscosity to the density of lubricant. Therefore,

$$z_k = \frac{z}{\rho} \tag{16.5}$$

where ρ is the density of the lubricant in g/cm^3. A similar principle is used in Redwood viscometer, where 50 cm^3 of lubricating oil is passed through a capillary tube of specific dimensions and the time is measured in terms of Redwood seconds. In the Engler viscometer, the viscosity is measured in terms of Engler degrees (°E), which is the ratio of time taken by the oil to the time taken by water at the same temperature.

16.4 VISCOSITY INDEX

The viscous resistance of lubricating oil is due to intermolecular forces. As the temperature increases, the oil expands and the molecules move further apart, decreasing the intermolecular force in consequence. Therefore, the viscosity of the lubricating oil decreases with increasing temperature. The viscosity–temperature curves for some lubricating oils are shown in Fig. 16.8. The approximate relationship between viscosity and temperature is as follows:

$$\log\mu = A + \frac{B}{T} \tag{16.6}$$

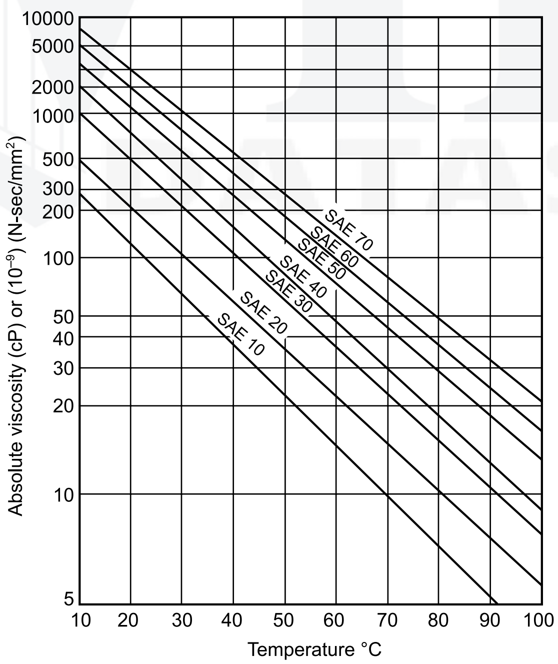


Fig. 16.8 *Viscosity–Temperature Relationship*

where A and B are constants and T is the absolute temperature.

The rate of change of viscosity with respect to temperature is indicated by a number called Viscosity Index (VI). *The viscosity index is defined as an arbitrary number used to characterize the variation of the kinematic viscosity of lubricating oil with temperature.* In order to find out viscosity index of the oil, two groups of reference oils are considered. One group consists of oils having VI = 100 and these oils have very small change of viscosity with temperature. The other group consists of oils having VI = 0 and these oils have very large change of viscosity with temperature. The given oil is compared with these two reference oils, one with a viscosity index of 100 and the other of zero. Figure 16.9 shows the readings to be taken and values of reference oils required to find out the viscosity index. The procedure consists of the following steps:

(i) Measure the viscosities of the given sample of oil at 100° F and 212° F. Suppose the viscosities of the given sample of oil at 100° F and 212° F are y and x respectively. They are plotted in the figure at points B and A respectively. The line BA shows the variation of viscosity for the given sample of oil.

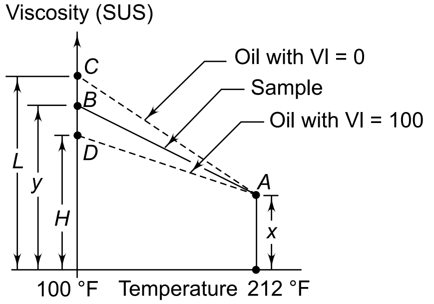


Fig. 16.9 *Viscosity Index*

(ii) Among the first reference group of oils with VI = 100, there will be one oil whose viscosity at 212° F will be x. Select this as the reference oil for comparison. Suppose the viscosity of this reference oil at 100° F is H. The line DA shows the variation of viscosity for the reference oil with VI = 100.

(iii) Among the second reference group of oils with VI = 0, there will be one oil whose viscosity at 212° F will be x. Select this as reference oil for comparison. Suppose the viscosity of this reference oil at 100° F is L. The line CA shows the variation of viscosity for the reference oil with VI = 0.

According to ASTM standards, the viscosity index is given by,

$$VI = \left(\frac{L - y}{L - H} \right) \times 100\%$$

Therefore, the viscosity index indicates the rate of change of viscosity with temperature, as compared to oils with very small or very large rates of change of viscosity with temperature.

An oil with VI = 70 has less rate of change of viscosity with temperature compared with an oil with VI = 60.

16.5 PETROFF'S EQUATION

Petroff's equation is used to determine the coefficient of friction in journal bearings. It is based on the following assumptions:

(i) The shaft is concentric with the bearing.

(ii) The bearing is subjected to light load.

In practice, such conditions do not exist. However, Petroff's equation is important because it defines the group of dimensionless parameters that govern the frictional properties of the bearing. A vertical shaft rotating in the bearing is shown in Fig. 16.10(a). The following notations are used:

r = radius of the journal (mm)

l = length of the bearing (mm)

c = radial clearance (mm)

n_s = journal speed (rev/sec)

The velocity at the surface of the journal is given by,

$$U = (2\pi r)\, n_s \qquad \text{(a)}$$

Refer to Fig. 16.7 of Newton's law of viscosity and using Eq. 16.1,

$$P = \mu A \left(\frac{U}{h} \right) \qquad \text{(b)}$$

We will apply the above equation for viscous flow through the annular portion between the journal and the bearing in the circumferential direction.

P = tangential frictional force

A = area of journal surface = $(2\pi r)l$

U = surface velocity = $(2\pi r)n_s$

h = distance between journal and bearing surfaces = c

Substituting above values in Eq. (b),

$$P = \mu(2\pi rl)(2\pi rn_s)\left(\frac{1}{c}\right) = \frac{4\pi^2 r^2 l\mu n_s}{c} \tag{c}$$

The frictional torque is given by,

$$(M_t)_f = Pr = \frac{4\pi^2 r^3 l\mu n_s}{c} \tag{d}$$

Let us consider a radial force (W), acting on the bearing as shown in Fig. 16.10(b). The unit bearing pressure (p) acting on the bearing is given by,

$$p = \frac{W}{\text{projected area of bearing}} = \frac{W}{(2rl)}$$

or $$W = 2\,prl \tag{e}$$

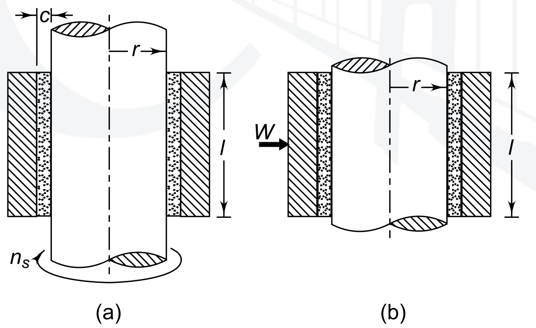


Fig. 16.10

The frictional force will be ($f\,W$) and frictional torque will be ($f\,Wr$). Therefore,

$$(M_t)_f = f\,Wr = f\,(2prl)r = f(2p\,r^2 l) \tag{f}$$

where f is the coefficient of friction.

From (d) and (f),

$$\frac{4\pi^2 r^3 l\mu n_s}{c} = f\,(2\,pr^2\,l)$$

$$f = (2\pi^2)\left(\frac{r}{c}\right)\left(\frac{\mu n_s}{p}\right) \quad \text{(Petroff's equation)}$$

Petroff's equation indicates that there are two important dimensionless parameters, namely, $\left(\frac{r}{c}\right)$ and $\left(\frac{\mu n_s}{p}\right)$ that govern the coefficient of friction and other frictional properties like frictional torque, frictional power loss and temperature rise in the bearing.

16.6 MCKEE'S INVESTIGATION

In hydrodynamic bearings, initially the journal is at rest. There is no relative motion and no hydrodynamic film. Therefore, there is metal to metal contact between the surfaces of the journal and the bearing. As the journal starts to rotate, it takes some time for the hydrodynamic film to build sufficient pressure in the clearance space. During this period, there is partial metal to metal contact and a partial lubricant film. This is thin film lubrication. As the speed is increased, more and more lubricant is forced into the wedge-shaped clearance space and sufficient pressure is built up, separating the surfaces of the journal and the bearing. This is thick film lubrication. Therefore, there is a transition from thin film lubrication to thick film lubrication as the speed increases.

The transition from thin film lubrication to thick film hydrodynamic lubrication can be better visualized by means of a curve called $\mu N/p$ curve. This curve is shown in Fig. 16.11. The $\mu N/p$ curve

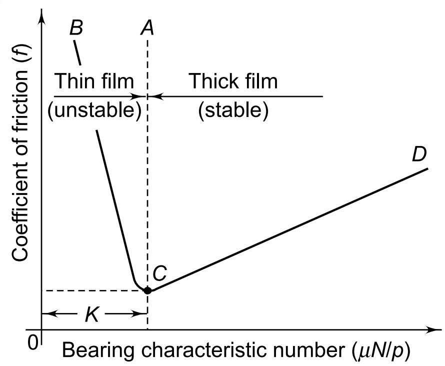


Fig. 16.11 *$\mu N/p$ Curve*

is an experimental curve developed by McKee brothers. A *bearing characteristic number* is a dimensionless group of parameters given by,

$$\text{Bearing characteristic number} = \left(\frac{\mu N}{p}\right)$$

where,

μ = absolute viscosity of the lubricant
N = speed of the journal
p = unit bearing pressure (load per unit of projected area of bearing)

The bearing characteristic number is plotted on the abscissa. The coefficient of friction f is plotted on the ordinate. The coefficient of friction f is the ratio of tangential frictional force to the radial load acting on the bearing. As seen in Fig. 16.11, there are two distinct parts of the curve—*BC* and *CD*.

(i) In the region *BC*, there is partial metal to metal contact and partial patches of lubricant. This is the condition of thin film or boundary lubrication.

(ii) In the region *CD*, there is relatively thick film of lubricant and hydrodynamic lubrication takes place.

(iii) *AC* is the dividing line between these two modes of lubrication. The region to the left of the line *AC* is the thin film zone while the region to the right of the line *AC* is the thick film zone.

(iv) It is observed that the coefficient of friction is minimum at *C* or at the transition between these two modes. The value of the bearing characteristic number corresponding to this minimum coefficient is called the *bearing modulus*. It is denoted by *K* in the figure.

The bearing should not be operated near the critical value *K* at the point *C*. A slight drop in the speed (N) or a slight increase in the load (p) will reduce the value of $\mu N/p$ resulting in boundary lubrication. The guidelines for hydrodynamic lubrication are as follows:

(i) In order to avoid seizure, the operating value of the bearing characteristic number ($\mu N/p$) should be at least 5 to 6 times that when the coefficient of friction is minimum. (5 *K* to 6 *K* or 5 to 6 times the bearing modulus).

(ii) If the bearing is subjected to fluctuating loads or impact conditions, the operating value of the bearing characteristic number ($\mu N/p$) should be at least 15 times that when the coefficient of friction is minimum. (15 K or 15 times the bearing modulus).

It is observed from the ($\mu N/p$) curve that when viscosity of the lubricant is very low, the value of ($\mu N/p$) parameter will be low and boundary lubrication will result. Therefore, if the viscosity of the lubricant is very low then the lubricant will not separate the surfaces of the journal and the bearing and metal to metal contact will occur resulting in excessive wear at the contacting surfaces.

The ($\mu N/p$) curve is important because it defines the stability of hydrodynamic journal bearings and helps to visualize the transition from boundary lubrication to thick film lubrication.

16.7 VISCOUS FLOW THROUGH RECTANGULAR SLOT

The flow of lubricating oil through a rectangular slot is shown in Fig. 16.12(a). l is the length of the slot in the direction of flow, while b and h are

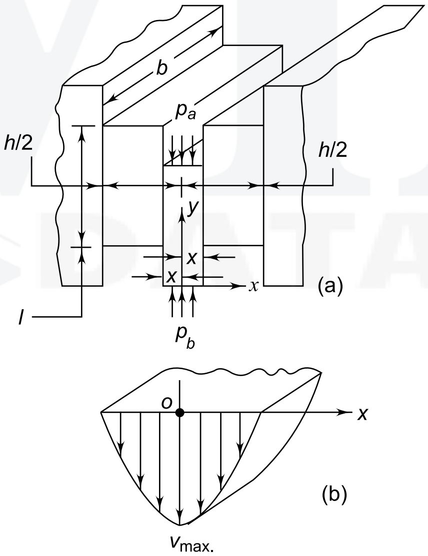


Fig. 16.12 *Viscous flow through slot: (a) Rectangular slot (b) Velocity distribution*

dimensions of the slot in a plane perpendicular to the direction of flow. The dimensions b is very large compared with h so that losses at the sides are neglected. The pressure difference between the two sides of the central slice is ($p_a - p_b$) or Δp. The downward force due to this pressure difference is area ($2xb$) multiplied by the pressure difference (Δp) or ($2xb\ \Delta p$). On account of this force, the rectangular slice of width ($2x$) is extruded down. The shear resistance on both surfaces of the slice is due to the viscosity of the lubricant. According to Newton's law,

$$P = \mu A\left(\frac{dU}{dh}\right)$$

$$= \mu(2lb)\left(\frac{dv}{dx}\right)$$

where v is the velocity in the Y direction.

Considering equilibrium of forces in the vertical direction,

$$2xb\Delta p = -\mu(2lb)\left(\frac{dv}{dx}\right)$$

or
$$dv = -\left(\frac{\Delta p}{\mu l}\right)xdx$$

The negative sign is introduced in the above equation because velocity v decreases as x increases.

Integrating the expression,

$$v = -\left(\frac{\Delta p}{\mu l}\right)\frac{x^2}{2} + C \qquad \text{(a)}$$

The constant C of integration is evaluated from the boundary condition,

$$v = 0 \quad \text{when} \quad x = \pm\left(\frac{h}{2}\right)$$

Therefore,

$$C = \left(\frac{\Delta p}{\mu l}\right)\frac{h^2}{8} \qquad \text{(b)}$$

From (a) and (b),

$$v = \frac{\Delta p}{2\mu l}\left[\frac{h^2}{4} - x^2\right] \qquad \text{(c)}$$

It is observed from Eq. (c), that the velocity distribution along the X-axis is parabolic, which is shown in Fig. 16.12(b). The maximum velocity at the centre ($x = 0$) is given by,

$$v_{\text{max.}} = \frac{\Delta p h^2}{8\mu l}$$

For parabolic profile, the average height is two-third of the maximum height. Therefore,

$$(v_{\text{ave}}) = \left(\frac{2}{3}\right)v_{\text{max.}} = \frac{\Delta p h^2}{12\mu l}$$

The flow Q of the lubricant through the slot is given by,

$$Q = (v_{\text{ave}}) \times (\text{area}) = \left(\frac{\Delta p h^2}{12\mu l}\right) \times (bh)$$

$$\therefore \qquad Q = \frac{\Delta p b h^3}{12\mu l} \qquad (16.7)$$

This is the fundamental equation for viscous flow through the rectangular slot.

16.8 HYDROSTATIC STEP BEARING

A hydrostatic step bearing is shown in Fig. 16.13. The following notations are used in the analysis:

W = thrust load (N)
R_o = outer radius of the shaft (mm)
R_i = radius of the recess or the pocket (mm)
P_i = supply of inlet pressure (N/mm^2) or (MPa)
P_o = outlet or atmospheric pressure (N/mm^2) or (MPa)
h_o = fluid film thickness (mm)
Q = flow of the lubricant (mm^3/s)
μ = viscosity of the lubricant (N-s/mm^2) or (MPa-s)

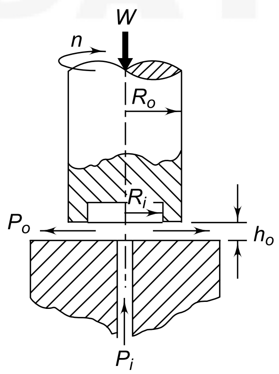


Fig. 16.13 *Hydrostatic Step Bearing*

The lubricant is flowing radially outward through the annulus of radii R_i and R_o and leaves at the periphery of the shaft. Consider an elemental ring of radius r and thickness (dr) as shown in Fig. 16.14(a). The flow of the lubricant through this elemental ring is given by Eq. (16.7). Therefore,

$$Q = \frac{\Delta p b h^3}{12 \mu l} \tag{a}$$

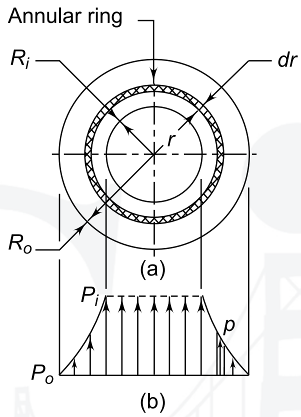


Fig. 16.14 *Pressure Distribution in Hydrostatic Bearing*

The length l in the direction of flow is (dr) while the width b is $(2\pi r)$ and

$$h = h_o \qquad \Delta p = dp$$

Substituting these quantities in Eq. (a),

$$Q = -\left(\frac{\pi r h_o^3}{6\mu}\right)\frac{dp}{dr}$$

The negative sign is introduced in the equation because pressure decreases as the radius r increases or (dp/dr) is negative.

Rearranging the terms,

$$dp = -\left(\frac{6\mu Q}{\pi h_o^3}\right)\frac{dr}{r}$$

Integrating,

$$p = -\left(\frac{6\mu Q}{\pi h_o^3}\right)\log_e r + C \tag{b}$$

The constant C of integration is evaluated from the boundary condition,

$$p = 0 \quad \text{when} \quad r = R_o$$

Therefore,

$$C = \left(\frac{6\mu Q}{\pi h_o^3}\right)\log_e R_o$$

Substituting the value of C in Eq. (b),

$$p = \left(\frac{6\mu Q}{\pi h_o^3}\right)\log_e\left(\frac{R_o}{r}\right) \tag{16.8}$$

The second boundary condition is

$$p = P_i \quad \text{when} \quad r = R_i$$

Substituting these values in Eq. (16.8),

$$P_i = \left(\frac{6\mu Q}{\pi h_o^3}\right)\log_e\left(\frac{R_o}{R_i}\right)$$

$$\therefore \qquad Q = \frac{\pi P_i h_o^3}{6\mu \log_e\left(\frac{R_o}{R_i}\right)} \tag{16.9}$$

Equation (16.9) is used to calculate the flow requirement of the bearing. The distribution of pressure is shown in Fig. 16.14(b). The load carrying capacity of the bearing is the sum of the load supported by the central recess area, where the pressure P_i is constant and the load supported by the annular area from the radius R_i to radius R_o, where pressure p varies. Therefore,

$$W = P_i(\pi R_i^2) + \int_{R_i}^{R_o} p(2\pi r\, dr) \tag{c}$$

Substituting Eq. (16.8) in the above expression,

$$W = \pi P_i R_i^2 + \frac{12\mu Q}{h_o^3}\int_{R_i}^{R_o} \log_e\left(\frac{R_o}{r}\right) r dr \tag{d}$$

We will integrate by parts. Suppose,

$$u = \log_e\left(\frac{R_o}{r}\right) \quad \text{and} \quad dv = r dr$$

$$\therefore \qquad du = \left(\frac{r}{R_o}\right)(R_o)\left(-\frac{1}{r^2}\right)dr = -\left(\frac{1}{r}\right)dr$$

$$\left[\because \frac{d}{dx}(\log_e x) = \frac{1}{x}\right]$$

$$v = \int r dr = \left(\frac{r^2}{2}\right)$$

$$\int u\,dv = uv - \int v\,du$$

Substituting values of u and v,

$$\int \log_e\left(\frac{R_o}{r}\right) r dr = \left[\frac{r^2}{2}\log_e\left(\frac{R_o}{r}\right) + \frac{r^2}{4}\right]$$

Therefore,

$$\int_{R_i}^{R_o} \log_e\left(\frac{R_o}{r}\right) r dr = \left[\frac{r^2}{2}\log_e\left(\frac{R_o}{r}\right) + \frac{r^2}{4}\right]_{R_i}^{R_o}$$

$$= \frac{(R_o^2 - R_i^2)}{4} - \left(\frac{R_i^2}{2}\right)\log_e\left(\frac{R_o}{R_i}\right)$$

Substituting this value and Eq. (16.9) in (d), we have

$$W = \frac{\pi P_i}{2}\left[\frac{R_o^2 - R_i^2}{\log_e\left(\frac{R_o}{R_i}\right)}\right] \qquad (16.10)$$

The above equation can be used even if there is no recess, in which case, R_i will be the radius of the oil-supply pipe.

16.9 ENERGY LOSSES IN HYDROSTATIC BEARING

The total energy loss in a hydrostatic step bearing consists of two factors—the energy required to pump the lubricating oil and energy loss due to viscous friction. The energy E_P required to pump the oil is given by,

$$E_p = Q(P_i - P_o)\frac{\text{mm}^3}{s} \times \frac{N}{\text{mm}^2}$$

$$E_p = Q(P_i - P_o) \quad \text{N-mm/s}$$

$$= Q(P_i - P_o)\,(10^{-3}) \text{ N-m/s or W}$$

Therefore,

$$(\text{kW})_p = Q\,(P_i - P_o)\,(10^{-6}) \qquad (16.11)$$

where $(\text{kW})_p$ is the power loss in pumping (in kW). The frictional power loss is determined by considering the elemental ring of radius (r) and radial thickness (dr) illustrated in Fig. 16.14(a). The viscous resistance for this ring is (dF). It is determined by Newton's law of viscosity. According to this law,

$$dF = \mu A\left(\frac{U}{h}\right)$$

Substituting

$$A = 2\pi r\,dr \quad U = \omega r = \left(\frac{2\pi n}{60}\right) r \text{ and } h = h_o$$

we have $$dF = \left(\frac{4\pi^2}{60}\right)\left(\frac{\mu n}{h_o}\right) r^2 dr$$

The frictional torque $d(M_t)_f$ is given by

$$d(M_t)_f = r \times dF = \left(\frac{4\pi^2}{60}\right)\left(\frac{\mu n}{h_o}\right) r^3 dr$$

Integrating,

$$(M_t)_f = \left(\frac{4\pi^2}{60}\right)\left(\frac{\mu n}{h_o}\right)\int_{R_i}^{R_o} r^3 dr$$

$$= \left(\frac{4\pi^2}{60}\right)\left(\frac{\mu n}{h_o}\right)\left[\frac{r^4}{4}\right]_{R_i}^{R_o}$$

$$= \left(\frac{4\pi^2}{60}\right)\left(\frac{\mu n}{h_o}\right)\frac{(R_o^4 - R_i^4)}{4}$$

$$= \left(\frac{\pi^2}{60}\right)\frac{\mu n(R_o^4 - R_i^4)}{h_o}$$

The unit of $(M_t)_f$ is (N-mm).

$$(\text{kW})_f = \frac{2\pi n (M_t)_f}{60 \times 10^6}$$

$$= \frac{2\pi n}{60 \times 10^6}\left(\frac{\pi^2}{60}\right)\frac{\mu n(R_o^4 - R_i^4)}{h_o}$$

Therefore,

$$(\text{kW})_f = \left(\frac{2\pi^3}{3600 \times 10^6}\right)\frac{\mu n^2 (R_o^4 - R_i^4)}{h_o}$$

or $$(\text{kW})_f = \left(\frac{1}{58.05 \times 10^6}\right)\frac{\mu n^2 (R_o^4 - R_i^4)}{h_o} \qquad (16.12)$$

The total power loss $(\text{kW})_t$ is given by

$$(\text{kW})_t = (\text{kW})_p + (\text{kW})_f \qquad (16.13)$$

Example 16.1 *The following data is given for a hydrostatic thrust bearing:*

thrust load = 500 kN

shaft speed = 720 rpm
shaft diameter = 500 mm
recess diameter = 300 mm
film thickness = 0.15 mm
viscosity of lubricant = 160 SUS
specific gravity = 0.86

Calculate

(i) supply pressure;
(ii) flow requirement in litres/min;
(iii) power loss in pumping; and
(iv) frictional power loss.

Solution

Given $W = 500$ kN $\quad n = 720$ rpm $\quad D_o = 500$ mm
$D_i = 300$ mm $\quad h_o = 0.15$ mm $\quad \rho = 0.86$
viscosity = 160 SUS

Step I *Supply pressure*

From Eq. (16.10),

$$P_i = \frac{2W \log_e\left(\frac{R_o}{R_i}\right)}{\pi(R_o^2 - R_i^2)} = \frac{2(500\times10^3)\log_e\left(\frac{250}{150}\right)}{\pi(250^2 - 150^2)}$$

$= 4.065$ N/mm^2 or MPa (a)

Step II *Flow requirement*

From Eq. (16.4)

$$z_k = \left[0.22t - \frac{180}{t}\right] = \left[0.22(160) - \frac{180}{(160)}\right]$$

$= 34.075$ cSt

$z = \rho z_k = 0.86(34.075) = 29.3$ cP

$$\mu = \frac{z}{10^9} = (29.3)(10^{-9}) \text{ N-s/mm}^2$$

From Eq. (16.9),

$$Q = \frac{\pi P_i h_o^3}{6\mu \log_e\left(\frac{R_o}{R_i}\right)} = \frac{\pi(4.065)(0.15)^3}{6(29.3)(10^{-9})\log_e\left(\frac{250}{150}\right)}$$

$= (0.48 \times 10^6)$ mm^3/s

$Q = (0.48 \times 10^6)$ mm^3/s
$= (0.48 \times 10^6)(10^{-3})$ cc/s
$= (0.48 \times 10^6)(10^{-3})(10^{-3})$ litres/s
(1000 cc = 1 litre)
$= (0.48 \times 10^6)(10^{-6})(60)$ l/min
$= 28.8$ l/min (b)

Step III *Power loss in pumping*

From Eq. (16.11),

$$(\text{kW})_p = Q(P_i - P_o)(10^{-6})$$
$$= 0.48 \times 10^6 (4.065 - 0)(10^{-6}) = 1.95 \quad \text{(c)}$$

Step IV *Frictional power loss*

From Eq. (16.12),

$$(\text{kW})_f = \left(\frac{1}{58.05\times10^6}\right)\frac{\mu n^2(R_o^4 - R_i^4)}{h_o}$$

$$= \left(\frac{1}{58.05\times10^6}\right)\frac{(29.3\times10^{-9})(720)^2[(250)^4 - (150)^4]}{(0.15)}$$

$= 5.93$ (d)

Example 16.2 *The pad of a square hydrostatic thrust bearing, with four pockets of 150 × 150 mm, is shown in Fig. 16.15. The thrust load is 500 kN and the film thickness is 0.15 mm. The viscosity of the lubricant is 250 cP. The pressure in the area A bordering the pockets can be assumed to be uniform and equal to the supply pressure. The pressure distribution in the area B (shown by hatching lines) is assumed to be linear, varying from supply pressure at the inner edge to atmospheric pressure at the outer edge. For calculating the flow of the lubricant, it can be assumed that the area B is straightened out and has length equal to the mean length shown by the dotted line. Calculate*

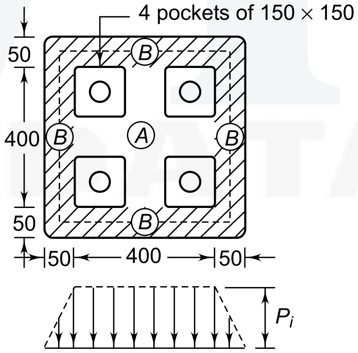


Fig. 16.15

(i) supply pressure; and
(ii) flow requirement in litres/min.

Solution

Given $W = 500$ kN $h_o = 0.15$ mm $z = 250$ cP

Step I *Supply pressure*

The pressure in the area A is the supply pressure P_i while the average pressure in the area B is $\left(\frac{1}{2}P_i\right)$. Therefore,

$$W = (\text{area } A)(P_i) + (\text{area } B)(0.5\, P_i)$$

or $500 \times 10^3 =$

$$= (400 \times 400)\, P_i + (500 \times 500 - 400 \times 400)(0.5\, P_i)$$

$$\therefore \quad P_i = 2.44 \text{ N/mm}^2 \text{ or MPa} \qquad \text{(a)}$$

Step II *Flow requirement*

When the area B is straightened out, its length is equal to (450×4) or 1800 mm.

From Eq. (16.7),

$$Q = \frac{\Delta p b h^3}{12\mu l} = \frac{2.44(1800)(0.15)^3}{12(250 \times 10^{-9})(50)}$$

$$= 98\,820 \text{ mm}^3\text{/s}$$

$$Q = (98\,820) \text{ mm}^3\text{/s}$$

$$= (98\,820)(10^{-3}) \text{ cc /s}$$

$$= (98\,820)(10^{-3})(10^{-3}) \text{ litres/s}$$

(1000 cc = 1 litre)

$$= (98\,820)\,(10^{-6})\,(60) \text{ l/min}$$

$$= 5.93 \text{ l/min} \qquad \text{(b)}$$

Example 16.3 *The dimensions of a hydrostatic thrust bearing, with a rectangular oil-groove A, are given in Fig. 16.16. The pressure distribution can be assumed to be linear, varying from supply pressure at the inner edge of the groove to atmospheric pressure at the outer edge of the pad. The flow over the corners can be neglected. The thrust load is 100 kN and the film thickness is 0.02 mm. The viscosity of the lubricant is 300 cP. Calculate*

(a) supply pressure; and

(b) requirement of flow.

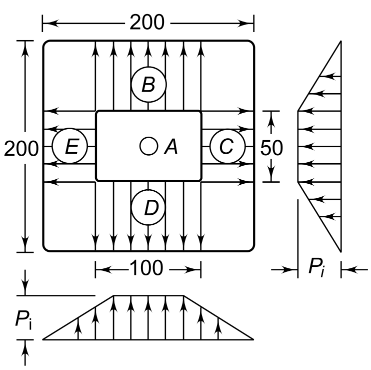


Fig. 16.16

Solution

Given $W = 100$ kN $h_o = 0.02$ mm $z = 300$ cP

Step I *Supply pressure*

$$W = (\text{area } A)(P_i) + [(2 \times \text{area } B) + (2 \times \text{area } C)]\,(0.5\, P_i)$$

or $(100 \times 10^3) =$

$$= (100 \times 50)\,(P_i) + [(2 \times 100 \times 75) + (2 \times 50 \times 50)]\,(0.5\, P_i)$$

$$\therefore \quad P_i = 6.667 \text{ N/mm}^2 \quad \text{or} \quad 6.667 \text{ MPa} \qquad \text{(a)}$$

Step II *Flow requirement*

The flow over area B is

$$Q_B = \frac{\Delta p b h^3}{12\mu l} = \frac{6.667(100)(0.02)^3}{12(300 \times 10^{-9})(75)}$$

$$= 19.75 \text{ mm}^3\text{/s}$$

Similarly,

$$Q_C = \frac{\Delta p b h^3}{12\mu l} = \frac{6.667\,(50)(0.02)^3}{12(300 \times 10^{-9})(50)}$$

$$= 14.82 \text{ mm}^3\text{/s}$$

Therefore, the total flow Q is given by,

$$Q = 2(Q_B + Q_C) = 2(19.75 + 14.82)$$

$$= 69.14 \text{ mm}^3\text{/s} \qquad \text{(b)}$$

Example 16.4 *The hydrostatic thrust bearing of a generator consists of six pads as shown in Fig. 16.17(a). The total thrust load is 900 kN and the film thickness is 0.05 mm. The viscosity of the lubricant is 300 SUS. Neglecting the flow over corners, each pad can be approximated as a circular area of 500 mm and 100 mm as outer and inner diameters respectively. This is shown in Fig. 16.17(b). The density of the lubricating oil is 0.9 g/cc. Calculate*

(i) the supply pressure; and

(ii) the flow requirement.

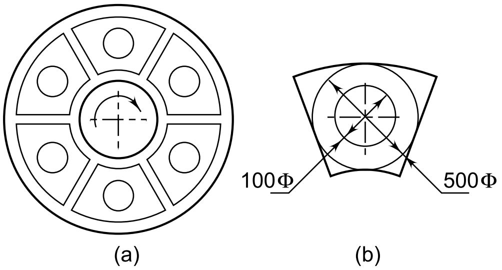


Fig. 16.17 *(a) Six pad Bearing (b) Dimensions of Pad*

Solution

Given $W = (900/6)$ kN $D_o = 500$ mm $D_i = 100$ mm
$h_o = 0.05$ mm $\rho = 0.9$ viscosity $= 300$ SUS

Step I *Supply pressure*

The load acting on each pad is given by,

$$W = \frac{900 \times 10^3}{6} = 150 \times 10^3 \text{ N}$$

From Eq. (16.10),

$$P_i = \frac{2W \log_e\left(\dfrac{R_o}{R_i}\right)}{\pi(R_o^2 - R_i^2)} = \frac{2(150 \times 10^3)\log_e\left(\dfrac{250}{50}\right)}{\pi(250^2 - 50^2)}$$

$$= 2.56 \text{ N/mm}^2 \text{ or MPa} \qquad \text{(a)}$$

Step II *Flow requirement*

From Eq. (16.4)

$$z_k = \left[0.22t - \frac{180}{t}\right] = \left[0.22(300) - \frac{180}{(300)}\right]$$

$$= 65.4 \text{ cSt}$$

$$z = \rho z_k = 0.9(65.4) = 58.86 \text{ cP}$$

$$\mu = \frac{z}{10^9} = (58.86 \times 10^{-9}) \text{ N-s/mm}^2$$

From Eq. (16.9),

$$Q = \frac{\pi P_i h_o^3}{6\mu \log_e\left(\dfrac{R_o}{R_i}\right)} = \frac{\pi(2.56)(0.05)^3}{6(58.86 \times 10^{-9})\log_e\left(\dfrac{250}{50}\right)}$$

$$= 1830.91 \text{ mm}^3\text{/s}$$

There are six pads in the bearing. Therefore,

Total flow $= 6Q = 6\ (1830.91)$

$$= 10\,985.47 \text{ mm}^3\text{/s} \qquad \text{(b)}$$

Example 16.5 *A hydrostatic conical thrust bearing is shown in Fig. 16.18(a). Show that the load carrying capacity of the bearing is given by,*

$$W = \frac{\pi P_i}{2}\left[\frac{R_o^2 - R_i^2}{\log_e\left(\dfrac{R_o}{R_i}\right)}\right]$$

and the flow requirement is given by,

$$Q = \frac{\pi P_i h_o^3 \sin\alpha}{6\mu \log_e\left(\dfrac{R_o}{R_i}\right)}$$

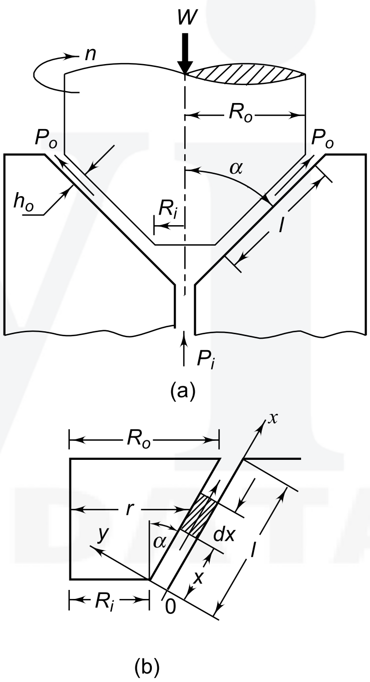


Fig. 16.18 *Conical Thrust Bearing*

Solution

Step I *Flow requirement*

Consider an elemental ring of thickness dx at a distance x from the origin, as shown in Fig. 16.18(b).

$$r = R_i + (R_o - R_i)\left(\frac{x}{l}\right) = R_i + x \sin\alpha$$

From Eq. (16.7), $\quad Q = \dfrac{\Delta p b h^3}{12\mu l}$

where, $l = dx \quad b = 2\pi r = 2\pi(R_i + x \sin \alpha)$ and $h = h_o$

Substituting these values, we have

$$Q = \frac{-dp[2\pi(R_i + x\sin\alpha)](h_o^3)}{12\mu(dx)}$$

The negative sign is introduced in the above expression because the pressure p decreases as x increases.

Rearranging the terms,

$$dp = -\left(\frac{6\mu Q}{\pi h_o^3}\right)\frac{dx}{(R_i + x\sin\alpha)}$$

$$\left[\int \frac{dx}{(ax+b)} = \frac{1}{a}\log_e(ax+b)\right]$$

Integrating the above expression,

$$p = -\left(\frac{6\mu Q}{\pi h_o^3}\right)\frac{1}{\sin\alpha}\log_e(R_i + x\sin\alpha) + C_1$$

The constant C_1 of the integration is evaluated from the boundary condition,

when $\quad x = l, \quad p = 0$

Therefore,

$$C_1 = \left(\frac{6\mu Q}{\pi h_o^3}\right)\frac{\log_e R_o}{\sin\alpha}$$

Substituting this value in the expression for p, we get

$$p = \left(\frac{6\mu Q}{\pi h_o^3}\right)\frac{1}{\sin\alpha}\log_e\left[\frac{R_o}{R_i + x\sin\alpha}\right] \qquad \text{(a)}$$

Substituting the second boundary condition,

$$x = 0, \quad p = P_i$$

in the expression (a),

$$P_i = \left(\frac{6\mu Q}{\pi h_o^3}\right)\frac{1}{\sin\alpha}\log_e\left(\frac{R_o}{R_i}\right)$$

$$\therefore \qquad Q = \frac{\pi P_i h_o^3 \sin\alpha}{6\mu\log_e\left(\dfrac{R_o}{R_i}\right)}$$

Step II *Load carrying capacity*

The load carrying capacity (W) is given by

$$W = \pi R_i^2 P_i + \int_0^l [2\pi(R_i + x\sin\alpha)dx](p\sin\alpha)$$

Substituting Eq. (a) in the above expression,

$$W = \pi R_i^2 P_i + \frac{12\mu Q}{h_0^3}\int_0^l \log_e\left[\frac{R_o}{R_i + x\sin\alpha}\right](R_i + x\sin\alpha)dx \qquad \text{(b)}$$

Substitute,

$$y = (R_i + x \sin \alpha),$$

when $\quad x = 0 \quad y = R_i \quad$ and when $\quad x = l \quad y = R_o$

also $\quad dy = \sin\alpha\, dx$

Therefore,

$$I = \int_0^l \log_e\left[\frac{R_o}{R_i + x\sin\alpha}\right](R_i + x\sin\alpha)dx$$

$$= \int_{R_i}^{R_o} \log_e\left(\frac{R_o}{y}\right) y\left(\frac{dy}{\sin\alpha}\right)$$

$$I = \frac{1}{\sin\alpha}\int_{R_i}^{R_o}\log_e\left(\frac{R_o}{y}\right) ydy \qquad \text{(c)}$$

We will integrate by parts. Suppose,

$$u = \log_e\left(\frac{R_o}{y}\right) \quad \text{and} \quad dv = ydy$$

$$du = \left(\frac{y}{R_o}\right)(R_o)\left(-\frac{1}{y^2}\right)dy = -\left(\frac{1}{y}\right)dy$$

$$v = \int ydy = \left(\frac{y^2}{2}\right)$$

Substituting the above values in the following expression,

$$\int u\,dv = uv - \int v\,du$$

We get,

$$\int_{R_i}^{R_o}\log_e\left(\frac{R_o}{y}\right)ydy$$

$$= \left(\frac{y^2}{2}\right)\log_e\left(\frac{R_o}{y}\right) - \int\left(\frac{y^2}{2}\right)\left(-\frac{1}{y}\right)dy$$

$$= \left(\frac{y^2}{2}\right)\log_e\left(\frac{R_o}{y}\right) + \frac{1}{2}\int ydy$$

$$= \left[\left(\frac{y^2}{2}\right)\log_e\left(\frac{R_o}{y}\right) + \frac{y^2}{4}\right]_{R_i}^{R_o}$$

or

$$I = \frac{1}{\sin\alpha}\left[-\frac{1}{2}R_i^2 \log_e\left(\frac{R_o}{R_i}\right) + \frac{1}{4}(R_o^2 - R_i^2)\right]$$

Substituting the value of the integral in the expression for W,

$$W = \frac{\pi P_i}{2}\left[\frac{R_o^2 - R_i^2}{\log_e\left(\frac{R_o}{R_i}\right)}\right]$$

Example 16.6 *The following data is given for the hydrostatic step bearing of a vertical turbo generator:*

thrust load = 450 kN
shaft diameter = 400 mm
recess diameter = 250 mm
shaft speed = 750 rpm
viscosity of lubricant = 30 cP

Draw a neat sketch showing the effect of film thickness on energy losses. Calculate the optimum film thickness for minimum power loss.

Solution

Given $W = 450$ kN $n = 750$ rpm $D_o = 400$ mm $D_i = 250$ mm $z = 30$ cP

Step I *Variation of energy losses against film thickness*

From Eq. (16.10),

$$P_i = \frac{2W \log_e\left(\frac{R_o}{R_i}\right)}{\pi(R_o^2 - R_i^2)}$$

$$= \frac{2(450\times10^3)\log_e\left(\frac{200}{125}\right)}{\pi(200^2 - 125^2)} = 5.52 \text{ N/mm}^2$$

From Eq. (16.9),

$$Q = \frac{\pi P_i h_o^3}{6\mu \log_e\left(\frac{R_o}{R_i}\right)}$$

$$= \frac{\pi(5.52)h_o^3}{6(30\times10^{-9})\log_e\left(\frac{200}{150}\right)}$$

$$= (205\times10^6)h_0^3 \quad \text{mm}^3/\text{s}$$

From Eq. (16.11),

$$(\text{kW})_p = Q(P_i - P_o)(10^{-6})$$

$$= [(205\times10^6)\,h_o^3]\,[5.52 - 0]\,(10^{-6})$$

$$= 1131.6\,h_o^3 \qquad \text{(a)}$$

From Eq. (16.12),

$$(\text{kW})_f = \left(\frac{1}{58.05\times10^6}\right)\frac{\mu n^2(R_o^4 - R_i^4)}{h_o}$$

$$= \left(\frac{1}{58.05\times10^6}\right)\frac{(30\times10^{-9})(750)^2[(200)^4 - (125)^4]}{h_o}$$

$$= \left(\frac{0.394}{h_o}\right) \qquad \text{(b)}$$

The total energy loss is given by

$$(\text{kW})_t = (\text{kW})_p + (\text{kW})_f$$

or

$$(\text{kW})_t = 1131.6h_o^3 + \left(\frac{0.394}{h_o}\right) \qquad \text{(c)}$$

Substituting various values of h_o in Eqs (a), (b) and (c), the results are tabulated in the following way:

h_o (mm)	$(kW)_p$	$(kW)_f$	$(kW)_t$
0.05	0.14	7.88	8.02
0.10	1.13	3.94	5.07
0.1038	1.27	3.80	5.07
0.15	3.82	2.63	6.45
0.20	9.05	1.97	11.02

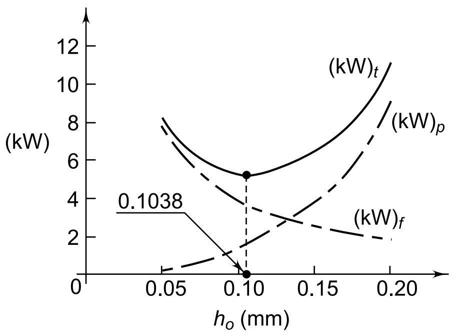


Fig. 16.19 *Variation of Power Losses*

Figure 16.19 shows the variation of power losses against the film thickness.

Step II Optimum film thickness for minimum power loss

Differentiating $(kW)_t$ in the expression (c) with respect to h_o and setting the result to zero,

$$\frac{d}{dh_o}(kW)_t = 0$$

or $$1131.6(3h_o^2) - \frac{0.394}{h_o^2} = 0$$

$\therefore$ $$h_o = 0.1038 \text{ mm}$$

Example 16.7 *Assume the data given in Example 16.6. The specific gravity and specific heat of lubricating oil are 0.86 and 2 kJ/kg°C respectively. For the optimum film thickness calculated in the above example, find out the temperature rise. Assume that the total power loss is converted into frictional heat.*

Solution

Given $\rho = 0.86$ $c_p = 2$ kJ/kg°C

Step I Total power loss

From the previous problem

$$h_o = 0.1038 \text{ mm}$$

$$(kW)_t = (kW)_p + (kW)_f = 1131.6h_o^3 + \left(\frac{0.394}{h_o}\right)$$

$$\therefore (kW)_t = 1131.6(0.1038)^3 + \left(\frac{0.394}{0.1038}\right) = 5.06 \text{ kW}$$

Step II Temperature rise

Heat generated = $(kW)_t$ = 5.06 kW = 5.06 kJ/s (a)

Also,

$$Q = (205 \times 10^6)\, h_o^3 = (205 \times 10^6)(0.1038)^3 = (229.27 \times 10^3) \text{ mm}^3/\text{s}$$

The mass of lubricating oil passing through the bearing is given by

$$m = \rho Q = 0.86(229.27 \times 10^3)(10^{-6}) \text{ kg/s} = 0.197 \text{ kg/s}$$

If Δt is the temperature rise then

$$\text{Heat generated} = mc_p\,\Delta t \qquad \text{(b)}$$

From (a) and (b),

$$5.06 = 0.197\,(2)\,\Delta t$$
$$\Delta t = 12.84° \text{ C}$$

Example 16.8 *A helical pinion is mounted on a vertical shaft and the shaft is supported on a hydrostatic foot step bearing. The trust component of the helical pinion is transmitted to the bearing through the shaft. The pinion transmits 75 kW power at 500 rpm. The normal module and the transverse module are 6 and 8 mm respectively. The pinion has 14 teeth. The nominal bearing pressure is 0.6 MPa. The ratio of recess diameter to shaft diameter is 0.70. Calculate*

(i) the diameter of the shaft;
(ii) the diameter of the recess; and
(iii) the supply pressure.

Select the viscosity of lubricating oil so that the minimum oil film thickness is 20 microns and the power lost in pumping and friction taken together is minimum.

Solution

Given kW = 75 $n = 500$ rpm

For helical pinion, $m_n = 6$ mm $m = 8$ mm $z = 14$

For bearing, $R_i/R_o = 0.7$ $h_o = 20$ microns

$p_b = 0.6$ MPa

Step I Thrust load on bearing

The analysis of forces acting on helical gear is discussed in Chapter 18 on helical gears. The torque transmitted by the pinion is given by,

$$M_t = \frac{(60 \times 10^6)\text{kW}}{2\pi n_p} = \frac{(600 \times 10^6)(75)}{2\pi(500)}$$
$$= 1\,432\,394.49 \text{ N-mm} \qquad \text{(a)}$$

From Eq. (18.6),

$$d = mz = \frac{zm_n}{\cos\psi}$$

where,

m = transverse module (mm)
m_n = normal module (mm)
d = pitch circle diameter of gear (mm)
z = number of teeth

Therefore,

$$d = mz = 8\,(14) = 112 \text{ mm} \qquad \text{(b)}$$

From Eq. (18.17),

$$P_t = \frac{2M_t}{d}$$

where (P_t) is the tangential component of gear force. Substituting values from (a) and (b),

$$P_t = \frac{2M_t}{d} = \frac{2(1\,432\,394.49)}{(112)} = 25\,578.47 \text{ N} \quad \text{(c)}$$

From Eq. (18.3),

$$m_n = m \cos \psi$$

where (ψ) is helix angle. Therefore,

$$\cos \psi = \frac{m_n}{m} = \frac{6}{8} = 0.75 \quad \text{or} \quad \psi = 41.41° \quad \text{(d)}$$

From Eq. (18.15),

$$P_a = P_t \tan \psi$$

where (P_a) is the thrust component or load (W) acting on the bearing. Substituting values from (c) and (d),

$$W = P_a = P_t \tan \psi = (25\,578.47) \tan (41.41°)$$
$$= 22\,558.39 \text{ N} \quad \text{(e)}$$

Step II *Diameter of shaft and recess*
The ratio of recess diameter to shaft diameter is given as 0.70.

$$\therefore \quad \frac{R_i}{R_o} = \frac{D_i}{D_o} = 0.70 \quad \text{(f)}$$

The bearing pressure (p_b) is given as 0.6 MPa or 0.6 N/mm^2. The bearing area is the annular area between the shaft circumference and recess circumference. Therefore,

$$W = \pi(R_o^2 - R_i^2)p_b$$

Substituting values from (e) and (f),

$$22\,558.39 = \pi[R_o^2 - (0.70R_o)^2](0.6)$$

$$\therefore \quad R_o = 153.19 \text{ mm}$$

and $R_i = 0.70\,(153.19) = 107.23$ mm

Rounding the values,

$$R_o = 155 \text{ mm} \quad \text{and} \quad R_i = 107.5 \text{ mm}$$

Shaft diameter = 2 (155) = 310 mm (i)

Recess diameter = 2 (107.5) = 215 mm (ii)

Step III *Supply pressure*
From Eq. (16.10),

$$P_i = \frac{2W \log_e\left(\frac{R_o}{R_i}\right)}{\pi(R_o^2 - R_i^2)}$$

$$= \frac{2(22\,558.39)\log_e\left(\frac{155}{107.5}\right)}{\pi(155^2 - 107.5^2)}$$

$$= 0.42 \text{ N/mm}^2$$

or $$P_i = 0.42 \text{ MPa} \quad \text{(iii)}$$

Step IV *Optimum viscosity for minimum power loss*
From Eq. (16.9),

$$Q = \frac{\pi P_i h_o^3}{6\mu \log_e\left(\frac{R_o}{R_i}\right)}$$

$$= \frac{\pi(0.42)(20\times10^{-3})^3}{6\mu\log_e\left(\frac{155}{107.5}\right)} = \left[\frac{4.8077(10^{-6})}{\mu}\right] \text{mm}^3\text{/s}$$

From Eq. (16.11),

$$(\text{kW})_p = Q(P_i - P_o)(10^{-6})$$

$$= \left[\frac{4.8077(10^{-6})}{\mu}\right](0.42 - 0.0)(10^{-6})$$

$$= \left[\frac{2.019(10^{-12})}{\mu}\right] \quad \text{(a)}$$

From Eq. (16.12),

$$(\text{kW})_f = \left(\frac{1}{58.05\times10^6}\right)\frac{\mu n^2(R_o^4 - R_i^4)}{h_o}$$

$$= \left(\frac{1}{58.05\times10^6}\right)\frac{\mu(500)^2\left[(155)^4 - (107.5)^4\right]}{(20\times10^{-3})}$$

$$= [95.53(10^6)\mu] \quad \text{(b)}$$

The total energy loss is given by

$$(\text{kW})_t = (\text{kW})_p + (\text{kW})_f$$

or $$(\text{kW})_t = \left[\frac{2.019\,(10^{-12})}{\mu}\right] + \left[95.53\,(10^6)\mu\right] \quad \text{(c)}$$

Differentiating (kW)$_t$ with respect to (μ) and setting the result to zero,

$$\frac{d}{d\mu}(\text{kW})_t = 0$$

or $$0 = 2.019(10^{-12})\left(-\frac{1}{\mu^2}\right) + 95.53(10^6)$$

$\therefore$ $$\mu = 0.145(10^{-9}) \text{ N-s/mm}^2 = 0.145 \text{ cP}$$

16.10 REYNOLD'S EQUATION

The theory of hydrodynamic lubrication is based on a differential equation derived by Osborne Reynold. This equation is based on the following assumptions:

(i) The lubricant obeys Newton's law of viscosity.
(ii) The lubricant is incompressible.
(iii) The inertia forces in the oil film are negligible.
(iv) The viscosity of the lubricant is constant.
(v) The effect of curvature of the film with respect to film thickness is neglected. It is assumed that the film is so thin that the pressure is constant across the film thickness.
(vi) The shaft and the bearing are rigid.
(vii) There is a continuous supply of lubricant.

An element having dimensions dx, dy and dz is considered in this analysis, and is shown in Figs 16.20 and 16.21. X is the axis in the direction of motion, Y is the axis in the radial plane and Z is the axis parallel to the axis of the journal. u, v and w are velocities in X, Y, Z directions respectively. τ_x and τ_z are shear stresses along X and Z directions, while p is the fluid film pressure.

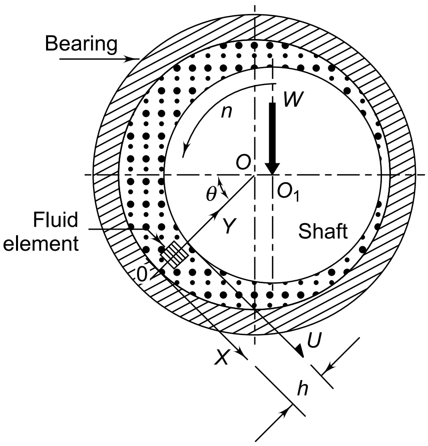


Fig. 16.20 *Fluid Element in X–Y Plane*

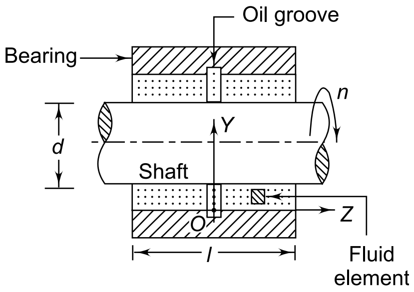


Fig. 20.21 *Fluid Element in Y–Z Plane*

The forces acting on the element in X direction are shown in Fig. 16.22. Considering equilibrium of forces,

$$\left(\frac{\partial \tau_x}{\partial y} dy\right)(dxdz) = \left(\frac{\partial p}{\partial x} dx\right)(dydz) \qquad \text{(a)}$$

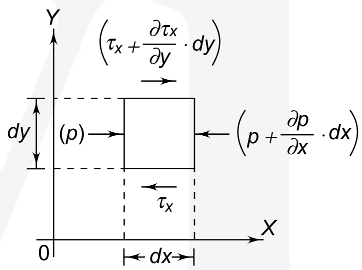


Fig. 16.22 *Equilibrium of Forces in X Direction*

The product ($dx\ dy\ dz$) indicates the volume of the element. Since the element has positive volume,

$$(dx\ dy\ dz) \neq 0$$

Therefore, Eq. (a) is written as,

$$\left(\frac{\partial \tau_x}{\partial y}\right) = \left(\frac{\partial p}{\partial x}\right) \qquad \text{(b)}$$

According to Newton's law of viscosity [Eq. (16.2)],

$$\tau_x = \mu \frac{\partial u}{\partial y} \qquad \text{(c)}$$

From (b) and (c),

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial p}{\partial x}$$

Integrating twice,

$$u = \frac{1}{\mu}\frac{\partial p}{\partial x}\frac{y^2}{2} + C_1 y + C_2 \qquad \text{(d)}$$

The constants C_1 and C_2 of integration are evaluated from following two boundary conditions:

$$u = 0 \quad \text{when} \quad y = 0$$
$$u = U \quad \text{when} \quad y = h$$

Substituting these boundary conditions in Eq. (d),

$$C_2 = 0 \quad \text{and} \quad C_1 = \frac{U}{h} - \frac{1}{\mu}\frac{\partial p}{\partial x}\frac{h}{2} \qquad \text{(e)}$$

Substituting these values in Eq. (d),

$$u = \frac{Uy}{h} + \frac{1}{2\mu}\frac{\partial p}{\partial x}(y^2 - hy) \qquad \text{(f)}$$

The forces acting on the element in Z direction are shown in Fig. 16.23. Considering equilibrium of forces,

$$\left(\frac{\partial \tau_z}{\partial y} dy\right)(dz\, dx) = \left(\frac{\partial p}{\partial z} dz\right)(dx\, dy) \qquad \text{(g)}$$

Since, $\quad (dx\, dy\, dz) \neq 0$

Equation (g) is written as,

$$\left(\frac{\partial \tau_z}{\partial y}\right) = \left(\frac{\partial p}{\partial z}\right) \qquad \text{(h)}$$

According to Newton's law of viscosity [Eq. (16.2)],

$$\tau_z = \mu \frac{\partial w}{\partial y} \qquad \text{(j)}$$

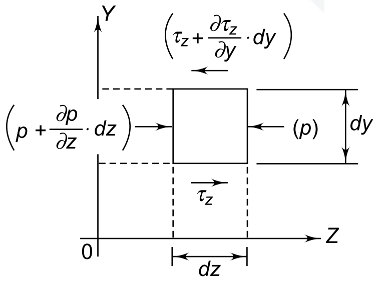


Fig. 16.23 *Equilibrium of Forces in Z Direction*

From (j) and (h),

$$\frac{\partial^2 w}{\partial y^2} = \frac{1}{\mu}\frac{\partial p}{\partial z}$$

Integrating twice,

$$w = \frac{1}{\mu}\frac{\partial p}{\partial z}\frac{y^2}{2} + C_3 y + C_4 \qquad \text{(k)}$$

The constants C_3 and C_4 of integration are evaluated from following two boundary conditions:

$$w = 0 \quad \text{when} \quad y = 0$$
$$w = 0 \quad \text{when} \quad y = h$$

Substituting these boundary conditions in Eq. (k),

$$C_4 = 0 \quad \text{and} \quad C_3 = -\frac{1}{\mu}\frac{\partial p}{\partial z}\frac{h}{2}$$

Substituting the above values in Eq. (k),

$$w = \frac{1}{2\mu}\frac{\partial p}{\partial z}(y^2 - hy) \qquad \text{(l)}$$

The general continuity equation for incompressible flow is given by,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad \text{(m)}$$

Despite there is no flow in Y direction; the local continuity equation in three directions must be satisfied. Therefore,

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} - \frac{\partial w}{\partial z}$$

Integrating the above equation with respect to y, within limits 0 to h,

$$\int_0^h \frac{\partial v}{\partial y} dy = -\int_0^h \frac{\partial u}{\partial x} dy - \int_0^h \frac{\partial w}{\partial z} dy \qquad \text{(n)}$$

The left hand side of the above equation is expressed as,

$$\int_0^h \frac{\partial v}{\partial y} dy = \{(v) \text{ at } (y = h)\} - \{(v) \text{ at } (y = 0)\} \qquad \text{(o)}$$

Figure 16.24 shows the fluid film in the X–Y plane. When ($y = 0$), it indicates stationary bearing

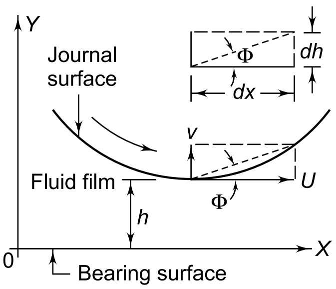


Fig. 16.24

surface and velocity in Y direction (v) is zero. When ($y = h$), it indicates journal surface and velocity in Y direction (v) is given by,

$$\tan\phi = \frac{v}{U} = \frac{dh}{dx} \quad \text{or} \quad v = U\frac{dh}{dx}$$

In the above expression, the curvature effect is neglected. Substituting the above values in Eq. (o),

$$\int_0^h \frac{\partial v}{\partial y}dy = U\frac{dh}{dx} - 0 = U\frac{dh}{dx} \tag{p}$$

From Eqs (n) and (p),

$$\int_0^h \frac{\partial u}{\partial x}dy + \int_0^h \frac{\partial w}{\partial z}dy = -U\frac{dh}{dx} \tag{q}$$

We will apply Leibnitz's theorem[1] for interchanging the signs of integration and differentiation of the first term of the above equation, because the upper limit h is a function of x. According to Leibnitz's theorem,

$$\frac{d}{dx}\int_{h_1(x)}^{h_2(x)} u(x,y)dy = \int_{h_1(x)}^{h_2(x)} \frac{\partial}{\partial x}u(x,y)dy +$$

$$\left(u[h_2(x),x]\left(\frac{dh_2(x)}{dx}\right)\right) - \left(u[h_1(x),x]\left(\frac{dh_1(x)}{dx}\right)\right)$$

Substituting following values,

$$h_1(x) = 0 \quad h_2(x) = h \quad u(x,y) = u$$

$$u\,[h_1\,(x), x] = u \text{ at } [h_1\,(x), x] = 0$$

$$u\,[h_2\,(x), x] = u \text{ at } [h_2\,(x), x] = U$$

We get,

$$\frac{d}{dx}\int_0^h udy = \int_0^h \frac{\partial u}{\partial x}dy + U\frac{dh}{dx}$$

Therefore, the first term of Eq. (q) is given by,

$$\int_0^h \frac{\partial u}{\partial x}dy = \frac{\partial}{\partial x}\int_0^h udy - U\frac{dh}{dx} \tag{r}$$

In the second term of Eq. (q), the upper limit h is constant with respect to y or z. Therefore, the signs of integration and differentiation can be interchanged. Or

$$\int_0^h \frac{\partial w}{\partial z}dy = \frac{\partial}{\partial z}\int_0^h wdy \tag{s}$$

Substituting Eqs (r) and (s) in Eq. (q),

$$\frac{\partial}{\partial x}\int_0^h udy - U\frac{dh}{dx} + \frac{\partial}{\partial z}\int_0^h w\,dy = -U\frac{dh}{dx}$$

$$\frac{\partial}{\partial x}\int_0^h udy + \frac{\partial}{\partial z}\int_0^h wdy = 0 \tag{t}$$

Substituting the value of u from Eq. (f) in the first expression of Eq. (t),

$$\frac{\partial}{\partial x}\int_0^h udy = \frac{\partial}{\partial x}\int_0^h \left[\frac{Uy}{h} + \frac{1}{2\mu}\frac{\partial p}{\partial x}(y^2 - hy)\right]dy$$

$$= \frac{\partial}{\partial x}\left[\frac{Uy^2}{2h} + \frac{1}{2\mu}\frac{\partial p}{\partial x}\left(\frac{y^3}{3} - \frac{hy^2}{2}\right)\right]_0^h$$

$$= \frac{\partial}{\partial x}\left[\frac{Uh}{2} + \frac{1}{2\mu}\frac{\partial p}{\partial x}\left(-\frac{h^3}{6}\right)\right]$$

$$= \frac{U}{2}\frac{\partial h}{\partial x} - \frac{1}{12\mu}\frac{\partial}{\partial x}\left[h^3\frac{\partial p}{\partial x}\right] \tag{u}$$

Substituting the value of w from Eq. (l) in the second expression of Eq. (t),

$$\frac{\partial}{\partial z}\int_0^h wdy = \frac{\partial}{\partial z}\int_0^h \left[\frac{1}{2\mu}\frac{\partial p}{\partial z}(y^2 - hy)\right]dy$$

$$= \frac{\partial}{\partial z}\left[\frac{1}{2\mu}\frac{\partial p}{\partial z}\left(\frac{y^3}{3} - \frac{hy^2}{2}\right)\right]_0^h$$

$$= \frac{\partial}{\partial z}\left[\frac{1}{2\mu}\frac{\partial p}{\partial z}\left(-\frac{h^3}{6}\right)\right]$$

$$= -\frac{1}{12\mu}\frac{\partial}{\partial z}\left[h^3\frac{\partial p}{\partial z}\right] \tag{v}$$

Substituting Eqs (u) and (v) in Eq. (t),

$$\frac{U}{2}\frac{\partial h}{\partial x} - \frac{1}{12\mu}\frac{\partial}{\partial x}\left[h^3\frac{\partial p}{\partial x}\right] - \frac{1}{12\mu}\frac{\partial}{\partial z}\left[h^3\frac{\partial p}{\partial z}\right] = 0$$

or,

$$\frac{\partial}{\partial x}\left[h^3\frac{\partial p}{\partial x}\right] + \frac{\partial}{\partial z}\left[h^3\frac{\partial p}{\partial z}\right] = 6\mu U\left(\frac{\partial h}{\partial x}\right) \tag{16.14}$$

The above equation is known as Reynold's equation. There is no exact analytical solution for this equation for bearings with finite length. Theoretically, exact solutions can be obtained

[1] Courtesy: Dr. AK Chattopadhyay, Professor of Mech. Engg., Bengal Engineering College, Howrah.

if the bearing is assumed to be either infinitely long or very short. These two solutions are called *Sommerfeld's solutions*. Approximate solutions using numerical methods are available for bearings with finite length.

16.11 RAIMONDI AND BOYD METHOD

There is no exact solution to Reynold's equation for a journal bearing having a finite length. However, AA Raimondi and John Boyd of Westinghouse Research Laboratory solved this equation on computer using the iteration technique. The results of this work are available in the form of charts and tables. In the Raimondi and Boyd method, the performance of the bearing is expressed in terms of dimensionless parameters. Table 16.1 gives values of these parameters[2,3] for a full journal bearing with side flow.

In Fig. 16.25, O and O' are the axes of bearing and journal respectively. The distance OO' is called eccentricity and denoted by the letter e. The radial clearance c is given by,

$$c = R - r \tag{16.15}$$

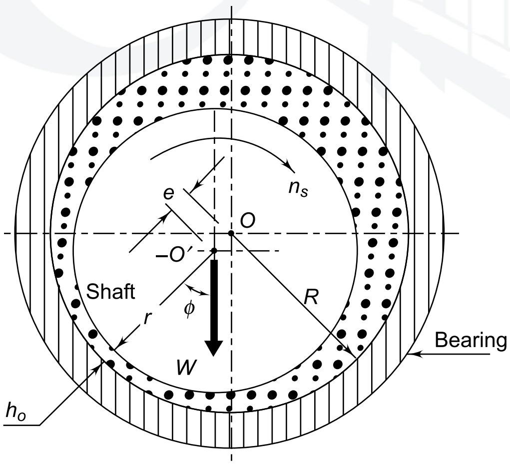


Fig. 16.25

where,

c = radial clearance (mm)

R = radius of bearing (mm)

r = radius of journal (mm)

The eccentricity ratio (ε) is defined as the ratio of eccentricity to radial clearance.

Therefore,

$$\varepsilon = \frac{e}{c} \tag{16.16}$$

where, ε is the eccentricity ratio.

Referring to Fig. 16.25,

$$R = e + r + h_o \tag{a}$$

where,

h_o = minimum film thickness (mm)

Substituting Eq. (16.16) in expression (a),

$$c = R - r = e + h_o = c\varepsilon + h_o$$

or $\quad c(1 - \varepsilon) = h_o$

$$\therefore \quad \varepsilon = 1 - \left(\frac{h_o}{c}\right) \tag{16.17}$$

The quantity $\left(\frac{h_o}{c}\right)$ is called the minimum film thickness variable.

The Sommerfeld number is given by

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu n_s}{p} \tag{16.18}$$

where,

S = Sommerfeld number (dimensionless)

μ = viscosity of the lubricant (N-s/mm^2) or (MPa-s)

n_s = journal speed (rev./s)

p = unit bearing pressure, i.e., load per unit of the projected area (N/mm^2)

The Sommerfeld number contains all variables, which are controlled by the designer.

The angle ϕ shown in Fig. 16.25 is called the angle of eccentricity or attitude angle. It locates the position of minimum film thickness with respect to the direction of load. The values of ϕ given in Table 16.1 are in degrees. The coefficient of friction variable (CFV) is given by,

[2] AA Raimondi and John Boyd—'A solution for the finite journal bearing and its application to analysis and design'—ASLE Transactions, Vol. 1, No. 1, April 1958 (in three parts).

[3] The values are reproduced with the permission of Mr AA Raimondi, Westinghouse Electric Corporation, USA.

Table 16.1 *Dimensionless performance parameters for full journal bearing with side flow*

$\left(\frac{l}{d}\right)$	ε	$\left(\frac{h_o}{c}\right)$	S	ϕ	$\left(\frac{r}{c}\right)f$	$\left(\frac{Q}{rcn_s l}\right)$	$\left(\frac{Q_s}{Q}\right)$	$\left(\frac{p}{p_{max.}}\right)$
∞	0	1.0	∞	(70.92)	∞	π	0	–
	0.1	0.9	0.240	69.10	4.80	3.03	0	0.826
	0.2	0.8	0.123	67.26	2.57	2.83	0	0.814
	0.4	0.6	0.0626	61.94	1.52	2.26	0	0.764
	0.6	0.4	0.0389	54.31	1.20	1.56	0	0.667
	0.8	0.2	0.021	42.22	0.961	0.760	0	0.495
	0.9	0.1	0.0115	31.62	0.756	0.411	0	0.358
	0.97	0.03	–	–	–	–	0	–
	1.0	0	0	0	0	0	0	0
1	0	1.0	∞	(85)	∞	π	0	–
	0.1	0.9	1.33	79.5	26.4	3.37	0.150	0.540
	0.2	0.8	0.631	74.02	12.8	3.59	0.280	0.529
	0.4	0.6	0.264	63.10	5.79	3.99	0.497	0.484
	0.6	0.4	0.121	50.58	3.22	4.33	0.680	0.415
	0.8	0.2	0.0446	36.24	1.70	4.62	0.842	0.313
	0.9	0.1	0.0188	26.45	1.05	4.74	0.919	0.247
	0.97	0.03	0.00474	15.47	0.514	4.82	0.973	0.152
	1.0	0	0	0	0	0	1.0	0
$\left(\frac{1}{2}\right)$	0	1.0	∞	(88.5)	∞	π	0	–
	0.1	0.9	4.31	81.62	85.6	3.43	0.173	0.523
	0.2	0.8	2.03	74.94	40.9	3.72	0.318	0.506
	0.4	0.6	0.779	61.45	17.0	4.29	0.552	0.441
	0.6	0.4	0.319	48.14	8.10	4.85	0.730	0.365
	0.8	0.2	0.0923	33.31	3.26	5.41	0.874	0.267
	0.9	0.1	0.0313	23.66	1.60	5.69	0.939	0.206
	0.97	0.03	0.00609	13.75	0.610	5.88	0.980	0.126
	1.0	0	0	0	0	–	1.0	0
$\left(\frac{1}{4}\right)$	0	1.0	∞	(89.5)	∞	π	0	–
	0.1	0.9	16.2	82.31	322.0	3.45	0.180	0.515
	0.2	0.8	7.57	75.18	153.0	3.76	0.330	0.489
	0.4	0.6	2.83	60.86	61.1	4.37	0.567	0.415
	0.6	0.4	1.07	46.72	26.7	4.99	0.746	0.334
	0.8	0.2	0.261	31.04	8.8	5.60	0.884	0.240
	0.9	0.1	0.0736	21.85	3.50	5.91	0.945	0.180
	0.97	0.03	0.0101	12.22	0.922	6.12	0.984	0.108
	1.0	0	0	0	0	–	1.0	0

$$(\text{CFV}) = \left(\frac{r}{c}\right) f \tag{16.19}$$

where f is the coefficient of friction. The frictional torque is given by,

$$(M_t)_f = fWr \text{ N-mm}$$

$$\begin{aligned}\text{Frictional power} &= (2\pi n_s)(fWr) \text{ N-mm/s} \\ &= (2\pi n_s)(fWr)(10^{-3}) \text{ W} \\ &= (2\pi n_s)(fWr)(10^{-6}) \text{ kW}\end{aligned}$$

Therefore,

$$(\text{kW})_f = \frac{2\pi n_s fWr}{10^6} \tag{16.20}$$

The flow variable (FV) is given by,

$$(\text{FV}) = \frac{Q}{rcn_s l} \tag{16.21}$$

where,

l = length of the bearing (mm)

Q = flow of the lubricant (mm^3/s)

In this case, Q represents the total flow of the lubricating oil, a part of which is circulated around the periphery of the journal, while the remaining oil flows out as side leakage. Q_s represents the side leakage, which can be calculated from the values of parameter $\left(\frac{Q_s}{Q}\right)$ given in the table.

The maximum pressure ($p_{\max}$) developed in the film is calculated from the ratio $\left(\frac{p}{p_{\max.}}\right)$ given in the last column of the table. This value is based on the assumption that the oil is supplied at the atmospheric pressure. If the oil is supplied at a higher pressure, the maximum pressure ($p_{\max.}$) will also increase by the corresponding value.

16.12 TEMPERATURE RISE

Heat is generated in the bearing due to viscosity of the lubricating oil. The frictional work is converted into heat, which increases the temperature of the lubricant. Assuming that the total heat generated in the bearing is carried away by the total oil flow in the bearing, the expression for temperature rise can be determined. From Eq. (16.20),

$$(\text{kW})_f = (2\pi n_s)(fWr)(10^{-6})$$

The heat generated (H_g) is given by,

$$H_g = (\text{kW})_f = (2\pi n_s)(fWr)(10^{-6}) \text{ kW or kJ/s}$$

Substituting,

$$f = \left(\frac{c}{r}\right)(\text{CFV}) \quad \text{and} \quad W = 2\,plr$$

in the above expression

$$H_g = (4\pi)(10^{-6}) rcn_s lp\,(\text{CFV}) \tag{a}$$

The heat carried away by oil flow (H_c) is given by,

$$H_c = m\, C_p\, \Delta t \tag{b}$$

where,

m = mass of the lubricating oil passing through the bearing (kg/s)

C_p = specific heat of lubricating oil (kJ/kg°C)

Δt = temperature rise (°C)

The mass of the lubricating oil is given by,

$$m = \rho Q\,(10^{-6}) \text{ kg/s}$$

Substituting

$$Q = rcn_s l\,(\text{FV})$$

the mass is given by

$$m = \rho(rcn_s l)\,(FV)\,(10^{-6}) \text{ kg/s} \tag{c}$$

Substituting Eq. (c) in Eq. (b),

$$H_c = C_p\,\Delta t\,\rho(rcn_s l)\,(\text{FV})\,(10^{-6}) \tag{d}$$

Equating the expressions for H_g and H_c,

$$\Delta t = \left(\frac{4\pi p}{\rho C_p}\right)\frac{(\text{CFV})}{(\text{FV})} \tag{16.22}$$

For most lubricating oils,

$$\rho = 0.86 \quad \text{and} \quad C_p = 1.76 \text{ kJ/kg°C}$$

Substituting these values in Eq. (16.22),

$$\Delta t = \frac{8.3\,p(\text{CFV})}{(\text{FV})} \tag{16.23}$$

The average temperature of the lubricating oil is given by

$$T_{\text{av}} = T_i + \left(\frac{\Delta t}{2}\right) \tag{16.24}$$

where T_i is the inlet temperature.

16.13 BEARING DESIGN—SELECTION OF PARAMETERS

Very often, in the preliminary stages of journal bearing design, it is required to select suitable values for the following parameters:

(i) length-to-diameter ratio;
(ii) unit bearing pressure;
(iii) start-up load;
(iv) radial clearance;
(v) minimum oil film thickness; and
(vi) maximum oil film temperature.

(i) Length to Diameter Ratio In the design of hydrodynamic bearings, the diameter of the shaft is determined by strength or rigidity considerations and not on the basis of bearing capacity. The shaft diameter is usually determined by using the criteria such as permissible stress, permissible lateral deflection or permissible angle of twist. Therefore, it is the bearing length that the designer has to decide to obtain a given bearing capacity.

The length to diameter ratio (l/d) affects the performance of the bearing. As the ratio increases, the resulting film pressure increases as shown in Fig. 16.26. A long bearing, therefore, has more load carrying capacity compared with a short bearing. A short bearing, on the other hand, has greater side flow, which improves heat dissipation.

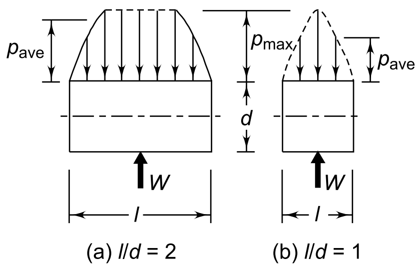


Fig. 16.26 *Effect of (l/d) ratio on Average Bearing Pressure*

The long bearings are more susceptible to metal to metal contact at the two edges, when the shaft is deflected under load. The longer the bearing, the more difficult it is to get sufficient oil flow through the passage between the journal and the bearing. Therefore, the design trend is to use (l/d) ratio as 1 or less than 1. When the shaft and the bearing are precisely aligned, the shaft deflection is within the limit and cooling of lubricant and bearing does not present a serious problem, the (l/d) ratio can be taken as more than 1. In practice, the (l/d) ratio varies from 0.5 to 2.0, but in the majority of applications, it is taken as 1 or less than 1. The following terminology is used in relation to (l/d) ratio,

(a) When (l/d) ratio is more than 1, the bearing is called '*long*' bearing.
(b) When (l/d) ratio is less than 1, the bearing is called '*short*' bearing.
(c) When (l/d) ratio is equal to 1, the bearing is called '*square*' bearing.

(ii) Unit Bearing Pressure The unit bearing pressure is the load per unit of projected area of the bearing in running condition. It depends upon a number of factors, such as bearing material, operating temperature, the nature and frequency of load and service conditions. The values of unit bearing pressure, based on past experience, are given in Table 16.2.

Table 16.2 *Permissible bearing pressures*

Application	*Unit bearing pressure (p) (N/mm^2)*
(i) *Diesel engines*	
Main bearing	5–10
Crank pin	7–14
Gudgeon pin	13–14
(ii) *Automotive engines*	
Main bearing	3–4
Crank pin	10–14
(iii) *Air compressors*	
Main bearing	1–1.5
Crank pin	1.5–3.0
(iv) *Centrifugal pumps*	
Main bearing	0.5–0.7
(v) *Electric motors*	
Main bearing	0.7–1.5
(vi) *Transmission shafting*	
Light duty	0.15
Heavy duty	1.00
(vii) *Machine tools*	
Main bearing	2

(iii) Start-up Load The unit bearing pressure for starting conditions should not exceed 2 N/mm^2. The start-up load is the static load when the shaft is stationary. It mainly consists of the dead weight of the shaft and its attachments. The start-up load can be used to determine the minimum length of the bearing on the basis of starting conditions.

(iv) Radial Clearance The radial clearance should be small to provide the necessary velocity gradient. However, this requires costly finishing operations, rigid mountings of the bearing assembly and clean lubricating oil without any foreign particles. This increases the initial and maintenance costs. The practical value of radial clearance is 0.001 mm per mm of the journal radius. Or,

$$c = (0.001)\,r$$

The practical values of radial clearances for commonly used bearing materials are given in Table 16.3.

Table 16.3 *Radial clearance*

Material	*Radial clearance*
Babbitts	(0.001) r to (0.00167) r
Copper–lead	(0.001) r to (0.01) r
Aluminium–alloy	(0.002) r to (0.0025) r

(v) Minimum Oil Film Thickness The surface finish of the journal and the bearing is governed by the value of the minimum oil film thickness selected by the designer and vice versa. There is a lower limit for the minimum oil film thickness, below which metal to metal contact occurs and the hydrodynamic film breaks. This lower limit is given by,

$$h_0 = (0.0002)r$$

(vi) Maximum Oil Film Temperature The lubricating oil tends to oxidise when the operating temperature exceeds 120°. Also, the surface of babbitt bearing tends to soften at 125°C (for bearing pressure of 7 N/mm^2) and at 190°C (for bearing pressure of 1.4 N/mm^2). Therefore, the operating temperature should be kept within these limits. In general, the limiting temperature is 90°C for bearings made of babbitts.

Bearings can be designed for two different conditions—bearings for maximum load carrying capacity and bearings for minimum frictional loss. The optimum values of $\left(\frac{h_o}{c}\right)$ for full journal bearing for these conditions are as follows:

$\left(\frac{l}{d}\right)$ *ratio*	$\left(\frac{h_o}{c}\right)$ *for maximum load*	$\left(\frac{h_o}{c}\right)$ *for minimum friction*
∞	0.66	0.60
1	0.53	0.30
0.5	0.43	0.12
0.25	0.27	0.03

The designer can use the above values for design of bearings under optimum conditions.

Example 16.9 *The following data is given for a 360° hydrodynamic bearing:*

radial load = 3.2 kN
journal speed = 1490 rpm
journal diameter = 50 mm
bearing length = 50 mm
radial clearance = 0.05 mm
viscosity of lubricant = 25 cP

Assuming that the total heat generated in the bearing is carried by the total oil flow in the bearing, calculate

(i) coefficient of friction;
(ii) power lost in friction;
(iii) minimum oil film thickness;
(iv) flow requirement in 1itres/min; and
(v) temperature rise.

Solution

Given $W = 3.2$ kN $n = 1490$ rpm $d = 50$ mm $l = 50$ mm $c = 0.05$ mm $z = 25$ cP

Step I *Performance parameters*

$$p = \frac{W}{ld} = \frac{(3.2)(1000)}{(50)(50)} = 1.28 \text{ N/mm}^2$$

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu n_s}{p} = \left(\frac{25}{0.05}\right)^2 \left(\frac{25}{10^9}\right)\left(\frac{1490}{60}\right)\left(\frac{1}{1.28}\right)$$
$$= 0.121$$

$$\left(\frac{l}{d}\right) = \left(\frac{50}{50}\right) = 1$$

From Table 16.1,

$$\left(\frac{r}{c}\right) f = 3.22 \qquad \left(\frac{h_o}{c}\right) = 0.4 \qquad \frac{Q}{rcn_s l} = 4.33$$

Step II *Coefficient of friction*

$$f = 3.22\left(\frac{c}{r}\right) = 3.22\left(\frac{0.05}{25}\right) = 0.00644 \qquad \text{(i)}$$

Step III *Power lost in friction*

From Eq. (16.20),

$$(\text{kW})_f = \frac{2\pi n_s\, fWr}{10^6}$$

$$= \frac{2\pi\,(1490/60)\,(0.00644)\,(3.2)(1000)\,(25)}{10^6}$$

$$= 0.08 \qquad \text{(ii)}$$

Step IV *Minimum oil film thickness*

$$h_o = 0.4c = 04(0.05) = 0.02 \text{ mm} \qquad \text{(iii)}$$

Step V *Flow requirement*

$$Q = 4.33\, rcn_s\, l = 4.33(25)(0.05)(1490/60)(50)$$
$$= 6720.5 \text{ mm}^3/\text{s}$$
$$Q = (6720.5)\,(10^{-3}) \text{ cc/s}$$
$$Q = (6720.5)\,(10^{-3})\,(10^{-3}) \text{ litres/s}\,(1000 \text{ cc} = 1 \text{litre})$$
$$Q = (6720.5)(10^{-6})\,(60) \text{ litre/min}$$
$$= 0.403 \text{ litre/min} \qquad \text{(iv)}$$

Step VI *Temperature rise*

From Eq. 16.23,

$$\Delta t = \frac{8.3p(\text{CFV})}{(\text{FV})}$$

$$= \frac{8.3(1.28)(3.22)}{(4.33)} = 7.9°C \qquad \text{(v)}$$

Example 16.10 *The following data is given for a full hydrodynamic bearing used for electric motor:*

radial load = 1200 N
journal speed = 1440 rpm
journal diameter = 50 mm
static load on the bearing = 350 N

The values of surface roughness (cla) of the journal and the bearing are 2 and 1 micron respectively. The minimum oil film thickness should be five times the sum of surface roughness of the journal and the bearings. Determine

(i) length of the bearing;
(ii) radial clearance;
(iii) minimum oil film thickness;
(iv) viscosity of lubricant; and
(v) flow of lubricant.

Select a suitable oil for this application assuming the operating temperature as 65°C.

Solution

Given $W = 1200$ N $\quad n = 1440$ rpm
$d = 50$ mm $\quad$ static load = 350 N
$h_o = 5$ (sum of surface roughness)

Step I *Length of the bearing*

Starting condition

The starting load on the bearing is the static load, i.e., 350 N. As discussed in Article 16.13, the start-up bearing pressure is usually taken as 2 N/mm^2.

$$p = \frac{W}{ld} \text{ or } l = \frac{W}{pd} = \frac{350}{(2)\,(50)} = 3.5 \text{ mm} \qquad \text{(a)}$$

Running condition

During running condition, the radial load on the bearing is 1200 N. From Table 16.2, the range of permissible bearing pressure in the application of an electric motor is from 0.7 to 1.5 N/mm^2. We will assume the permissible bearing pressure as 1 N/mm^2 in this range. Therefore, during running conditions,

$$p = \frac{W}{ld} \text{ or } l = \frac{W}{pd} = \frac{1200}{(1)(50)} = 24 \text{ mm} \qquad \text{(b)}$$

From (a) and (b), the minimum length of the bearing is 24 mm.

$$\therefore \qquad \left(\frac{l}{d}\right) = \frac{24}{50} = 0.48$$

We will assume the standard value for (l/d) ratio as 0.5.

$$\left(\frac{l}{d}\right) = 0.50$$

$$l = 0.5\, d = 0.5\,(50) = 25 \text{ mm} \qquad \text{(i)}$$

Step II *Radial clearance*

The standard value of radial clearance in case of Babbitt bearing is given by,

$$c = (0.001)\,r = (0.001)(25) = 0.025 \text{ mm} \qquad \text{(ii)}$$

Step III *Minimum oil film thickness*

The minimum oil film thickness is given by,

h_o = 5 (sum of values of surface roughness of journal and bearing)

$$h_o = 5\,(2 + 1) = 15 \text{ microns} = 0.015 \text{ mm} \qquad \text{(iii)}$$

Step IV *Viscosity of lubricant*

Refer to Table 16.1,

$$\left(\frac{l}{d}\right) = \frac{1}{2} \quad \text{and} \quad \left(\frac{h_o}{c}\right) = \frac{0.015}{0.025} = 0.6$$

For the above mentioned values,

$$S = 0.779 \quad \text{and} \quad \frac{Q}{rcn_s l} = 4.29$$

$$n_s = \frac{1440}{60} = 24 \text{ rps}$$

$$p = \frac{W}{ld} = \frac{1200}{(25)(50)} = 0.96 \text{ N/mm}^2$$

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu n_s}{p} \quad \text{or} \quad 0.779 = (1000)^2 \frac{\mu(24)}{(0.96)}$$

or $\quad \mu = (31.16)(10^{-9})$ N-sec/mm^2 or 31.16 cP (iv)

Step V *Selection of lubricant*

Referring to Fig. 16.8, it is observed that the values of viscosity for SAE-30 and SAE-40 oils are 30 and 38 cP respectively at the operating temperature of 65°C. We will select SAE-40 oil for this application, which will satisfy the minimum viscosity of 31.16 cP.

Step VI *Flow of lubricant*

$$Q = 4.29\, rcn_s l = 4.29\,(25)(0.025)\,(24)(25) = 1608.75 \text{ mm}^3\text{/s} \qquad \text{(v)}$$

Example 16.11 *Design a full hydrodynamic journal bearing with the following specification for machine tool application:*

journal diameter = 75 mm
radial load = 10 kN
journal speed = 1440 rpm
minimum oil film thickness = 22.5 microns
inlet temperature = 40°C
bearing material = babbitt

Determine the length of the bearing and select a suitable oil for this application.

Solution

Given $W = 10$ kN $\quad n = 1440$ rpm $\quad d = 75$ mm
$h_o = 22.5$ microns $\quad T_i = 40$°C

Step I *Length of bearing*

From Table 16.2, the permissible bearing pressure for machine tool applications is 2 N/mm^2. Therefore, during running conditions,

$$p = \frac{W}{ld} \quad \text{or} \quad l = \frac{W}{pd} = \frac{10\,000}{(2)(75)} = 66.67 \text{ mm}$$

$$\therefore \qquad \left(\frac{l}{d}\right) = \frac{66.67}{75} = 0.89$$

We will assume standard value for (l/d) ratio as 1

$$\left(\frac{l}{d}\right) = 1$$

$$l = d = 75 \text{ mm} \qquad \text{(i)}$$

Step II *Selection of lubricant*

$$p = \frac{W}{ld} = \frac{10\,000}{(75)(75)} = 1.78 \text{ N/mm}^2$$

The standard value of radial clearance in case of babbitt bearing is given by,

$$c = (0.001)\,r = (0.001)(75/2) = 0.0375 \text{ mm}$$

$$h_o = 22.5 \text{ microns} = 0.0225 \text{ mm}$$

Refer to Table 16.1,

$$\left(\frac{l}{d}\right) = 1 \quad \text{and} \quad \left(\frac{h_o}{c}\right) = \frac{0.0225}{0.0375} = 0.6$$

For the above mentioned values,

$$S = 0.264 \quad \left(\frac{r}{c}\right) f = 5.79 \quad \frac{Q}{rcn_s l} = 3.99$$

$$n_s = \frac{1440}{60} = 24 \text{ rps}$$

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu n_s}{p} \quad \text{or} \quad 0.264 = (1000)^2 \frac{\mu(24)}{(1.78)}$$

or

$\mu = (19.58)(10^{-9})$ N-sec mm^2 or 19.58 cP (a)

From Eq. (16.23),

$$\Delta t = \frac{8.3p(\text{CFV})}{(\text{FV})} = \frac{8.3(1.78)(5.79)}{(3.99)} = 21.44\text{°C}$$

From Eq. (16.24),

$$T_{av} = T_i + \frac{\Delta t}{2} = 40 + \frac{21.44}{2} = 50.72°C \qquad \text{(b)}$$

From (a) and (b), it is observed that the lubricating oil should have minimum viscosity of 19.58 cP at 50.72°C. Referring to Fig. 16.8, the viscosity of SAE-10 oil is 22 cP at 50°C. We will select SAE-10 oil for this application, which will satisfy the minimum viscosity of 19.58 cP.

Example 16.12 *The following data is given for a 360° hydrodynamic bearing:*

length to diameter ratio = 1
journal speed = 1350 rpm
journal diameter = 100 mm
diametral clearance = 100 μm
external load = 9 kN

The value of minimum film thickness variable is 0.3. Find the viscosity of oil that need be used.

Solution

Given $W = 9$ kN $\quad n = 1350$ rpm $\quad d = 100$ mm
diametral clearance = 100 μm $\quad l/d = 1 \quad h_o/c = 0.3$

Step I *Sommerfeld number*

$$\left(\frac{l}{d}\right) = 1 \quad \text{and} \quad \left(\frac{h_o}{c}\right) = 0.3$$

From Table 16.1,

$$S = \frac{0.0446 + 0.121}{2} = 0.0828$$

Step II *Viscosity of lubricant*

$$p = \frac{W}{ld} = \frac{9000}{(100)(100)} = 0.9 \text{ N/mm}^2$$

$$c = \left(\frac{1}{2}\right)(\text{diametral clearance}) = \left(\frac{1}{2}\right)(100\ \mu\text{m})$$

$$= 50 \text{ microns} = (50)(10^{-3}) \text{ mm}$$

The Sommerfeld number is given by,

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu n_s}{p}$$

or

$$0.0828 = \left[\frac{50}{(50)(10^{-3})}\right]^2 \frac{\mu(1350/60)}{(0.9)}$$

$$\mu = 3.312(10^{-9}) \text{ N-s/mm}^2 \text{ or } 3.312 \text{ cP}$$

Example 16.13 *The following data is given for a 360° hydrodynamic bearing:*

radial load = 10 kN
journal speed = 1440 rpm
unit bearing pressure = 1000 kPa
clearance ratio (r/c) = 800
viscosity of lubricant = 30 mPa s

Assuming that the total heat generated in the bearing is carried by the total oil flow in the bearing, calculate:

(i) dimensions of bearing;
(ii) coefficient of friction;
(iii) power lost in friction;
(iv) total flow of oil;
(v) side leakage; and
(vi) temperature rise.

Solution

Given $W = 10$ kN $\quad n = 1440$ rpm $\quad p = 1000$ kPa
$r/c = 800 \quad \mu = 30$ mPa s

Step I *Dimensions of bearing*

We will assume,

$$\left(\frac{l}{d}\right) = 1$$

$$p = 1000 \text{ kPa} = 1000\ (10^3)$$
$$\text{Pa} = 1000\ (10^3)(10^{-6}) \text{ MPa} = 1 \text{ MPa} = 1 \text{ N/mm}^2$$

$$p = \frac{W}{ld} = \frac{W}{d^2} \quad \text{or} \quad 1 = \frac{10\,000}{d^2} \quad \therefore d = 100 \text{ mm}$$

$$d = l = 100 \text{ mm} \qquad \text{(i)}$$

Step II *Performance parameters*

$\mu = 30$ m Pa s = 30 milli Pascals s = 30 (10^{-3}) Pa s = 30 (10^{-9}) MPa s

or $\qquad \mu = 30\ (10^{-9})$ N-s/mm^2

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu n_s}{p} = (800)^2 \frac{(30)(10^{-9})(1440/60)}{(1)}$$
$$= 0.4608$$

The values of dimensionless performance parameters are obtained by linear interpolation from Table (16.1). The principal of linear interpolation is illustrated in Fig. 16.27. For (l/d =1)

$$\left(\frac{r}{c}\right)f = 5.79 + (12.8 - 5.79)\frac{(0.4608 - 0.264)}{(0.631 - 0.264)}$$

$$= 9.55$$

$$\left(\frac{Q}{rcn_s l}\right) = 3.99 - (3.99 - 3.59)\frac{(0.4608 - 0.264)}{(0.631 - 0.264)}$$

$$= 3.78$$

$$\frac{Q_s}{Q} = 0.497 - (0.497 - 0.28)\frac{(0.4608 - 0.264)}{(0.631 - 0.264)}$$

$$= 0.38$$

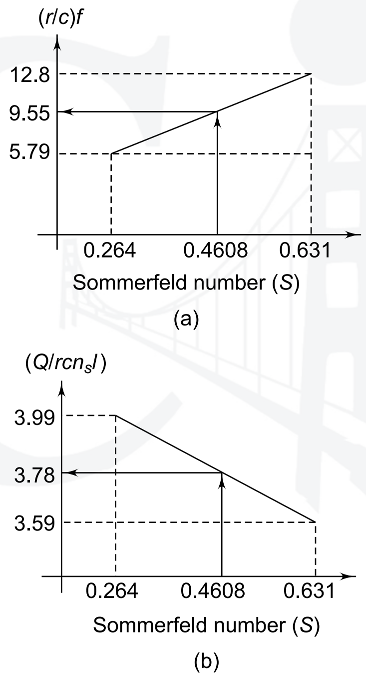


Fig. 16.27

Step III *Coefficient of friction*

$$f = 9.55\left(\frac{c}{r}\right) = 9.55\left(\frac{1}{800}\right) = 0.0119 \qquad \text{(ii)}$$

Step IV *Power lost in friction*

From Eq. (16.20),

$$(\text{kW})_f = \frac{2\pi n_s f W r}{10^6}$$

$$= \frac{2\,\pi\,(1440/60)(0.0119)(10000)(50)}{10^6} = 0.9 \qquad \text{(iii)}$$

Step V *Total flow of oil*

$$Q = 3.78 rcn_s l$$

$$= 3.78(50)\left(\frac{50}{800}\right)(1440/60)(100)$$

$$= 28\,350 \text{ mm}^3/\text{s} \qquad \text{(vi)}$$

Step VI *Side leakage*

$$Q_s = 0.38\,Q = 0.38(28\,350) = 10\,773 \text{ mm}^3/\text{s} \qquad \text{(v)}$$

Step VII *Temperature rise*

From Eq. 16.23,

$$\Delta t = \frac{8.3\,p(\text{CFV})}{(\text{FV})} = \frac{8.3(1)(9.55)}{(3.78)} = 22.97°C \qquad \text{(vi)}$$

Example 16.14 *The following data is given for a 360° hydrodynamic bearing:*

radial load = 6.5 kN
journal speed = 1200 rpm
journal diameter = 60 mm
bearing length = 60 mm
minimum oil film thickness = 0.009 mm
The class of fit is H7e7 (fine) normal running fit.

Specify the viscosity of the lubricating oil that you will recommend for this application.

Solution

Given $W = 6.5$ kN $\quad n = 1200$ rpm $\quad d = 60$ mm
$l = 60$ mm $\quad h_o = 0.009$ mm

Step I *Sommerfeld number (Tables 3.2 and 3.3a)*

The hole and shaft limits for H7e7 running fit are as follows:

Hole limits (60 + 0.00) and (60 + 0.03) mm
Shaft limits (60 – 0.09) and (60 – 0.06) mm

If the manufacturing processes are centered, the average diameter of the bearing and journal will be 60.015 and 59.925 mm respectively.

$$c = \left(\frac{1}{2}\right)(60.015 - 59.925) = 0.045 \text{ mm}$$

$$\left(\frac{h_o}{c}\right) = \left(\frac{0.009}{0.045}\right) = 0.2$$

$$\left(\frac{l}{d}\right) = \left(\frac{60}{60}\right) = 1$$

From Table 16.1,

$$S = 0.0446$$

Step II *Viscosity of lubricating oil*

From Eq. (16.18),

$$\mu = S\left(\frac{c}{r}\right)^2\left(\frac{p}{n_s}\right)$$

$$= (0.0446)\left(\frac{0.045}{30}\right)^2\left(\frac{6500}{(60)(60)}\right)\frac{1}{(1200/60)}$$

$$= (9.06\times10^{-9})\text{N-s/mm}^2$$

$$\therefore \quad \mu = 9.06 \text{ cP}$$

Example 16.15 *An oil ring bearing of a transmission shaft is shown in Fig. 16.28. There is no hydrodynamic action over the width of 4 mm of the oil ring. The total radial load acting on the journal is 20 kN and the journal rotates at 1450 rpm. The radial clearance and minimum film thickness are 20 to 5 microns respectively. Calculate*

(i) viscosity of the lubricant; and
(ii) required quantity of oil.

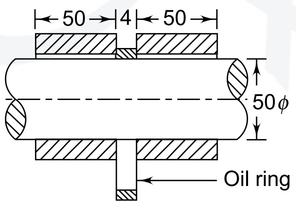


Fig. 16.28

Solution

Given total load = 20 kN $n = 1450$ rpm
$h_o = 5$ microns $c = 20$ microns

Step I *Performance parameters*

The oil ring divides the bearing into two parts. Each part can be treated as separate hydrodynamic bearing carrying a load of (20/2) or 10 kN. For each part,

$$\left(\frac{l}{d}\right) = \left(\frac{50}{50}\right) = 1$$

$$\left(\frac{h_o}{c}\right) = \left(\frac{5}{20}\right) = 0.25$$

From Table 16.1, the value of Sommerfeld number can be obtained by linear interpolation.

$$S = 0.0446 + (0.121 - 0.0446)\frac{(0.25-0.20)}{(0.40-0.20)}$$

$$= 0.0637$$

$$\left(\frac{Q}{rcn_s l}\right) = 4.62 - (4.62 - 4.33)\frac{(0.25-0.20)}{(0.40-0.20)}$$

$$= 4.5475$$

Step II *Viscosity of the lubricant*

$$p = \frac{W}{ld} = \frac{10\,000}{(50)(50)} = 4 \text{ N/mm}^2$$

From Eq. (16.18),

$$\mu = S\left(\frac{c}{r}\right)^2\left(\frac{p}{n_s}\right)$$

$$= (0.0637)\left(\frac{0.020}{25}\right)^2\left(\frac{4}{(1450/60)}\right)$$

$$= (6.7478\times10^{-9})\text{ N-s/mm}^2$$

$$\therefore \quad \mu = 6.75 \text{ cP} \qquad \text{(i)}$$

Step III *Required quantity of oil*

$$Q = 4.5475 rcn_s l$$
$$= 4.5475(25)(0.02)(1450/60)(50)$$
$$= 2747 \text{ mm}^3/\text{s}$$

Since there are two such parts, the total oil flow is given by,

$$Q_t = 2Q = 2(2747) = 5494 \text{ mm}^3/\text{s}$$
$$Q_t = 5494(60)(10^{-6}) \text{ litre/min} = 0.33 \text{ litre/min}$$

(ii)

Example 16.16 *A hardened and ground journal, 50 mm in diameter, rotates at 1440 rpm in a lathe turned bronze bushing which is 50 mm long. For hydrodynamic lubrication, the minimum oil film thickness should be five times the sum of surface* *roughness (cla values) of the journal and the bearing. The data about machining methods is as follows:*

	Machining method	*Surface roughness (cla)*
shaft	*grinding*	*0.8 micron*
bearing	*turning / boring*	*1.6 microns*

The class of fit is H8d8 and the viscosity of the lubricant is 18 cP. Determine the maximum radial load that the journal can carry and still operate under hydrodynamic conditions.

Solution

Given $n = 1440$ rpm $d = 50$ mm $l = 50$ mm
$h_o = 5$ (sum of surface roughness) $z = 18$ cP

Step I *Sommerfeld number*

$h_o = 5(0.8 + 1.6) = 12$ microns or 0.012 mm

From Table 3.2 and 3.3a,

The limits for the hole and shaft are

Hole limits (50 + 0.000) and (50 + 0.039) mm

Shaft limits (50 – 0.119) and (50 – 0.080) mm

When the process of manufacturing is centered, the average size of the hole and shaft will be 50.0195 and 49.9005 mm respectively.

∴ diametral clearance = 50.0195 – 49.9005
= 0.119 mm

$$c = \left(\frac{1}{2}\right)(0.119) = 0.0595 \text{ mm}$$

$$\left(\frac{h_o}{c}\right) = \left(\frac{0.012}{0.0595}\right) = 0.2$$

$$\left(\frac{l}{d}\right) = \left(\frac{50}{50}\right) = 1$$

From Table 16.1,

$$S = 0.0446$$

Step II *Load carrying capacity*

From Eq. (16.18),

$$p = \left(\frac{r}{c}\right)^2 \frac{\mu n_s}{S}$$

$$= \left(\frac{25}{0.0595}\right)^2 \left(\frac{18}{10^9}\right)\left(\frac{1440}{60}\right)\left(\frac{1}{0.0446}\right)$$

$$= 1.71 \text{ N/mm}^2$$

$$W = pld = 1.71(50)(50) = 4275 \text{ N}$$

Example 16.17 *The following data is given for a 360° hydrodynamic bearing:*

radial load = 2 kN
journal diameter = 50 mm
bearing length = 50 mm
viscosity of oil = 20 m Pa s

Specify radial clearance that need be provided so that when the journal is rotating at 2800 rpm, the minimum film thickness is 30 microns. Evaluate the corresponding coefficient of friction.

Solution

Given $W = 2$ kN $n = 2800$ rpm $d = 50$ mm
$l = 50$ mm $h_o = 30$ microns $\mu = 20$ mPa s

Step I *Minimum film thickness variable*

$$n_s = \frac{2800}{60} = 46.67 \text{ rps}$$

$$\mu = 20 \text{ m Pa s} = 20 \text{ milli Pascals sec}$$
$$= 20\,(10^{-3}) \text{ Pa s} = 20\,(10^{-9}) \text{ MPa s}$$

or $\mu = 20\,(10^{-9})$ N-sec/mm^2

$$h_0 = 30 \text{ microns} = 0.03 \text{ mm}$$

$$p = \frac{W}{ld} = \frac{2000}{(50)(50)} = 0.8 \text{ N/mm}^2$$

The Sommerfeld number is given by,

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu n_s}{p} = \frac{25^2}{c^2}\frac{(20)(10^{-9})(46.67)}{(0.8)}$$

$$= \frac{(0.73)(10^{-3})}{c^2}$$

or $$c = \sqrt{\frac{(0.73)(10^{-3})}{S}} \quad \text{(a)}$$

From Table 16.1, it is observed that Sommerfeld number depends upon the ratio $\left(\frac{h_0}{c}\right)$ and the radial clearance c is unknown at this stage. Therefore it is a vicious circle. The problem is to be solved by trial and error.

Trial 1 Let us assume that $\left(\frac{h_0}{c}\right) = 0.9$

∴ assumed value of $c = \frac{h_0}{0.9} = \frac{0.03}{0.9} = 0.0333$ mm (i)

From Table 16.1, $S = 1.33$

$$c = \sqrt{\frac{(0.73)(10^{-3})}{S}} = \sqrt{\frac{(0.73)(10^{-3})}{1.33}} = 0.0234 \text{ mm}$$

∴ calculated value of $c = 0.0234$ mm (ii)

From (i) and (ii), it is observed that there is a difference between assumed and calculated values of radial clearance c.

Trial 2 Let us assume that $\left(\frac{h_0}{c}\right) = 0.8$

∴ assumed value of $c = \frac{h_0}{0.8} = \frac{0.03}{0.8} = 0.0375$ mm (i)

From Table 16.1, $S = 0.631$

$$c = \sqrt{\frac{(0.73)(10^{-3})}{S}}$$

$$= \sqrt{\frac{(0.73)(10^{-3})}{0.631}} = 0.034 \text{ mm}$$

∴ calculated value of $c = 0.034$ mm (ii)

From (i) and (ii), it is observed that there is a difference between assumed and calculated values of the radial clearance c.

Trial 3 Let us assume that $\left(\frac{h_0}{c}\right) = 0.6$

∴ assumed value of $c = \frac{h_0}{0.6} = \frac{0.03}{0.6} = 0.05$ mm (i)

From Table 16.1, $S = 0.264$

$$c = \sqrt{\frac{(0.73)(10^{-3})}{S}} = \sqrt{\frac{(0.73)(10^{-3})}{0.264}} = 0.0525 \text{ mm}$$

∴ calculated value of $c = 0.0525$ mm (ii)

From (i) and (ii), it is observed that the assumed value and calculated value of radial clearance c are approximately same. Therefore,

∴ $c = 0.05$ mm and $\left(\frac{h_0}{c}\right) = 0.6$

Step II *Radial clearance*

$$c = 0.05 \text{ mm}$$

Step III *Coefficient of friction*

From Table 16.1,

$$\left(\frac{r}{c}\right) f = 5.79$$

$$f = 5.79\left(\frac{c}{r}\right) = 5.79\left(\frac{0.05}{25}\right) = 0.01158$$

Example 16.18 *The following data is given for a 360° hydrodynamic bearing:*

radial load = 30 kN
journal diameter = 75 mm
bearing length = 75 mm
journal speed = 3600 rpm
radial clearance = 0.15 mm
inlet temperature = 40°C

The temperature–viscosity relationship is as follows:

T(°C)	40	41	42	43	44	45	46	47	48	49	50
z(cP)	52.5	50	47.5	45	43	41	39	37.5	36	34	33

Assume that the total heat produced in the bearing is carried by the total oil flow. The specific gravity and specific heat of the lubricant are 0.86 and 1.76 kJ/kg°C respectively.

Calculate the power lost in friction and the requirement of oil flow.

Solution

Given $W = 30$ kN $n = 3600$ rpm $d = 75$ mm $l = 75$ mm $c = 0.15$ mm $T_i = 40°$ $\rho = 0.86$ $c_p = 1.76$ kJ/kg°C

Step I *Sommerfeld number–viscosity relationship*

In practice, this type of problem occurs when the inlet temperature and lubricant are known. To determine the Sommerfeld number, the viscosity of the lubricant is to be specified. However, it varies with temperature and the average working temperature is unknown at this stage. Such problems are solved by the trial and error procedure.

$$p = \frac{W}{ld} = \frac{30\,000}{(75)(75)} = 5.333 \text{ N/mm}^2$$

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu n_s}{p}$$

$$= \left(\frac{37.5}{0.15}\right)^2 \left(\frac{z}{10^9}\right)\left(\frac{3600}{60}\right)\left(\frac{1}{5.333}\right) = 7.03\,(10^{-4}) z$$

Step II *Viscosity at operating temperature*

The viscosity of the lubricant at an inlet temperature of 40°C is 52.5 cP. The average temperature will be more than 40°C. As a first trial, the viscosity is assumed as 45 cP.

Trial 1

$z = 45$ cP

$S = 7.03(10^{-4})\, z = 7.03(10^{-4})(45) = 0.0316$

From Table 16.1, $(l/d = 1)$

$$\left(\frac{r}{c}\right) f = 1.05 + (1.7 - 1.05)\frac{(0.0316 - 0.0188)}{(0.0446 - 0.0188)} = 1.37$$

$$\left(\frac{Q}{rcn_s l}\right) = 4.74 - (4.74 - 4.62)\frac{(0.0316 - 0.0188)}{(0.0446 - 0.0188)} = 4.68$$

From Eq. (16.23),

$$\Delta t = \frac{8.3p(\text{CFV})}{(\text{FV})} = \frac{8.3(5.333)(1.37)}{(4.68)} = 12.96°\text{C}$$

$$T_{av} = T_i + \frac{\Delta t}{2} = 40 + \left(\frac{12.96}{2}\right) = 46.48°\text{C}$$

It will be seen from the viscosity–temperature table that the viscosity corresponding to 46.5°C is approximately 38 cP, while the assumed viscosity for this trial is 45 cP. As a second trial, the viscosity is assumed as 39 cP.

Trial 2

$z = 39$ cP

$S = 7.03\,(10^{-4})\, z = 7.03(10^{-4})\,(39) = 0.0274$

From Table 16.1,

$$\left(\frac{r}{c}\right) f = 1.05 + (1.7 - 1.05)\frac{(0.0274 - 0.0188)}{(0.0446 - 0.0188)} = 1.267$$

$$\left(\frac{Q}{rcn_s l}\right) = 4.74 - (4.74 - 4.62)\frac{(0.0274 - 0.0188)}{(0.0446 - 0.0188)} = 4.7$$

$$\Delta t = \frac{8.3p(\text{CFV})}{(\text{FV})} = \frac{8.3(5.333)(1.267)}{(4.7)} = 11.93°\text{C}$$

$$T_{av} = T_i + \frac{\Delta t}{2} = 40 + \left(\frac{11.93}{2}\right) = 45.97°\text{C} \cong 46°\text{C}$$

It is seen from the viscosity–temperature table that viscosity at 46°C is 39 cP. Therefore, the assumption is correct.

Step III *Power lost in friction*

Therefore,

$$f = 1.267\left(\frac{c}{r}\right) = 1.267\left(\frac{0.15}{37.5}\right) = 5.068(10^{-3})$$

From Eq. (16.20),

$$(\text{kW})_f = \frac{2\pi n_s f W r}{10^6} = \frac{2\pi(60)(5.068)(10^{-3})(30\,000)(37.5)}{10^6} = 2.15$$

Step IV *Requirement of oil flow*

$Q = 4.7\, rcn_s l = 4.7(37.5)(0.15)(60)(75)$
$= 118\,968.75$ mm³/s

$Q = (118\,968.75)(10^{-6})(60)$ litres/min
$= 7.138$ litres/min

16.14 BEARING CONSTRUCTIONS

There are two types of bearing construction—solid bushing and lined bushing, as shown in Fig. 16.29. A solid bushing is made either by casting or by machining from a bar. It is then

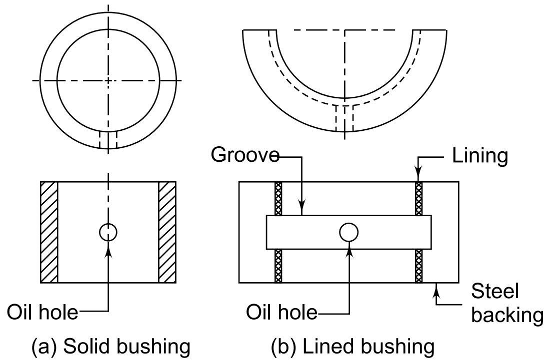


Fig. 16.29 *Bearing Constructions*

finished by grinding and reaming operations. A typical example of this type of bushing is bronze bearing. A lined bushing consists of steel backing with a thin lining of bearing material like babbitt. It is usually split into two halves.

There are two basic types of patterns of oil grooves—circumferential and cylindrical. A circumferential oil-groove bearing, as shown in Fig. 16.30(a), in effect divides the bearing into two short bearings, each of length (l/2). The pressure developed in the bearing along the axis is reduced due to the groove, as shown in Fig. 16.30(b). This results in a lower load carrying capacity. The centrifugal force acting on the oil in the circumferential groove may build pressure higher than the supply pressure, which may restrict the flow of the lubricant. Circumferential oil-groove bearings are used for the connecting rods and crankshafts of automotive engines.

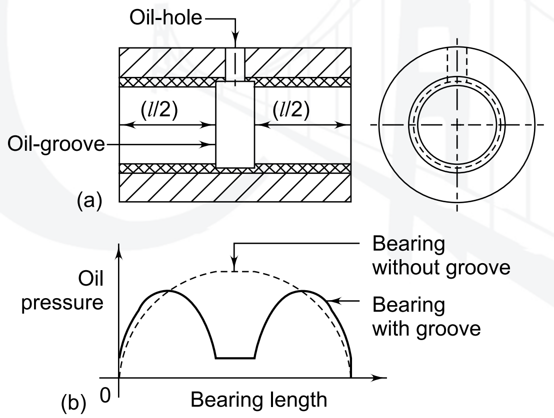


Fig. 16.30 *Circumferential Oil Groove Bearing*

The cylindrical oil-groove bearing, as shown in Fig. 16.31, has an axial groove almost along the full length of the bearing. It has a higher load carrying capacity compared with the circumferential oil-groove bearing; however, it is more susceptible to vibrations, known as oil-whip. Cylindrical oil-groove bearings are used for gearboxes and high speed applications. In practice, different patterns of oil-groove are obtained by the combination of cylindrical and circumferential passages of the oil.

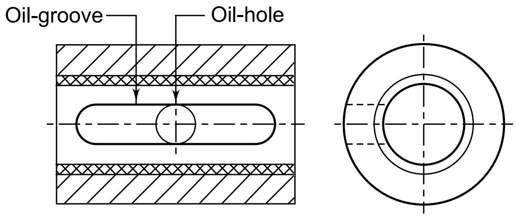


Fig. 16.31 *Cylindrical Oil Groove Bearing*

The principle of an oil-ring bearing is illustrated in Fig. 16.32. It consists of an oil ring in contact with the shaft and dipping in the oil bath below. The diameter of the oil ring is large compared with the diameter of the shaft. As the shaft rotates, the oil ring also rotates, although at a considerably lower speed and carries along with it the oil from the oil bath to the shaft. There are spreader grooves, which carry the oil to the entire surface of the shaft. The use of the oil ring bearing is restricted to horizontal shafts.

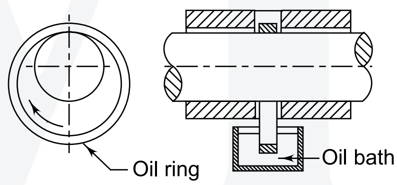


Fig. 16.32 *Principle of Oil ring Bearing*

16.15 BEARING MATERIALS

The desirable properties of a good bearing material are as follows:

(i) When metal to metal contact occurs, the bearing material should not damage the surface of the journal. It should not stick or weld to the journal surface.

(ii) It should have high compressive strength to withstand high pressures without distortion.

(iii) In certain applications like connecting rods or crankshafts, bearings are subjected to fluctuating stresses. The bearing material, in these applications, should have sufficient endurance strength to avoid failure due to pitting.

(iv) The bearing material should have the ability to yield and adopt its shape to that of the journal. This property is called *conformability*. When the load is applied, the journal is deflected resulting in contact at the edges. A conformable material adjusts its shape under these circumstances.

(v) The dirt particles in lubricating oil tend to jam in the clearance space and, if hard, may cut scratches on the surfaces of the journal and bearing. The bearing material should be soft to allow these particles to get embedded in the lining and avoid further trouble. This property of the bearing material is called *embeddability*.

(vi) In applications like engine bearings, the excessive temperature causes oxidation of lubricating oils and forms corrosive acids. The bearing material should have sufficient corrosion resistance under these conditions.

(vii) The bearing material should have reasonable cost and should be easily available in the market.

The most popular bearing material is babbitt. Due to its silvery appearance, babbitt is called 'white' metal. There are two varieties of babbitts—lead-base and tin-base, depending upon the major alloying element. They are used in the form of a strip or thin lining—about 0.5 mm thick—bonded to steel shells. Babbitts have excellent conformability and embeddability. Tin-base babbitts have better corrosion resistance and can be easily bonded to steel shells. High cost and shortage of tin are their main limitations. Babbitts, whether lead-base or tin-base, are inherently weak and their strength decreases rapidly with increasing temperature. Their use is further restricted due to poor fatigue strength. There are ten grades of white metals[4] and their typical applications are as follows:

(i) Grade 90 and Grade 84 are used for bearings in petrol and diesel engines and in crossheads of steam engines.

(ii) Grade 75 is used for repair jobs in mills and marine installations.

(iii) Grade 69 is used in underwater applications and gland packing.

(iv) Grade 60 is used in dynamos, electric motors, centrifugal pumps and other medium speed applications.

(v) Grade 20 and Grade 10 are used in low speed applications, like concrete mixers and rope conveyors.

(vi) Grade 6 is used in heavy duty applications like engine bearings, turbines and cross-heads of steam engines.

(vii) Grade 5 is used for line shafting and in railway carriages.

The chemical composition of these grades is given in Table 16.4.

Table 16.4 *Chemical composition of antifriction bearing alloys (%)*

Grade	*Sn*	*Sb*	*Pb*	*Cu*	*Zn*
90	90	7	–	3	–
84	84	10	R	5.5	–
75	75	11	R	3	–
69	69	–	–	1	30
60	60	11.5	R	3	–
20	20	15	R	1.5	–
10	10	14	R	0.75	–
6	6	15	R	1	–
5	5	15	R	0.5	–

(*R* = remainder)

The other bearing materials are bronze, copper–lead, aluminium alloys and plastics. Compared with babbitts, bronze is cheaper, stronger and can withstand high pressures. It has got excellent casting and machining characteristics. Bronze bearing is made as a single solid unit. The main drawback of bronze bearing is its tendency to stick to the surface of the journal at high temperatures. Copper–lead bearings (70% Cu and 30% Pb) are used in the form of a thin lining like white metal. They have more hardness and fatigue strength and are used in heavy duty applications at high temperatures. Tin–aluminium alloys have higher fatigue strength and they retain their strength even at high temperatures. They are used in engine bearings.

[4] IS 25–1979: Specifications for antifriction bearing alloy

There are certain non-metallic bearings like graphite, plastics (Teflon) and rubber. For high temperature applications, conventional bearings with lubricating oils cannot be used. In such cases, bearings made of pure carbon (graphite) are employed. Teflon has an extremely low coefficient of friction and requires no external lubricant like an oil. They are particularly useful where the bearing is located at an inaccessible position or where the lubricating oil is likely to cause contamination such as bearings for food processing machines. Rubber is used as bearing material in marine applications.

16.16 SINTERED METAL BEARINGS

Sintered metal powder bearings are made from compressed metal powder by the sintering process. They are porous bearings impregnated with lubricating oils. They can absorb lubricating oil to the extent of 20% to 30% of their volume. This oil serves as a reservoir enabling the bearing to run for a long period without any attention. They can be refilled with oil by soaked felts or wick-feed lubricators at periodic intervals without dismantling. There are two grades—copper-base and iron-base—of sintered bearings. A copper-base material has more corrosion resistance compared with an iron-base material. The permissible bearing pressures (load per unit of projected area) for copper-base and iron-base bearings are 60 N/mm^2 and 100 N/mm^2 respectively. They give a satisfactory performance of up to 80°C. Sintered metal powder bearings are used in automobiles, textile machinery and machine tools.

16.17 LUBRICATING OILS

The desirable properties of lubricating oil are as follows:

(i) It should be available in a wide range of viscosities.

(ii) There should be little change in viscosity of the oil with change in temperature.

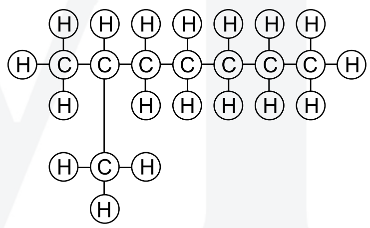
(iii) The oil should be chemically stable with the bearing material and atmosphere at all temperatures encountered in the application.

(iv) The oil should have sufficient specific heat to carry away frictional heat, without abnormal rise in temperature.

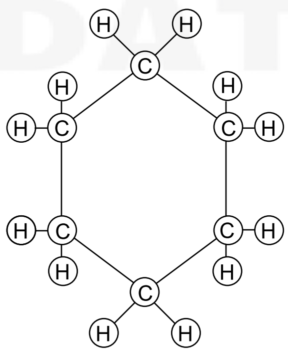
(v) It should be commercially available at reasonable cost.

Lubricating oils are divided into two groups—mineral oils and vegetable or animal oils.

Mineral oils consist of hydrocarbons, which are obtained by the distillation of crude oil. There are two different classes of mineral oils—those with a paraffinic series and those with a naphthenic series. A *paraffinic* oil is composed of straight and branched chains of hydrocarbons defined by the general formula $C_nH_{(2n+2)}$. A *naphthenic oil* is composed of a saturated single-ring formation of hydrocarbons defined by the general formula C_nH_{2n}. Their structures are illustrated in Fig. 16.33.



(a) Paraffinic oil (n = 8) (C_8H_{18})



(b) Naphthenic oil (n = 6) (C_6H_{12})

Fig. 16.33 *Structure of Lubricating Oils*

The Society of Automotive Engineers (SAE) of USA has classified lubricating oils by a number, which is related to the viscosity of the oil in Saybolt

Universal Seconds. This classification is based on only one property of lubricating oils, namely, viscosity. According to this system, automotive oils are classified in accordance with Table 16.5.

Table 16.5 *SAE oils*

SAE number	*Saybolt universal viscosity at 210°F*	
	Minimum	*Maximum*
20	45	less than 58
30	58	less than 70
40	70	less than 85
50	85	less than 110

It can be observed from this table that the SAE number corresponds to approximately one-half of the viscosity of oil at 210°F measured in terms of SUS. When the SAE number is more, it indicates more viscous oil. The viscosities in the above table are measured at only one fixed temperature, i.e. 210°F. It does not indicate the full picture of the viscosity–temperature relationship. It is, therefore, necessary to specify viscosities of these oils at a low temperature. This is achieved by specifying viscosities at 0°F as shown in the following table:

SAE number	*Saybolt universal viscosity at 0°F*	
	Minimum	*Maximum*
5 W	–	6000
10 W	6000	less than 12000
20 W	12000	48000

The letter W in the above table indicates winter grading of oils.

Compared with vegetable or animal oils, mineral oils offer the following advantages:

(i) Mineral oils are chemically inert.
(ii) They have a wide range of viscosities, corresponding to different values of *n* in the general formula.
(iii) They have little tendency to oxidise or form corrosive acids.
(iv) After periodic filtration, they can be reused without any loss or change of their properties.
(v) At normal temperature, they are not liable to spontaneous ignition.

Vegetable oils used for lubrication are castor oil, rapeseed oil, palm oil and olive oil. Lubricating oils of animal origin are lard oil, tallow oil and certain oils obtained from marine species, such as whales, sperms or dolphin jaws. The advantages of vegetable and animal oils are as follows:

(i) These oils are sometimes referred to as *fixed oils* because they are non-volatile, unless there is chemical decomposition. This property prevents them from being expelled from intimate contacts of solid surfaces by frictional heat.
(ii) They retain their viscosities at high temperature much better than mineral oils.
(iii) These oils are called '*polar*' compounds. They have a long chain of molecules with positive and negative charges at the two ends. One end of the polar molecule adheres to the surface of the journal or bearing and the long chain of molecules extends into clearance space. They form 'clusters' which prevents metal to metal contact in boundary lubrication.

The main drawbacks of vegetable or animal oils are as follows:

(i) At low temperature, these oils solidify and become '*fats*'. The fat is melted at about 65°C and becomes oil.
(ii) These oils react with oxygen in the atmosphere and become acidic. In some cases they change from the liquid state to an elastic solid form. Due to this reason, they are termed '*drying oils*'.
(iii) They are subjected to saponification either by contact with base metals or with hot water. They produce glycerol and some organic acids, which attack metallic surfaces and form metallic soaps.

Castor oil was used in the past as a lubricant in racing cars and aero-engines. Rapeseed oil is added to mineral oil to increase viscosity. Cottonseed oil is mainly used as a thickener in mineral oils. Lard oil is used as cutting oil, while tallow oil is used as cylinder oil. In light machine tools, sperm oil is used for spindle lubrication.

16.18 ADDITIVES FOR MINERAL OILS

An 'additive' is a substance added to mineral oil in order to improve a particular property of that oil. Mineral oil in which additives are mixed is called '*doped oil*' or '*base oil*'. The additives must be soluble in mineral oils. When the lubricating oil is likely to come in contact with water, the additive should not be soluble in water; otherwise it is likely to be washed out of the oil. Such a situation arises in case of steam engine or turbine lubricants. Most mineral oils depend upon additives. The additives are classified according to the property they are intended to improve, e.g., oxidation inhibitor, VI improver, pour-point depressant or anti-foam additive. Some additives perform more than one function and are called multi-functional additives, e.g., an oxidation inhibitor also reduces corrosion of bearing surfaces.

Oxidation inhibitors are widely used in automotive lubricants. It is observed that lubricating oils tend to oxidise on the walls of the piston and cylinder. This oxidation is accelerated as the temperature increases. The product of oxidation is a gummy substance. The piston ring may stick to the surface of the cylinder wall due to this substance. The mechanism of oxidation is not fully understood; however, it appears to be a chain reaction resulting in the formation of peroxides, which attack the bearing surfaces. Oil-soluble compounds containing sulphur and phosphorus are used as oxidation inhibitors. These additives act in two different ways. They compete with the base oil for oxygen and thus retard the formation of peroxides. They also decompose and break peroxides and prevent further corrosion of bearing surfaces.

The function of a '*detergent*' *additive* is similar to that of soap. Hard abrasive piston deposits find their way into lubricating oil. These particles have a tendency to form aggregates. These aggregates wear out bearing surfaces and clog oil passages. Metallic soaps, such as calcium phenylstearate, are used as detergent additives. The detergent acts by coating the complete surface of the particle just like soap. The coating prevents the individual particles from combining, thus preventing formation of aggregate. Since the particles are small, they remain suspended in lubricating oil and do not cause wearing of bearing surfaces or clogging of oil passages.

A *viscosity index improver* is a substance which when added to mineral oils, increases viscosity at high temperatures. These additives are polymerized resins of high molecular weight, such as polyisobutylene, polymethacrylate ester and polyfumarite ester. The popular VI improver is '*paratone*' which is a polymer of butylene having the formula $(C_4H_8)_n$ where n varies from 180 to 270. Due to the large size of the molecule, the additive has extremely high viscosity. When added to the base oil, it increases viscosity at high temperature. At low temperature, the molecules are coiled and remain as colloidal suspension in the base oil. As the temperature increases, they uncoil, go into solution and increase its viscosity. This improves the viscosity index of the base oil. Normally, 1 to 2 per cent VI improver is added to the base oil.

Pour point is the lowest temperature at which the oil can flow. This is an important characteristic when the lubricant is used in refrigeration and air-conditioning equipment. Mineral oils of paraffinic origin contain some amount of wax. When the temperature decreases below the pour point, the wax is separated in the form of needle shaped crystals. These crystals join with each other and form a matrix. The oil is held in the matrix in much the same fashion as water is retained in sponge. The mineral oil, therefore, ceases to function as a fluid. 'Paraflow' is used as a *pour point depressant*. This type of additive surrounds the wax crystals in the form of a coating and prevents them from forming a large size matrix. The small-size crystals remain in suspension, but do not affect the fluidity of the base oil.

Anti-foam additives are used in lubricants for aero-engines. In aircraft engines, a continuous formation of foam on the surface of the lubricating oil in the supply tank is often observed. Foaming is a serious problem at high altitudes. This is caused due to the excess capacity of the scavenge pump over that of the pressure pump. This causes the scavenge pump to introduce a large quantity of air into the lubricant. The excess capacity of the scavenge pump is necessary and cannot be avoided. When air is mixed in lubricating oil, foam is formed in the storage tank. When the storage tank is completely filled, the foam overflows into the crankcase through the connecting pipeline. Due to the compressibility of entrained air, the lubricating quality of the oil is impaired. Silicon polymer is used as an anti-foam additive. It acts as a chemical foam-breaker and reduces the stability of the foam.

Oiliness is considered as an important property under the conditions of boundary lubrication. Oiliness is distinct from viscosity, and refers to friction-reducing capacity. Oiliness is a joint property of the lubricant and the metallic surfaces in contact. It is a measure of lubricating qualities under boundary lubrication where the metal to metal contact is prevented by the 'absorbed' oil film. If there are two lubricants having same viscosities and if one results in lower friction under identical conditions then that lubricant is said to have better oiliness. There is no absolute measure of oiliness. It is observed that lard oil has better oiliness than mineral oils. Also, babbitts favour establishment of the 'absorbed' oil film improving oiliness. Oleic and stearic acids are used as *oiliness additives*. These additives have polar molecules. One end of the long chain of molecules has a strong affinity for the journal or bearing surface and sticks to it. These molecules orient in a particular fashion and form a cluster. The cluster of molecules prevents metal to metal contact. Vegetable or animal oils have more oiliness than mineral oils.

Extreme Pressure (EP) additives are used in applications, such as gears, where the lubricant is subjected to extremely high local pressures, at which oiliness additives are ineffective. These additives react with the surfaces of the journal and bearing and form a thin protective film under intense frictional heat at local spots. There are two types of EP additives—*active* and *mild*. Active additives include compounds of sulphur or chlorine. Sulphur reacts with steel surface and forms a tough film of iron sulphide. The melting point of this film is more than that of steel, which prevents local welding and formation of junctions. Mild additives include compounds of phosphorus and metallic soaps like lead naphthenate. EP additives are used for lubrication of all types of gears and cams.

16.19 SELECTION OF LUBRICANTS

Lubricating oils are commercially available under different trade names. Indian Oil Corporation[5] manufactures a wide range of lubricating oils and greases. These commercial lubricants contain a base oil and a group of additives, which are suitable for a given application. The properties of lubricating oils used for automotive applications are given in Table 16.6. There are five grades from SAE 10 to SAE 50. These base oils are blended with viscosity index improver, detergent additives and oxidation inhibitors. These oils are used for engine lubrication of petrol and diesel vehicles. They are also used for generators and pumping sets operating on diesel engines. There are two different classes of crankcase oil—Servo Engine Oil and Servo Super. The second is superior and costly. It is used for heavy duty internal combustion engines.

Table 16.6 *Properties of lubricating oils for automotive crankcase applications*

Properties	*Servo Engine Oil/Servo Supper*				
	10	*20*	*30*	*40*	*50*
1. SAE Grade	10 W	20	30	40	50
2. Kinematic viscosity (cSt at 100°C)	5 (min)	6–8	10–12	13–15	18–20
3. Viscosity index (min)	100	95	95	90	90
4. Flash point (°C) (min)	190	200	220	225	230
5. Pour point (°C) (max)	–27	–21	–6	–6	–6

2T oil is a popular variety of lubricating oil used on two-stroke engines in scooters, mopeds

[5] Servo Products, Indian Oil Corp. Ltd., Mumbai

and motorcycles. The advantages of 2T oil are as follows:

(i) It is self-mixing with petrol.

(ii) It keeps the piston, piston rings, cylinder, plugs and exhaust port clean.

(iii) It prevents corrosion of engine components.

(iv) It controls deposits in the combustion chamber and reduces pre-ignition.

It consists of SAE 30 as base oil, and detergent additives. The recommended fuel/oil ratio varies from 12:1 to 50:1 depending upon the engine. The properties of 2T oil are as follows:

1. Kinematic viscosity (cSt at 40°C)	27–33
2. Flash point (°C) (min)	66
3. Pour point (°C) (min)	–6

Commercial lubricating oils for gears consist of SAE 80, SAE 90 and SAE 140 as base oils and a mixture of extreme-pressure additives, oxidation inhibitors and oiliness additives. The properties of these oils are given in Table 16.7. The following are the advantages of these oils:

(i) They have excellent chemical stability even at high temperatures.

(ii) They can withstand extremely high local pressures in high-torque and low-speed conditions.

(iii) They protect gear assemblies against rust and corrosion.

They are used for the lubrication of all types of gears.

Table 16.7 *Properties of gear oils*

Properties	*Servo Gear HP / Gear Super*		
	80	*90*	*140*
1. SAE Grade	80	90	140
2. Kinematic viscosity (cSt at 100°C)	9–11	16.5–18.5	31–33
3. Viscosity index (min.)	85	85	80
4. Flash point (°C) (min.)	165	180	190
5. Pour point (°C) (max.)	–27	–9	0

16.20 GREASES

Grease is a semisolid substance, composed of mineral oil and soap. Sometimes additives are added to this mixture to achieve specific properties, such as chemical stability or oilness. The soap is present in the form of fibres, which form a matrix for the oil by the swelling mechanism. The type and amount of soap determines the texture and properties of the resulting grease. Grease is thixotropic, i.e., it undergoes a change in apparent viscosity with the amount of shearing. When the journal is stationary, the grease in the clearance space is quite rigid and immobile but when the journal starts rotating, the viscosity of the grease approaches to that of the base oil in the grease. Grease is normally recommended for inaccessible parts, where leakage of oil is objectionable. It is also used in applications where clearance is large due to rough machining.

Greases are classified on the basis of the soap employed. *Lime-base grease* consists of calcium soap in mineral oils of grades SAE 10 to SAE 40. It is insoluble in water. It is buttery and offers resistance to flow. However, it has a tendency to channel and separate out from the machine component by centrifugal action. It is, therefore, not suitable for ball bearings. It is used for chassis parts including suspension, and for steering systems of vehicles. It is also used for open or semi-enclosed gears and chains. *Soda-base grease* is produced from sodium soap. It has more resistance to decomposition at high temperatures and pressures. Soda-base grease is water-soluble and possesses a sponge-like structure. It does not have a tendency to channel. It is mainly used for the lubrication of automotive wheel bearings. Lithium soap grease has excellent resistance to oxidation, and is used for water pumps, wheel bearings and chassis fittings.

16.21 BEARING FAILURE—CAUSES AND REMEDIES

Fatigue failures are not common in journal bearings unlike ball bearings. The failures in journal bearings are mainly associated with insufficient lubricant, contamination of lubricant and faulty assembly. The principal types of bearing failure are as follows:

(i) Abrasive Wear Abrasive wear on the surface of the bearing is a common type of bearing failure. It is in the form of scratches in the direction of motion often with embedded particles. Abrasive wear occurs when the lubricating oil is contaminated with dust, foreign particles, rust or spatter. Proper enclosures for the bearing and the housing, cleanliness of lubricating oil and use of high viscosity oil are some of the remedies against this type of wear.

(ii) Wiping of Bearing Surface When the rotating journal touches the bearing, excessive rubbing occurs resulting in melting and smearing of the surface of the bearing. This type of failure is in the form of surface melting and flow of bearing material. The main causes for this type of wear are inadequate clearance, excessive transient load and insufficient oil supply. The remedy is to keep these factors under control.

(iii) Corrosion The corrosion of bearing surface is caused by the chemical attack of reactive agents that are present in the lubricating oil. These oxidation products corrode materials such as lead, copper, cadmium and zinc. Lead reacts rapidly with all oxidation agents. The remedy is to use oxidation inhibitors as additive in the lubricating oil.

(iv) Distortion Misalignment and incorrect type of fit are the major sources of difficulties in journal bearings. When the fit is too tight, bore distortion occurs. When foreign particles are trapped between the bearing and the housing during the assembly, local bore distortion occurs. Correct selection of the fit and proper assembly procedure are the remedies against this type of wear.

16.22 COMPARISON OF ROLLING AND SLIDING CONTACT BEARINGS

In this and the previous chapters, the characteristics of ball and roller bearings and hydrostatic and hydrodynamic bearings are discussed. The factors, which govern the selection between these two basic types of bearings, are load carrying capacity, frictional loss, space requirement, accuracy, noise and cost.

The load carrying capacity of a hydrodynamic bearing is linearly proportional to speed, as shown in Fig. 16.34. Any point below this curve, such as the point P_1, indicates that the life corresponding to this load–speed combination is infinity. When the load exceeds, such as the point P_2, the fluid film will break, resulting in metal to metal contact. In hydrostatic bearings, the load capacity is independent of speed. The rolling contact bearings have finite life for a given combination of load and speed. Hydrodynamic bearings are suitable for high load–high speed conditions, particularly from considerations of a long life. Rolling contact bearings are vulnerable to shock loads due to poor damping capacity. The balls and raceways are subjected to plastic deformation under shock loads or fluctuating loads leading to noise, heat and fatigue failure. On the other hand, hydrodynamic bearings are better suited for these conditions, which occur in connecting rod or crankshaft applications.

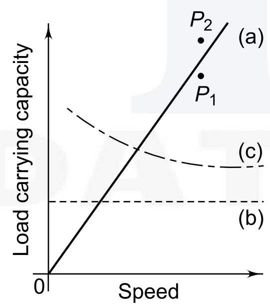


Fig. 16.34 *Load Characteristics of Bearings: (a) Hydrodynamic Bearing (b) Hydrostatic Bearing (c) Rolling Contact Bearing*

Rolling-contact bearings require a lower starting torque compared to hydrodynamic bearings. In hydrodynamic bearings, metal to metal contact occurs at the beginning, which results in higher starting friction. However, under running conditions, when full hydrodynamic film has developed, the power losses due to friction are lower than that of rolling contact bearings. Ball bearings are, therefore, suitable for applications where there are frequent starts. On the other hand, if there is comparatively light load at the start and if the load increases gradually with speed, hydrodynamic bearing is a better choice.

Rolling contact bearings require considerable radial space, while hydrodynamic bearings require more axial space. Hydrodynamic bearings require a lubricating system consisting of a pump, filter, sump, pipelines, etc., which requires considerable additional space. From space considerations, rolling contact bearings are better. For the precise location of the journal axis, rolling contact bearings are preferred. In case of rolling contact bearings, the axes of the journal and the bearing are collinear. In hydrodynamic bearings, the journal moves eccentrically with respect to the bearing and the eccentricity varies with load and speed.

Rolling contact bearings, due to metal to metal contact, generate more noise compared with hydrodynamic bearings. The cost of hydrodynamic bearing is much more than that of rolling contact bearing due to additional accessories, like pump, filter and pipelines. The maintenance cost of hydrodynamic bearing is also more. From cost considerations, rolling contact bearings are cheaper.

Short-Answer Questions

16.1 What are the four objectives of lubrication?
16.2 What is thick film lubrication?
16.3 What is a zero film bearing?
16.4 What is hydrodynamic lubrication?
16.5 What is hydrostatic lubrication?
16.6 Why is hydrodynamic journal bearing called 'self acting' bearing?
16.7 Why is hydrostatic bearing called 'externally pressurized' bearing?
16.8 Give two applications of hydrodynamic journal bearings.
16.9 Give two applications of hydrostatic bearings.
16.10 State any two advantages of hydrodynamic bearings over hydrostatic bearings.
16.11 State any two advantages of hydrostatic bearings over hydrodynamic bearings.
16.12 Give two examples of thin film bearings.
16.13 What is full journal bearing?
16.14 What is partial bearing?
16.15 Define viscosity.
16.16 State Newton's law of viscosity.
16.17 What are the units of absolute viscosity?
16.18 Define kinematic viscosity.
16.19 Why does viscosity decrease with increasing temperature?
16.20 What is viscosity index?
16.21 Write down Petroff's equation.
16.22 Write down the expression for Sommerfeld's number.
16.23 What is bearing characteristic number as applied to the journal bearing?
16.24 What is bearing modulus as applied to the journal bearing?
16.25 What is meant by 'square' bearing?
16.26 What are the advantages and disadvantages of long bearings over short bearings?
16.27 What are the advantages and disadvantages of circumferential oil-groove bearing over cylindrical oil-groove bearing?
16.28 Give two applications of circumferential oil-groove bearings.
16.29 Give two applications of cylindrical oil-groove bearings.
16.30 State any four desirable properties of a good bearing material.
16.31 Define conformability.
16.32 Define embeddability.
16.33 What are the advantages and disadvantages of babbitt as bearing material?
16.34 Where do you use sintered metal bearings?
16.35 State any four desirable properties of a good lubricant.
16.36 What is SAE?
16.37 Define 'additive' for mineral oil.
16.38 What is the purpose of additive?
16.39 What is 'doped' oil?
16.40 What are EP additives? Where do you use them?
16.41 What is 2T oil? What are its advantages? Where do you use 2T oil?
16.42 What is grease? State its applications.
16.43 Grease is 'thixotropic'. What does it mean?

Problems for Practice

16.1 The following data is given for a hydrostatic thrust bearing:
shaft speed = 720 rpm
shaft diameter = 400 mm
recess diameter = 250 mm
film thickness = 0.15 mm
viscosity of lubricant = 30 cP
specific gravity = 0.86
specific heat = 1.75 kJ/kg°C
supply pressure = 5 MPa
Calculate
(i) load carrying capacity of the bearing;
(ii) flow requirement;
(iii) pumping power loss;
(iv) frictional power loss; and
(v) temperature rise.
Assume that the total power loss in the bearing is converted into frictional heat.

[*(i) 407.32 kN (ii) 37.6 litres/min (iii) 3.13 kW (iv) 2.42 kW (v) 5.88°C*]

16.2 The developed view of a hydrostatic bearing is shown in Fig. 16.35(a). Consider the flow in the direction shown by arrows and neglect the flow in the other direction and over corners. The pressure distribution is linear as shown in Fig. 16.35(b). The thrust load is 500 kN and the film thickness is 0.2 mm. The viscosity of the lubricant is 500 cP. Calculate the supply pressure and flow requirement.

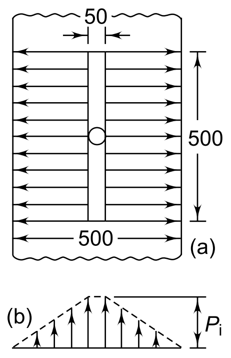


Fig. 16.35

[*(i) 3.64 MPa (ii) 1.29 litres/min*]

16.3 A hydrostatic thrust bearing consists of four pads as shown in Fig. 16.36(a). Neglecting the flow over corners, each pad can be approximated as a circular area of outer and inner diameters of 200 mm and 50 mm respectively, as shown in Fig. 16.36(b). The thrust load is 300 kN and the film thickness is 0.1 mm. The viscosity and specific gravity of the lubricating oil are 250 SUS and 0.88 respectively. Calculate the supply pressure and flow requirement.

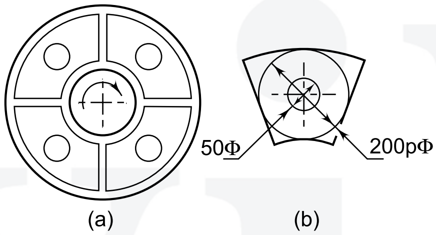


Fig. 16.36

[*(i) 7.06 MPa (ii) 13.4 litres/min*]

16.4 A hydrostatic spherical step bearing is shown in Fig. 16.37. Show that the load carrying capacity of the bearing is given by

$$W = \frac{\pi P_i R^2 (\cos\phi_2 - \cos\phi_1)}{\log_e \left[\dfrac{\tan\left(\dfrac{\phi_1}{2}\right)}{\tan\left(\dfrac{\phi_2}{2}\right)} \right]}$$

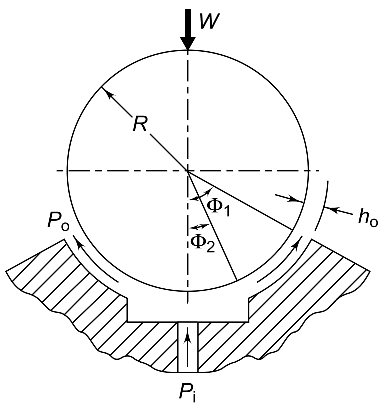


Fig. 16.37 *Spherical step bearing*

and the flow requirement is given by,

$$Q = \frac{\pi P_i h_o^3}{6\mu \log_e \left[\dfrac{\tan\left(\dfrac{\phi_1}{2}\right)}{\tan\left(\dfrac{\phi_2}{2}\right)} \right]}$$

16.5 A 360° hydrodynamic bearing operates under the following conditions:
radial load = 50 kN
journal diameter = 150 mm
bearing length = 150 mm
radial clearance = 0.15 mm
minimum film thickness = 0.03 mm
viscosity of lubricant = 8 cP
What is the minimum speed of operation for the journal to work under hydrodynamic conditions?

[*2973 rpm*]

16.6 The following data is given for a 360° hydrodynamic bearing:
journal diameter = 100 mm
bearing length = 100 mm
radial load = 50 kN
journal speed = 1440 rpm
radial clearance = 0.12 mm
viscosity of lubricant = 16 cP
Calculate
(i) minimum film thickness;
(ii) coefficient of friction; and
(iii) power lost in friction.

[*(i) 0.0087 mm (ii) 2.016 × 10^{-3} (iii) 0.76 kW*]

16.7 The following data is given for a full hydrodynamic bearing:
radial load = 25 kN
journal speed = 900 rpm
unit bearing pressure = 2.5 MPa
(l/d) ratio = 1
viscosity of lubricant = 20 cP
class of fit = H7e7
Calculate
(i) dimensions of the bearing ,
(ii) minimum film thickness, and
(iii) requirement of oil flow.

[*(i) 100 × 100 mm (ii) 0.0191 mm (iii) 1.057 litre/min*]

16.8 The following data is given for a 360° hydrodynamic bearing:
bearing diameter = 50.02 mm
journal diameter = 49.93 mm
bearing length = 50 mm
journal speed = 1440 rpm
radial load = 8 kN
viscosity of lubricant = 12 cP
The bearing is machined on the lathe from bronze casting, while the steel journal is hardened and ground. The surface roughness (cla) values for turning and grinding are 0.8 and 0.4 microns respectively. For thick film hydrodynamic lubrication, the minimum film thickness should be five times the sum of surface roughness values for the journal and bearing. Calculate
(i) the permissible minimum film thickness;
(ii) the actual film thickness under operating conditions; and
(iii) power lost in friction.

[*(i) 6 microns (ii) 6.07 microns (iii) 0.069 kW*]

The McGraw-Hill Companies

Spur Gears

Chapter 17

17.1 MECHANICAL DRIVES

Belt, chain and gear drives are often called '*mechanical*' drives. *A mechanical drive is defined as a mechanism, which is intended to transmit mechanical power over a certain distance, usually involving a change in speed and torque.* In general, the mechanical drive is required between the prime mover, such as electric motor and the part of the operating machine. A mechanical drive is used on account of the following reasons:

(i) The torque and speed of the machine are always different than that of electric motor or engine. Machines usually run at low speed and require high torque. For example, in case of overhead travelling crane, the motor runs at 1440 rpm while the speed of the rope drum is as low as 20 rpm.

(ii) In certain machines, variable speeds are required for the operation, whereas the prime mover runs at constant speed. For example, in case of lathe, the motor runs at constant speed, while different speeds are required for the spindle of the chuck to turn the jobs of different materials and with different feeds and depth of cut.

(iii) Standard electric motors are designed for uniform rotary motion. However, in some machines like shaper or planer, linear motions with varying velocities are required.

Mechanical drives are classified into two groups according to their principle of operation. The two broad groups are as follows:

(i) Mechanical drives that transmit power by means of friction, e.g., belt drive and rope drive

(ii) Mechanical drives that transmit power by means of engagement, e.g., chain drives and gear drives

The selection of a proper mechanical drive for a given application depends upon a number of factors such as centre distance, velocity ratio, shifting arrangement, maintenance considerations and cost. The guidelines for selection of suitable mechanical drive for the given application are as follows:

(i) Flat belts and roller chains are suitable for long centre distances. V-belts have comparatively short centre distances. Gear drives have the smallest centre distance between two shafts.

(ii) In flat belt drives, the belt slips over the pulley. Therefore, the driven pulley rotates at a speed which is less than that calculated by the ratio of diameters of the driving and driven pulleys. Due to slip, the velocity ratio is not constant. Therefore, flat belt drive is not recommended where constant speed is desirable. In case of chain drives, the velocity ratio is not constant during one revolution of the sprocket wheel due to '*polygonal*' effect.

Gear drives are preferred in applications which require constant speed.

(iii) In some applications, shifting mechanism is required to obtain different speeds such as headstock of lathe or automotive gearbox. Flat belts with relatively long centre distances can be shifted from tight to loose pulleys. Spur gears can be shifted on splined shaft. In case of V-belts or chain drives, it is not possible to use the shifting mechanism.

(iv) Maintenance of belt drives is relatively simple. It usually consists of periodic adjustment of centre distance in order to compensate the stretch of the belt. In chain and gear drives, lubrication is an important consideration in maintenance.

(v) Flat belt drive is the cheapest, V-belt and chain drives are comparatively costly, and gear drives are costliest.

17.2 GEAR DRIVES

Gears are defined as toothed wheels or multilobed cams, which transmit power and motion from one shaft to another by means of successive engagement of teeth. Gear drives offer the following advantages compared with chain or belt drives:

(i) It is a positive drive and the velocity ratio remains constant.

(ii) The centre distance between the shafts is relatively small, which results in compact construction.

(iii) It can transmit very large power, which is beyond the range of belt or chain drives.

(iv) It can transmit motion at very low velocity, which is not possible with the belt drives.

(v) The efficiency of gear drives is very high, even up to 99 per cent in case of spur gears.

(vi) A provision can be made in the gearbox for gear shifting, thus changing the velocity ratio over a wide range.

Gear drives are, however, costly and their maintenance cost is also higher. The manufacturing processes for gears are complicated and highly specialized. Gear drives require careful attention for lubrication and cleanliness. They also require precise alignment of the shafts.

17.3 CLASSIFICATION OF GEARS

Gears are broadly classified into four groups, viz., spur, helical, bevel and worm gears. A pair of spur gears is shown in Fig. 17.1. In case of *spur gears*, the teeth are cut parallel to the axis of the shaft. As the teeth are parallel to the axis of the shaft, spur gears are used only when the shafts are parallel. The profile of the gear tooth is in the shape of an involute curve and it remains identical along the entire width of the gear wheel. Spur gears impose radial loads on the shafts.

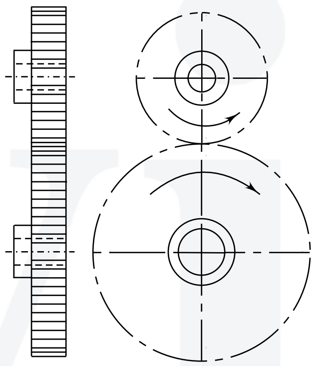


Fig. 17.1 *Spur Gears*

A pair of *helical gears* is shown in Fig. 17.2. The teeth of these gears are cut at an angle with the axis of the shaft. Helical gears have an involute profile similar to that of spur gears. However, this involute profile is in a plane, which is perpendicular to the tooth element. The magnitude of the helix angle

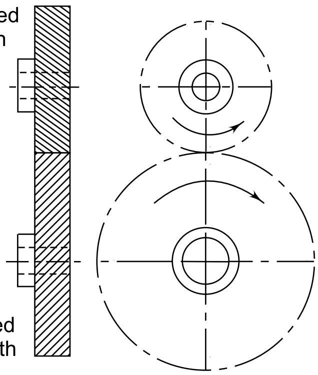


Fig. 17.2 *Helical Gears*

of pinion and gear is same; however, the hand of the helix is opposite. A right-hand pinion meshes with a left-hand gear and vice versa. Helical gears impose radial and thrust loads on shafts. There is a special type of helical gear, consisting of two helical gears with the opposite hand of helix, as shown in Fig. 17.3. It is called *herringbone gear*. The construction results in equal and opposite thrust reactions, balancing each other and imposing no thrust load on the shaft. Herringbone gears are used only for parallel shafts.

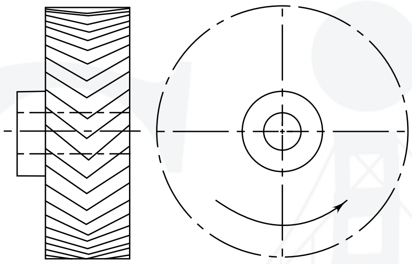


Fig. 17.3 *Herringbone Gear*

Bevel gears, as shown in Fig. 17.4, have the shape of a truncated cone. The size of the gear tooth, including the thickness and height, decreases towards the apex of the cone. Bevel gears are normally used for shafts, which are at right angles to each other. This, however, is not a rigid condition and the angle can be slightly more or less than 90 degrees. The tooth of the bevel gears can be cut straight or spiral.

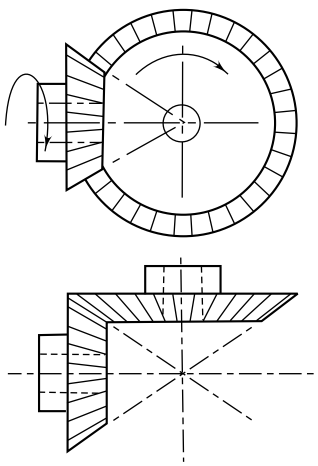


Fig. 17.4 *Bevel Gears*

Bevel gears impose radial and thrust loads on the shafts.

The *worm gears*, as shown in Fig. 17.5, consist of a worm and a worm wheel. The worm is in the form of a threaded screw, which meshes with the matching wheel. The threads on the worm can be single or multi-start and usually have a small lead. Worm gear drives are used for shafts, the axes of which do not intersect and are perpendicular to each other. The worm imposes high thrust load, while the worm wheel imposes high radial load on the shafts. Worm gear drives are characterized by high speed reduction ratio.

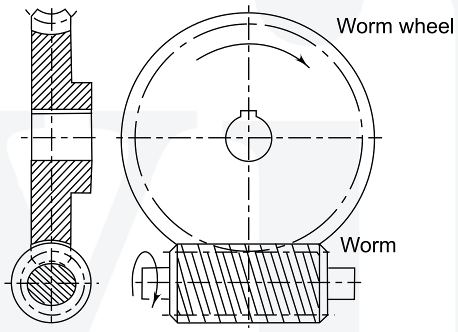


Fig. 17.5 *Warm Gears*

17.4 SELECTION OF TYPE OF GEARS

The first step in the design of the gear drive is the selection of a proper type of gear for a given application. The factors that are considered for deciding the type of gear are general layout of shafts, speed reduction, power to be transmitted, input speed and cost. Spur and helical gears are used when the shafts are parallel. When the shafts intersect at right angles, bevel gears are used. Worm gears are recommended when the axes of shafts are perpendicular and non-intersecting. When the axes of two shafts are neither perpendicular not intersecting, crossed helical gears are employed.

The speed reduction or velocity ratio for a single pair of spur or helical gears is normally taken as 6 : 1. On rare occasions, this can be raised to 10 : 1. When the velocity ratio increases, the size of the gear wheel increases. This results in increase in the size of the gearbox and the material cost increases.

For high speed reduction, two-stage or three-stage constructions are used. The normal velocity ratio for a pair of bevel gears is 1 : 1, which can be increased to 3: 1 under certain circumstances. For high speed reduction, worm gears offer the best choice. The velocity ratio in their case is 60 : 1, which can be increased to 100 : 1. They are widely used in material handling equipment due to this advantage.

Spur gears generate noise in high speed applications, due to sudden contact over the entire face width between two meshing teeth. In helical gears, the contact between the two meshing teeth begins with a point and gradually extends along the tooth, resulting in quiet operations. Helical gears are, therefore, preferred for high speed power transmission. From cost considerations, spur gears are the cheapest. They are not only easy to manufacture but there exist a number of methods to manufacture them. The manufacturing of helical, bevel and worm gears is a specialized and costly operation.

17.5 LAW OF GEARING

The fundamental law of gearing states *'The common normal to the tooth profile at the point of contact should always pass through a fixed point, called the pitch point, in order to obtain a constant velocity ratio'*. Referring to Fig. 17.6, O_1 and O_2 are centres of the two gears rotating with angular velocities ω_1 and ω_2 respectively. C is the point of contact between the teeth of the two gears and NN is the common normal at the point of contact.

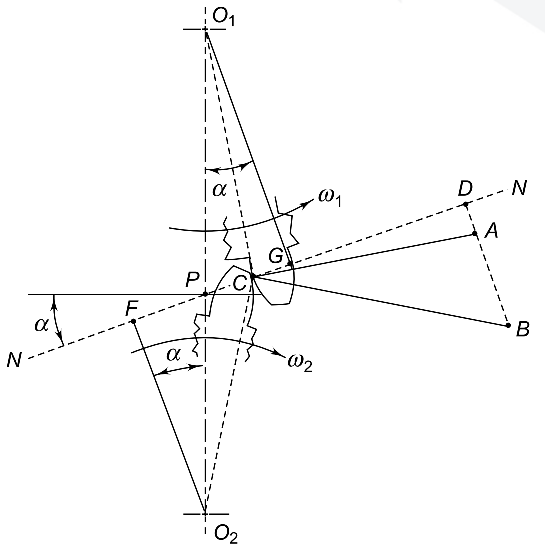


Fig. 17.6 *Law of Gearing*

$\overrightarrow{CA}$ is the velocity of the point C, when it is considered on the gear 1, while $\overrightarrow{CB}$ is the velocity of the point C, when it is considered on the gear 2. Also,

$$CA \perp O_1C \quad \text{and} \quad CB \perp O_2C$$

The projections of the two vectors $\overrightarrow{CA}$ and $\overrightarrow{CB}$, i.e., $\overrightarrow{CD}$, along the common normal NN must be equal, otherwise the teeth will not remain in contact and there will be a slip.

$$CA = \omega_1 \times O_1C$$

$$CB = \omega_2 \times O_2C$$

$$\frac{\omega_1}{\omega_2} = \frac{O_2C}{O_1C} \times \frac{CA}{CB} \quad \text{(a)}$$

Since ΔO_1CG and ΔCAD are similar, hence

$$\frac{O_1C}{CA} = \frac{O_1G}{CD} \quad \text{(b)}$$

Similarly, ΔO_2FC and ΔCDB are similar, and thus

$$\frac{O_2C}{CB} = \frac{O_2F}{CD} \quad \text{(c)}$$

From (b) and (c),

$$\frac{CA}{CB} = \frac{O_1C}{O_2C} \times \frac{O_2F}{O_1G} \quad \text{(d)}$$

From (a) and (d),

$$\frac{\omega_1}{\omega_2} = \frac{O_2F}{O_1G} \quad \text{(e)}$$

Similarly, ΔO_2FP and ΔO_1GP are similar, therefore,

$$\frac{O_2F}{O_1G} = \frac{O_2P}{O_1P} \quad \text{(f)}$$

From (e) and (f),

$$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} \quad \text{(g)}$$

Also,

$$O_1P + O_2P = O_1O_2 = \text{constant} \quad \text{(h)}$$

Therefore, for a constant velocity ratio (ω_1/ω_2), P should be a fixed point. This point P is called the pitch point.

It has been found that only involute and cycloidal curves satisfy the fundamental law of gearing. The meaning of these curves is as follows:

(i) An involute is a curve traced by a point on a line as the line rolls without slipping on a circle.

(ii) A cycloid is a curve traced by a point on the circumference of a generating circle as it rolls without slipping along the inside and outside of another circle. The cycloid profile consists of two curves, namely, epicycloid and hypocycloid. An *epicycloid* is a curve traced by a point on the circumference of a generating circle as it rolls without slipping on the outside of the pitch circle. A *hypocycloid* is a curve traced by a point on the circumference of a generating circle as it rolls without slipping on the inside of the pitch circle.

Cycloidal tooth offers the following advantages compared with involute tooth:

(i) In case of cycloidal gears, a convex flank on one tooth comes in contact with the concave flank of the mating tooth. This increases the contact area and also the wear strength. In involute gears, the contact is between two convex surfaces on mating teeth, resulting in smaller contact area and lower wear strength.

(ii) The phenomenon of interference does not occur at all in cycloidal gears.

However, cycloidal teeth are rarely used in practice due to the following disadvantages:

(i) Cycloidal tooth is made of two curves—hypocycloid curve below the pitch circle and epicycloid curve above the pitch circle. It is very difficult to manufacture an accurate profile consisting of two curves. The profile of an involute tooth is made of a single curve and only one cutter is necessary to manufacture one complete set of pinion and gear. This results in reduction in manufacturing cost.

(ii) In case of an involute profile, the common normal at the point of contact always passes through the pitch point P and maintains a constant inclination α with the common tangent to the two pitch circles. The angle α is called the *pressure angle*. Therefore, the pressure angle remains constant in involute tooth. In case of cycloidal tooth, the pressure angle varies. The pressure angle has maximum value at the beginning of engagement and reduces to zero when the point of contact coincides with the pitch point. It again increases to maximum value in the reverse direction.

It is due to these reasons that cycloidal curves have become obsolete. However, they are still in use in some of applications, such as spring driven watches and clocks, and some instruments. In these applications, their ability to provide satisfactory operation with very small number of teeth is used to advantage.

17.6 TERMINOLOGY OF SPUR GEARS

The terminology of gears includes a number of terms peculiar to gears and it forms the basis of *gear language*. The terminology applied to spur gears is illustrated in Figs 17.7 to 17.9. The notations used

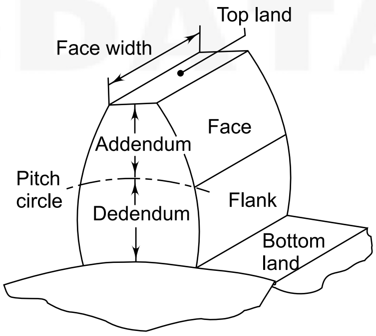


Fig. 17.7 *Gear Nomenclature*

in this chapter are recommended by the Bureau of Indian Standards[1, 2]. Gear terminology consists of following terms:

[1] IS 2458–1965: Glossary of terms for toothed gearing.
[2] IS 2467–1963: Notations for toothed gearing.

(i) Pinion A pinion is the smaller of the two mating gears.

(ii) Gear A gear is the larger of the two mating gears.

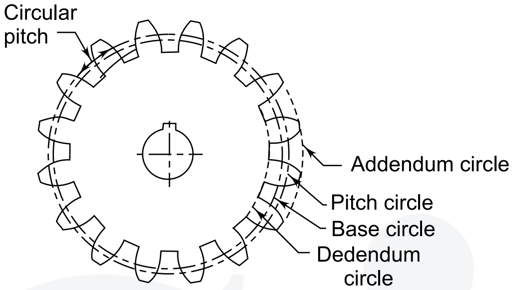


Fig. 17.8 *Gear Nomenclature*

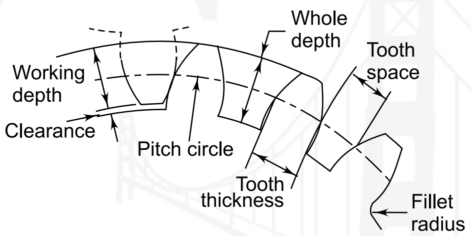


Fig. 17.9 *Terminology of Gear*

(iii) Velocity Ratio (i) Velocity ratio is the ratio of angular velocity of the driving gear to the angular velocity of the driven gear. It is also called the speed ratio.

(iv) Transmission Ratio (i′) The transmission ratio (i') is the ratio of the angular speed of the first driving gear to the angular speed of the last driven gear in a gear train.

(v) Pitch Surface The pitch surfaces of the gears are imaginary planes, cylinders or cones that roll together without slipping.

(vi) Pitch Circle The pitch circle is the curve of intersection of the pitch surface of revolution and the plane of rotation. It is an imaginary circle that rolls without slipping with the pitch circle of a mating gear. The pitch circles of a pair of mating gears are tangent to each other.

(vii) Pitch Circle Diameter The pitch circle diameter is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also called *pitch diameter*. The pitch circle diameter is denoted by d'.

(viii) Pitch Point The pitch point is a point on the line of centres of two gears at which two pitch circles of mating gears are tangent to each other.

(ix) Top Pand The top land is the surface of the top of the gear tooth.

(x) Bottom Pand The bottom land is the surface of the gear between the flanks of adjacent teeth.

(xi) Involute An involute is a curve traced by a point on a line as the line rolls without slipping on a circle.

(xii) Base Circle The base circle is an imaginary circle from which the involute curve of the tooth profile is generated. The base circles of two mating gears are tangent to the pressure line.

(xiii) Addendum Circle The addendum circle is an imaginary circle that borders the tops of gear teeth in the cross section.

(xiv) Addendum (h_a) The addendum (h_a) is the radial distance between the pitch and the addendum circles. Addendum indicates the height of the tooth above the pitch circle.

(xv) Dedendum Circle The dedendum circle is an imaginary circle that borders the bottom of spaces between teeth in the cross section. It is also called *root* circle.

(xvi) Dedendum (h_f) The dedendum (h_f) is the radial distance between pitch and the dedendum circles. The dedendum indicates the depth of the tooth below the pitch circle.

(xvii) Clearance (c) The clearance is the amount by which the dedendum of a given gear exceeds the addendum of its mating tooth.

(xviii) Face of Tooth The surface of the gear tooth between the pitch cylinder and the addendum cylinder is called the face of tooth.

(xix) Flank of Tooth The surface of the gear tooth between the pitch cylinder and the root cylinder is called flank of the tooth.

(xx) Face Width (b) Face width is the width of the tooth measured parallel to the axis.

(xxi) Fillet Radius The radius that connects the root circle to the profile of the tooth is called fillet radius.

(xxii) Circular Tooth Thickness The length of the arc on the pitch circle subtending a single gear tooth is called circular tooth thickness. Theoretically, circular tooth thickness is half of the circular pitch.

(xxiii) Tooth Space The width of the space between two adjacent teeth measured along the pitch circle is called the tooth space. Theoretically, tooth space is equal to circular tooth thickness or half the circular pitch.

(xxiv) Working Depth (h_k) The working depth is the depth of engagement of two gear teeth, that is, the sum of their addendums.

(xxv) Whole Depth (h) The whole depth is the total depth of the tooth space, that is, the sum of the addendum and dedendum. Whole depth is also equal to working depth plus clearance.

(xxvi) Centre Distance The centre distance is the distance between centres of pitch circles of mating gears. It is also the distance between centres of base circles of mating gears.

(xxvii) Pressure Angle The pressure angle is the angle which the line of action makes with the common tangent to the pitch circles. The pressure angle is also called the angle of obliquity. It is denoted by α.

(xxviii) Line of Action The line of action is the common tangent to the base circles of mating gears. The contact between the involute surfaces of mating teeth must be on this line to give a smooth operation. The force is transmitted from the driving gear to the driven gear on this line.

(xxix) Arc of Contact The arc of contact is the arc of the pitch circle through which a tooth moves from the beginning to the end of contact with mating tooth.

(xxx) Arc of Approach The arc of approach is the arc of the pitch circle through which a tooth moves from its beginning of contact until the point of contact arrives at the pitch point.

(xxxi) Arc of Recess The arc of recess is the arc of the pitch circle through which a tooth moves from the contact at the pitch point until the contact ends.

(xxxii) Contact Ratio (m_p) The number of pairs of teeth that are simultaneously engaged is called contact ratio. If there are two pairs of teeth in contact all the time, the contact ratio is 2. As the two gears rotate, smooth and continuous transfer of power from one pair of meshing teeth to the following pair is achieved when the contact of the first pair continues until the following pair has established contact. Some overlapping is essential for this purpose. Therefore, the contact ratio is always more than 1. Other things being, the greater the contact ratio, the smoother the action of gears. The contact ratio for smooth transfer of motion is usually taken as 1.2. In industrial gearboxes for power transmission, the contact ratio is usually more than 1.4 (1.6 to 1.7).

(xxxiii) Circular Pitch The circular pitch (p) is the distance measured along the pitch circle between two similar points on adjacent teeth. Therefore,

$$p = \frac{\pi d'}{z} \tag{17.1}$$

where z is the number of teeth.

(xxxiv) Diametral Pitch The diametral pitch (P) is the ratio of the number of teeth to the pitch circle diameter. Therefore,

$$P = \frac{z}{d'} \tag{17.2}$$

From Eqs (17.1) and (17.2),

$$P \times p = \pi \tag{17.3}$$

(xxxv) Module The module (m) is defined as the inverse of the diametral pitch. Therefore,

$$m = \frac{1}{P} = \frac{d'}{z}$$

or

$$d' = mz \tag{17.4}$$

The centre to centre distance between two gears having z_p and z_g teeth is given by

$$a = \frac{1}{2}(d'_p + d'_g) = \frac{1}{2}(mz_p + mz_g)$$

$$\therefore \quad a = \frac{m(z_p + z_g)}{2} \tag{17.5}$$

where,

a = centre to centre distance (mm)

z_p = number of teeth on pinion

z_g = number of teeth on gear

The gear ratio (i) that is, the ratio of the number of teeth on gear to that on pinion is given by,

$$i = \frac{n_p}{n_g} = \frac{z_g}{z_p} \tag{17.6}$$

where n_p = speed of pinion (rpm)

n_g = speed of gear (rpm)

There are a number of methods to manufacture gears. They include casting, blanking and machining. However, power transmitting gears are made of steel and made by the following methods:

(i) Milling
(ii) Rack generation
(iii) Hobbing
(iv) Fellow gear shaper method

The hobbing process accounts for the manufacture of a major quantity of gears that are used for power transmission.

17.7 STANDARD SYSTEMS OF GEAR TOOTH

All standard systems prescribe the involute profile for gear tooth. The reasons are as follows:

(i) The involute profile satisfies the fundamental law of gearing at any centre distance.
(ii) All involute gears of a given module and pressure angle are completely interchangeable.
(iii) All involute gears of a given module and pressure angle can be machined from one single tool.
(iv) The basic rack of an involute profile has straight sides. It is comparatively easy to machine straight sides. Further, straight sides can be more accurately machined compared with a curved surface.
(v) A slight change in the centre distance, which might be caused by incorrect mounting, has no effect upon the shape of the involute. In addition, the pitch point is still fixed and the law of gearing is satisfied. Therefore, the velocity ratio remains constant.

There are three standard systems for the shape of gear teeth. They are as follows:

(i) 14.5° full depth involute system
(ii) 20° full depth involute systems
(iii) 20° stub involute system

As the number of teeth on the gear is increased, the involute outline becomes straighter and straighter. When the number of teeth is infinity or when the pitch circle radius approaches infinity, the gear becomes a rack with straight-sided teeth. This rack is called the '*basic*' rack, which is standardized in each system of gearing. The basic racks for three standard systems are shown in Fig. 17.10. The shapes of tooth profile for these systems are illustrated in Fig. 17.11. The standard proportions for these systems are given in Table 17.1.

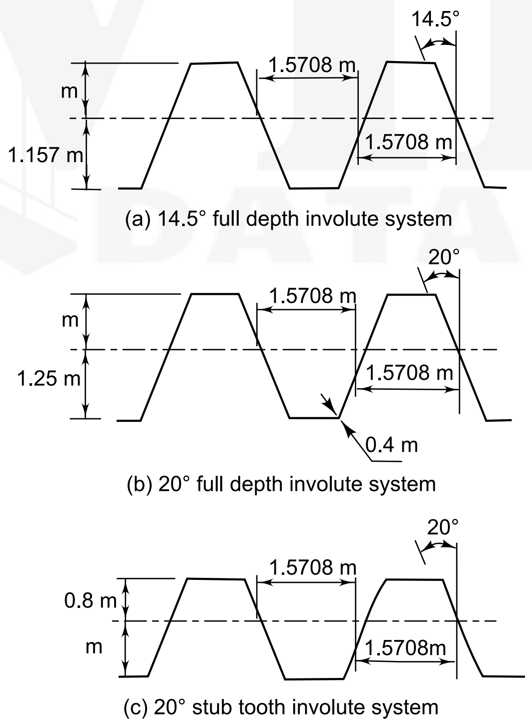


Fig. 17.10 *Basic racks for Standard Gear Systems*

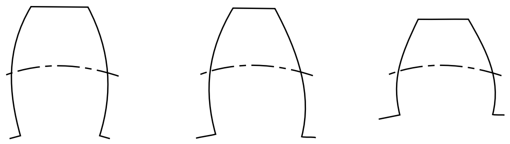


Fig. 17.11 *Standard Tooth Profiles*

Table 17.1 *Proportions of standard involute teeth (in terms of module m)*

	14.5° full depth system	*20° full depth system*	*20° stub system*
Pressure angle	14.5°	20°	20°
Addendum	m	m	0.8 m
Dedendum	1.157 m	1.25 m	m
Clearance	0.157 m	0.25 m	0.2 m
Working depth	2 m	2 m	1.6 m
Whole depth	2.157 m	2.25 m	1.8 m
Tooth thickness	1.5708 m	1.5708 m	1.5708 m

(i) 14.5° Full Depth Involute system The basic rack for this system is composed of straight sides except for the fillet arcs. In this system, interference occurs when the number of teeth on the pinion is less than 23. This system is satisfactory when the number of teeth on the gears is large. If the number of teeth is small and if the gears are made by generating process, undercutting is unavoidable.

(ii) 20° Full Depth Involute System The basic rack for this system is also composed of straight sides except for the fillet arcs. In this system, interference occurs when the number of teeth on the pinion is less than 17. The 20° pressure angle system with full depth involute teeth is widely used in practice. It is also recommended by the Bureau of Indian Standards and adopted in this chapter.

Increasing pressure angle improves the tooth strength but shortens the duration of contact. Decreasing pressure angle requires more number of teeth on the pinion to avoid undercutting. The 20° pressure angle is a good compromise for most of the power transmission as well as precision gearboxes.

The 20° pressure angle system has the following advantages over the 14.5° pressure angle system:

(a) It reduces the risk of undercutting.
(b) It reduces interference.
(c) Due to the increased pressure angle, the tooth becomes slightly broader at the root. This makes the tooth stronger and increases the load carrying capacity.
(d) It has greater length of contact.

The main advantage of the 14.5° pressure angle system is its quietness of operation.

(iii) 20° Stub Involute System The gears in this system have shorter addendum and shorter dedendum. The interfering portion of the tooth, that is, a part of the addendum, is thus removed. Therefore, these teeth have still smaller interference. This also, reduces the undercutting. In this system, the minimum number of teeth on the pinion, to avoid interference, is 14. Since the pinion is small, the drive becomes more compact. Stub teeth are stronger than full depth teeth because of the smaller moment arm of the bending force. Therefore, the stub system transmits very high load. Stub teeth results in lower production cost, as less metal must be cut away. The main drawback of this system is that the contact ratio is reduced due to short addendum. Due to insufficient overlap, vibrations are likely to occur.

The module specifies the size of the gear tooth. Figure 17.12 shows the actual sizes of gear teeth with four different modules. It is observed that as the module increases, the size of gear tooth also increases. It can be said that module is the '*index*' of the size of gear tooth. The standard values of module[3] are given in Table 17.2. The module given

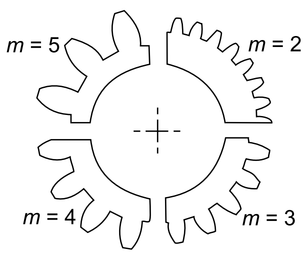


Fig. 17.12 *Size of Gear Tooth for Various Modules*

[3] IS 2535–1963: Basic rack and module for cylindrical gears.

under Choice-1 is always preferred. If that is not possible under certain circumstances, the module under Choice-2 can be selected.

Table 17.2 *Recommended series of module (mm)*

Choice-1 Preferred	1.0	1.25	1.5	2.0	2.5	3.0	4.0
	5.0	6.0	8.0	10	12	16	20
Choice-2	1.125	1.375	1.75	2.25	2.75	3.5	4.5
	5.5	7	9	11	14	18	

The standard proportions of the gear tooth in terms of module *m*, for 20° full depth system are rewritten here.

addendum (h_a) = (m)
dedendum (h_f) = (1.25 m)
clearance (c) = (0.25 m)
working depth (h_k) = (2 m)
whole depth (h) = (2.25 m)
tooth thickness (s) = (1.5708 m)
tooth space = (1.5708 m)
fillet radius = (0.4 m)

One of the methods of strengthening the gear tooth is '*crowning*'. During operation, there is uneven distribution of pressure along the face width of the tooth due to the following reasons:

(i) Inaccuracies of tooth profile caused by machining errors and distortion during heat treatment
(ii) Errors in assembly
(iii) Elastic deflection of shaft due to gear tooth forces and bearing reactions

This results in shifting the maximum pressure to the end of the tooth along the face width. This load can be shifted towards the middle of the face width by crowning the tooth as illustrated in Fig. 17.13.

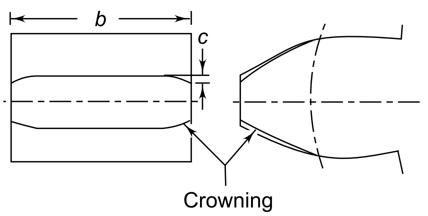


Fig. 17.13 *Crowning of Gear Tooth*

In the crowning process, the ends of the tooth are made slightly thinner by an amount *c*. The crowning is done by shaving cutters. The crown *c* is very small and depends upon the elastic deflection of teeth in operation. In practice, the crown *c* is usually taken as (0.0003 *b*) to (0.0005 *b*).

In the design of gears, the number of teeth is decided from the speed ratio. The module is calculated from strength and wear equations. Knowing these two parameters, the other dimensions can be calculated by the above proportions. The conventional representation of a pair of spur gears is illustrated in Fig. 17.14. The pitch circles of the pinion and gear are important in drawing, because pitch surfaces roll together without slip during engagement. The pitch circles are shown by centrelines. In addition to pitch circles, the addendum circles are drawn with full lines.

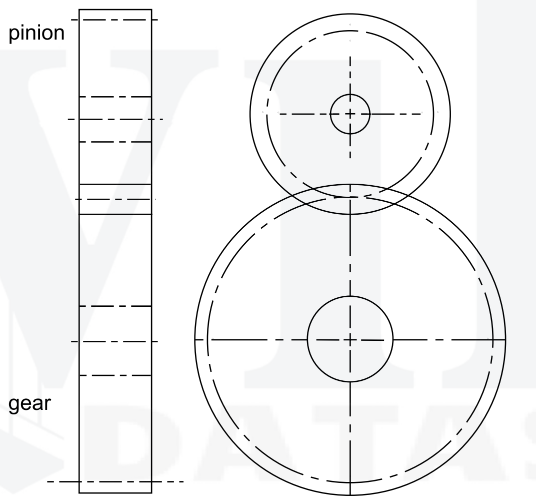


Fig. 17.14 *Conventional Representation*

Example 17.1 *A pair of spur gears consists of a 20 teeth pinion meshing with a 120 teeth gear. The module is 4 mm. Calculate*

(i) the centre distance;
(ii) the pitch circle diameters of the pinion and the gear;
(iii) the addendum and dedendum;
(iv) the tooth thickness;
(v) the bottom clearance; and
(vi) the gear ratio.

Solution

Given $z_p = 20 \quad z_g = 120 \quad m = 4$ mm

Step I *Centre distance*

From Eq. (17.5),

$$a = \frac{m(z_p + z_g)}{2} = \frac{4(20+120)}{2} = 280 \text{ mm} \qquad \text{(i)}$$

Step II *Pitch circle diameters of pinion and gear*

$$d'_p = m\, z_p = 4(20) = 80 \text{ mm}$$

$$d'_g = m\, z_g = 4(120) = 480 \text{ mm} \qquad \text{(ii)}$$

Step III *Addendum and dedendum*

$$\text{addendum } (h_a) = \text{m} = 4 \text{ mm}$$

$$\text{dedendum } (h_f) = 1.25 \text{ m} = 1.25(4) = 5 \text{ mm} \qquad \text{(iii)}$$

Step IV *Tooth thickness*

$$\text{tooth thickness} = 1.5708 \text{ m} = 1.5708(4) = 6.2832 \text{ mm} \qquad \text{(iv)}$$

Step V *Bottom clearance*

$$\text{clearance } (c) = 0.25 \text{ m} = 0.25(4) = 1 \text{ mm} \qquad \text{(v)}$$

Step VI *Gear ratio*

$$i = \frac{z_g}{z_p} = \frac{120}{20} = 6 \qquad \text{(vi)}$$

17.8 GEAR TRAINS

A gear train consists of two or more gears transmitting power from the driving shaft to the driven shaft. The gear trains are classified into the following categories:

(i) Simple gear train
(ii) Compound gear train
(iii) Reverted gear train
(iv) Epicyclic gear train

Simple gear trains are illustrated in Fig. 17.15. A simple gear train is one in which each shaft carries only one gear. In this type of train, the velocity ratio is equal to the number of teeth on the last driven gear to the number of teeth on the first driving gear. For example, the velocity ratio for the gear train illustrated in Fig. 17.15(c), is given by,

$$\frac{n_1}{n_4} = \frac{z_4}{z_1}$$

The gears other than driving and driven gears are called *idler gears*. The functions of idler gear are as follows:

(i) Idler gears fill the space between the driving and driven gears.
(ii) Idler gears change the direction of rotation of the last driven shaft relative to the first driving shaft.

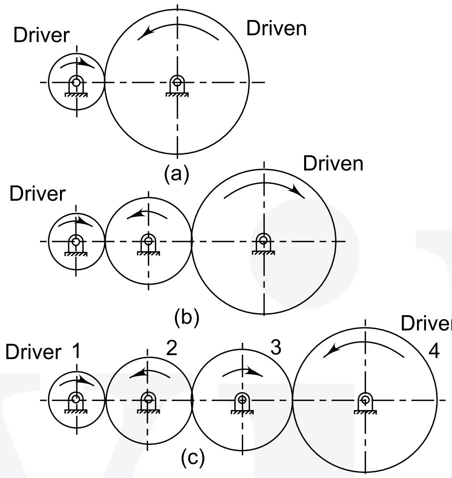


Fig. 17.15 *Simple Gear Trains*

The rules regarding direction of rotation are as follows:

(i) If an odd number of idler gears is used, the first and last shafts rotate in the same direction.
(ii) If even (or zero) number of idler gears is used, the first and last shafts rotate in the opposite direction.

The main drawback of simple gear train is its large overall dimensions and weight. A compound gear train is one in which at least one shaft carries two gears. A compound gear train is illustrated in Fig. 17.16. In this figure, the intermediate shaft has two gears, one meshing with the gear on the driving shaft and the other meshing with the gear on the driven shaft. The angular velocity of two gears mounted on the intermediate shaft is the same. The velocity reduction is done in two stages. The velocity ratio is given by,

$$\frac{n_1}{n_4} = \left(\frac{z_2}{z_1}\right)\left(\frac{z_4}{z_3}\right)$$

Compound gear train is compact in construction compared with simple gear train.

When the number of teeth on various gears in compound gear train are selected in such a way that the centre distance between gears 1 and 2 is equal to the centre distance between gears 3 and 4 then the driving and driven shafts can be located on the same centre line. This type of arrangement is called '*reverted*' gear train, which is illustrated in Fig. 17.17. In this case,

$$m_1\,(z_1 + z_2) = m_2\,(z_3 + z_4)$$

where,

m_1 = module for gears 1 and 2

m_2 = module for gears 3 and 4

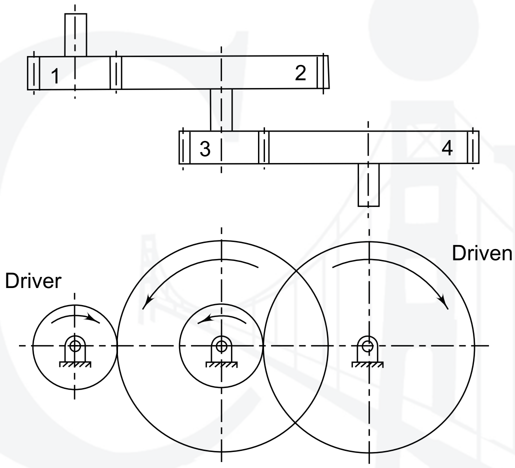


Fig. 17.16 *Compound Gear Train*

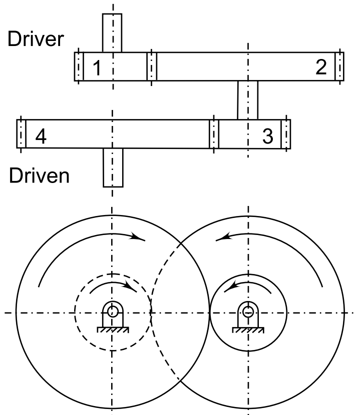


Fig. 17.17 *Reverted Gear Train*

Reverted gear train is very useful in clocks and instruments where it is desirable to have two pointers on concentric shafts moving with specific velocity ratio. Reverted gear train is the most compact gearbox.

An epicyclic gear train is illustrated in Fig. 17.18. It is a gear train in which one gear is fixed and the meshing gear has a motion composed of two parts, namely, a rotation about its own axis and a rotation about the axis of the fixed gear. This type of train is also called a '*planetary*' gear train. The fixed gear is called the *sun gear,* while the revolving gear is called the *planet gear*. In the arrangement shown in the figure, the sun gear is the driving gear and the crank is connected to the driven shaft. The crank is also called the *planet carrier*. In some arrangements, there are three planet gears and a fixed ring gear. The epicyclic gear train has compact construction.

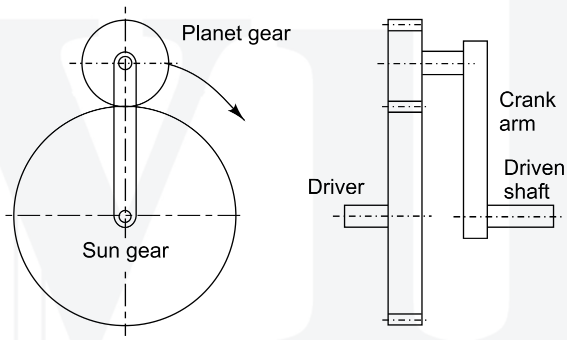


Fig. 17.18 *Epicyclic Gear Train*

17.9 INTERFERENCE AND UNDERCUTTING

A gear tooth has involute profile only outside the base circle. In fact, the involute profile begins at the base circle. In some cases, the dedendum is so large that it extends below this base circle. In such situations, the portion of the tooth below the base circle is not involute. The tip of the tooth on the mating gear, which is involute, interferes with this non-involute portion of the dedendum. This phenomenon of tooth profiles overlapping and cutting into each other is called '*interference*'. In this case, the tip of the tooth overlaps and digs into the root section of its mating gear. Interference is non-conjugate action and results in excessive wear, vibrations and jamming.

When the gears are generated by involute rack cutters, this interference is automatically eliminated because the cutting tool removes the interfering portion of the flank. This is called '*undercutting*'. Undercutting solves the problem of interference. However, an undercut tooth is considerably weaker. Undercutting not only weakens the tooth, but also removes a small involute portion adjacent to the base circle. This loss of involute profile may cause a serious reduction in the length of the contact.

Interference is the main disadvantage of involute gears. It is maximum when the smallest pinion is in mesh with the largest gear. The following methods can eliminate interference:

(i) Increase the Number of Teeth on the Pinion Increasing the number of teeth increases the size of the gearbox and also increases the pitch line velocity. This is not desirable. The minimum number of teeth to avoid interference and undercutting is as follows:

For 14.5° full depth system	32 teeth
For 20° full depth system	17 teeth
For 20° stub system	14 teeth

In case of generated gears, the number of teeth can be further reduced due to undercutting.

(ii) Increase Pressure Angle This results in smaller base circle so that more portion of the tooth profile becomes involute.

(iii) Use Long and Short Addendum Gearing In this method, the addendum of the pinion is made longer than the standard addendum. Also, the addendum of the mating gear is made shorter than the standard addendum. However, this results in non-standard and non-interchangeable gears.

The best course to avoid interference is to avoid the theoretical conditions that result in overlapping profile of mating teeth, instead of undercutting.

17.10 BACKLASH

Backlash is defined as the amount by which the width of tooth space exceeds the thickness of the engaging tooth measured along the pitch circle. It is illustrated in Fig. 17.19. In general, backlash is the play between mating teeth and it occurs only when teeth are in mesh. The objectives for providing backlash are as follows:

(i) Backlash prevents the mating teeth from jamming together. The mating teeth do not make contact on both sides simultaneously. This makes the teeth roll together freely and smoothly.

(ii) Backlash compensates for machining errors.

(iii) Backlash compensates for thermal expansion of teeth.

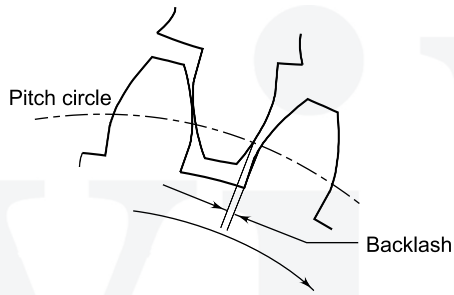


Fig. 17.19 *Backlash*

There are two methods to provide backlash. They are as follows:

(i) The teeth of the gear are cut slightly thinner. This is obtained by setting the cutting tool deeper into the blank resulting in thinner tooth and wider space.

(ii) The centre distance between mating gears is slightly increased.

The magnitude of recommended backlash depends upon the diametral pitch or module and the centre distance. The magnitude of backlash is very small for gear trains used in precision equipment and instruments.

The backlash and variation in centre distance have no effect on tooth action or velocity ratio.

17.11 FORCE ANALYSIS

In gears, power is transmitted by means of a force exerted by the tooth of the driving gear on the meshing tooth of the driven gear. Figure 17.20 shows the tooth of the driving pinion exerting a force P_N on the tooth of the driven gear. According to the fundamental law of gearing, this resultant force P_N

always acts along the pressure line. The resultant force P_N can be resolved into two components—tangential component P_t and radial component P_r at the pitch point as shown in Fig. 17.21. The tangential component P_t is a useful load because it determines the magnitude of the torque and consequently the power, which is transmitted. The radial component

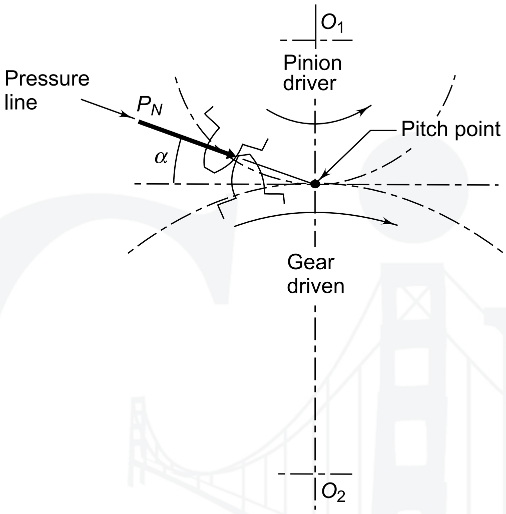


Fig. 17.20 *Gear Tooth Force*

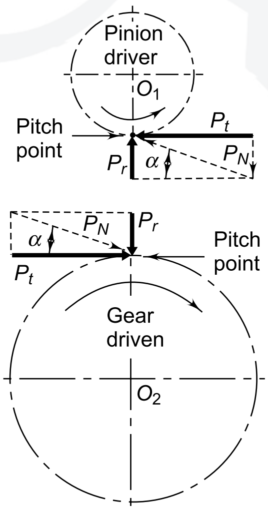


Fig. 17.21 *Components of Tooth Force*

P_r is a separating force, which is always directed towards the centre of the gear. The torque transmitted by the gears is given by,

$$M_t = \frac{60 \times 10^6 \,(\text{kW})}{2\pi n} \qquad (17.7)$$

where,

M_t = torque transmitted by gears (N-mm)

kW = power transmitted by gears (kW)

n = speed of rotation (rpm)

The tangential component P_t acts at the pitch circle radius. Therefore,

$$P_t \times \left(\frac{d'}{2}\right) = M_t$$

or

$$P_t = \frac{2M_t}{d'} \qquad (17.8)$$

From Fig. 17.21,

$$P_r = P_t \tan \alpha \qquad (17.9)$$

The resultant force P_N is given by

$$P_N = \frac{P_t}{\cos \alpha} \qquad (17.10)$$

The above analysis of the gear tooth force is based on the following assumptions:

(i) As the point of contact moves, the magnitude of the resultant force P_N changes. This effect is neglected in the above analysis.

(ii) It is assumed that only one pair of teeth takes the entire load. At times there are two pairs, which are simultaneously in contact and share the load. This aspect is neglected in the analysis.

(iii) The analysis is valid under static conditions, i.e., when the gears are running at very low velocities. In practice, there is dynamic force in addition to force due to power transmission. The effect of this dynamic force is neglected in the analysis.

In examples of gear tooth forces, it is always required to find out the magnitude and direction of two components. The magnitudes are determined by using the following three equations:

$$M_t = \frac{60 \times 10^6 \,(\text{kW})}{2\pi n_p}$$

$$P_t = \frac{2M_t}{d'_p}$$

$$P_r = P_t \tan \alpha$$

where the suffix p is used for the pinion.

The directions of two components P_t and P_r are decided by constructing the free-body diagram as shown in Fig. 17.22.

Refer to Fig.17.22(a):

(i) It is assumed that the pinion is the driving element while gear is the driven element.

(ii) It is assumed that the pinion rotates in anti-clockwise direction. Therefore, the gear will rotate in clockwise direction.

(iii) In running condition, the point 2 on the pinion and the point 1 on gear are in contact with each other.

Refer to Fig.17.22(b):

(i) The gear G is the driven element. It is made to rotate in clockwise direction. Therefore, at point 1 on the gear G, the tangential component P_t will act towards the left.

(ii) There will be equal and opposite reaction at point 2 on the pinion P. It is observed that the direction of tangential component P_t on the driving element, that is, pinion is opposite to the direction of rotation.

Refer to Fig.17.22(c):

(i) The radial component acts towards the centre of the respective gear. For pinion P, the radial component P_r acts at point 2 towards the centre O_2.

(ii) For the gear G, the radial component P_r acts at point 1 towards the centre O_1.

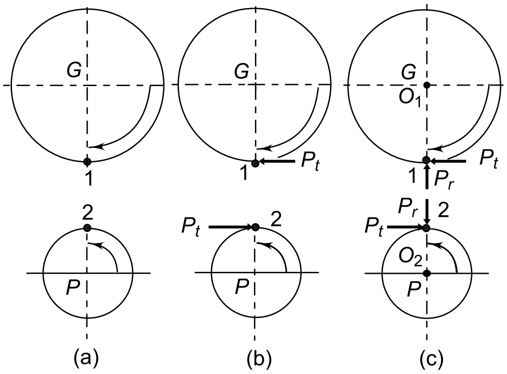


Fig. 17.22 *Free-body Diagram of Forces*

Example 17.2 *The pitch circles of a train of spur gears are shown in Fig. 17.23. Gear A receives 3.5 kW of power at 700 rpm through its shaft and rotates in the clockwise direction. Gear B is the idler gear while the gear C is the driven gear. The number of teeth on gears A, B and C are 30, 60 and 40 respectively, while the module is 5 mm. Calculate*

(i) the torque on each gear shaft; and

(ii) the components of gear tooth forces.

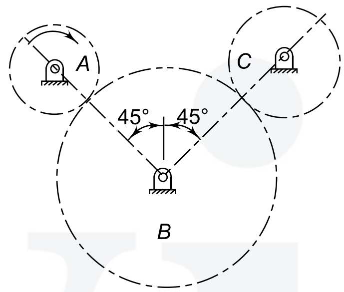


Fig. 17.23

Draw a free-body diagram of forces and determine the reaction on the idler gear shaft. Assume 20° involute system for the gears.

Solution

Given kW = 3.5 $n = 700$ rpm $z_A = 30$ $z_B = 60$ $z_C = 40$ $m = 5$ mm $\alpha = 20°$

Step I *Torque acting on shafts A, B and C*

$$d'_A = mz_A = 5(30) = 150 \text{ mm}$$

$$d'_B = mz_B = 5\,(60) = 300 \text{ mm}$$

$$d'_c = mz_c = 5(40) = 200 \text{ mm}$$

$$(M_t)_A = \frac{60 \times 10^6 (\text{kW})}{2\pi n_A} = \frac{60 \times 10^6 (3.5)}{2\pi(700)}$$

$$= 47\,746.48 \text{ N-mm}$$

Gear B is the idler gear and does not transmit any torque to its shaft. Therefore,

$$(M_t)_B = 0$$

Since the same power is transmitted from the gear A to the gear C,

$$(M_t)_A \times n_A = (M_t)_C \times n_C$$

$$(M_t)_C = (M_t)_A \times \left(\frac{n_A}{n_C}\right) = (47\,746.48) \times \left(\frac{40}{30}\right)$$

$$= 63\,661.98 \text{ N-mm} \qquad \text{(i)}$$

Step II *Components of gear tooth forces*

The components of the gear tooth force between gears A and B are given by,

$$(P_t)_{AB} = \frac{2(M_t)_A}{d'_A} = \frac{2(47\,746.48)}{150} = 636.62 \text{ N}$$

$$(P_r)_{AB} = (P_t)_{AB} \tan\alpha = 636.62 \tan(20)$$

$$= 231.71 \text{ N} \qquad \text{(ii)}$$

Since the gear B is the idler, whatever torque it receives from the gear A is transmitted to the gear C. Therefore,

$$(P_t)_{AB} \times \frac{d'_B}{2} = (P_t)_{BC} \times \frac{d'_B}{2}$$

or, $$(P_t)_{AB} = (P_t)_{BC} = P_t$$

The tangential component between gears B and C must be equal to the tangential component between gears A and B. Since the tangential components are equal, the radial components ($P_t \tan\alpha$) must be equal.

Step III *Free-body diagram of forces*

The free-body diagram of forces is shown in Fig. 17.24. Gear A is rotating in the clockwise direction. It is a driving gear and the direction of tangential component is opposite to that of rotation. Therefore, the tangential component at the point-1 on the gear A will act towards the upper right-hand corner of the page. Since the action and reaction are equal and opposite, the tangential component at the point-2 on the gear B will act towards the lower left-hand corner of the page. Between gears B and C, B is the driving gear and C is the driven gear. Gear C is rotating in the clockwise direction. The direction of tangential component for driven gear is same as that of rotation. Therefore, at the point-4 on the gear C, the tangential component will act towards the upper left-hand corner of the page. Since the action and reaction are equal and opposite, the tangential component at point-3 on the gear B will act towards lower right-hand corner of the page. The radial components at points 1, 2, 3 and 4 will act towards the centres of respective gears.

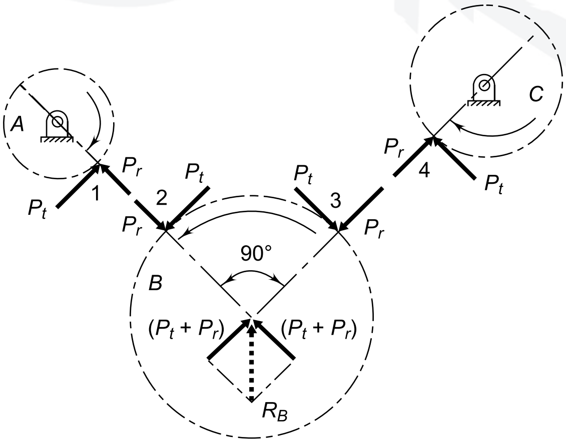


Fig. 17.24 *Free-body Diagram of Forces*

Step IV *Reaction on idler gear shaft*

Consider the equilibrium of forces acting on the gear B, in two planes inclined at 45° to the vertical. The forces are acting at points 2 and 3 and their reactions at the shaft. Each component of the reaction R_B on these planes is equal to $(P_t + P_r)$. Therefore, the reaction R_B on the idler gear shaft is given by,

$$R_B = \sqrt{(P_t + P_r)^2 + (P_t + P_r)^2}$$

$$= \sqrt{2(636.62 + 231.71)^2} = 1228 \text{ N}$$

Example 17.3 *A planetary gear train is shown in Fig. 17.25. The sun gear A rotates in a clockwise direction and transmits 5 kW of power at 1440 rpm to the gear train. The number of teeth on the sun gear A, the planet gear B and the fixed ring gear C are 30, 60 and 150 respectively. The module is 4 mm and the pressure angle is 20°. Draw a free-body diagram of forces and calculate the torque that the arm D can deliver to its output shaft.*

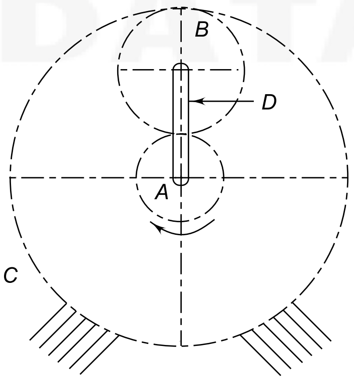


Fig. 17.25 *Planetary Gear Train*

Solution

Given kW = 5 $n = 1440$ rpm $z_A = 30$ $z_B = 60$ $z_C = 150$ $m = 4$ mm $\alpha = 20°$

Step I *Free-body diagram of forces*

$$d'_A = mz_A = 4(30) = 120 \text{ mm}$$

$$d'_B = mz_B = 4(60) = 240 \text{ mm}$$

$$d'_C = mz_C = 4(150) = 600 \text{ mm}$$

The length L_D of the arm D is given by,

$$L_D = \frac{d'_A + d'_B}{2} = \frac{120 + 240}{2} = 180 \text{ mm}$$

The tangential component of the gear tooth force on the sun gear A is calculated in the following way.

$$(M_t)_A = \frac{60 \times 10^6 (\text{kW})}{2\pi n_A} = \frac{60 \times 10^6 (5)}{2\pi(1440)}$$

$$= 33\,157.28 \text{ N-mm}$$

$$P_t = \frac{2(M_t)_A}{d'_A} = \frac{2(33\,157.28)}{120} = 552.62 \text{ N}$$

The free-body diagram of forces is shown in Fig. 17.26. Gear A is rotating in the clockwise direction. It is a driving gear and the direction of the tangential component is opposite to that of rotation. Therefore, the tangential component at point-1 on the gear A will act towards the left side of the page.

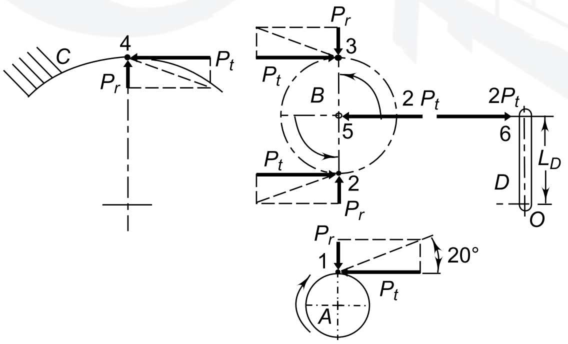


Fig. 17.26

Since the action and reaction are equal and opposite, the tangential component at point-2 on the gear B will act towards the right side of the page. Gear C is fixed in the housing and will resist the motion of the gear B. Therefore, in relative terms, the gear B is the driving gear in relation to the gear C. The direction of tangential component for the driving gear is opposite to that of rotation. Therefore, at point-3 on the gear B, the tangential component will act towards the right-hand side of the page. The radial components at points 1, 2 and 3 will act towards the centres of respective gears.

Step II *Torque that the arm D can deliver to its output shaft*

Considering the equilibrium of horizontal forces acting on the gear B, the magnitude of reaction at point 5 will be ($2P_t$) and it will act towards the left side of the page. Since the action and reaction are equal and opposite, the tangential force at point-6 on the arm D will be ($2P_t$) and it will act towards the right side of the page.

The torque about the axis O is given by,

$$\text{Torque} = 2P_t(L_D) = 2(552.62)(180)$$

$$= 198\,943 \text{ N-mm}$$

or Torque = 198.94 N-m

Example 17.4 *The gearbox for the rotating drum of a concrete mixer is shown in Fig.17.27. The mixing drum receives 5 kW of power and rotates at 250 rpm. Two pins are rigidly fixed to the drum and each carries an identical planetary spur gear F. The spur gears E and C are integral with the shaft and rotate at the same speed. The spur gear A is the fixed ring gear. The number of teeth on gears A, B, C and E are 65, 20, 80 and 35 respectively. The module is 5 mm for all gears. The pressure angle is 20°. Assume that each planetary gear shares an equal part of load and neglect frictional losses. Calculate:*

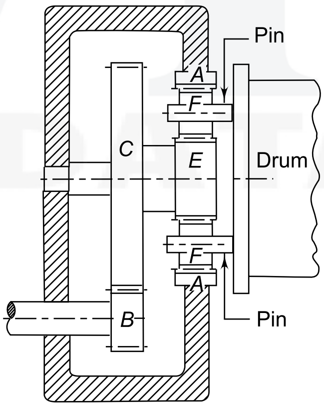


Fig. 17.27

(i) Components of tooth force between gears E and F.

(ii) Components of tooth force between gears B and C.

Solution

Given kW = 5 $n = 250$ rpm $\alpha = 20°$

$z_A = 65$ $z_B = 20$ $z_C = 80$ $z_E = 35$ $m = 5$ mm

Step I *Free-body diagram of forces*

$$d'_A = mz_A = 5(65) = 325 \text{ mm}$$

$$d'_E = mz_E = 5(35) = 175 \text{ mm}$$

$$d'_C = mz_C = 5(80) = 400 \text{ mm}$$

The centre distance between two pins, which are fixed to the drum, is denoted by a.

$$a = \frac{d'_A + d'_E}{2} = \frac{325 + 175}{2} = 250 \text{ mm}$$

The output torque on the mixing drum is given by,

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n} = \frac{60 \times 10^6 (5)}{2\pi(250)}$$

$$= 190\ 985.93 \text{ N-mm} \qquad \text{(a)}$$

Refer to Fig. 17.27 for the flow of power. B is the input pinion. Power is supplied from the pinion B to the gear C and then from the gear E to the gear F. Therefore, between gears E and F, the gear E is the driving gear and the gear F is the driven gear. The construction of a free-body diagram of forces is illustrated in Fig. 17.28. Let us denote F_1 and F_2 as upper and lower planetary gears F.

Refer to Fig.17.28(a):

(i) It is assumed that the gear E is rotating in the clockwise direction. Therefore, gears F_1 and F_2 will rotate in anti-clockwise direction. A is a fixed internal gear.

(ii) During working operation, the point 1 on the gear E and the point 2 on the gear F_1 will be in contact with each other. Similarly, points 3 and 4, points 5 and 6, and points 7 and 8 will be in contact with each other.

Refer to Fig.17.28(b):

(i) Gear F_1 is the driven gear as compared to the gear E. It is made to rotate in the anti-clockwise direction. Therefore, at the point 2, the tangential component $(P_t)_{EF}$ will act towards the right. For this component, there will be an equal and opposite reaction on the gear E at the point 1. Therefore, the tangential component $(P_t)_{EF}$ will act towards left at the point 1 on gear E.

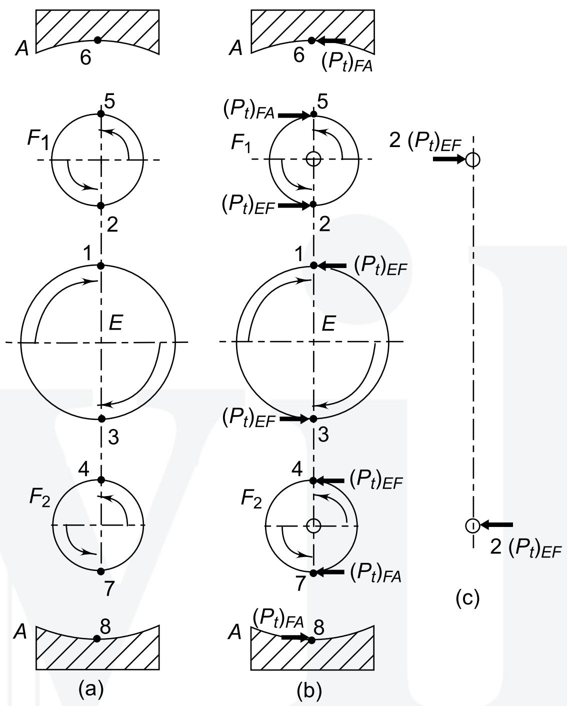


Fig. 17.28 *Free Body Diagram of Forces*

(ii) Gear F_2 is also a driven gear as compared to the gear E. It is made to rotate in anti-clockwise direction. Therefore, at the point 4, the tangential component $(P_t)_{EF}$ will act towards the left. For this component, there will be equal and opposite reaction on the gear E at the point 3. Therefore, the tangential component $(P_t)_{EF}$ will act towards the right at the point 3 on the gear E.

(iii) Gear A is fixed in the housing and will resist the motion of planetary gears F_1 and F_2. Therefore, in relative terms, gears F_1 and F_2 are driving gears with respect to the gear A. For the driving gear, the tangential component acts in opposite direction of the motion. Therefore, at the point 5 on the gear F_1, $(P_t)_{FA}$ will act towards the right. Similarly, at the point 7 on the gear F_2, $(P_t)_{FA}$ will act towards the left.

(iv) Gears F_1 and F_2 are planetary gears and do not transmit any torque to their pins. Considering the forces on the gear F_1,

$$\therefore \quad (P_t)_{EF} \times \text{radius of } F_1 = (P_t)_{FA} \times \text{radius of } F_1$$

or
$$(P_t)_{EF} = (P_t)_{FA}$$

The pin of the gear F_1 is subjected to a force $2(P_t)_{EF}$ towards the right. Similarly, it can be proved that the pin of the gear F_2 is subjected to a force $2(P_t)_{EF}$ towards the left. The forces acting on the two pins are illustrated in Fig. 17.28(c). The pins are fixed to the drum. Therefore, the torque acting on the drum is given by,

$$M_t = 2(P_t)_{EF} \times \left(\frac{a}{2}\right) + 2(P_t)_{EF} \times \left(\frac{a}{2}\right)$$

$$= 2(P_t)_{EF} \times a \qquad \text{(b)}$$

Step II *Components of tooth force between gears E and F*

From (a) and (b),

$$190\ 985.93 = 2(P_t)_{EF} \times (250)$$

$$\therefore \quad (P_t)_{EF} = 381.97 \text{ N}$$

$$(P_r)_{EF} = (P_t)_{EF} \tan \alpha = 381.97 \tan (20)$$
$$= 139.03 \text{ N} \qquad \text{(a)}$$

Step III *Components of tooth force between gears B and C*

Considering the tangential components on gears E and C,

$$2(P_t)_{EF} \times \left(\frac{d'_E}{2}\right) = (P_t)_{CB} \times \left(\frac{d'_C}{2}\right)$$

$$2(381.97) \times \left(\frac{175}{2}\right) = (P_t)_{CB} \times \left(\frac{400}{2}\right)$$

$$(P_t)_{CB} = 334.22 \text{ N}$$

$$(P_r)_{CB} = (P_t)_{CB} \tan \alpha = 334.22 \tan (20)$$
$$= 121.65 \text{ N} \qquad \text{(b)}$$

Example 17.5 *The layout of a two-stage gear box is shown in Fig. 17.29. The number of teeth on the gears are as follows:*

$$z_1 = 20 \quad z_2 = 50 \quad z_3 = 20 \quad z_4 = 50$$

Pinion 1 rotates at 1440 rpm in the anti-clockwise direction when observed from the left side of the page and transmits 10 kW of power to the gear train. The pressure angle is 20°. Draw a free-body diagram of the gear tooth forces and determine the reactions at bearings E and F.

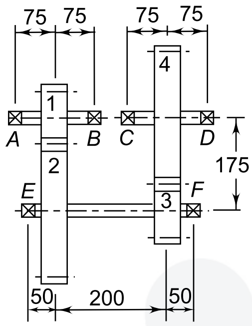


Fig. 17.29

Solution

Given kW = 10 $n = 1440$ rpm $\alpha = 20°$
$z_1 = 20$ $z_2 = 50$ $z_3 = 20$ $z_4 = 50$ $a = 175$ mm

Step I *Free-body diagram of forces*

From Eq. (17.5),

$$m = \frac{2a}{(z_1 + z_2)} = \frac{2(175)}{(20 + 50)} = 5 \text{ mm}$$

$$d'_1 = m\, z_1 = 5(20) = 100 \text{ mm}$$

$$d'_2 = m\, z_2 = 5(50) = 250 \text{ mm}$$

The forces between gears 1 and 2 are calculated in the following way

$$(M_t)_1 = \frac{60 \times 10^6 (\text{kW})}{2\pi n_1} = \frac{60 \times 10^6 (10)}{2\pi (1440)}$$

$$= 66\ 314.56 \text{ N-mm}$$

$$P_t = \frac{2(M_t)_1}{d'_1} = \frac{2(66\ 314.56)}{100} = 1326.29 \text{ N}$$

$$P_r = P_t \tan \alpha = 1325.29 \tan (20) = 482.73 \text{ N}$$

The corresponding forces between gears 3 and 4 are denoted by P'_t and P'_r. Since,

$$P_t \times \frac{d'_2}{2} = P'_t \times \frac{d'_3}{2}$$

$$\therefore \quad P'_t = P_t \times \left(\frac{d'_2}{d'_3}\right) = 1326.29 \times \left(\frac{250}{100}\right)$$

$$= 3315.73 \text{ N}$$

$$P'_r = P'_t \tan \alpha = 3315.73 \tan (20) = 1206.83 \text{ N}$$

The free-body diagram of forces acting on the shaft *EF* is shown in Fig. 17.30. The pinion-1 is rotating in anti-clockwise direction when observed from the left side of the page. Therefore, the gear-2 and the shaft *EF* are rotating in clockwise direction. Power is transmitted from the gear-1 to the gear-2, and then from gear-3 to the gear-4. Therefore, the gear-2 is driven gear and the gear-3 is the driving gear. The direction of tangential component for the driven gear is the same as that of rotation. Therefore, at the point of contact on the gear-2, the tangential component will act towards the lower right-hand corner of the page. The direction of tangential component for driving gear is opposite to that of rotation. Therefore, at the point of contact on the gear-3, the tangential component will act towards the upper left-hand corner of the page. The radial components will act towards the centres of respective gears.

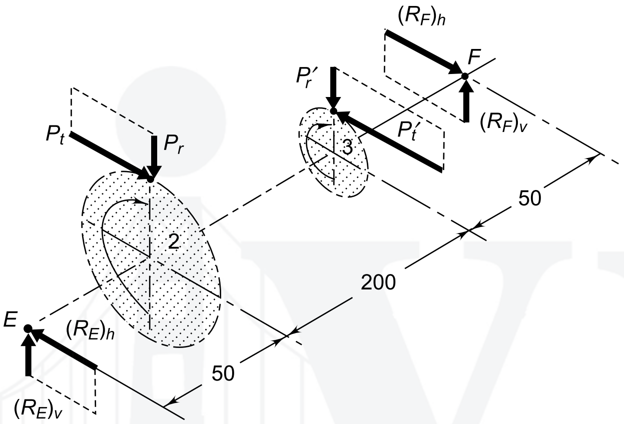


Fig. 17.30

Step II *Reactions at bearings E and F*

The forces acting in vertical and horizontal planes are shown in Fig. 17.31.

Considering vertical forces and taking moments about the bearing *E*,

$$P_r \times 50 + P'_r \times 250 = (R_F)_v \times 300$$

$$482.73 \times 50 + 1206.83 \times 250 = (R_F)_v \times 300$$

$$\therefore \quad (R_F)_V = 1086.15 \text{ N} \qquad \text{(i)}$$

$$P_r + P'_r = (R_F)_v + (R_E)_v$$

$$482.73 + 1206.83 = 1086.15 + (R_E)_v$$

$$\therefore \quad (R_E)_v = 603.41 \text{ N} \qquad \text{(ii)}$$

Considering horizontal forces and taking moments about the bearing *E*,

$$P'_t \times 250 - P_t \times 50 = (R_F)_h \times 300$$

$$3315.73 \times 250 - 1326.29 \times 50 = (R_F)_h \times 300$$

$$\therefore \quad (R_F)_h = 2542.06 \text{ N} \qquad \text{(iii)}$$

$$(R_F)_h + P_t = (R_E)_h + P'_t$$

$$2542.06 + 1326.29 = (R_E)_h + 3315.73$$

$$\therefore \quad (R_E)_h = 552.62 \text{ N} \qquad \text{(iv)}$$

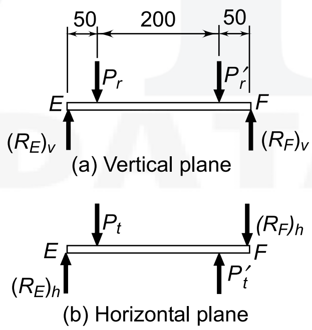


Fig. 17.31

17.12 GEAR TOOTH FAILURES

There are two basic modes of gear tooth failure—breakage of the tooth due to static and dynamic loads and the surface destruction[4, 5]. The complete breakage of the tooth can be avoided by adjusting the

[4] RH Pearson–'Gear overdesign and how to avoid it'–Machine Design—May 9, 1968, vol. 40, no. 11, p. 153.

[5] Eugene E Shipley – 'Gear failures—how to recognize them, what causes them and how to avoid them?'—Machine Design—Dec 7, 1967, vol. 39, no. 28, p. 152.

parameters in the gear design, such as the module and the face width, so that the beam strength of the gear tooth is more than the sum of static and dynamic loads. The surface destruction or tooth wear is classified according to the basis of their primary causes. The principal types of gear tooth wear are as follows:

(i) Abrasive Wear Foreign particles in the lubricant, such as dirt, rust, weld spatter or metallic debris can scratch or brinell the tooth surface. Remedies against this type of wear are provision of oil filters, increasing surface hardness and use of high viscosity oils. A thick lubricating film developed by these oils allows fine particles to pass without scratching.

(ii) Corrosive Wear The corrosion of the tooth surface is caused by corrosive elements, such as extreme pressure additives present in lubricating oils and foreign materials due to external contamination. These elements attack the tooth surface, resulting in fine wear uniformly distributed over the entire surface. Remedies against this type of wear are, providing complete enclosure for the gears free from external contamination, selecting proper additives and replacing the lubricating oil at regular intervals.

(iii) Initial Pitting The initial or corrective pitting is a localized phenomenon, characterized by small pits at high spots. Such high spots are progressively worn out and the load is redistributed. Initial pitting is caused by the errors in tooth profile, surface irregularities and misalignment. The remedies against initial pitting are precise machining of gears, adjusting the correct alignment of gears so that the load is uniformly distributed across the full face width, and reducing the dynamic loads.

(iv) Destructive Pitting Destructive pitting is a surface fatigue failure, which occurs when the load on the gear tooth exceeds the surface endurance strength of the material. This type of failure is characterized by pits, which continue to grow resulting in complete destruction of the tooth surface and, in some cases, even premature breakage of the tooth. Destructive pitting depends upon the magnitude of the Hertz' contact stress and the number of stress cycles. This type of failure can be avoided by designing the gears in such a way that the wear strength of the gear tooth is more than the sum of static and dynamic loads. The surface endurance strength can be improved by increasing the surface hardness.

(v) Scoring Excessive surface pressure, high surface speed and inadequate supply of lubricant result in the breakdown of the oil film. This results in excessive frictional heat and overheating of the meshing teeth. Scoring is a stick-slip phenomenon, in which alternate welding and shearing takes place rapidly at the high spots. Here, the rate of wear is faster. Scoring can be avoided by selecting the parameters, such as surface speed, surface pressure and the flow of lubricant in such a way that the resulting temperature at the contacting surfaces is within permissible limits. The bulk temperature of the lubricant can be reduced by providing fins on the outside surface of the gear box and a fan for forced circu1ation of air over the fins.

17.13 SELECTION OF MATERIAL

The desirable properties of gear material are as follows:

(i) The load carrying capacity of the gear tooth depends upon the ultimate tensile strength or the yield strength of the material. When the gear tooth is subjected to fluctuating forces, the endurance strength of the tooth is the deciding factor. The gear material should have sufficient strength to resist failure due to breakage of the tooth.

(ii) In many cases, it is '*wear rating*' rather than '*strength rating*' which decides the dimensions of the gear tooth. The resistance to wear depends upon alloying elements, grain size, percentage of carbon, and surface hardness. The gear material should have sufficient surface endurance strength to avoid failure due to destructive pitting.

(iii) For high-speed power transmission, the sliding velocities are very high and the material should have low coefficient of friction to avoid failure due to scoring.

(iv) The amount of thermal distortion or warping during the heat treatment process is a major problem in gear applications. Due to warping, the load gets concentrated at one corner of the gear tooth. Alloy steels are superior to plain carbon steels in this respect, due to consistent thermal distortion.

Gears are made of cast iron, steel, bronze and phenolic resins. Large size gears are made of grey cast iron of Grades FG 200, FG 260 or FG 350. They are cheap and generate less noise compared with steel gears. They have good wear resistance. Their main drawback is poor strength. Case-hardened steel gears offer the best combination of a wear-resisting hard surface together with a ductile and shock-absorbing core. The plain carbon steels used for medium duty applications are 50C8, 45C8, 50C4 and 55C8. For heavy duty applications, alloy steels 4OCrl, 30Ni4Cr1 and 4ONi3Cr65Mo55 are used. For planetary gear trains, alloy steel 35NilCr60 is recommended. Although steel gears are costly, they have higher load carrying capacity. Bronze is mainly used for worm wheels due to its low coefficient of friction and excellent conformability. It is also suitable where resistance to corrosion is an important consideration in applications like water pumps. Their main drawback is excessive cost.

Non-metallic gears are used under the following conditions:

(i) The load is light and the pitch line velocity is low.

(ii) A long life is expected.

(iii) It is required to have quiet operation free from noise and vibrations.

(iv) The gears are likely to be affected by water and oil.

In non-metallic gear drives, only the pinion is made of non-metals such as molded nylon, laminated phenolics like Bakelite or Celoron. The non-metallic pinions generally run with cast iron gears. Gears made of phenolic resins have low modulus of elasticity and work on marginal lubrication. They can tolerate errors in the tooth profile.

17.14 GEAR BLANK DESIGN

Depending on the purpose and the size, there are different constructions for gears. These constructions are broadly classified into the following three groups:

(i) Small size gears;

(ii) Medium size gears; and

(iii) gears with large diameter.

In this article, we will consider the salient features of these constructions.

(i) Small Sized Gears A pinion with root diameter near to the required diameter of the shaft is made integral with the shaft. This type of construction is shown in Fig. 17.32. The rule of thumb for making integral gear is as follows:

'If the diameter of dedendum circle (d_f) exceeds the diameter of the shaft (d_s), at the point where the pinion is fitted, by less than (d_s/2), the pinion is made integral with the shaft'.

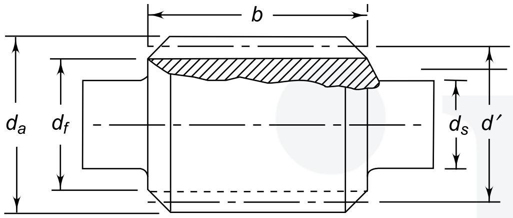


Fig. 17.32 *Integral Gear*

The dimensions of the pinion are as follows:

(a) Pitch circle diameter = $d' = mz$

(b) Addendum circle diameter = $d_a = d' + 2h_a = mz + 2(m) = m(z + 2)$

(c) Dedendum circle diameter = $d_f = d' - 2h_f = mz - 2(1.25\text{ m}) = m(z - 2.5)$

(d) Shaft diameter = d_s

(e) Width of gear = face width = b

The advantages of integral construction are as follows:

(a) It reduces the amount of machining since there is no need to cut keyways on the shaft and the pinion.

(b) It reduces the number of parts since there is no key. This reduces the cost.

(c) It increases the rigidity of the shaft and also increases the accuracy of contact.

However, integral construction can be used only when the size of pinion is small. If the diameter of dedendum circle is considerably larger than the shaft diameter, the pinion is made separate from the shaft.

(ii) Medium Sized Gears There are two methods to manufacture medium size gears. They include machining from bar stock and forging. Gears of addendum circle diameter up to 150 mm are machined on rolled steel bars. The gear blanks in this case are turned on lathe. Gears of addendum circle diameter from 150 mm to 400 mm are mostly forged in open or closed dies. It again depends upon the volume of production. Even small diameter gears are forged, if the volume of production is large. Forged gears offer the following advantages:

(a) The factor of material utilisation is equal to (1/3) when the gear is machined from bar stock. In case of forgings, material utilization factor is (2/3), which is twice. This reduces the cost of the material.

(b) Forged gear has lightweight construction which reduces inertia and centrifugal forces.

(c) The fibre lines of the forged gears are arranged in a predetermined way to suit the direction of external force. In case of gears, prepared by machining methods, the original fibre lines of rolled stock are broken. Therefore, the forged gear is inherently strong compared with machined gear.

The limiting factor governing the choice of forged gears is their high cost. The equipment and tooling required to make forged gears is costly. Forged gears become economical only when they are manufactured on large scale.

Figure. 17.33 shows a gear in the form of a flat circular disk, which is machined from bar stock. The gear blank in this case is turned on the lathe. This type of construction is used for gears with addendum circle diameter up to 150 mm or when the gears are manufactured on small scale. It is more economical than forged gear.

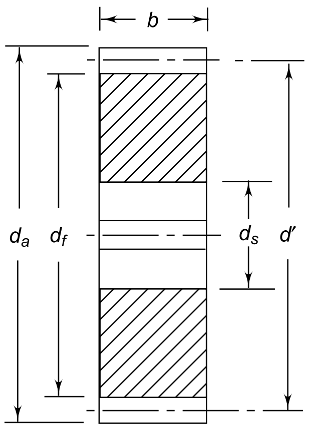


Fig. 17.33 *Machined Gear*

The main dimensions of the gear blank are as follows:

(a) Pitch circle diameter = $d' = mz$

(b) Addendum circle diameter = $d_a = m(z + 2)$

(c) Dedendum circle diameter = $d_f = m(z - 2.5)$

(d) Shaft diameter = d_s

(e) Width of gear = face width = b

The construction of a forged gear is shown in Fig. 17.34. It is made of three parts—hub, web and rim. The web connects the hub with the rim. Holes are provided in the web for clamping the gear blank during machining. It also reduces the weight. The inner surface of the rim and the outer surface of the hub are provided with draft for easy removal from the dies. The dimensions of forged gears are based on the following thumb rules:

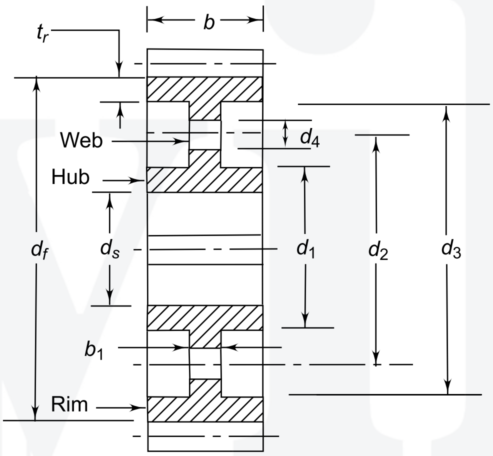


Fig. 17.34 *Forged Gear*

(a) The thickness of the rim (t_r) up to the root circle diameter is taken as (2m) to (3m).

(b) The thickness of web (b_1) is taken as ($0.2b$) to ($0.3b$).

(c) The outer diameter of hub (d_1) is taken as ($1.5d_s$) to ($2.0d_s$).

The main dimensions of the gear blank of forged gear are as follows:

(a) Pitch circle diameter = $d' = mz$

(b) Addendum circle diameter = $d_a = m(z + 2)$

(c) Dedendum circle diameter = $d_f = m(z - 2.5)$

(d) Shaft diameter = d_s

(e) Outer diameter of hub = $d_1 = (1.5d_s)$ to $(2.0d_s)$

(f) Length of hub = length of key or width of gear (maximum value)
(g) Width of web = b_1 = (0.2b) to (0.3b)
(h) Inner diameter of rim = $d_3 = (d_f - 2t_r)$
(i) Outer diameter of rim = dedendum circle diameter = d_f
(j) Thickness of rim = t_r = 2m to 3m
(k) Diameter of holes in web = $d_4 = (d_3 - d_1)/4$
(l) Pitch circle diameter of holes = $d_2 = (d_3 + d_1)/2$
(m) Width of rim = face width = b

Whatever the computed dimensions, care should be taken to make the gear a '*sound*' forging and modify the dimensions, if required. The guidelines for designing a forging are explained in Section 3.3 on Design considerations of Forgings. These rules related to uniform thickness, minimum section thickness, fillet and corner radii and provision of draft must be applied to final drawing of gear blank.

(iii) Gears with Large Diameter There are two varieties of large size gears—solid cast gears and rimmed gears. When the addendum circle diameter is up to 900 mm, a solid cast iron gear with one web is recommended. When the addendum circle diameter is more than 1000 mm, two webs are provided. A solid cast gear with two webs is shown in Fig. 17.35. Solid cast iron gears are extensively used due to low cost. Though cast iron gears are cheaper than steel gears, their torque transmitting capacity is low. The dimensions of cast iron gears are determined by thumb rules and principles of casting design.

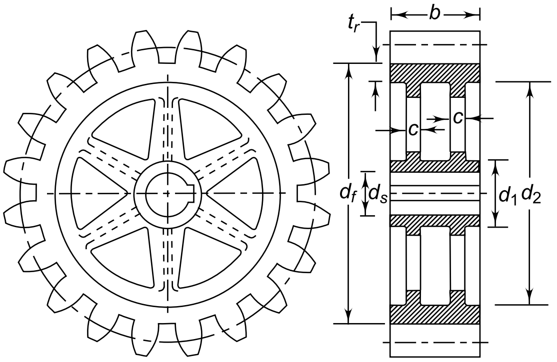


Fig. 17.35 *Cast Iron Web Type Gear*

The main dimensions of cast iron gear, illustrated in Fig. 17.35 are as follows:

(a) Pitch circle diameter = $d' = mz$
(b) Addendum circle diameter = $d_a = m(z + 2)$
(c) Dedendum circle diameter = $d_f = m(z - 2.5)$
(d) Shaft diameter = d_s
(e) Outer diameter of hub = $d_1 = 2.0d_s$
(f) Length of hub = (1.25 d_s) to (2.0 d_s) = length of key
(g) Width of web = c = 0.5 to 0.6 of circular pitch
(h) Inner diameter of rim = $d_2 = (d_f - 2t_r)$
(i) Outer diameter of rim = dedendum circle diameter = d_f
(j) Thickness of rim = t_r = 0.56 × circular pitch
(k) Width of rim = face width = b

Whatever the computed values of dimensions, care should be taken to make the gear a '*sound*' casting and modify the dimensions, if required. The guidelines for designing a casting are explained in Section 3.2 on Design considerations of Casting. These rules related to uniform thickness, minimum section thickness, fillet and corner radii and avoiding concentration of metal at any junction must be applied to the final drawing of gear blank.

A rimmed gear consists of a steel rim fitted on the central casting with hub, arms or webs. The rim is forged from alloy steel. There are two varieties of rimmed gears, which are illustrated in Figs 17.36 (a) and (b). In the first case, the rim is press fitted on the casting and setscrews are used to prevent displacement of the rim with respect to casting. In the second type of construction, the rim is bolted to the central casting. Rimmed gears save costly high strength material, but they are more expensive to manufacture. The thickness of the rim from the inside diameter to the root circle diameter of tooth is usually taken as (7m) to (8m).

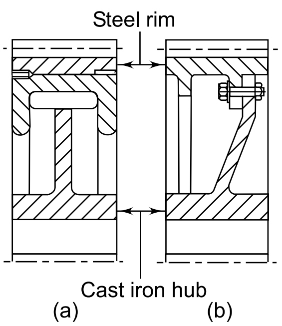


Fig. 17.36 *Rimmed Gears*

When the pitch circle diameter of cast iron gear is large, arms are used in place of web. The arms are also called *spokes*. The cross-section of an arm can be elliptical, I-section, H-section or cross (+). The recommended number of arms is as follows:

Pitch circle diameter (mm)	*Number of arms (n)*
300 – 500	4
500 – 1500	6
1500 – 2400	8
> 2400	10–12

In some cases, it is required to design the cross-section of the arm. Figure 17.37 shows the arm of a cast iron gear. The analysis is based on the following assumptions:

(a) The arm is assumed as a cantilever beam, fixed at the hub and subjected to a force P at the pitch line.

(b) The rim is rigid so that each arm takes equal share of total tooth load.

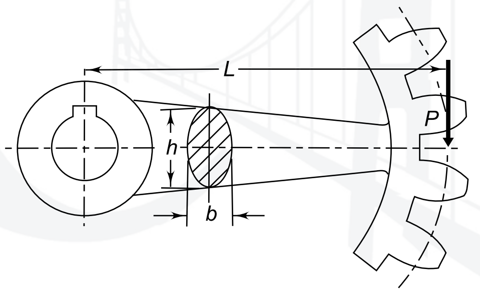


Fig. 17.37

The transmitted torque produces bending moment on arm. Suppose,

M_t = transmitted torque (N-mm)
n = number of arms
P = force acting on each arm (N)
L = length of cantilever beam (mm)

The bending moment (M_b) on each arm is given by,

$$M_b = P \times L \tag{a}$$

The torque transmitted by each arm is given by,

$$\frac{M_t}{n} = P \times L \tag{b}$$

From (a) and (b),

$$M_b = \frac{M_t}{n} \tag{c}$$

Also,

$$\sigma_b = \frac{M_b y}{I} = \left(\frac{M_t}{n}\right)\frac{y}{I} \tag{d}$$

For the elliptical cross-section,

$$I = \frac{\pi b h^3}{64} \quad \text{and} \quad y = h/2 \tag{e}$$

The major axis is usually taken as twice the minor axis. Also, the major axis is in the plane of rotation. Therefore,

$$b = h/2$$

Substituting this value in Eq. (e),

$$I = \frac{\pi h^4}{128} \quad \text{and} \quad y = h/2 \tag{f}$$

From (d) and (f),

$$\left(\frac{M_t}{n}\right) = \sigma_b \left(\frac{\pi h^3}{64}\right) \tag{17.11}$$

The above equation is used for finding out the dimensions (h) of cross-section of the arm. The permissible bending stress is given by,

$$\sigma_b = \frac{S_{ut}}{(fs)}$$

A higher factor of safety should be used because the above method is approximate, the assumptions are not exactly valid and stress concentration is neglected. The cross-section of arm is given a taper 1:16 from the hub to the rim.

17.15 NUMBER OF TEETH

In the design of gears, it is required to decide the number of teeth on the pinion and gear. There is a limiting value of the minimum number of teeth on the pinion. As the number of teeth decreases, a point is reached when there is interference and the standard tooth profile requires modification. The minimum number of teeth to avoid interference is given by,

$$z_{\text{min.}} = \frac{2}{\sin^2 \alpha} \tag{17.12}$$

In practice, giving a slight radius to the tip of tooth can further reduce the value of z_{min}. Theoretical and practical values of the minimum number of teeth on the pinion are as follows:

Pressure angle (α)	14.5°	20°	25°
$z_{min.}$ (theoretical)	32	17	11
$z_{min.}$ (practical)	27	14	9

For the 20° full-depth involute tooth system, it is always safe to assume the number of teeth on the pinion as 18 or 20. This does not require any modification in the profile. Once the number of teeth on the pinion is decided, the number of teeth on the gear is calculated by the velocity ratio ($i = z_g/z_p$).

There is a concept of '*hunting*' tooth for uniform distribution of tooth wear. Suppose ($z_p = 20$) and ($z_g = 40$), then after every two revolutions of the pinion, the same pair of teeth will engage. If however, we take ($z_p = 20$) and ($z_g = 41$), the pinion will rotate 41 times before the same pair of teeth will engage again. This extra tooth is called the *hunting tooth*. It results in more even distribution of wear. For the provision of hunting tooth, it should be permissible to alter the velocity ratio slightly.

In a multi-stage gearbox consisting of two or three stages, the velocity ratio at each stage should not exceed 6 : 1. The intermediate speeds, in this case, are arranged in geometric progression. If i' is the total transmission ratio, i.e., the ratio of angular velocity of the first driving gear to that of the last driven gear, then the speed reduction at each stage (i) is obtained in the following way.

For two stages,

$$i = \sqrt{i'} \tag{17.13}$$

For three stages,

$$i = \sqrt[3]{i'} \tag{17.14}$$

As the number of teeth increases, the pitch circle diameter and the size of the gear wheel also increase, thus increasing the cost. Therefore, the number of teeth on the pinion as well as on the gear should be kept as small as possible.

17.16 FACE WIDTH

In the design of gears, it is required to express the face width in terms of the module. In the Lewis equation, it is assumed that the tangential force P_t is uniformly distributed over the entire face width. If the face width is too large, there is a possibility of concentration of load at one end of the gear tooth due to a number of factors, like misalignment, elastic deformation of shafts, and warping of gear tooth. On the other hand, gears with a small face width have a poor capacity to resist the shock and absorb vibrations. They also wear at a faster rate. A narrow face width results in a coarse pitch. In practice, the optimum range of the face width is

$$(8 \text{ m}) < b < (12 \text{ m}) \tag{17.15}$$

In the preliminary stages of gear design, the face width is assumed as ten times of module.

Example 17.6 *The layout of a two-stage gearbox is shown in Fig. 17.38. The number of teeth on gears 1,2,3 and 4 are z_1, z_2, z_3 and z_4 respectively. The module for all gears is m. The total transmission ratio, i.e., the ratio of angular velocity of the gear-1 to that of the gear-4, is i. The velocity ratios of the first and second stages are i_1 and i_2 respectively. Show that the condition for designing a compact gearbox is given by,*

$$i_1 = i_2 = \sqrt{i}$$

Assume the two pinions to be exactly identical.

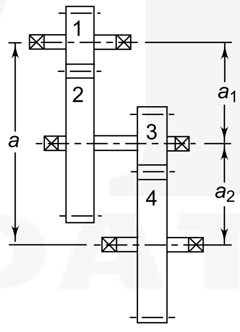


Fig. 17.38

Solution

$$d_1' = mz_1 \quad \text{and} \quad d_2' = mz_2$$

$$\therefore \qquad \frac{d_2'}{d_1'} = \frac{z_2}{z_1} = i_1 \quad \text{or} \quad d_2' = i_1 d_1'$$

The centre distance a_1 is given by,

$$a_1 = \frac{1}{2}(d_1' + d_2') = \frac{1}{2}(d_1' + i_1 d_1') = \frac{1}{2} d_1'(1 + i_1) \tag{a}$$

Similarly, it can be shown that

$$a_2 = \frac{1}{2} d_3'(1 + i_2) \tag{b}$$

Since the two pinions are identical,

$$d_1' = d_3' \quad \text{(c)}$$

From (b) and (c),

$$a_2 = \frac{1}{2} d_1'(1 + i_2) \quad \text{(d)}$$

The total centre distance is given by,

$$a = a_1 + a_2$$

Substituting (a) and (d) in above expression,

$$a = \frac{1}{2} d_1'[(1 + i_1) + (1 + i_2)]$$

$$= \frac{1}{2} d_1'[2 + i_1 + i_2] \quad \text{(e)}$$

The total velocity ratio is given by,

$$i = i_1\, i_2 \quad \text{or} \quad i_2 = \frac{i}{i_1} \quad \text{(f)}$$

From (e) and (f),

$$a = \frac{d_1'}{2}\left(2 + i_1 + \frac{i}{i_1}\right) \quad \text{(g)}$$

The centre distance a is a measure of the size of the gearbox. When the centre distance is more, the size and the volume of the gearbox is more. Therefore, the condition for compact gearbox is written as,

$$\frac{\partial a}{\partial i_1} = 0 \quad \text{or} \quad \frac{\partial}{\partial i_1}\left[2 + i_1 + \frac{i}{i_1}\right] = 0$$

Differentiating,

$$\left[0 + 1 - \frac{i}{i_1^2}\right] = 0 \quad \text{or} \quad 1 = \frac{i}{i_1^2}$$

or $$i_1 = \sqrt{i} \quad \text{(h)}$$

From (f) and (h),

$$i_2 = \frac{i}{i_1} = \frac{i}{\sqrt{i}} = \sqrt{i}$$

$$\therefore \quad i_1 = i_2 = \sqrt{i}$$

17.17 BEAM STRENGTH OF GEAR TOOTH

The analysis of bending stresses in gear tooth was done by Wilfred Lewis in his paper, 'The investigation of the strength of gear tooth' submitted at the Engineer's Club of Philadelphia in 1892. Even today, the Lewis equation is considered as the basic equation in the design of gears. In the Lewis analysis, the gear tooth is treated as a cantilever beam as shown in Fig. 17.39. The tangential component (P_t) causes the bending moment about the base of the tooth. The Lewis equation is based on the following assumptions:

(i) The effect of the radial component (P_r), which induces compressive stresses, is neglected.

(ii) It is assumed that the tangential component (P_t) is uniformly distributed over the face width of the gear. This is possible when the gears are rigid and accurately machined.

(iii) The effect of stress concentration is neglected.

(iv) It is assumed that at any time, only one pair of teeth is in contact and takes the total load.

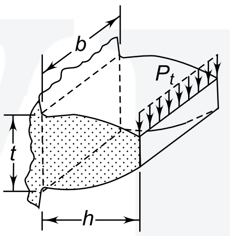


Fig. 17.39 *Gear Tooth as Cantilever*

It is observed that the cross-section of the tooth varies from the free end to the fixed end. Therefore, a parabola is constructed within the tooth profile and shown by a dotted line in Fig. 17.40. The advantage of parabolic outline is that it is a beam of uniform strength. For this beam, the stress at any cross-section is uniform or same. The weakest section of the gear tooth is at the section XX, where the parabola is tangent to the tooth profile.

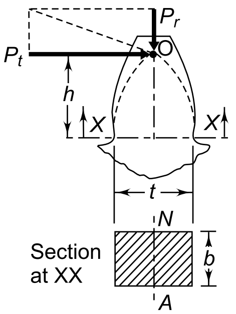


Fig. 17.40 *Gear Tooth as Parabolic Beam*

At the section *XX*,

$$M_b = P_t \times h$$

$$I = \left(\frac{1}{12}\right) bt^3$$

$$y = \frac{t}{2}$$

The bending stresses are given by,

$$\sigma_b = \frac{M_b y}{I} = \frac{(P_t \times h)\left(\frac{t}{2}\right)}{\left[\left(\frac{1}{12}\right) b\,t^3\right]}$$

Rearranging the terms,

$$P_t = b\sigma_b \left(\frac{t^2}{6h}\right)$$

Multiplying the numerator and denominator of the right-hand side by *m*,

$$P_t = mb\sigma_b \left(\frac{t^2}{6hm}\right)$$

Defining a factor *Y*,

$$Y = \left(\frac{t^2}{6hm}\right)$$

the equation is rewritten as,

$$P_t = mb\sigma_b Y \qquad \text{(a)}$$

In the above equation, *Y* is called the *Lewis form factor*. Equation (a) gives the relationship between the tangential force (P_t) and the corresponding stress σ_b. When the tangential force is increased, the stress also increases. When the stress reaches the permissible magnitude of bending stresses, the corresponding force (P_t) is called the beam strength. *Therefore, the beam strength (S_b) is the maximum value of the tangential force that the tooth can transmit without bending failure.* Replacing (P_t) by (S_b), Eq. (a) is modified in the following way:

$$S_b = mb\sigma_b Y \qquad (17.16)$$

where,

S_b = beam strength of gear tooth (N)

σ_b = permissible bending stress (N/mm^2)

Equation 17.16 is known as the Lewis Equation. The values of the Lewis form factor *Y* for 20° full-depth involute system, are given in Table 17.3. In order to avoid the breakage of gear tooth due to bending, the beam strength should be more than the effective force between the meshing teeth.

Therefore,

$$S_b \geq P_{\text{eff}}$$

The method of calculating of P_{eff} is discussed in Section 17.19. In the design of gears, it is required to decide the weaker between the pinion and gear.

Rewriting the Lewis equation,

$$S_b = mb\sigma_b Y$$

It is observed that *m* and *b* are same for pinion as well as for gear. When different materials are used, the product ($\sigma_b \times Y$) decides the weaker between pinion and gear. The Lewis form factor *Y* is always less for a pinion compared with gear. When the same material is used for the pinion and gear, the pinion is always weaker than the gear.

Table 17.3 *Values of the Lewis form factor Y for 20° full-depth involute system*

z	Y	z	Y	z	Y
15	0.289	27	0.348	55	0.415
16	0.295	28	0.352	60	0.421
17	0.302	29	0.355	65	0.425
18	0.308	30	0.358	70	0.429
19	0.314	32	0.364	75	0.433
20	0.320	33	0.367	80	0.436
21	0.326	35	0.373	90	0.442
22	0.330	37	0.380	100	0.446
23	0.333	39	0.386	150	0.458
24	0.337	40	0.389	200	0.463
25	0.340	45	0.399	300	0.471
26	0.344	50	0.408	Rack	0.484

17.18 PERMISSIBLE BENDING STRESS

The tooth of the gear is subjected to fluctuating bending stress as it comes in contact with the meshing tooth. The stress–time diagrams for gear teeth are illustrated in Fig. 17.41. The following observations are made from the figure:

(i) The teeth of the driving and driven gears are subjected to stress in one direction only as shown in Fig. 17.41(b). It is called '*repeated*' stress. For this type of stress distribution,

$$\sigma_m = \left(\frac{1}{2}\right)\sigma_{\text{max.}} \quad \text{and} \quad \sigma_a = \left(\frac{1}{2}\right)\sigma_{\text{max.}}$$

where,

$\sigma_{\text{max.}}$ = maximum bending stress

σ_m = mean stress

σ_a = stress amplitude

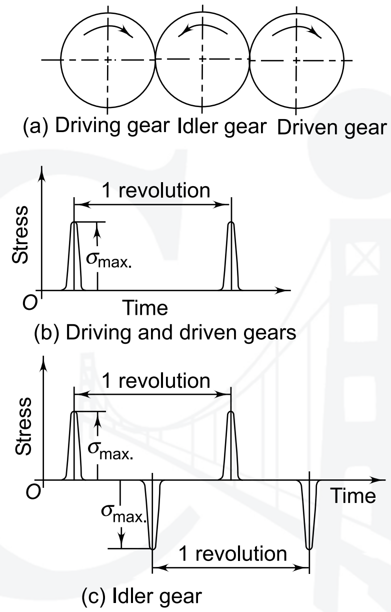


Fig. 17.41 *Stress–time Diagram in Gear Teeth*

(ii) The teeth of the idler gear or planetary pinion are subjected to stress in both directions as shown in Fig. 17.41(c). It is called '*reversed*' stress. For this type of stress distribution,

$$\sigma_m = 0 \quad \text{and} \quad \sigma_a = \sigma_{\text{max.}}$$

Since the teeth are subjected to fluctuating stresses, endurance limit stress (S_e) is the criterion of design. Therefore, the maximum bending stress is equal to the endurance limit stress of the gear tooth. The endurance limit stress of the gear tooth depends upon the following factors:

(i) Surface finish of the gear tooth
(ii) Size of the gear tooth
(iii) Reliability used in design
(iv) Stress concentration in the gear tooth
(v) Gears rotating in one direction or both directions
(vi) Gears tooth subjected to stress in one direction or both directions

In practice, it is difficult to get the above-mentioned data for each and every case of gear design. *Earle Buckingham has suggested that the endurance limit stress of gear tooth is approximately one-third of the ultimate tensile strength of the material.* In this chapter, we will use this approximate value. Therefore,

$$\sigma_b = S_e = \left(\frac{1}{3}\right)S_{ut} \qquad (17.17)$$

In case of bronze gears, the endurance limit stress is taken as 40% of the ultimate tensile strength.

17.19 EFFECTIVE LOAD ON GEAR TOOTH

In Section 17.11, a method to determine the tangential component of the resultant force between two meshing teeth is discussed. The component is calculated by using the following two equations:

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n}$$

$$P_t = \frac{2M_t}{d'}$$

The value of the tangential component, therefore, depends upon the rated power and rated speed. In practical applications, the torque developed by the source of power varies during the work cycle. Similarly, the torque required by the driven machine also varies. The two sides are balanced by means of a flywheel. In gear design, the maximum force (due to maximum torque) is the criterion. This is accounted by means of a service factor. The service factor C_s is defined as

$$C_s = \frac{\text{maximum torque}}{\text{rated torque}}$$

Therefore,

$$C_s = \frac{(M_t)_{\text{max.}}}{M_t} = \frac{(P_t)_{\text{max.}}}{P_t}$$

where (P_t) is the tangential force due to rated torque (M_t). Rearranging the terms,

$$(P_t)_{max.} = C_s\, P_t \tag{17.18}$$

For electric motors,

$$C_s = \frac{\text{starting torque}}{\text{rated torque}} \tag{17.19}$$

The values of the service factor are given in Tables 17.4. The examples of driving and driven machines with different working characteristics are given in Tables 17.5 and 17.6

Table 17.4 *Service factor for speed reduction gearboxes*

Working characteristics of Driving machine (Table 17.5)	*Working characteristics of Driven machine (Table 17.6)*		
	Uniform	*Moderate shock*	*Heavy shock*
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.5	1.75	2.25

Table 17.5 *Examples of Driving machines with different working characteristics*

Characteristic of operation	*Driving machines*
Uniform	Electric motor, steam turbine, gas turbine
Light shock	Multi-cylinder internal combustion engine
Medium shock	Single cylinder internal combustion engine

Table 17.6 *Examples of Driven machines with different working characteristics*

Characteristic of operation	*Driven machines*
Uniform	Generator, belt conveyor, platform conveyor, light elevator, electric hoist, feed gears of machine tools, ventilators, turbo-blower, mixer for constant density material
Medium shock	Main drive to machine tool, heavy elevator, turning gears of crane, mine ventilator, mixer for variable density material, multi-cylinder piston pump, feed pump
Heavy shock	Press, shear, rubber dough mill, rolling mill drive, power shovel, heavy centrifuge, heavy feed pump, rotary drilling apparatus, briquette press, pug mill

When gears rotate at very low speed, almost at zero velocity, the transmitted load (P_t) can be considered to be the actual force present between two meshing teeth. However, in most of the cases, the gears rotate at an appreciable speed and it becomes necessary to consider the dynamic force resulting from the impact between mating teeth. The dynamic force is induced due to the following factors:

(i) inaccuracies of the tooth profile;
(ii) errors in tooth spacing;
(iii) misalignment between bearings;
(iv) elasticity of parts; and
(v) inertia of rotating disks.

There are two methods to account for the dynamic load—approximate estimation by the *velocity factor* in the preliminary stages of gear design and precise calculation by *Buckingham's equation* in the final stages of gear design.

It is difficult to calculate the exact magnitude of dynamic load in the preliminary stages of gear design. To overcome this difficulty, a velocity factor C_v developed by Barth is used. The values of the velocity factor are as follows:

(i) For ordinary and commercially cut gears made with form cutters and with $v < 10$ m/s,

$$C_v = \frac{3}{3+v} \tag{17.20}$$

(ii) For accurately hobbed and generated gears with $v < 20$ m/s,

$$C_v = \frac{6}{6+v} \tag{17.21}$$

(iii) For precision gears with shaving, grinding and lapping operations and with $v > 20$ m/s,

$$C_v = \frac{5.6}{5.6+\sqrt{v}} \tag{17.22}$$

where v is the pitch line velocity in m/s.

The pitch line velocity is given by,

$$v = \frac{\pi d'n}{60 \times 10^3} \tag{17.23}$$

The effective load between two meshing teeth is given by,

$$P_{\text{eff}} = \frac{C_S P_t}{C_v} \tag{17.24}$$

The velocity factor is an empirical relationship developed by past experience. This method of calculating dynamic load has the following advantages:

(i) It is much easier to calculate the velocity factor and design gears.

(ii) Velocity factors have sanctions of the American Gear Manufacturing Association (AGMA)

(iii) They were used in the past for many years and given satisfactory results.

The disadvantages of the velocity factor method are as follows:

(i) The dynamic load depends upon a number of factors such as the mass of gears, mass connected to the gear shaft and properties of the gear material, like modulus of a elasticity. A gear tooth of a material with low modulus of elasticity will deflect more than the gear tooth of a material with higher modulus of elasticity, and other things being equal, will absorb the energy of impact and reduce the dynamic load. Velocity factor method neglects these factors. It assumes that dynamic load depends only upon pitch line velocity.

(ii) Use of velocity factor is restricted to a limited range of pitch line velocities. It is not possible to extrapolate the values.

In the final stages of gear design, when gear dimensions are known, errors specified and the quality of gears determined, the dynamic load is calculated by equations derived by Earle Buckingham. The effective load is given by,

$$P_{\text{eff}} = (C_s P_t + P_d) \tag{17.25}$$

where, (P_d) is the dynamic load or additional load due to dynamic conditions between two meshing teeth. Buckingham has used a term '*incremental dynamic load*' for (P_d). The dynamic load[6] is given by,

$$P_d = \frac{21v(Ceb + P_t)}{21v + \sqrt{(Ceb + P_t)}} \tag{17.26}$$

where,

P_d = dynamic load or incremental dynamic load (N)

v = pitch line velocity (m/s)

C = deformation factor (N/mm^2)

e = sum of errors between two meshing teeth (mm)

b = face width of tooth (mm)

P_t = tangential force due to rated torque (N)

The deformation factor C depends upon the modulii of elasticity of materials for pinion and gear and the form of tooth or pressure angle. It is given by,

$$C = \frac{k}{\left[\dfrac{1}{E_p} + \dfrac{1}{E_g}\right]} \tag{17.27}$$

where,

k = constant depending upon the form of tooth

E_p = modulus of elasticity of pinion material (N/mm^2)

E_g = modulus of elasticity of gear material (N/mm^2)

[6] Buckingham Earle – 'Analytical Mechanics of Gears' – McGraw-Hill Inc.,1949.

The values of k for various tooth forms are as follows:

$k = 0.107$ (for 14.5° full depth teeth)

$k = 0.111$ (for 20° full depth teeth)

$k = 0.115$ (for 20° stub teeth)

The ready made values of C for various conditions are given in Table 17.7.

Table 17.7 *Values of deformation factor C (N/mm²)*

Materials		*14.5° full depth teeth*	*20° full depth teeth*	*20° stub teeth*
Pinion material	Gear material			
Grey CI	Grey CI	5500	5700	5900
Steel	Grey CI	7600	7900	8100
Steel	Steel	11 000	11 400	11 900

The incremental dynamic load (P_d) calculated by Buckingham's equation is far more than the corresponding load calculated by the velocity factor method. Very often, it is three to four times the load (P_t) due to power transmission. It has been observed that in most of the practical cases, the actual dynamic load is less than that of the calculated values by Buckingham's equation. This is because the equation is mainly applicable to large gears with connected masses that rotate at moderate speeds. The actual dynamic load is less in the following cases:

(i) small gears transmitting low power;
(ii) high-speed light-load gears;
(iii) small gears on small shaft;
(iv) gear shafts up to 50 mm diameter; and
(v) small gears transmitting less than 15 kW of power.

Gears on small-diameter shaft easily twist through an angle equivalent to the effective tooth error and the dynamic load is reduced.

The error e in Eq. (17.26) is given by,

$$e = e_p + e_g \tag{17.28}$$

where,

e_p = error for pinion
e_g = error for gear

The error depends upon the quality of the gear and the method of manufacture. There are twelve different grades from Gr. 1 to Gr. 12 in decreasing order of precision[7]. The expected error on the gear tooth is considered to be equal to tolerance.

The tolerances for adjacent pitch error (e) are given in Table 17.8. These tolerances are calculated by using the following basic equation:

$$\phi = m + 0.25\sqrt{d'} \tag{17.29}$$

where,

ϕ = tolerance factor
m = module (mm)
d' = pitch circle diameter (mm)

Table 17.8 *Tolerances on the adjacent pitch*

Grade	*e (microns)*
1	$0.80 + 0.06\,\phi$
2	$1.25 + 0.10\,\phi$
3	$2.00 + 0.16\,\phi$
4	$3.20 + 0.25\,\phi$
5	$5.00 + 0.40\,\phi$
6	$8.00 + 0.63\,\phi$
7	$11.00 + 0.90\,\phi$
8	$16.00 + 1.25\,\phi$
9	$22.00 + 1.80\,\phi$
10	$32.00 + 2.50\,\phi$
11	$45.00 + 3.55\,\phi$
12	$63.00 + 5.00\,\phi$

The method of manufacture for gears depends upon the grade of the gear. Gears of Grade 11 and Grade 12 are manufactured by casting. Gears of Grade 8 and Grade 9 require rough and fine hobbing. Gears of Grade 6 are obtained by hobbing and rough grinding, while Grade 4 requires shaving and finish grinding.

17.20 ESTIMATION OF MODULE BASED ON BEAM STRENGTH

In order to avoid failure of gear tooth due to bending,

$$S_b > P_{\text{eff}}$$

[7] IS 3681—1966: General plan for spur and helical gears.

Introducing a factor of safety,

$$S_b = P_{\text{eff}}\,(fs) \qquad \text{(a)}$$

The recommended factor of safety is from 1.5 to 2. The tangential component is given by,

$$P_t = \frac{2M_t}{d'} = \frac{2M_t}{mz} = \frac{2}{mz}\left\{\frac{60\times10^6(\text{kW})}{2\pi n}\right\}$$

From Eq. (17.24),

$$P_{\text{eff}} = \frac{C_s}{C_v}P_t = \frac{60\times10^6}{\pi}\left\{\frac{(\text{kW})C_s}{mznC_v}\right\} \qquad \text{(b)}$$

From Eq. (17.16),

$$S_b = mb\sigma_b Y = m^2\left(\frac{b}{m}\right)\left(\frac{S_{ut}}{3}\right)Y \qquad \text{(c)}$$

From (a), (b) and (c),

$$m = \left[\frac{60\times10^6}{\pi}\left\{\frac{(\text{kW})C_s(fs)}{znC_v\left(\frac{b}{m}\right)\left(\frac{S_{ut}}{3}\right)Y}\right\}\right]^{1/3} \qquad (17.30)$$

The above equation is used in the preliminary stages of gear design.

17.21 WEAR STRENGTH OF GEAR TOOTH

The failure of the gear tooth due to pitting occurs when the contact stresses between two meshing teeth exceed the surface endurance strength of the material. Pitting is a surface fatigue failure, characterized by small pits on the surface of the gear tooth. In order to avoid this type of failure, the proportions of the gear tooth and surface properties, such as surface hardness, should be selected in such a way that the wear strength of the gear tooth is more than the effective load between the meshing teeth. The analysis of wear strength was done by Earle Buckingham, in his paper 'The relation of load to wear of gear teeth', which was submitted before the American Gear Manufacturing Association (AGMA) in 1926. Buckingham's equation gives the wear strength of the gear tooth.

Buckingham's equation is based on Hertz theory of contact stresses. When two cylinders are pressed together as shown in Fig. 17.42(a), the contact stress is given by,

$$\sigma_c = \frac{2P}{\pi b l} \qquad \text{(a)}$$

and
$$b = \left[\frac{2P(1-\mu^2)\left(\frac{1}{E_1}+\frac{1}{E_2}\right)}{\pi l\left(\frac{1}{d_1}+\frac{1}{d_2}\right)}\right]^{1/2} \qquad \text{(b)}$$

where,

σ_c = maximum value of the compressive stress (N/mm^2)

P = force pressing the two cylinders together (N)

b = half width of deformation (mm)

l = axial length of the cylinder (mm)

d_1, d_2 = diameters of the two cylinders (mm)

E_1, E_2 = modulii of elasticity of two cylinder materials (N/mm^2)

μ = Poisson's ratio

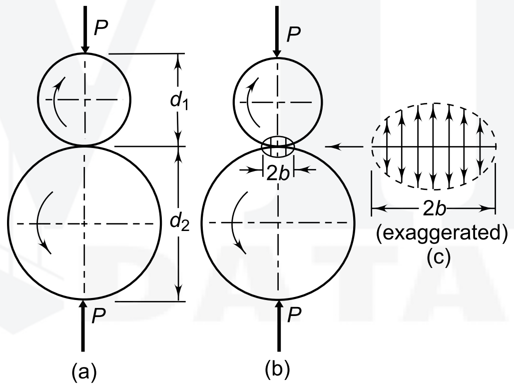


Fig. 17.42 *Contact Stresses*

Due to deformation under the action of load P, a rectangular surface of width (2b) and length (l) is formed between the two cylinders. The elliptical stress distribution across the width (2b) is shown in Fig. 17.42(b) and (c).

Substituting Eq. (b) in Eq. (a) and squaring both sides,

$$\sigma_c^2 = \frac{1}{\pi\,(1-\mu^2)}\left(\frac{P}{l}\right)\left\{\frac{\left(\frac{1}{r_1}+\frac{1}{r_2}\right)}{\left(\frac{1}{E_1}+\frac{1}{E_2}\right)}\right\} \qquad \text{(c)}$$

where r_1, r_2 are the radii of two cylinders.

Substituting ($\mu = 0.3$),

$$\sigma_c^2 = 0.35\left(\frac{P}{l}\right)\left\{\frac{\left(\frac{1}{r_1}+\frac{1}{r_2}\right)}{\left(\frac{1}{E_1}+\frac{1}{E_2}\right)}\right\} \qquad \text{(d)}$$

The above equation of the contact stress is based on the following assumptions:

(i) The cylinders are made of isotropic materials.

(ii) The elastic limit of the material is not exceeded.

(iii) The dimensions r_1, r_2 are very large when compared to the width ($2b$) of the deformation.

Figure 17.43 shows the contact between two meshing teeth at the pitch point. The radii r_1 and r_2

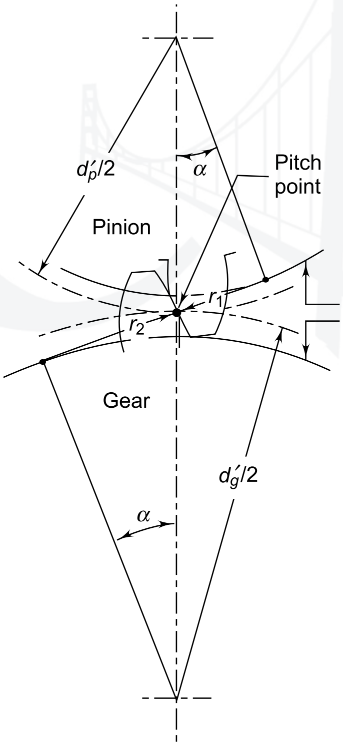


Fig. 17.43

in Eq. (d) are to be replaced by the radii of curvature at the pitch point. Therefore,

$$r_1 = \frac{d'_p \sin\alpha}{2} \quad \text{and} \quad r_2 = \frac{d'_g \sin\alpha}{2}$$

There are two reasons for taking the radii of curvature at the pitch point. The wear on the gear tooth generally occurs at or near the pitch line. When only one pair of teeth carries the entire load, the contact occurs at the pitch point. When the contact takes place at the top or at the bottom of the tooth profile, usually two pairs of meshing teeth share the load. The dynamic load is imposed on the gear tooth near the pitch line area. Therefore, it is more reasonable to select the radii of curvature at the pitch point.

$$\left(\frac{1}{r_1}+\frac{1}{r_2}\right) = \frac{2}{\sin\alpha}\left[\frac{1}{d'_p}+\frac{1}{d'_g}\right] \qquad \text{(e)}$$

A ratio factor Q is defined as,

$$Q = \frac{2z_g}{z_g + z_p} \qquad (17.31)$$

Substituting, ($d'_p = mz_p$) and ($d'_g = mz_g$) in Eq. (17.31),

$$Q = \frac{2d'_g}{d'_g + d'_p} \qquad \text{(f)}$$

Therefore,

$$\left[\frac{1}{d'_p}+\frac{1}{d'_g}\right] = \frac{d'_p + d'_g}{d'_p\, d'_g} = \frac{2}{Qd'_p} \qquad \text{(g)}$$

From (e) and (g),

$$\left(\frac{1}{r_1}+\frac{1}{r_2}\right) = \frac{4}{Qd'_p \sin\alpha} \qquad \text{(h)}$$

The force acting along the pitch line in Fig. 17.43 is P_N. Therefore,

$$P = P_N = \frac{P_t}{\cos\alpha} \qquad \text{(j)}$$

The axial length of the gears is the face width b, i.e.,

$$l = b \qquad \text{(k)}$$

Substituting Eqs (h), (j) and (k) in Eq. (d),

$$\sigma_c^2 = \frac{1.4P_t}{bQd'_p \sin\alpha\cos\alpha(1/E_1 + 1/E_2)} \qquad \text{(l)}$$

A load-stress factor K is defined as,

$$K = \frac{\sigma_c^2 \sin\alpha\cos\alpha(1/E_1 + 1/E_2)}{1.4} \qquad (17.32)$$

Substituting the above equation in Eq. (l),

$$P_t = bQd'_p K \qquad \text{(m)}$$

This equation gives a relationship between the tangential force P_t and the corresponding contact stress σ_c (or K). When the tangential force is increased, the contact stress also increases. Pitting occurs when the contact stress reaches the magnitude of the surface endurance strength. The corresponding value of P_t is called *wear strength*. *Therefore, the wear strength is the maximum value of the tangential force that the tooth can transmit without pitting failure.* Replacing (P_t) by (S_w), Eq. (m) is written as,

$$S_w = bQd'_p K \qquad (17.33)$$

where,

S_w = wear strength of the gear tooth (N)

σ_c = surface endurance strength of the material (N/mm^2)

Equation (17.33) is known as *Buckingham's equation for wear*. The ratio factor for internal gears is defined as,

$$Q = \frac{2z_g}{z_g - z_p} \qquad (17.34)$$

The expression for the load-stress factor K can be simplified when both the gears are made of steel with a 20° pressure angle. In this special case,

$E_1 = E_2 = 206\,000$ N/mm^2

$\alpha = 20°$

According to G Niemann,[8]

$\sigma_c = 0.27$ (BHN) kgf/mm^2

$= 0.27$ (9.81)(BHN) N/mm^2

where BHN is the Brinell Hardness Number.

Therefore,

$$K = \frac{\sigma_c^2 \sin\alpha \cos\alpha\,(1/E_1 + 1/E_2)}{1.4}$$

$$= \frac{(0.27 \times 9.81)^2 (\text{BHN})^2 \sin(20)\cos(20)(2/206\,000)}{1.4}$$

$$= 0.156\left(\frac{\text{BHN}}{100}\right)^2$$

or

$$K = 0.16\left(\frac{\text{BHN}}{100}\right)^2 \qquad (17.35)$$

The above equation is applicable only when both the gears are made of steel with a 20° pressure angle. In other cases, Eq. (17.32) should be used. Table 17.9 gives the material properties for the calculation of K. In order to avoid failure of the gear tooth due to pitting, the wear strength should be more than the effective force between the meshing teeth.

Table 17.9 *Values of modulus of elasticity and Poisson's ratio for gear materials*

Material	*Modulus of elasticity (N/mm^2)*	*Poisson's ratio*
Steel	206 000	0.3
Cast steel	202 000	0.3
Spheroidal cast iron	173 000	0.3
Cast tin bronze	103 000	0.3
Tin bronze	113 000	0.3
Grey cast iron	118 000	0.3

17.22 ESTIMATION OF MODULE BASED ON WEAR STRENGTH

In order to avoid failure of gear tooth due to pitting,

$$S_w > P_{\text{eff}}$$

Introducing a factor of safety,

$$S_w = P_{\text{eff}}\,(fs) \qquad \text{(a)}$$

The recommended factor of safety is from 1.5 to 2. As discussed in Section 17.20(Eq. b), the effective load is given by,

$$P_{\text{eff}} = \frac{60 \times 10^6}{\pi}\left\{\frac{(\text{kW})C_s}{mznC_v}\right\} \qquad \text{(b)}$$

From Eq. (17.33),

$$S_w = bQd'_p K = m\left(\frac{b}{m}\right) Q(mz_p)K$$

[8] Gustav Niemann—'Machine Elements—Design and Calculations in Mechanical Engineering'—Springer-Verlag, Berlin, Vol. 1, page 240.

or $$S_w = m^2 \left(\frac{b}{m}\right) Q z_p K \qquad \text{(c)}$$

Substituting (b) and (c) in (a), we have

$$m = \left[\frac{60 \times 10^6}{\pi} \left\{ \frac{(\text{kW}) C_s (fs)}{z_p^2 n_p C_v \left(\frac{b}{m}\right) Q K} \right\} \right]^{1/3} \qquad (17.36)$$

Example 17.7 *It is required to design a pair of spur gears with 20° full-depth involute teeth based on the Lewis equation. The velocity factor is to be used to account for dynamic load. The pinion shaft is connected to a 10 kW, 1440 rpm motor. The starting torque of the motor is 150% of the rated torque. The speed reduction is 4 : 1. The pinion as well as the gear is made of plain carbon steel 40C8 (S_{ut} = 600 N/mm²). The factor of safety can be taken as 1.5. Design the gears, specify their dimensions and suggest suitable surface hardness for the gears.*

Solution

Given kW = 10 n = 1440 rpm i = 4
S_{ut} = 600 N/mm² (fs) = 1.5
starting torque = 150% (rated torque)

Step I *Estimation of module based on beam strength*

Since both gears are made of the same material, the pinion is weaker than the gear. The minimum number of teeth for 20° pressure angle is 18. Therefore,

$$z_p = 18$$
$$z_g = i\, z_p = 4(18) = 72$$

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n_p} = \frac{60 \times 10^6 (10)}{2\pi (1440)}$$
$$= 66\ 314.56 \text{ N-mm}$$

The Lewis form factor is 0.308 for 18 teeth. (Table 17.3)

$$Y = 0.308$$

$$C_s = \frac{\text{starting torque}}{\text{rated torque}} = 1.5$$

The velocity factor is unknown at this stage. Assuming a trial value for the pitch line velocity as 5 m/s,

$$C_v = \frac{3}{3+v} = \frac{3}{3+5} = \frac{3}{8}$$

It is assumed that the ratio (b/m) is 10. From Eq. (17.30),

$$m = \left[\frac{60 \times 10^6}{\pi} \left\{ \frac{(\text{kW}) C_s (fs)}{z_p n_p C_v \left(\frac{b}{m}\right)\left(\frac{S_{ut}}{3}\right) Y} \right\} \right]^{1/3}$$

$$= \left[\frac{60 \times 10^6}{\pi} \left\{ \frac{(10)(1.5)(1.5)}{(18)(1440)\left(\frac{3}{8}\right)(10)\left(\frac{600}{3}\right)(0.308)} \right\} \right]^{1/3}$$

= 4.16 mm

Step II *Selection of module*

The first preference value of the module is 5 mm.

Trial 1

$$m = 5 \text{ mm}$$
$$d'_p = m z_p = 5(18) = 90 \text{ mm}$$
$$d'_g = m z_g = 5(72) = 360 \text{ mm}$$
$$b = 10 \text{ m} = 10(5) = 50 \text{ mm}$$

Check for design

$$P_t = \frac{2M_t}{d'_p} = \frac{2(66\ 314.56)}{90} = 1473.66 \text{ N}$$

$$v = \frac{\pi\, d'_p\, n_p}{60 \times 10^3} = \frac{\pi (90)(1440)}{60 \times 10^3} = 6.7858 \text{ m/s}$$

$$C_v = \frac{3}{3+v} = \frac{3}{3+6.7858} = 0.3066$$

$$P_{\text{eff}} = \frac{C_s}{C_v} P_t = \frac{1.5(1473.66)}{0.3066} = 7209.69 \text{ N}$$

From Eq. (17.16),

$$S_b = m b \sigma_b Y = 5\,(50)\,(200)\,(0.308) = 15\,400 \text{ N}$$

$$(fs) = \frac{S_b}{P_{\text{eff}}} = \frac{15\,400}{7209.69} = 2.14$$

The design is satisfactory and the module should be 5 mm.

Step III *Surface hardness for gears*

$$Q = \frac{2z_g}{z_g + z_p} = \frac{2(72)}{72+18} = 1.6$$

$$S_w = bQd'_p\ K = 50(1.6)(90)(0.16)\left(\frac{\text{BHN}}{100}\right)^2$$

$$= 1152\left(\frac{\text{BHN}}{100}\right)^2$$

Since, $S_w = P_{\text{eff}}\,(fs)$

$$\therefore \quad 1152\left(\frac{\text{BHN}}{100}\right)^2 = 7209.69\ (1.5)$$

$$\therefore \quad \text{BHN} = 306.39 \quad \text{or} \quad 310$$

Example 17.8 *It is required to design a pair of spur gears with 20° full-depth involute teeth consisting of a 20-teeth pinion meshing with a 50 teeth gear. The pinion shaft is connected to a 22.5 kW, 1450 rpm electric motor. The starting torque of the motor can be taken as 150% of the rated torque. The material for the pinion is plain carbon steel Fe 410 (S_{ut} = 410 N/mm²), while the gear is made of grey cast iron FG 200 (S_{ut} =200 N/mm²). The factor of safety is 1.5. Design the gears based on the Lewis equation and using velocity factor to account for the dynamic load.*

Solution

Given kW = 22.5 n = 1450 rpm $z_p = 20$
$z_g = 50$ starting torque = 150% (rated torque)
$(fs) = 1.5$ For pinion, $S_{ut} = 410$ N/mm²
For gear, $S_{ut} = 200$ N/mm²

Step I *Deciding weaker between pinion and gear*
The materials of pinion and gear are different. From Table 17.3, the Lewis form factors for 20 and 50 teeth are 0.32 and 0.408 respectively.

For pinion,

$$\sigma_b = \frac{S_{ut}}{3} = \frac{410}{3} = 136.67 \text{ N/mm}^2$$

$$(\sigma_b \times Y) = 136.67 \times 0.32 = 43.73$$

For gear,

$$\sigma_b = \frac{S_{ut}}{3} = \frac{200}{3} = 66.67 \text{ N/mm}^2$$

$$(\sigma_b \times Y) = 66.67 \times 0.408 = 27.20$$

The product ($\sigma_b \times Y$) is less for the gear. The gear is weaker than the pinion and it is necessary to design the gear.

Step II *Estimation of module based on beam strength*
For gear,

$$n_g = \frac{1450(20)}{50} = 580 \text{ rpm}$$

The velocity factor is unknown at this stage. Assuming the pitch line velocity as 5 m/s,

$$C_v = \frac{3}{3+v} = \frac{3}{3+5} = \frac{3}{8}$$

From Eq. (17.30),

$$m = \left[\frac{60\times10^6}{\pi}\left\{\frac{(\text{kW})C_s\,(fs)}{z_g n_g C_v \left(\frac{b}{m}\right)\left(\frac{S_{ut}}{3}\right) Y}\right\}\right]^{1/3}$$

$$= \left[\frac{60\times10^6}{\pi}\left\{\frac{(22.5)(1.5)(1.5)}{(50)(580)\left(\frac{3}{8}\right)(10)\left(\frac{200}{3}\right)(0.408)}\right\}\right]^{1/3}$$

$= 6.89$ mm

Step III *Selection of module*
The standard value of the module under Choice 2 of Table 17.2 is 7 mm.

Trial 1

$$m = 7 \text{ mm}$$

$$d'_p = mz_p = 7(20) = 140 \text{ mm}$$

$$d'_g = mz_g = 7(50) = 350 \text{ mm}$$

$$b = 10\text{ m} = 10(7) = 70 \text{ mm}$$

Check for design

$$M_t = \frac{60\times10^6\,(\text{kW})}{2\pi n_g} = \frac{60\times10^6\,(22.5)}{2\pi(580)}$$

$$= 370\ 446.85 \text{ N-mm}$$

$$P_t = \frac{2M_t}{d'_g} = \frac{2(370\ 446.85)}{350} = 2116.84 \text{ N}$$

$$v = \frac{\pi\, d'_p\, n_p}{60\times10^3} = \frac{\pi(140)(1450)}{60\times10^3} = 10.63 \text{ m/s}$$

$$C_v = \frac{6}{6+v} = \frac{6}{6+10.63} = 0.3608$$

$$P_{\text{eff}} = \frac{C_s}{C_v} P_t = \frac{1.5\ (2116.84)}{0.3608} = 8800.61 \text{ N}$$

$$S_b = mb\sigma_b Y = 7(70)\left(\frac{200}{3}\right)(0.408) = 13\,328 \text{ N}$$

$$(fs) = \frac{S_b}{P_{\text{eff}}} = \frac{13\,328}{8800.61} = 1.51$$

The design is satisfactory and the module should be 7 mm.

Example 17.9 *A pair of spur gears with 20° full-depth involute teeth consists of a 19 teeth pinion meshing with a 40 teeth gear. The pinion is mounted on a crankshaft of 7.5 kW single cylinder diesel engine running at 1500 rpm. The driven shaft is connected to a two-stage compressor. Assume the service factor as 1.5. The pinion as well as the gear is made of steel 40C8 (S_{ut} = 600 N/mm^2). The module and face width of the gears are 4 and 40 mm respectively.*

(i) Using the velocity factor to account for the dynamic load, determine the factor of safety.
(ii) If the factor of safety is two for pitting failure, recommend surface hardness for the gears.
(iii) If the gears are machined to meet the specifications of Grade 8, determine the factor of safety for bending using Buckingham's equation for dynamic load.
(iv) Is the gear design satisfactory? If not, what is the method to satisfy the design conditions? How will you modify the design?

Solution

Given kW = 7.5 n = 1500 rpm z_p = 19
z_g = 40 m = 4 mm b = 40 mm C_s = 1.5
S_{ut} = 600 N/mm^2 Grade of machining - 8

Step I *Factor of safety based on dynamic load by velocity factor*

Beam strength

Since both gears are made of the same material, the pinion is weaker than the gear. The Lewis form factor for 19 teeth is 0.314 (Table 17.3). The permissible bending stress is one-third of the ultimate tensile strength or 200 N/mm^2. Therefore,

$$S_b = mb\sigma_b Y = 4(40)(200)(0.314) = 10\ 048 \text{ N}$$

Tangential force due to rated torque

$$d'_p = mz_p = 4(19) = 76 \text{ mm}$$

$$M_t = \frac{60\times10^6 (\text{kW})}{2\ \pi n_p} = \frac{60\times10^6 (7.5)}{2\pi(1500)}$$

$$= 47\ 746.48 \text{ N-mm}$$

$$P_t = \frac{2M_t}{d'_p} = \frac{2(47\ 746.48)}{76} = 1256.49 \text{ N}$$

Effective load

$$v = \frac{\pi\ d'_p n_p}{60\times10^3} = \frac{\pi(76)(1500)}{60\times10^3} = 5.969 \text{ m/s}$$

$$C_v = \frac{3}{3+v} = \frac{3}{3+5.969} = 0.3345$$

$$P_{\text{eff}} = \frac{C_s}{C_v} P_t = \frac{1.5(1256.49)}{0.3345} = 5634.48 \text{ N}$$

Factor of safety

$$(fs) = \frac{S_b}{P_{\text{eff}}} = \frac{10\,048}{5634.48} = 1.78$$

Step II *Surface hardness for gears with (fs) as 2*

$$S_w = P_{\text{eff}}\,(fs) = 5634.48(2.0) = 11\,268.96 \text{ N}$$

$$Q = \frac{2z_g}{z_g + z_p} = \frac{2(40)}{40+19} = 1.356$$

$$S_w = bQd'_pK$$

or
$$11\,268.96 = 40\ (1.356)(76)(0.16)\left(\frac{\text{BHN}}{100}\right)^2$$

$\therefore$ BHN = 413.35 or 420

Step III *Factor of safety based on dynamic load by Buckingham's equation*

For Grade 8,

$$e = 16 + 1.25\ \phi$$

For pinion,

$$\phi = m + 0.25\ \sqrt{d'_p} = 4 + 0.25\ \sqrt{76}$$

$$e_p = 16 + 1.25\ \phi = 23.72\ \mu\text{m}$$

For gear,

$$d'_g = m\ z_g = 4(40) = 160 \text{ mm}$$

$$\phi = m + 0.25\ \sqrt{d'_g} = 4 + 0.25\ \sqrt{160}$$

$$e_g = 16 + 1.25\phi = 24.95\ \mu\text{m}$$

$$\therefore\quad e = e_p + e_g = 23.72 + 24.95 = 48.67\ \mu\text{m}$$
$$= (48.67 \times 10^{-3}) \text{ mm}$$

From Table 17.7, the value of the deformation factor C is 11 400 N/mm^2. Also,

$v = 5.969$ m/s $\quad b = 40$ mm $\quad P_t = 1256.49$ N

From Eq. (17.26),

$$P_d = \frac{21v(Ceb + P_t)}{21v + \sqrt{(Ceb + P_t)}}$$

$$= \frac{21(5.969)\left[11400\,(48.67 \times 10^{-3})(40) + 1256.49\right]}{21(5.969) + \sqrt{\left[11400\,(48.67 \times 10^{-3})(40) + 1256.49\right]}}$$

$= 10\,555.17$ N

$P_{\text{eff}} = (C_s P_t + P_d) = 1.5(1256.49) + 10\,555.17$
$= 12\,439.91$ N

Since,

$$S_b = 10\,048 \text{ N} \quad \text{and} \quad S_w = 11\,268.96 \text{ N}$$

$$\therefore \quad S_b < P_{\text{eff}} \quad \text{and} \quad S_w < P_{\text{eff}}$$

The design is unsatisfactory both from the standpoint of strength and wear.

Step IV *Modification of design*

We will select a finer grade for the manufacture to reduce the dynamic load. It is assumed that the gears are manufactured according to Grade 6. For this grade,

$$e = 8 + 0.63\,\phi$$

For pinion,

$$\phi = m + 0.25\,\sqrt{d'_p} = 4 + 0.25\,\sqrt{76}$$

$$e_p = 8 + 0.63\,\phi = 11.893\ \mu\text{m}$$

For gear,

$$\phi = m + 0.25\,\sqrt{d'_g} = 4 + 0.25\,\sqrt{160}$$

$$e_g = 8 + 0.63\,\phi = 12.512\ \mu\text{m}$$

$$\therefore \quad e = e_p + e_g = 11.893 + 12.512$$
$$= 24.405\ \mu\text{m or } (24.405 \times 10^{-3}) \text{ mm}$$

From Eq. (17.26),

$$P_d = \frac{21v\,(Ceb + P_t)}{21v + \sqrt{(Ceb + P_t)}}$$

$$= \frac{21(5.969)\,[11\,400\,(24.405 \times 10^{-3})(40) + 1256.49]}{21(5.969) + \sqrt{[11400\,(24.405 \times 10^{-3})(40) + 1256.49]}}$$

$= 6560.53$ N

$P_{\text{eff}} = (C_s P_t + P_d) = 1.5(1256.49) + 6560.53$
$= 8445.26$ N

The beam strength is lower than the wear strength. Therefore,

$$(fs) = \frac{S_b}{P_{\text{eff}}} = \frac{10048}{8445.26} = 1.19$$

The design is satisfactory.

Example 17.10 *A pair of spur gears with 20° full-depth involute teeth consists of a 20 teeth pinion meshing with a 41 teeth gear. The module is 3 mm while the face width is 40 mm. The material for pinion as well as gear is steel with an ultimate tensile strength of 600 N/mm^2. The gears are heat-treated to a surface hardness of 400 BHN. The pinion rotates at 1450 rpm and the service factor for the application is 1.75. Assume that velocity factor accounts for the dynamic load and the factor of safety is 1.5.*

Determine the rated power that the gears can transmit.

Solution

Given $n = 1450$ rpm $\quad z_p = 20 \quad z_g = 41$
$m = 3$ mm $\quad b = 40$ mm $\quad C_s = 1.75 \quad (fs) = 1.5$
BHN = 400 $\quad S_{ut} = 600$ N/mm^2

Step I *Beam strength*

Since the same material is used for the pinion and the gear, the pinion is weaker than the gear. From Table 17.3, the Lewis form factor is 0.32 for 20 teeth.

$$\sigma_b = \left(\frac{1}{3}\right) S_{ut} = \left(\frac{1}{3}\right)(600) = 200 \text{ N/mm}^2$$

$$S_b = mb\sigma_b\,Y = 3(40)(200)(0.32) = 7680 \text{ N}$$

Step II *Wear strength*

$$Q = \frac{2z_g}{z_g + z_p} = \frac{2(41)}{41 + 20} = 1.344$$

$$K = 0.16\left(\frac{\text{BHN}}{100}\right)^2 = 0.16\left(\frac{400}{100}\right)^2 = 2.56$$

$$d'_p = mz_p = 3(20) = 60 \text{ mm}$$

$$S_\omega = bQd'_pK = 40(1.344)(60)(2.56)$$
$$= 8257.54 \text{ N}$$

Step III *Effective Load*

$$v = \frac{\pi d'_p n_p}{60 \times 10^3} = \frac{\pi(60)(1450)}{60 \times 10^3} = 4.5553 \text{ m/s}$$

$$C_v = \frac{3}{3+v} = \frac{3}{3+4.5553} = 0.397$$

$$P_{\text{eff}} = \frac{C_s}{C_v} P_t = \frac{1.75}{0.397} P_t = (4.41\, P_t) \text{ N}$$

Step IV *Static load*

In this example, the beam strength is lower than the wear strength. Therefore, beam strength is the criterion of design.

$$S_b = P_{\text{eff}}\,(fs) \quad \text{or} \quad 7680 = (4.41\, P_t)\,(1.5)$$

$$\therefore \quad P_t = 1161 \text{ N}$$

Step V *Rated power*

$$M_t = \frac{P_t d'_p}{2} = \frac{1161(60)}{2} = 34\;830 \text{ N-mm}$$

$$kW = \frac{2\pi n_p M_t}{60 \times 10^6} = \frac{2\pi\,(1450)(34\;830)}{60 \times 10^6} = 5.29$$

Example 17.11 *It is required to design a spur gear speed reducer for a compressor running at 250 rpm driven by a 7.5 kW, 1000 rpm electric motor. The centre distance between the axes of the gear shafts should be exactly 250 mm. The starting torque of the motor can be assumed to be 150% of the rated torque. The gears are made of carbon steel 50C4 (S_{ut} = 700 N/mm^2). The pressure angle is 20°. The factor of safety is 2 for preliminary design based on the use of velocity factor.*

(i) Design the gears and specify their dimensions.
(ii) Assume that the gears are manufactured to meet the requirements of Grade 6 and calculate the dynamic load by using Buckingham's equation.
(iii) Calculate the effective load.
(iv) What is the actual factor of safety against bending failure?
(v) Using the same factor of safety against pitting failure, specify suitable surface hardness for the gears.

Solution

Given $kW = 7.5 \quad n_p = 1000$ rpm
$n_g = 250$ rpm $\quad a = 250$ mm
starting torque = 150% (rated torque)
$S_{ut} = 700$ N/mm^2 $\quad (fs) = 2$
Grade of machining-6

Step I *Estimation of module based on beam strength*

$$a = \frac{1}{2}(d'_p + d'_g) \quad \text{or} \quad 250 = \frac{1}{2}(d'_p + d'_g)$$

$$(d'_p + d'_g) = 500 \text{ mm} \qquad \text{(a)}$$

Also,
$$\frac{d'_g}{d'_p} = \frac{n_p}{n_g} = \frac{1000}{250} = 4 \qquad \text{(b)}$$

From (a) and (b),

$$d'_p = 100 \text{ mm} \quad \text{and} \quad d'_g = 400 \text{ mm}$$

$$v = \frac{\pi d'_p n_p}{60 \times 10^3} = \frac{\pi(100)(1000)}{60 \times 10^3} = 5.236 \text{ m/s}$$

$$C_v = \frac{3}{3+v} = \frac{3}{3+5.236} = 0.3643$$

The face width is assumed to be ten times that of the module. The Lewis form factor is unknown at this stage. It varies from 0.32 (for 20 teeth) to 0.358 (for 30 teeth). Assuming an intermediate value,

$$Y = 0.34$$

$$S_b = mb\sigma_b Y = m(10\,m)\left(\frac{700}{3}\right)(0.34)$$

$$= (793.33\, m^2) \text{ N} \qquad \text{(c)}$$

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n_p} = \frac{60 \times 10^6 (7.5)}{2\pi(1000)}$$

$$= 71\;619.93 \text{ N-mm}$$

$$P = \frac{2M_t}{d'_p} = \frac{2(71\;619.93)}{100} = 1432.39 \text{ N}$$

$$P_{\text{eff}} = \frac{C_s}{C_v} P_t = \frac{1.5\,(1432.39)}{0.3643} = 5897.85 \text{ N} \qquad \text{(d)}$$

From (c) and (d),

$$P_{\text{eff}}\,(fs) = S_b \quad \text{or} \quad 5897.85(2) = 793.33\, m^2$$

$$\therefore \quad m = 3.86 \text{ mm}$$

Step II *Gear dimensions*

The first preference value of the module is 4 mm.

$$m = 4 \text{ mm}$$
$$b = 10 \text{ m} = 10(4) = 40 \text{ mm}$$

$$z_p = \frac{d'_p}{m} = \frac{100}{4} = 25$$

$$z_g = \frac{d'_g}{m} = \frac{400}{4} = 100$$

Step III *Beam strength*

From Table 17.3, the Lewis form factor for 25 teeth is 0.34.

$$S_b = mb\sigma_b Y = 4(40)\left(\frac{700}{3}\right)(0.34) = 12\,693.33 \text{ N}$$

Step IV *Dynamic load by Buckingham's equation*

For Grade 6,

$$e = 8 + 0.63\,\phi$$

For pinion,

$$\phi = m + 0.25\sqrt{d'_p} = 4 + 0.25\sqrt{100}$$
$$e_p = 8 + 0.63\,\phi = 12.095\ \mu\text{m}$$

For gear,

$$\phi = m + 0.25\sqrt{d'_g} = 4 + 0.25\sqrt{400}$$
$$e_g = 8 + 0.63\,\phi = 13.67\ \mu\text{m}$$

$$\therefore \quad e = e_p + e_g = 12.095 + 13.67$$
$$= 25.765\ \mu\text{m or } (25.765 \times 10^{-3}) \text{ mm}$$

From Table 17.7, the value of deformation factor C is 11 400 N/mm^2. Also,

$$v = 5.236 \text{ m/s} \quad b = 40 \text{ mm} \quad P_t = 1432.39 \text{ N}$$

From Eq. (17.26),

$$P_d = \frac{21v\,(Ceb + P_t)}{21v + \sqrt{(Ceb + P_t)}}$$

$$= \frac{21\,(5.236)[11400(25.765 \times 10^{-3})(40) + 1432.39]}{21(5.236) + \sqrt{[11400(25.765 \times 10^{-3})(40) + 1432.39]}}$$

$$= 6448.30 \text{ N}$$

Step V *Effective load*

$$P_{\text{eff}} = (C_s P_t + P_d) = 1.5(1432.39) + 6448.30$$
$$= 8596.89 \text{ N}$$

Step VI *Actual factor of safety against bending failure*

$$\therefore \quad (fs) = \frac{S_b}{P_{\text{eff}}} = \frac{12\,693.33}{8596.89} = 1.48$$

The design is satisfactory and the module should be 4 mm.

Step VII *Surface hardness for gears*

$$Q = \frac{2z_g}{z_g + z_p} = \frac{2(100)}{100 + 25} = 1.6$$

$$P_{\text{eff}}\,(fs) = S_w \quad \text{or} \quad P_{\text{eff}}\,(fs) = bQd'_pK$$

$$\therefore \quad 8596.89\,(1.48) = 40(1.6)(100)(0.16)\left(\frac{\text{BHN}}{100}\right)^2$$

$$\text{BHN} = 352.49 \text{ or } 360$$

Step VIII *Dimensions of gears*

(i) number of teeth on pinion = 25
(ii) number of teeth on gear = 100
(iii) module = 4 mm
(iv) face width = 40 mm
(v) pitch circle diameter of pinion = 100 mm
(vi) pitch circle diameter of gear = 400 mm
(vii) addendum (m) = 4 mm
(viii) dedendum (1.25 m) = 5 mm
(ix) clearance (0.25 m) = 1 mm
(x) tooth thickness (1.5708 m) = 6.2832 mm
(xi) fillet radius (0.4 m) = 1.6 mm

Example 17.12 *It is required to design a two-stage spur gear reduction unit with 20° full-depth involute teeth. The input shaft rotates at 1440 rpm and receives 10 kW power through a flexible coupling. The speed of the output shaft should be approximately 180 rpm. The gears are made of plain carbon steel 45C8 (S_{ut} = 700 N/mm^2) and heat-treated to a surface hardness of 340 BHN. The gears are to be machined to the requirement, of Grade 6. The service factor can be taken as 1.5.*

(i) Assuming that the dynamic load to be proportional to the pitch-line velocity, estimate the required value of the module. The factor of safety is 1.5.

(ii) Select the first preference value of the module and determine the correct value of factor of safety for bending, using Buckingham's equation.

(iii) Determine the factor of safety against pitting.

(iv) Give a list of gear dimensions.

Solution

Given $kW = 10$ $n = 1440$ rpm $C_s = 1.5$
$S_{ut} = 700$ N/mm^2 $(fs) = 1.5$ BHN = 340
Grade of machining = 6
speed of output shaft = 180 rpm

Step I *Estimation of module based on beam strength*

The total transmission ratio i' is given by,

$$i' = \frac{\text{angular velocity of the first driving gear}}{\text{angular velocity of the last driven gear}}$$

$$= \frac{1440}{180} = 8$$

As discussed in Section 17.15, the speed reduction at each stage (i) is given by,

$$i = \sqrt{i'} = \sqrt{8} = 2.8284$$

For a 20° pressure angle, the minimum number of teeth to avoid interference is 18.

$\therefore$ $z_p = 18$ and $z_g = i\,z_p = 2.8284\,(18)$
$= 50.91$ or 51

The layout of gears is shown in Fig. 17.44. For ease of manufacturing, the pinions 1 and 3 are made identical, while gears 2 and 4 are also made identical.

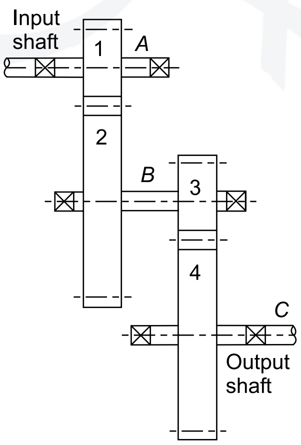


Fig. 17.44

$z_1 = z_3 = 18$ and $z_2 = z_4 = 51$

The speeds of the shafts are as follows,

$$n_A = 1440 \text{ rpm}$$

$$n_B = n_A \times \left(\frac{z_1}{z_2}\right) = 1440 \times \left(\frac{18}{51}\right) = 508.235 \text{ rpm}$$

$$n_C = n_B \times \left(\frac{z_3}{z_4}\right) = 508.235 \times \left(\frac{18}{51}\right) = 179.38 \text{ rpm}$$

The speed of the output shaft is 179.38 or approximately 180 rpm. The velocity factor is unknown at this stage. Assuming the pitch line velocity as 5 m/s.

$$C_v = \frac{3}{3+v} = \frac{3}{3+5} = \frac{3}{8}$$

The pair at the second stage, i.e., pinion 3 and gear 4, transmits more torque than the pair consisting of gears 1 and 2. Therefore, pinion 3 and gear 4 are to be designed. From Table 17.3, the Lewis form factor for 18 teeth is 0.308.

From Eq. (17.30),

$$m = \left[\frac{60\times10^6}{\pi}\left\{\frac{(\text{kW})C_s(fs)}{z_3 n_3 C_v \left(\frac{b}{m}\right)\left(\frac{S_{ut}}{3}\right)Y}\right\}\right]^{1/3}$$

$$= \left[\frac{60\times10^6}{\pi}\left\{\frac{(10)(1.5)(1.5)}{(18)(508.235)\left(\frac{3}{8}\right)(10)\left(\frac{700}{3}\right)(0.308)}\right\}\right]^{1/3}$$

$= 5.59$ mm

Step II *Gear dimensions*

The first preference value of the module is 6 mm.

$$m = 6 \text{ mm}$$
$$d_3' = mz_3 = 6(18) = 108 \text{ mm}$$
$$d_4' = mz_4 = 6(51) = 306 \text{ mm}$$
$$b = 10\text{ m} = 10(6) = 60 \text{ mm}$$

Step III *Factor of safety using Buckingham's equation*

Beam strength

$$S_b = mb\sigma_b Y = 6(60)\left(\frac{700}{3}\right)(0.308) = 25\,872 \text{ N}$$

Tangential force due to rated torque

$$M_t = \frac{60\times10^6\,(\text{kW})}{2\ \pi n_B} = \frac{60\times10^6\,(10)}{2\pi(508.235)}$$

$$= 187\ 891.36\ \text{N-mm}$$

$$P_t = \frac{2M_t}{d_3'} = \frac{2(187\ 891.36)}{108} = 3479.47\ \text{N}$$

Dynamic load

For Grade 6,

$$e = 8 + 0.63\ \phi$$

For pinion,

$$\phi = m + 0.25\ \sqrt{d_3'} = 6 + 0.25\ \sqrt{108}$$

$$e_p = 8 + 0.63\ \phi = 13.4168\ \mu\text{m}$$

For gear,

$$\phi = m + 0.25\ \sqrt{d_4'} = 6 + 0.25\ \sqrt{306}$$

$$e_g = 8 + 0.63\ \phi = 14.5351\ \mu\text{m}$$

$$\therefore \quad e = e_p + e_g = 13.4168 + 14.5351$$

$$= 27.9519\ \mu\text{m or } (27.9519\times10^{-3})\ \text{mm}$$

From Table 17.7, the value of deformation factor C is 11 400 N/mm^2.

$$v = \frac{\pi d_3' n_3}{60\times10^3} = \frac{\pi(108)(508.235)}{60\times10^3} = 2.874\ \text{m/s}$$

Also, $b = 60$ mm $P_t = 3479.47$ N

From Eq. (17.26),

$$P_d = \frac{21v(Ceb + P_t)}{21v + \sqrt{(Ceb + P_t)}}$$

$$= \frac{21(2.874)\left[11400\ (27.9519\times10^{-3})(60) + 3479.47\right]}{21(2.874) + \sqrt{\left[11400(27.9519\times10^{-3})(60) + 3479.47\right]}}$$

$$= 6473.80\ \text{N}$$

Effective load

$$P_{\text{eff}} = (C_s\,P_t + P_d) = 1.5\,(3479.47) + 6473.80$$

$$= 11\,693\ \text{N}$$

Factor of safety

$$\therefore \quad (fs) = \frac{S_b}{P_{\text{eff}}} = \frac{25\,872}{11\,693} = 2.21$$

The design is satisfactory and the module should be 6 mm.

Step IV *Factor of safety against pitting*

$$Q = \frac{2z_g}{z_g + z_p} = \frac{2(51)}{51+18} = 1.4783$$

$$S_w = bQd_p'K = 60(1.4783)(108)(0.16)\left(\frac{340}{100}\right)^2$$

$$= 17\,718.03\ \text{N}$$

$$\therefore \quad (fs) = \frac{S_w}{P_{\text{eff}}} = \frac{17\,718.03}{11693} = 1.52$$

The design is satisfactory and the factor of safety is adequate against pitting failure.

Step V *Gear dimensions*

(i) module = 6 mm
(ii) face width = 60 mm
(iii) addendum = 6 mm
(iv) dedendum = 7.5 mm
(v) fillet radius = 2.4 mm

Gear 1 and 3

(i) pitch circle diameter = 108 mm
(ii) addendum circle diameter = 120 mm
(iii) dedendum circle diameter = 93 mm
(iv) number of teeth = 18

Gear 2 and 4

(i) pitch circle diameter = 306 mm
(ii) addendum circle diameter = 318 mm
(iii) dedendum circle diameter = 291 mm
(iv) number of teeth = 51

17.23 INTERNAL GEARS

An internal gear has teeth which are cut on the inside of the rim as shown in Fig. 17.45. It is also called *annular gear* or *ring gear*. The shape of the tooth of internal gear corresponds to the tooth space of external gear of the same pitch circle diameter. Similarly, the tooth space of internal gear corresponds to the shape of the tooth of external gear.

Internal gears offer the following advantages:

(i) The centre to centre distance between the axes of external pinion and internal gear is small. This results in compact construction as compared to a pair of external gears.

(ii) There are more number of teeth in contact between external pinion and internal gear.

Each pair shares the load simultaneously. The load is not shifted abruptly from one pair to another like external spur gears. This results in smooth and quiet operation.

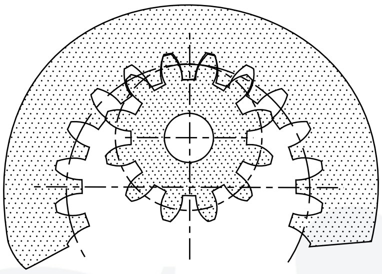


Fig. 17.45 *Internal Gear*

The disadvantages of internal gears are as follows:

(i) The internal gears are costly as compared with external gears.

(ii) The assembly of internal gears is more difficult than that of external spur gears.

Internal gears are used in compact speed reducer. They are mainly used in planetary gear trains.

In the design of internal gears, the following guidelines should be used:

(i) In order to avoid interference, the number of teeth on internal gear should be considerably more than the number of teeth on the external pinion. The minimum difference (z_g–z_p) between the teeth of internal gear and external pinion should be as follows:

8 teeth for 20° stub tooth form
10 teeth for 20° full-depth form
12 teeth for 14.5° full-depth form

(ii) As shown in Fig. 17.46, the tooth of the internal gear corresponds to the tooth space of a similar external gear. This results in relatively thick and strong tooth. Therefore, when the same material is used, the internal gear tooth is always stronger than the tooth of the external pinion. It is not necessary to calculate the beam strength of the internal gear tooth. All that is required is to design external pinion tooth.

(iii) As shown in Fig. 17.46, the concave surface on the tooth of the internal gear is in contact with the convex surface on the tooth of the pinion. This increases the contact area between two teeth. This is more than the contact area between two convex surfaces on external gears. Therefore, the limit load in wear is more in case of internal gears. There is another reason for higher wear strength of internal gears. The ratio factor Q for internal gears is given by,

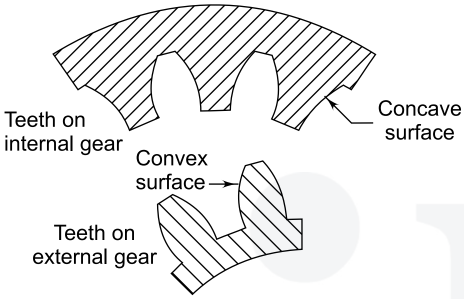


Fig. 17.46

$$Q = \frac{2z_g}{z_g - z_p} = \frac{2(z_g/z_p)}{(z_g/z_p) - 1} = \frac{2i}{i-1}$$

The ratio factor for external gears is given by,

$$Q = \frac{2z_g}{z_g + z_p} = \frac{2(z_g/z_p)}{(z_g/z_p) + 1} = \frac{2i}{i+1}$$

Therefore, the ratio factor for internal gears is more than that of external gears. The wear strength is given by,

$$S_w = bQd'_pK$$

It is observed from the above expressions that wear strength of the tooth of internal gear is more than the tooth of external pinion. It is not necessary to calculate the wear strength of internal gear tooth.

Both from bending and wear considerations, the tooth of internal gear is stronger than the tooth of external pinion. It is not necessary to design the internal gear after the satisfactory design of external pinion.

17.24 GEAR LUBRICATION

Proper lubrication of the gear teeth is essential for the satisfactory performance and durability of the gears. Gears are lubricated by grease, straight mineral oils or EP (extreme pressure) lubricants[9, 10]. Grease is used as lubricant for the gears of hand-operated mechanisms, where the pitch line velocity is low and the operation is intermittent. For medium velocities, the gears are enclosed in a box and dipped in a bath of mineral oil. This is known as *splash lubrication*. In some cases, gears are lubricated by spraying the lubricating oil, in which a jet of oil is directed towards the meshing teeth. When the pitch line velocities are medium, mineral oils of grades SAE 8O, SAE 90 or SAE 140 are used. For heavy duty applications, EP lubricants are used. EP lubricants are mineral oils with additives for the purpose of improving the performance of the oil. They are used in the automobile gear box and heavy duty industrial gearboxes. For splash or spray lubrication, the gearbox has additional features, like oil seals for the shaft, gaskets for the cover, plug for inserting the oil, a drain plug at the bottom and, sometimes, an oil level indicator too.

Short-Answer Questions

17.1 State applications of gear drives.
17.2 State any four advantages of gear drive over other types of drives.
17.3 State any two disadvantages of gear drive over other types of drives.
17.4 In a gear speed reducer, why is the diameter of an output shaft greater than input shaft?
17.5 In which gear drive is self-locking possible?
17.6 What is herringbone gear?
17.7 What are the advantages of cycloidal teeth gears?
17.8 What are the advantages of involute teeth gears?
17.9 State two important reasons for adopting involute curve for gear tooth profile.
17.10 What are the advantages of 14.5° full-depth involute teeth gears?
17.11 What are the advantages of 20° full-depth involute teeth gears?
17.12 What are the advantages of 20° stub involute teeth gears?
17.13 What is full depth involute gear tooth system?
17.14 What is the stub involute gear tooth system?
17.15 Why is the tangential component of gear tooth force called ‘useful’ component?
17.16 Why is the radial component of gear tooth force called ‘separating’ component?
17.17 What is pitting?
17.18 What is scoring?
17.19 What is the minimum number of teeth on spur gear? Why?
17.20 What is a ‘hunting’ tooth?
17.21 Why is the pinion weaker than the gear made of same material?
17.22 State two advantages of internal gears.
17.23 State two disadvantages of internal gears.
17.24 What are the advantages of planetary reduction gears as compared to ordinary gearboxes?
17.25 Where do you use grease as gear lubricant?
17.26 Where do you use oil as gear lubricant?

Problems for Practice

17.1 In a pair of spur gears, the number of teeth on the pinion and the gear are 20 and 100 respectively. The module is 6 mm. Calculate
(i) the centre distance;
(ii) the pitch circle diameters of the pinion and the gear;
(iii) addendum and dedendum;
(iv) tooth thickness and bottom clearance; and
(v) the gear ratio.
[*(i) 360 mm (ii) 120 and 600 mm, (iii) 6 and 7.5 mm, (iv) 9.4248 and 1.5 mm (v) 5*]

[9] IS 1277: Specifications for gear lubricants (regular).
[10] IS 1118: Specifications for gear lubricants-multipurpose (EP gear oils).

17.2 A pinion with 25 teeth and rotating at 1200 rpm drives a gear which rotates at 200 rpm. The module is 4 mm. Calculate the centre distance between the gears.

[*350 mm*]

17.3 A pair of spur gears with a centre distance of 495 mm is used for a speed reduction of 4.5: 1. The module is 6 mm. Calculate the number of teeth on the pinion and the gear.

[*30 and 135*]

17.4 A train of spur gears is shown in Fig. 17.47. Gear 1 is the driving gear and transmits 5 kW power at 720 rpm. The number of teeth on gears 1, 2, 3 and 4 are 20, 50, 30 and 60 respectively. The module for all gears is 4 mm. The gears have a 20° full-depth involute profile. Calculate the tangential and radial components of the tooth force between

(i) Gears 1 and 2 and

(ii) Gears 3 and 4

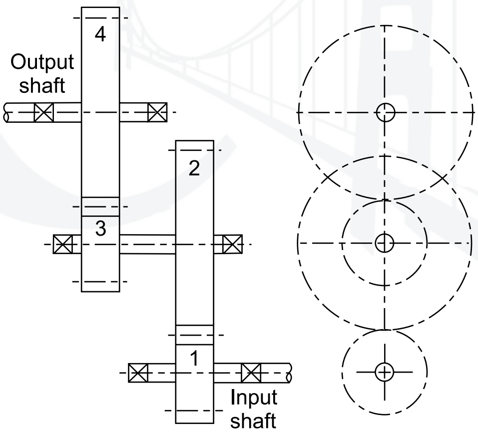


Fig. 17.47

[*(i) 1657.86 and 603.41 N*
(ii) 2763.11 and 1005.69 N]

17.5 A train of gears transmitting power from a 10 kW, 1440 rpm motor to a rope drum is shown in Fig. 17.48. The number of teeth on the various gears is as follows:

$z_1 = 20$ $z_2 = 100$ $z_3 = 25$

$z_4 = 150$ $z_5 = 25$ $z_6 = 150$

The module of gears 1 and 2 is 5 mm, while that of all other gears is 6 mm. The pressure angle is 20°. Calculate

(i) torques acting on shafts *A*, *B*, *C* and *D*;

(ii) tangential and radial components of tooth forces between gears 1 and 2, gears 3 and 4 and gears 5 and 6;

(iii) resultant reactions at bearings B_1 and B_2; and

(iv) resultant reactions at bearings C_1 and C_2.

[*(i) 66 314.56, 331 572.8, 1 989 436.79 and 11 936 620.73 N-mm (ii) 1326.29 and 482.73 N, 4420.97 and 1609.1 N, 26 525.82 and 9654.61 N (iii) 3678.56 and 2213.47 N (iv) 21 210.32 and 10856.55 N*]

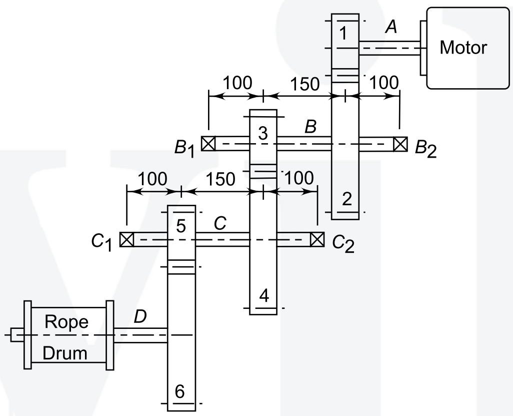


Fig. 17.48

17.6 A train of spur gears with 20° full-depth involute teeth is shown in Fig. 17.49. Gear 1 is the driving gear and transmits 50 kW power at 300 rpm to the gear train. The number teeth on gears 1, 2, 3 and 4 are 30, 60, 25 and 50 respectively, while the module for all gears is 8 mm. Gears 2 and 3 are mounted on the same shaft. Gear 1

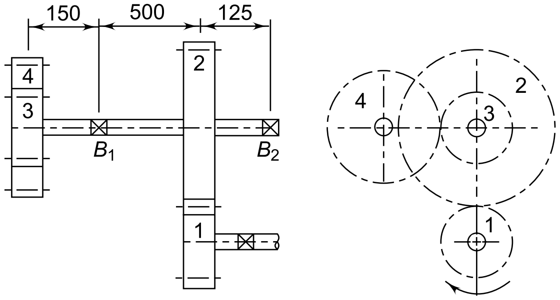


Fig. 17.49

is rotating in the clockwise direction when seen from the left side of the page. Calculate

(i) tangential and radial components of tooth forces between gears 1 and 2 and gears 3 and 4; and
(ii) resultant reactions at bearing B_1 and B_2.

[*(i) 13262.91 and 4827.31 N, 31830.99 and 11585.53 N, (ii) 43871.35 and 8693.44 N*]

17.7 The following data is given for a pair of spur gears with 20° full-depth involute teeth:

number of teeth on pinion = 24
number of teeth on gear = 56
speed of pinion = 1200 rpm
module = 3 mm
service factor = 1.5
face width = 30 mm

Both gears are made of steel with an ultimate tensile strength of. 600 N/mm^2. Using the velocity factor to account for the dynamic load, calculate

(i) beam strength;
(ii) velocity factor; and
(iii) rated power that the gears can transmit without bending failure, if the factor of safety is 1.5.

[*(i) 6066 N (ii) 0.3987 (iii) 4.86 kW*]

17.8 The pitch circle diameters of the pinion and gear are 100 and 300 mm respectively. The pinion is made of plain carbon steel 4OC8 (S_{ut} = 600 N/mm^2) while the gear is made of grey cast iron FG 300 (S_{ut} = 300 N/mm^2). The pinion receives 5 kW power at 500 rpm through its shaft. The service factor and factor of safety can be taken as 1.5 each. The face width of the gear can be taken as ten times that of the module. Assume that the velocity factor accounts for the dynamic load. Calculate

(i) module; and (ii) the number of teeth on the pinion and gear.

[*(i) 5 mm, (ii) 20 and 60*]

17.9 A steel pinion with 20° full depth involute teeth is transmitting 7.5 kW power at 1000 rpm from an electric motor. The starting torque of the motor is twice the rated torque. The number of teeth on the pinion is 25, while the module is 4 mm. The face width is 45 mm. Assuming that velocity factor accounts for the dynamic load, calculate

(i) the effective load on the gear tooth; and
(ii) the bending stresses in the gear tooth.

[*(i) 7863.79 N, (ii) 128.49 N/mm^2*]

17.10 A pair of spur gears with 20° pressure angle, consists of a 25 teeth pinion meshing with a 60 teeth gear. The module is 5 mm, while the face width is 45 mm. The pinion rotates at 500 rpm. The gears are made of steel and heat treated to a surface hardness of 220 BHN. Assume that dynamic load is accounted by means of the velocity factor. The service factor and the factor of safety are 1.75 and 2 respectively. Calculate

(i) wear strength of gears;
(ii) the static load that the gears can transmit without pitting; and
(iii) rated power that can be transmitted by gears.

[*(i) 6149.8 N, (ii) 840.41 N, (iii) 2.75 kW*]

17.11 A pair of spur gears consists of a 24 teeth pinion, rotating at 1000 rpm and transmitting power to a 48 teeth gear. The module is 6 mm, while the face width is 60 mm. Both gears are made of steel with an ultimate tensile strength of 450 N/mm^2. They are heat treated to a surface hardness of 250 BHN. Assume that velocity factor accounts for the dynamic load. Calculate

(i) beam strength;
(ii) wear strength; and
(iii) the rated power that the gears can transmit, if service factor and the factor of safety are 1.5 and 2, respectively.

[*(i) 18 198 N (ii) 11 517.12 N (iii) 8.24 kW*]

17.12 It is required to design a pair of spur gears with 20° full-depth involute teeth. The input shaft rotates at 720 rpm and receives 5 kW power through a flexible coupling. The speed of the output shaft should be 144 rpm. The pinion as well as the gear are made of steel Fe 410 (S_{ut} = 410 N/mm^2). The service factor for the application is 1.25. The gears are machined to meet the specifications of Grade 6.

(i) Assume suitable number of teeth for the pinion and the gear.

(ii) For preliminary calculations, assume the pitch line velocity as 5 m/s and the factor of safety as 2. Estimate the module and select the first preference value of the module.

(iii) Using this value of the module, calculate the pitch circle diameters of the pinion and gear and the face width.

(iv) Determine static load and the dynamic load by Buckingham's equation. Also, calculate the beam strength and the correct value of factor of safety based on beam strength.

(v) Using a factor of safety of 2 for wear strength, specify the surface hardness for gears.

[*(i) 18 and 90 teeth (ii) 4.89 or 5 mm (iii) 90, 450 and 50 mm (iv) 1473.66, 5993 and 10523.33 N; (fs) = 1.34 (v) 361.33 or 370 BHN*]

The McGraw-Hill Companies

Helical Gears

Chapter 18

18.1 HELICAL GEARS

There is a basic difference between spur and helical gears. While the teeth of spur gears are cut parallel to the axis of the shaft, the teeth of helical gears are cut in the form of a helix on the pitch cylinder. In spur gears, the contact between meshing teeth occurs along the entire face width of the tooth, resulting in sudden application of the load, which in turn, results in impact conditions and generates noise in high speed applications. In helical gears, the contact between meshing teeth begins with a point on the leading edge of the tooth and gradually extends along the diagonal line across the tooth. There is a gradual pick-up of load by the tooth, resulting in smooth engagement and quiet operation even at high speeds. Helical gears are used in automobiles, turbines and high-speed applications even up to 3000 m/min. There are two basic types of helical gears, parallel and crossed. *Parallel helical gears* operate on two parallel shafts. In this case, the magnitude of the helix angle is the same for the pinion and the gear, however, the hand of the helix is opposite. A right-hand pinion meshes with a left-hand gear and vice versa. Crossed helical gears are mounted on shafts with crossed axes. Their teeth may have the same or opposite hand of the helix. The discussion in this chapter is mainly limited to the design of parallel helical gears.

18.2 TERMINOLOGY OF HELICAL GEARS

The conventional representation of a pair of helical gears on technical drawings is illustrated in Fig. 18.1. It is necessary to show the hand of the helix or the direction of teeth on the drawing. The hand of the helix is indicated by drawing three thin continuous lines. The lines are slopping downward to the right side of the page for right-handed helical teeth. The lines are slopping upward to the right side of the page for left-handed helical teeth.

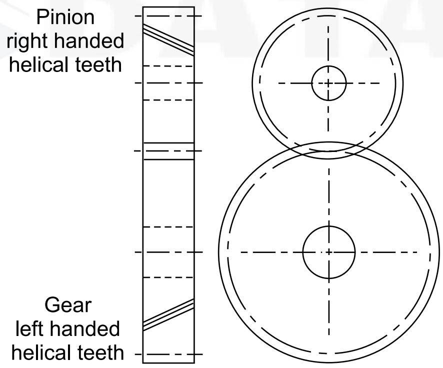


Fig. 18.1 *Conventional Representation*

A portion of the top view of a parallel helical gear is shown in Fig. 18.2. $\overline{A_1B_1}$ and $\overline{A_2B_2}$ are centre lines of the adjacent teeth taken on the pitch plane. The angle $A_1B_2A_2$ is the helix angle ψ. It is defined as the angle between the axis of the shaft and the centre line of the tooth taken on the pitch plane. *XX* is the plane of rotation, while *YY* is a plane perpendicular to the tooth elements. The distance $\overline{A_1A_2}$ is called the transverse circular pitch (p), which is measured in the plane of rotation. The distance $\overline{A_1C}$ is called the normal circular pitch (p_n), which is measured in a plane perpendicular to the tooth elements.

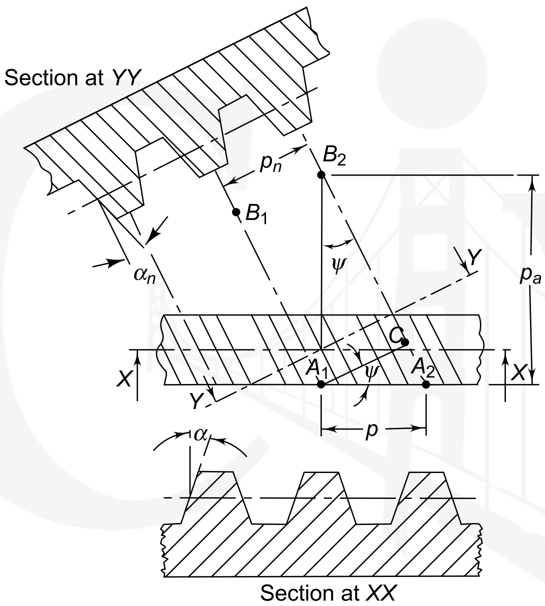


Fig. 18.2 *Tooth Relationships*

From triangle A_1A_2C,

$$\frac{p_n}{p} = \frac{\overline{A_1C}}{\overline{A_1A_2}} = \cos\psi$$

$$\therefore \qquad p_n = p\cos\psi \qquad (18.1)$$

Substituting,

$$pP = \pi$$

in the above expression, we get

$$P_n = \frac{P}{\cos\psi} \qquad (18.2)$$

where P_n and P are normal and transverse diametral pitches respectively.

Substituting ($P = 1/m$) in the above expression, we have

$$m_n = m\cos\psi \qquad (18.3)$$

where,

m_n = normal module (mm)

m = transverse module (mm)

The distance $\overline{A_1B_2}$ is called the axial pitch (p_a). From triangle $A_1A_2B_2$,

$$p_a = \frac{p}{\tan\psi} \qquad (18.4)$$

There are two pressure angles, transverse pressure angle α and normal pressure angle α_n in their respective planes. It can be proved that they are related by the following expression,

$$\cos\psi = \frac{\tan\alpha_n}{\tan\alpha} \qquad (18.5)$$

The normal pressure angle is usually 20°. The pitch circle diameter d of the helical gear is given by,

$$d = \frac{zp}{\pi} = \frac{z}{P} = zm = \frac{zm_n}{\cos\psi}$$

$$\therefore \qquad d = \frac{zm_n}{\cos\psi} \qquad (18.6)$$

The centre to centre distance a between the two helical gears having z_1 and z_2 as the number of teeth is given by,

$$a = \frac{d_1}{2} + \frac{d_2}{2} = \frac{z_1 m_n}{2\cos\psi} + \frac{z_2 m_n}{2\cos\psi}$$

$$\therefore \qquad a = \frac{m_n(z_1 + z_2)}{2\cos\psi} \qquad (18.7)$$

The speed ratio i for helical gears is determined in the same manner as for the spur gears, i.e.,

$$i = \frac{\omega_p}{\omega_g} = \frac{z_g}{z_p} \qquad (18.8)$$

where suffix p and g refer to the pinion and gear respectively.

18.3 VIRTUAL NUMBER OF TEETH

The pitch cylinder of the helical gear is cut by the plane *A–A*, which is normal to the tooth elements as shown in Fig. 18.3. The intersection of the plane *A–A* and the pitch cylinder (extended) produces an ellipse. This ellipse is shown by a dotted line. The semi-major and semi-minor axes of this ellipse

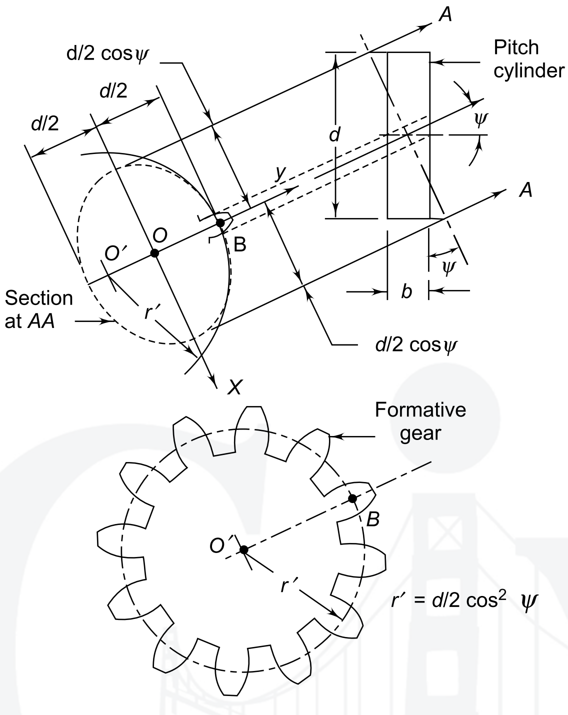


Fig. 18.3 *Formative Gear*

are $\left(\dfrac{d}{2\cos\psi}\right)$ and $\left(\dfrac{d}{2}\right)$ respectively. It can be proved from analytical geometry that the radius of curvature r' at the point B is given by,

$$r' = \frac{a^2}{b}$$

where a and b are semi-major and semi-minor axes respectively. Substituting the values of a and b in the expression for r',

$$r' = \frac{d}{2\cos^2\psi} \tag{18.9}$$

In the design of helical gears, an imaginary spur gear is considered in the plane A–A with centre at O' having a pitch circle radius of r' and module m_n. It is called a '*formative*' or '*virtual*' spur gear. The pitch circle diameter d' of the virtual gear is given by,

$$d' = 2r' = \frac{d}{\cos^2\psi} \tag{18.10}$$

The number of teeth z' on this imaginary spur gear is called the *virtual number of teeth*. It is given by

$$z' = \frac{2\pi r'}{p_n} = \frac{2\pi(d/2\cos^2\psi)}{\pi m_n} = \frac{d}{m_n\cos^2\psi}$$

Substituting Eq. (18.6) in the above expression,

$$z' = \frac{z}{\cos^3\psi} \tag{18.11}$$

where z is the actual number of teeth.

18.4 TOOTH PROPORTIONS

In helical gears, the normal module m_n should be selected from standard values. The first preference values of the normal module are

m_n (in mm) = 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8 and 10

The standard proportions of the addendum and the dedendum are,

addendum (h_a) = m_n
dedendum (h_f) = 1.25 m_n
clearance (c) = 0.25 m_n

The addendum circle diameter d_a is given by

$$d_a = d + 2h_a = \frac{zm_n}{\cos\psi} + 2m_n$$

or

$$d_a = m_n\left[\frac{z}{\cos\psi} + 2\right] \tag{18.12}$$

Similarly, the dedendum circle diameter d_f is given by

$$d_f = d - 2h_f = \frac{z\,m_n}{\cos\psi} - 2.5\,m_n$$

or

$$d_f = m_n\left[\frac{z}{\cos\psi} - 2.5\right] \tag{18.13}$$

The normal pressure angle α_n is always 20°. The helix angle varies from 15 to 25°. A portion of the top view of a helical gear is shown in Fig. 18.4.

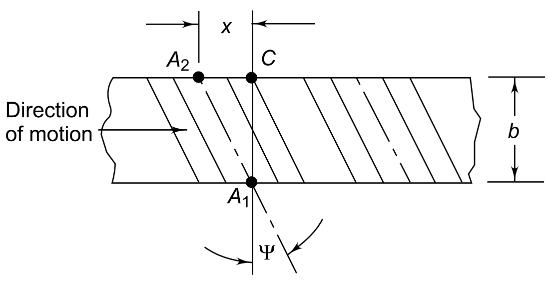


Fig. 18.4

The gear rotates from left to right as indicated by the arrow. For this rotation, the point A_1 will be the first point to come in contact with its meshing tooth on the other gear. It is called the 'leading' edge of the tooth. Also, the point A_2 will be the last point to come in contact with its meshing tooth on the other gear. It is called the 'trailing' edge of the tooth. In order that the contact on the face of the tooth shall always contain at least one point, the leading edge of the tooth should be advanced ahead of the trailing end by a distance greater than the circular pitch. Or,

$$x \geq p \tag{a}$$

From the triangle A_1A_2C,

$$\tan\psi = \frac{A_2C}{A_1C} = \frac{x}{b}$$

$$\therefore \quad x = b\tan\psi$$

Substituting the above expression in Eq. (a)

$$b\tan\psi \geq p$$

or

$$b \geq \frac{p}{\tan\psi}$$

From Eq. (18.3),

$$\frac{p}{\tan\psi} = \frac{\pi m}{\tan\psi} = \frac{\pi m_n}{\tan\psi\cos\psi} = \frac{zm_n}{\sin\psi}$$

Therefore,

$$b \geq \frac{\pi m_n}{\sin\psi} \tag{18.14}$$

This is the minimum face width.

Example 18.1 *A pair of parallel helical gears consists of a 20 teeth pinion meshing with a 40 teeth gear. The helix angle is 25° and the normal pressure angle is 20°. The normal module is 3 mm. Calculate*

(i) the transverse module;
(ii) the transverse pressure angle;
(iii) the axial pitch;
(iv) the pitch circle diameters of the pinion and the gear;
(v) the centre distance; and
(vi) the addendum and dedendum circle diameters of the pinion.

Solution

Given $z_p = 20 \quad z_g = 40 \quad m_n = 3$ mm
$\psi = 25° \quad \alpha_n = 20°$

Step I *Transverse module*

$$m = \frac{m_n}{\cos\psi} = \frac{3}{\cos(25)} = 3.31 \text{ mm} \tag{i}$$

Step II *Transverse pressure angle*

$$\tan\alpha = \frac{\tan\alpha_n}{\cos\psi} = \frac{\tan(20)}{\cos(25)} \quad \text{or} \quad \alpha = 21.88° \tag{ii}$$

Step III *Axial pitch*

$$p_a = \frac{p}{\tan\psi} = \frac{\pi m}{\tan\psi} = \frac{\pi(3.31)}{\tan(25)} = 22.3 \text{ mm} \tag{iii}$$

Step IV *Pitch circle diameters of pinion and gear*

$$d_p = \frac{z_p m_n}{\cos\psi} = \frac{20(3)}{\cos(25)} = 66.2 \text{ mm}$$

$$d_g = \frac{z_g m_n}{\cos\psi} = \frac{40(3)}{\cos(25)} = 132.4 \text{ mm} \tag{iv}$$

Step V *Centre distance*

$$a = \frac{d_p + d_g}{2} = \frac{66.2 + 132.4}{2} = 99.3 \text{ mm} \tag{v}$$

Step VI *Addendum and dedendum circle diameters of pinion*

$$d_a = m_n\left[\frac{z}{\cos\psi} + 2\right]$$

$$= 3\left[\frac{20}{\cos(25)} + 2\right] = 72.2 \text{ mm}$$

$$d_f = m_n\left[\frac{z}{\cos\psi} - 2.5\right]$$

$$= 3\left[\frac{20}{\cos(25)} - 2.5\right] = 58.7 \text{ mm} \tag{vi}$$

18.5 FORCE ANALYSIS

The resultant force P acting on the tooth of a helical gear is resolved into three components, P_t, P_r and P_a as shown in Fig. 18.5(a), where

P_t = tangential component (N)
P_r = radial component (N)
P_a = axial or thrust component (N)

The normal pressure angle α_n is in the plane ABC shaded by dots, while helix angle Ψ is in the lower plane BCD.

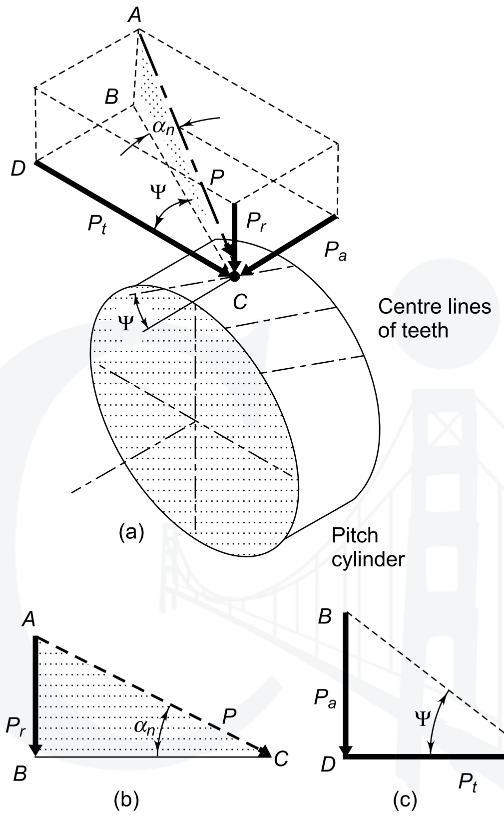


Fig. 18.5 *Components of Tooth Force*

From the triangle ABC,

$$P_r = P \sin \alpha_n \tag{a}$$

$$\overline{BC} = P \cos \alpha_n \tag{b}$$

From the triangle BDC,

$$P_a = \overrightarrow{BC} \sin \psi = P \cos \alpha_n \sin \psi \tag{c}$$

$$P_t = \overrightarrow{BC} \cos \psi = P \cos \alpha_n \cos \psi \tag{d}$$

From Eqs (c) and (d),

$$P_a = P_t \tan \psi \tag{18.15}$$

From Eqs (a) and (d),

$$P_r = P_t \left[\frac{\tan \alpha_n}{\cos \psi} \right] \tag{18.16}$$

The tangential component is calculated from the relationship

$$P_t = \frac{2M_t}{d} \tag{18.17}$$

where,

M_t = transmitted torque (N-mm)

d = pitch circle diameter (mm)

In examples of gear tooth forces, it is often required to find out the magnitude and direction of the three components. The magnitudes are determined by using the following four equations,

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n_p}$$

$$P_t = \frac{2M_t}{d_p}$$

$$P_r = P_t \left[\frac{\tan \alpha_n}{\cos \psi} \right]$$

$$P_a = P_t \tan \psi$$

where the suffix p is used for pinion.

The following information is required in order to decide the direction of the three components:

(i) Which is the driving element? Which is the driven element?

(ii) Is the pinion rotating in clockwise or anti-clockwise direction?

(iii) What is the hand of the helix? Is it right handed or left handed?

The directions of tangential and radial components are decided by the same method that is used for spur gears.

(i) Tangential Component (P_t)

(a) The direction of tangential component for a driving gear is opposite to the direction of rotation.

(b) The direction of tangential component for a driven gear is same as the direction of rotation.

(ii) Radial Component (P_r)

(a) The radial component on the pinion acts towards the centre of the pinion.

(b) The radial component on the gear acts towards the centre of the gear.

(iii) Thrust Component (P_a) The following guidelines can be used to determine the direction of the thrust component:

(a) Select the driving gear from the pair.

(b) Use right hand for RH-helix and left hand for LH-helix.
(c) Keep the fingers in the direction of rotation of the gear and the thumb will indicate the direction of the thrust component for the driving gear.
(d) The direction of the thrust component for the driven gear will be opposite to that for the driving gear.

Example 18.2 *A pair of parallel helical gears is shown in Fig. 18.6. A 5 kW power at 720 rpm is supplied to the pinion A through its shaft. The normal module is 5 mm and the normal pressure angle is 20°. The pinion has right-hand teeth, while the gear has left-hand teeth. The helix angle is 30°. The pinion rotates in the clockwise direction when seen from the left side of the page. Determine the components of the tooth force and draw a free-body diagram showing the forces acting on the pinion and the gear.*

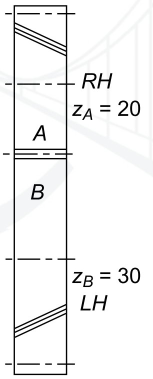


Fig. 18.6

Solution

Given kW = 5 $n_A = 720$ rpm $z_A = 20$ $z_B = 30$ $m_n = 5$ mm $\psi = 30°$ $\alpha_n = 20°$

Step I *Components of tooth force*

$$(M_t)_A = \frac{60 \times 10^6 (\text{kW})}{2\pi n_A} = \frac{60 \times 10^6 (5)}{2\pi (720)}$$

$$= 66\ 314.56 \text{ N-mm}$$

$$d_A = \frac{z_A\ m_n}{\cos\ \psi} = \frac{20(5)}{\cos(30)} = 115.47 \text{ mm}$$

$$P_t = \frac{2(M_t)_A}{d_A} = \frac{2\ (66\ 314.56)}{115.47} = 1148.6 \text{ N}$$

$$P_a = P_t \tan \psi = 1148.6 \tan (30) = 663.14 \text{ N}$$

$$P_r = P_t \left[\frac{\tan \alpha_n}{\cos \psi}\right] = 1148.6 \left[\frac{\tan(20°)}{\cos(30°)}\right] = 482.73 \text{ N}$$

Step II *Free-body diagram of forces*

The free-body diagram of forces is shown in Fig. 18.7. The pinion is the driving element. It is rotating in clockwise direction. The direction of tangential component for the driving element is opposite to that of rotation. Therefore, the tangential component on the pinion at the point-1 will act towards the lower right-hand corner of the page. The radial component acts towards the centre of respective gear. Therefore, the radial component at the point-1 will act in the upward direction. The pinion has right-handed teeth. Use the right hand and keep the fingers in the direction of rotation, i.e., in the clockwise direction. The thumb will point towards the upper right-hand corner of the page. Therefore, the axial component at the point-1 will

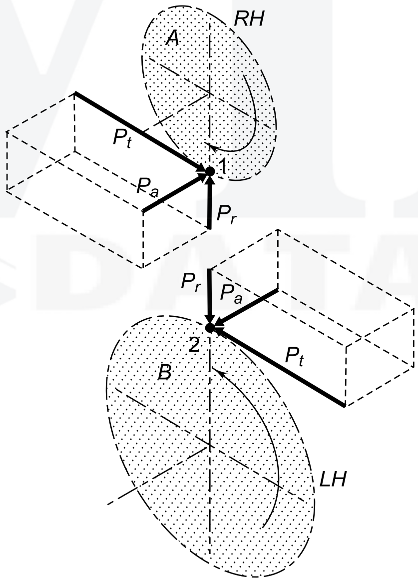


Fig. 18.7

act towards the upper right-hand corner of the page. The action and reaction are equal and opposite. Therefore, the direction of three components on the gear at the point-2 will be opposite to that of the pinion.

Example 18.3 *The layout of a double-reduction helical gearbox is shown in Fig. 18.8. Pinion A is the driving gear and 10 kW power at 720 rpm is supplied to it through its shaft no. 1. The number of teeth on different helical gears are as follows:*

$$z_A = 20 \quad z_B = 50 \quad z_C = 20 \quad z_D = 60$$

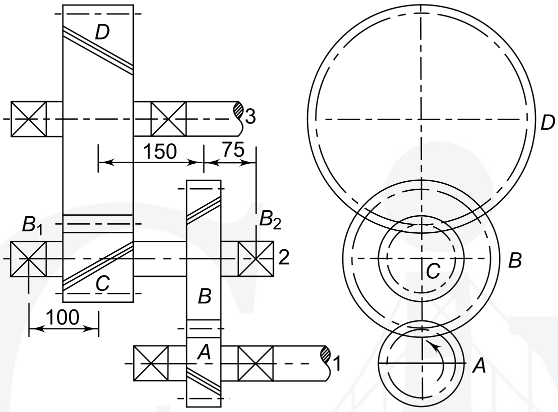


Fig. 18.8

The normal pressure angle for all gears is 20°. For the pair of helical gears A and B, the helix angle is 30°, and the normal module is 3 mm. For the pair C and D, the helix angle is 25° and the normal module is 5 mm. Pinion A has right-handed helical teeth, while the pinion C has left-handed helical teeth. The bearings B_1 and B_2 are mounted on shaft no.2 in such a way that bearing B_1 can take only radial load, while the bearing B_2 can take both radial as well as thrust load. Determine the magnitude and direction of bearing reactions on shaft no. 2.

Solution

Given $kW = 10 \quad n_A = 720$ rpm $\quad z_A = 20$
$z_B = 50 \quad z_C = 20 \quad z_D = 60 \quad \alpha_n = 20°$
For gears A and B, $\quad m_n = 3$ mm $\quad \psi = 30°$
For gears C and D, $\quad m_n = 5$ mm $\quad \psi = 25°$

Step I *Components of tooth force between gears A and B*
From Eq. (18.6),

$$d_A = \frac{z_A\, m_n}{\cos\psi} = \frac{20(3)}{\cos(30)} = 69.28 \text{ mm}$$

$$d_B = \frac{z_B\, m_n}{\cos\ \psi} = \frac{50\ (3)}{\cos\ (30)} = 173.21 \text{ mm}$$

$$d_C = \frac{z_C m_n}{\cos\psi} = \frac{20(5)}{\cos(25)} = 110.34 \text{ mm}$$

$$(M_t)_1 = \frac{60\times10^6(\text{kW})}{2\ \pi n_1} = \frac{60\times10^6(10)}{2\pi(720)}$$

$$= 132\ 629.12 \text{ N-mm}$$

$$(P_t)_{AB} = \frac{2(M_t)_1}{d_A} = \frac{2(132\ 629.12)}{69.28} = 3828.78 \text{ N}$$

$$(P_a)_{AB} = (P_t)_{AB} \tan\psi$$

$$= 3828.78\ \tan(30) = 2210.55 \text{ N}$$

$$(P_r)_{AB} = (P_t)_{AB}\left[\frac{\tan\alpha_n}{\cos\ \psi}\right]$$

$$= 3828.78\left[\frac{\tan(20)}{\cos(30)}\right] = 1609.15 \text{ N}$$

Step II *Components of tooth force between gears C and D*

$$(M_t)_2 = (P_t)_{AB}\left(\frac{d_B}{2}\right) = 3828.78\left(\frac{173.21}{2}\right)$$

$$= 331\ 591.49 \text{ N-mm}$$

$$(P_t)_{CD} = \frac{2(M_t)_2}{d_C}$$

$$= \frac{2\ (331\ 591.49)}{110.34} = 6010.36 \text{ N}$$

$$(P_a)_{CD} = (P_t)_{CD} \tan\psi$$

$$= 6010.36 \tan(25) = 2802.68 \text{ N}$$

$$(P_r)_{CD} = (P_t)_{CD}\left[\frac{\tan\alpha_n}{\cos\psi}\right]$$

$$= 6010.36\left[\frac{\tan(20)}{\cos(25)}\right] = 2413.74 \text{ N}$$

Step III *Free-body diagram of forces*
The forces acting on the gears C and B and the components of the bearing reactions are shown in Fig. 18.9. Driving and driven gears are relative terms. A is the driving gear, while the gear B is the driven gear. C is the driving gear, while D is the driven gear. Therefore, on shaft-2, the driving gear C and the driven gear B are mounted.

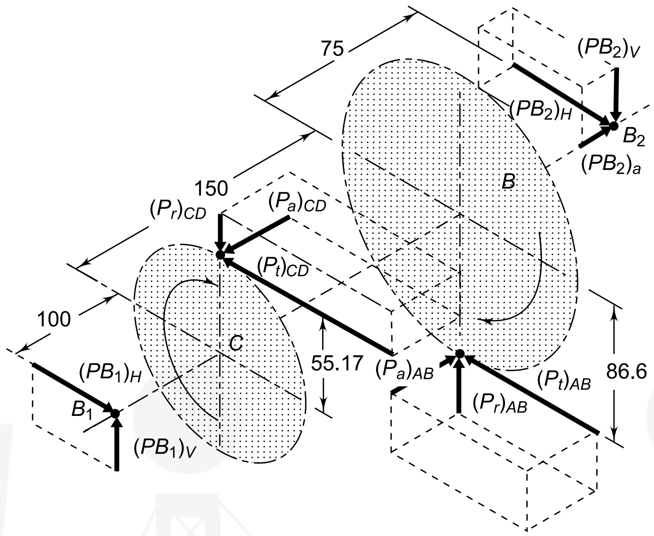


Fig. 18.9

Gear C is the driving element. It is rotating in clockwise direction. The direction of tangential component for the driving element is opposite to that of rotation. Therefore, the tangential component on the gear C will act towards the upper left-hand corner of the page. The radial component always acts towards the centre of the gear. Therefore, the radial component on the gear C will act in the downward direction at the point of contact. The gear C has left-handed teeth. Use the left hand and keep fingers in the direction of rotation, i.e., in clockwise direction. The thumb will point towards the lower left-hand corner of the page. Therefore, the axial component on the gear C will act towards the lower left corner of the paper.

Gear B is the driven element. It is rotating in the clockwise direction. The direction of tangential component for the driven element is same as that of rotation. Therefore, the tangential component on the gear B at the point of contact will act towards the upper left-hand corner of the page. The radial component always acts towards the centre of gear. Therefore, the radial component on the gear B will act in the upward direction at the point of contact. The gear B is the driven gear and the axial component is to be decided in relation to the driving gear A. It can be shown that the axial component on the gear B acts towards the upper right-hand corner of the page.

Step VI *Bearing reactions on shaft-2*

Considering the forces in vertical plane only and taking moment of these forces about the bearing B_1,

$$(P_r)_{CD} \times 100 - (P_a)_{CD} \times 55.17 - (P_r)_{AB} \times 250 - (P_a)_{AB} \times 86.6 + (PB_2)_v \times 325 = 0$$

$$2413.74 \times 100 - 2802.68 \times 55.17 - 1609.15 \times 250 - 2210.55 \times 86.6 + (PB_2)_v \times 325 = 0$$

or $$(PB_2)_v = 1559.91 \text{ N} \qquad \text{(i)}$$

Considering the equilibrium of vertical forces,

$$(PB_1)_v - (P_r)_{CD} + (P_r)_{AB} - (PB_2)_V = 0$$

or $$(PB_1)_v - 2413.74 + 1609.15 - 1559.91 = 0$$

or $$(PB_1)_v = 2364.5 \text{ N} \qquad \text{(ii)}$$

Considering the forces in the horizontal plane only and taking the moment of these forces about the bearing B_1,

$$(P_t)_{CD} \times 100 + (P_t)_{AB} \times 250 - (PB_2)_H \times 325 = 0$$

or $$6010.36 \times 100 + 3828.78 \times 250 - (PB_2)_H \times 325 = 0$$

$\therefore$ $$(PB_2)_H = 4794.56 \text{ N} \qquad \text{(iii)}$$

Considering the equilibrium of horizontal forces,

$$(PB_1)_H - (P_t)_{CD} - (P_t)_{AB} + (PB_2)_H = 0$$

$$(PB_1)_H - 6010.36 - 3828.78 + 4794.56 = 0$$

$\therefore$ $$(PB_1)_H = 5044.58 \text{ N} \qquad \text{(iv)}$$

Considering the equilibrium of axial forces,

$$(P_a)_{CD} - (P_a)_{AB} - (PB_2)_a = 0$$

or $\quad 2802.68 - 2210.55 - (PB_2)_a = 0$

$$(PB_2)_a = 592.13 \text{ N} \qquad \text{(v)}$$

18.6 BEAM STRENGTH OF HELICAL GEARS

In order to determine beam strength, the helical gear is considered to be equivalent to a formative spur gear, which is discussed in Section 18.3. The formative gear is an imaginary spur gear in a plane perpendicular to the tooth element. The pitch circle diameter of this gear is d', the number of teeth is z' and the module m_n. From Eq. (17.16), the beam strength of the spur gear is given by,

$$S_b = mb\sigma_b Y \qquad \text{(a)}$$

This equation is also applicable to the formative spur gear.

Referring to Fig. 18.10,

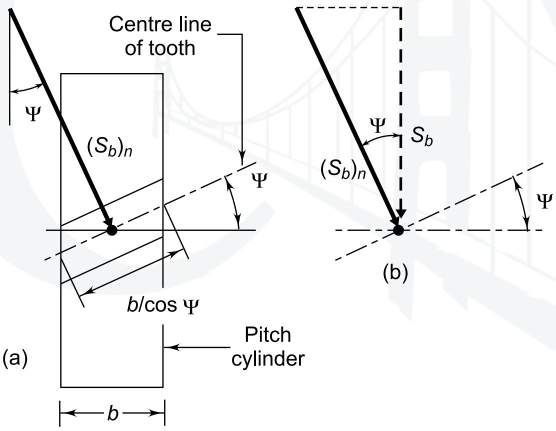


Fig. 18.10

$S_b = (S_b)_n$ = beam strength perpendicular to the tooth element

$m = m_n$ = normal module

$$b = \frac{b}{\cos \psi}$$

Y = Lewis form factor based on virtual number of teeth z'.

Substituting these values in Eq. (a),

$$(S_b)_n = \frac{m_n\, b\, \sigma_b\, Y}{\cos\ \psi} \qquad \text{(b)}$$

In Fig. 18.10, S_b is the component of $(S_b)_n$ in the plane of rotation. Thus,

$$S_b = (S_b)_n \cos \psi \qquad \text{(c)}$$

From (b) and (c),

$$S_b = m_n\, b\, \sigma_b\, Y \qquad (18.18)$$

Equation (18.18) is known as Lewis Equation for helical gears. In this equation, the form factor Y is based on the virtual number of teeth. Equation (18.18) gives beam strength in the plane of rotation. *Therefore, beam strength (S_b) indicates the maximum value of tangential force that the tooth can transmit without bending failure.* It should be always more than the effective force between the meshing teeth.

18.7 EFFECTIVE LOAD ON GEAR TOOTH

In Section 18.5, a method to determine the tangential component of the resultant force between two meshing teeth of helical gears is discussed. The component is calculated by using the following two equations:

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n}$$

$$P_t = \frac{2M_t}{d}$$

The above value of the tangential component, therefore, depends upon rated power and rated speed. In addition to this, there is the dynamic load as discussed in Section 17.19. There are two methods to account for the dynamic load—approximate estimation by means of the *velocity factor* in the preliminary stages of gear design and precise calculation by *Buckingham's equation* in the final stages.

In the preliminary stages, the effective load P_{eff} between two meshing teeth is given by,

$$P_{\text{eff}} = \frac{C_s P_t}{C_v} \qquad (18.19)$$

where,

C_s = service factor (Table 17.4)

C_v = velocity factor

The velocity factor for helical gears is given by

$$C_v = \frac{5.6}{5.6 + \sqrt{v}} \qquad (18.20)$$

where v is the pitch line velocity in m/s.

In the final stages of gear design, when gear dimensions are known, errors specified and the quality of gears determined, the dynamic load is calculated by equation derived by Earle Buckingham. The dynamic load[1] is given by,

$$P_d = \frac{21v(Ceb\cos^2\psi + P_t)\cos\psi}{21v + \sqrt{(Ceb\cos^2\psi + P_t)}} \tag{18.21}$$

where,

P_d = dynamic load or incremental dynamic load (N)

v = pitch line velocity (m/s)

C = deformation factor (N/mm^2)

e = sum of errors between two meshing teeth (mm)

b = face width of tooth (mm)

P_t = tangential force due to rated torque (N)

ψ = helix angle (degrees)

The deformation factor C depends upon the modulii of elasticity of materials for pinion and gear and the form of tooth or pressure angle. The values of C for various conditions are given in Table 17.7. Most of the helical gears are made of steel with 20° full depth involute system. In this case, the deformation factor C is 11400 N/mm^2.

The effective load is given by,

$$P_{\text{eff}} = (C_s P_t + P_d) \tag{18.22}$$

where, (P_d) is the dynamic load or additional load due to dynamic conditions between two meshing teeth.

In order to avoid failure of gear tooth due to bending,

$$S_b > P_{\text{eff}}$$

Introducing a factor of safety,

$$S_b = P_{\text{eff}}\,(fs) \tag{18.23}$$

18.8 WEAR STRENGTH OF HELICAL GEARS

The wear strength equation of the spur gear is modified to suit helical gears. For this purpose, a pair of helical gears is considered to be equivalent to a formative pinion and a formative gear in a plane perpendicular to the tooth element. From Eq. (17.33), the wear strength of the spur gear is given by,

$$S_w = bQd'_pK \tag{a}$$

Referring to Fig. 18.10,

$S_w = (S_w)_n$ = wear strength perpendicular to the tooth element

$b = \dfrac{b}{\cos\psi}$ = face width along the tooth element

$d'_p = \dfrac{d_p}{\cos^2\psi}$ = pitch circle diameter of the formative pinion.

Substituting these values in Eq. (a),

$$(S_w)_n = \frac{bQd_pK}{\cos^3\psi} \tag{b}$$

The component of $(S_w)_n$ in the plane of rotation is denoted by S_w. Therefore,

$$S_w = (S_w)_n\cos\psi \tag{c}$$

From (b) and (c),

$$S_w = \frac{bQd_pK}{\cos^2\psi} \tag{18.24}$$

Equation (18.24) is known as Buckingham's equation of wear strength. Equation (18.24) gives wear strength in the plane of rotation. *Therefore, wear strength (S_w) indicates the maximum tangential force that the tooth can transmit without pitting failure.* It should be always more than the effective force between the meshing teeth. The virtual number of teeth on the pinion and gear are z'_p and z'_g respectively. The ratio factor Q for external helical gears is given by,

$$Q = \frac{2z'_g}{z'_g + z'_p} \tag{d}$$

Substituting

$$z'_g = \frac{z_g}{\cos^3\psi} \quad\text{and}\quad z'_p = \frac{z_p}{\cos^3\psi}$$

in Eq. (d), we have

$$Q = \frac{2z_g}{z_g + z_p} \tag{18.25}$$

Similarly, for a pair of internal helical gears, it can be proved that

$$Q = \frac{2z_g}{z_g - z_p} \tag{18.26}$$

where z_g and z_p are the actual number of teeth on the gear and the pinion respectively. The pressure

[1] Buckingham Earle – 'Analytical Mechanics of Gears' – McGraw-Hill Inc.,1949.

angle in a plane perpendicular to the tooth element is α_n. The K factor in Eq. (18.24) is given by

$$K = \frac{\sigma_c^2 \sin\alpha_n \cos\alpha_n \left(\frac{1}{E_1} + \frac{1}{E_2}\right)}{1.4} \tag{18.27}$$

where,

σ_c = surface endurance strength (N/mm²)

E_1, E_2 = modulii of elasticity of materials for pinion and gear (N/mm²)

α_n = normal pressure angle (20°)

Referring to Section 17.21, Eq. (17.35),

$$K = 0.16\left(\frac{\text{BHN}}{100}\right)^2 \tag{18.28}$$

The above equation is applicable for steel gears with 20° normal pressure angle. In order to avoid failure of gear tooth due to pitting,

$$S_w > P_{\text{eff}}$$

Introducing a factor of safety,

$$S_w = P_{\text{eff}}\,(fs) \tag{18.29}$$

Example 18.4 *A pair of parallel helical gears consists of a 20 teeth pinion meshing with a 100 teeth gear. The pinion rotates at 720 rpm. The normal pressure angle is 20°, while the helix angle is 25°. The face width is 40 mm and the normal module is 4 mm. The pinion as well as the gear is made of steel 40C8 (S_{ut} = 600 N/mm²) and heat treated to a surface hardness of 300 BHN. The service factor and the factor of safety are 1.5 and 2 respectively. Assume that the velocity factor accounts for the dynamic load and calculate the power transmitting capacity of gears.*

Solution

Given $n_p = 720$ rpm $z_p = 20$ $z_g = 100$
$m_n = 4$ mm $b = 40$ mm $\psi = 25°$ $\alpha_n = 20°$
$S_{ut} = 600$ N/mm² BHN = 300 $C_s = 1.5$ $(fs) = 2$

Since both gears are made of the same material, the pinion is weaker than the gear.

Step I *Beam strength*

$$z'_p = \frac{z_p}{\cos^3\psi} = \frac{20}{\cos^3(25)} = 26.87$$

From Table 17.3,

$$Y = 0.344 + \frac{(0.348 - 0.344)(26.87 - 26)}{(27 - 26)} = 0.3475$$

$$\sigma_b = \frac{S_{ut}}{3} = \frac{600}{3} = 200 \text{ N/mm}^2$$

$$S_b = m_n b \sigma_b Y = 4(40)(200)(0.3475) = 11\,120 \text{ N}$$

Step II *Wear strength*

$$Q = \frac{2z_g}{z_g + z_p} = \frac{2(100)}{100 + 20} = 1.667$$

$$d_p = \frac{z_p m_n}{\cos\psi} = \frac{20(4)}{\cos(25)} = 88.27 \text{ mm}$$

$$K = 0.16\left(\frac{\text{BHN}}{100}\right)^2$$

$$= 0.16\left(\frac{300}{100}\right)^2 = 1.44 \text{ N/mm}^2$$

$$S_w = \frac{bQd_pK}{\cos^2\psi} = \frac{40(1.667)(88.27)(1.44)}{\cos^2(25)}$$

$$= 10\,318.58 \text{ N}$$

Since wear strength is lower than beam strength, pitting is the criterion of failure.

Step III *Tangential force due to rated torque*

$$v = \frac{\pi d_p n_p}{60 \times 10^3} = \frac{\pi(88.27)(720)}{60 \times 10^3} = 3.328 \text{ m/s}$$

$$C_v = \frac{5.6}{5.6 + \sqrt{v}} = \frac{5.6}{5.6 + \sqrt{3.328}} = 0.7543$$

From Eqs (18.29) and (18.19),

$$S_w = \frac{C_s P_t (fs)}{C_v}$$

or

$$10\,318.58 = \frac{1.5 P_t (2)}{0.7543}$$

$$\therefore \quad P_t = 2594.43 \text{ N}$$

Step IV *Power transmitting capacity of gears*

$$M_t = \frac{P_t d_p}{2} = \frac{2594.43\,(88.27)}{2}$$

$$= 114\,505.39 \text{ N-mm}$$

$$kW = \frac{2\pi n_p M_t}{60 \times 10^6} = \frac{2\pi(720)(114\,505.39)}{60 \times 10^6} = 8.63$$

Example 18.5 *A pair of parallel helical gears consists of 24 teeth pinion rotating at 5000 rpm and supplying 2.5 kW power to a gear. The speed reduction is 4 : 1. The normal pressure angle and* *helix angle are 20° and 23° respectively. Both gears are made of hardened steel (S_{ut} = 750 N/mm^2). The service factor and the factor of safety are 1.5 and 2 respectively. The gears are finished to meet the accuracy of Grade 4.*

(i) In the initial stages of gear design, assume that the velocity factor accounts for the dynamic load and that the face width is ten times the normal module. Assuming the pitch line velocity to be 10 m/s, estimate the normal module.

(ii) Select the first preference value of the normal module and calculate the main dimensions of the gears.

(iii) Determine the dynamic load using Buckingham's equation and find out the effective load for the above dimensions. What is the correct factor of safety for bending?

(iv) Specify surface hardness for the gears, assuming a factor of safety of 2 for wear consideration.

Solution

Given $kW = 2.5$ $\quad n_p = 5000$ rpm $\quad z_p = 24$
$i = 4$ $\quad \psi = 23°$ $\quad \alpha_n = 20°$ $\quad S_{ut} = 750$ N/mm^2
$C_s = 1.5$ $\quad (fs) = 2$ $\quad$ Grade of machining = 4

Step I *Estimation of module based on dynamic load by velocity factor*

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n_p}$$

$$= \frac{60 \times 10^6 (2.5)}{2\pi (5\,000)} = 4774.648 \text{ N-mm}$$

$$d_p = \frac{z_p\, m_n}{\cos\psi} = \frac{24\, m_n}{\cos(23)}$$

$$= (26.073\, m_n) \text{ mm}$$

$$P_t = \frac{2M_t}{d_p} = \frac{2(4774.648)}{26.073 m_n} = \left(\frac{366.25}{m_n}\right) \text{N}$$

$$C_v = \frac{5.6}{5.6 + \sqrt{v}} = \frac{5.6}{5.6 + \sqrt{10}} = 0.6391$$

$$P_{\text{eff}} = \frac{C_s P_t}{C_v} = \frac{1.5}{0.6391}\left(\frac{366.25}{m_n}\right) = \left(\frac{859.61}{m_n}\right) \text{N} \quad \text{(a)}$$

$$z_p' = \frac{z_p}{\cos^3\psi} = \frac{24}{\cos^3(23)} = 30.77$$

From Table 17.3,

$$Y = 0.358 + \frac{(0.364 - 0.358)(30.77 - 30)}{(32 - 30)}$$

$$= 0.36$$

$$\sigma_b = \frac{S_{ut}}{3} = \frac{750}{3} = 250 \text{ N/mm}^2$$

$$S_b = m_n b \sigma_b Y = m_n (10\, m_n)(250)(0.36)$$
$$= (900\, m_n^2) \text{ N} \quad \text{(b)}$$

Since, $\quad S_b = P_{\text{eff}}(fs)$

From (a) and (b),

$$900\, m_n^2 = \left(\frac{859.61}{m_n}\right)(2.0)$$

$$m_n = 1.24 \text{ mm}$$

The first preference value of the module is taken as 1.5 mm.

Step II *Main dimensions of gears*

$$m_n = 1.5 \text{ mm}$$

$$b = 10\, m_n = 10(1.5) = 15 \text{ mm}$$

$$d_p = \frac{z_p m_n}{\cos\psi} = \frac{24\ (1.5)}{\cos\ (23)} = 39.11 \text{ mm}$$

$$d_g = \frac{z_g m_n}{\cos\psi} = \frac{(24 \times 4)(1.5)}{\cos(23)} = 156.44 \text{ mm}$$

Step III *Effective load based on dynamic load by Buckingham's equation*

Beam strength

$$S_b = m_n b \sigma_b Y = 1.5\ (15\ )(250)(0.36) = 2025 \text{ N}$$

Tangential force due to rated torque

$$P_t = \frac{2M_t}{d_p} = \frac{2(4774.648)}{39.11} = 244.16 \text{ N}$$

Dynamic load

From Table 17.8, the error for Grade 4 is given by,

$$e = 3.20 + 0.25\phi \quad \text{where} \quad \phi = m_n + 0.25\sqrt{d}$$

For pinion,

$$\phi = m_n + 0.25\sqrt{d_p} = 1.5 + 0.25\sqrt{39.11}$$

$$\therefore \quad e_p = 3.20 + 0.25\,(1.5 + 0.25\sqrt{39.11})$$
$$= 3.9659\ \mu\text{m}$$

For gear,

$$\phi = m_n + 0.25\sqrt{d_g} = 1.5 + 0.25\sqrt{156.44}$$

$$\therefore \quad e_g = 3.20 + 0.25\,(1.5 + 0.25\sqrt{156.44})$$
$$= 4.3567\ \mu\text{m}$$
$$e = e_p + e_g = 3.9659 + 4.3567$$
$$= 8.3226\ \mu\text{m or } (8.3226 \times 10^{-3})\ \text{mm}$$

Also, $C = 11400\ \text{N/mm}^2 \quad b = 15\ \text{mm}$

$$P_t = 244.16\ \text{N}$$

$$v = \frac{\pi d_p n_p}{60 \times 10^3} = \frac{\pi(39.11)(5000)}{60 \times 10^3} = 10.24\ \text{m/s}$$

From Eq. (18.21),

$$P_d = \frac{21v(Ceb\cos^2\psi + P_t)\cos\psi}{21v + \sqrt{(Ceb\cos^2\psi + P_t)}}$$

$$= \frac{21(10.24)[11400(8.3226\times10^{-3})(15)\cos^2(23) + 244.16]\cos\ (23)}{21(10.24) + \sqrt{[11400(8.3226\times10^{-3})(15)\cos^2(23) + 244.16]}}$$

$$= 1133.94\ \text{N}$$

From Eq. (18.22),

$$P_{\text{eff}} = (C_s P_t + P_d) = 1.5\,(244.16) + 1133.94$$
$$= 1500.18\ \text{N}$$

Step IV *Correct factor of safety*

$$\therefore \quad (fs) = \frac{S_b}{P_{\text{eff}}} = \frac{2025}{1500.18} = 1.35 \qquad \text{(OK)}$$

Step V *Surface hardness for gears*

$$S_w = P_{\text{eff}}\,(fs) = 1500.18\,(2.0) = 3000.36\ \text{N}$$

$$Q = \frac{2z_g}{z_g + z_p} = \frac{2(96)}{96 + 24} = 1.6$$

$$S_w = \frac{bQd_pK}{\cos^2\psi}$$

$$\text{or} \quad 3000.36 = \frac{15(1.6)(39.11)\left[0.16\left(\frac{\text{BHN}}{100}\right)^2\right]}{\cos^2(23)}$$

$$\therefore \quad \text{BHN} = 411.44 \text{ or } 420$$

18.9 HERRINGBONE GEARS

When a pair of helical gears transmits power, both the input and the output shafts are subjected to a thrust load. These thrust forces impose reactions on the bearings. This increases the size and cost of bearings. The thrust forces on input and output shafts can be eliminated by using herringbone or double helical gears as shown in Fig. 18.11. Both types of gear are constructed by joining two identical helical gears of the same module; number of teeth and pitch circle diameter, but with teeth having opposite hand of helix.

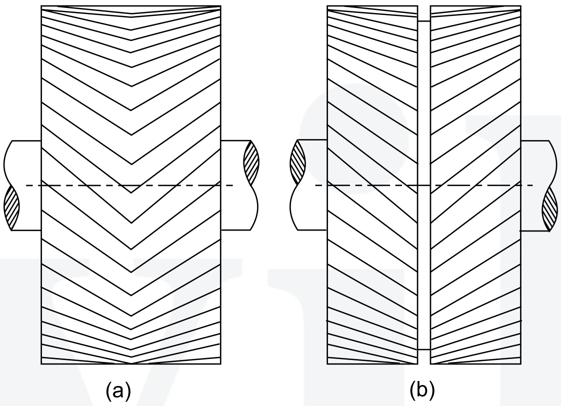


Fig. 18.11 *(a) Herringbone Gear (b) Double Helical Gear*

There is a basic difference between herringbone gear and double helical gear. There is a groove between two helical gears in case of double helical gear, while a gear without a groove is called herringbone gear. Double helical gears are cut on a single gear blank, by a hob with a tool run-out groove between the hands of helices. Herringbone gear is cut by two cutters, which reciprocate 180° out of phase to avoid clashing.

Herringbone and double helical gears offer the following advantages:

(i) They develop opposite thrust reactions and thus cancel out the thrust force within the gear itself. The net axial force that acts on the bearings is zero.

(ii) The power transmitting capacity is high.

(iii) High pitch line velocities can be attained with herringbone and double helical gears.

The disadvantages of herringbone and double helical gears are as follows:

(i) Herringbone and double helical gears are very expensive compared to helical gears.

(ii) Balancing of thrust forces depends upon the equal distribution of load between the right and left part of the gear. Therefore, a high degree of precision is required to locate herringbone and double helical gears axially on the shaft. They must be aligned accurately, if each half is to take exactly one half of the load. One method of alignment is to use thrust bearing for one shaft only and allow the other shaft to float axially. In this case, the gear tooth forces automatically position the other gear so that no external forces are present.

Herringbone and double helical gears are used in high power applications such as ship drives and turbines.

The helix angle for single helical gear is from 15° to 25°. Herringbone and double helical gears permit higher helix angle because there is no thrust force. The helix angle for herringbone and double helical gears is from 20° to 45°. The design procedure and design equations for herringbone and double helical gears are the same as for single helical gear. In design, a herringbone or double helical gear is considered as equal to two identical helical gears, each transmitting one half power.

Example 18.6 *A herringbone speed reducer consists of a 26 teeth pinion driving a 104 teeth gear. The gears have a normal module of 2 mm. The pressure angle is 20° and the helix angle is 25°. The pinion receives 100 kW power through its shaft and rotates at 3600 rpm. The face width of each half is 35 mm. The gears are made of alloy steel 30Ni4Cr1 (S_{ut} = 1500 N/mm^2) and heat treated to a surface hardness of 450 BHN. The service factor is 1.25.*

Determine the factor of safety against bending failure and against pitting failure.

Solution

Given $kW = 100$ $n_p = 3600$ rpm $z_p = 26$ $z_g = 104$ $m_n = 2$ mm $b = 35$ mm $\psi = 25°$ $\alpha_n = 20°$ $S_{ut} = 1500$ N/mm^2 BHN = 450 $C_s = 1.25$

Since a herringbone gear consists of two identical pairs of helical gears, the power transmitted by each pair is (100/2) or 50 kW.

Step I *Tangential force due to rated torque*

$$M_t = \frac{60\times10^6(\text{kW})}{2\pi n_p} = \frac{60\times10^6(50)}{2\pi(3600)}$$

$$= 132\ 629.12 \text{ N-mm}$$

$$d_p = \frac{z_p m_n}{\cos\psi} = \frac{26(2)}{\cos(25)} = 57.38 \text{ mm}$$

$$P_t = \frac{2M_t}{d_P} = \frac{2\ (132\ 629.12)}{57.38} = 4622.83 \text{ N}$$

Step II *Effective load*

$$v = \frac{\pi d_p n_p}{60\times10^3} = \frac{\pi(57.38)(3600)}{60\times10^3} = 10.82 \text{ m/s}$$

$$C_v = \frac{5.6}{5.6+\sqrt{v}} = \frac{5.6}{5.6+\sqrt{10.82}} = 0.63$$

$$P_{\text{eff}} = \frac{C_s P_t}{C_v} = \frac{1.25\,(4622.83)}{0.63} = 9172.28 \text{ N}$$

Step III *Beam strength*

$$z'_p = \frac{z_p}{\cos^3\psi} = \frac{26}{\cos^3(25)} = 34.93$$

From Table17.3,

$$Y = 0.367 + \frac{(0.373-0.367)(34.93-33)}{(35-33)}$$

$$= 0.37279$$

$$\sigma_b = \frac{S_{ut}}{3} = \frac{1500}{3} = 500 \text{ N/mm}^2$$

$$S_b = m_n b \sigma_b Y = 2(35)(500)(0.37279)$$

$$= 13\,047.65 \text{ N}$$

Step IV *Wear strength*

$$Q = \frac{2z_g}{z_g+z_p} = \frac{2(104)}{104+26} = 1.6$$

$$K = 0.16\left(\frac{\text{BHN}}{100}\right)^2$$

$$= 0.16\left(\frac{450}{100}\right)^2 = 3.24 \text{ N/mm}^2$$

$$S_w = \frac{bQd_pK}{\cos^2\psi} = \frac{35(1.6)(57.38)(3.24)}{\cos^2(25)}$$

$$= 12\,674.83 \text{ N}$$

Step V *Factor of safety*

$$(fs) = \frac{S_b}{P_{\text{eff}}} = \frac{13\,047.65}{9172.28} = 1.42$$

$$(fs) = \frac{S_w}{P_{\text{eff}}} = \frac{12\,674.83}{9172.28} = 1.38$$

18.10 CROSSED HELICAL GEARS

Helical gears, which are mounted on non-parallel shafts, are called crossed helical gears. In these gears, the axes of two shafts are neither parallel nor intersecting like worm gears. The pitch cylinders of a pair of crossed helical gears are illustrated in Fig. 18.12. The action of crossed helical gears differs fundamentally from that of parallel helical gears. There is a line contact between meshing teeth of parallel helical gears. It is observed from the figure, that kinematically, there is a point contact between the meshing teeth of crossed helical gears. Since the contact area of a point is very small, the contact

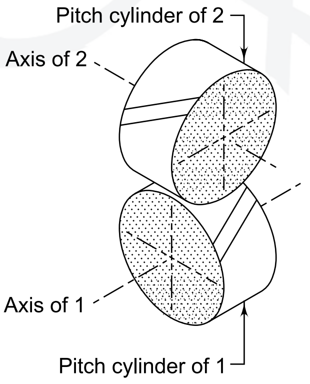


Fig. 18.12 *Pitch Cylinders of Crossed Helical Gears*

pressure is high and wear is comparatively rapid. Therefore, crossed helical gears have very low load carrying capacity. They are not recommended for high power transmission. They are particularly useful in light duty applications. They are used in small internal combustion engines to drive the speedometer cable and oil pump and distribution system. Their other applications include feed mechanisms on machine tools, water pumps and instruments.

Figure 18.13(a) shows the contact between meshing teeth of crossed helical gears, mounted on two shafts—1 and 2. The contact point lies between the lower surface of gear-2 and the upper surface of gear-1. The gears are rotating in the directions shown. At the point of contact,

V_1 = pitch line velocity of contact point when considered on shaft-1.

V_2 = pitch line velocity of contact point when considered on shaft-2.

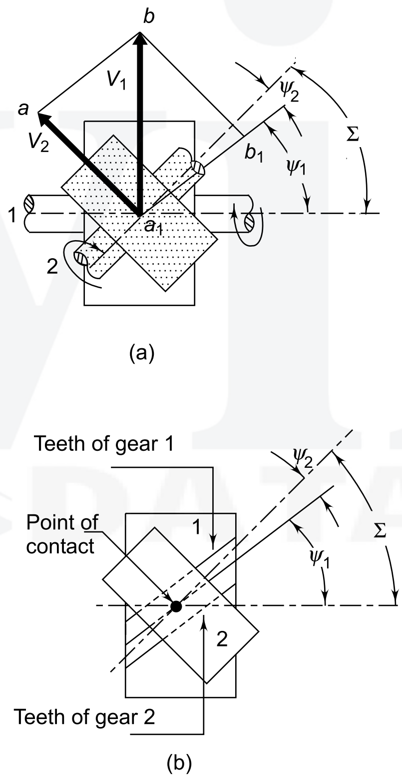


Fig. 18.13 *Crossed Helical Gears*

It should be noted that the velocity vector V_2 shown in the figure is on the lower side of gear-2. The vectors V_1 and V_2 act as tangents to their respective pitch cylinders at the point of contact. The velocity of sliding is the vector difference between V_1 and V_2. The line **ab** indicates the vector difference. Therefore, the velocity of sliding must be directed parallel to the line **ab** at the point of contact. Therefore, if the teeth are to be machined on these pitch cylinders, the teeth must permit this sliding to take place and must be parallel to the line **ab** at the point of contact. Figure 18.13(b) shows the orientation of teeth at the point of contact in this particular case. The teeth on gear-2 at the point of contact are on lower side. Therefore, they are shown by dotted lines. It is observed that in this case, both gears have teeth with left-hand helix and that the angle between the shafts is the sum of the helix angles. Therefore,

$$\Sigma = \psi_1 + \psi_2 \quad \text{(for same hand of helix)} \tag{18.30}$$

Similarly, it can be proved that,

$$\Sigma = \psi_1 - \psi_2 \quad \text{(for opposite hand of helix)} \tag{18.31}$$

where,

ψ_1 = helix angle of gear-1
ψ_2 = helix angle of gear-2
Σ = shaft angle

When the shaft angle is small, opposite hand of helix is used. On the other hand, the same hand of helix is used when the shaft angle is more. For a particular case, when the shaft angle is 90°, the gears must have the same hand of helix and each helix angle is 45°. This is the most common case of crossed helical gears. There is no difference between a crossed helical gear and a parallel helical gear until they are mounted in a particular position. They are manufactured in the same way. A pair of crossed helical gears has the same normal module and pressure angle. From Eq. (18.6),

$$d_p = \frac{z_p m_n}{\cos\psi_p} \quad \text{and} \quad d_g = \frac{z_g m_n}{\cos\psi_g} \tag{a}$$

where,

z_p, z_g = number of teeth on pinion and gear
d_p, d_g = pitch circle diameters of pinion and gear (mm)
m_n = normal module (mm)
ψ_p, ψ_g = helix angles for pinion and gear (degrees)

The speed ratio i for crossed helical gears is given by,

$$i = \frac{\omega_p}{\omega_g} = \frac{z_g}{z_p} \tag{b}$$

From (a) and (b),

$$i = \frac{\omega_p}{\omega_g} = \frac{z_g}{z_p} = \frac{d_g \cos\psi_g}{d_p \cos\psi_p} \tag{18.32}$$

The centre distance a between axes of crossed helical gears is given by,

$$a = \frac{d_p}{2} + \frac{d_g}{2} = \frac{z_p m_n}{2\cos\psi_p} + \frac{z_g m_n}{2\cos\psi_g}$$

$$\therefore \quad a = \frac{m_n}{2}\left[\frac{z_p}{\cos\psi_p} + \frac{z_g}{\cos\psi_g}\right] \tag{18.33}$$

Crossed helical gears are a special category. They have limited applications due to poor load carrying capacity, considerable amount of sliding between meshing teeth and poor efficiency. There are no standards for the proportions of crossed helical gears.

Short-Answer Questions

18.1 Compare the contact between mating teeth of spur and helical gears.
18.2 What are the advantages of helical gears over spur gears?
18.3 Where do you use helical gears?
18.4 What is a parallel helical gear?
18.5 What is a crossed helical gear?
18.6 What is virtual or formative helical gear?
18.7 What is the relationship between actual and virtual number of teeth and the helix angle?
18.8 What is the main disadvantage of a single helical gear? What is the remedy?
18.9 What is a double helical gear?
18.10 What is a herringbone helical gear?
18.11 What is the difference between double and herringbone helical gears?
18.12 State two advantages of herringbone and double helical gears.
18.13 State two disadvantages of herringbone and double helical gears.
18.14 Where do you use herringbone and double helical gears.

18.15 What is the relationship between ψ_1, ψ_2 and Σ in crossed helical gears?

18.16 What hands of helix are used for crossed helical gear?

18.17 Compare the contact between mating teeth of parallel and crossed helical gears.

18.18 Why are crossed helical gears not used for high power transmission?

18.19 State the applications of crossed helical gears.

Problems for Practice

18.1 A pair of helical gears consists of a 25 teeth pinion meshing with a 50 teeth gear. The normal module is 4 mm. Find the required value of the helix angle, if the centre distance is exactly 165 mm.

[*24.62°*]

18.2 A pair of parallel helical gears consists of a 20 teeth pinion and the velocity ratio is 3 : 1. The helix angle is 15° and the normal module is 5 mm. Calculate

(i) the pitch circle diameters of the pinion and the gear; and

(ii) the centre distance.

[*(i) 103.53 and 310.58 mm (ii) 207.06 mm*]

18.3 A pair of parallel helical gears consists of 18 teeth pinion meshing with a 63 teeth gear. The normal module is 3 mm. The helix angle is 23° while the normal pressure angle is 20°. Calculate

(i) the transverse module;

(ii) the transverse pressure angle; and

(iii) the axial pitch.

[*(i) 3.26 mm (ii) 21.57° (iii) 24.13 mm*]

18.4 A pair of parallel helical gears consists of an 18 teeth pinion meshing with a 45 teeth gear. 7.5 kW power at 2000 rpm is supplied to the pinion through its shaft. The normal module is 6 mm, while the normal pressure angle is 20°. The helix angle is 23°. Determine the tangential, radial and axial components of the resultant tooth force between the meshing teeth.

[*610.41, 241.36 and 259.10 N*]

18.5 The following data is given for a pair of parallel helical gears made of steel:

power transmitted = 20 kW
speed of pinion = 720 rpm
number of teeth on pinion = 35
number of teeth on gear = 70
centre distance = 285 mm
normal module = 5 mm
face width = 50 mm
normal pressure angle = 20°
ultimate tensile strength = 600 N/mm^2
surface hardness = 300 BHN
grade of machining = Gr. 6
service factor = 1.25

Calculate

(i) the helix angle;

(ii) the beam strength;

(iii) the wear strength;

(iv) the static load;

(v) the dynamic load by Buckingham's equation;

(vi) the effective load;

(vii) the effective factor of safety against bending failure; and

(viii) the effective factor of safety against pitting failure.

[*(i) 22.92° (ii) 19 930 N (iii) 21495.64 N (iv) 2792.19 N (v) 8047.29 N (vi) 11 537.53 N (vii) 1.73 (viii) 1.86*]

Bevel Gears

Chapter 19

19.1 BEVEL GEARS

Bevel gears are used to transmit power between two intersecting shafts. There are two common types of bevel gears—*straight* and *spiral*, as shown in Fig. 19.1. The elements of the teeth of the straight bevel gears are straight lines, which converge into a common apex point. The elements of the teeth of the spiral bevel gears are spiral curves,

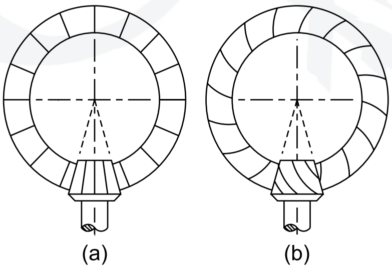


Fig. 19.1 *Types of Bevel Gears: (a) Straight Bevel Gear (b) Spiral Bevel Gear*

which also converge into a common apex point. Involute profile is used for the form of the teeth for both types of gears. Straight bevel gears are easy to design and manufacture and give reasonably good service when properly mounted on shafts. However, they create noise at high-speed conditions. Spiral bevel gears, on the other hand, are difficult to design and costly to manufacture, for they require specialized and sophisticated machinery for their manufacture. Spiral bevel gears have smooth teeth engagement, which results in quiet operation, even at high speeds. They have better strength and are thus used for high speed–high power transmission.

In some cases, bevel gears are classified on the basis of pitch angle. Three types of bevel gears that are based on pitch angle are as follows:

(i) When the pitch angle is less than 90°, it is called *external* bevel gear.

(ii) When the pitch angle is equal to 90°, it is called *crown* bevel gear.

(iii) When the pitch angle is more than 90°, it is called *internal* bevel gear.

There are certain specific categories of bevel gears. They are as follows:

(i) Miter Gears When two identical bevel gears are mounted on shafts, which are intersecting at right angles, they are called '*miter*' gears. They are shown in Fig. 19.2. Miter gears have the following characteristics:

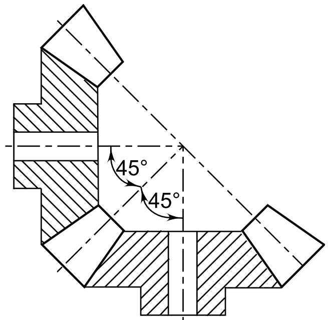


Fig. 19.2 *Miter Gears*

(a) The pitch angles of pinion and gear of miter gears are same and each is equal to 45°.
(b) The pinion and gear of miter gears rotate at the same speed.
(c) The pinion and gear have same dimensions, namely, addendum, dedendum, pitch circle diameter, number of teeth and module.
(d) The pinion and gear of miter gears are always mounted on shafts, which are perpendicular to each other.

(ii) Crown Gear In a pair of bevel gears, when one of the gears has a pitch angle of 90° then that gear is called '*crown*' gear. Such bevel gears are mounted on shafts, which are intersecting at an angle that is more than 90°. The crown gear is equivalent to the rack in spur gearing. The pitch cone of the crown gear becomes plane. They are shown in Fig. 19.3. Crown gear arrangement has the following characteristics:

(a) The pitch angle of crown gear is 90°.
(b) The bevel pinion and crown gear are always mounted on shafts, which are intersecting at angle more than 90°.

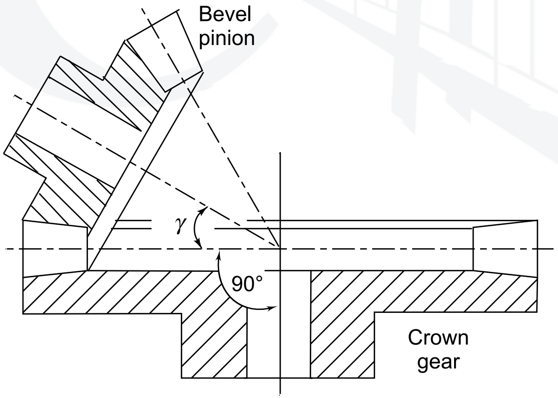


Fig. 19.3 *Crown Gear*

(iii) Internal Bevel Gears When the teeth of bevel gear are cut on the inside of the pitch cone, it is called *internal* bevel gear. In this case, the pitch angle of internal gear is more than 90° and the apex point is on the backside of the teeth on that gear. They are shown in Fig. 19.4. Internal bevel gears are used in planetary gear trains.

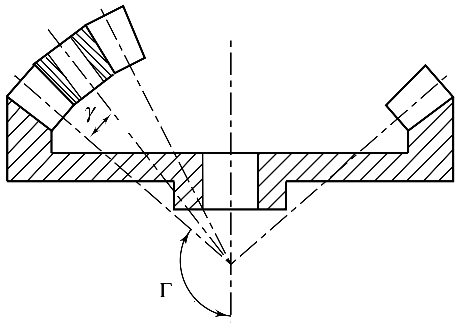


Fig. 19.4 *Internal Bevel Gears*

(iv) Skew Bevel Gears When two straight bevel gears are mounted on shafts, which are non-parallel and non-intersecting, they are called '*skew*' bevel gears. They are shown in Fig. 19.5(b). It is seen that the apex point of pinion is offset with respect to that of gear. Skew bevel gears have following characteristics:

(a) Skew bevel gears have straight teeth.
(b) Skew bevel gears are mounted on non-parallel and non-intersecting shafts.

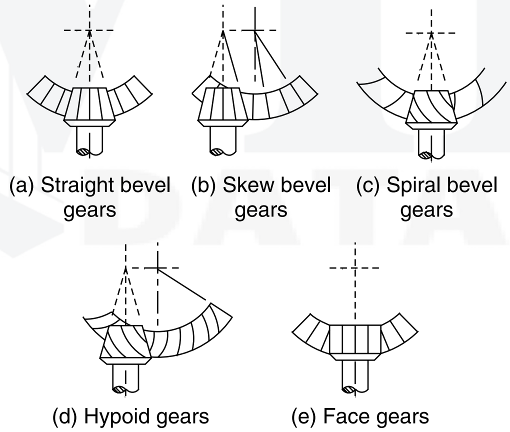


Fig. 19.5 *Types of Bevel Gears*

(v) Hypoid Gears Hypoid gears are similar to spiral bevel gears that are mounted on shafts, which are non-parallel and non-intersecting. They are shown in Fig. 19.5(d). Hypoid gears are based upon pitch surfaces, which are hyperboloids of revolution. When two hyperboloids are rotated together, the resultant motion is a combination of rolling and sliding. The sliding is along the length of teeth and results in increased friction. On this account, Extreme Pressure (EP) lubricants are used for these gears. Sliding friction reduces the efficiency of hypoid gears. The efficiency of bevel gear drive is 98 to 99 per cent, whereas that of hypoid gears is 96 to 98 per cent. Hypoid gears have the following characteristics:

(a) Hypoid gears have curved teeth.
(b) Hypoid gears are mounted on non-parallel and non-intersecting shafts.

Hypoid bevel gears are mainly used in automobile differentials. These gears allow the drive shaft to be placed well below the centreline of the rear axle and thereby lower the centre of gravity of the vehicle. Another advantage of hypoid gears is that the offset of the shaft is so great that the shafts may continue past each other. Therefore, multiple power take-offs from a single shaft with several pinions is possible. Hypoid gears give noiseless operation even at high speeds.

(vi) Zerol Gears Zerol gears are spiral bevel gears with zero spiral angle. These gears theoretically give more gradual contact and a slightly larger contact ratio.

(vii) Face Gears Face gears consist of a spur or helical pinion mating with a conjugate gear of disk form. They are shown in Fig. 19.5(e). Face gears have the following characteristics:

(a) The pinion of face gears is either a spur gear or a helical gear.
(b) Face gears are mounted on intersecting shafts that are at right angles to each other.
(c) The teeth of face gears can be straight or curved.

Face gears have the following advantages:

(a) They can be cut with spur gear cutters and gear shapers.
(b) The axial position of the pinion is not critical as in case of bevel pinion.

The disadvantage is that the width of the tooth face is small.

Bevel gears, straight, spiral or hypoid, are not interchangeable and are always designed in pairs. In majority of cases, the angle between the axes of intersecting shafts is 90°. The discussion in this chapter is mainly restricted to straight bevel gears, connecting two intersecting shafts at right angles.

19.2 TERMINOLOGY OF BEVEL GEARS

A bevel gear is in the form of the frustum of a cone. The dimensions of a bevel gear are illustrated in Fig. 19.6(a) and (b). The following terms are important in terminology of bevel gears:

(i) Pitch Cone Pitch cone is an imaginary cone, the surface of which contains the pitch lines of all teeth in the bevel gear.

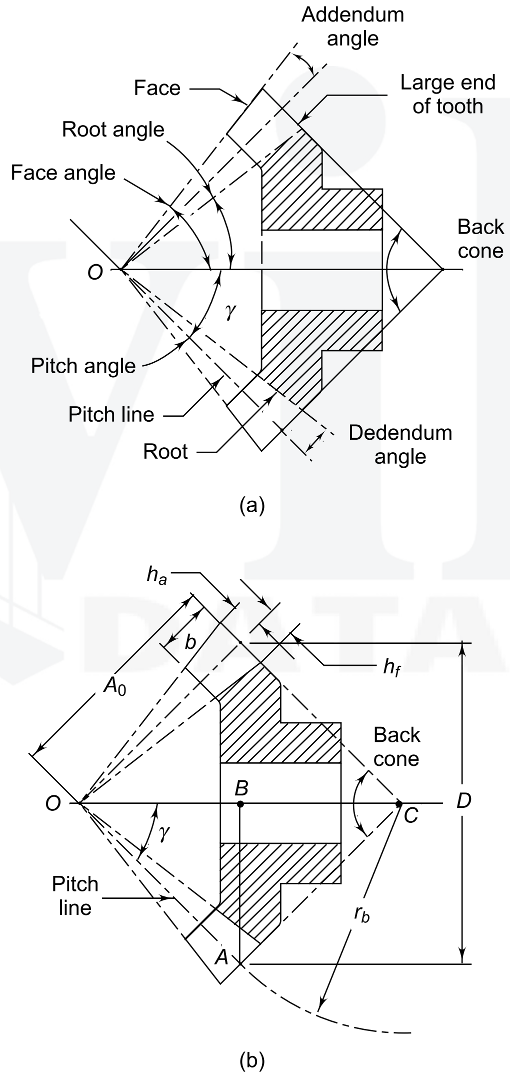


Fig. 19.6 *Terminology of Bevel Gear*

(ii) Cone Centre The apex of the pitch cone is called the cone centre. It is denoted by O.

(iii) Cone Distance Cone distance is the length of the pitch-cone element. It is also called pitch-cone radius. It is denoted by A_0.

(iv) Pitch Angle The angle that the pitch line makes with the axis of the gear, is called the pitch angle. It is denoted by γ. The pitch angle is also called centre angle.

(v) Addendum Angle It is the angle subtended by the addendum at the cone centre. It is denoted by α.

(vi) Dedendum Angle It is the angle subtended by the dedendum at the cone centre. It is denoted by δ.

(vii) Face Angle It is the angle subtended by the face of the tooth at the cone centre.

Face angle = pitch angle + addendum angle = $\gamma + \alpha$.

(viii) Root Angle It is the angle subtended by the root of the tooth at the cone centre.

Root angle = pitch angle – dedendum angle = $\gamma - \delta$.

(ix) Back Cone The back cone is an imaginary cone and its elements are perpendicular to the elements of the pitch cone.

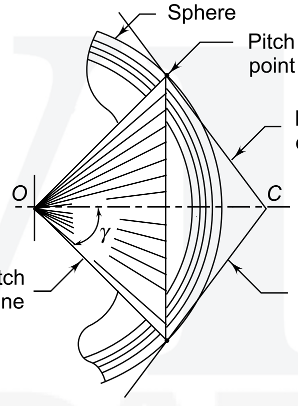
(x) Back Cone Distance It is the length of the back cone element. It is also called back cone radius. It is denoted by r_b.

It is observed from the figure that the cross-section of the tooth decreases in size as it approaches towards the apex point O. Therefore, the pitch circle diameter, module, addendum, and dedendum decreases and there is no single value for these parameters. In practice, these dimensions are measured at the largest tooth section called the large end of the tooth. The dimensions of the bevel gear are always specified and measured at the large end of the tooth. The addendum h_a, the dedendum h_f and the pitch circle diameter D are specified at the large end of the tooth as shown in Fig. 19.6(b).

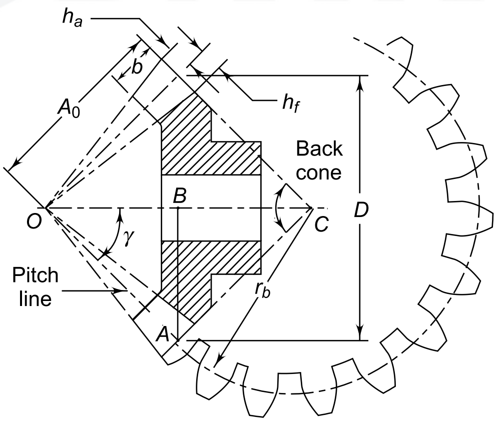
The analysis for a bevel gear will show that a true section of involutes profile of tooth lies on the surface of a sphere. It is not possible to represent on a plane surface, the exact profile of a bevel gear tooth that actually lies on the surface of a sphere. Since the beam strength and wear strength equations are based on upon tooth profile, it is necessary to approximate the tooth profile as accurately as possible. The approximation is called '*Tredgold's approximation*'. As shown in Fig. 19.7(a), a tangent to the sphere at the pitch point will closely approximate the surface of the sphere for a short distance on either side of the pitch point. Therefore, the back cone can be developed as a plane surface for constructing a spur gear.

As shown in Fig. 19.7(b), an imaginary spur gear is considered in a plane perpendicular to the tooth at the large end. r_b is the pitch circle radius of this imaginary spur gear and z' is the number of teeth on this gear. The gear is called the formative gear and the number of teeth z' on this gear is called the *virtual* or the *formative number of teeth*. The formative number of teeth is given by,

$$z' = \frac{2r_b}{m} \qquad \text{(a)}$$



(a) Back cone



(b) Equivalent spur gear

Fig. 19.7

where m is the module at the large end of the tooth. If z is the actual number of teeth on the bevel gear, then

$$z = \frac{D}{m} \tag{b}$$

From (a) and (b),

$$\frac{z'}{z} = \frac{2r_b}{D} \tag{c}$$

In Fig. 19.7(b), a line AB is drawn perpendicular to the the line OC. From ΔABC,

$$\sin \angle BCA = \frac{AB}{AC} \quad \text{or} \quad \sin(90 - \gamma) = \frac{D/2}{r_b}$$

Therefore,

$$r_b = \frac{D}{2\cos\gamma} \tag{19.1}$$

Substituting the above expression in Eq. (c),

$$z' = \frac{z}{\cos\gamma} \tag{19.2}$$

A pair of bevel gears is shown in Fig. 19.8. D_p and D_g are the pitch circle diameters of pinion and gear respectively. γ is the pitch angle of the pinion, while Γ is the pitch angle of the gear. Line AB is perpendicular to the line OB. Consider the triangle OAB,

$$\tan\gamma = \frac{\overline{AB}}{\overline{OB}} = \frac{D_p/2}{D_g/2} = \frac{D_p}{D_g} = \frac{mz_p}{mz_g}$$

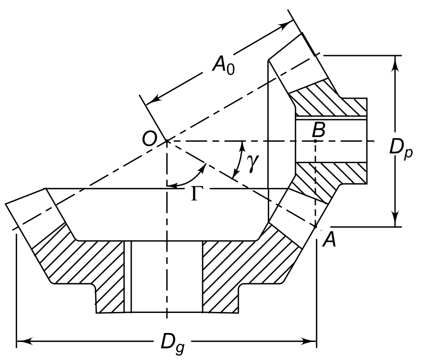


Fig. 19.8 *Pair of bevel gears*

Therefore

$$\tan\gamma = \frac{z_p}{z_g} \tag{19.3}$$

Similarly, it can be proved that

$$\tan\Gamma = \frac{z_g}{z_p} \tag{19.4}$$

Also

$$\gamma + \Gamma = \frac{\pi}{2} \tag{19.5}$$

The cone distance A_0 is given by,

$$A_0 = \overline{OA} = \sqrt{(\overline{AB})^2 + (\overline{BO})^2}$$

$$= \sqrt{\left(\frac{D_p}{2}\right)^2 + \left(\frac{D_g}{2}\right)^2} \tag{19.6}$$

19.3 FORCE ANALYSIS

In force analysis, it is assumed that the resultant tooth force between two meshing teeth of a pair of bevel gears is concentrated at the midpoint along the face width of the tooth. This is illustrated in Fig. 19.9(a), where the resultant force P, shown by the dotted line, acts at the midpoint D of the face width of the pinion. The resultant force has following three components:

P_t = tangential or useful component (N)

P_r = radial component (N)

P_a = axial or thrust component (N)

Consider the plane $ABCD$ shaded by dots in Fig. 19.9(a) and again shown in Fig. 19.9(b). From the triangle BCD,

$$\tan\alpha = \frac{BC}{CD} = \frac{P_s}{P_t} \tag{a}$$

where,

P_s = separating component (N)

α = pressure angle (degrees)

From (a),

$$P_s = P_t \tan\alpha \tag{b}$$

Refer to Fig. 19.9(a). The separating force P_s is perpendicular to the pitch line OD. Therefore,

$$AD \perp OD$$

Line FD is vertical, while the line OX is horizontal. Therefore,

$$FD \perp OX$$

There are two pairs of perpendicular lines and their included angle should be equal. The angle between lines OD and OX should be equal to the angle between the lines AD and FD. The angle between lines OD and OX is the pitch angle γ. Therefore, the angle between lines AD and FD should be equal to the pitch angle γ. Consider the plane $DEAF$ shown in Fig. 19.9(c). From triangle ADF,

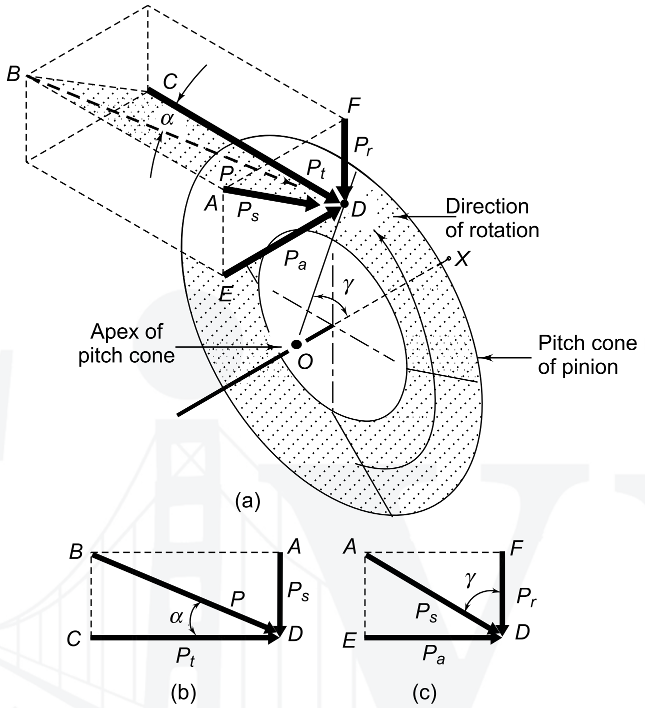


Fig. 19.9 *Components of Tooth Face*

$$P_r = P_s \cos \gamma$$
$$P_a = P_s \sin \gamma$$

Substituting Eq. (b) in the above expressions,

$$P_r = P_t \tan \alpha \cos \gamma \qquad (19.7)$$
$$P_a = P_t \tan \alpha \sin \gamma \qquad (19.8)$$

The tangential component is determined from the following relationship:

$$P_t = \frac{M_t}{r_m} \qquad (19.9)$$

where,

M_t = torque transmitted by gears (N-mm)

r_m = radius of the pinion at midpoint along the face width (mm)

Equations (19.7), (19.8) and (19.9) are used to determine the components of the tooth force on the pinion. The components of the tooth force acting on the gear can be determined by considering actions and reactions as equal and opposite. A two-dimensional representation of force components is illustrated in Fig. 19.10. As shown in Fig. 19.10(b), the mean radius (r_m) where the resultant force acts, is given by,

$$r_m = \overline{AC} - \overline{AB} \qquad (c)$$

$$\overline{AC} = \frac{D_p}{2} \qquad (d)$$

$$\frac{\overline{AB}}{(b/2)} = \sin \gamma \quad \text{or} \quad \overline{AB} = \frac{b}{2} \sin \gamma \qquad (e)$$

From (c), (d) and (e),

$$r_m = \left[\frac{D_p}{2} - \frac{b \sin \gamma}{2} \right] \qquad (19.10)$$

where

b = face width of the tooth (mm)

As seen in Fig.19.10(c), the radial component on the gear is equal to the axial component (P_a) on the pinion. Similarly, the axial component on the gear is equal to the radia1 component (P_r) on the pinion.

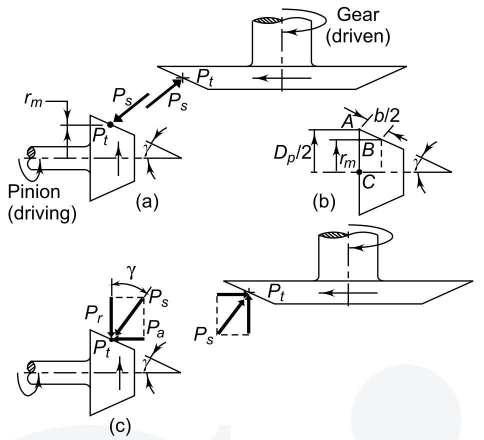


Fig. 19.10 *Force Analysis*

In two-dimensional representation of forces, very often (•) and (×) are used to indicate forces perpendicular to the plane of the paper. (•) indicates a force that is perpendicular to the plane of paper and which is towards the observer. (×) indicates a force that is perpendicular to the plane of the paper and which is away from the observer. In examples of bevel gear tooth forces, it is often required to find out the magnitude and direction of three components acting on the pinion and gear.

Force Components on Pinion

The magnitudes are determined by using the following five equations:

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n_p}$$

$$r_m = \left[\frac{D_p}{2} - \frac{b \sin\gamma}{2}\right]$$

$$P_t = \frac{M_t}{r_m}$$

$$P_r = P_t \tan\alpha \cos\gamma$$

$$P_a = P_t \tan\alpha \sin\gamma$$

Note that the notations (P_t), (P_r) and (P_a) are used for components of force on pinion only.

The directions of tangential and radial components acting on the pinion and gear are decided by same method that is used for spur gears.

(i) Tangential Component (P_t)

(a) The direction of tangential component for the driving gear is opposite to the direction of rotation.

(b) The direction of tangential component for the driven gear is same as the direction of rotation.

(ii) Radial Component (P_r)

(a) The radial component on the pinion acts towards the centre of the pinion.

(b) The radial component on the gear acts towards the centre of the gear.

(iii) Thrust Component (P_a) The following guidelines can be used to determine the direction of the thrust component:

(a) The thrust component on the pinion is equal and opposite of the radial component on the gear.

(b) The thrust component on the gear is equal and opposite of the radial component on the pinion.

The tendency of thrust components is to separate the meshing teeth. This fact is useful in deciding their directions.

Example 19.1 *A pair of bevel gears transmitting 7.5 kW at 300 rpm is shown in Fig. 19.11(a). The pressure angle is 20°. Determine the components of the resultant gear tooth force and draw a free-body diagram of forces acting on the pinion and the gear.*

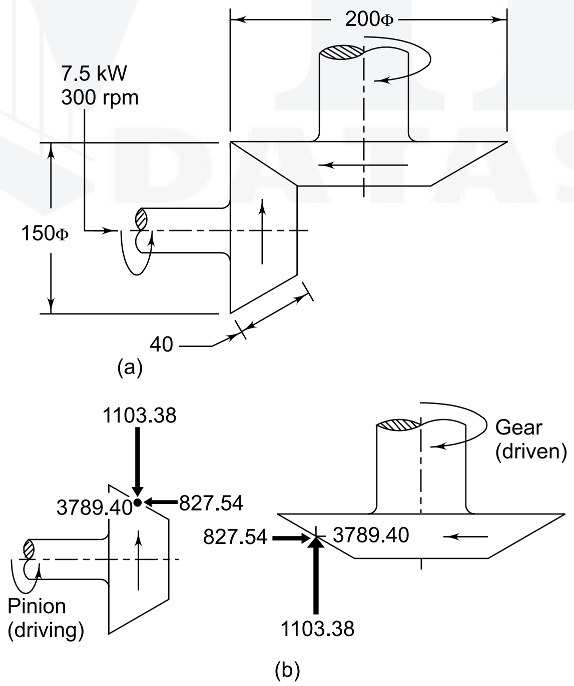


Fig. 19.11

Solution

Given $kW = 7.5$ $n_p = 300$ rpm $a = 20°$
$D_p = 150$ mm $D_g = 200$ mm $b = 40$ mm

Step I *Components of gear tooth force*

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n_p}$$

$$= \frac{60 \times 10^6 (7.5)}{2\pi (300)} = 238\ 732.41 \text{ N-mm}$$

$$\tan\gamma = \frac{z_p}{z_g} = \frac{D_p}{D_g} = \frac{150}{200} = 0.75 \text{ or } \gamma = 36.87°$$

$$r_m = \left[\frac{D_p}{2} - \frac{b\sin\gamma}{2}\right]$$

$$= \left[\frac{150}{2} - \frac{40\sin(36.87)}{2}\right] = 63 \text{ mm}$$

$$P_t = \frac{M_t}{r_m} = \frac{238\,732.41}{63} = 3789.40 \text{ N}$$

$P_r = P_t \tan\alpha \cos\gamma$
$= 3789.40 \tan (20) \cos (36.87) = 1103.38$ N
$P_a = P_t \tan\alpha \sin\gamma$
$= 3789.40 \tan (20) \sin (36.87) = 827.54$ N

Step II *Free-body diagram of forces*

The free-body diagram of the forces is shown in Fig. 19.11(b).

Example 19.2 *The dimensions of a pair of bevel gears are given in Fig 19.12. The gear G delivers 5 kW of power at 500 rpm to the output shaft. The bearings A and B are mounted on the output shaft in such a way that the bearing B can take radial as well as thrust load, while the bearing A can take only radial load. Determine the reactions at the two bearings.*

Solution

Given $kW = 5$ $n_g = 500$ rpm $D_p = 100$ mm
$D_g = 200$ mm $b = 20$ mm

Step I *Components of gear tooth force on pinion*

$$\frac{n_p}{n_g} = \frac{D_g}{D_p} \quad \text{or} \quad \frac{n_p}{500} = \frac{200}{100}$$

$\therefore$ $n_p = 1000$ rpm

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n_p}$$

$$= \frac{60 \times 10^6 (5)}{2\pi (1000)} = 47\,746.48 \text{ N-mm}$$

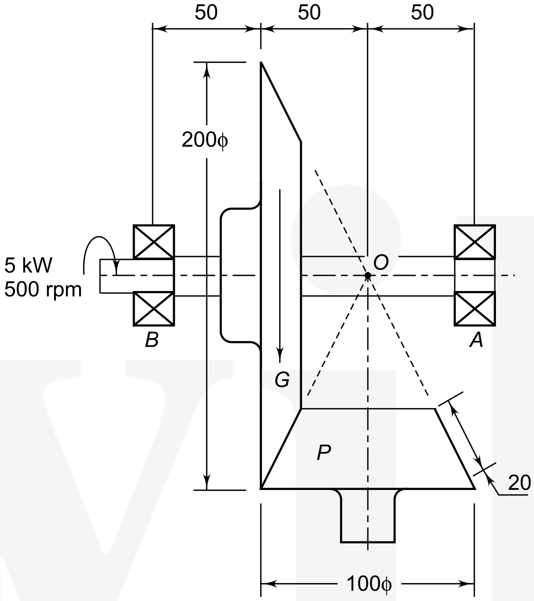


Fig. 19.12

For pinion,

$$\tan\gamma = \frac{z_p}{z_g} = \frac{D_p}{D_g} = \frac{100}{200} = 0.5 \text{ or } \gamma = 26.57°$$

$$r_m = \left[\frac{D_p}{2} - \frac{b\sin\gamma}{2}\right]$$

$$= \left[\frac{100}{2} - \frac{20\sin(26.57)}{2}\right] = 45.53 \text{ mm}$$

$$P_t = \frac{M_t}{r_m} = \frac{47\,746.48}{45.53} = 1048.75 \text{ N}$$

$P_r = P_t \tan\alpha \cos\gamma$
$= 1048.75 \tan (20) \cos (26.57) = 341.40$ N
$P_a = P_t \tan\alpha \sin\gamma$
$= 1048.75 \tan (20) \sin (26.57) = 170.74$ N

Step II *Components of gear tooth force on gear*

The radial component on the gear is equal to the axial component (P_a) of the pinion. Also, the axial component on the gear is equal to the radial component (P_r) of the pinion. Therefore, the force components acting on the gear tooth are as follows:

$$P_t = 1048.75 \text{ N}$$
$$P_r = 170.74 \text{ N}$$
$$P_a = 341.40 \text{ N}$$

For gear,

$$\Gamma = 90 - \gamma = 90 - 26.57 = 63.43°$$

$$r_m = \left[\frac{D_g}{2} - \frac{b \sin \Gamma}{2}\right]$$

$$= \left[\frac{200}{2} - \frac{20 \sin (63.43)}{2}\right] = 91.06 \text{ mm}$$

Step III *Reactions at bearings A and B*

The components of tooth force on the gear act at a mean radius of 91.06 mm. The forces acting on the gear shaft are shown in Fig. 19.13.

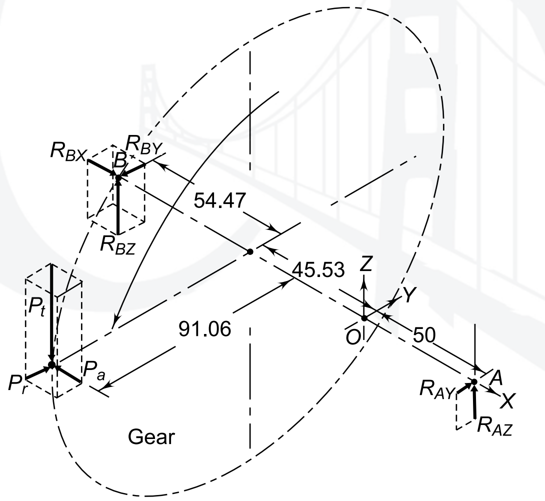


Fig. 19.13

Forces in the XZ plane

Taking moment of forces about the bearing B,

$$R_{AZ} \times (50 + 45.53 + 54.47) = P_t \times (54.47)$$

or $$R_{AZ} \times (150) = 1048.75 \times (54.47)$$

$\therefore$ $$R_{AZ} = 380.84 \text{ N} \qquad \text{(i)}$$

Considering the equilibrium of forces,

$$R_{AZ} + R_{BZ} = P_t$$
$$380.84 + R_{BZ} = 1048.75 \text{ N}$$

$\therefore$ $$R_{BZ} = 667.91 \text{ N} \qquad \text{(ii)}$$

Forces in the XY plane

Taking moments of forces about the bearing B,

$$R_{AY} \times (50 + 45.53 + 54.47) + P_r \times (54.47)$$
$$= P_a \times (91.06)$$
$$R_{AY} \times (150) + 170.74 \times (54.47)$$
$$= 341.40 \times (91.06)$$

$\therefore$ $$R_{AY} = 145.25 \text{ N} \qquad \text{(iii)}$$

Considering equilibrium of forces,

$$R_{BY} = R_{AY} + P_r \quad \text{or} \quad R_{BY} = 145.25 + 170.74$$

$\therefore$ $$R_{BY} = 315.99 \text{ N} \qquad \text{(iv)}$$

and $$R_{BX} = P_a = 341.40 \text{ N} \qquad \text{(v)}$$

Example 19.3 *A differential planetary gear train is shown in Fig. 19.14. The input shaft receives 10 kW power at 500 rpm. The pitch circle diameters of bevel gears A, B, C and D at the midpoint along the face width are 250, 125, 250 and 500 mm respectively. The pitch circle diameters of spur gears E and F are 250 and 350 mm respectively. The gears rotate at constant speed. Draw a free-body diagram of forces acting on various gears and calculate the torque on each of the two output shafts.*

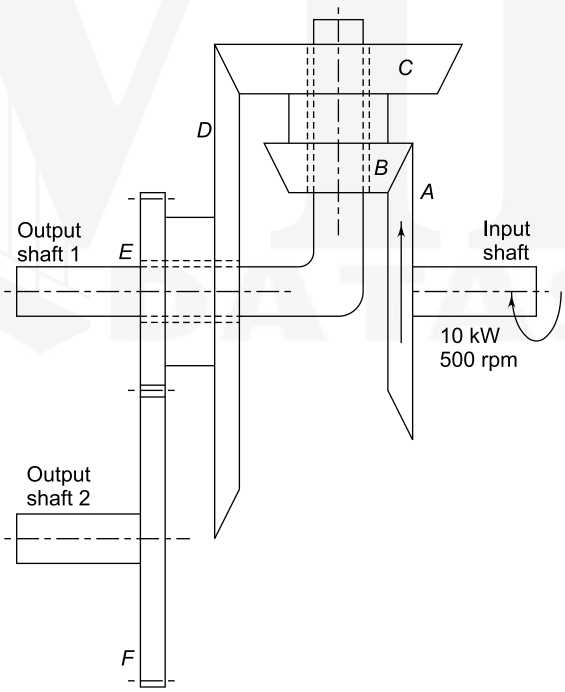


Fig. 19.14 *Differential Planetary Gear System*

Solution

Given kW = 10 $n_A = 500$ rpm $D_A = 250$ mm
$D_B = 125$ mm $D_C = 250$ mm $D_D = 500$ mm
$D_E = 250$ mm $D_F = 350$ mm

Step I *Tangential component of tooth force between gears A and B*

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n_A}$$

$$= \frac{60 \times 10^6 (10)}{2\pi(500)} = 190\ 985.93 \text{ N-mm}$$

$$(P_t)_{AB} = \frac{M_t}{(r_m)_A} = \frac{190\ 985.93}{125} = 1527.89 \text{ N}$$

Step II *Tangential component of tooth force between gears C and D*

The tangential component of the tooth force between gears C and D is $(P_t)_{CD}$. For constant speed,

$$(P_t)_{AB} \times (r_m)_B = (P_t)_{CD} \times (r_m)_C$$

$$(1527.89) \times \left(\frac{125}{2}\right) = (P_t)_{CD} \times \left(\frac{250}{2}\right)$$

$$(P_t)_{CD} = 763.95 \text{ N}$$

Step III *Tangential component of tooth force between gears E and F*

Considering forces on gears D and E,

$$(P_t)_{CD} \times (r_m)_D = (P_t)_{EF} \times (r_m)_E$$

$$(763.95) \times \left(\frac{500}{2}\right) = (P_t)_{EF} \times \left(\frac{250}{2}\right)$$

$$(P_t)_{EF} = 1527.89 \text{ N}$$

Step IV *Free-body diagram of forces*

The free-body diagram of forces is shown in Fig. 19.15. In the figure, (•) and (×) represent forces perpendicular to the plane of the paper, (•) is towards the observer, while (×) is away from the observer.

Step V *Torque on output shaft 1 and 2*

The torque on the output shaft 2 is given by

$$(M_t)_2 = (P_t)_{EF} \times (r_m)_F$$

$$= 1527.89 \left(\frac{350}{2}\right) = 267\ 380.75 \text{ N} \qquad \text{(i)}$$

Referring to Fig. 19.16, the torque on the output shaft 1 is given by

$$(M_t)_1 = (P_t)_{AB} \times (r_m)_A + (P_t)_{CD} \times (r_m)_D$$

$$= 1527.89 \times \left(\frac{250}{2}\right) + 763.95 \times \left(\frac{500}{2}\right)$$

$$= 381\ 973.5 \text{ N-mm} \qquad \text{(ii)}$$

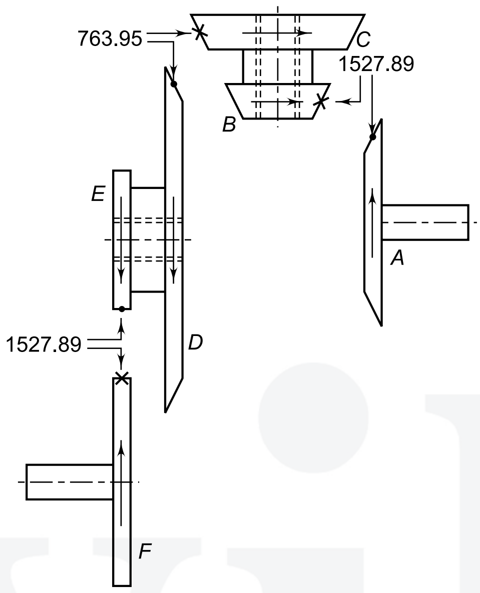


Fig. 19.15 *Free-body Diagram of Forces*

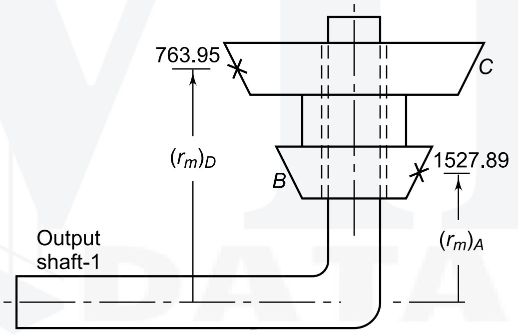


Fig. 19.16

19.4 BEAM STRENGTH OF BEVEL GEARS

The size of the cross-section of the tooth of a bevel gear varies along the face width as shown in Fig. 19.17. In order to determine the beam strength of the tooth of a bevel gear, it is considered to be equivalent to a formative spur gear in a plane perpendicular to the tooth element. Consider an elemental section of the tooth at a distance x from the apex O and having a width dx. Applying the Lewis equation to a formative spur gear at a distance x from the apex,

$$\delta(S_b) = m_x b_x \sigma_b Y \qquad \text{(a)}$$

where,

$\delta(S_b)$ = beam strength of the elemental section (N)
m_x = module of the section (mm)
b_x = face width of elemental section (mm)
Y = Lewis form factor based on virtual number of teeth

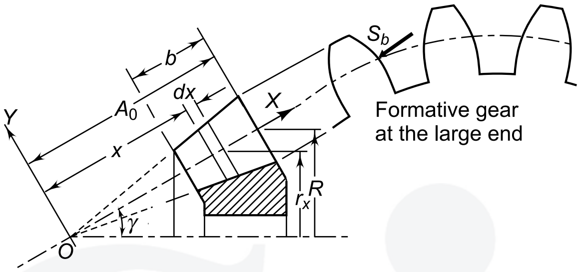


Fig. 19.17 *Beam Strength of Bevel Gear Tooth*

From the figure,

$$\frac{r_x}{R} = \frac{x}{A_o} \quad \text{or} \quad r_x = \frac{xR}{A_o} \tag{b}$$

At the elemental section,

$$m_x = \frac{2r_x}{z} = \frac{2xR}{zA_o} \tag{c}$$

At the large end of the tooth,

$$m = \frac{2R}{z} \tag{d}$$

From (c) and (d),

$$m_x = m\left(\frac{x}{A_o}\right) \tag{e}$$

Also, $\quad b_x = dx \quad$ (f)

Substituting (e) and (f) in (a), we have

$$\delta(S_b) = \frac{m\,\sigma_b\,Yxdx}{A_o} \tag{g}$$

From (b) and (g),

$$\int [r_x\,\delta(S_b)] = \left(\frac{m\,\sigma_b\,YR}{A_o^2}\right)\int x^2 dx \tag{h}$$

The left-hand side indicates the torque M_t, i.e.,

$$M_t = \left(\frac{m\,\sigma_b\,YR}{A_o^2}\right)\int_{(A_o-b)}^{A_o} x^2 dx$$

$$= \left(\frac{m\,\sigma_b\,YR}{A_o^2}\right)\left[\frac{x^3}{3}\right]_{(A_o-b)}^{A_o}$$

$$M_t = m\,b\,\sigma_b\,YR\left[1 - \frac{b}{A_o} + \frac{b^2}{3A_o^2}\right] \tag{i}$$

Assuming beam strength (S_b) as the tangential force at the large end of tooth,

$$M_t = S_b\,R \tag{j}$$

From (h) and (j),

$$S_b = m\,b\,\sigma_b\,Y\left[1 - \frac{b}{A_o} + \frac{b^2}{3A_o^2}\right]$$

The face width of the bevel gear is limited to one-third of the cone distance. Therefore, the last term in the bracket will never be more than (1/27). Neglecting the last term,

$$S_b = m\,b\,\sigma_b\,Y\left[1 - \frac{b}{A_o}\right] \tag{19.11}$$

where,

S_b = beam strength of the tooth (N)
m = module at the large end of the tooth (mm)
b = face width (mm)
σ_b = permissible bending stress ($S_{ut}/3$) (N/mm^2)
Y = Lewis form factor based on formative number of teeth
A_o = cone distance (mm)

The term $\left[1 - \frac{b}{A_0}\right]$ is called the *bevel factor*.

Equation (19.11) is known as the Lewis equation for bevel gears. In the above analysis, the beam strength (S_b) is determined using the pitch radius R at the large end of the tooth ($S_b = M_t/R$). *Therefore, the beam strength indicates the maximum value of the tangential force at the large end of the tooth that the tooth can transmit without bending failure.* This is not similar to the force analysis in Section 19.3, where the force components are determined at the mean radius r_m. It is necessary to compare the beam strength with an imaginary force P_t, considered to be acting at the large end of the tooth. This component is given by,

$$P_t = \frac{2M_t}{D} \tag{19.12}$$

The beam strength should always be more than the effective force between the meshing teeth at the large end of the tooth.

The face width of the bevel gear is generally taken as 10 m or (A_o/3), whichever is smaller, i.e.,

$$b = 10 \text{ m} \quad \text{or} \quad b = A_o/3 \text{ (whichever is smaller)} \tag{19.13}$$

19.5 WEAR STRENGTH OF BEVEL GEARS

The contact between two meshing teeth of straight bevel gears is a line contact, which is similar to that of spur gears. In order to determine the wear strength, the bevel gear is considered to be equivalent to a formative spur gear in a plane which is perpendicular to the tooth at the large end. Applying Buckingham's equation to these formative gears,

$$S_w = b\,Q\,d_p'\,K \tag{a}$$

where,

b = face width of gears (mm)
Q = ratio factor
d_p' = pitch circle diameter of formative pinion (mm)
K = material constant (N/mm^2)

The pitch circle diameter of the formative pinion is given by

$$d_p' = 2r_b$$

Substituting Eq. (19.1) in the above expression,

$$d_p' = \frac{D_p}{\cos\gamma} \tag{b}$$

where D_p is the pitch circle diameter of the pinion at the large end of the tooth. From (a) and (b),

$$S_w = \frac{bQD_pK}{\cos\gamma} \tag{c}$$

In case of bevel gears, either the pinion or the gear is generally overhanging. It is subjected to deflection under the action of tooth forces and it has been found that to transmit the load, only three-quarters of the face width is effective. Modifying Eq. (c) to account for this effect,

$$S_w = \frac{0.75bQD_pK}{\cos\gamma} \tag{19.14}$$

Equation (19.14) is known as Buckingham's equation for the wear strength of bevel gears. The equation is derived for the formative pair of pinion and gear. The ratio factor Q is, therefore, given by

$$Q = \frac{2z_g'}{z_g' + z_p'} \tag{d}$$

From Eq. (19.2),

$$z_p' = \frac{z_p}{\cos\gamma}$$

and
$$z_g' = \frac{z_g}{\cos\Gamma} = \frac{z_g}{\cos(90° - \gamma)} = \frac{z_g}{\sin\gamma}$$

Substituting these values in Eq. (d),

$$Q = \frac{2z_g}{z_g + z_p\tan\gamma} \tag{19.15}$$

The material constant K is the same as for spur gears and is given by

$$K = \frac{\sigma_c^2 \sin\alpha\cos\alpha\left[\dfrac{1}{E_p} + \dfrac{1}{E_g}\right]}{1.4} \tag{19.16}$$

When the pinion as well as the gear is made of steel and the pressure angle is 20° the value of K is given by

$$K = 0.16\left(\frac{\text{BHN}}{100}\right)^2 \tag{19.17}$$

The wear strength (S_w) indicates the maximum value of the tangential force at the large end of the tooth that the tooth can transmit without pitting failure. It should be more than the effective force between the meshing teeth.

19.6 EFFECTIVE LOAD ON GEAR TOOTH

The tangential component due to power transmission, considered to be acting at the large end of the tooth, is determined by using the following two equations:

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n_p}$$

$$P_t = \frac{2M_t}{D}$$

In addition to the tangential component due to power transmission, there is the dynamic load. There are two methods to account for the dynamic load—an approximate estimation by means of the *velocity factor* in the preliminary stages of gear design and a precise calculation by *Buckingham's equation* in the final stages. The effective load P_{eff} between two meshing teeth is given by,

$$P_{\text{eff}} = \frac{C_sP_t}{C_v} \tag{19.18}$$

where C_s = service factor (Table 17.4)
C_v = velocity factor

The velocity factor for cut teeth is given by,

$$C_v = \frac{6}{6+v} \tag{19.19}$$

The velocity factor for generated teeth is given by,

$$C_v = \frac{5.6}{5.6+\sqrt{v}} \tag{19.20}$$

where v is the pitch line velocity in m/s. In the final stages of gear design, the dynamic load is determined by the equation derived by Earle Buckingham[1]. The equation for dynamic load in bevel gears is as follows,

$$P_d = \frac{21v(Ceb+P_t)}{21v+\sqrt{(Ceb+P_t)}} \tag{19.21}$$

where,

P_d = dynamic load or incremental dynamic load (N)
v = pitch line velocity (m/s)
C = deformation factor (N/mm^2) (Table 17.7)
e = sum of errors between two meshing teeth (mm)
b = face width of tooth (mm)
P_t = tangential force due to power transmission (N)

Bevel gears are usually made of steel and the deformation factor C is 11 400 N/mm^2.

The method used to calculate the error e between two meshing teeth in chapters 17 and 18 is limited to spur and helical gears. For bevel gears, such precise information is difficult to get. In practice, it is necessary to contact the manufacturer and find out the expected error between meshing teeth. In the absence of such information. Table 19.1 may be used to get the values of error e. The classes of gears mentioned in the table indicate the following manufacturing methods,

Class-1 Well cut commercial gear teeth
Class-2 Gear teeth cut with great care
Class-3 Ground and lapped precision gear teeth

The effective load is given by

$$P_{\text{eff}} = C_s P_t + P_d \tag{19.22}$$

In order to avoid failure of the gear tooth due to bending,

$$S_b = P_{\text{eff}}\,(fs) \tag{19.23}$$

Table 19.1 *Maximum expected error between two meshing teeth (mm)*

Module (m) (mm)	*Class - 1*	*Class - 2*	*Class - 3*
Up to 4	0.050	0.025	0.0125
5	0.056	0.025	0.0125
6	0.064	0.030	0.0150
7	0.072	0.035	0.0170
8	0.080	0.038	0.0190
9	0.085	0.041	0.0205
10	0.090	0.044	0.0220

In order to avoid failure of the gear tooth due to pitting,

$$S_w = P_{\text{eff}}\,(fs) \tag{19.24}$$

Equations (19.23) and (19.24) are used to design bevel gears.

Example 19.4 *A pair of bevel gears, with 20° pressure angle, consists of a 20 teeth pinion meshing with a 30 teeth gear. The module is 4 mm, while the face width is 20 mm. The material for the pinion and gear is steel 50C4 (S_{ut} = 750 N/mm^2). The gear teeth are lapped and ground (Class-3) and the surface hardness is 400 BHN. The pinion rotates at 500 rpm and receives 2.5 kW power from the electric motor. The starting torque of the motor is 150% of the rated torque. Determine the factor of safety against bending failure and against pitting failure.*

Solution

Given $kW = 2.5$ $n_p = 500$ rpm $z_p = 20$
$z_g = 30$ $m = 4$ mm $b = 20$ mm
$S_{ut} = 750$ N/mm^2 BHN = 400
starting torque = 150% (rated torque)
machining grade = Class-3 $\alpha = 20°$

Step I *Beam strength*

Since the same material is used for both pinion and gear, the pinion is weaker than the gear.

[1] Buckingham Earle – 'Analytical Mechanics of Gears' – McGraw-Hill Inc., 1949.

$$\tan\gamma = \frac{z_p}{z_g} = \frac{20}{30} \quad \text{or} \quad \gamma = 33.69°$$

$$z'_p = \frac{z_p}{\cos\gamma} = \frac{20}{\cos(33.69)} = 24.04$$

From Table 17.3,

$$Y = 0.337 + \frac{(0.340 - 0.337)(24.04 - 24)}{(25 - 24)}$$

$$= 0.33712$$

$$\sigma_b = \frac{S_{ut}}{3} = \frac{750}{3} = 250 \text{ N/mm}^2$$

$$D_p = m\,z_p = 4(20) = 80 \text{ mm}$$
$$D_g = m\,z_g = 4(30) = 120 \text{ mm}$$

$$A_o = \sqrt{\left(\frac{D_p}{2}\right)^2 + \left(\frac{D_g}{2}\right)^2} = \sqrt{\left(\frac{80}{2}\right)^2 + \left(\frac{120}{2}\right)^2}$$

$$= 72.11 \text{ mm}$$

$$S_b = mb\,\sigma_b\,Y\left[1 - \frac{b}{A_o}\right]$$

$$= 4(20)(250)(0.33712)\left[1 - \frac{20}{72.11}\right]$$

$$= 4872.37 \text{ N}$$

Step II *Wear strength*

$$Q = \frac{2z_g}{z_g + z_p\tan\gamma} = \frac{2(30)}{30 + 20\tan(33.69)} = 1.385$$

$$K = 0.16\left(\frac{\text{BHN}}{100}\right)^2 = 0.16\left(\frac{400}{100}\right)^2 = 2.56 \text{ N/mm}^2$$

$$S_w = \frac{0.75bQD_pK}{\cos\gamma} = \frac{0.75(20)(1.385)(80)(2.56)}{\cos(33.69)}$$

$$= 5113.53 \text{ N}$$

Step III *Tangential force due to rated torque*

$$M_t = \frac{60\times10^6(\text{kW})}{2\pi n_p} = \frac{60\times10^6(2.5)}{2\pi(500)}$$

$$= 47\,746.48 \text{ N-mm}$$

$$P_t = \frac{2M_t}{D_p} = \frac{2(47746.48)}{80} = 1193.66 \text{ N}$$

Step IV *Dynamic load by Buckingham's equation*

From Table 19.1, the error for Class-3 gear teeth with 4 mm module is 0.0125 mm.

$$v = \frac{\pi D_p n_p}{60\times10^3} = \frac{\pi(80)(500)}{60\times10^3} = 2.094 \text{ m/s}$$

Also, $C = 11400 \text{ N/mm}^2$ $\quad b = 20$ mm

$P_t = 1193.66$ N

From Eq. (19.21),

$$P_d = \frac{21v(Ceb + P_t)}{21v + \sqrt{(Ceb + P_t)}}$$

$$= \frac{21(2.094)[11400(0.0125)(20) + 1193.66)]}{21(2.094) + \sqrt{[11400(0.0125)(20) + 1193.66)]}}$$

$$= 1653.25 \text{ N}$$

Step V *Effective load*

From Eq. (19.22),

$$P_{\text{eff}} = C_s P_t + P_d = 1.5\,(1193.66) + 1653.25$$
$$= 3443.74 \text{ N}$$

Step VI *Factor of safety*

Against bending failure,

$$(fs) = \frac{S_b}{P_{\text{eff}}} = \frac{4872.37}{3443.74} = 1.41$$

Against pitting failure,

$$(fs) = \frac{S_w}{P_{\text{eff}}} = \frac{5113.53}{3443.74} = 1.48$$

Example 19.5 *A pair of straight bevel gears, mounted on shafts which are intersecting at right angles, consists of a 24 teeth pinion meshing with a 32 teeth gear. The pinion shaft is connected to an electric motor developing 12.5 kW rated power at 1440 rpm. The starting torque of the motor is 150% of the rated torque. The pressure angle is 20°. Both gears are made of case hardened steel (S_{ut} = 750 N/mm^2). The teeth on gears are generated and finished by grinding and lapping processes to meet the requirements of Class-3 Grade. The factor of safety in the preliminary stages of gear design is 2.*

(i) In the initial stages of gear design, assume that velocity factor accounts for the dynamic load and that the pitch line velocity is

7.5 m/s. Estimate the module based on beam strength.

(ii) Select the first preference value of the module and calculate the main dimensions of the gears.

(iii) Determine the dynamic load using Buckingham's equation and find out the effective load for the above dimensions. What is the correct factor of safety for bending?

(iv) Specify the surface hardness for the gears assuming a factor of safety of 2 for wear consideration.

Solution

Given kW = 12.5 $n_p = 1440$ rpm $z_p = 24$
$z_g = 32$ $S_{ut} = 750$ N/mm^2 BHN = 400
starting torque = 150% (rated torque)
machining grade = Class-3 $\alpha = 20°$

Step I *Estimation of module based on beam strength*

$$M_t = \frac{60\times10^6(\text{kW})}{2\pi n_p} = \frac{60\times10^6(12.5)}{2\pi(1440)}$$

$$= 82\,893.2 \text{ N-mm}$$

$$D_p = m\,z_p = m\,(24) = (24m) \text{ mm}$$

$$P_t = \frac{2M_t}{D_p} = \frac{2(82\,893.2)}{24\,m} = \left(\frac{6907.77}{m}\right)\text{N}$$

For generated teeth,

$$C_v = \frac{5.6}{5.6+\sqrt{v}} = \frac{5.6}{5.6+\sqrt{7.5}} = 0.6716$$

$$P_{\text{eff}} = \frac{C_s P_t}{C_v} = \frac{1.5}{0.6716}\left(\frac{6907.77}{m}\right) = \left(\frac{15\,428.32}{m}\right)\text{N}$$

It is assumed that

$$\frac{b}{A_o} = \frac{1}{3} \quad \text{and} \quad b = 10\text{ m}$$

$$\tan\gamma = \frac{z_p}{z_g} = \frac{24}{32} \quad \text{or} \quad \gamma = 36.87°$$

$$z'_p = \frac{z_p}{\cos\gamma} = \frac{24}{\cos(36.87)} = 30$$

From Table 17.3, the value of the Lewis form factor is 0.358.

From Eq. (19.11),

$$S_b = m\,b\,\sigma_b\,Y\left[1-\frac{b}{A_o}\right]$$

$$= m\,(10m)\left(\frac{750}{3}\right)(0.358)\left[1-\frac{1}{3}\right]$$

$$= (596.67\,\text{m}^2)\text{ N}$$

$$S_b = P_{\text{eff}}\,(fs) \quad \text{or} \quad 596.67\text{m}^2 = \left(\frac{15\,428.31}{m}\right)(2)$$

$\therefore$ $m = 3.73$ mm

The first preference value of the module is 4 mm. However, to account for higher dynamic load, the module is increased to 5 mm.

Step II *Main gear dimensions*

$m = 5$ mm

$D_p = m\,z_p = 5(24) = 120$ mm

$D_g = m\,z_g = 5(32) = 160$ mm

$$A_o = \sqrt{\left(\frac{D_p}{2}\right)^2 + \left(\frac{D_g}{2}\right)^2} = \sqrt{\left(\frac{120}{2}\right)^2 + \left(\frac{160}{2}\right)^2}$$

$$= 100\text{ mm}$$

$10\,m = 10\,(5) = 50$ mm and

$$\frac{A_o}{3} = \frac{100}{3} = 33.33\text{ mm} \quad \therefore\ b = 33\text{ mm}$$

Step III *Correct factor of safety*

Tangential force due to rated torque

$$P_t = \frac{2M_t}{D_p} = \frac{2(82\,893.2)}{120} = 1381.55\text{ N}$$

Dynamic load by Buckingham's equation

From Table 19.1, the error for Class-3 gear teeth with a 5 mm module is 0.0125 mm.

$$v = \frac{\pi D_p n_p}{60\times10^3} = \frac{\pi\,(120)\,(1440)}{60\times10^3} = 9.048\text{ m/s}$$

Also, $C = 11400$ N/mm^2 $b = 33$ mm

$P_t = 1381.55$ N

From Eq. (19.21),

$$P_d = \frac{21v(Ceb+P_t)}{21v + \sqrt{(Ceb+P_t)}}$$

$$= \frac{21(9.048)[11400(0.0125)(33)+1381.55)]}{21(9.048)+\sqrt{[11400(0.0125)(33)+1381.55)]}}$$

$$= 4313.31\text{ N}$$

Effective load

From Eq. (19.22),

$$P_{\text{eff}} = C_s P_t + P_d = 1.5(1381.55) + 4313.31$$
$$= 6385.64 \text{ N}$$

Beam strength,

$$S_b = mb\sigma_b Y\left[1 - \frac{b}{A_o}\right]$$

$$= 5(33)\left(\frac{750}{3}\right)(0.358)\left[1 - \frac{33}{100}\right]$$

$$= 9894.23 \text{ N}$$

Factor of safety

For bending consideration,

$$(fs) = \frac{S_b}{P_{\text{eff}}} = \frac{9894.23}{6385.64} = 1.55$$

Step IV *Surface hardness for gears*

$$S_w = P_{\text{eff}} \times (fs) = 6385.64(2) = 12\,771.28 \text{ N}$$

$$Q = \frac{2z_g}{z_g + z_p \tan\gamma} = \frac{2(32)}{32 + 24\tan(36.87)} = 1.28$$

$$S_w = \frac{0.75bQD_pK}{\cos\gamma}$$

$$\therefore\ 12\,771.28 = \frac{0.75(33)(1.28)(120)(0.16)(\text{BHN})^2}{\cos(36.87)(100)^2}$$

$$\therefore \quad \text{BHN} = 409.84 \text{ or } 410$$

Example 19.6 *A pair of straight bevel gears is mounted on shafts, which are intersecting at right angles. The number of teeth on the pinion and gear are 21 and 28 respectively. The pressure angle is 20°. The pinion shaft is connected to an electric motor developing 5 kW rated power at 1440 rpm. The service factor can be taken as 1.5. The pinion and the gear are made of steel (S_{ut} = 750 N/mm²) and heat-treated to a surface hardness of 380 BHN. The gears are machined by a manufacturing process, which limits the error between the meshing teeth to 10 μm. The module and face width are 4 mm and 20 mm respectively.*

Determine the factor of safety against bending as well as against pitting failure.

Solution

Given kW = 5 $n_p = 1440$ rpm $z_p = 21$ $z_g = 28$

$S_{ut} = 750$ N/mm² BHN = 380 $m = 4$ mm

$b = 20$ mm $C_s = 1.5$

error between meshing teeth = 10 μm $\alpha = 20°$

Step I *Beam strength*

$$D_p = m\,z_p = 4(21) = 84 \text{ mm}$$
$$D_g = m\,z_g = 4(28) = 112 \text{ mm}$$

$$A_o = \sqrt{\left(\frac{D_p}{2}\right)^2 + \left(\frac{D_g}{2}\right)^2} = \sqrt{\left(\frac{84}{2}\right)^2 + \left(\frac{112}{2}\right)^2}$$
$$= 70 \text{ mm}$$

$$\tan\gamma = \frac{z_p}{z_g} = \frac{21}{28} \quad \text{or} \quad \gamma = 36.87°$$

$$z'_p = \frac{z_p}{\cos\gamma} = \frac{21}{\cos(36.87)} = 26.25$$

From Table 17.3,

$$Y = 0.344 + \frac{(0.348 - 0.344)(26.25 - 26)}{(27 - 26)}$$
$$= 0.345$$

$$\sigma_b = \frac{S_{ut}}{3} = \frac{750}{3} = 250 \text{ N/mm}^2$$

$$S_b = mb\sigma_b Y\left[1 - \frac{b}{A_o}\right]$$
$$= 4(20)(250)(0.345)\left[1 - \frac{20}{70}\right]$$
$$= 4928.57 \text{ N}$$

Step II *Wear strength*

$$Q = \frac{2z_g}{z_g + z_p\tan\gamma} = \frac{2(28)}{28 + 21\tan(36.87)} = 1.28$$

$$K = 0.16\left(\frac{\text{BHN}}{100}\right)^2 = 0.16\left(\frac{380}{100}\right)^2$$
$$= 2.3104 \text{ N/mm}^2$$

$$S_w = \frac{0.75bQD_pK}{\cos\gamma} = \frac{0.75(20)(1.28)(84)(2.3104)}{\cos(36.87)}$$
$$= 4657.77 \text{ N}$$

Step III *Tangential force due to rated torque*

$$M_t = \frac{60\times10^6\,(\text{kW})}{2\pi n_p} = \frac{60\times10^6\,(5)}{2\pi(1440)}$$

$$= 33157.28\text{ N-mm}$$

$$P_t = \frac{2M_t}{D_p} = \frac{2(33157.28)}{84} = 789.46\text{ N}$$

Step IV *Dynamic load by Buckingham's equation*

$$v = \frac{\pi D_p n_p}{60\times10^3} = \frac{\pi(84)(1440)}{60\times10^3} = 6.3335\text{ m/s}$$

Also, $C = 11400$ N/mm², $b = 20$ mm,
$P_t = 789.46$ N, $e = 0.01$ mm

From Eq. (19.21),

$$P_d = \frac{21v\,(Ceb + P_t)}{21v + \sqrt{(Ceb + P_t)}}$$

$$= \frac{21\,(6.3335)[11400\,(0.01)(20) + 789.46]}{21(6.3335) + \sqrt{[11400(0.01)(20) + 789.46]}}$$

$$= 2166.84\text{ N}$$

Step V *Effective load*

From Eq. (19.22),

$$P_{\text{eff}} = C_s P_t + P_d = 1.5\,(789.46) + 2166.84$$
$$= 3351.03\text{ N}$$

Step VI *Factor of safety*

For bending consideration,

$$(fs) = \frac{S_b}{P_{\text{eff}}} = \frac{4928.57}{3351.03} = 1.47$$

For wear consideration,

$$(fs) = \frac{S_w}{P_{\text{eff}}} = \frac{4657.77}{3351.03} = 1.39$$

19.7 SPIRAL BEVEL GEARS

A pair of spiral bevel gears is illustrated in Fig. 19.18. In this type of gear, the teeth lie in the form of a spiral curve on the pitch cones. The spiral angle (ψ) is measured at the mean radius of the gear. In the figure, P is a point of the spiral tooth on the circle having mean radius. The line XY is drawn tangent to the tooth profile at the point P. The inclination of the line XY is the spiral angle (ψ). The salient features of the design of spiral bevel gears are as follows:

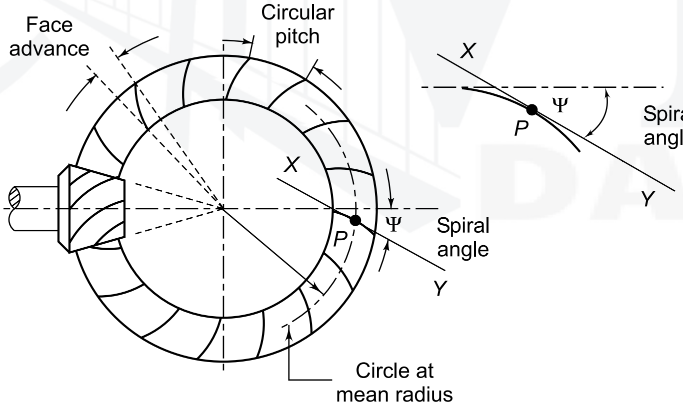


Fig. 19.18 *Spiral Gear Terminology*

(i) The spiral angle is usually 35°. The pressure angle varies from 14.5° to 20°.

(ii) In order to obtain smooth spiral tooth action, the face-contact ratio should be more than 1.25. The face-contact ratio is a ratio of face advance to circular pitch. Both these terms are illustrated in the figure.

(iii) The pinion should have minimum 12 teeth in general industrial applications. However, spiral bevel gears can be designed with the pinion having as few as 6 teeth, provided the sum of the number of teeth on the pinion and the gear is more than 40.

(iv) The choice of hand for spiral is important in the design. Under certain conditions, the induced axial force, with one direction of rotation, draws the gears tightly together, whereas with opposite direction of rotation, the axial force pushes the gears apart. The hand of the spiral should be selected in such a way so as to cause the gears to separate from each other.

(v) The teeth of the spiral bevel gear are always ground.

The comparison between straight and spiral bevel gears is similar to the comparison between spur and helical gears. Spiral bevel gears offer the following advantages over straight bevel gears:

(i) There is a gradual contact between mating teeth. This results in smooth and quiet operation.

(ii) The spiral bevel gears have more load carrying capacity because more teeth are in contact simultaneously and there is a greater arc of contact.

(iii) Spiral bevel gears permit higher operational speeds.

The spiral bevel gears are used where high speeds are encountered.

The analysis of forces in spiral gears is difficult compared with straight tooth bevel gears because a number of parameters are involved such as hand of spiral for pinion as well as for gear and direction of rotation for pinion.

Short-Answer Questions

19.1 Where do you use bevel gear?

19.2 What are the advantages of straight bevel gears over spiral bevel gears?

19.3 What are the disadvantages of straight bevel gears over spiral bevel gears?

19.4 Where do you use spiral bevel gears?

19.5 What is zerol bevel gear?

19.6 What is the magnitude of the spiral angle in zerol bevel gear?

19.7 What are skew gears? Where do you use them?

19.8 What is crown gear?

19.9 What is miter gear?

19.10 What is hypoid gear? Why is it used in automobiles?

19.11 What is virtual or formative bevel gear?

19.12 What is the relationship between actual and virtual number of teeth and the pitch angle in bevel gears?

Problems for Practice

19.1 A pair of bevel gears consists of a 30 teeth pinion meshing with a 48 teeth gear. The gears are mounted on shafts, which are intersecting at right angles. The module at the large end of the tooth is 4 mm. Calculate

(i) the pitch circle diameters of the pinion and the gear;

(ii) the pitch angles for the pinion and gear; and

(iii) the cone distance.

[*(i) 120 and 192 mm (ii) 32° and 58° (iii) 113.21 mm*]

19.2 A pair of straight bevel gears has a velocity ratio of 2:1. The pitch circle diameter of the pinion is 80 mm at the large end of the tooth. 5 kW power is supplied to the pinion, which rotates at 800 rpm. The face width is 40 mm and the pressure angle is 20°. Calculate the tangential, radial and axial components of the resultant tooth force acting on the pinion.

[*1921.79 N, 625.63 N and 312.81 N*]

19.3 A pair of straight bevel gears consists of a 30 teeth pinion meshing with a 45 teeth gear. The module and the face width are 6 mm and 50 mm respectively. The pinion as well as the gear is made of steel (S_{ut} = 600 N/mm^2). Calculate the beam strength of the tooth.

[*15 636.82 N*]

19.4 A pair of straight bevel gears consists of a 24-teeth pinion meshing with a 48 teeth gear. The module at the outside diameter is 6 mm, while the face width is 50 mm. The gears are made of grey cast iron FG 220 (S_{ut} = 220 N/mm^2). The pressure angle is 20°. The teeth are generated and assume that velocity factor accounts for the dynamic load. The pinion rotates at 300 rpm and the service factor is 1.5. Calculate

(i) the beam strength of the tooth;

(ii) the static load that the gears can transmit with a factor of safety of 2 for bending consideration; and

(iii) the rated power that the gears can transmit.

[*(i) 5267.74 N (ii) 1384.19 N (iii) 3.13*]

19.5 A pair of straight bevel gears is made of grey cast iron FG 200 (E = 114000 N/mm^2). The surface endurance strength is 90 N/mm^2. The number of teeth on the pinion and gear are 30 and 40 respectively. The module and the face width are 6 mm and 50 mm respectively. The pressure angle is 20°. Determine the wear strength of the tooth.

[*352.08 N*]

19.6 A pair of straight bevel gears is mounted on shafts, which are intersecting at right angles. The gears are made of steel and the surface hardness is 300 BHN. The number of teeth on the pinion and gear are 40 and 65 respectively. The module at the outside diameter is 3 mm, while the face width of the tooth is 35 mm. Calculate the wear strength of the tooth.

[*7723.02 N*]

19.7 A pair of straight bevel gears is mounted on shafts, which are intersecting at right angles. The number of teeth on the pinion and gear are 30 and 45 respectively. The pressure angle is 20°. The pinion shaft is connected to an electric motor developing 16.5 kW rated power at 500 rpm. The service factor can be taken as 1.5. The pinion and the gear are made of steel (S_{ut} = 570 N/mm^2) and heat-treated to a surface hardness of 350 BHN. The gears are manufactured in such a way that the error between two meshing teeth is limited to 20 μm. The module and face width are 6 mm and 50 mm respectively.
Determine the factor of safety against bending as well as pitting.

[*1.25 and 1.85*]

Worm Gears

Chapter 20

20.1 WORM GEARS

Worm gear drives are used to transmit power between two non-intersecting shafts, which are, in general, at right angles to each other. The worm gear drive consists of a worm and a worm wheel. The worm is a threaded screw, while the worm wheel is a toothed gear. The teeth on the worm wheel envelope the threads on the worm and give line contact between mating parts. The advantages of worm gear drives are as follows:

(i) The most important characteristic of worm gear drives is their high speed reduction. A speed reduction as high as 100 : 1 can be obtained with a single pair of worm gears.

(ii) The worm gear drives are compact with small overall dimensions, compared with equivalent spur or helical gear drives having same speed reduction.

(iii) The operation is smooth and silent.

(iv) Provision can be made for self locking operation, where the motion is transmitted only from the worm to the worm wheel. This is advantageous in applications like cranes and lifting devices.

The drawbacks of the worm gear drives are as follows:

(i) The efficiency is low compared with other types of gear drives.

(ii) The worm wheel, in general, is made of phosphor bronze, which increases the cost.

(iii) Considerable amount of heat is generated in worm gear drives, which is required to be dissipated by a lubricating oil to the housing walls and finally to the surroundings.

(iv) The power transmitting capacity is low. Worm gear drives are used for up to 100 kW of power transmission.

It is necessary to consider the relationship between the number of starts on the worm and the efficiency to decide the suitability of worm gear drive for a particular application. Two guidelines are as follows:

(i) Single-threaded worm gives large speed reduction, however, the efficiency is low. The large velocity ratio is obtained at the cost of efficiency.

(ii) Multi-threaded worm gives high efficiency, however, the speed reduction is low. The high efficiency is obtained at the cost of speed reduction.

The above guidelines lead to four major areas of application of worm gear drives. They are as follows:

(i) Manually Operated Intermittent Mechanisms In these applications, large mechanical advantage is required and efficiency is of minor importance. The examples of these mechanisms include steering mechanism and opening and closing of gate valves by means of hand wheels.

(ii) Motorized Operated Intermittent Mechanisms In these applications, a small capacity low-cost motor drives the mechanism and the efficiency is of minor importance. The examples of these mechanisms include drive for small hoists and opening and closing of large gate valves by means of electric motor.

(iii) Motorized Continuous Operations In these applications, worm gear drives are used in place of other gear drives due to space limitations and silent operation. The efficiency is more important in these applications. Multi-threaded worms are used in these applications to obtain higher efficiency. The examples of this type include drives for machine tools and elevators.

(iv) Motorized Speed Increasing Applications In these applications, worm gear drives are preferred due to high velocity ratio and silent operation. The efficiency is more important in these applications. Speed increasing applications include drives for automotive supercharger and centrifugal cream charger. In order to increase efficiency, the automotive supercharger is provided with six-threaded worm having lead angle of about 45°.

20.2 TERMINOLOGY OF WORM GEARS

A pair of worm gears is specified and designated by four quantities in the following manner:

$$z_1/z_2/q/m$$

where,

z_1 = number of starts on the worm
z_2 = number of teeth on the worm wheel
q = diametral quotient
m = module (mm)

The diametral quotient is given by,

$$q = \frac{d_1}{m} \tag{20.1}$$

where d_1 is the pitch circle diameter of the worm. A schematic diagram of the worm and worm wheel is shown in Fig. 20.1(a). d_1 and d_2 are pitch circle diameters of the worm and the worm wheel respectively. The worm is similar to a screw with single-start or multi-start threads. The threads of the worm have an involute helicoid profile. The following terms are used in terminology of worm gear drives:

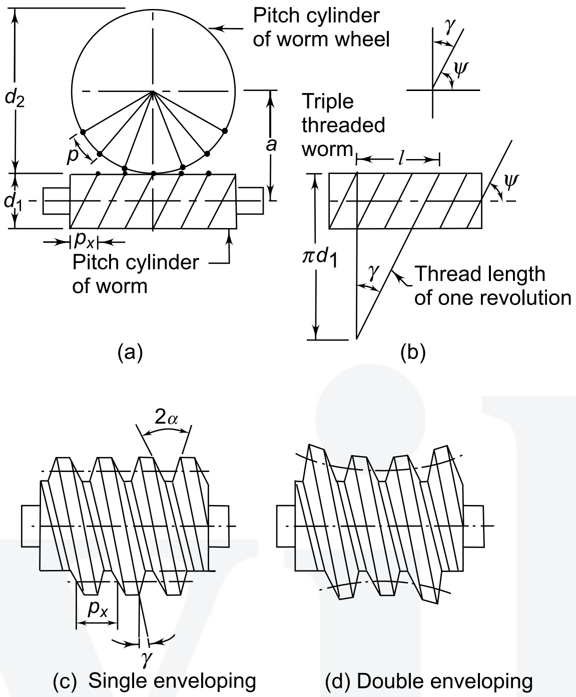


Fig. 20.1 *Worm Gear Terminology*

(i) Axial Pitch The axial pitch (p_x) of the worm is defined as the distance measured from a point on one thread to the corresponding point on the adjacent thread, measured along the axis of the worm.

(ii) Lead The lead (l) of the worm is defined as the distance that a point on the helical profile will move when the worm is rotated through one revolution. It is the thread advance in one turn. For single-start threads, the lead is equal to the axial pitch. For double-start threads, the lead is twice the axial pitch, and so on. Therefore,

$$l = p_x z_1 \tag{20.2}$$

The recommended number of starts on the worm is as follows:

Velocity ratio	*Number of starts*
20 and above	Single-start
12–36	Double-start
8–12	Triple-start
6–12	Quadruple-start
4–10	Sextuple-start

The pitch circle diameter of the worm wheel is given by

$$d_2 = mz_2 \tag{20.3}$$

As seen in the figure, the axial pitch of the worm should be equal to the circular pitch of the worm wheel. Therefore,

$$p_x = \frac{\pi d_2}{z_2} = \frac{\pi(mz_2)}{z_2}$$

$$\therefore \quad p_x = \pi m \tag{20.4}$$

From Eqs (20.2) and (20.4),

$$l = \pi m z_1 \tag{20.5}$$

When one thread of the worm is developed, it becomes the hypotenuse of a triangle as shown in Fig. 20.1(b). The base of this triangle is equal to the lead of the worm, while the altitude is equal to the circumference of the worm. There are two angles related to this triangle, namely, lead angle and helix angle.

(iii) Lead Angle The lead angle (γ) is defined as the angle between a tangent to the thread at the pitch diameter and a plane normal to the worm axis. From the triangle in Fig. 20.1(b),

$$\tan\gamma = \frac{l}{\pi d_1} \tag{20.6}$$

From Eqs (20.1) and (20.5),

$$\tan\gamma = \frac{\pi m z_1}{\pi(qm)}$$

$$\therefore \quad \tan\gamma = \frac{z_1}{q} \tag{20.7}$$

(iv) Helix Angle The helix angle (ψ) is defined as the angle between a tangent to the thread at the pitch diameter and the axis of the worm. The worm helix angle is the complement of the worm lead angle.

$$\gamma + \psi = \frac{\pi}{2}$$

The helix angle should be limited to 6° per thread. For example, if $\psi = 30°$ then the worm should have at least five threads.

(v) Pressure Angle The tooth pressure angle (α) is measured in a plane containing the axis of the worm and it is equal to one-half of the thread angle. It is illustrated in Fig. 20.1(c). The pressure angle should not be less than 20° for single and double start worms and 25° for triple and multi-start worms.

From Fig. 20.1(a), the centre distance is given by

$$a = \frac{1}{2}(d_1 + d_2) \tag{20.8}$$

where a is the centre to centre distance. Substituting Eqs (20.1) and (20.3) in the above expression, we get,

$$a = \frac{1}{2} m (q + z_2) \tag{20.9}$$

When the worm wheel is rotated through one revolution, the worm will complete z_2 revolutions for single-start threads. For double-start threads, the number of revolutions of the worm will be $(z_2/2)$. The speed ratio (i) is, therefore, given by,

$$i = \frac{z_2}{z_1} \tag{20.10}$$

There are two classes of worm gear drives in common use, namely, single enveloping and double enveloping, as shown in Fig. 20.1(c) and (d).

(i) Single-enveloping Worm Gear Drive A single-enveloping worm gear set is one in which the gear wraps around or partially encloses the worm. This results in line contact between the threads of the worm and the teeth of the worm wheel. In this case, the worm is also called '*cylindrical*' or '*straight*' *cylindrical* worm. The single-enveloping worm gear drive is more widely used.

(i) Double-enveloping Worm Gear Drive A double-enveloping gear set is one in which the gear wraps around the worm and the worm also wraps around the gear. This results in area contact between the threads of the worm and the teeth of the worm wheel. In this case, the worm is also called '*hourglass*' worm. This drive is also called '*cone*' gearing.

Double-enveloping worm gear drive has the following advantages:

(i) The contact pressure between the threads of the worm and the teeth of the worm wheel is low. This reduces wear.

(ii) The drive occupies less space for a given capacity. Double-enveloping worm gear drive needs only about two-thirds of the space and has about one-third of the weight compared with a single-enveloping worm gear drive.

The main drawback of double-enveloping worm gear drive is the requirement of precise alignment. It is much more critical than in case of single-enveloping worm gear drive. A small deviation from the correct centre distance results in the loss of theoretical area of contact.

The preferred values of $z_1/z_2/q/m$ for worm gears[1] are given in Table 20.1. While designing the worm gears, it is better to use the following values of q,

Preferred values of q 8, 10, 12.5, 16, 20 and 25.

The number of starts (z_1) on the worm is usually taken as 1, 2 or 4.

Table 20.1 *Preferred values of* $(z_1/z_2/q/m)$ *for worm gears*

Transmission ratio (Approx.)	*Centre distance (mm)*				
	100	*125*	*160*	*200*	*250*
20	2/40/10/4	2/40/10/5	–	2/40/10/8	2/40/10/10
25	–	2/52/10/4	2/54/10/5	–	2/52/10/8
30	1/30/10/5	1/31/10/6	1/30/10/8	1/30/10/10	–
40	1/40/10/4	1/40/10/5	–	1/40/10/8	1/40/10/10
50	–	1/52/10/4	1/54/10/5	–	1/52/10/8

20.3 PROPORTIONS OF WORM GEARS

The basic dimensions of the worm and the worm wheel are shown in Fig. 20.2. For an involute helicoidal tooth form,

$$h_{a1} = m \tag{20.11}$$

$$h_{f1} = (2.2 \cos \gamma - 1)m \tag{20.12}$$

$$c = 0.2\, m \cos \gamma \tag{20.13}$$

where,

h_{a1} = addendum (mm)

h_{f1} = dedendum (mm)

c = clearance (mm)

The outside and root diameters of the worm are expressed as follows:

$$d_{a1} = d_1 + 2h_{a1} = qm + 2m$$

$$\therefore \quad d_{a1} = m(q + 2) \tag{20.14}$$

$$d_{f1} = d_1 - 2h_{f1} = qm - 2m\,(2.2 \cos \gamma - 1)$$

$$\therefore \quad d_{f1} = m(q + 2 - 4.4 \cos \gamma) \tag{20.15}$$

where

d_{a1} = outside diameter of the worm (mm)

d_{f1} = root diameter of the worm (mm)

Similarly, the dimensions of the worm wheel can be expressed. The addendum and dedendum of the worm wheel are expressed at the throat. They are given by,

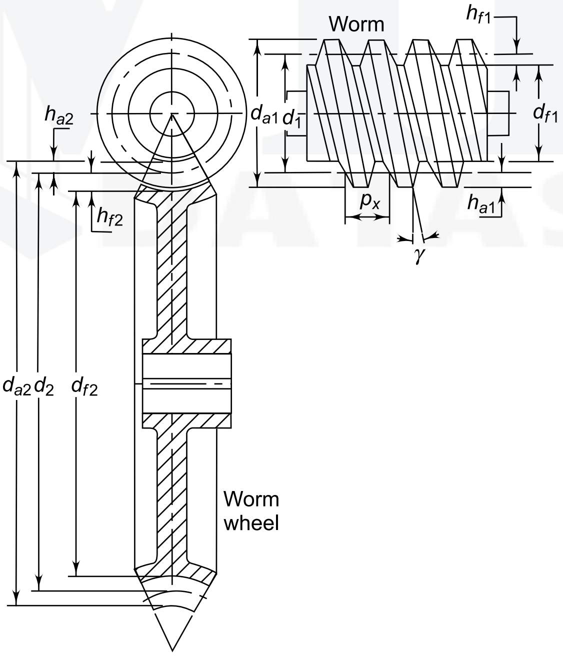


Fig. 20.2 *Dimension of Worm Gears*

[1] IS 3734–1966: Dimensions of worm gearing.

$$h_{a2} = m\,(2\cos\gamma - 1) \qquad (20.16)$$
$$h_{f2} = m\,(1 + 0.2\cos\gamma) \qquad (20.17)$$

where,

h_{a2} = addendum at the throat (mm)

h_{f2} = dedendum in the median plane (mm)

The dimensions of the worm wheel are as follows:

$$d_{a2} = d_2 + 2h_{a2} = mz_2 + 2m(2\cos\gamma - 1)$$
$$\therefore \quad d_{a2} = m\,(z_2 + 4\cos\gamma - 2) \qquad (20.18)$$
$$d_{f2} = d_2 + 2h_{f2} = mz_2 - 2m(1 + 0.2\cos\gamma)$$
$$\therefore \quad d_{f2} = m(z_2 - 2 - 0.4\cos\gamma) \qquad (20.19)$$

where,

d_{a2} = throat diameter of the worm wheel (mm)

d_{f2} = root diameter of the worm wheel (mm)

The effective face width F of the worm wheel is shown in Fig. 20.3. It is obtained by drawing a tangent AB to the pitch circle diameter of the worm. A and B are the points of intersection of this tangent and the outside diameter of the worm.

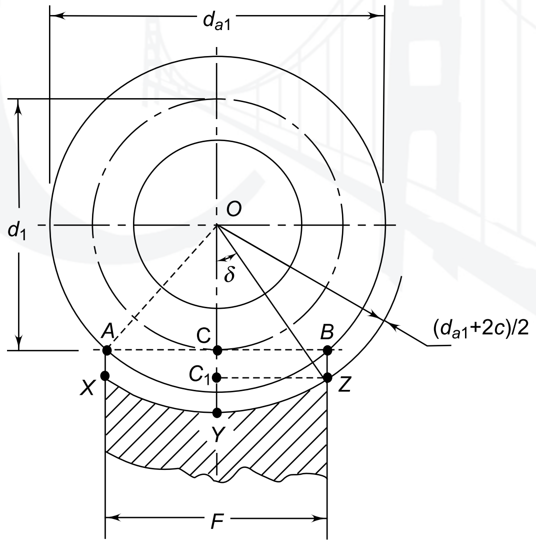


Fig. 20.3 *Face Width of Worm Wheel*

From triangle AOC,

$$(\overline{AC})^2 = (\overline{AO})^2 - (\overline{OC})^2$$

or
$$\left(\frac{F}{2}\right)^2 = \left(\frac{d_{a1}}{2}\right)^2 - \left(\frac{d_1}{2}\right)^2$$
$$= \left[\frac{m(q+2)}{2}\right]^2 - \left[\frac{qm}{2}\right]^2$$

$$\therefore \quad F = 2m + \sqrt{(q+1)} \qquad (20.20)$$

From triangle OZC_1,

$$\sin\delta = \frac{C_1Z}{OZ} = \frac{F/2}{(d_{a1}+2c)/2}$$

or
$$\delta = \sin^{-1}\left(\frac{F}{d_{a1}+2c}\right)$$

The length of the root of the worm wheel teeth is the arc XYZ, which is denoted by (l_r).

$$l_r = \text{arc } XYZ = \left(\frac{2\delta}{2\pi}\right)[\pi\,(d_{a1}+2c)] = (d_{a1}+2c)\delta$$

$$l_r = (d_{a1}+2c)\sin^{-1}\left[\frac{F}{(d_{a1}+2c)}\right] \qquad (20.21)$$

Example 20.1 *A pair of worm gears is designated as, 1/30/10/8*

Calculate

(i) the centre distance;
(ii) the speed reduction;
(iii) the dimensions of the worm; and
(iv) the dimensions of the worm wheel

Solution

Given $z_1 = 1$ $\quad z_2 = 30$ teeth $\quad q = 10$ $\quad m = 8$ mm

Step I *Centre distance*

From Eq. (20.9),

$$a = \frac{1}{2}m\,(q+z_2)\frac{1}{2}(8)(10+30) = 160 \text{ mm} \qquad \text{(i)}$$

Step II *Speed reduction*

$$i = \frac{z_2}{z_1} = 30 \qquad \text{(ii)}$$

Step III *Dimensions of worm*

$$d_1 = qm = 10(8) = 80 \text{ mm} \qquad \text{(a)}$$
$$d_{a1} = m(q+2) = 8\,(10+2) = 96 \text{ mm} \qquad \text{(b)}$$
$$\tan\gamma = \frac{z_1}{q} = \frac{1}{10} \quad \text{or} \quad \gamma = 5.71°$$
$$d_{f1} = m(q + 2 - 44\cos\gamma)$$
$$= 8[10 + 2 - 4.4\cos(5.71)]$$
$$= 60.9747 \text{ mm} \qquad \text{(c)}$$
$$p_x = \pi m = \pi(8) = 25.1327 \text{ mm} \qquad \text{(d)}$$

Step IV *Dimensions of worm wheel*

$$d_2 = mz_2 = 8\,(30) = 240 \text{ mm} \qquad \text{(a)}$$

$$d_{a2} = m(\,z_2 + 4\cos\gamma - 2)$$
$$= 8\,[30 + 4\cos(5.71) - 2]$$
$$= 255.8412 \text{ mm} \qquad \text{(b)}$$

$$d_{f2} = m\,(z_2 - 2 - 0.4\cos\gamma)$$
$$= 8\,[30 - 2 - 0.4\cos(5.71)]$$
$$= 220.8159 \qquad \text{(c)}$$

20.4 FORCE ANALYSIS

The analysis of three components of the resultant tooth force between the meshing teeth of worm and worm wheel is based on the following assumptions:

(i) The worm is the driving element, while the worm wheel is the driven element.

(ii) The worm has right-handed threads.

(iii) The worm rotates in anti-clockwise directions as shown in Fig. 20.4.

The three components of the gear tooth force between the worm and the worm wheel are shown in Fig. 20.4. Suffix 1 is used for the worm, while suffix 2 for the worm wheel. The components of the resultant force acting on the worm are as follows:

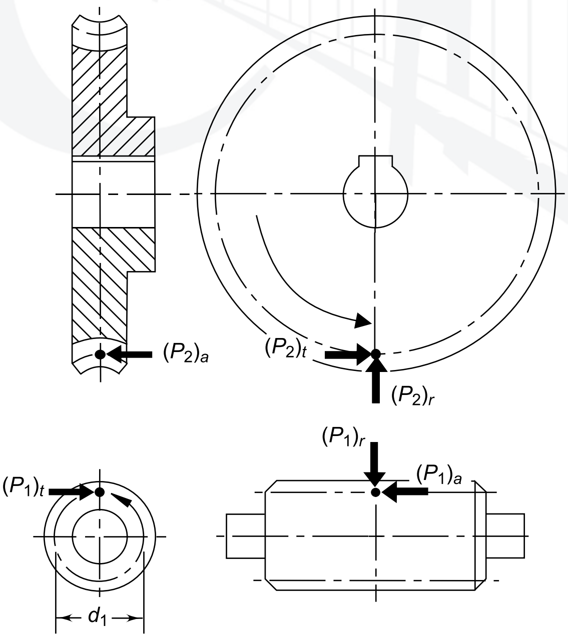


Fig. 20.4 *Components of Tooth Force*

$(P_1)_t$ = tangential component on the worm (N)

$(P_1)_a$ = axial component on the worm (N)

$(P_1)_r$ = radial component on the worm (N)

The components $(P_2)_t$, $(P_2)_a$ and $(P_2)_r$ acting on the worm wheel are defined in a similar way. The force acting on the worm wheel is the equal and opposite reaction of the force acting on the worm.

Therefore,

$$(P_2)_t = (P_1)_a \qquad (20.22)$$

$$(P_2)_a = (P_1)_t \qquad (20.23)$$

$$(P_2)_r = (P_1)_r \qquad (20.24)$$

In the present analysis, the expressions are derived for the components of force acting on the worm. The components of force acting on the worm wheel can be determined by the above relationship.

It is difficult to understand the directions of components with the help of two views shown in Fig. 20.4. It is better to construct an isometric sketch of the worm and worm wheel for understanding the directions. The directions can be decided with the help of such isometric sketches illustrated in Fig. 20.5.

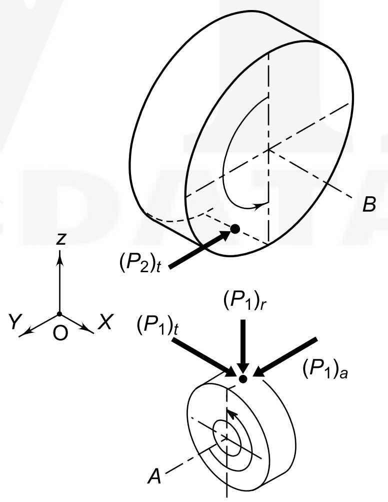


Fig. 20.5 *Direction of Force Components*

(i) Tangential Component $(P_1)_t$ The worm is the driving element. It is rotating in an anti-clockwise direction, when viewed from A. For the driving element, the direction of tangential component is opposite to the direction of rotation. Therefore, $(P_1)_t$ will act in the positive X direction at the point of contact.

(ii) Axial Component $(P_1)_a$ The worm has right-hand threads and when the right-hand thumb rule is applied, by keeping the fingers in the direction of rotation, the thumb will be projecting along the positive Y-axis. Therefore, if we treat the worm as 'screw' and the worm wheel as 'nut', the screw will have a tendency to move in the direction of the thumb or along the positive Y-axis. The nut or the worm wheel will have a tendency to move in the opposite direction, i.e., along the negative Y-axis. Therefore, the worm wheel will rotate in the anti-clockwise direction when observed from the bearing B. The worm wheel is the driven member and the direction of $(P_2)_t$ will be the same as the direction of rotation or along the negative Y-axis. Since, $(P_1)_a$ and $(P_2)_t$ are equal and opposite, the axial component $(P_1)_a$ will act in the positive Y direction at the point of contact.

(iii) Radial Component $(P_1)_r$ The radial component always acts towards the centre of gear. Therefore, $(P_1)_r$ will act towards the centre of the worm or along the negative Z-direction at the point of contact.

The resultant force acting on the worm consists of two components—components of normal reaction between the meshing teeth and components of frictional force. The two components are superimposed to get the resultant components.

The components of the normal reaction P acting on the worm are shown in Fig. 20.6. Here, α is the normal pressure angle, while γ is the lead angle. Note that the angle α is in the plane $ABCD$ shaded by dots, while the angle γ is in the top plane $AEBF$. Resolving the normal reaction P in the plane $ABCD$ shown in Fig. 20.6(b),

$$P_N = P \cos \alpha \qquad \text{(a)}$$

$$P_r = P \sin \alpha \qquad \text{(b)}$$

Resolving the component P_N in the plane $AEBF$ shown in Fig. 20.6(c),

$$P_a = P_N \cos \gamma \qquad \text{(c)}$$

$$P_t = P_N \sin \gamma \qquad \text{(d)}$$

From relationships (a), (b), (c) and (d),

$$P_t = P \cos \alpha \sin \gamma$$

$$P_a = P \cos \alpha \cos \gamma$$

$$P_r = P \sin \alpha \qquad (20.25)$$

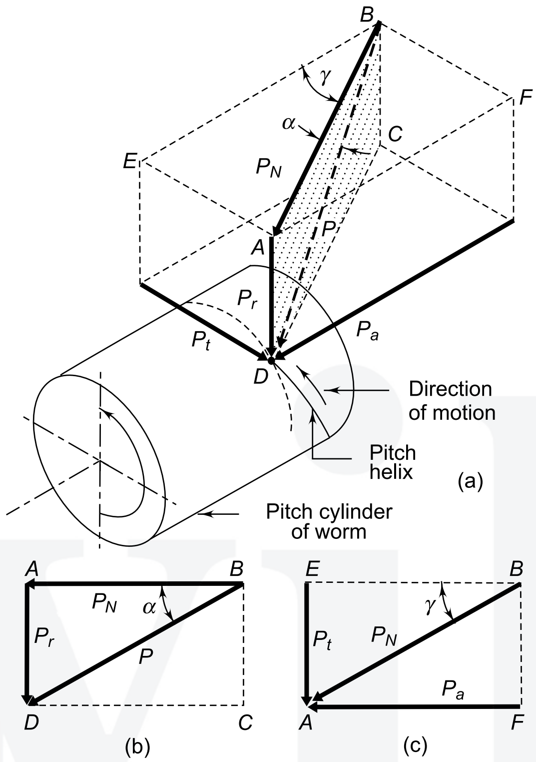


Fig. 20.6 *Components of Normal Reaction*

The frictional force is significant in worm gear drives, because there is sliding motion between the threads of the worm and the teeth of the worm wheel, as compared with the rolling motion between the teeth of the pinion and gear in other types of gears. The resultant frictional force is (μP) where μ is the coefficient of friction. The direction of the frictional force will be along the pitch helix and opposite to the direction of rotation, as shown in Fig. 20.7. There are two components of the frictional force:

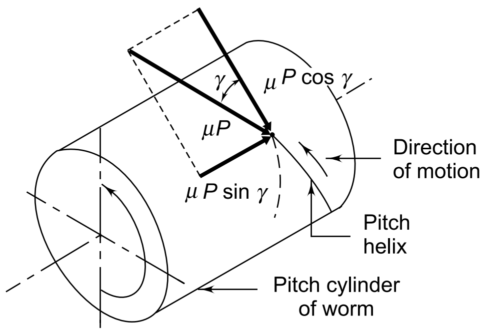


Fig. 20.7 *Components of Frictional Force*

(i) Component (μP cos γ) in the tangential direction. The direction of this component is same as that of P_t.

(ii) Component (μP sin γ) in the axial direction. The direction of this component is opposite to that of P_a.

Superimposing the components of normal reaction and frictional force, we have

$$(P_1)_t = P \cos \alpha \sin \gamma + \mu P \cos \gamma$$

$$\therefore \quad (P_1)_t = P (\cos \alpha \sin \gamma + \mu \cos \gamma) \quad (20.26)$$

Similarly, $(P_1)_a = P \cos \alpha \cos \gamma - \mu P \sin \gamma$

$$\therefore \quad (P_1)_a = P (\cos \alpha \cos \gamma - \mu \sin \gamma) \quad (20.27)$$

and $\quad (P_1)_r = P \sin \alpha \quad (20.28)$

In practice, the tangential component $(P_1)_t$ on the worm is determined from the torque that is transmitted from the worm to the worm wheel. Therefore,

$$(P_1)_t = \frac{2M_t}{d_1} \quad (20.29)$$

From Eqs (20.26) and (20.27),

$$(P_1)_a = (P_1)_t \times \frac{(\cos \alpha \cos \gamma - \mu \sin \gamma)}{(\cos \alpha \sin \gamma + \mu \cos \gamma)} \quad (20.30)$$

From Eqs (20.26) and (20.28),

$$(P_1)_r = (P_1)_t \times \frac{\sin \alpha}{(\cos \alpha \sin \gamma + \mu \cos \gamma)} \quad (20.31)$$

Equations (20.29), (20.30) and (20.31) are used to determine the magnitude of components of the resultant tooth force.

Example 20.2 *A pair of worm and worm wheel is designated as*

3/60/10/6

The worm is transmitting 5 kW power at 1440 rpm to the worm wheel. The coefficient of friction is 0.1 and the normal pressure angle is 20°. Determine the components of the gear tooth force acting on the worm and the worm wheel.

Solution

Given kW = 5 $\quad n = 1440$ rpm $\quad \mu = 0.1$
$\alpha = 20°$ $\quad z_1 = 3$ $\quad z_2 = 60$ teeth $\quad q = 10$
$m = 6$ mm

Step I *Components of tooth force acting on worm*

$$d_1 = q\,m = 10(6) = 60 \text{ mm}$$

$$\tan \gamma = \frac{z_1}{q} = \frac{3}{10} = 0.3 \quad \text{or} \quad \gamma = 16.7°$$

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n_1} = \frac{60 \times 10^6 (5)}{2\pi (1440)}$$

$$= 33157.28 \text{ N-mm}$$

From Eq. (20.29),

$$(P_1)_t = \frac{2M_t}{d_1} = \frac{2(33157.28)}{60} = 1105.24 \text{ N} \quad \text{(a)}$$

From Eqs (20.30),

$$(P_1)_a = (P_1)_t \times \frac{(\cos \alpha \cos \gamma - \mu \sin \gamma)}{(\cos \alpha \sin \gamma + \mu \cos \gamma)}$$

$$= 1105.24 \times \frac{[\cos(20)\sin(16.7) - 0.1\sin(16.7)]}{[\cos(20)\sin(16.7) + 0.1\cos(16.7)]}$$

$$= 2632.55 \text{ N} \quad \text{(b)}$$

From Eq. (20.31),

$$(P_1)_r = (P_1)_t \times \frac{\sin \alpha}{(\cos \alpha \sin \gamma + \mu \cos \gamma)}$$

$$= 1105.24 \times \frac{\sin(20)}{[\cos(20)\sin(16.7) + 0.1\cos(16.7)]}$$

$$= 1033.35 \text{ N} \quad \text{(c)}$$

Step II *Components of tooth force acting on worm wheel*

The force components acting on the worm wheel are as follows (Eqs. 20.22 to 20.24):

$$(P_2)_t = (P_1)_a = 2632.55 \text{ N}$$

$$(P_2)_a = (P_1)_t = 1105.24 \text{ N}$$

$$(P_2)_r = (P_1)_r = 1033.35 \text{ N}$$

20.5 FRICTION IN WORM GEARS

It has been observed that the coefficient of friction in worm gear drives depends upon the rubbing speed. The rubbing speed is the relative velocity between the worm and the wheel. The velocity triangle is shown in Fig. 20.8.

In this velocity triangle,

V_1 = pitch line ve1ocity of the worm (m/s)

V_2 = pitch line velocity of the worm wheel (m/s)

V_s = rubbing velocity (m/s)

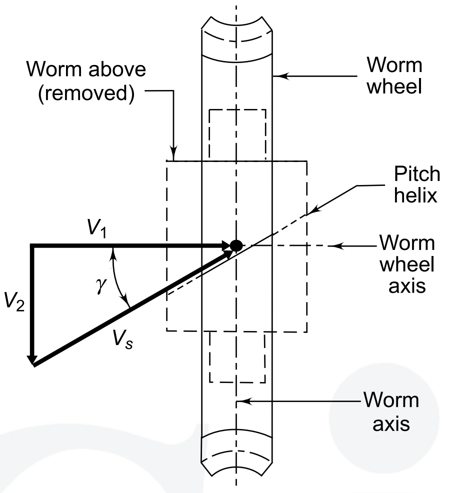


Fig. 20.8 *Valocity of Sliding*

The pitch line velocity of the worm is given by,

$$V_1 = \frac{\pi d_1 n_1}{60(1000)} \tag{20.32}$$

From the velocity triangle,

$$V_s = \frac{V_1}{\cos \gamma}$$

$$\therefore \quad V_s = \frac{\pi d_1 n_1}{60\,000 \cos \gamma} \tag{20.33}$$

The variation of the coefficient of friction with respect to rubbing velocity is shown in Fig. 20.9. The values of the coefficient of friction in this figure are based on the following two assumptions:

(i) The worm wheel is made of phosphor-bronze, while the worm is made of case-hardened steel.

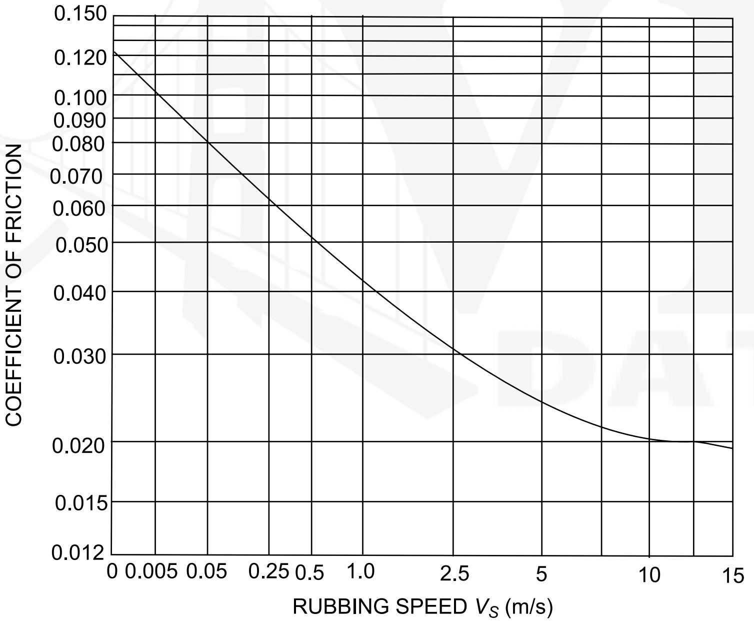


Fig. 20.9 *Coefficient of Friction of Worm Gears*

(ii) The gears are lubricated with a mineral oil having a viscosity of 16 to 130 centiStokes at 60°C.

The efficiency of the worm gear drive is given by,

$$\eta = \frac{\text{power output}}{\text{power input}} = \frac{(P_2)_t \times (d_2)/2 \times (n_2)}{(P_1)_t \times (d_1)/2 \times (n_1)} \tag{a}$$

Since

$$\frac{n_2}{n_1} = \frac{1}{i} \tag{b}$$

and

$$\frac{d_2}{d_1} = \frac{m z_2}{m q} = \frac{z_2}{q} = \frac{z_2/z_1}{q/z_1} = i \tan \gamma \tag{c}$$

From (a), (b) and (c),

$$\eta = \frac{(P_2)_t}{(P_1)_t} \times \tan\gamma = \frac{(P_1)_a}{(P_1)_t} \times \tan\gamma$$

From Eq. (20.30),

$$\eta = \tan\gamma \times \frac{(\cos\alpha\cos\gamma - \mu\sin\gamma)}{(\cos\alpha\sin\gamma + \mu\cos\gamma)}$$

$$\therefore \quad \eta = \frac{(\cos\alpha - \mu\tan\gamma)}{(\cos\alpha + \mu\cot\gamma)} \qquad (20.34)$$

The efficiency of spur or helical gears is very high and virtually constant in the range of 98% to 99%. On the other hand, the efficiency of worm gears is low and varies considerably in the range of 50% to 98%. In general, the efficiency is inversely proportional to speed ratio, provided the coefficient of friction is constant.

In general, the worm is the driver and the worm wheel is the driven member and the reverse motion is not possible. This is called '*self-locking*' drive, because the worm wheel cannot drive the worm. As for screw threads, the criterion for self-locking is the relationship between the coefficient of friction and lead angle. A worm gear drive is said to be self-locking if the coefficient of friction is greater than tangent of lead angle, i.e., the friction angle is more than the lead angle. This approximate condition is rewritten as,

$$\mu > \tan\gamma$$

There is another term, '*reversible*' or '*overrunning*' or '*back-driving*' worm gear drive. In this type of drive, the worm and worm wheel can drive each other. In general, the worm is the driver and the worm wheel is the driven member. If the driven machinery has large inertia and if the driving power supply is cut off suddenly, the worm is freely driven by the worm wheel. This prevents the damage to the drive and source of power. A worm-gear drive is said to be reversible if the coefficient of friction is less than tangent of lead angle, i.e., the friction angle is less than the lead angle. This approximate condition is rewritten as,

$$\mu < \tan\gamma$$

Example 20.3 *1 kW power at 720 rpm is supplied to the worm shaft. The number of starts for threads of the worm is four with a 50 mm pitch–circle diameter. The worm wheel has 30 teeth with 5 mm module. The normal pressure angle is 20°. Calculate the efficiency of the worm gear drive and the power lost in friction.*

Solution

Given $kW = 1$ $\quad n = 720$ rpm $\quad \alpha = 20°$
$d_1 = 50$ mm $\quad z_1 = 4 \quad z_2 = 30$ teeth $\quad m = 5$ mm

Step I *Efficiency of worm gear drive*

From Eq. (20.5),

$$l = \pi m z_1 = \pi(5)(4) = (20\pi) \text{ mm}$$

$$\tan\gamma = \frac{l}{\pi d_1} = \frac{20\pi}{\pi(50)} = 0.4 \quad \text{or} \quad \gamma = 21.8°$$

From Eq. (20.33),

$$V_s = \frac{\pi d_1 n_1}{60\,000\cos\gamma} = \frac{\pi(50)(720)}{60\,000\cos(21.8)} = 2.03 \text{ m/s}$$

From Fig. 20.9, the coefficient of friction is 0.035.

From Eq. (20.34),

$$\eta = \frac{(\cos\alpha - \mu\tan\gamma)}{(\cos\alpha + \mu\cot\gamma)} = \frac{[\cos(20) - 0.035\tan(21.8)]}{[\cos(20) + 0.035\cot(21.8)]}$$

$$= 0.9012 = 90.12\%$$

Step II *Power lost in friction*

Power lost in friction = $(1 - \eta)$ kW = $(1 - 0.9012)(1)$
= 0.0988 kW or 98.8 W

Example 20.4 *5 kW of power at 720 rpm is supplied to the worm shaft, as shown in Fig. 20.10. The worm gear drive is designated as,*

2/40/10/5

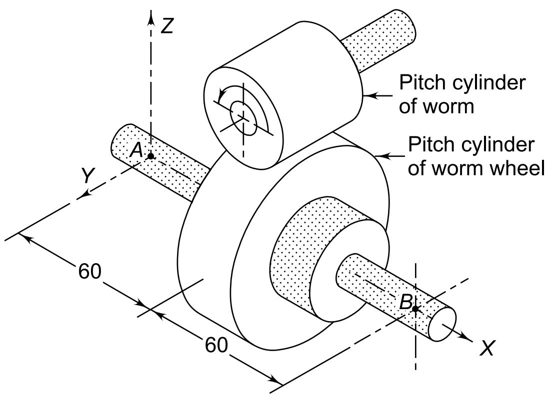


Fig. 20.10

The worm has right-hand threads and the pressure angle is 20°. The worm wheel is mounted between two bearings A and B. It can be assumed that the bearing A is located at the origin of the co-ordinate system and the bearing B takes complete thrust load. Determine the reactions at the two bearings.

Solution

Given $kW = 5$ $n = 720$ rpm $\alpha = 20°$ $z_1 = 2$ $z_2 = 40$ teeth $q = 10$ $m = 5$ mm

Step I *Components of tooth force acting on worm*

$$d_1 = q\,m = 10(5) = 50 \text{ mm}$$

$$\tan\gamma = \frac{z_1}{q} = \frac{2}{10} = 0.2 \quad \text{or} \quad \gamma = 11.31°$$

$$M_t = \frac{60\times10^6(\text{kW})}{2\pi n_1} = \frac{60\times10^6(5)}{2\pi(720)}$$

$$= 66\,314.56 \text{ N-mm}$$

$$(P_1)_t = \frac{2M_t}{d_1} = \frac{2(66\,314.56)}{50} = 2653 \text{ N}$$

$$V_s = \frac{\pi d_1 n_1}{60\,000\cos\gamma} = \frac{\pi(50)(720)}{60\,000\cos(11.31)}$$

$$= 1.92 \text{ m/s}$$

From Fig. 20.9, the coefficient of friction is 0.035.

From Eqs (20.30),

$$(P_1)_a = (P_1)_t \times \frac{(\cos\alpha\cos\gamma - \mu\sin\gamma)}{(\cos\alpha\sin\gamma + \mu\cos\gamma)}$$

$$= (2653)\frac{[\cos(20)\cos(11.31) - 0.035\sin(11.31)]}{[\cos(20)\sin(11.31) + 0.035\cos(11.31)]}$$

$$= 11\,097 \text{ N}$$

From Eq. (20.31),

$$(P_1)_r = (P_1)_t \times \frac{\sin\alpha}{(\cos\alpha\sin\gamma + \mu\cos\gamma)}$$

$$= (2653)\frac{\sin(20)}{[\cos(20)\sin(11.31) + 0.035\cos(11.31)]}$$

$$= 4150 \text{ N}$$

Step II *Components of tooth force acting on worm wheel*

The force components acting on the worm wheel are as follows (Eqs 20.22 to 20.24):

$$(P_2)_t = (P_1)_a = 11097 \text{ N}$$
$$(P_2)_a = (P_1)_t = 2653 \text{ N}$$
$$(P_2)_r = (P_1)_r = 4150 \text{ N}$$

The directions of the three components is decided with reference to Fig. 20.11.

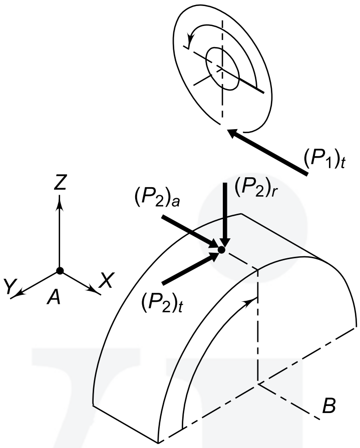


Fig. 20.11

(i) Tangential Component $(P_2)_t$ The worm has right-hand threads and when the right-hand thumb rule is applied by keeping the fingers in the direction of rotation, the thumb will be projecting along the positive Y-axis. Therefore, if we treat the worm as a '*screw*' and the worm wheel as '*nut*', the screw will have a tendency to move in the direction of the thumb or along the positive Y-axis. The nut or the worm wheel will have a tendency to move in the opposite direction, i.e., along the negative Y-axis. Therefore, the worm wheel will rotate in the clockwise direction when observed from the bearing B. The worm wheel is the driven member and the direction of $(P_2)_t$ will be the same as the direction of rotation or along the negative Y-axis.

(ii) Axial Component $(P_2)_a$ The worm is the driving element. It is rotating in an anti-clockwise direction as shown in the figure. The direction of tangential component for the driving element is opposite to the direction of rotation. Therefore, $(P_1)_t$ will act in the negative X-direction. Since $(P_1)_t$ and $(P_2)_a$ are equal and opposite, the direction of $(P_2)_a$ will be along the positive X-direction.

(iii) Radial Component (P_2)$_r$ The radial component always acts towards the centre of gear. Therefore, $(P_2)_r$ will act towards the centre of worm wheel or along negative Z-direction.

Step III *Reactions at two bearings*

The forces acting on the worm wheel shaft are shown in Fig. 20.12. For the sake of convenience, we will call the *XZ* plane as the vertical plane and the *XY* plane as the horizontal plane. The forces in the vertical and horizontal planes are shown in Fig. 20.13.

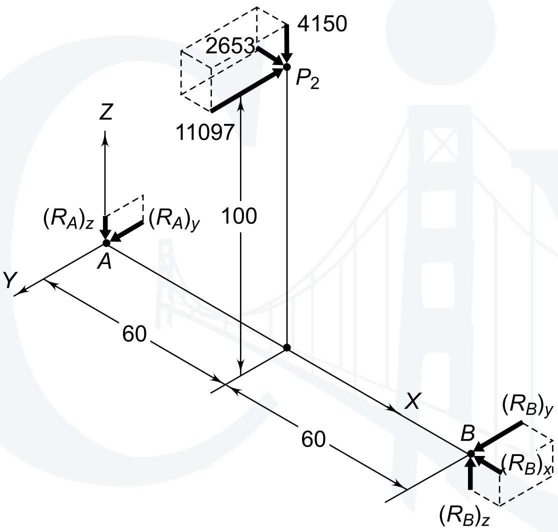


Fig. 20.12

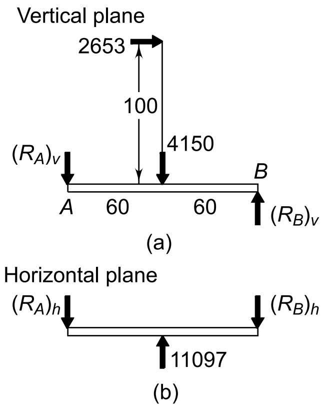


Fig. 20.13

Taking moment of forces in the vertical plane about the bearing *A*,

$$4150 \times 60 + 2653 \times 100 = (R_B)_v \times 120$$

$$(R_B)_v = 4285.83 \text{ N}$$

Considering equilibrium of vertical forces,

$$(R_A)_v + 4150 = 4285.83$$

$$(R_A)_v = 135.83 \text{ N}$$

Considering forces in the horizontal plane,

$$(R_A)_h = (R_B)_h = 11097/2 = 5548.5 \text{ N}$$

Considering equilibrium of axial forces,

$$(R_B)_a = 2653 \text{ N}$$

20.6 SELECTION OF MATERIALS

The selection of materials for the worm and the worm wheel is more limited than it is for other types of gears. The threads of the worm are subjected to fluctuating stresses and the number of stress cycles is fairly large. Therefore, the surface endurance strength is an important criterion in the selection of the worm material. The core of the worm should be kept ductile and tough to ensure maximum energy absorption. The worms are, therefore, made of case hardened steel with a surface hardness of 60 HRC and a case depth of 0.75 to 4.5 mm. The following varieties of steel are used for the worm:

Normalized carbon steels—40C8, 55C8
Case-hardened carbon steels—10C4, 14C6
Case-hardened alloy steels–16Ni80Cr60, 20Ni2Mo25
Nickel–chromium steels—13Ni3Cr80, 15Ni4Cr1

The magnitude of contact stresses on the worm wheel teeth is the same as that on the worm threads. However, the number of stress cycles is reduced by a factor equal to the speed reduction. The worm wheel cannot be accurately generated in the hobbing process. The final profile and finish of the worm wheel teeth is the result of plastic deformation during the initial stages of service. Therefore, the worm wheel material should be soft and conformable. Phosphor-bronze, with a surface hardness of 90–120 BHN, is widely used for the worm wheel. Phosphor-bronze worm wheels are sand-cast, sand-cast and chilled, or centrifugally cast. Phosphor-bronze is costly and in case of worm wheels with large dimensions, only the outer rim is made of phosphor-bronze. It is then bolted to the cast iron wheel. There are two reasons for using 'dissimilar' or 'heterogeneous' materials for worms and worm wheels:

(i) The coefficient of friction is reduced.

(ii) The conformability of worm wheel with respect to the worm is improved.

In spur, helical or bevel gears, the same material can be used for pinion and gear. In worm gear drive, however, dissimilar materials are used for the worm and worm wheel.

20.7 STRENGTH RATING OF WORM GEARS

Since the teeth of worm wheel are weaker than the threads of worm, the design for strength can be based on Lewis' equation as applied to worm wheel teeth. In this case, it is not necessary to design the worm on the basis of strength.

The worm gears are usually designed according to national and international codes. There are two basic equations: beam strength and wear strength equations. The maximum permissible torque that the worm wheel can withstand without bending failure is given by the lower of the following two values[2] :

$$(M_t)_1 = 17.65\, X_{b1} S_{b1} m l_r\, d_2 \cos\gamma \qquad (20.35)$$

$$(M_t)_2 = 17.65\, X_{b2} S_{b2} m l_r\, d_2 \cos\gamma \qquad (20.36)$$

where,

$(M_t)_1$, $(M_t)_2$ = permissible torque on the worm wheel (N-mm)

X_{b1}, X_{b2} = speed factors for strength of worm and worm wheel

S_{b1}, S_{b2} = bending stress factors of worm and worm wheel

m = module (mm)

l_r = length of the root of worm wheel teeth (mm) [Eq. (20.21)]

d_2 = pitch circle diameter of worm wheel (mm)

γ = lead angle of the worm

The bending stress factor (S_b) for various materials is given in Table 20.2. The speed factor for strength (X_b) of worm gears is obtained from Fig. 20.14. The power transmitting capacity based on beam strength is given by,

$$\text{kW} = \frac{2\pi n M_t}{60 \times 10^6} \qquad (20.37)$$

where (M_t) is the lower value between $(M_t)_1$ and $(M_t)_2$.

Table 20.2 *Values of bending stress factor S_b*

Material	S_b
Phosphor-bronze (centrifugally cast)	7.00
Phosphor-bronze (sand-cast and chilled)	6.40
Phosphor-bronze (sand-cast)	5.00
0.4% Carbon steel-normalized (40C8)	14.10
0.55% Carbon steel-normalized (55C8)	17.60
Case-hardened carbon steels (10C4, 14C6)	28.20
Case-hardened alloy steels (16Ni80Cr60 and 20Ni2Mo25)	33.11
Nickel-chromium steels (13Ni3Cr80 and 15Ni4Crl)	35.22

Example 20.5 *A pair of worm and worm wheel is designated as,*

1/30/10/10

The input speed of the worm is 1200 rpm. The worm wheel is made of centrifugally cast, phosphor-bronze and the worm is made of case-hardened carbon steel 14C6. Determine the power transmitting capacity based on the beam strength.

Solution

Given $n_1 = 1200$ rpm $z_1 = 1$ $z_2 = 30$ teeth $q = 10$ $m = 10$ mm

Step I *Permissible torque on worm wheel*

$$i = \frac{z_2}{z_1} = \frac{30}{1} = 30$$

$$n_1 = 1200 \text{ rpm} \quad n_2 = \frac{1200}{i} = \frac{1200}{30} = 40 \text{ rpm}$$

$$d_2 = mz_2 = 10(30) = 300 \text{ mm}$$

$$\tan\gamma = \frac{z_1}{q} = \frac{1}{10} = 0.1 \quad \text{or} \quad \gamma = 5.71°$$

[2] IS 7443–1974: Method for load rating of worm gears

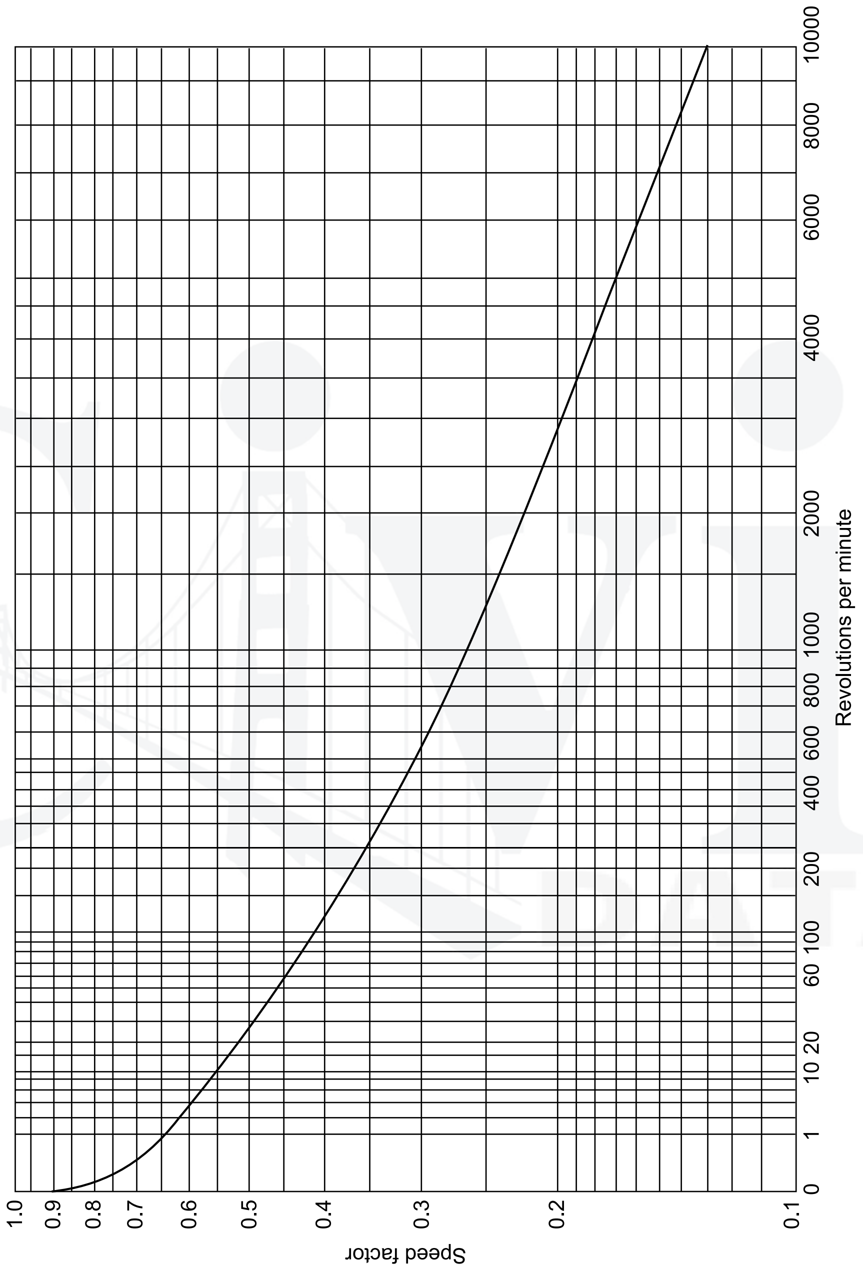


Fig. 20.14 *Speed Factor for Worm Gears for Strenght (X_b)*

From Eq. (20.20),

$$F = 2m\sqrt{(q+1)} = 2(10)\sqrt{(10+1)}$$
$$= 66.33 \text{ mm}$$

From Eqs (20.13) and (20.14),

$$c = 0.2\ m \cos\gamma = 0.2(10)\cos(5.71) = 1.99 \text{ mm}$$
$$d_{a1} = m\,(q+2) = 10\,(10+2) = 120 \text{ mm}$$

From Eq. (20.21),

$$l_r = (d_{a1} + 2c)\sin^{-1}\left[\frac{F}{(d_{a1}+2c)}\right]$$
$$= (120 + 2\times 1.99)\sin^{-1}\left[\frac{66.33}{(120+2\times 1.99)}\right]$$
$$= 69.988 \text{ mm}$$

For case-hardened carbon steel 14C6 (Table 20.2),

$$S_{b1} = 28.2$$

For centrifugally cast phosphor-bronze,

$$S_{b2} = 7.0$$

From Fig. 20.14,

$$X_{b1} = 0.25 \quad \text{for} \quad n_1 = 1200 \text{ rpm}$$
$$X_{b2} = 0.48 \quad \text{for} \quad n_2 = 40 \text{ rpm}$$

From Eqs (20.35) and (20.36),

$$(M_t)_1 = 17.65\, X_{b1} S_{b1} m\, l_r\, d_2 \cos\gamma$$
$$= 17.65(0.25)(28.2)(10)(69.988)(300)\cos(5.71)$$
$$= 25\,996\,711 \text{ N-mm} \qquad \text{(a)}$$
$$(M_t)_2 = 17.65\, X_{b2}\, S_{b2}\, m l_r\, d_2 \cos\gamma$$
$$= 17.65\,(0.48)\,(7.0)(10)(69.998)\,(300)\cos(5.71)$$
$$= 12\,389\,922 \text{ N-mm} \qquad \text{(b)}$$

The lower value of the torque on the worm wheel is 12 389 922 N-mm.

Step II *Power transmitting capacity based on beam strength*

$$\text{kW} = \frac{2\pi n_2 (M_t)}{60\times 10^6} = \frac{2\pi(40)(12\,389\,922)}{60\times 10^6} = 51.9$$

20.8 WEAR RATING OF WORM GEARS

The maximum permissible torque that the worm wheel can withstand without pitting failure, is given by the lower of the following two values:

$$(M_t)_3 = 18.64\, X_{c1}\, S_{c1}\, Y_z\, (d_2)^{1.8}\ \text{m} \qquad (20.38)$$
$$(M_t)_4 = 18.64\, X_{c2}\, S_{c2}\, Y_z\, (d_2)^{1.8}\ \text{m} \qquad (20.39)$$

where,

$(M_t)_3$, $(M_t)_4$ = permissible torque on the worm wheel (N-mm)

X_{c1}, X_{c2} = speed factors for the wear of worm and worm wheel

S_{c1}, S_{c2} = surface stress factors of the worm and worm wheel

Y_z = zone factor

The values of the surface stress factor (S_c) for the various materials are given in Table 20.3. The values of the worm gear zone factor are given in Table 20.4. The speed factors (X_c) for wear depend upon the rotational speed and the rubbing speed V_s. They can be determined from Fig. 20.15.

Table 20.3 *Values of the Surface Stress Factor S_c*

	Materials	*Values of S_c when running with*			
		A	*B*	*C*	*D*
A	Phosphor-bronze (centrifugally cast)	–	0.85	0.92	1.55
	Phosphor-bronze (sand cast and chilled)	–	0.63	0.70	1.27
	Phosphor-bronze (sand-cast)	–	0.47	0.54	1.06
B	0.4% carbon steel-normalized (40C8)	1.1	–	–	–
C	0.55% carbon steel-normalized (55C8)	1.55	–	–	–
D	Case-hardened carbon steel (10C4 , 14C6)	4.93	–	–	–
	Case-hardened alloy steel (16Ni80Cr60 , 20Ni2Mo25)	5.41	–	–	–
	Nickel–chromium steel (13Ni3Cr80 , 15Ni4Cr1)	6.19	–	–	–

Table 20.4 *Values of the zone factor Y_z*

z_1	$q = 8$	$q = 9$	$q = 10$	$q = 12$	$q = 16$	$q = 20$
1	1.084	1.128	1.143	1.202	1.374	1.508
2	1.114	1.214	1.231	1.280	1.418	1.575
4	1.204	1.380	1.460	1.515	1.634	1.798

Example 20.6 *Assume the data of Example 20.5 for a pair of worm gears. Determine the power transmitting capacity based on wear strength.*

Solution

Given $n_1 = 1200$ rpm $z_1 = 1$ $z_2 = 30$ teeth
$q = 10$ $m = 10$ mm

Step I Permissible torque on worm wheel

For the given pair of worm gears,

$$d_2 = 300 \text{ mm}$$

For ($q = 10$) and ($z_1 = 1$), the zone factor Y_z from Table 20.4 is given by

$$Y_z = 1.143$$

For case-hardened carbon steel 14C6 (Table 20.3),

$$S_{c1} = 4.93$$

For centrifugally cast phosphor-bronze,

$$S_{c2} = 1.55$$

From Eq. (20.33),

$$V_s = \frac{\pi d_1 n_1}{60\,000 \cos \gamma} = \frac{\pi (10 \times 10)(1200)}{60\,000 \cos(5.71)} = 6.315 \text{ m/s}$$

For $V_s = 6.315$ m/s and $n_1 = 1200$ rpm (Fig. 20.15),

$$X_{c1} = 0.112$$

For $V_s = 6.315$ m/s and $n_1 = 40$ rpm

$$X_{c2} = 0.26$$

From Eqs (20.38) and (20.39),

$$(M_t)_3 = 18.64\, X_{c1}\, S_{c1}\, Y_z (d_2)^{1.8}\, m$$
$$= 18.64\,(0.112)\,(4.93)(1.143)(300)^{1.8}\,(10)$$
$$= 3\,383\,570.4 \text{ N-mm} \quad \text{(a)}$$

$$(M_t)_4 = 18.64\, X_{c2}\, S_{c2}\, Y_z\, (d_2)^{1.8}\, m$$
$$= 18.64\,(0.26)(1.55)(1.143)(300)^{1.8}\,(10)$$
$$= 2\,469\,535.8 \text{ N-mm} \quad \text{(b)}$$

The lower value of torque on worm wheel is 2 469 535.8 N-mm.

Step II Power transmitting capacity based on wear strength

$$\text{kW} = \frac{2\pi n_2 (M_t)}{60 \times 10^6} = \frac{2\pi (40)(2\,469\,535.8)}{60 \times 10^6} = 10.34$$

20.9 THERMAL CONSIDERATIONS

The efficiency of a worm gear drive is low and the work done by friction is converted into heat. When the worm gears operate continuously, considerable amount of heat is generated. The rate of heat generated (H_g) is given by,

$$H_g = 1000\,(1 - \eta) \text{ kW} \quad \text{(a)}$$

where,

H_g = rate of heat generation (W)
η = efficiency of worm gears (fraction)
kW = power transmitted by gears (kW)

The heat is dissipated through the lubricating oil to the housing wall and finally to the surrounding air. The rate of heat dissipated (H_d) by the housing walls to the surrounding air is given by,

$$H_d = k\,(t - t_o)A \quad \text{(b)}$$

where,

H_d = the rate of heat dissipation (W)
k = overall heat transfer coefficient of housing walls (W/m^2 °C)
t = temperature of the lubricating oil (°C)
t_o = temperature of the surrounding air (°C)
A = effective surface area of housing (m^2)

Equating (a) and (b),

$$1000(1 - \eta) \text{ kW} = k(t - t_o)\,A$$

The above equation is written in the following two ways:

$$\text{kW} = \frac{k(t - t_o)A}{1000(1 - \eta)} \quad (20.40)$$

$$t = t_o + \frac{1000(1 - \eta)\text{ kW}}{kA} \quad (20.41)$$

Equation (20.40) gives the power transmitting capacity based on thermal considerations. Equation (20.41) gives the resultant temperature of the lubricating oil for a given power transmitting capacity.

The overall heat transfer coefficient under normal working conditions with natural air circulation is 12 to 18 W/m^2 °C. This value can be further increased by providing a fan on the worm shaft and arranging the fins horizontally along the stream of air. In such cases, the value of the overall heat transfer coefficient can be taken as 20 to 28 W/m^2 °C. It has been observed that the maximum permissible

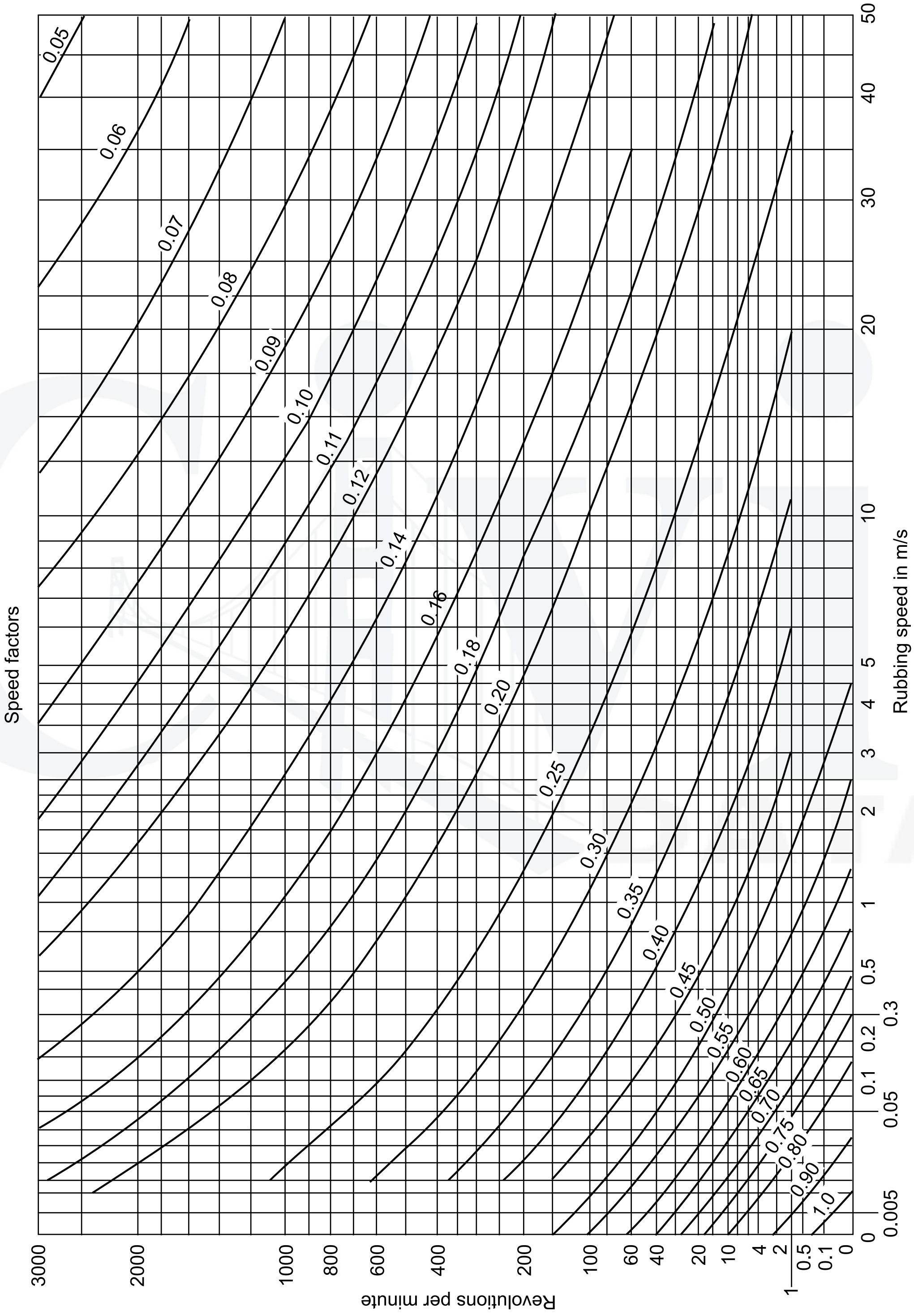


Fig. 20.15 *Speed Factor for Worm Gears for Wear (X_c)*

temperature for commonly used lubricating oils is 95°C, above which it loses its properties and there is a danger of gear tooth failure due to seizure.

Example 20.7 *A worm gear box with an effective surface area of 1.5 m² is operating in still air with a heat transfer coefficient of 15 W/m²°C. The temperature rise of the lubricating oil above the atmospheric temperature is limited to 50°C. The worm gears are designated as,*

1/30/10/8

The worm shaft is rotating at 1440 rpm and the normal pressure angle is 20°. Calculate the power transmitting capacity based on the thermal considerations.

Solution

Given $A = 1.5\ \text{m}^2$ $k = 15\ \text{W/m}^2\ °\text{C}$
$(t - t_o) = 50°\text{C}$ $z_1 = 1$ $z_2 = 30$ teeth $q = 10$
$m = 8$ mm $n = 1440$ rpm

Step I *Efficiency of worm gear drive*

$$\tan\gamma = \frac{z_1}{q} = \frac{1}{10} = 0.1 \quad \text{or} \quad \gamma = 5.71°$$

$$d_1 = mq = 8\,(10) = 80 \text{ mm}$$

$$V_s = \frac{\pi d_1 n_1}{60\,000 \cos\gamma} = \frac{\pi(80)(1440)}{60\,000\cos(5.71)} = 6.06 \text{ m/s}$$

From Fig. 20.9, the coefficient of friction is 0.024. From Eq. (20.34),

$$\eta = \frac{(\cos\alpha - \mu\tan\gamma)}{(\cos\alpha + \mu\cot\gamma)} = \frac{[\cos(20) - 0.024(0.1)]}{[\cos(20) + 0.024(1/0.1)]}$$
$$= 0.7945$$

Step II *Power transmitting capacity based on thermal considerations*

From Eq. (20.40),

$$\text{kW} = \frac{k(t - t_o)A}{1000(1-\eta)} = \frac{15(50)(1.5)}{1000(1 - 0.7945)} = 5.47$$

Short-Answer Questions

20.1 Where do you use worm gear drive?
20.2 What are the advantages of worm gear drives?
20.3 What are the drawbacks of worm gear drives?
20.4 What kind of contact occurs between worm and worm wheel? How does it differ from other types of gears?
20.5 Why are worm gear reduction units not preferred over other types of gearboxes for transmitting large powers?
20.6 What are single-enveloping and double-enveloping worm gear drives? Where do you use them?
20.7 What are the advantages of double-enveloping worm-gear drives over single-enveloping worm gear drives?
20.8 What are the four important parameters that are required to specify the worm gear drive?
20.9 What is the material for worm? Why?
20.10 What is the material for worm wheel? Why?
20.11 Why is the efficiency of worm gear drive low?

Problems for Practice

20.1 A pair of worm gears is designated as

2/54/10/5

Calculate

(i) the centre distance;
(ii) the speed reduction;
(iii) the dimensions of the worm; and
(iv) the dimensions of the worm wheel.

[(i) 160 mm (ii) 27 (iii) d_1 = 50 mm; d_{a1} = 60 mm d_{f1} = 38.427 mm; p_x = 15.708 mm (iv) d_2 = 270 mm; d_{a2} = 279.612 mm; d_{f2} = 258.039 mm]

20.2 A pair of worm and worm wheel is designated as

2/52/10/4

10 kW power at 720 rpm is supplied to the worm shaft. The coefficient of friction is 0.04 and the pressure angle is 20°. Calculate the tangential, axial and radial components of the resultant gear tooth force acting on the worm wheel.

[27 105.78 N, 6631.46 N and 10 147.47 N]

20.3 A pair of worm gears is designated as

1/52/10/8

The worm rotates at 1000 rpm and the normal pressure angle is 20°. Determine the coefficient of friction and the efficiency of the worm gears.

[*0.027 and 77.45%*]

20.4 A pair of worm gears is designated as

1/40/10/4

The input speed of the worm shaft is 1000 rpm The worm wheel is made of phosphor-bronze (sand cast), while the worm of case-hardened carbon steel 1OC4. Determine the power transmitting capacity based on beam strength.

[*2.1 kW*]

20.5 Assume the data of Example 20.4 and determine the power transmitting capacity based on wear strength.

[*0.77 kW*]

20.6 The gear box for the worm gears of examples 20.4 and 20.5 has an effective surface area of 0.25 m^2. A fan is mounted on the worm shaft to circulate air over the surface of the fins. The coefficient of heat transfer can be taken as 25 W/m^2 °C. The permissible temperature rise of the lubricating oil above the atmospheric temperature is 45°C. The coefficient of friction is 0.035 and the normal pressure angle is 20°. Calculate the power transmitting capacity based on thermal considerations.

[*1.03 kW*]

Flywheel

Chapter 21

21.1 FLYWHEEL

A flywheel is a heavy rotating body that acts as a reservoir of energy. The energy is stored in the flywheel in the form of kinetic energy. The flywheel acts as an energy bank between the source of power and the driven machinery. Depending upon the source of power and type of driven machinery, there are two distinct applications of the flywheel.

In certain cases, the power is supplied at uniform rate, while the demand for power from the driven machinery is variable, e.g., a punch press driven by an electric motor. In punching and shearing machines, maximum power is required only during a small part of the cycle, when actual punching or shearing takes place. During the remaining part of the cycle, negligible power is required to overcome friction. If these machines are directly driven by an electric motor, a higher capacity motor corresponding to maximum power requirement during actual punching or shearing will be required. Such a motor will run almost idle during the remaining part of the cycle. It is obviously wasteful to provide such a large motor when its full capacity is needed but a small fraction of the time. Providing a flywheel to these machines allow a much smaller motor to be used. During the actual punching or shearing operations, energy will be taken from the flywheel, slowing it down. During the relatively long period between two punching or shearing operations, the motor will accelerate the flywheel back to its original speed. Thus, the flywheel stores the kinetic energy during the idle portion of the work cycle by increasing its speed and delivers this kinetic energy during the peak-load period of punching or shearing. Therefore, when a flywheel is used between the motor and these machines, a smaller capacity motor is sufficient.

In other applications, the power is supplied at variable rate, while the requirement of the driven machinery is at a uniform rate, e.g., machinery driven by an internal combustion engine. In IC engines, the power is generated at a variable rate. The flywheel absorbs the excess energy during the expansion stroke, when power developed in the cylinder exceeds the demand. This energy is delivered during suction, compression and exhaust strokes. The flywheel, therefore, enables the engine to supply the power at a practically uniform rate.

The functions of the flywheel are as follows:

(i) To store and release energy when needed during the work cycle

(ii) To reduce the power capacity of the electric motor or engine

(iii) To reduce the amplitude of speed fluctuations

The construction of a solid one-piece flywheel made of grey cast iron is shown in Fig. 21.1. The arms have an elliptical cross-section. In small flywheels, the arms are replaced by a solid web. In large flywheels, stresses are induced in the arms during the casting process. There is a heavy concentration of mass at the rim and at the hub, which results in unequal cooling rates for the rim, the hub and the arms. The resulting stresses, called *cooling stresses*, are sometimes of such a magnitude as to cause the breakage of arms. Such cooling stresses can be avoided by using a split-type construction as illustrated in Fig. 21.2. In this case, the rim and the hub are cut through the centre. The arms are, therefore, free to contract during the cooling process in the mould and residual cooling stresses are avoided.

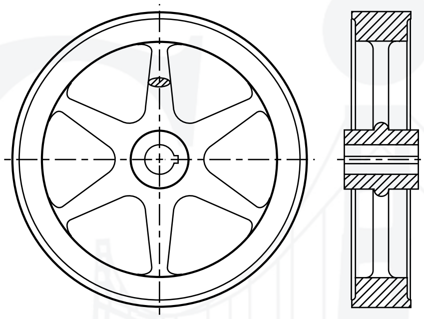


Fig. 21.1 *Solid One Piece Flywheel*

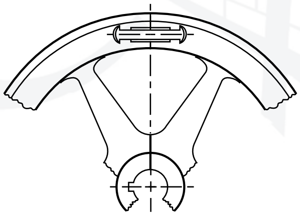


Fig. 21.2 *Split Flywheel*

21.2 FLYWHEEL AND GOVERNOR

Both flywheel and governor, are used in internal combustion engines to control the speed. The function of the engine governor is to control the mean speed of the engine. If the mean speed varies due to variation of load, the governor adjusts the fuel supply to the engine and restores the speed to its former value. If the load on the engine shaft increases, the speed of the engine decreases and the supply of fuel is increased by increasing the opening of the throttle valve. This increases the engine speed to its former value. On the other hand, if the load on the engine shaft decreases, the speed of the engine increases and the supply of fuel is decreased by reducing the opening of the throttle valve. This decreases the engine speed to its former value. The governor adjusts the opening of the throttle valve through a suitable mechanism. In petrol engine, the governor adjusts the opening of the throttle valve. In diesel engine, the governor adjusts the opening of the fuel pump.

There is a basic difference between the functions of flywheel and governor. It is as follows:

(i) The flywheel limits the inevitable fluctuations of speed during '*each cycle*', which arise from fluctuations of turning moment on the crankshaft. The governor controls the '*mean*' *speed* of the engine by varying the fuel supply to the engine.

(ii) The flywheel has no influence on the '*mean*' speed of the engine. It does not maintain a constant speed. The governor has no influence on '*cyclic*' speed fluctuations.

(iii) If the load on the engine is constant, the mean speed will be constant from cycle to cycle and the governor will not operate. On the other hand, the flywheel will be always acting. The operation of flywheel is continuous while that of governor is more or less intermittent.

(iv) A flywheel may not be used if the cyclic fluctuations of energy output are small or negligible. A governor is essential for all types of engines to adjust the fuel supply as per the demand.

(v) The kind of energy stored in flywheel is kinetic energy. The kinetic energy is all available, 100% convertible into work without friction. The governor mechanism involves frictional losses.

21.3 FLYWHEEL MATERIALS

Traditionally, flywheels are made of cast iron. From design considerations, cast iron flywheels offer the following advantages:

(i) Cast iron flywheels are the cheapest.

(ii) Cast iron flywheel can be given any complex shape without involving machining operations.

(iii) Cast iron flywheel has excellent ability to damp vibrations.

However, cast iron has poor tensile strength compared to steel. The failure of cast iron flywheel is sudden and total. The machinability of cast iron flywheel is poor compared to steel flywheel. The mass density of materials used for the flywheel is given in Table 21.1.

More recently, flywheels are made of high strength steels and composites in vehicle applications. Graphite-Fiber Reinforced Polymer (GFRP) is considered as an excellent choice for flywheels fitted on modern car engines.

Table 21.1 *Mass density of flywheel materials*

Material	*Mass density (kg/m³) (ρ)*
Grey cast iron	
FG 150	7050
FG 200	7100
FG 220	7150
FG 260	7200
FG 300	7250
Steels	
Carbon steels	7800

21.4 TORQUE ANALYSIS

A flywheel mounted on a relatively stiff shaft is shown in Fig. 21.3. Considering the equilibrium of torques,

$$I\left(\frac{d\omega}{dt}\right) = T_i - T_o \qquad (21.1)$$

where,

T_i = driving or input torque (N-m)

T_o = load or output torque (N-m)

I = mass moment of inertia of flywheel (kg-m^2)

ω = angular velocity of shaft (rad/s) ($d\theta/dt$)

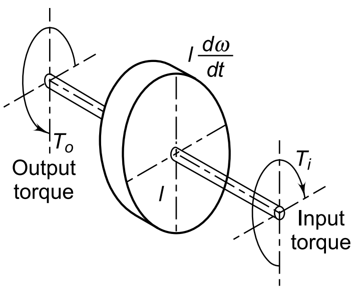


Fig. 21.3 *Torque on Flywheel Shaft*

When the driving torque is more than the load torque, the term ($T_i - T_o$) is positive and the flywheel is accelerated. When the driving torque is less than the load torque, the term ($T_i - T_o$) is negative indicating retardation of flywheel. A T–θ diagram for a particular application is shown in Fig. 21.4(a). In this case, the power is supplied to the machine by a constant-torque motor, developing a torque T_m. The work cycle of the driven machine consists of three parts—*AB*, *BC* and *CD*. During the elements *AB* and *CD*, the torque required by the machine remains constant at T_1. At the point *B*, it increases to T_2 and remains constant during the element *BC*. The cycle is repeated for each revolution.

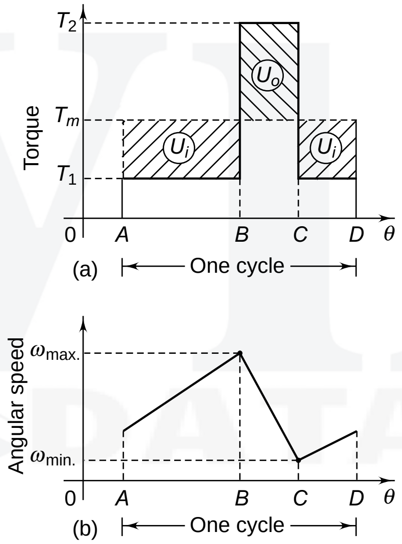


Fig. 21.4 *(a) Torque Diagram (b) Speed Diagram*

During the first element *AB*, the torque required by the machine is less than that supplied by the motor ($T_1 < T_m$). The flywheel is, therefore, accelerated and its angular velocity increases to $\omega_{max.}$ at the point *B*. During the element *BC*, the torque required by the machine is more than the torque supplied by the motor ($T_2 > T_m$). The flywheel is retarded and its angular velocity decreases to $\omega_{min.}$ at point *C*. This variation in angular velocity is shown in Fig. 21.4(b).

During elements AB and CD, the energy is supplied to the flywheel. The input energy U_i is given by,

$$U_i = \int_A^B (T_m - T_1)d\theta + \int_C^D (T_m - T_1)d\theta$$

During the element BC, the energy is taken from the flywheel and the energy output U_o from the flywheel is given by,

$$U_o = \int_B^C (T_2 - T_m)d\theta$$

The change in kinetic energy from point B to C is given by

$$U_o = \frac{1}{2} I(\omega_{max.}^2 - \omega_{min.}^2)$$

or

$$U_o = \frac{1}{2} I(\omega_{max.} + \omega_{min.})(\omega_{max.} - \omega_{min.}) \qquad \text{(a)}$$

The difference between the maximum and minimum speeds ($\omega_{max.} - \omega_{min.}$) during a cycle is called the maximum fluctuation of speed. *The ratio of maximum fluctuation of speed to the mean speed is called coefficient of fluctuation of speed*. Therefore, the coefficient of fluctuation of speed, denoted by C_s is given by,

$$C_s = \frac{\omega_{max.} - \omega_{min.}}{\omega} \qquad (21.2)$$

where ω is the average or mean angular velocity of the flywheel. It is given by,

$$\omega = \frac{\omega_{max.} + \omega_{min.}}{2} \qquad (21.3)$$

Substituting Eqs (21.2) and (21.3) in expression (a),

$$U_o = I\omega^2 C_s \qquad (21.4)$$

The values of the coefficients of fluctuation of speed, used in practice are given in Table 21.2.

The coefficient of fluctuation of speed can be expressed in the following ways:

$$C_s = \frac{\omega_{max.} - \omega_{min.}}{\omega} = \frac{\omega_{max.} - \omega_{min.}}{(\omega_{max.} + \omega_{min.})/2}$$

$$= \frac{2(\omega_{max.} - \omega_{min.})}{(\omega_{max.} + \omega_{min.})}$$

or

$$C_s = \frac{2(\omega_{max.} - \omega_{min.})}{(\omega_{max.} + \omega_{min.})} \qquad (21.5)$$

Similarly,

$$C_s = \frac{\omega_{max.} - \omega_{min.}}{\omega} = \frac{n_{max.} - n_{min.}}{n}$$

or

$$C_s = \frac{n_{max.} - n_{min.}}{n} \qquad (21.6)$$

$$C_s = \frac{n_{max.} - n_{min.}}{n} = \frac{n_{max.} - n_{min.}}{(n_{max.} + n_{min.})/2}$$

$$= \frac{2(n_{max.} - n_{min.})}{(n_{max.} + n_{min.})}$$

or

$$C_s = \frac{2(n_{max.} - n_{min.})}{(n_{max.} + n_{min.})} \qquad (21.7)$$

where n is the speed in rpm.

Sometimes, a term called coefficient of 'steadiness' is used. *The coefficient of steadiness is defined as the reciprocal of the coefficient of fluctuation of speed*. It is denoted by the letter m. Therefore,

$$m = \frac{1}{C_s} = \frac{\omega}{\omega_{max.} - \omega_{min.}} = \frac{n}{n_{max.} - n_{min.}} \qquad (21.8)$$

Table 21.2 *Coefficients of fluctuations of speed*

Type of Equipment	C_s
Punching, shearing and forming presses	0.200
Compressor (belt driven)	0.120
Compressor (gear driven)	0.020
Machine tools	0.025
Reciprocating pumps	0.040
Geared drives	0.020
Internal combustion engines	0.030
D.C. generators (direct drive)	0.010
A.C. generator (direct drive)	0.005

21.5 COEFFICIENT OF FLUCTUATION OF ENERGY

The turning moment diagram for a multi-cylinder engine is shown in Fig. 21.5. The intercepted areas between the torque developed by the engine and the mean torque line, taken in order from one end are $-a_1, +a_2, -a_3, +a_4, -a_5, +a_6$, and $-a_7$ respectively.

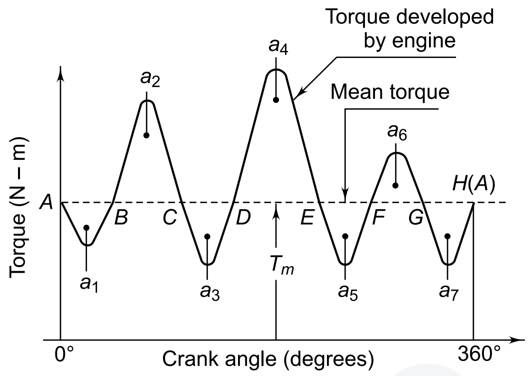


Fig. 21.5 *Turning Moment Diagram*

It is assumed that the energy stored in the flywheel is U at the point A. Therefore,

Energy at $B = U - a_1$
Energy at $C = U - a_1 + a_2$
Energy at $D = U - a_1 + a_2 - a_3$
Energy at $E = U - a_1 + a_2 - a_3 + a_4$
Energy at $F = U - a_1 + a_2 - a_3 + a_4 - a_5$
Energy at $G = U - a_1 + a_2 - a_3 + a_4 - a_5 + a_6$
Energy at $H = U - a_1 + a_2 - a_3 + a_4 - a_5 + a_6 - a_7 = U$

Suppose, the maximum and minimum energy occurs at points E and B. The angular velocity of the flywheel will be maximum at the point E and minimum at the point B.

Maximum energy = Energy at E
$= U - a_1 + a_2 - a_3 + a_4$

Minimum energy = Energy at $B = U - a_1$

The maximum fluctuation of energy is defined as the difference between the maximum kinetic energy and minimum kinetic energy in the cycle. It is denoted by U_o.

$$U_o = U_E - U_B = (U - a_1 + a_2 - a_3 + a_4) - (U - a_1)$$
$$= a_2 - a_3 + a_4$$

The coefficient of fluctuation of energy is defined as the ratio of the maximum fluctuation of energy to the work done per cycle. It is denoted by C_e.

$$C_e = \frac{\text{maximum fluctuations of energy}}{\text{work done per cycle}}$$

$$= \frac{U_0}{\text{work done per cycle}} \qquad (21.9)$$

The work done per cycle is given by,

Work done/cycle = area below mean torque line from 0° to 360°

Work done/cycle = $(2\pi)T_m$ (21.10)
(for two stroke engine)

In case of two-stroke internal combustion engine, the crank rotation is (2π) radians per cycle. In case of a four-stroke internal combustion engine, the crank rotation is (4π) radians per cycle. Therefore,

Work done/cycle = $(4\,\pi)T_m$ (21.11)
(for four stroke engine)

The values of the coefficient of fluctuations of energy are given in Table 21.3.

Table 21.3 *Coefficient of fluctuations of energy*

Type of engine	C_e
Single-cylinder, double-acting steam engine	0.21
Cross-compound steam engine	0.096
Single-cylinder, four-stroke petrol engine	1.93
Four-cylinder, four-stroke petrol engine	0.066
Six-cylinder, four-stroke petrol engine	0.031

21.6 SOLID DISK FLYWHEEL

The simple type of flywheel is a solid circular disk as shown in Fig. 21.6. The mass moment of inertia of this disk is given by,

$$I = \frac{mR^2}{2} \qquad (a)$$

where,

I = mass moment of inertia of disk (kg-m²)
m = mass of disk (kg)
R = outer radius of disk (m)

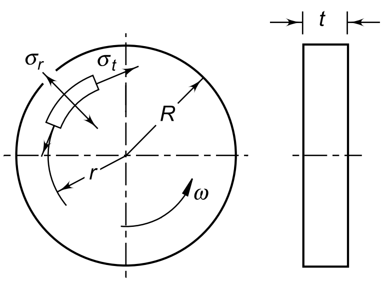


Fig. 21.6 *Solid Disk Flywheel*

The mass of the disk is given by

$$m = \pi R^2 t\rho \qquad \text{(b)}$$

where,

t = thickness of disk (m)

ρ = mass density of flywheel material (kg/m^3)

From (a) and (b),

$$I = \frac{\pi}{2}\rho t R^4 \qquad (21.12)$$

There are two principal stresses in the rotating disk—the tangential stress σ_t, and radial stress σ_r. The general equations for these stresses at a radius r are as follows[1]:

$$\sigma_t = \frac{\rho v^2}{10^6}\left(\frac{\mu+3}{8}\right)\left[1-\left(\frac{3\mu+1}{\mu+3}\right)\left(\frac{r}{R}\right)^2\right]$$

$$\sigma_r = \frac{\rho v^2}{10^6}\left(\frac{\mu+3}{8}\right)\left[1-\left(\frac{r}{R}\right)^2\right]$$

where,

σ_t = tangential stress at radius r (N/mm^2)

σ_r = radial stress at radius r (N/mm^2)

μ = Poisson's ratio (0.3 for steel and 0.27 for cast iron)

v = peripheral velocity (m/s) ($R\omega$)

The maximum tangential stress and maximum radial stress are equal and both occur at (r = 0).

Therefore,

$$(\sigma_t)_{\text{max.}} = (\sigma_r)_{\text{max.}} = \frac{\rho v^2}{10^6}\left(\frac{\mu+3}{8}\right) \qquad (21.13)$$

Example 21.1 *A machine is driven by a motor, which exerts a constant torque. The resisting torque of the machine increases uniformly from 500 N-m to 1500 N-m through a 360° rotation of the driving shaft and drops suddenly to 500 N-m again at the beginning of the next revolution. The mean angular velocity of the machine is 30 rad/s and the coefficient of speed fluctuations is 0.2. A solid circular steel disk, 25 mm thick, is used as flywheel. The mass density of steel is 7800 kg/m^3 while Poisson's ratio is 0.3. Calculate the outer radius of the flywheel disk and the maximum stresses induced in it.*

Solution

Given ω = 30 rad/s C_s = 0.2 t = 25 mm μ = 0.3 ρ = 7800 kg/m^3

Step I *Energy output from flywheel (U_o)*

The turning moment diagram is shown in Fig. 21.7. The mean torque T_m supplied by the motor is given by,

$$T_m = \frac{\text{Area } OABG}{\text{Length } OG} = \frac{\left(\frac{500+1500}{2}\right)(2\pi)}{(2\pi)}$$

$$= 1000 \text{ N-m}$$

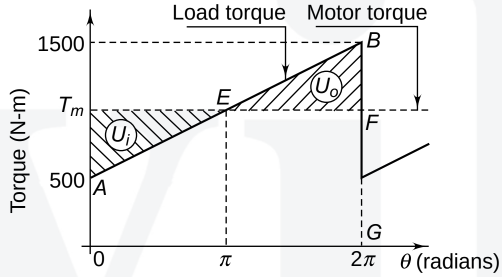


Fig. 21.7 *Turning Moment Diagram*

The maximum and minimum angular velocities occur at points E and B respectively. Therefore,

$$U_o = \text{Area of } \Delta EBF = \frac{1}{2} BF \times EF$$

$$= \frac{1}{2}(1500-1000)(2\pi-\pi)$$

$$= (250\,\pi) \text{ N-m or J}$$

Step II *Outer radius of flywheel*

From Eq. 21.4,

$$I = \frac{U_o}{\omega^2 C_s} = \frac{(250\,\pi)}{(30)^2(0.2)} = 4.3633 \text{ kg-m}^2$$

From Eq. 21.12,

$$I = \frac{\pi}{2}\rho t R^4 \quad \text{or} \quad (4.3633) = \frac{\pi}{2}(7800)\left(\frac{25}{1000}\right)R^4$$

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[1] S Timoshenko—*Strength of Materials*—Part II, MacMillan and Co. Ltd.

$\therefore \quad R = 0.345 \text{ m} = 345 \text{ mm}$

or $\quad R = 350 \text{ mm}$

Step III *Stresses in flywheel*

The tangential and radial stresses are maximum at the centre of the disk. From Eq. 21.13,

$$(\sigma_t)_{\max.} = (\sigma_r)_{\max.} = \frac{\rho v^2}{10^6}\left(\frac{\mu+3}{8}\right)$$

$$= \frac{(7800)(30 \times 0.35)^2}{10^6}\left(\frac{0.3+3}{8}\right)$$

$$= 0.35 \text{ N/mm}^2$$

Example 21.2 *The torque developed by an engine is given by the following equation:*

$$T = 14\,250 + 2200 \sin 2\theta - 1800 \cos 2\theta$$

where T is the torque in N-m and θ is the crank angle from the inner dead centre position. The resisting torque of the machine is constant throughout the work cycle. The coefficient of speed fluctuations is 0.01. The engine speed is 150 rpm. A solid circular steel disk, 50 mm thick, is used as a flywheel. The mass density of steel is 7800 kg/m³. Calculate the radius of the flywheel disk.

Solution

Given $n = 150$ rpm $\quad C_s = 0.01 \quad t = 50$ mm $\quad \rho = 7800 \text{ kg/m}^3$

Step I *Energy output from flywheel (U_o)*

The fluctuating terms (sin 2θ) and (cos 2θ) have a zero mean. Therefore, the mean torque is given by,

$$T_m = 14\,250 \text{ N-m}$$

When $T = T_m$,

$$2200 \sin 2\theta - 1800 \cos 2\theta = 0$$

$$\tan 2\theta = \frac{1800}{2200}$$

or $\quad 2\theta = 39.29° \quad$ or $\quad (180 + 39.29)°$

Therefore,

$$\theta = 19.645° \quad \text{or} \quad 109.645°$$

The turning moment diagram is shown in Fig. 21.8. The maximum and minimum angular velocities will occur at points A and B respectively.

Therefore,

$$U_o = \int_A^B (T - T_m)\, d\theta$$

$$= \int_{19.645}^{109.645} (2200 \sin 2\theta - 1800 \cos 2\theta)\, d\theta$$

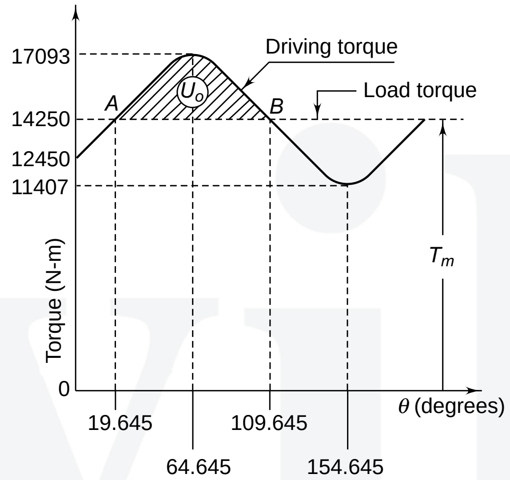


Fig. 21.8 *Turning Moment Diagram*

$$= [-1100 \cos 2\theta - 900 \sin 2\theta]_{19.645}^{109.645}$$

$$= -1100\,(-0.774 - 0.774) - 900\,(-0.633 - 0.633)$$

$$= 2842.2 \text{ N-m or J}$$

Step II *Outer radius of flywheel disk*

From Eq. 21.4,

$$I = \frac{U_o}{\omega^2 C_s} = \frac{(2842.2)}{\left(\frac{2\pi \times 150}{60}\right)^2 (0.01)} = 1151.9 \text{ kg-m}^2$$

From Eq. 21.12,

$$R^4 = \frac{2I}{\pi \rho t} = \frac{2(1151.9)}{\pi (7800)\left(\frac{50}{1000}\right)}$$

$\therefore \quad R = 1.171 \text{ m} = 1171 \text{ mm}$

or $\quad R = 1175 \text{ mm}$

21.7 RIMMED FLYWHEEL

The solid disk flywheel discussed in Section 21.6 is rarely used in practice. In most cases, the flywheel consists of a rim, a hub and four to six spokes as shown in Fig. 21.1. Due to the complicated geometric shapes of its component parts, it is difficult to determine the exact moment of inertia of this flywheel. Therefore, the analysis of such a flywheel is done by using anyone of the following two assumptions:

(i) The spokes, the hub and the shaft do not contribute any moment of inertia, and the entire moment of inertia is due to the rim alone.

(ii) The effect of spokes, the hub and the shaft is to contribute 10% of the required moment of inertia, while the rim contributes 90%.

The moment of inertia of the rim is, therefore, given by

$$I_r = KI \tag{a}$$

where,

I_r = moment of inertia of rim (kg-m^2)

I = required moment of inertia (kg-m^2)

The factor K is equal to 1 when the entire moment of inertia is due to the rim alone. The factor K is taken as 0.9, when it is assumed that the rim contributes 90% of the required moment of inertia. From Eq. 21.4,

$$I = \frac{U_o}{\omega^2 C_s} \tag{b}$$

From (a) and (b),

$$I_r = \frac{U_o K}{\omega^2 C_s} \tag{21.14}$$

The thickness of the rim is usually very small compared with the mean radius. Therefore, it is assumed that the radius of gyration of the rim is equal to the mean radius and

$$I_r = m_r R^2 \tag{21.15}$$

where,

m_r = mass of the rim (kg)

R = mean radius of the rim (m)

In the design of a flywheel, many times it is required to decide the mean radius of the rim. From Eq. 21.15, it is seen that the mass of the flywheel can be considerably reduced by increasing the mean radius for a required amount of moment of inertia. The aim should be to use the largest possible radius because it reduces the weight. However, there are two limiting factors—speed and availability of space. In some cases, the diameter of the flywheel is governed by the amount of space, which is available for the flywheel. Flywheels are usually made of grey cast iron, for which the limiting mean rim velocity is 30 m/s. When the velocity exceeds this limit, there is a possibility of bursting due to centrifugal force, resulting in an explosion. Since,

$$v = \omega R$$

$$R < \frac{30}{\omega} \tag{21.16}$$

For high-speed applications, cast steel is used as flywheel material.

21.8 STRESSES IN RIMMED FLYWHEEL

A portion of a rimmed flywheel made of grey cast iron is shown in Fig. 21.9. The rotating rim is subjected to a uniformly distributed centrifugal force P_c, which acts in a radially outward direction. This induces a tensile force P in the cross-section of the rim acting in tangential direction and a bending moment M. Under the action of centrifugal force, the tendency of the rim is to fly outward, which is prevented due to the tensile force P_1 acting in each spoke. The tensile stress in the spoke is given by

$$\sigma_t = \frac{P_1}{A_1} \tag{a}$$

where,

P_1 = tensile force in each spoke (N)

A_1 = cross-sectional area of spoke (mm^2)

σ_t = tensile stress in spokes (N/mm^2)

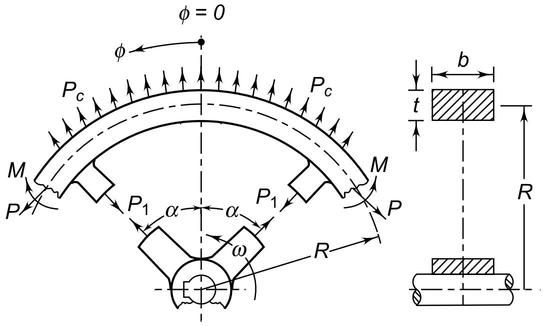


Fig. 21.9 *Forces and Moments in Rimmed Flywheel*

The rim is subjected to direct tensile stress due to P, and bending stresses due to M. The resultant stresses in the rim are given by,

$$\sigma_t = \frac{P}{A} \pm \frac{My}{I}$$

or $$\sigma_t = \frac{P}{bt} \pm \frac{6M}{bt^2} \tag{b}$$

where,

P = tensile force in rim (N)

M = bending moment on rim section (N-mm)

b = width of rim (mm)

t = thickness of rim (mm)

The expressions for P_1, P and M have been derived by S Timoshenko[1]. His analysis is based on the following assumptions:

(i) the thickness of the rim is small in comparison with the mean radius and stresses due to bending moment and tension alone are taken into account; and

(ii) the length of the spoke is considered to be equal to the mean radius, although, in practice, it is somewhat less than the mean radius.

The equations derived by S Timoshenko are as follows:

$$P_1 = \frac{2}{3}\left[\frac{(1000)\,mv^2}{C}\right]$$

$$C = 12(10^6)\left(\frac{R^2}{t^2}\right) f_2(\alpha) + f_1(\alpha) + \frac{A}{A_1}$$

$$f_1(\alpha) = \frac{1}{2\sin^2\alpha}\left[\frac{\sin 2\alpha}{4} + \frac{\alpha}{2}\right]$$

$$f_2(\alpha) = \frac{1}{2\sin^2\alpha}\left[\frac{\sin 2\alpha}{4} + \frac{\alpha}{2}\right] - \frac{1}{2\alpha}$$

$$(M)\text{ at }\phi = \frac{(1000)P_1 R}{2}\left[\frac{\cos\phi}{\sin\alpha} - \frac{1}{\alpha}\right]$$

$$(P)\text{ at }\phi = (1000)mv^2 - \frac{P_1\cos\phi}{2\sin\alpha}$$

where,

m = mass of the rim per mm of circumference (kg/mm)

v = velocity at the mean radius (m/s)

R = mean radius of rim (m)

t = thickness of the rim (mm)

b = width of the rim (mm)

A = cross-sectional area of rim (mm^2) (bt)

A_1 = cross-sectional area of spokes (mm^2)

2α = angle between two consecutive spokes (rad)

C = constant

Substituting these expressions in Eqs (a) and (b), the stresses in the spokes are given by,

$$\sigma_t = \frac{2}{3}\left[\frac{(1000)\,mv^2}{CA_1}\right] \tag{21.17}$$

The stresses in the rim are given by,

$$\sigma_t = \frac{(1000)\,mv^2}{bt}\left[1 - \frac{\cos\phi}{3C\sin\alpha} \pm \frac{2(1000)R}{Ct}\left(\frac{1}{\alpha} - \frac{\cos\phi}{\sin\alpha}\right)\right] \tag{21.18}$$

The numerical value of the constant C for different number of spokes is calculated as follows:

For 4 Spokes

$$2\alpha = \frac{\pi}{2}$$

$$f_1(\alpha) = 0.643 \qquad f_2(\alpha) = 0.00608$$

Therefore,

$$C = \left[\frac{72\,960\,R^2}{t^2} + 0.643 + \frac{A}{A_1}\right] \tag{21.19}$$

For 6 Spokes

$$2\alpha = \frac{\pi}{3}$$

$$f_1(\alpha) = 0.957 \qquad f_2(\alpha) = 0.00169$$

Therefore,

$$C = \left[\frac{20\,280\,R^2}{t^2} + 0.957 + \frac{A}{A_1}\right] \tag{21.20}$$

The mass m of the rim per millimetre of the circumference is given by,

$$m = bt\rho \tag{21.21}$$

where ρ is mass density in kg/mm^3.

Example 21.3 *The turning moment diagram of a multi-cylinder engine is drawn with a scale of (1 mm = 1°) on the abscissa and (1 mm = 250 N-m) on the ordinate. The intercepted areas between the torque developed by the engine and the mean resisting torque of the machine, taken in order from one end are – 350, + 800, – 600, + 900, – 550, + 450 and –650 mm². The engine is running at a mean speed of 750 rpm and the coefficient of speed fluctuations is limited to 0.02. A rimmed flywheel made of grey cast iron FG 200 (ρ = 7100 kg/m³) is provided. The spokes, hub and shaft are assumed to contribute 10% of the required moment of inertia. The rim has rectangular cross-section and the ratio of width to thickness is 1.5.*

Determine the dimensions of the rim.

Solution

Given $n = 750$ rpm $C_s = 0.02$ $b/t = 1.5$
$K = 0.9$ $\rho = 7100$ kg/m^3

Step I *Energy output from flywheel (U_o)*

The turning moment diagram is shown in Fig. 21.10.

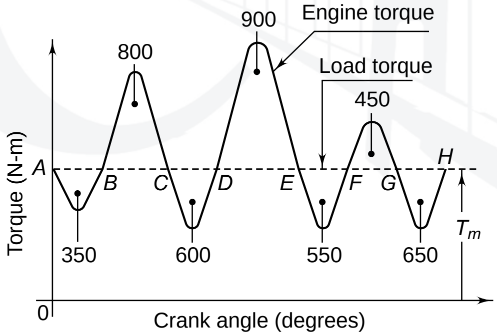


Fig. 21.10 *Turning Moment Diagram*

It is assumed that the energy stored in the flywheel is U at point A. Therefore,

Energy at $B = U - 350$
Energy at $C = U - 350 + 800 = U + 450$
Energy at $D = U + 450 - 600 = U - 150$
Energy at $E = U - 150 + 900 = U + 750$
Energy at $F = U + 750 - 550 = U + 200$
Energy at $G = U + 200 + 450 = U + 650$
Energy at $H = U + 650 - 650 = U$

The maximum and minimum energy occurs at points E and B. The angular velocity of the flywheel will be maximum at the point E and minimum at the point B.

$$U_o = U_E - U_B = (U + 750) - (U - 350)$$
$$= 1100 \text{ mm}^2$$
$$U_o = 1100(250)\left(\frac{\pi}{180}\right) \text{ N-m} \quad \text{or} \quad \text{J}$$
$$= 4799.66 \text{ N-m}$$

Step II *Dimensions of rim*

$$\omega = \frac{2\pi n}{60} = \frac{2\pi(750)}{60} = (25\pi) \text{ rad/s}$$

From Eq. 21.14,

$$I_r = \frac{U_o K}{\omega^2 C_s} = \frac{(4799.66)(0.9)}{(25\pi)^2(0.02)} = 35 \text{ kg-m}^2$$

From Eq. 21.16, the mean radius R of the rim is given by,

$$R < \frac{30}{\omega} \quad \text{or} \quad R < \frac{30}{(25\pi)} \quad \text{or} \quad R < 0.38 \text{ m}$$

Therefore, $R = 0.35$ m

The mass of the rim is given by (Eq. 21.15)

$$m_r = \frac{I_r}{R^2} = \frac{35}{(0.35)^2} = 285.71 \text{ kg}$$

The mass of the flywheel rim is given by,

$$m_r = 2\pi R\left(\frac{b}{1000}\right)\left(\frac{t}{1000}\right)\rho$$

or
$$285.71 = 2\pi(0.35)\left(\frac{1.5t}{1000}\right)\left(\frac{t}{1000}\right)(7100)$$

$t = 110.45$ mm $b = 1.5(110.45) = 165.67$ mm

Therefore,

$b = 170$ mm and $t = 110$ mm

Example 21.4 *It is required to design a rimmed flywheel for an engine consisting of three single acting cylinders with their cranks set equally at 120° to each other. The torque-crank angle diagram for each cylinder consists of a triangle with the following values:*

Crank angle (deg.)	*0*	*60*	*180*	*180 to 360*
Torque (N-m)	*0*	*300*	*0*	*0*

The engine runs at a mean speed of 240 rpm and the coefficient of speed fluctuations is limited to

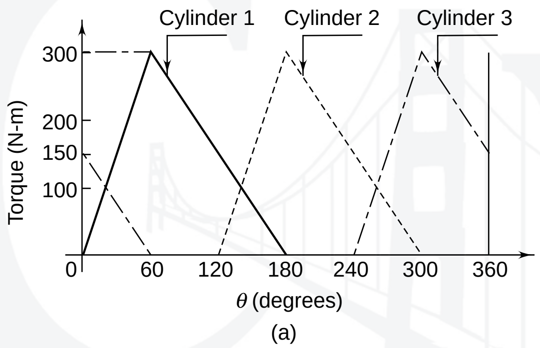
0.03. The resisting torque of the machine is constant throughout the work cycle. From considerations of space, the mean radius of the rim should not exceed 0.25 m. It can be assumed that the rim contributes 90% of the required moment of inertia. The flywheel is made of grey cast iron FG 220 (ρ = 7150 kg/m^3). The cross-section of the rim is square. Determine the dimensions of the cross-section of the rim.

Solution

Given $n = 240$ rpm $C_s = 0.03$ $b/t = 1$
$K = 0.9$ $\rho = 7150$ kg/m^3 $R \leq 0.25$ m

Step I *Energy output from flywheel (U_o)*

Figure. 21.11(a) shows the torque diagram of each cylinder.



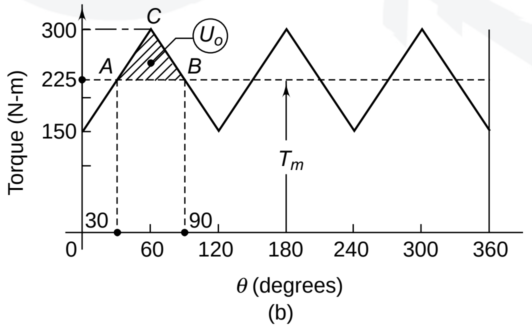


Fig. 21.11 *(a) Torque Developed by each Cylinder*
(b) Resultant Torque Diagram

Work done per revolution = area of three triangles

$$= 3\left[\frac{1}{2}(300)(\pi)\right]$$

$$= (450\pi) \text{ N-m or J}$$

$$\text{Mean torque } T_m = \frac{\text{work done per revolution}}{\text{crank angle per revolution}} = \frac{(450\pi)}{(2\pi)} = 225 \text{ N-m}$$

The resultant torque diagram is shown in Fig. 21.11(b). It is obtained by taking the sum of the torques developed by each cylinder at a given crank angle. It follows a linear variation from 150 N-m to 300 N-m. A and B are points of intersection of this diagram with the mean torque line. From the geometry of straight lines, it can be easily proved that the crank angles at points A and B are 30° and 90° respectively. The angular velocity of the flywheel is maximum at the point B and minimum at the point A. Therefore,

$$U_o = \text{area of } \Delta ABC = \frac{1}{2} AB(300 - 225) \qquad \text{(a)}$$

Also,

$$AB = \left(\frac{90 - 30}{180}\right)\pi = \left(\frac{\pi}{3}\right) \text{ rad} \qquad \text{(b)}$$

From (a) and (b),

$$U_o = \frac{1}{2}\left(\frac{\pi}{3}\right)(300 - 225) = 39.27 \text{ N-m}$$

Step II *Dimensions of cross-section of rim*

$$\omega = \frac{2\pi\, n}{60} = \frac{2\pi\,(240)}{60} = (8\,\pi) \text{ rad/s}$$

From Eq. 21.14,

$$I_r = \frac{U_o K}{\omega^2 C_s} = \frac{(39.27)(0.9)}{(8\pi)^2(0.03)} = 1.865 \text{ kg-m}^2$$

$$m_r = \frac{I_r}{R^2} = \frac{1.865}{(0.25)^2} = 29.84 \text{ kg}$$

The mass of the flywheel rim is given by,

$$m_r = 2\pi R\left(\frac{b}{1000}\right)\left(\frac{t}{1000}\right)\rho$$

or
$$29.84 = 2\pi(0.25)\left(\frac{t}{1000}\right)\left(\frac{t}{1000}\right)(7150)$$

$\therefore$ $b = t = 51.54$ mm or 55 mm

Example 21.5 *The torque developed by a three-crank engine is given by the following expression:*

T_i = *19 000 + 7000 sin (3θ) N-m*

The resisting torque of the machine is given by the following expression,

T_o = *19 000 + 3000 sin θ N-m*

where θ is the crank angle. The engine is running at a mean speed of 300 rpm and the coefficient of speed fluctuations is limited to 0.03. It can be assumed that the rim contributes 90% of the required moment of inertia. The rimmed flywheel is made of grey cast iron FG 200 (ρ = 7100 kg/m³). The cross-section of the rim is rectangle and the ratio of width to thickness is 1.5.

Determine the dimensions of the rim.

Solution

Given $n = 300$ rpm $C_s = 0.03$ $b/t = 1.5$
$K = 0.9$ $\rho = 7100$ kg/m^3

Step I *Energy output from flywheel (U_o)*

The torque diagram is shown in Fig. 21.12. The fluctuating terms (sin 3θ) and (sinθ) have a zero mean. Therefore,

$$T_m = 19\,000 \text{ N-m}$$

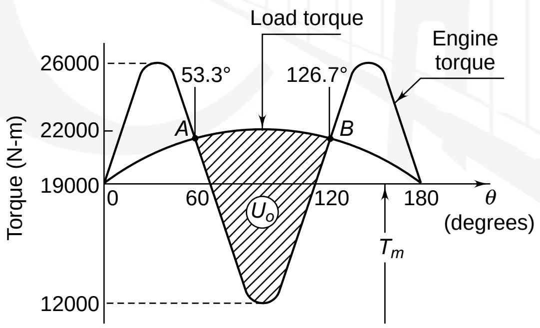


Fig. 21.12 *Torque Diagram*

A and *B* are points of intersection of the two curves. At these points,

$$19\,000 + 7000 \sin 3\theta = 19\,000 + 3000 \sin \theta$$

$$\frac{\sin(3\theta)}{\sin\theta} = \frac{3}{7} \quad \text{or} \quad \frac{3\sin\theta - 4\sin^3\theta}{\sin\theta} = \frac{3}{7}$$

$$\sin^2\theta = \frac{9}{14} \quad \text{or} \quad \theta = 53.3°$$

By symmetry, the value of θ at the point *B* will be [90 + (90 – 53.3)] or 126.7°. The areas between the two curves indicate the increase or decrease of energy. The angular velocity is maximum and minimum at points *A* and *B* respectively. The shaded area indicates U_o.

Therefore,

$$U_o = \int_{53.3°}^{126.7°} (3000 \sin\theta - 7000 \sin 3\theta)\, d\theta$$

$$= 3000[-\cos\theta]_{53.3°}^{126.7°} - 7000\left[\frac{-\cos\theta}{3}\right]_{53.3°}^{126.7°}$$

$$= 3585.75 + 4382.44$$
$$= 7968.19 \text{ N-m or J}$$

Step II *Dimensions of rim*

$$\omega = \frac{2\pi n}{60} = \frac{2\pi(300)}{60} = (10\pi) \text{ rad/s}$$

From Eq. 21.14,

$$I_r = \frac{U_o K}{\omega^2 C_s} = \frac{(7968.19)(0.9)}{(10\pi)^2(0.03)} = 242.2 \text{ kg-m}^2$$

From Eq. 21.16,

$$R < \frac{30}{\omega} \quad \text{or} \quad R < \frac{30}{(10\pi)} \quad \text{or} \quad R < 0.954 \text{ m}$$

Therefore, $R = 0.9$ m

The mass of the rim is given by Eq. (21.15)

$$m_r = \frac{I_r}{R^2} = \frac{242.2}{(0.9)^2} = 299 \text{ kg}$$

The mass of the flywheel rim is also given by,

$$m_r = 2\pi R\left(\frac{b}{1000}\right)\left(\frac{t}{1000}\right)\rho$$

or

$$299 = 2\pi(0.9)\left(\frac{1.5t}{1000}\right)\left(\frac{t}{1000}\right)(7100)$$

$t = 70.46$ mm $b = 1.5(70.46) = 105.69$ mm

Therefore,

$t = 75$ mm and $b = 110$ mm

Example 21.6 *The following data is given for a rimmed flywheel made of grey cast iron FG 200:*

mean radius of rim = 1.5 m
thickness of rim = 200 mm
width of rim = 300 mm
number of spokes = 6
cross-sectional area of each spoke = 10 000 mm²
speed of rotation = 720 rpm

Calculate:

(i) the tensile stress in rim at $\phi = 30°$ and $\phi = 0°$, and

(ii) the axial stress in each spoke.

The mass density of cast iron FG 200 is 7100 kg/m^3.

Solution

Given $n = 720$ rpm $R = 1.5$ m $t = 200$ mm $b = 300$ mm For spokes, $A_1 = 10\,000$ mm^2 number of spokes = 6 $\rho = 7100$ kg/m^3

Step I *Tensile stresses in rim*

From Eq. 21.21,

$$m = bt\rho = (300)(200)(7100 \times 10^{-9}) = 0.426 \text{ kg/mm}^2$$

$$A = bt = (300)(200) = 60\,000 \text{ mm}^2$$

From Eq. 21.20,

$$C = \left[\frac{20\,280\,R^2}{t^2} + 0.957 + \frac{A}{A_1}\right] = \left[\frac{20\,280\,(1.5)^2}{(200)^2} + 0.957 + \frac{60\,000}{10\,000}\right] = 8.098$$

$$v = \omega R = \left(\frac{2\pi n}{60}\right)R = \left(\frac{2\pi(720)}{60}\right)(1.5) = 113.097 \text{ m/s}$$

$$\left(\frac{1000mv^2}{bt}\right) = \frac{(1000)(0.426)(113.097)^2}{(300)(200)} = 90.816$$

$$2\alpha = \frac{360}{\text{Number of spokes}} = \frac{360}{6}$$

$$\therefore \quad \alpha = 30° = \frac{\pi}{6} \text{ rad}$$

From Eq. 21.18, the stresses in the rim are given by

At $\phi = 30°$,

$$\sigma_t = \frac{(1000)\,mv^2}{bt}\left[1 - \frac{\cos\phi}{3C\sin\alpha} \pm \frac{2(1000)R}{Ct}\left(\frac{1}{\alpha} - \frac{\cos\phi}{\sin\alpha}\right)\right]$$

$$= (90.816)\left[1 - \frac{\cos(30)}{3(8.098)\sin(30)} \pm \frac{2(1000)(1.5)}{(8.098)(200)}\left(\frac{6}{\pi} - \frac{\cos(30)}{\sin(30)}\right)\right]$$

$$= 114.25 \text{ N/mm}^2 \quad \text{(using the positive sign)}$$

At $\phi = 0°$,

$$\sigma_t = (90.816)\left[1 - \frac{1}{3(8.098)\sin(30)} \pm \frac{2(1000)(1.5)}{(8.098)(200)}\left(\frac{6}{\pi} - \frac{1}{\sin(30)}\right)\right]$$

$$= 98.5 \text{ N/mm}^2 \text{ (using the negative sign)}$$

Step II *Axial stress in spokes*

From Eq. 21.17, the stress in the spoke is given by,

$$\sigma_t = \frac{2}{3}\left[\frac{(1000)mv^2}{CA_1}\right] = \frac{2}{3}\left[\frac{(1000)(0.426)(113.097)^2}{(8.098)(10\,000)}\right] = 44.86 \text{ N/mm}^2$$

Example 21.7 *A rimmed flywheel made of grey cast iron FG 200 (ρ = 7100 kg/m^3) is required to keep down fluctuations in speed from 200 to 220 rpm. The cyclic fluctuations in energy is 30 000 N-m, while the maximum torque during the cycle is 75 000 N-m. The outside diameter of the flywheel should not exceed 2 m. It can be assumed that there are six spokes and the rim contributes 90% of the required moment of inertia. The cross-section of the rim is rectangular and the ratio of width to thickness is 2. Determine the dimensions of the rim. Assuming suitable cross-section for spokes, calculate the stresses in the rim and the spokes using Timoshenko's expressions.*

Solution

Given $n = 200$ to 220 rpm $b/t = 2$ $K = 0.9$ $\rho = 7100$ kg/m^3 $U_o = 30\,000$ N-m outer diameter < 2 m number of spokes = 6

Step I *Dimensions of rim*

The average speed of the flywheel is 210 rpm. Therefore,

$$\omega = \frac{2\pi n}{60} = \frac{2\pi(210)}{60} = 21.99 \text{ rad/s}$$

$$C_s = \frac{\omega_{max.} - \omega_{min.}}{\omega} = \frac{n_{max.} - n_{min.}}{n}$$

$$= \frac{220 - 200}{210} = 0.095$$

From Eq. 21.14,

$$I_r = \frac{U_o K}{\omega^2 C_s} = \frac{(30\,000)(0.9)}{(21.99)^2 (0.095)} = 587.75 \text{ kg-m}^2$$

Since the outer diameter of flywheel is limited to 2 m, the mean radius of the rim (R) is assumed as 0.9 m.

The mass of the rim is given by Eq. (21.15)

$$m_r = \frac{I_r}{R^2} = \frac{587.75}{(0.9)^2} = 725.61 \text{ kg}$$

The mass of the flywheel rim is also given by,

$$m_r = 2\pi R \left(\frac{b}{1000}\right)\left(\frac{t}{1000}\right)\rho$$

or $$725.61 = 2\pi(0.9)\left(\frac{2t}{1000}\right)\left(\frac{t}{1000}\right)(7100)$$

$t = 95.06$ mm or 100 mm $b = 2(100) = 200$ mm

The cross-section of the rim is 200×100 mm.

Step II *Stresses in rim*

It is assumed that the spokes have elliptical cross-section with 200 mm as the major axis and 100 mm as the minor axis. The cross-sectional area (A_1) of the spokes is given by,

$$A_1 = \pi ab$$

where a and b are semi-major and semi-minor axes respectively.

$$A_1 = \pi(100)(50) = 15\,707.96 \text{ mm}^2$$

The cross-sectional area (A) of the rim is given by,

$$A = (200)(100) = 20\,000 \text{ mm}^2$$

$$\frac{A}{A_1} = \frac{20\,000}{15\,707.96} = 1.27$$

From Eq. 21.20,

$$C = \left[\frac{20\,280\,R^2}{t^2} + 0.957 + \frac{A}{A_1}\right]$$

$$= \left[\frac{20\,280\,(0.9)^2}{(100)^2} + 0.957 + 1.27\right] = 3.87$$

$$v = \omega R = 21.99(0.9) = 19.79 \text{ m/s}$$

The mass of the rim per millimetre of the circumference is given by,

$$m = bt\rho = \left(\frac{200}{1000}\right)\left(\frac{100}{1000}\right)(7100)(10^{-3})$$

$$= 0.142 \text{ kg/mm}$$

$$\left(\frac{1000 mv^2}{bt}\right) = \frac{(1000)(0.142)(19.79)^2}{(200)(100)} = 2.78$$

There are six spokes and (2α) is the angle between two consecutive spokes.

$$(2\alpha) = 360/6 = 60° \quad \therefore \ \alpha = 30° = \left(\frac{\pi}{6}\right) \text{ rad.}$$

From Eq. 21.18, the stresses in the rim are given by

At $\phi = 30°$,

$$\sigma_t = \frac{(1000)mv^2}{bt}\left[1 - \frac{\cos\phi}{3C\sin\alpha} \pm \frac{2(1000)R}{Ct}\left(\frac{1}{\alpha} - \frac{\cos\phi}{\sin\alpha}\right)\right]$$

$$= (2.78)\left[1 - \frac{\cos(30)}{3(3.87)\sin(30)} \pm \frac{2(1000)(0.9)}{(3.87)(100)}\left(\frac{6}{\pi} - \frac{\cos(30)}{\sin(30)}\right)\right]$$

$= 4.66$ N/mm^2 (using the positive sign)

At $\phi = 0°$,

$$\sigma_t = (2.78)\left[1 - \frac{\cos(0)}{3(3.87)\sin(30)} \pm \frac{2(1000)(0.9)}{(3.87)(100)}\left(\frac{6}{\pi} - \frac{\cos(0)}{\sin(30)}\right)\right]$$

$= 1.14$ N/mm^2 (using the negative sign)

Step III *Stresses in spoke*

From Eq. 21.17, the stress in the spokes is given by,

$$\sigma_t = \frac{2}{3}\left[\frac{(1000)mv^2}{CA_1}\right] = \frac{2}{3}\left[\frac{(1000)(0.142)(19.79)^2}{(3.87)(15\,707.96)}\right]$$

$= 0.61$ N/mm^2

Example 21.8 *The layout of a punching press using slider-crank mechanism is illustrated in Fig. 21.13. It consists of a pinion P_1 mounted on a motor shaft and the driving gear G_1. The motor speed is 720 rpm. The pinion P_2 mounted on the intermediate shaft drives the gear G_2. The crank of the slider-crank mechanism is fixed to the gear G_2. The speed reduction of the gear train is such that 20 holes are punched per minute. The gear G_1, pinion P_2 and flywheel are mounted on the same intermediate shaft that rotates at 100 rpm. The following data is given:*

diameter of the hole to be punched = 25 mm
thickness of the plate = 10 mm
shear resistance of plate material = 300 MPa
mean diameter of rim = 1 m
coefficient of speed fluctuations = 0.1
efficiency of mechanism = 98%

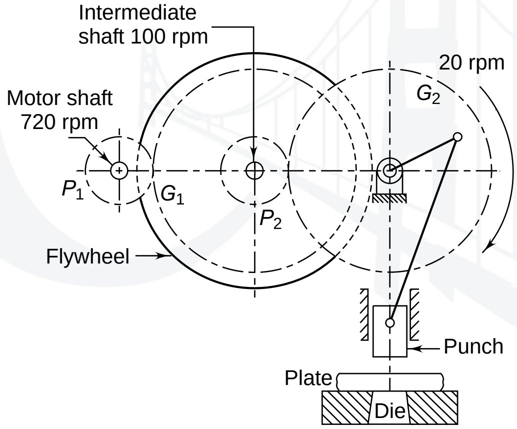


Fig. 21.13 *Mechanism of Punch Press*

The actual punching operation takes 25% of the time interval between two consecutive punching operations. The force–displacement curve during punching can be assumed to be triangular in shape. Assume that the rim contributes 90% of the required moment of inertia. The rimmed flywheel is made of grey cast iron FG 200 (ρ = 7100 kg/m^3). The cross-section of the rim is a rectangle with a width to thickness ratio of 1.5. Determine

(i) power of electric motor to drive the press; and
(ii) dimensions of the rim.

Solution

Given n = 100 rpm b/t = 1.5 C_s = 0.1
K = 0.9 ρ = 7100 kg/m^3 $2R$ = 1 m η = 0.98
For punched holes, d = 25 mm t = 10 mm
τ = 300 MPa

Step I *Power of electric motor to drive press*

τ = shear resistance = 300 MPa = 300 N/mm^2

The force P required to shear the hole is given by,

$$P = \pi d t \tau = \pi (25)(10)(300) = 235\,619.45 \text{ N}$$

The force–displacement diagram during the punching operation is shown in Fig. 21.14.

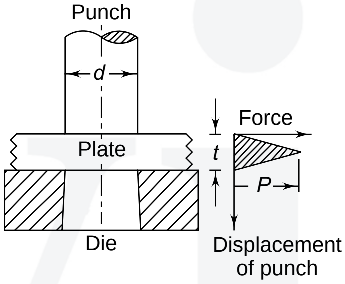


Fig. 21.14 *Force-displacement Diagram*

W = Work done during one punching operation
= Average force × Distance

$$\therefore \quad W = \frac{1}{2} Pt = \frac{1}{2}(235\,619.45)(10)$$

$$= 1\,178\,097.25 \text{ N-mm} = 1178.1 \text{ N-m}$$

Since 20 holes are punched per minute, the work cycle of one punching operation consists of (60/20) or 3 seconds. Therefore,

$$\text{Power output} = \frac{1178.1}{3} = 392.7 \text{ W}$$

$$\text{Power input} = \frac{\text{Power output}}{\eta} = \frac{392.7}{0.98}$$

$$= 400.7 \text{ W or } 0.4 \text{ kW} \quad \text{(i)}$$

Step II *Energy output from flywheel (U_o)*

The force–displacement diagram during one work cycle is shown in Fig. 21.15. The actual punching takes 25% of the time interval between two consecutive punching operations. Therefore,

Actual punching time = 0.25 (3) = 0.75 s

The work done during one punching operation (1178.1 N-m) is shown by the triangular area *ABCDE* in the figure. It should be supplied in 0.75 seconds. When the flywheel is used, the same amount of energy (1178.1 N-m) is supplied per cycle , which is shown by the rectangular area *OFGE*. During actual punching operation,

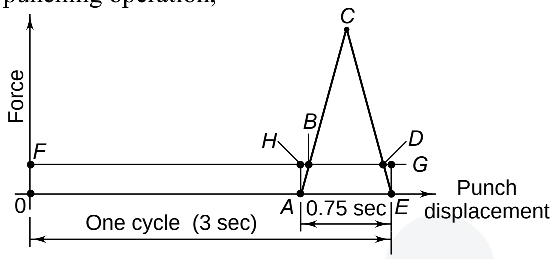


Fig. 21.15 *Force–displacement Diagram*

Energy required = area *ABCDE* = 1178.1 N-m

Energy supplied by motor = area *AHGE*

$$= \left(\frac{0.75}{3}\right)(1178.1) = 294.5 \text{ N-m}$$

∴ Energy supplied by flywheel = U_o

$$= 1178.1 - 294.5 = 883.6 \text{ N-m}$$

Step III *Dimensions of cross-section of rim*

$$\omega = \frac{2\pi n}{60} = \frac{2\pi(100)}{60} = 10.47 \text{ rad/s}$$

From Eq. 21.14,

$$I_r = \frac{U_o K}{\omega^2 C_s} = \frac{(883.6)(0.9)}{(10.47)^2(0.1)} = 72.54 \text{ kg-m}^2$$

The mass of the rim is given by (Eq. 21.15)

$$m_r = \frac{I_r}{R^2} = \frac{72.54}{(0.5)^2} = 290.16 \text{ kg}$$

The mass of the flywheel rim is also given by,

$$m_r = 2\pi R\left(\frac{b}{1000}\right)\left(\frac{t}{1000}\right)\rho$$

or $$290.16 = 2\pi(0.5)\left(\frac{1.5t}{1000}\right)\left(\frac{t}{1000}\right)(7100)$$

$$t = 93.13 \text{ mm or } 100 \text{ mm}$$

$$b = 1.5(100) = 150 \text{ mm}$$

Example 21.9 *Treating the rim of a rimmed flywheel as a free rotating ring and neglecting the effect of spokes, prove that the tensile stress in the rim cross-section is given by,*

$$\sigma_t = \rho v^2$$

where,

σ_t = tensile stress in rim cross-section (N/m²)

ρ = mass density (kg/m³)

v = velocity at mean radius (m/s)

Solution Consider the forces acting on the half rim as shown in Fig. 21.16. The rim is in equilibrium under the action of two forces—inertia force which is uniformly distributed in the radial direction and the tensile force *P* at the rim cross-section. At an angle θ, consider an elemental section shown by the

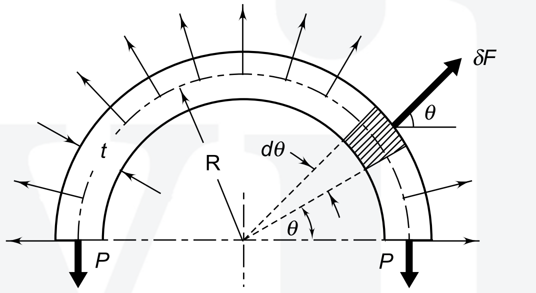


Fig. 21.16 *Rim as Free Rotating Body*

shaded area. The volume of this element is ($btR\,d\theta$) and the elemental mass is given by,

$$\delta_m = btRd\theta\rho$$

where the thickness *t*, width *b* and mean radius *R* are in metres. The elemental inertia force δ_F acting on this element is given by,

$$\delta_F = \text{mass} \times \text{acceleration}$$

$$\delta_F = (btRd\theta\rho)(R\omega^2)$$

Since, $$v = R\omega$$

$$\delta_F = bt\rho v^2 d\theta$$

The vertical component of this elemental force is ($\delta_F \sin\theta$). Considering equilibrium of vertical forces,

$$2P = \int_0^\pi bt\rho v^2 \sin\theta\, d\theta$$

$$= bt\rho v^2 \int_0^\pi \sin\theta\, d\theta = bt\rho v^2 [-\cos\theta]_0^\pi$$

$$= 2bt\rho v^2$$

∴ $$\sigma_t = \frac{P}{bt} = \rho v^2$$

Example 21.10 *A geared crank type press consists of a mechanism similar to one illustrated in Fig. 21.13. It absorbs 25 kN-m of work from flywheel during the last 30° of crank rotation. The crank rotates at 120 rpm. The flywheel rotates three times faster. The flywheel has rimmed type construction with four spokes and a hub. It is made of grey cast iron FG 200 (S_{ut} = 200 N/mm² and ρ = 7100 kg/m³). The outer diameter of the flywheel should be within 1200 mm. The flywheel is to keep down the fluctuations in speed within 12% of normal speed. Assume linear variation of the torque during 30° of crank rotation when punching takes place.*

(i) The rim contributes 90% of the required moment of inertia. Determine the dimensions of the cross-section of the rim, if it has a square cross-section.

(ii) Determine the stress in the rim treating it as a free rotating ring.

(iii) The flywheel is keyed to a shaft made of plain carbon steel 40C8 (S_{ut}= 650 N/mm² and S_{yt} = 380 N/mm²). Neglecting bending moment, determine the shaft diameter as per ASME code if

$$K_t = 2.0$$

(iv) Determine the hub diameter.

(v) The spokes have elliptical cross-section with major axis of the ellipse equal to twice of the minor axis. Each spoke transmits equal load from the mean radius of the rim to the surface of the hub. Assume that the spoke acts as a cantilever beam fixed at the hub surface and subjected to force at the mean radius of the rim. Determine the dimensions of the cross-section of the spokes based on simple bending theory if the factor of safety is 2.5.

Solution

Given For flywheel, outer diameter < 1200 mm

U_o = 25 kN-m n = 3(120) rpm b/t = 1

C_s = 0.12 K = 0.9 ρ = 7100 kg/m³

S_{ut} = 200 N/mm² For shaft, S_{ut} = 650 N/mm²

S_{yt} = 380 N/mm² K_t = 2.0 For spokes, $b = 2a$

(fs) = 2.5 Number of spokes = 4

Step I *Dimensions of cross-section of rim*

The outer diameter of the rim is to be within 1200 mm. Therefore, it is assumed that the mean radius of the rim is 0.5 m.

$$R = 0.5 \text{ m}$$

$$U_o = 25 \text{ kN-m} = 25000 \text{ N-m}$$

Flywheel speed = 3 (crank speed) = 3(120)
= 360 rpm

$$\omega = \frac{2\pi n}{60} = \frac{2\pi(360)}{60} = (12\pi) \text{ rad/s}$$

The speed fluctuations are limited to 12% of the normal speed. Therefore,

$$C_s = 0.12$$

From Eq. 21.14,

$$I_r = \frac{U_o K}{\omega^2 C_s} = \frac{(25000)(0.9)}{(12\pi)^2(0.12)} = 131.93 \text{ kg-m}^2$$

The mass of the rim is given by Eq. (21.15)

$$m_r = \frac{I_r}{R^2} = \frac{131.93}{(0.5)^2} = 527.71 \text{ kg}$$

The mass of the flywheel rim is also given by,

$$m_r = 2\pi R\left(\frac{b}{1000}\right)\left(\frac{t}{1000}\right)\rho$$

or $$527.71 = 2\pi(0.5)\left(\frac{t}{1000}\right)\left(\frac{t}{1000}\right)(7100)$$

$$t = b = 153.81 \text{ mm}$$

Therefore,

$$b = t = 155 \text{ mm} \qquad \text{(i)}$$

Check for outer diameter:

Outer diameter of rim = mean diameter + $(t/2)$ + $(t/2)$

= 1000 + (155/2) + (155/2)

= 1155 mm (less than 1200 mm)

Step II *Stresses in rim*

$$v = \omega R = (12\pi)(0.5) = 18.85 \text{ m/s}$$

$$\sigma_t = \rho v^2 = (7100)(18.85)^2 = (2.52 \times 10^6) \text{ N/m}^2$$

or σ_t = 2.52 N/mm² or MPa (ii)

Step III *Shaft diameter*

The linear variation of torque during 30° or $(\pi/6)$ radians of crank rotation is shown in Fig. 21.17. The punching takes place in this period. The area *OAB* indicates the work 25 kN-m, absorbed in punching.

Therefore,

$$W = \text{area } OAB \quad \text{or} \quad (25\,000) = \frac{1}{2}(T_{max.})\left(\frac{\pi}{6}\right)$$

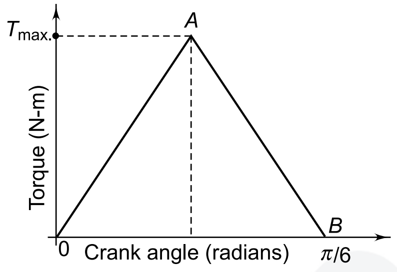


Fig. 21.17 *Turning Moment Diagram*

Therefore, $T_{max.} = M_t = 95\,492.96$ N-m

For shaft material (40C8),

$$0.30\, S_{yt} = 0.30(380) = 114 \text{ N/mm}^2$$
$$0.18\, S_{ut} = 0.18(650) = 117 \text{ N/mm}^2$$

The lower of the above two values is 114 N/mm^2. Since there is keyway, the permissible shear stress is given by,

$$\tau_{max.} = 0.75\,(114) = 85.5 \text{ N/mm}^2$$

Using ASME code and neglecting bending moment M_b,

$$d^3 = \frac{16}{\pi \tau_{max.}}\sqrt{(k_b M_b)^2 + (k_t M_t)^2}$$

$$= \frac{16}{\pi(85.5)}(k_t M_t)$$

$$= \frac{16}{\pi(85.5)}[(2)(95\,492.96 \times 10^3)]$$

$\therefore \quad d = 224.91$ or 225 mm (iii)

Step IV *Hub diameter*

The hub diameter is taken as twice of the shaft diameter. Therefore,

Hub diameter = 2(225) = 450 mm (iv)

Step V *Dimensions of cross-section of spokes*

It is assumed that each spoke takes a tangential force P at the mean radius of the rim as shown in Fig. 21.18.

Torque = PR (number of spokes)

$$95\,492.96 = P\,(0.5)(4) \quad \text{or} \quad P = 47\,746.48 \text{ N}$$

The bending moment diagram for the spoke is shown in Fig. 21.19. The spoke is assumed as a cantilever beam fixed at the hub surface and subjected to a tangential force at the mean radius. The maximum bending moment at the hub surface is given by,

$$M_b = P\,(R - 225) = 47\,746.48\,(500 - 225)$$
$$= (13.13 \times 10^6) \text{ N-mm}$$

$$\sigma_b = \frac{S_{ut}}{(fs)} = \frac{200}{2.5} = 80 \text{ N/mm}^2$$

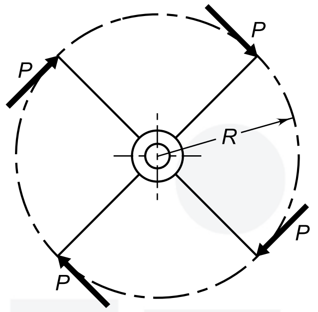


Fig. 21.18

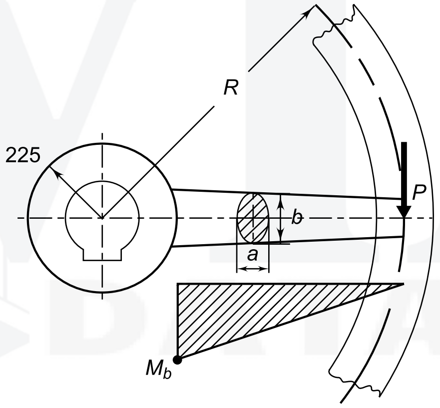


Fig. 21.19 *Spoke as Cantilever Beam*

For elliptical cross-section with a and b as minor and major axes respectively and ($b = 2a$),

$$I = \frac{\pi a b^3}{64} = \frac{\pi a (2a)^3}{64} = \frac{\pi a^4}{8}$$

$$y = \frac{b}{2} = a$$

Substituting the above values in the bending equation,

$$\sigma_b = \frac{M_b y}{I} \quad \text{or} \quad (80) = \frac{(13.13 \times 10^6)(a)}{\left(\frac{\pi a^4}{8}\right)}$$

$a = 74.77$ or 75 mm

$b = 2(75) = 150$ mm (v)

Short-Answer Questions

21.1 What is the function of flywheel?

21.2 What are the applications of flywheel?

21.3 Why flywheels are used in presses?

21.4 What is cooling stress?

21.5 What is the advantage of a split-type flywheel over solid one-piece flywheel?

21.6 What is the coefficient of speed fluctuation?

21.7 What is the coefficient of steadiness?

21.8 What is the coefficient of fluctuation of energy?

Problems for Practice

21.1 A rimmed flywheel made of grey cast iron (mass density = 7100 kg/m^3) is used on a punching press running at a mean speed of 200 rpm. The punching operation consists of one-quarter revolution during which the flywheel is required to supply 3000 N-m of energy. The coefficient of speed fluctuations is limited to 0.2. The rim, which contributes 90% of the required moment of inertia, has a mean radius of 0.5 m due to space limitations. The cross-section of the rim is square. Determine its dimensions.

[*75 × 75 mm*]

21.2 The turning moment diagram of a multi-cylinder engine is drawn with a scale of (1 mm = 2°) on abscissa and (1 mm = 1250 N-m) on ordinate. The intercepted areas between the torque developed by the engine and the mean resisting torque of the machine taken in order from one end are – 30, + 400, – 270, + 330, – 310, + 230, – 380, + 270, and – 240 mm^2. The engine is running at a mean speed of 240 rpm and the coefficient of speed fluctuations is limited to 0.02. A rimmed flywheel made of grey cast iron FG 200 (mass density = 7100 kg/m^3) is provided. The rim contributes 90% of the required moment of inertia. The rim has rectangular cross-section with width to thickness ratio of 1.5. Determine the dimensions of the rim.

[*R = 1.15 m and 120 × 180 mm*]

21.3 The following data is given for a rimmed flywheel made of grey cast iron:

Mean radius of the rim	= 1 m
Thickness of the rim	= 100 mm
Width of the rim	= 200 mm
Number of spokes	= 4
Cross-sectional area of each spoke	= 6500 mm^2
Speed of rotation	= 720 rpm
Mass density of flywheel	= 7200 kg/m^3

Calculate

(i) the maximum tensile stress in the rim; and

(ii) the axial stress in each spoke.

[*(i) 60 N/mm^2 at $\phi = 45°$ (ii) 7.62 N/mm^2*]

Cylinders and Pressure Vessels

Chapter 22

22.1 THIN CYLINDERS

Cylindrical pressure vessels are divided into two groups—*thin* and *thick* cylinders. *A cylinder is considered thin when the ratio of its inner diameter to the wall thickness is more than 15.* Boiler shells, pipes, tubes, and storage tanks are treated as thin cylinders. As shown in Fig. 22.1, there are two principal stresses in thin cylinder—the circumferential or tangential stress (σ_t) and longitudinal stress (σ_1). It is assumed that the stresses are uniformly distributed over the wall thickness. Considering equilibrium of forces acting on the half portion of cylinder of unit length [Fig. 22.1(a)],

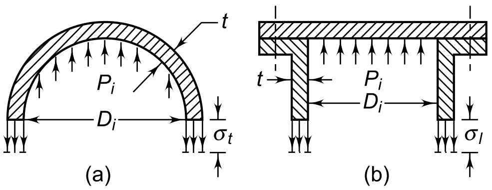


Fig. 22.1 *Stresses in Thin Cylinder*

$$D_i P_i = 2\sigma_t t$$

$$\therefore \qquad \sigma_t = \frac{P_i D_i}{2t} \qquad (22.1)$$

where,

P_i = internal pressure (N/mm^2)
D_i = internal diameter of cylinder (mm)
t = cylinder wall thickness (mm)

Considering equilibrium of forces in the longitudinal direction [Fig. 22.1(b)],

$$P_i\left(\frac{\pi}{4}D_i^2\right) = \sigma_1(\pi D_i t)$$

$$\therefore \qquad \sigma_1 = \frac{P_i D_i}{4t} \qquad (22.2)$$

From Eqs 22.1 and 22.2, it is seen that the circumferential stress (σ_t) is twice the longitudinal stress (σ_1). Therefore, we have the following criteria:

(i) When the circumferential stress exceeds the yield strength, failure will occur lengthwise. Also, when the longitudinal stress exceeds the yield strength, failure will occur in the transverse section. It can be concluded that in case of thin cylinders subjected to internal pressure, the tendency to burst lengthwise is twice as great as at transverse section.

(ii) In case of thin cylinders subjected to internal pressure, the circumferential stress should be the criterion for determining the cylinder wall thickness.

Equation 22.1 is rewritten as,

$$t = \frac{P_i D_i}{2\sigma_t} \qquad (22.3)$$

where (σ_t) is the permissible tensile stress for cylinder material. In this analysis, it is assumed that there are no longitudinal or circumferential joints in the cylinder.

Example 22.1 *A seamless cylinder with a storage capacity of 0.025 m³ is subjected to an internal pressure of 20 MPa. The length of the cylinder is twice its internal diameter. The cylinder is made of plain carbon steel 20C8 (S_{ut} = 390 N/mm²) and the factor of safety is 2.5. Determine the dimensions of the cylinder.*

Solution

Given $V = 0.025\ \text{m}^3$ $P_i = 20$ MPa $L = 2D_i$
$S_{ut} = 390\ \text{N/mm}^2$ $(fs) = 2.5$

Step I *Diameter and length of cylinder*

$$V = \frac{\pi}{4} D_i^2 L \quad \text{and} \quad L = 2D_i$$

$$V = \frac{\pi}{4} D_i^2 (2D_i) = \frac{\pi}{2} D_i^3$$

Therefore,

$$D_i = \sqrt[3]{\frac{2V}{\pi}} = \sqrt[3]{\frac{2(0.025)}{\pi}} = 0.25\ \text{m or 250 mm} \qquad \text{(i)}$$

$$L = 2D_i = 2(250) = 500\ \text{mm} \qquad \text{(ii)}$$

Step II *Thickness of cylinder*

$$\sigma_t = \frac{S_{ut}}{(fs)} = \frac{390}{2.5} = 156\ \text{N/mm}^2$$

$$t = \frac{P_i D_i}{2\sigma_t} = \frac{(20)(250)}{2(156)} = 16\ \text{mm} \qquad \text{(iii)}$$

22.2 THIN SPHERICAL VESSELS

A spherical pressure vessel with a thin wall, cut into two halves, is shown in Fig. 22.2. Considering equilibrium of forces for each half,

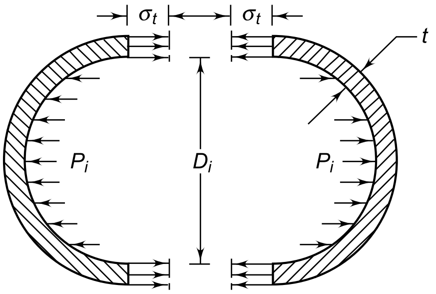


Fig. 22.2 *Stresses in Spherical Shell*

$$P_i\left(\frac{\pi}{4} D_i^2\right) = \sigma_t (\pi D_i t)$$

$$\therefore \qquad t = \frac{P_i D_i}{4\sigma_t} \qquad (22.4)$$

where,

P_i = internal pressure (N/mm²)
D_i = inner diameter of spherical shell (mm)
t = thickness of wall (mm)
(σ_t) = permissible tensile stress (N/mm²)

The volume V of the shell is given by,

$$V = \frac{\pi}{6} D_i^3$$

or

$$D_i = \left(\frac{6V}{\pi}\right)^{1/3} \qquad (22.5)$$

Equation (22.4) is based on the assumption that there are no welded joints in the shell.

Example 22.2 *An air receiver consisting of a cylinder closed by hemispherical ends is shown in Fig. 22.3. It has a storage capacity of 0.25 m³ and an operating internal pressure of 5 MPa. It is made of plain carbon steel 10C4 (S_{ut} = 340 N/mm²) and the factor of safety is 4. Neglecting the effect of welded joints, determine the dimensions of the receiver.*

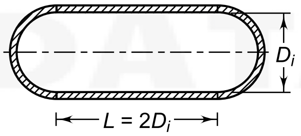


Fig. 22.3

Solution

Given $V = 0.25\ \text{m}^3$ $P_i = 5$ MPa $L = 2D_i$
$S_{ut} = 340\ \text{N/mm}^2$ $(fs) = 4$

Step I *Diameter and length of cylindrical portion*

$$V = \frac{\pi}{4} D_i^2 L + \frac{\pi}{6} D_i^3 \quad \text{and} \quad L = 2D_i$$

$$V = \frac{\pi}{4} D_i^2 (2D_i) + \frac{\pi}{6} D_i^3 = \pi D_i^3 \left[\frac{1}{2} + \frac{1}{6}\right] = \frac{2}{3}\pi D_i^3$$

Therefore,

$$D_i = \sqrt[3]{\frac{3V}{2\pi}} = \sqrt[3]{\frac{3(0.25)}{2\pi}} = 0.492 \text{ m or } 500 \text{ mm} \quad \text{(i)}$$

$$L = 2D_i = 2(500) = 1000 \text{ mm} \quad \text{(ii)}$$

Step II *Thickness of cylindrical portion*

$$\sigma_t = \frac{S_{ut}}{(fs)} = \frac{340}{4} = 85 \text{ N/mm}^2$$

The thickness of the cylinder wall is given by,

$$t = \frac{P_i D_i}{2\sigma_t} = \frac{(5)(500)}{2(85)} = 14.7 \text{ mm or } 15 \text{ mm} \quad \text{(iii)}$$

Step III *Thickness of hemispherical ends*

The thickness of the hemispherical ends is given by,

$$t = \frac{P_i D_i}{4\sigma_t} = \frac{(5)(500)}{4(85)} = 7.35 \text{ mm or } 8 \text{ mm} \quad \text{(iv)}$$

22.3 THICK CYLINDERS—PRINCIPAL STRESSES

When the ratio of the inner diameter of the cylinder to the wall thickness is less than 15, the cylinder is called a 'thick-walled' cylinder or simply 'thick' cylinder. Hydraulic cylinders, high-pressure pipes and gun barrels are examples of thick cylinders. The difference between the analysis of stresses in thin and thick cylinders is as follows:

(i) In thin cylinders, it is assumed that the tangential stress (σ_t) is uniformly distributed over the cylinder wall thickness. In thick cylinders, the tangential stress (σ_t) has highest magnitude at the inner surface of the cylinder and gradually decreases towards the outer surface.

(ii) The radial stress (σ_r) is neglected in thin cylinders, while it is of significant magnitude in case of thick cylinders.

In this section, it is assumed that the cylinder is subjected to only internal pressure as shown in Fig. 22.4(a). Consider an elemental ring of radius r and radial thickness dr as shown in Fig. 22.4(b). (σ_r) and (σ_t) are radial and tangential stresses respectively. Considering the equilibrium of vertical forces,

$$2\sigma_t dr + 2(r + dr)(\sigma_r + d\sigma_r) = 2r\sigma_r$$

Neglecting the term ($dr \times d\sigma_r$), the above expression is written as,

$$\sigma_t + \sigma_r + r\frac{d}{dr}(\sigma_r) = 0 \quad \text{(a)}$$

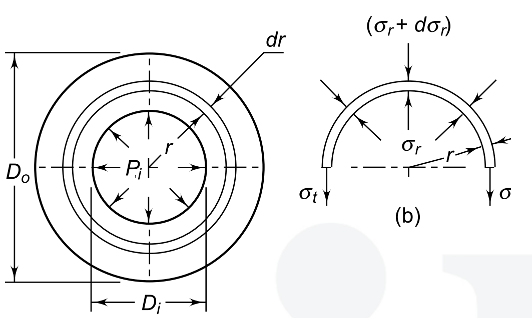


Fig. 22.4

It is further assumed that the axial stress (σ_1) is uniformly distributed over the cylinder wall thickness. Therefore, the strain (ε_1) is constant.

$$\varepsilon_1 = \frac{\sigma_1}{E} + \mu\frac{\sigma_r}{E} - \mu\frac{\sigma_t}{E}$$

or

$$\sigma_r - \sigma_t = \frac{E}{\mu}\left[\varepsilon_1 - \frac{\sigma_1}{E}\right] \quad \text{(b)}$$

where E is the modulus of elasticity and (μ) is Poisson's ratio. The right-hand side of Eq. (b) is constant and is denoted by ($-2C_1$).

$$\therefore \quad \sigma_r - \sigma_t = -2C_1 \quad \text{(c)}$$

Adding Eqs. (a) and (c),

$$2\sigma_r + r\frac{d}{dr}(\sigma_r) = -2C_1$$

Multiplying both sides of the above equation by (r),

$$2\sigma_r r + r^2\frac{d}{dr}(\sigma_r) = -2C_1 r \quad \text{(d)}$$

Since,

$$\frac{d}{dr}(r^2\sigma_r) = 2r\sigma_r + r^2\frac{d}{dr}(\sigma_r) \quad \text{(e)}$$

From (d) and (e),

$$\frac{d}{dr}(r^2\sigma_r) = -2C_1 r$$

Integrating with respect to r,

$$r^2\sigma_r = -C_1 r^2 + C_2$$

or $$\sigma_r = -C_1 + \frac{C_2}{r^2} \quad \text{(f)}$$

Substituting the above value in the Eq. (c),

$$\sigma_t = C_1 + \frac{C_2}{r^2} \quad \text{(g)}$$

The constants C_1 and C_2 are evaluated from the following two boundary conditions:

[Fig. 22.5(a)]

$$\sigma_r = P_i \qquad \text{when} \qquad r = \frac{D_i}{2}$$

$$\sigma_r = 0 \qquad \text{when} \qquad r = \frac{D_o}{2}$$

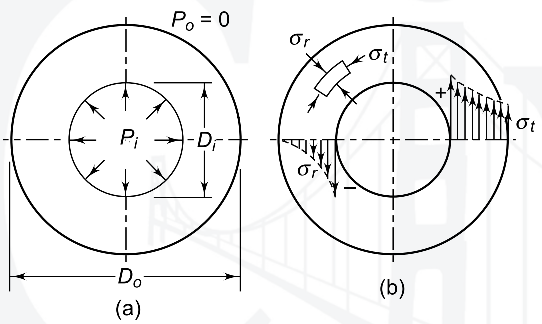


Fig. 22.5 *Variation of Principal Stresses (Cylinder with Internal Pressure)*

Substituting the boundary conditions,

$$C_1 = \frac{P_i D_i^2}{(D_o^2 - D_i^2)}$$

$$C_2 = \frac{P_i D_i^2 D_o^2}{4(D_o^2 - D_i^2)}$$

Substituting the constants in Eqs. (f) and (g),

$$\sigma_r = -\frac{P_i D_i^2}{(D_o^2 - D_i^2)}\left[\frac{D_o^2}{4r^2} - 1\right] \quad (22.6)$$

$$\sigma_t = +\frac{P_i D_i^2}{(D_o^2 - D_i^2)}\left[\frac{D_o^2}{4r^2} + 1\right] \quad (22.7)$$

The negative sign is introduced in the expression for (σ_r) since it denotes compressive stress.

At the inner surface of the cylinder,

$$r = \frac{D_i}{2}$$

and the stresses from Eqs 22.6 and 22.7 are given by,

$$\sigma_r = -P_i \quad (22.8)$$

$$\sigma_t = +\frac{P_i(D_o^2 + D_i^2)}{(D_o^2 - D_i^2)} \quad (22.9)$$

At the outer surface of the cylinder,

$$r = \frac{D_o}{2}$$

and the stresses are given by,

$$\sigma_r = 0 \quad (22.10)$$

$$\sigma_t = +\frac{2P_i D_i^2}{(D_o^2 - D_i^2)} \quad (22.11)$$

The variation of principal stresses (σ_r) and (σ_t) across the cylinder thickness is shown in Fig. 22.5(b).

The principal stress in axial direction (σ_1) is assumed to be uniform over the cylinder wall thickness. Considering equilibrium of forces in the axial direction,

$$\left(\frac{\pi}{4} D_i^2\right) P_i = \frac{\pi}{4}(D_o^2 - D_i^2)\sigma_1$$

$$\therefore \quad \sigma_1 = \frac{P_i D_i^2}{(D_o^2 - D_i^2)} \quad (22.12)$$

There are a number of equations for the design of thick cylinders. The choice of equation depends upon two parameters—cylinder material (whether brittle or ductile) and condition of cylinder ends (open or closed). In this chapter, we will discuss only important equations.

22.4 LAME'S EQUATION

When the material of the cylinder is brittle, such as cast iron or cast steel, Lame's equation is used to determine the wall thickness. It is based on the maximum principal stress theory of failure, where maximum principal stress is equated to permissible stress for the material. As discussed in the preceding section, the three principal stresses at the inner surface of the cylinder are as follows:

$$\sigma_r = -P_i \quad \text{(a)}$$

$$\sigma_t = +\frac{P_i(D_o^2 + D_i^2)}{(D_o^2 - D_i^2)} \quad \text{(b)}$$

$$\sigma_1 = +\frac{P_i D_i^2}{(D_o^2 - D_i^2)} \quad \text{(c)}$$

Therefore,

$$\sigma_t > \sigma_1 > \sigma_r$$

and (σ_t) is the criterion of design. From Eq. (b),

$$\frac{\sigma_t}{P_i} = \frac{(D_o^2 + D_i^2)}{(D_o^2 - D_i^2)}$$

$$\frac{\sigma_t + P_i}{\sigma_t - P_i} = \frac{(D_o^2 + D_i^2) + (D_o^2 - D_i^2)}{(D_o^2 + D_i^2) - (D_o^2 - D_i^2)}$$

$$\frac{\sigma_t + P_i}{\sigma_t - P_i} = \frac{D_o^2}{D_i^2}$$

or

$$\frac{D_o}{D_i} = \sqrt{\frac{\sigma_t + P_i}{\sigma_t - P_i}}$$

Substituting ($D_o = D_i + 2t$) in the above equation,

$$\frac{D_i + 2t}{D_i} = \sqrt{\frac{\sigma_t + P_i}{\sigma_t - P_i}}$$

$$1 + 2\left(\frac{t}{D_i}\right) = \sqrt{\frac{\sigma_t + P_i}{\sigma_t - P_i}}$$

$\therefore$

$$t = \frac{D_i}{2}\left[\sqrt{\frac{\sigma_t + P_i}{\sigma_t - P_i}} - 1\right]$$

(Lame's equation) (22.13)

where

$$\sigma_t = \frac{S_{ut}}{(fs)}$$

22.5 CLAVARINO'S AND BIRNIE'S EQUATIONS

When the material of the cylinder is ductile, such as mild steel or alloy steel, maximum strain theory of failure is used as a criterion to indicate failure. This theory is also called *St Venant's theory. According to this theory, the material begins to yield or begins to fail when the maximum strain at a point equals the yield point strain in a simple tension test*. The three principal stresses at the inner surface of the cylinder are as follows:

$$\sigma_r = -P_i \quad \text{(a)}$$

$$\sigma_t = +\frac{P_i(D_o^2 + D_i^2)}{(D_o^2 - D_i^2)} \quad \text{(b)}$$

$$\sigma_1 = +\frac{P_i D_i^2}{(D_o^2 - D_i^2)} \quad \text{(c)}$$

Also,

$$\varepsilon_t = \frac{1}{E}[\sigma_t - \mu(\sigma_r + \sigma_1)] \quad \text{(d)}$$

and

$$\varepsilon_t = \frac{\sigma}{E} = \frac{\left(\frac{S_{yt}}{(fs)}\right)}{E} = \frac{S_{yt}}{E(fs)} \quad \text{(e)}$$

From expressions (d) and (e),

$$\frac{1}{E}[\sigma_t - \mu(\sigma_r + \sigma_1)] = \frac{\sigma}{E}$$

or

$$\sigma = \sigma_t - \mu(\sigma_r + \sigma_1)$$

Substituting the values of principal stresses,

$$\frac{\sigma}{P_i} = \frac{(1+\mu)D_o^2 + (1-2\mu)D_i^2}{(D_o^2 - D_i^2)}$$

Rearranging the terms,

$$\frac{D_o}{D_i} = \sqrt{\frac{\sigma + (1-2\mu)P_i}{\sigma - (1+\mu)P_i}}$$

Substituting ($D_o = D_i + 2t$) in the above equation,

$$\frac{D_i + 2t}{D_i} = \sqrt{\frac{\sigma + (1-2\mu)P_i}{\sigma - (1+\mu)P_i}}$$

$$1 + 2\left(\frac{t}{D_i}\right) = \sqrt{\frac{\sigma + (1-2\mu)P_i}{\sigma - (1+\mu)P_i}}$$

Therefore,

$$t = \frac{D_i}{2}\left[\sqrt{\frac{\sigma + (1-2\mu)P_i}{\sigma - (1+\mu)P_i}} - 1\right]$$

(Clavarino's equation) (22.14)

where,

$$\sigma = \frac{S_{yt}}{(fs)}$$

Clavarino's equation is applicable to cylinders with closed ends and made of ductile materials. When the cylinder ends are open,

$$\sigma_1 = 0$$

and $\sigma = \sigma_t - \mu(\sigma_r + \sigma_1) = \sigma_t - \mu\sigma_r$

Substituting the values of principal stresses,

$$\frac{\sigma}{P_i} = \frac{(1+\mu)D_o^2 + (1-\mu)D_i^2}{(D_o^2 - D_i^2)}$$

Rearranging the terms,

$$\frac{D_o}{D_i} = \sqrt{\frac{\sigma + (1-\mu)P_i}{\sigma - (1+\mu)P_i}}$$

Substituting $(D_o = D_i + 2t)$ in the above equation,

$$\frac{D_i + 2t}{D_i} = \sqrt{\frac{\sigma + (1-\mu)P_i}{\sigma - (1+\mu)P_i}}$$

$$1 + 2\left(\frac{t}{D_i}\right) = \sqrt{\frac{\sigma + (1-\mu)P_i}{\sigma - (1+\mu)P_i}}$$

Therefore,

$$t = \frac{D_i}{2}\left[\sqrt{\frac{\sigma + (1-\mu)P_i}{\sigma - (1+\mu)P_i}} - 1\right]$$

(Birnie's equation) (22.15)

Birnie's equation is applicable to open cylinders made of ductile material.

In addition to the above mentioned equations, one more equation called Barlow's equation is sometimes used. It is used for high-pressure oil and gas pipes. Barlow's equation is given by,

$$t = \frac{P_i D_o}{2\sigma_t} \quad \text{(Barlow's equation)}$$

where D_o is the outer diameter of the cylinder. Barlow's equation is analogous to thin cylinder equation except that the outer diameter (D_o) is used instead of inner the diameter (D_i). Barlow's equation is approximate.

Example 22.3 *The piston rod of a hydraulic cylinder exerts an operating force of 10 kN. The friction due to piston packing and stuffing box is equivalent to 10% of the operating force. The pressure in the cylinder is 10 MPa. The cylinder is made of cast iron FG 200 and the factor of safety is 5. Determine the diameter and the thickness of the cylinder.*

Solution

Given $P_i = 10$ MPa $S_{ut} = 200$ N/mm^2 $(fs) = 5$ piston force = 10 kN effect of friction = 10% of piston force

Step I *Diameter of cylinder*

The total force P on the piston is given by,

$P = 10 + 10\%$ of $10 = 11$ kN $= 11\,000$ N

$$P = \left(\frac{\pi}{4}D_i^2\right)P_i \quad \text{or} \quad 11\,000 = \left(\frac{\pi}{4}D_i^2\right)(10)$$

$D_i = 37.4$ or 40 mm

Step II *Thickness of cylinder*

$$\sigma_t = \frac{S_{ut}}{(fs)} = \frac{200}{5} = 40 \text{ N/mm}^2$$

The cylinder material is brittle and Lame's equation is applicable.

Using Lame's equation,

$$t = \frac{D_i}{2}\left[\sqrt{\frac{\sigma_t + P_i}{\sigma_t - P_i}} - 1\right] = \frac{40}{2}\left[\sqrt{\frac{40+10}{40-10}} - 1\right]$$

$= 5.82$ or 6 mm

Example 22.4 *The inner diameter of a cylindrical tank for liquefied gas is 250 mm. The gas pressure is limited to 15 MPa. The tank is made of plain carbon steel 10C4 (S_{ut} = 340 N/mm^2 and μ = 0.27) and the factor of safety is 5. Calculate the cylinder wall thickness.*

Solution

Given $D_i = 250$ mm $P_i = 15$ MPa
$S_{ut} = 340$ N/mm^2 $(fs) = 5$ $\mu = 0.27$

Step I *Selection of equation*
The cylindrical tank is made of ductile material and the ends are closed. Therefore, Clavarino's equation is applicable.

Step II *Thickness of cylinder*

$$\sigma = \frac{S_{ut}}{(fs)} = \frac{340}{5} = 68 \text{ N/mm}^2$$

Using Clavarino's equation,

$$t = \frac{D_i}{2}\left[\sqrt{\frac{\sigma + (1-2\mu)P_i}{\sigma - (1+\mu)P_i}} - 1\right]$$

$$= \frac{250}{2}\left[\sqrt{\frac{68 + [1-2(0.27)](15)}{68 - (1+0.27)(15)}} - 1\right]$$

$$= 29.62 \text{ or } 30 \text{ mm}$$

Example 22.5 *A seamless steel pipe of 100 mm internal diameter is subjected to internal pressure of 12 MPa. It is made of steel (S_{yt} = 230 N/mm^2 and μ = 0.27) and the factor of safety is 2.5. Determine the thickness of the pipe.*

Solution

Given $D_i = 100$ mm $P_i = 12$ MPa
$S_{yt} = 230$ N/mm^2 $(fs) = 2.5$ $\mu = 0.27$

Step I *Selection of equation*
The pipe has open ends and it is made of ductile material. Therefore, Birnie's equation is applicable.

Step II *Thickness of pipe*

$$\sigma = \frac{S_{yt}}{(fs)} = \frac{230}{2.5} = 92 \text{ N/mm}^2$$

Using Birnie's equation,

$$t = \frac{D_i}{2}\left[\sqrt{\frac{\sigma + (1-\mu)P_i}{\sigma - (1+\mu)P_i}} - 1\right]$$

$$= \frac{100}{2}\left[\sqrt{\frac{92 + (1-0.27)(12)}{92 - (1+0.27)(12)}} - 1\right] = 7.29 \text{ or } 8 \text{ mm}$$

22.6 CYLINDERS WITH EXTERNAL PRESSURE

A thick cylinder subjected to external pressure P_o is shown in Fig. 22.6(a). As discussed in Sec. 22.3, the two principal stresses are

$$\sigma_t = C_1 + \frac{C_2}{r^2} \qquad \text{(a)}$$

$$\sigma_r = -C_1 + \frac{C_2}{r^2} \qquad \text{(b)}$$

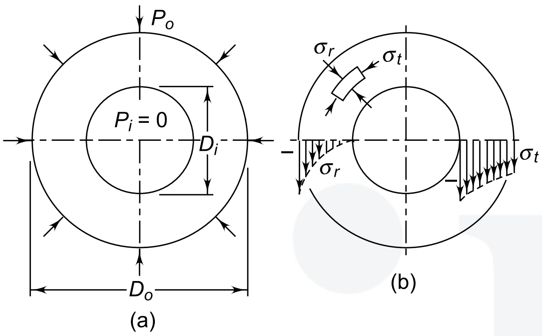


Fig. 22.6 *Variation of Principal Stresses (Cylinder with External Pressure)*

The constants C_1 and C_2 are evaluated from the following two boundary conditions:

$$\sigma_r = P_o \quad \text{when} \quad r = \frac{D_o}{2}$$

$$\sigma_r = 0 \quad \text{when} \quad r = \frac{D_i}{2}$$

Substituting the boundary conditions,

$$C_1 = -\frac{P_o D_o^2}{(D_o^2 - D_i^2)}$$

$$C_2 = -\frac{P_o D_i^2 D_o^2}{4(D_o^2 - D_i^2)}$$

Substituting the constants in Eqs. (a) and (b),

$$\sigma_r = -\frac{P_o D_o^2}{(D_o^2 - D_i^2)}\left[1 - \frac{D_i^2}{4r^2}\right] \qquad (22.16)$$

$$\sigma_t = -\frac{P_o D_o^2}{(D_o^2 - D_i^2)}\left[1 + \frac{D_i^2}{4r^2}\right] \qquad (22.17)$$

The negative sign is introduced in the expression of (σ_r) since it is compressive stress.

At the inner surface of the cylinder,

$$r = \frac{D_i}{2}$$

From Eqs (22.16) and (22.17), the stresses are as follows:

$$\sigma_r = 0 \tag{22.18}$$

$$\sigma_t = -\frac{2P_o D_o^2}{(D_o^2 - D_i^2)} \tag{22.19}$$

At the outer surface of the cylinder,

$$r = \frac{D_o}{2}$$

and the stresses are given by,

$$\sigma_r = -P_o \tag{22.20}$$

$$\sigma_t = -\frac{P_o(D_o^2 + D_i^2)}{(D_o^2 - D_i^2)} \tag{22.21}$$

The variation of principal stresses across the cylinder wall thickness is shown in Fig. 22.6(b).

22.7 AUTOFRETTAGE

Autofrettage is a process of pre-stressing the cylinder before using it in service. It is used in case of high-pressure cylinders and gun barrels. When the cylinder is subjected to internal pressure, the circumferential stress (σ_t) at the inner surface limits the pressure capacity of the cylinder. In the pre-stressing process, residual compressive stresses are developed at the inner surface. When the cylinder is loaded in service, the residual compressive stresses at the inner surface begin to decrease, become zero and finally become tensile as the pressure is gradually increased. There are three methods of pre-stressing the cylinder. They are as follows:

(i) A compound cylinder consists of two concentric cylinders with the outer cylinder shrunk onto the inner one. This induces compressive stresses in the inner cylinder. The compound cylinder is extensively used in practice.

(ii) The second method consists of overloading the cylinder before it is put into service. The overloading pressure is adjusted in such a way that a portion of the cylinder near the inner diameter is subjected to stresses in the plastic range, while the outer portion is still in the elastic range. When the pressure is released, the outer portion contracts exerting pressure on the inner portion which has undergone permanent deformation. This induces residual compressive stresses at the inner surface.

(iii) In the third method, a wire under tension is closely wound around the cylinder, which results in residual compressive stresses.

Autofrettage increases the pressure capacity of the cylinder. It has another advantage. The residual compressive stresses close the cracks within the cylinder resulting in increased endurance strength.

22.8 COMPOUND CYLINDER

A compound cylinder, consisting of a cylinder and a jacket is shown in Fig. 22.7(a). The inner diameter of the jacket is slightly smaller than the outer diameter of the cylinder. When the jacket is heated, it expands sufficiently to move over the cylinder. As the jacket cools, it tends to contract onto the inner cylinder, which induces residual compressive stresses. There is a shrinkage pressure P between the cylinder and the jacket. The pressure P tends to contract the cylinder and expand the jacket as shown in Fig. 22.7 (b) and (c).

Let,

δ_j = increase in inner diameter of jacket

δ_c = decrease in outer diameter of cylinder

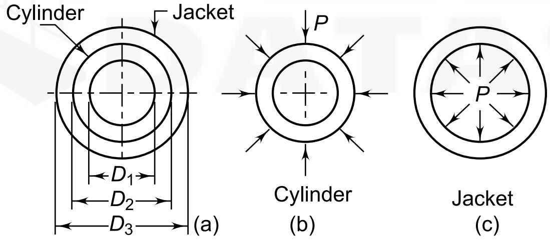


Fig. 22.7 *Compound Cylinder*

The tangential strain $(\varepsilon_t)_j$ for the jacket is given by,

$$(\varepsilon_t)_j = \frac{\text{change in circumference}}{\text{original circumference}}$$

$$= \frac{\pi(D_2 + \delta_j) - \pi D_2}{\pi D_2}$$

or $$(\varepsilon_t)_j = \frac{\delta_j}{D_2} \tag{a}$$

The tangential strain $(\varepsilon_t)_c$ for the cylinder is given by,

$$(\varepsilon_t)_c = \frac{\pi D_2 - \pi(D_2 - \delta_c)}{\pi D_2}$$

or $$(\varepsilon_t)_c = \frac{\delta_c}{D_2} \quad \text{(b)}$$

also, $$(\varepsilon_t)_j = \frac{1}{E}[\sigma_t - \mu\sigma_r] \quad \text{(c)}$$

From (a) and (c),

$$\frac{\delta_j}{D_2} = \frac{1}{E}[\sigma_t - \mu\sigma_r]$$

or $$\delta_j = \frac{D_2}{E}[\sigma_t - \mu\sigma_r] \quad \text{(d)}$$

where

$$\sigma_t = +\frac{P(D_3^2 + D_2^2)}{(D_3^2 - D_2^2)} \quad \text{(from Eq. 22.9)}$$

$$\sigma_r = -P \quad \text{(from Eq. 22.8)}$$

Substituting the above values in Eq. (d),

$$\delta_j = \frac{D_2 P}{E}\left[\frac{(D_3^2 + D_2^2)}{(D_3^2 - D_2^2)} + \mu\right] \quad \text{(e)}$$

Similarly,

$$(\varepsilon_t)_c = \frac{1}{E}[\sigma_t - \mu\sigma_r] \quad \text{(f)}$$

From (b) and (f),

$$\frac{\delta_c}{D_2} = \frac{1}{E}[\sigma_t - \mu\sigma_r]$$

or $$\delta_c = \frac{D_2}{E}[\sigma_t - \mu\,\sigma_r] \quad \text{(g)}$$

where

$$\sigma_t = -\frac{P(D_2^2 + D_1^2)}{(D_2^2 - D_1^2)} \quad \text{(Eq. 22.21)}$$

$$\sigma_r = -P \quad \text{(Eq. 22.20)}$$

Substituting the above values in Eq. (g),

$$\delta_c = -\frac{D_2 P}{E}\left[\frac{(D_2^2 + D_1^2)}{(D_2^2 - D_1^2)} - \mu\right] \quad \text{(h)}$$

The negative sign indicates contraction. Neglecting the positive and negative signs and considering only magnitudes, the total deformation (δ) is given by,

$$\delta = \delta_j + \delta_c$$

or

$$\delta = \frac{D_2 P}{E}\left[\frac{(D_3^2 + D_2^2)}{(D_3^2 - D_2^2)} + \mu\right] + \frac{D_2 P}{E}\left[\frac{(D_2^2 + D_1^2)}{(D_2^2 - D_1^2)} - \mu\right]$$

$$\therefore \quad \delta = \frac{PD_2}{E}\left[\frac{2D_2^2(D_3^2 - D_1^2)}{(D_3^2 - D_2^2)(D_2^2 - D_1^2)}\right] \quad (22.22)$$

The shrinkage pressure P can be evaluated from the above equation for a given amount of interference (δ). The resultant stresses in a compound cylinder are found by superimposing the two stresses—stresses due to shrink fit and those due to internal pressure.

Example 22.6 *A high-pressure cylinder consists of a steel tube with inner and outer diameters of 20 and 40 mm respectively. It is jacketed by an outer steel tube, having an outer diameter of 60 mm. The tubes are assembled by a shrinking process in such a way that maximum principal stress induced in any tube is limited to 100 N/mm². Calculate the shrinkage pressure and original dimensions of the tubes (E = 207 kN/mm²).*

Solution

Given $D_1 = 20$ mm $D_2 = 40$ mm $D_3 = 60$ mm $\sigma_{max.} = 100$ N/mm² $E = 207$ kN/mm²

Step I *Shrinkage pressure*

The maximum principal stress is the tangential stress at the inner surface of jacket. From Eq. 22.9,

$$\sigma_t = \frac{P(D_3^2 + D_2^2)}{(D_3^2 - D_2^2)} \text{ or } 100 = \frac{P(60^2 + 40^2)}{(60^2 - 40^2)}$$

$$\therefore \quad P = 38.46 \text{ N/mm}^2$$

Step II *Original dimensions of the tubes*

From Eq. 22.22,

$$\delta = \frac{PD_2}{E}\left[\frac{2D_2^2(D_3^2 - D_1^2)}{(D_3^2 - D_2^2)(D_2^2 - D_1^2)}\right]$$

or $$\delta = \frac{(38.46)(40)}{(207 \times 10^3)}\left[\frac{2(40)^2(60^2 - 20^2)}{(60^2 - 40^2)(40^2 - 20^2)}\right]$$

$$= 0.0317 \text{ mm}$$

The dimensions of the tubes are as follows:

Outer diameter of inner tube = 40 mm

Inner diameter of jacket = 40 – 0.0317

= 39.9683 mm

Example 22.7 *Assume the data of Example 22.6 and plot the distribution of stresses due to shrink fit. In service, the cylinder is further subjected to an internal pressure of 300 MPa. Plot the resultant stress distribution.*

Solution

Given $D_1 = 20$ mm $D_2 = 40$ mm $D_3 = 60$ mm

$\sigma_{max.} = 100$ N/mm^2 $E = 207$ kN/mm^2

$P_i = 300$ MPa

Step I *Stresses due to shrink fit*

(a) Jacket The jacket is subjected to an internal pressure of 38.46 N/mm^2 due to shrink fit. From Eqs (22.6) and (22.7),

$$\sigma_r = -\frac{PD_2^2}{(D_3^2 - D_2^2)}\left[\frac{D_3^2}{4r^2} - 1\right]$$

$$= -\frac{(38.46)(40)^2}{(60^2 - 40^2)}\left[\frac{60^2}{4r^2} - 1\right]$$

$$= -30.77\left[\left(\frac{30}{r}\right)^2 - 1\right]$$

and $$\sigma_t = +\frac{PD_2^2}{(D_3^2 - D_2^2)}\left[\frac{D_3^2}{4r^2} + 1\right]$$

$$= +30.77\left[\left(\frac{30}{r}\right)^2 + 1\right]$$

The stresses in the jacket are tabulated as follows:

r	20	22	24	26	28	30
σ_r	–38	–26	–17	–10	–5	0
σ_t	100	88	79	72	66	62

(b) Inner tube The inner tube is subjected to an external pressure of 38.46 N/mm^2 due to shrink fit. From Eqs (22.16) and (22.17),

$$\sigma_r = -\frac{PD_2^2}{(D_2^2 - D_1^2)}\left[1 - \frac{D_1^2}{4r^2}\right]$$

$$= -\frac{(38.46)(40)^2}{(40^2 - 20^2)}\left[1 - \frac{20^2}{4r^2}\right]$$

$$= -51.28\left[1 - \left(\frac{10}{r}\right)^2\right]$$

and $$\sigma_t = -\frac{PD_2^2}{(D_2^2 - D_1^2)}\left[1 + \frac{D_1^2}{4r^2}\right]$$

$$= -51.28\left[1 + \left(\frac{10}{r}\right)^2\right]$$

The stresses in the tube are tabulated as follows:

r	10	12	14	16	18	20
σ_r	0	–16	–25	–31	–35	–38
σ_t	–103	–87	–77	–71	–67	–64

Step II *Stresses due to internal pressure*

When the compound cylinder is subjected to an internal pressure of 300 MPa in service, the stresses are calculated from Eqs (22.6) and (22.7).

$$\sigma_r = -\frac{P_i D_1^2}{(D_3^2 - D_1^2)}\left[\frac{D_3^2}{4r^2} - 1\right]$$

$$= -\frac{(300)(20)^2}{(60^2 - 20^2)}\left[\frac{60^2}{4r^2} - 1\right]$$

$$= -37.5\left[\left(\frac{30}{r}\right)^2 - 1\right]$$

and $$\sigma_t = +\frac{P_i D_1^2}{(D_3^2 - D_1^2)}\left[\frac{D_3^2}{4r^2} + 1\right]$$

$$= +37.5\left[\left(\frac{30}{r}\right)^2 + 1\right]$$

The stresses in the compound cylinder are tabulated as follows:

r	10	12	14	16	18	20	22	24	26	28	30
σ_r	–300	–197	–135	–94	–67	–47	–32	–21	–12	–6	0
σ_t	375	272	210	169	142	122	107	96	87	81	75

Step III *Resultant stresses*

The resultant stresses in the compound cylinder are obtained by superimposition of stresses due to shrink fit and those due to internal pressure. They are tabulated as follows:

	Inner tube						Jacket					
r	10	12	14	16	18	20	20	22	24	26	28	30
σ_r	–300	–213	–160	–125	–102	–85	–85	–58	–38	–22	–11	0
σ_t	272	185	133	98	75	58	222	195	175	159	147	137

Figure 22.8 shows the variation of these stresses.

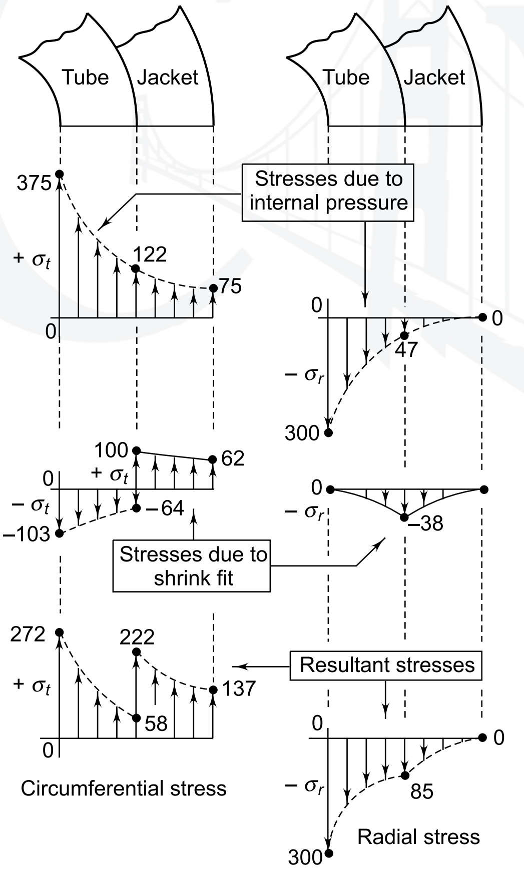


Fig. 22.8 *Stress Distribution in Compound Cylinder*

Example 22.8 *A tube, with 50 mm and 75 mm as inner and outer diameters respectively, is reinforced by shrinking a jacket with an outer diameter of 100 mm. The compound tube is to withstand an internal pressure of 35 MPa. The shrinkage allowance is such that the maximum tangential stress in each tube has same magnitude. Calculate*

(i) the shrinkage pressure; and

(ii) the original dimensions of tubes.

Show the distribution of tangential stresses. Assume $E = 207$ kN/mm^2.

Solution

Given $D_1 = 50$ mm $D_2 = 75$ mm $D_3 = 100$ mm $E = 207$ kN/mm^2 $P_i = 35$ MPa

The resultant stresses in the tubes are obtained by superimposing the stresses due to internal pressure and those due to shrinkage pressure.

Step I *Stresses due to internal pressure (Eq. 22.7)*

$$\sigma_t = +\frac{P_i D_1^2}{(D_3^2 - D_1^2)}\left[\frac{D_3^2}{4r^2} + 1\right]$$

$$= +\frac{(35)(50)^2}{(100^2 - 50^2)}\left[\frac{100^2}{4r^2} + 1\right]$$

$$= +11.67\left[\left(\frac{50}{r}\right)^2 + 1\right]$$

Substituting the values of (r), the stresses are as follows:

r	25	37.5	50
σ_t	58.35	32.42	23.34

Step II *Stresses due to shrinkage pressure*

Jacket (Eq. 22.7)

$$\sigma_t = +\frac{PD_2^2}{(D_3^2 - D_2^2)}\left[\frac{D_3^2}{4r^2} + 1\right]$$

$$= +\frac{P(75)^2}{(100^2 - 75^2)}\left[\frac{100^2}{4r^2} + 1\right]$$

$$= +1.286P\left[\left(\frac{50}{r}\right)^2 + 1\right]$$

The stresses are as follows:

r	37.5	50
σ_t	(3.57P)	(2.57P)

Inner tube (Eq. 22.17)

$$\sigma_t = -\frac{PD_2^2}{(D_2^2 - D_1^2)}\left[1 + \frac{D_1^2}{4r^2}\right]$$

$$= -\frac{P(75)^2}{(75^2 - 50^2)}\left[1 + \frac{50^2}{4r^2}\right] = -1.8P\left[1 + \left(\frac{25}{r}\right)^2\right]$$

The stresses are as follows:

r	25	37.5
σ_t	(–3.6 P)	(–2.6 P)

Step III *Shrinkage pressure*

Equating stresses at the inner surfaces of tube and jacket,

$$58.35 - 3.6\,P = 32.42 + 3.57\,P$$

or $$P = 3.62 \text{ MPa}$$

Step IV *Distribution of stresses*

The stresses are tabulated as follows:

	Inner tube		*Jacket*	
	r = 25	r = 37.5	r = 37.5	r = 50
Stresses due to P_i	58.35	32.42	32.42	23.34
Stresses due to P	(–3.6 P)	(–2.6 P)	(3.57 P)	(2.57 P)
(P = 3.62 MPa)	–13.03	–9.41	12.92	9.30
Resultant stresses	45.32	23.00	45.34	32.64

Figure. 22.9 shows the distribution of stresses.

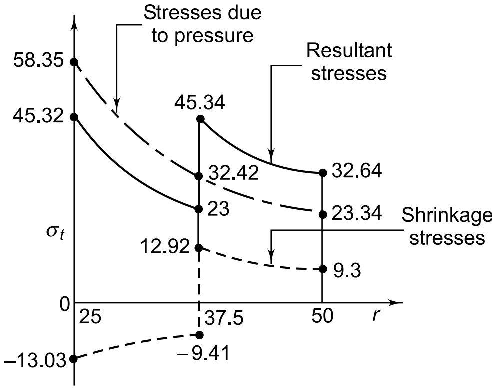


Fig. 22.9 *Distribution of Tangential Stresses*

Step V *Original dimensions of tubes*

From Eq. 22.22,

$$\delta = \frac{PD_2}{E}\left[\frac{2D_2^2(D_3^2 - D_1^2)}{(D_3^2 - D_2^2)(D_2^2 - D_1^2)}\right]$$

$$= \frac{(3.62)(75)}{(207\times 10^3)}\left[\frac{2(75)^2(100^2 - 50^2)}{(100^2 - 75^2)(75^2 - 50^2)}\right]$$

$$= 0.0081 \text{ mm}$$

The inner diameter of the jacket should be (75 – 0.0081) or 74.9919 mm.

22.9 GASKETS

A gasket is a device used to create and maintain a barrier against the transfer of fluid across the mating surfaces of a mechanical assembly. It is used in static joints, such as cylinder block and cylinder head. There are two types of gaskets—metallic and non-metallic. Metallic gaskets consist of sheets of lead, copper or aluminium. Non-metallic gaskets are made of asbestos, cork, rubber or plastics. Metallic gaskets are used for high-temperature and high-pressure applications. They can have corrugated construction or they can be made in the form of plain sheets. The limiting temperatures of metallic gaskets are as follows.

Lead	90°C
Copper/brass	250°C
Aluminium	400°C

The metallic gasket takes a permanent set when compressed in assembly and there is no recovery to compensate for separation of contact faces. They are also susceptible to corrosion and chemical atmosphere. Their performance also depends upon surface finish of the contacting surfaces.

Asbestos gaskets have excellent resistance to crushing loads and cutting action due to sharp edges of the flanges. Dimensional stability is another advantage. They are used in cylinder heads, water and steam-pipe fittings and manifold connections. Vulcanized compounds of rubber and cork are employed as gaskets in steam lines, combustion chambers and chemical environment. They are used for applications involving irregular surfaces. They are of low cost, but are affected by fungus and alkalis. Rubber compounds have excellent impermeability and ability to flow into joint imperfections when compressed. Asbestos gaskets can be used up to 250°C, while other non-metallic gaskets have a limiting temperature of 70°C. Different shapes of gasket for cylinder head are illustrated in Fig. 22.10.

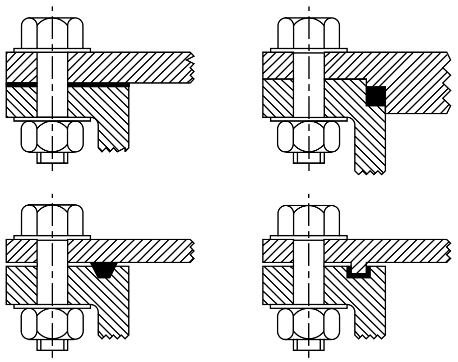


Fig. 22.10 *Shapes of Gasket*

22.10 GASKETED JOINT

A bolted assembly of cylinder, cylinder head and gasket is shown in Fig. 22.11. Initially, the bolt is tightened by means of a spanner to induce a preload P_1. The stiffness or spring constant k of a machine element is the load required to produce unit deflection. It is given by the ratio of the load to the deflection produced by that load. When machine member is loaded in tension or compression,

$$\delta = \frac{Pl}{AE}$$

or

$$k = \frac{P}{\delta} = \frac{AE}{l}$$

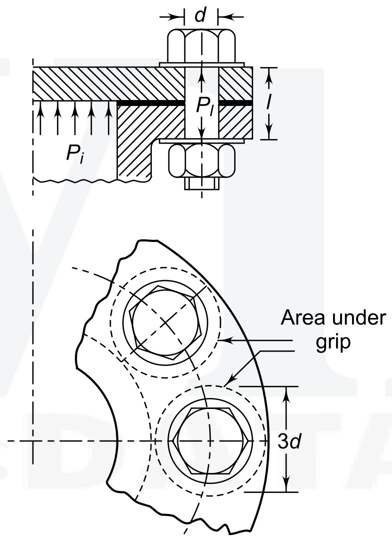


Fig. 22.11 *Bolted Assembly*

The stiffness of the bolt is given by,

$$k_b = \left(\frac{\pi}{4}d^2\right)\frac{E}{l} \qquad (22.23)$$

where ,

k_b = stiffness of the bolt (N/mm)
d = nominal diameter of the bolt (mm)
l = total thickness of the parts held together by the bolt (mm)
E = modulus of elasticity of bolt material (N/mm^2)

There are three members in the grip of the bolt—cylinder cover, cylinder flange and gasket. They act as three compression springs in series. Their combined stiffness (k_c) is given by,

$$\frac{1}{k_c} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_g} \qquad (22.24)$$

where k_1 and k_2 are the stiffness of the cylinder cover and the cylinder flange respectively and k_g is the stiffness of the gasket. It is difficult to predict the area of flanges compressed by the bolt. As shown in Fig. 22.11, it is assumed that a hollow circular area of (3*d*) and (*d*) as outer and inner diameters respectively is under the grip of the bolt.

$$A = \frac{\pi}{4}[(3d)^2 - d^2] = 2\pi d^2$$

and
$$k = \frac{AE}{l} = \frac{2\pi d^2 E}{t}$$

$$\therefore \qquad k = \frac{2\pi d^2 E}{t} \qquad (22.25)$$

where t is the thickness of the member under compression. When the gasket is very soft relative to the flanges, it is the gasket that is compressed during the tightening of the bolt. In such cases, the flanges are neglected and the stiffness of the gasket is considered to be k_c.

When the bolt is tightened with a preload P_1, the bolt is elongated by an amount (δ_b) and the two flanges with the gasket are compressed by an amount (δ_c). When the stresses are within the elastic limit,

$$\delta_b = \frac{P_l}{k_b} \qquad (a)$$

$$\delta_c = \frac{P_l}{k_c} \qquad (b)$$

The load-deflection diagram is shown in Fig. 22.12. Line $\overline{OA}$ represents elongation of the bolt, while line $\overline{CA}$ indicates the compression of the flanges. The slope of the line $\overline{CA}$ is negative because it indicates compression.

When the cylinder is assembled and put into service, it is further subjected to an external load P_i operating inside the vessel. The effect of P_i is as follows.

(i) The bolts are further elongated by an amount ($\Delta\delta$) and there is a corresponding increase in the bolt load which is denoted by (ΔP_i). This is represented by line $\overline{AB}$.

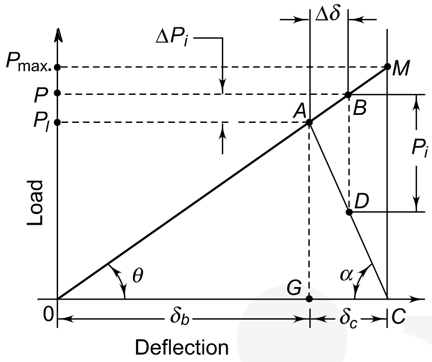


Fig. 22.12 *Deflection Diagram for Bolted Assembly*

(ii) The compression of two flanges and the gasket is relieved by a magnitude ($\Delta\delta$), and there is a corresponding reduction in load. The reduction in load is ($P_i - \Delta P_i$) which is represented by the line *AD*.

Since,

$$k_b = \frac{\Delta P_i}{\Delta\delta} \qquad (c)$$

and
$$k_c = \frac{(P_i - \Delta P_i)}{\Delta\delta} \qquad (d)$$

Dividing Eqs (c) by (d),

$$\Delta P_i = P_i\left[\frac{k_b}{k_b + k_c}\right] \qquad (22.26)$$

The resultant load on the bolt is given by,

$$P = P_1 + \Delta P_i \qquad (22.27)$$

The effect of the gasket on the bolted assembly can be explained with the help of the above equations, which can be expressed in the following manner:

$$P = P_l + P_i\left[\frac{1}{1 + \left(\frac{k_c}{k_b}\right)}\right]$$

When there is no gasket, the flanges are more rigid than the steel bolt, or

$$k_c > k_b$$

When (k_c) is extremely large compared to (k_b), the expression within bracket has a limiting value of zero, and

$$P = P_1$$

which indicates that almost all of the external load P_i is borne by the flanges to relieve their initial compression. This may lead to leakage between two flanges.

When there is a gasket of elastic material,

$$k_b > k_c$$

When (k_b) is too large compared with (k_c), the expression within bracket has a limiting value of one, and

$$P = P_l + P_i$$

which indicates that a major portion of the external load is borne by the bolt. This is desirable for leakproof joints.

Referring to Fig. 22.12, the elongation of the bolt will continue along the line $\overline{OA}$ as the operating pressure is gradually increased. The limiting point is M, where the compression of flanges becomes zero and the joint is on the verge of opening. The corresponding load $(P_{\max.})$ indicates the capacity of the cylinder to bear the load.

ΔOAG and ΔOMC are similar triangles. From the geometry of similar triangles,

$$\frac{\overline{AG}}{\overline{OG}} = \frac{\overline{MC}}{\overline{OC}}$$

$$\frac{P_l}{\delta_b} = \frac{P_{\max.}}{\delta_b + \delta_c}$$

$$P_{\max.} = P_l\left[\frac{\delta_b + \delta_c}{\delta_b}\right]$$

Substituting expressions (a) and (b),

$$P_{\max.} = P_l\left[\frac{k_b + k_c}{k_c}\right] \qquad (22.28)$$

Example 22.9 *The cover of a cylindrical pressure vessel made of cast iron is shown in Fig. 22.13. The inner diameter of the cylinder is 500 mm and the internal pressure is limited to 2 MPa. The cover is fixed to the cylinder by means of 16 bolts with a nominal diameter of 20 mm. Each bolt is initially tightened with a preload of 20 kN. The bolts are made of steel FeE 250 (S_{yt} = 250 N/mm²). Assume*

E for steel = 207 kN/mm²

E for cast iron =100 kN/mm²

E for zinc = 90 kN/mm²

Determine the factor of safety for bolts considering the effect of the gasket.

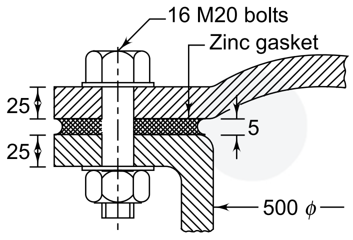


Fig. 22.13

Solution

Given For cylinder, D_i = 500 mm P_i = 2 MPa

For bolts, d = 20 mm n = 16 bolts

S_{yt} = 250 N/mm² P_1 = 20 kN

Step I *Stiffness of bolt*

From Eq. 22.23,

$$k_b = \left(\frac{\pi}{4}d^2\right)\frac{E}{l} = \left(\frac{\pi}{4}(20)^2\right)\frac{(207)}{(55)}$$

$$= 1182.38 \text{ kN/mm}$$

Step II *Combined stiffness of flanges and gasket*

For cast iron flanges,

$$k_1 = k_2 = \frac{2\pi d^2 E}{t} = \frac{2\pi(20)^2(100)}{(25)}$$

$$= 10\,053.10 \text{ kN/mm}$$

For a zinc gasket,

$$k_g = \frac{2\pi d^2 E}{t} = \frac{2\pi(20)^2(90)}{(5)} = 45\,238.93 \text{ kN/mm}$$

From Eq. 22.24,

$$\frac{1}{k_c} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_g} = \frac{2}{10\,053.10} + \frac{1}{45\,238.93}$$

$$\therefore \quad k_c = 4523.9 \text{ kN/mm}$$

Step III *Resultant bolt load*

$$\left[\frac{k_b}{k_b + k_c}\right] = \frac{1182.38}{1182.38 + 4523.90} = 0.2072$$

$$P_1 = 20 \text{ kN} = 20\,000 \text{ N}$$

$$(P_i) \text{ per bolt} = \frac{\pi}{4}(500)^2(2)\left(\frac{1}{16}\right) = 24\,543.69 \text{ N}$$

The resultant load on the bolt is given by (Eq. 22.27)

$$P = P_l + P_i\left[\frac{k_b}{k_b + k_c}\right]$$

$$= 20\,000 + 24\,543.69\,(0.2072)$$

$$= 25\,085.45 \text{ N}$$

Step IV *Factor of safety*

The resultant tensile stress in the bolt is given by,

$$\sigma_t = \frac{P}{\left(\frac{\pi}{4}d^2\right)} = \frac{25\,085.45}{\left(\frac{\pi}{4}(20)^2\right)} = 79.85 \text{ N/mm}^2$$

$$\text{Factor of safety} = \frac{S_{yt}}{\sigma_t} = \frac{250}{79.85} = 3.13$$

22.11 UNFIRED PRESSURE VESSELS

An unfired pressure vessel is defined as a vessel or a pipeline for carrying, storing or receiving steam, gases or liquids at pressures above the atmospheric pressure. Such pressure vessels are designed according to national and international codes[1]. The Indian standard code for pressure vessels gives the design procedure for welded pressure vessels that are made of ferrous materials and subjected to internal pressure from 1 kgf/cm^2 to 200 kgf/cm^2. (1 kgf = 9.81 N). Small pressure vessels with diameters less than 150 mm or water containers with capacities of less than 500 litres do not come under the scope of this code. The code does not include steam boilers, nuclear pressure vessels or hot water storage tanks.

The construction of a welded pressure vessel is shown in Fig. 22.14. There are four categories of welded joints—*A*, *B*, *C* and *D*. *The term category defines only the location of welded joint in the vessel and never implies the type of welded joint.* The welded joints included in four categories are as follows.

(i) Category A Longitudinal welded joints within the main shell, communicating chambers and nozzles, circumferential joints connecting the end closure to the main shell, any welded joint in spherical or formed head

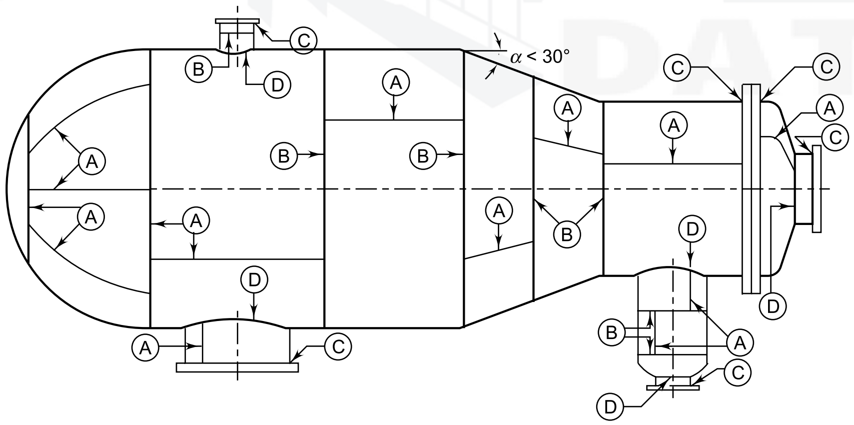


Fig. 22.14 *Welded Joints in Pressure Vessel*

(ii) Category B Circumferential welded joints in the main shell, communicating chambers or nozzles

(iii) Category C Welded joints connecting flanges and flat heads to the main shell

[1] IS 2825–1969: Code for unfired pressure vessels.

(iv) Category D Welded joints connecting communi-cating chambers and nozzles to the main shell

Pressure vessels are classified into three groups—Class 1, Class 2 and Class 3. *Class 1* pressure vessels are used to contain lethal and toxic substances. They include poisonous gases and liquids that are dangerous to human life, e.g., hydrocyanic acid, carbonyl chloride or mustard gas. Liquefied petroleum gas is not classified as a lethal substance. *Class 1* pressure vessels are also used when the operating temperature is less than –20°C. There are two types of welded joints used in these vessels—double welded butt joint with full penetration and single welded butt joint with backing strip. Welded joints of Class 1 pressure vessels are fully radiographed.

Class 2 pressure vessels are those which do not come under Class 1 or Class 3 categories. The maximum thickness of the main shell in this case is limited to 38 mm. The types of welded joints in Class 2 pressure vessels are the same as those in Class 1. However, in Class 2 pressure vessels, the welded joints are spot radiographed. *Class 3* pressure vessels are used for relatively light duties. They are not recommended for service when the operating temperature is less than 0°C or more than 250°C. The maximum pressure is limited to 17.5 kgf/cm^2 (1 kgf = 9.81 N) while the maximum shell thickness is limited to 16 mm. They are usually made from carbon and low alloy steels. Welded joints in Class 3 pressure vessels are not radiographed.

There are three terms related to pressure—working pressure, design pressure and hydrostatic test pressure. The maximum working pressure is that which is permitted for the vessel in operation. It is the pressure required for the processes that are carried out inside the pressure vessel. The design pressure (P) is the pressure used in design calculations for such quantities as the shell thickness and also in the design of other attachments like nozzles and openings. The design pressure is obtained by adding a minimum 5% to the maximum working pressure, or

Design pressure = 1.05 (maximum working pressure)

The pressure vessel is finally tested by the hydrostatic test. The hydrostatic test pressure is taken as 1.3 times the design pressure. Therefore,

Hydrostatic test pressure = 1.3 (design pressure)

Pressure vessels are fabricated from steel plates welded together by the fusion welding process. In fusion welding, the weld is made in a state of fusion without hammering. It includes arc welding, gas welding, thermit welding and electron beam welding. *The term weld joint efficiency is often used in pressure vessel design. It is defined as the ratio of the strength of the welded joint to the strength of the plates*. The magnitude of weld joint efficiency (η) depends upon two factors—the type of weld and the method of weld inspection. The three types of commonly used welded joints are shown in Fig. 22.15. The weld joint efficiency of these joints is given in Table 22.1.

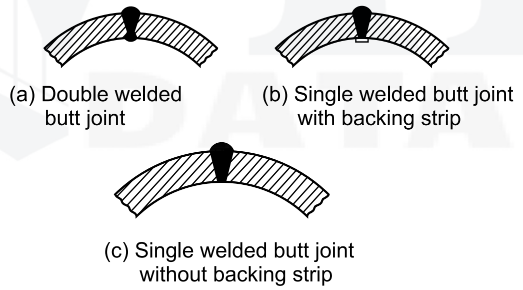


Fig. 22.15 *Welded Joints in Pressure Vessels*

Table 22.1 *Weld joint efficiency*

Type of welded joint	*Weld joint efficiency (η)*		
	Fully radiographed	*Spot radiographed*	*Not radiographed*
(a) Double welded butt joint with full penetration	1.0	0.85	0.70
(b) Single welded butt joint with backing strip	0.90	0.80	0.65
(c) Single welded butt joint without backing strip	–	–	0.60

22.12 THICKNESS OF CYLINDRICAL AND SPHERICAL SHELLS

The equations for the thickness of cylindrical or spherical shells are based on the theory of thin cylinders, with suitable modifications. The thickness of a cylindrical shell subjected to internal pressure, as shown in Fig. 22.16, is given by,

$$t = \frac{P_i D_i}{2\sigma_t \eta - P_i} + CA \tag{22.29}$$

The thickness of the spherical shell is given by

$$t = \frac{P_i D_i}{4\sigma_t \eta - P_i} + CA \tag{22.30}$$

where,

t = minimum thickness of the shell plate (mm)
P_i = design pressure (MPa or N/mm^2)
D_i = inner diameter of the shell (mm)
σ_t = allowable stress for the plate material (N/mm^2)
η = weld joint efficiency
CA = corrosion allowance (mm)

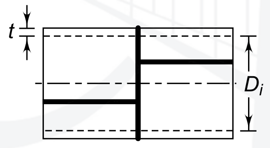


Fig. 22.16 *Cylindrical Shell*

The allowable stresses for the plate material are obtained from the following expressions:

$$\sigma_t = \frac{\text{Yield strength (or 0.2\% proof stress)}}{1.5}$$

$$\sigma_t = \frac{\text{Ultimate tensile strength}}{3.0}$$

The factor of safety of 1.5 or 3 in the above expressions is used under the following two operating conditions:

(i) the pressure vessel is operating at room temperature; and
(ii) the pressure inside the vessel is not fluctuating.

For high temperature applications, the values of allowable stresses at design temperature are given in the standard. The values of yield strength (S_{yt}) for commonly used carbon and low alloy steels are given in Table 22.2.

The walls of the pressure vessel are subjected to thinning due to corrosion, which reduces the useful life of the vessel. Corrosion in pressure vessels is of the following forms:

(i) Chemical attack where the metal is dissolved by a chemical reagent
(ii) Rusting due to air and moisture
(iii) Erosion where a reagent flows over the wall surface at high velocities
(iv) Scaling or high-temperature oxidation

Table 22.2 *Values of yield strength*

Material		S_{yt} *(N/mm^2)*
I S 2002–1962	I	200
	2A	205
	2B	255
IS 2041–1962	20Mo<u>55</u>	275
	20Mn2	290
IS 1570–1961	15Cr<u>90</u>Mo<u>55</u>	290
	C15Mn<u>75</u>	225

Provision has to be made by suitable increase in wall thickness to compensate for the thinning due to corrosion. *Corrosion Allowance (CA) is the additional metal thickness over and above that is required to withstand the internal pressure.* A minimum corrosion allowance of 1.5 mm is recommended unless a protective lining is employed.

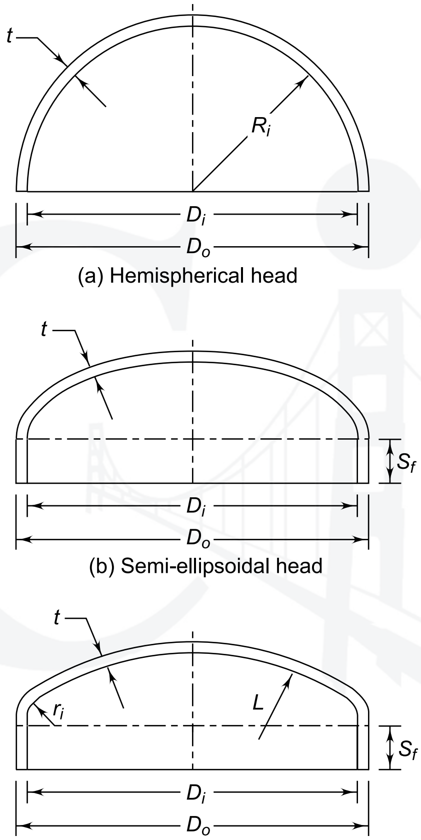
22.13 END CLOSURES

Formed heads are used as end closures for cylindrical pressure vessels. There are two types of end closures—domed heads and conical heads. The domed heads are further classified into three groups—hemispherical, semi-ellipsoidal and torispherical, as shown in Fig. 22.17. Hemispherical heads have minimum plate thickness, minimum weight and consequently lowest material cost. However, the amount of forming required to produce the hemispherical shape is more, resulting in increased forming cost. The thickness of the hemispherical head is given by,

$$t = \frac{P_i R_i}{2\sigma_t \eta - 0.2P_i} + CA \tag{22.31}$$

where R_i is the inner radius of the cylindrical shell. In a semi-ellipsoidal head, the ratio of the major axis to the minor axis is taken as 2 : 1. The thickness of a semi-ellipsoidal head is given by,

$$t = \frac{P_i D_i}{2\sigma_t \eta - 0.2 P_i} + CA \tag{22.32}$$



(a) Hemispherical head

(b) Semi-ellipsoidal head

(c) Torispherical head

Fig. 22.17 *Domed Heads*

From Eqs (22.31) and (22.32), it is observed that the thickness of the semi-ellipsoidal head is more (almost twice) than the corresponding hemispherical head, and to that extent, the material cost is more. However, due to the shallow dished shape, the forming cost is reduced.

The length of the straight portion (S_f) is given by,

$S_f = 3t$ or 20 mm (whichever is more)

Torispherical heads are extensively used as end closures for a large variety of cylindrical pressure vessels. They are shaped by using two radii—the crown radius L and knuckle radius r_i. The crown radius L is the radius of the dish, which constitutes the major portion of the head. The knuckle radius r_i is the corner radius joining the spherical crown with the cylindrical shell. Torispherical heads require less forming than semi-ellipsoidal heads. Their main drawback is the local stresses at the two discontinuities, namely, the junction between the crown and the knuckle radius and the junction between the knuckle radius and the cylindrical shell. The localised stresses may lead to failure due to brittle fracture. The thickness of a torispherical head is given by,

$$t = \frac{0.885 P_i L}{\sigma_t \eta - 0.1 P_i} + CA \tag{22.33}$$

where L is the inside crown radius.

The knuckle radius r_i is taken as 6% of the crown radius,

or $$r_i = 0.06 L$$

The crown radius L should not be greater than the outside diameter of the cylindrical shell. Therefore,

$$L < D_o$$

Hemispherical and semi-ellipsoidal heads are used for tall vertical towers because they are practically free from discontinuities. In such cases, the cost of the top end closure is only a small part of the total cost of the pressure vessel. Also, the space is not a limiting factor for vertical pressure vessels. Torispherical heads are more economical than other types of domed heads. Owing to their compact construction, they are used for horizontal pressure vessels such as tankers for water, milk, petrol, diesel and kerosene. They are also used for small vertical pressure vessels.

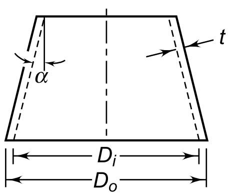


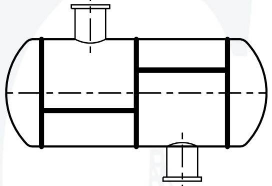
Fig. 22.18 *Conical Section*

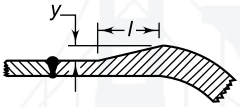
The thickness of a conical head or conical section, as shown in Fig. 22.18, is given by

$$t = \frac{P_i D_i}{2\cos\alpha\,(\sigma_t\,\eta - 0.6P_i)} + CA \qquad (22.34)$$

where (α) is half the apex angle. The half apex angle is usually less than 30°.

Example 22.10 *The cylindrical shell shown in Fig. 22.19(a) is subjected to an operating pressure of 0.75 MPa. The yield strength of the plate material is 200 N/mm² and the corrosion allowance is 3 mm. Spot radiographed double welded butt joints are used to fabricate the shell, whose internal diameter is 2.5 m. Torispherical heads, each with a crown radius of 2 m, are used as end closure. Determine the thickness of the cylindrical shell and the torispherical head.*


(a)



(b)

Fig. 22.19

Solution

Given Operating pressure = 0.75 MPa
S_{yt} = 200 N/mm² CA = 3 mm D_i = 2.5 m
L = 2 m

Step I *Thickness of cylindrical shell*

The design pressure P_i is given by,

$$P_i = 1.05(0.75) = 0.7875 \text{ MPa or N/mm}^2$$

From Table 22.1, the weld joint efficiency (η) for a spot radiographed double welded butt joint is 0.85.

$$\eta = 0.85$$

The allowable stress (σ_t) for the plate material is given by,

$$\sigma_t = \frac{S_{yt}}{1.5} = \frac{200}{1.5} = 133.33 \text{ N/mm}^2$$

From Eq. 22.29,

$$t = \frac{P_i D_i}{2\sigma_t\,\eta - P_i} + CA$$

$$= \frac{(0.7875)(2500)}{2(133.33)(0.85) - 0.7875} + 3$$

$$= 11.72 \text{ or } 12 \text{ mm}$$

Step II *Thickness of torispherical head*

From Eq. 22.33, the thickness of the torispherical head is given by,

$$t = \frac{0.885\,P_i L}{\sigma_t\eta - 0.1P_i} + CA$$

$$= \frac{0.885\,(0.7875)(2000)}{(133.33)(0.85) - 0.1(0.7875)} + 3$$

$$= 15.3 \text{ or } 16 \text{ mm}$$

The knuckle radius r_i is given by,

$$r_i = 0.06\,L = 0.06\,(2000) = 120 \text{ mm}$$

The thickness of the cylindrical shell is 12 mm while that of the head is 16 mm. As per recommendations, the minimum tapered transition l should be three times the offset y as shown in Fig. 22.19(b). Therefore,

$$l \geq 3y \quad \text{or} \quad l \geq 3\,(16 - 12) \quad \text{or} \quad l > 12 \text{ mm}$$

Therefore, l = 15 mm

Example 22.11 *A horizontal pressure vessel consists of a cylindrical shell enclosed by hemispherical ends. The volumetric capacity of the vessel should be approximately 2 m³ and the length should not exceed 3 m. Assuming the thickness negligibly small compared with overall dimensions of the vessel, determine the internal diameter and the length of the cylindrical shell.*

The pressure vessel is fabricated from steel plates with a yield strength of 255 N/mm². The weld joint efficiency factor is 0.85 and corrosion allowance 2 mm. The pressure vessel is subjected to an operating pressure of 2 MPa. Calculate the thickness of the cylindrical shell and the hemispherical end closures.

Solution

Given Operating pressure = 2 MPa η = 0.85
S_{yt} = 255 N/mm² CA = 2 mm V = 2 m³
L = 3 m

Step I *Diameter and length of cylindrical shell*

As shown in Fig. 22.20,

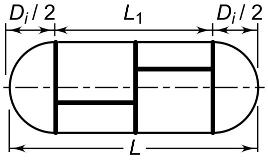


Fig. 22.20

$$L = \frac{D_i}{2} + L_1 + \frac{D_i}{2} = D_i + L_1$$

where L_1 is the length of cylindrical shell. The length is given as 3 m. Therefore,

$$D_i + L_1 = 3 \quad \text{or} \quad L_1 = 3 - D_i \qquad \text{(i)}$$

The volume of the pressure vessel is given by,

$$V = \frac{\pi D_i^2 L_1}{4} + \frac{\pi D_i^3}{6}$$

The volume is 2 m^3 and substituting the expression (i) in the above equation,

$$2 = \frac{\pi D_i^2 (3 - D_i)}{4} + \frac{\pi D_i^3}{6}$$

Simplifying the above expression,

$$D_i^3 - 9D_i^2 + \frac{24}{\pi} = 0$$

The above equation is solved by trial and error method and the values are given in the following table.

D_i	$\left[D_i^3 - 9D_i^2 + \frac{24}{\pi}\right]$
1	– 0.361
0.99	– 0.300
0.98	– 0.239
0.97	– 0.178
0.96	– 0.116
0.95	– 0.053
0.94	+0.010

The root of the cubic equation is between 0.94 and 0.95. Therefore,

$$D_i = 0.95 \text{ m}$$

and $$L_1 = 3 - D_i = 3 - 0.95 = 2.05 \text{ m}$$

$$V = \frac{\pi D_i^2 L_1}{4} + \frac{\pi D_i^3}{6}$$

$$= \frac{\pi (0.95)^2 (2.05)}{4} + \frac{\pi (0.95)^3}{6}$$

$$= 1.902 \text{ m}^3$$

Step II *Thickness of cylindrical shell*

The design pressure is given by,

$$P_i = 1.05\,(2) = 2.1 \text{ MPa or N/mm}^2$$

$$\sigma_t = \frac{S_{yt}}{1.5} = \frac{255}{1.5} = 170 \text{ N/mm}^2$$

From Eq. 22.29,

$$t = \frac{P_i D_i}{2\sigma_t \eta - P_i} + CA = \frac{(2.1)(950)}{2(170)(0.85) - 2.1} + 2$$

$$= 8.95 \text{ or } 9 \text{ mm}$$

Step III *Thickness of hemispherical head*

From Eq. 22.31,

$$t = \frac{P_i R_i}{2\sigma_t \eta - 0.2P_i} + CA$$

$$= \frac{(2.1)(475)}{2(170)(0.85) - 0.2(2.1)} + 2$$

$$= 5.46 \text{ or } 6 \text{ mm}$$

Example 22.12 *A pressure vessel consists of a cylindrical shell with torispherical ends. The crown and knuckle radii of torispherical end closure are* $\left(\frac{3}{4}D\right)$ *and* $\left(\frac{1}{8}D\right)$ *respectively, where D is the diameter of the cylindrical shell. Derive an expression for the volume of end closure in terms of diameter of the shell. Assume that the thickness is negligibly small compared with the overall dimensions of the shell and the end closures.*

The capacity of this vessel is 10 m^3 and the length is limited to 5 m. The vessel is subjected to an operating pressure of 0.5 MPa. The yield strength of the plate material is 200 N/mm^2 and the corrosion allowance is 2 mm. The weld joint efficiency can be taken as 0.6. Determine

(i) the diameter of the cylindrical shell;
(ii) the length of the cylindrical shell;
(iii) the crown radius;
(iv) the knuckle radius;
(v) the thickness of the cylindrical shell;
(vi) the thickness of the torispherical ends.

Solution

Given Operating pressure = 0.5 MPa $\eta = 0.6$
$S_{yt} = 200$ N/mm^2 $CA = 2$ mm $V = 10$ m^3
$L = 5$ m

Step I *Volume of end closure*

From Fig. 22.21,

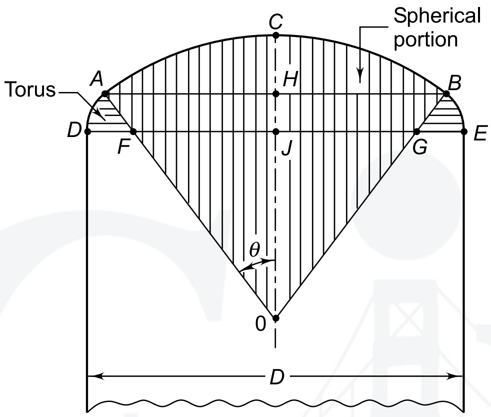


Fig. 22.21

Volume of end closure = (volume of spherical portion *OACB*)
+ (volume of torus shown by *AFD* or *BGE*)
– (volume of right circular cone shown by *OFG*). (i)

The crown radius (L) is shown by *OA* or *OB*

Suppose,

$$L = c_1 D$$

The knuckle radius (r_i) is shown by *FA* or *GB*.

Suppose,

$$r_i = c_2 D$$

(i) Volume of spherical portion OACB

Height of cap = $\overline{CH} = \overline{OH} - \overline{OH} = L - L\cos\theta$
$= L\,(1 - \cos\theta)$

From Fig. 22.22(a),

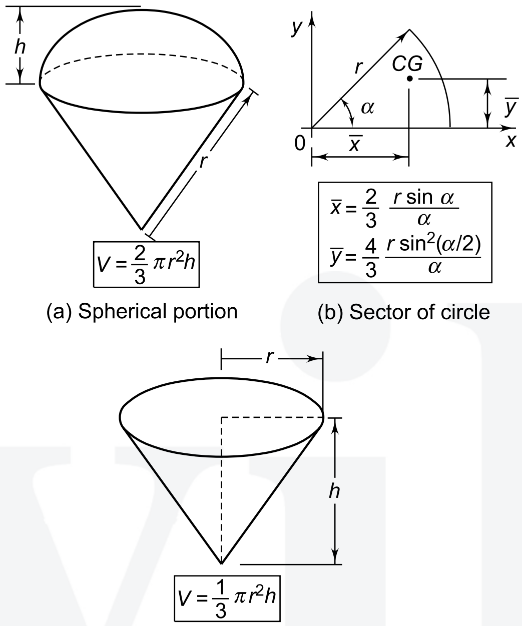
$$V = \frac{2}{3}\pi\,(\overline{OA})^2(\overline{CH})$$

$$= \frac{2}{3}\pi\,L^2[L\,(1-\cos\theta)]$$

$$= \frac{2}{3}\pi\,L^3\,(1-\cos\theta)$$

or

$$V = \frac{2}{3}\pi\,(1-\cos\theta)\,c_1^3\,D^3 \qquad \text{(ii)}$$



(a) Spherical portion (b) Sector of circle

(c) Right circular cone

Fig. 22.22

(ii) Volume of torus

From Fig. 22.21,

$$\angle AFD = \angle JFO = \left[\frac{\pi}{2} - \theta\right]$$

Area of sector *AFD*

$$= \left(\frac{\angle AFD}{2\pi}\right)\pi\,(\overline{FA})^2$$

$$= \left(\frac{(\pi/2-\theta)}{2\pi}\right)\pi c_2^2\,D^2$$

$$= \frac{1}{2}\left(\frac{\pi}{2}-\theta\right)c_2^2\,D^2$$

From Fig. 22.22(b),

$$\bar{x} = \frac{2}{3}\left[\frac{c_2 D\sin\left(\dfrac{\pi}{2}-\theta\right)}{\left(\dfrac{\pi}{2}-\theta\right)}\right]$$

Volume of torus = Area of sector *AFD* × $\pi[D - 2r_i + 2\bar{x}]$

$$= \frac{1}{2}\left(\frac{\pi}{2}-\theta\right) c_2^2 D^2 \pi \left\{ D - 2c_2 D + \frac{4}{3}\left[\frac{c_2 D \sin\left(\frac{\pi}{2}-\theta\right)}{\left(\frac{\pi}{2}-\theta\right)}\right]\right\}$$

$$= \frac{1}{2}\pi\left(\frac{\pi}{2}-\theta\right) c_2^2 D^3 \left[1 - 2c_2 + \frac{4}{3}\frac{c_2 \cos\theta}{\left(\frac{\pi}{2}-\theta\right)}\right] \qquad \text{(iii)}$$

(iii) Volume of right circular cone OFG

From Fig. 22.21,

$\overline{OF} = \overline{OA} - \overline{FA} = L - r_i = c_1 D - c_2 D = (c_1 - c_2)D$

$\overline{OJ} = \overline{OF} \cos\theta = (c_1 - c_2) D \cos\theta$

$\overline{JF} = \overline{OF} \sin\theta = (c_1 - c_2) D \sin\theta$

From Fig. 22.22(c),

$$V = \frac{1}{3}\pi (\overline{JF})^2 (\overline{OJ})$$

$$= \frac{1}{3}\pi [(c_1 - c_2)^2 D^2 \sin^2\theta][(c_1 - c_2) D \cos\theta]$$

$$= \frac{1}{3}\pi (c_1 - c_2)^3 D^3 \sin^2\theta \cos\theta \qquad \text{(iv)}$$

Substituting expressions (ii), (iii), and (iv) in the expression (i),

Volume of end closure

$$= \frac{2}{3}\pi (1 - \cos\theta) c_1^3 D^3$$

$$+ \frac{1}{2}\pi\left(\frac{\pi}{2}-\theta\right) c_2^2 D^3 \left[1 - 2c_2 + \frac{4}{3}\frac{c_2 \cos\theta}{\left(\frac{\pi}{2}-\theta\right)}\right]$$

$$- \frac{1}{3}\pi (c_1 - c_2)^3 D^3 \sin^2\theta \cos\theta \qquad \text{(v)}$$

Let us consider the special case,

$$L = \frac{3}{4} D \quad \text{or} \quad c_1 = \frac{3}{4}$$

$$r_i = \frac{1}{8} D \quad \text{or} \quad c_2 = \frac{1}{8}$$

From Fig. 22.21,

$$\sin\theta = \frac{\overline{FJ}}{\overline{OF}} = \frac{\left(\frac{D}{2} - r_i\right)}{(L - r_i)} = \frac{\left(\frac{D}{2} - \frac{D}{8}\right)}{\left(\frac{3D}{4} - \frac{D}{8}\right)} = 0.6$$

$$\cos\theta = \sqrt{1 - (0.6)^2} = 0.8$$

$$\theta = 0.6435 \text{ radian}$$

$$\frac{\pi}{2} - \theta = 0.9273 \text{ radian}$$

Substituting the above values in the expression (v),

Volume of end closure

$$= \frac{2}{3}\pi (1 - 0.8)\left(\frac{3}{4}\right)^3 D^3$$

$$+ \frac{1}{2}\pi (0.9273)\left(\frac{1}{8}\right)^2 D^3 \left[1 - \frac{2}{8} + \frac{4}{3}\frac{\left[\frac{1}{8}(0.8)\right]}{(0.9273)}\right]$$

$$- \frac{1}{3}\pi\left(\frac{3}{4} - \frac{1}{8}\right)^3 D^3 (0.6)^2 (0.8) \qquad \text{(v)}$$

or $\quad V = 0.1234\, D^3 \qquad$ (a)

Step II *Diameter of cylindrical shell*

From Fig. 22.21,

$$\overline{JC} = \overline{OC} - \overline{OJ} = \overline{OC} - \overline{OF}\cos\theta = L - (L - r_i)\cos\theta$$

or $\quad \overline{JC} = \frac{3}{4} D - \left[\frac{3}{4} - \frac{1}{8}\right] D (0.8) = 0.25\, D$

The length of the vessel is given as 5 m. From Fig. 22.23,

$$l_1 = (5 - 0.5D) \qquad \text{(b)}$$

The volume of the vessel is 10 m^3. Therefore,

$$10 = 2\,(0.1234\, D^3) + \frac{\pi}{4} D^2 (5 - 0.5D)$$

Rearranging the terms,

$$D^3 - 26.916\, D^2 + 68.54 = 0 \qquad \text{(c)}$$

The above equation is cubic and it is solved by the trial and error method. As a first trial,

$$D = 4$$

$$\text{LHS} = -298.116$$

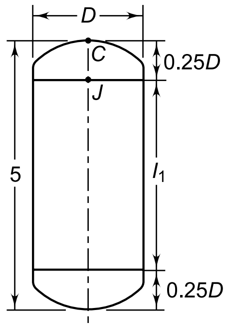


Fig. 22.23

The values of D and corresponding values of the left hand side expression of the equation (c) are as follows:

D	$(D^3 - 26.916\,D^2 + 68.54)$
4	–298.116
3	–146.704
2	–31.124
1.7	–4.334
1.65	–0.247
1.6	+3.731

Therefore,

$$D = 1.65 \text{ m} \quad \text{or} \quad D = 1650 \text{ mm} \tag{i}$$

Step III *Length of cylindrical shell*

$$l_1 = 5 - 0.5(1.65) = 4.175 \text{ m}$$

or $$l_1 = 4175 \text{ mm} \tag{ii}$$

Step IV *Crown radius*

$$L = \left(\frac{3}{4}\right) D = \left(\frac{3}{4}\right)(1650) = 1237.5 \text{ mm} \tag{iii}$$

Step V *Knuckle radius*

$$r_i = \left(\frac{1}{8}\right) D = \left(\frac{1}{8}\right)(1650) = 206.25 \text{ mm} \tag{iv}$$

Step VI *Thickness of cylindrical shell*

The design pressure P_i is given by,

$$P_i = 1.05(0.5) = 0.525 \text{ N/mm}^2$$

The allowable stress (σ_t) for plate material is given by,

$$\sigma_t = \frac{S_{yt}}{1.5} = \frac{200}{1.5} = 133.33 \text{ N/mm}^2$$

From Eq. 22.29,

$$t = \frac{P_i D_i}{2\sigma_t\,\eta - P_i} + CA$$

$$= \frac{(0.525)(1650)}{2(133.33)(0.60) - 0.525} + 2$$

$$= 7.43 \text{ or } 8 \text{ mm} \tag{v}$$

Step VII *Thickness of torispherical head*

From Eq. 22.33,

$$t = \frac{0.885\,P_i\,L}{\sigma_t \eta - 0.1P_i} + CA$$

$$= \frac{0.885(0.525)\,(1237.5)}{(133.33)(0.60) - 0.1\,(0.525)} + 2$$

$$= 9.19 \text{ or } 10 \text{ mm} \tag{vi}$$

22.14 OPENINGS IN PRESSURE VESSEL

Openings are provided in the pressure vessel; these could be an inlet and outlet pipe connections, manhole or hand hole, connections for pressure gauges, temperature gauges and safety valves. The openings are circular, elliptical or obround. The inner diameter of a manhole is generally 380 mm. Such openings are designed by the area compensation method.

The basic principle of the area compensation method is illustrated in Fig. 22.24. When the opening is cut in the pressure vessel, an area is removed from the shell. It must be reinforced by an equal amount of area near the opening. The area '*removed*' should be equal to the area '*added*'. The area is added by providing a reinforcing pad in the form of annular circular plate around the opening. It should be noted that in this method, we are considering cross-sectional area in the form of a rectangular strip. It is not the compensation of volume of metal that has been cut due to the opening by means of the reinforcing pad.

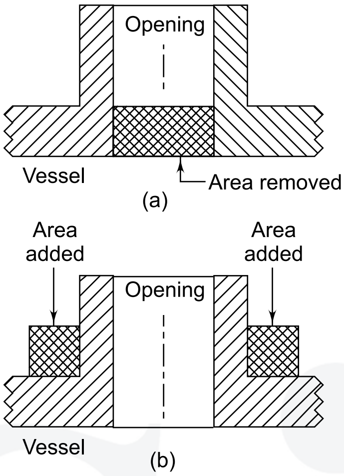


Fig. 22.24 *Principle of Area Compensation*

It is not always necessary to replace the actually removed area of the metal. The plate of the shell and nozzle are usually thicker than would be required to withstand pressure. This partially compensates for loss of area in the opening. As shown in Fig. 22.25,

$$A = d\, t_r \qquad \text{(i)}$$

where,

A = area of metal removed in corroded condition (mm^2)

d = inner diameter of opening in corroded condition

$= (d_i + 2CA)$ mm

d_i = inner diameter (nominal) of nozzle (mm)

t_r = required thickness of cylindrical shell (mm)

The required thickness t_r is given by (Eq. 22.29)

$$t_r = \frac{P_i D_i}{2\sigma_t \eta - P_i}$$

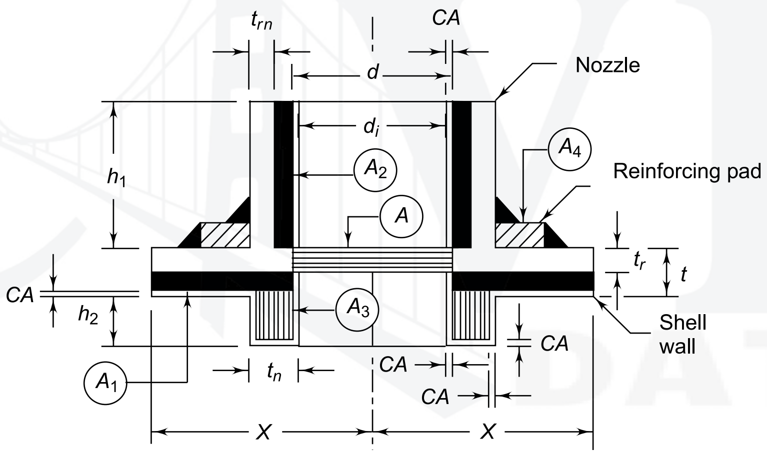


Fig. 22.25 *Area Compensation for Nozzle*

The metal used for reinforcement should be located in the vicinity of the opening. The limiting dimension X parallel to the wall of the cylindrical shell is given by

$$X = d \quad \text{or} \quad X = \left[\frac{d_i}{2} + t + t_n - 3CA\right]$$

(whichever is maximum)

The limiting dimensions h_1 and h_2 parallel to the nozzle wall are given by,

h_1 or $h_2 = 2.5\,(t - CA)$

h_1 or $h_2 = 2.5\,(t_n - CA)$ (whichever is minimum)

where,

t = total thickness of the wall of cylindrical shell (mm)

t_n = total thickness of nozzle wall (mm)

The area A_1 of excess thickness in the vessel wall, which is available for reinforcement, is given by

$$A_1 = (2X - d)\,(t - t_r - CA) \qquad \text{(ii)}$$

The area A_2 of excess thickness in the nozzle wall is given by,

$$A_2 = 2h_1\,(t_n - t_{rn} - CA) \qquad \text{(iii)}$$

where (t_{rn}) is the thickness required for the nozzle wall to be able to withstand the pressure, or

$$t_{rn} = \frac{P_i d_i}{2\sigma_t \eta - P_i}$$

The area A_3 of the inside extension of the nozzle is given by,

$$A_3 = 2h_2 (t_n - 2CA) \qquad \text{(iv)}$$

The total area available for reinforcement is $(A_1 + A_2 + A_3)$.

When,

$$(A_1 + A_2 + A_3) \geq A$$

the opening is adequately reinforced and no reinforcing pad is required. When this condition is not fulfilled, a reinforcing pad of area A_4 is required.

$$A_4 = A - (A_1 + A_2 + A_3) \qquad (22.35)$$

Sometimes a reinforcing pad of area equal to A is used for the opening to avoid detailed calculations. This results in oversized reinforcement.

Example 22.13 *A pressure vessel consists of a cylindrical shell with an inner diameter of 1500 mm, and thickness of 20 mm. It is provided with a nozzle with an inner diameter of 250 mm and thickness of 15 mm. The yield strength of the material for the shell and nozzle is 200 N/mm² and the design pressure is 2.5 MPa. The extension of the nozzle inside the vessel is 15 mm. The corrosion allowance is 2 mm, while the weld joint efficiency is 0.85. Neglecting the area of welds, determine whether or not a reinforcing pad is required for the opening. If so, determine the dimensions of pad made from a plate of 15 mm thickness.*

Solution

Given For cylindrical shell, $D_i = 1500$ mm
$t = 20$ mm For nozzle, $d_i = 250$ mm
$t_n = 15$ mm $h_2 = 15$ mm $P_i = 2.5$ MPa
$S_{yt} = 200$ N/mm² $CA = 2$ mm $\eta = 0.85$
For pad, $t = 15$ mm

Step I *Area of removed metal*

$$\sigma_t = \frac{S_{yt}}{1.5} = \frac{200}{1.5} = 133.33 \text{ N/mm}^2$$

$$t_r = \frac{P_i D_i}{2\sigma_t \eta - P_i} = \frac{(2.5)(1500)}{2(133.33)(0.85) - 2.5}$$
$$= 16.73 \text{ mm}$$

$$d = d_i + 2\,(CA) = 250 + 2(2) = 254 \text{ mm}$$
$$A = d\, t_r = 254(16.73) = 4249.42 \text{ mm}^2 \qquad \text{(a)}$$

Step II *Area available for reinforcement*

$$t_{rn} = \frac{P_i d_i}{2\sigma_t \eta - P_i} = \frac{(2.5)(250)}{2(133.33)(0.85) - 2.5}$$
$$= 2.79 \text{ mm}$$

The limiting dimension X is the higher of the following two values:

$$X = d = 254 \text{ mm}$$

$$X = \left[\frac{d_i}{2} + t + t_n - 3CA\right]$$
$$= (125 + 20 + 15 - 6) = 154 \text{ mm}$$

Therefore,

$$X = 254 \text{ mm}$$

The limiting dimension h_1 is the lower of the following two values:

$$h_1 = 2.5(t - CA) = 2.5\,(20 - 2) = 45 \text{ mm}$$
$$h_1 = 2.5(t_n - CA) = 2.5\,(15 - 2) = 32.5 \text{ mm}$$

Therefore,

$$h_1 = 32.5 \text{ mm and } \quad h_2 = 15 \text{ mm}$$

The areas available for reinforcement within the above limits are as follows:

$$A_1 = (2X - d)(t - t_r - CA)$$
$$= [2(254) - 254](20 - 16.73 - 2)$$
$$= 322.58 \text{ mm}^2$$
$$A_2 = 2h_1\,(t_n - t_{rn} - CA)$$
$$= 2(32.5)(15 - 2.79 - 2)$$
$$= 663.65 \text{ mm}^2$$
$$A_3 = 2h_2\,(t_n - 2CA) = 2\,(15)\,(15 - 4)$$
$$= 330 \text{ mm}^2$$
$$\therefore \quad (A_1 + A_2 + A_3) = 1316.23 \text{ mm}^2 \qquad \text{(b)}$$

Step III *Area of pad*

From (a) and (b),

$$A > (A_1 + A_2 + A_3)$$

Therefore, a reinforcing pad is necessary. The area of the reinforcing pad A_4 is given by

$$A_4 = A - (A_1 + A_2 + A_3) = 4249.42 - 1316.23$$
$$= 2933.19 \text{ mm}^2$$

Step IV *Dimensions of pad*

The thickness of the reinforcing pad is 15 mm. Therefore, the width of the pad is given by,

$$w = \frac{2933.19}{15} = 195.55 \text{ or } 200 \text{ mm}$$

The inner diameter of the pad is equal to the outer diameter of the nozzle, i.e., (250 + 30) or 280 mm.

Outer diameter of pad = 280 + 200 = 480 mm

Short-Answer Questions

22.1 What is thin cylinder?
22.2 Give practical examples of thin cylinder.
22.3 What are the types of stresses in thin cylinder?
22.4 What is thick cylinder?
22.5 Give practical examples of thick cylinder.
22.6 What are the types of stresses in thick cylinders?
22.7 What is the criterion to distinguish between thin and thick cylinders?
22.8 When do you use Lame's equation for cylinder wall thickness?
22.9 When do you use Clavarino's equation for cylinder wall thickness?
22.10 When do you use Birnie's equation for cylinder wall thickness?
22.11 What is autofrettage?
22.12 What are the methods of pre-stressing the cylinder?
22.13 What is compound cylinder?
22.14 What types of stresses are induced in the jacket and inner tube of compound cylinder?
22.15 What is the function of gasket?
22.16 Where do you use gasket?
22.17 Where do you use metallic gasket?
22.18 What are the advantages and disadvantages of metallic gasket over non-metallic gasket?
22.19 Where do you use asbestos gasket?
22.20 What is Class 1 pressure vessel? Where do you use it?
22.21 What is Class 2 pressure vessel?
22.22 What is Class 3 pressure vessel?
22.23 What do you understand by the term 'working pressure' in pressure vessel?
22.24 What do you understand by the term 'design pressure' in pressure vessel?
22.25 What do you understand by the term 'hydrostatic test pressure' in pressure vessel?
22.26 What is weld-joint efficiency?
22.27 What is corrosion allowance?
22.28 What are the types of end closure for cylindrical pressure vessel?
22.29 What are the advantages and disadvantages of hemispherical head for cylindrical pressure vessel?
22.30 What are the advantages and disadvantages of semi-ellipsoidal head for cylindrical pressure vessel?
22.31 What are the advantages and disadvantages of torispherical head for cylindrical pressure vessel?
22.32 Where do you use hemispherical head for cylindrical pressure vessel?
22.33 Where do you use torispherical head for cylindrical pressure vessel?
22.34 What are the objectives of providing openings in pressure vessel?

Problems for Practice

22.1 A gas cylinder with an internal diameter of 200 mm is subjected to an operating pressure of 10 MPa. It is made of plain carbon steel FeE 230 and the factor of safety is 2.5. Calculate the cylinder wall thickness assuming it to be a thin cylinder and neglecting the effect of welded joints.

[*10.87 mm*]

22.2 An air receiver, consisting of a 500 mm diameter cylinder closed by hemispherical ends, is made of steel FeE 200 and the factor of safety is 2.5. The operating pressure is limited to 3 MPa. Treating the receiver as a thin cylinder, calculate the thickness of the cylinder wall and the hemispherical ends. Neglect the effect of welded joints.

[*9.38 mm and 4.69 mm*]

22.3 According to the distortion energy theory of failure,

$$\sigma = \frac{S_{yt}}{(fs)} = \sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]}$$

where σ_1, σ_2 and σ_3 are principal stresses. Apply this theory to thick cylinder with closed ends and prove that the cylinder wall thickness (t) is given by,

$$t = \frac{D_i}{2}\left[\sqrt{\frac{\sigma}{(\sigma - \sqrt{3}P_i)}} - 1\right]$$

Assume that the cylinder is made of ductile material.

22.4 According to maximum shear stress theory of failure,

$$\tau = \frac{S_{sy}}{(fs)} = \frac{\sigma_1 - \sigma_2}{2}$$

where σ_1 and σ_2 are principal stresses. Apply this theory to thick cylinders with open ends and prove that the cylinder wall thickness t is given by,

$$t = \frac{D_i}{2}\left[\sqrt{\frac{\tau}{(\tau - P_i)}} - 1\right]$$

Assume that the cylinder is made of ductile material.

22.5 A hydraulic cylinder with closed ends is subjected to an internal pressure of 15 MPa. The inner and outer diameters of the cylinder are 200 mm and 240 mm respectively. The cylinder material is cast iron FG 300. Determine the factor of safety used in design. If the cylinder pressure is further increased by 50%, what will be the factor of safety?

[*3.61 and 2.4*]

22.6 A cast iron pipe used in a hydraulic circuit is subjected to an internal pressure of 50 MPa. The inner and outer diameters of the pipe are 20 mm and 40 mm respectively. Determine the distribution of principal stresses across the pipe thickness.

r (mm)	*10*	*12*	*14*	*16*	*18*	*20*
(σ_t) *(N\mm²)*	*83.34*	*62.96*	*50.68*	*42.71*	*37.24*	*33.33*
(σ_r) *(N\mm²)*	*50.00*	*29.63*	*17.35*	*9.38*	*3.91*	*0*

22.7 A high pressure cylinder consists of an inner cylinder of inner and outer diameters of 200 and 300 mm respectively. It is jacketed by an outer cylinder with an outside diameter of 400 mm. The difference between the outer diameter of the inner cylinder and the inner diameter of the jacket before assembly is 0.25 mm (E = 207 kN/mm^2). Calculate the shrinkage pressure and the maximum tensile stress induced in any of the cylinders.

[*27.95 MPa and 99.82 N/mm²*)

22.8 A cylinder with inner diameter of 300 mm and internal pressure of 1 MPa is closed by a cast iron cover with 12 bolts of 25 mm nominal diameter. The ratio of stiffness of the bolt (k_b) to the combined stiffness (k_c) of the two flanges and the asbestos gasket is 1.5. The initial pre-load of each bolt is 10 kN. Determine the maximum tensile load on each bolt.

[*13 534.29 N*]

22.9 A pressure vessel consists of a cylindrical shell with an inside diameter of 1650 mm, which is closed by torispherical heads with a crown radius of 1300 mm. The operating pressure inside the vessel is 1.5 MPa. The yield strength of the material used for the shell and head is 255 N/mm^2 and the weld joint efficiency may be assumed to be 0.8. The corrosion allowance is 2 mm. Determine the thickness of the cylindrical shell and the torispherical head.

[*11.61 mm and 15.34 mm*]

22.10 A pressure vessel, subjected to a design pressure of 0.75 MPa, consists of a cylindrical shell with 2 m inside diameter and 10 mm thickness. An opening with inner diameter of 300 mm and wall thickness of 10 mm is provided in the shell. The corrosion allowance is 2 mm and the weld joint efficiency is taken as 0.85. The extension of the opening inside the shell is 15 mm. The yield strength of the material used for the shell and the opening is 210 N/mm^2. A reinforcing pad made of a 10 mm thick plate is provided for the opening. Determine the inner and outer diameters of the pad.

[*320 mm and 415 mm*]

Miscellaneous Machine Elements

Chapter 23

23.1 OIL SEALS

An oil seal is a mechanical device, made of elastomer material, which is used to prevent leakage of fluid between two machine components. An oil seal has two important functions, namely, to prevent the leakage of expensive lubricating oils from the transmission system and to prevent the entry of foreign particles like dust or abrasive material into the operating medium. The construction of a typical commercial oil seal unit is shown in Fig. 23.1. It

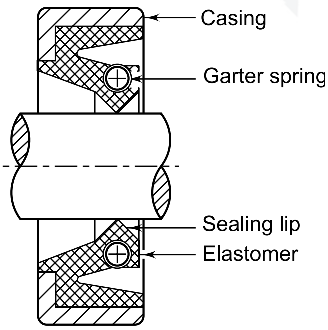


Fig. 23.1 *Construction of Oil Seal*

consists of an elastomer material, such as synthetic rubber and a circumferential spring called *garter spring*. Due to the radial pressure of the garter spring, the sealing lip rubs over the surface of the rotating shaft and prevents leakage. The diameter of the sealing lip is slightly less than the shaft diameter and consequently, the seal must be deformed slightly while it is mounted on the shaft. The commercial oil-seal units offer the following advantages:

(i) They are cheap, require small space and are easy to install.

(ii) They can be used over a wide range of lubricating oils and hydraulic fluids.

(iii) They can tolerate, to some extent, the misalignment of the shaft and vibrations.

The magnitude of contact pressure between the sealing lip and the rotating shaft is the most important parameter, affecting the performance of the seal. When the magnitude is too large, there is excessive friction, resulting in high temperature and rapid wear of the sealing lip. On the other hand, when the contact pressure is low, there is excessive leakage. The correct contact pressure results in a very thin film of lubricant between the lip and the shaft, which leads to reduced friction, prolonged life and good sealing.

The metallic casing of the oil seal is made of carbon steel, aluminium or brass. The garter spring material is usually carbon steel or stainless steel. The rubber compounds used for the lip are as follows:

(i) Nitrile compounds for general purpose lubricants up to a limiting temperature of 120°C. These compounds tend to harden in high-temperature applications.

(ii) Silicon compounds up to a limiting temperature of 175°C. These compounds, however, have poor mechanical properties and are subject to damage easily during rough handling and installation.

(iii) Fluoro elastomer compounds for a wide range of lubricants up to 200°C. They are costly compared with other rubber compounds.

The dimensions and tolerances for commercial oil seal units are given in Tables 23.1 and 23.2[1]. The housing in which the oil seals are mounted must provide a press fit for the seal and be machined to the tolerances given in Table 23.2. For proper functioning of the oil seal, the shaft should have a highly polished surface free from scratches and tool marks. Figure 23.2 shows the method of mounting the oil seal on the transmission shaft.

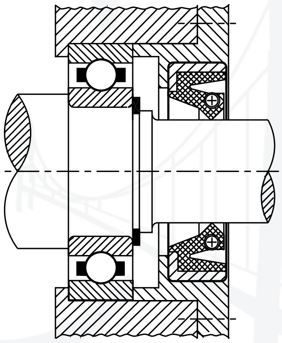


Fig. 23.2 *Mounting of Oil Seal*

Table 23.1 *Dimensions of oil seals (in mm)*

Shaft diameter	*Nominal bore diameter of housing*	*Width of seal*
10	19, 22, 24, 26	7
12	22, 24, 28, 30	7
14	24, 28, 30, 35	7
15	24, 26, 30, 32, 35	7
16	28, 30, 32, 35	7
18	30, 32, 35, 40	7
20	30, 32, 35, 40, 47	7
25	35, 40, 42, 47, 52	7
30	40, 42, 47, 52, 62	7
35	47, 50, 52, 62	7
40	52, 55, 62, 72	7
45	60, 62, 65, 72	8
50	65, 68, 72, 80	8
55	70, 72, 80, 85	8
60	75, 80, 85, 90	8
65	85, 90, 100	10
70	90, 100	10
75	95, 100	10
80	100, 110	10
90	110, 120	12
100	120, 125, 130	12

(Contd)

Table 23.1 *(Contd)*

[1] IS 5129–1979: Rotary shaft oil seal units.

Table 23.2 *Tolerances for bore diameter of housing (in mm)*

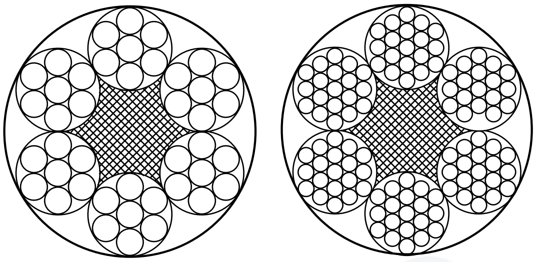
Nominal bore diameter of housing	*Tolerances for housing bore*	
	High limit	*Low limit*
Up to 50	Nominal	– 0.03
50 – 90	Nominal	– 0.03
90 – 115	+ 0.03	– 0.03
115 – 170	+ 0.03	– 0.03

23.2 WIRE ROPES

Wire ropes are extensively used in hoisting, haulage and material handling equipment. They are also used in stationary applications such as guy wires and stays. The advantages of wire ropes are as follows:

(i) high strength to weight ratio;
(ii) silent operation even at high velocities; and
(iii) greater reliability.

The constructions of the wire rope are shown in Fig. 23.3(a) and (b). The wire rope consists of a number of strands, each strand comprising several steel wires. The number of wires in each strand is generally 7, 19 or 37, while the number of strands is usually six. The individual wires are first twisted into the strand and then the strands are twisted around a fibre or steel core.



(a) 6×7 (6/1) (b) 6×19 (12/6/1)

Fig. 23.3 *Construction of Wire Rope*

The specification of wire ropes includes two numbers, such as 6 × 7 or 6 × 19. The first number indicates the number of strands in the wire rope, while the second gives the number of steel wires in each strand. The popular constructions of steel wire ropes are as follows:

6 × 7 (6/1)
6 × 19 (12/6/1)
6 × 37 (18/12/6/1)

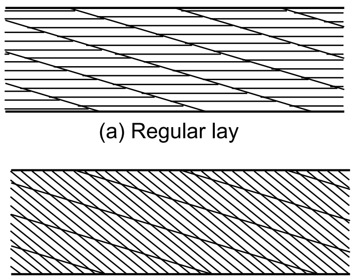
The mechanical properties of wire ropes[2] are given in Tables 23.3 and 23.4. In these tables, the nominal diameter (d_r) of the wire rope indicates the diameter of the smallest circle enclosing the wire rope. The tensile designation of wires, such as 1570 or 1770, indicates the minimum ultimate tensile strength (in N/mm^2) of the individual wires used for making the wire rope. The central portion of the wire rope is called the *core*. There are three types of cores—fibre, wire and synthetic material. The fibre core consists of natural fibres like sisal, hemp, jute or cotton. The fibre core is flexible and suitable for all conditions except when the rope is subjected to severe crushing, e.g., when working under high load. The steel core consists of another strand of fairly soft wires with lower tensile strength. The wire core is used where the wire rope is subjected to severe heat or crushing conditions. Plastic cores are used in special purpose wire ropes. It can be a plastic-impregnated fibre core, plastic covered fibre core or a solid plastic core.

Table 23.3 *Breaking load and mass for 6 × 7 (6/1) construction wire ropes*

Nominal diameter (mm) (d_r)	*Approximate mass (kg/100 m)*		*Minimum breaking load corresponding to tensile designation of wires of (kN)*					
			1570		*1770*		*1960*	
	Fibre core	*Steel core*	*Fibre core*	*Steel core*	*Fibre core*	*Steel core*	*Fibre core*	*Steel core*
8	22.9	25.2	33	36	38	41	42	45
9	28.9	31.8	42	46	48	51	53	57
10	35.7	39.1	52	56	59	64	65	70
11	43.2	47.6	63	68	71	77	79	85
12	51.5	56.6	75	81	85	91	94	101

There is one more term related to the construction of wire ropes, namely, rope-lay. *The lay of the rope refers to the manner in which the wires are helically laid into strands and the strands into the rope*. If the wires in the strand are twisted in the same direction as the strands, then the rope is called a *Lang's lay rope*. When the wires in the strand are twisted in a direction opposite to that of the strands, the rope is said to be *regular-lay* or *ordinary-lay*. The lays of wire rope are illustrated in Fig. 23.4.

[2] IS 2266–1989: Steel wire ropes for general engineering purposes-specification.



(a) Regular lay

(b) Lang's lay

Fig. 23.4 *Lays of Wire Rope*

Regular-lay ropes are more popular than the Lang's-lay ropes. The balance resulting from the opposite direction of twisting the strands to that of the wires is advantageous. Regular-lay ropes offer the following advantages:

(i) They have more structural stability.

(ii) They have more resistance to crushing and distortion.

(iii) They have less tendency to rotate under load.

(iv) There is less possibility of kinking.

(v) They are easy in handling during installation.

In Lang's-lay ropes, the same direction of twisting results in outer wires being bent on a larger arc of a circle. Lang's-lay ropes are difficult to handle and install. They are less resistant to crushing and distortion. Lang's-lay ropes are likely to untwist unless both ends are permanently fastened.

Table 23.4 *Breaking load and mass for 6 × 19 (12/6/1) construction wire ropes with fibre core*

Nominal diameter (d_r) (mm)	*Approximate mass (kg/100 m)*		*Minimum breaking load corresponding to tensile designation of (kN)*					
			1570		*1770*		*1960*	
	Fibre core	*Steel core*	*Fibre core*	*Steel core*	*Fibre core*	*Steel core*	*Fibre core*	*Steel core*
8	22.1	24.3	31	33	35	37.6	39	41.6
9	28.0	30.8	39	42	44	47.5	49	52.6
10	34.6	38.0	48	52	54	58.7	60	65
11	41.9	46	58	63	66	71.0	73	70.7
12	49.8	54	69	75	78	84.6	87	93.6
13	58.5	64.3	82	88	92	99	102	110
14	67.8	74.5	95	102	107	115	118	127
16	88.6	97.4	124	133	139	150	154	166
18	112	123.0	156	160	176	190	195	210
19	125	137	174	188	196	212	217	234
20	138	152.0	193	208	218	235	241	260
22	167	184.0	234	252	263	204	292	314
24	199	219.0	278	300	318	338	347	375
26	234	257	326	352	368	397	407	439

In the design of rope drives, it is required to select the wire rope from the manufacturer's catalogue. The guidelines for the selection of wire ropes are as follows:

(i) The strength of the wire rope depends upon the tensile strength of the individual wires. It is seen from Table 23.3 that wire ropes of tensile designation 1960 have higher load capacity than those of designation 1570. Use of steel cores in place of fibre cores increases the strength of the wire ropes to certain extent.

(ii) The flexibility of the wire rope is an important consideration where sheaves are small or where the rope makes many bends. Flexibility in wire ropes is achieved by using a large number of small-diameter wires. The wire rope of 6 × 7 construction consists of a few wires of relatively large size. It is too stiff for hoisting purposes. The 6 × 19 or 6 × 37 constructions are flexible wire ropes, and are commonly used in hoists. The 6 × 7 construction is suitable as a haulage and guy rope.

(iii) Where the wire rope is likely to drag through gritty material or across a stationary object, abrasion resistance is the governing factor. Large-diameter wires with 6 × 7 construction give better wear resistance.

The factors of safety for wire ropes for different applications are given in Tables 23.5 and 23.6.

Table 23.5 *Factors of safety for wire ropes in general applications*[3]

Application	*Class 1*	*Class 2 and 3*	*Class 4*
Fixed guys, jib cranes, ancillary applications like lifting beams	3.5	4.0	4.5
Hoisting and luffing systems of flexible cranes such as mobile derrick, guy derrick (where shock absorbing devices are incorporated in the system)	4.0	4.5	5.5
Cranes and hoists	4.5	5.0	6.0

Table 23.6 *Factor of safety for wire ropes in mining applications*

Application	*Factor of safety*
(a) Mining ropes	
For shafts of varying depths	
Up to 300 m	10
300–500	9
500–700	8
700–1000	7
(b) Haulage ropes	7

23.3 STRESSES IN WIRE ROPES

The analysis of stresses in wire rope is complicated, owing to a number of factors. The individual wires are subjected to direct tensile stress due to the load being raised, as well as to bending stresses. When the wire rope passes around the periphery of the sheave or the drum, the length of the wires in the outer portion of the rope increases, while that in the inner region decreases. This results in additional tensile stresses in outer wires. The bending stresses in one of the individual wires is given by,

$$\sigma_b = \frac{M_b y}{I} \quad \text{and} \quad y = \frac{d_w}{2}$$

Therefore,

$$\sigma_b = \frac{M_b d_w}{2I} \tag{a}$$

where,

d_w = diameter of individual wire (mm)

The elastic-curve equation is given by,

$$\frac{M_b}{EI} = \frac{1}{r}$$

The radius of curvature r in the above equation is equal to the radius of the sheave. Therefore,

$$\frac{M_b}{EI} = \frac{2}{D} \tag{b}$$

where,

D = diameter of the sheave (mm)

[3] IS 3973–1984: Code and practice for the selection, installation and maintenance of wire ropes.

From the expressions (a) and (b),

$$\sigma_b = \frac{Ed_w}{D} \tag{23.1}$$

In the above expression, E is the modulus of elasticity of the wire. However, the individual wires of the wire rope make a corkscrew figure in space, as they go around the periphery of the sheave. This configuration is different from that of the straight circular wire bending around the sheave. To account for this difference, the modulus of elasticity E is replaced by the effective modulus of elasticity E_r, called the *modulus of elasticity of the rope*. Replacing E by E_r,

$$\sigma_b = \frac{E_r d_w}{D} \tag{23.2}$$

In the design of wire ropes, it is convenient to convert the bending stresses into an equivalent bending load. It is a hypothetical tensile load that would induce the same bending stress. The equivalent bending load P_b is given by

$$P_b = \sigma_b A$$

or
$$P_b = \frac{AE_r d_w}{D} \tag{23.3}$$

where A is the area of the metallic cross-section in the wire rope. Table 23.7 gives the data for representative wire ropes.

The failure of the wire rope is mainly due to fatigue or wear, while passing around the sheave. The bending and straightening of the rope as it passes over the sheave results in fluctuating stresses leading to fatigue failure. The individual wires slide on each other and over the sheave resulting in gradual wearing of the load carrying material.

Table 23.7 *Wire-rope data*

Type of Construction	*Modulus of elasticity of rope (E_r) (N/mm^2)*	*Diameter of wire (d_w) (mm)*	*Metallic area of rope (A) (mm^2)*	*Sheave diameter (D) (mm)*	
				Minimum	*Recommended*
6 × 7	97 000	0.106 d_r	0.38 d_r^2	42 d_r	72 d_r
6 × 19	83 000	0.063 d_r	0.40 d_r^2	30 d_r	45 d_r
6 × 37	76 000	0.045 d_r	0.40 d_r^2	18 d_r	27 d_r

The amount of wear that occurs depends upon the pressure between the rope and the sheave. As shown in Fig. 23.5(a), the force per unit length of the wire rope is (pd_r). Considering equilibrium of forces in the vertical direction,

$$2P = pd_r D$$

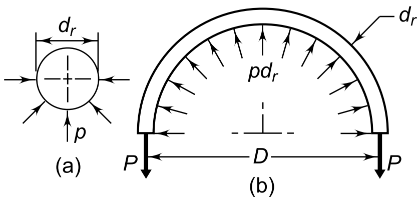


Fig. 23.5 *Forces Acting on Wire Rope around Sheave*

or
$$p = \frac{2P}{d_r D} \tag{23.4}$$

where,

P = tension in the rope (N)
d_r = nominal diameter of wire rope (mm)
D = sheave diameter (mm)

The fatigue diagram for 6 × 19 regular-lay rope is shown in Fig. 23.6[4]. The diagram is constructed by experiments. The ordinate represents a dimensionless quantity (p/S_{ut}), where S_{ut} is the ultimate tensile strength of the wire. The abscissa represents the number of bends that would cause fatigue failure of the rope. It has been observed from the fatigue diagram that the rope has long life if the

[4] DC Drucker and H Tachau—'A new design criterion for wire rope'—Trans. ASME Vol. 67, A-33.

ratio (p/S_{ut}) is less than 0.0015. The values of S_{ut} for wire materials are given in Table 23.8.

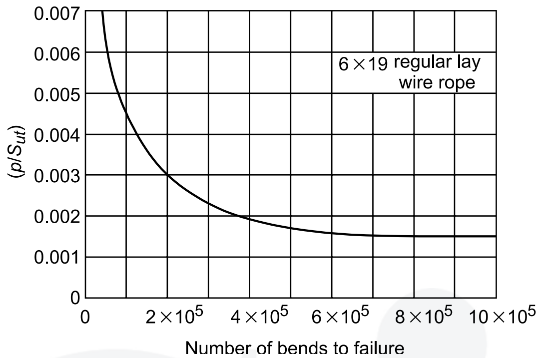


Fig. 23.6 *Relationship for Number of Bends to Failure (Experimental Data)*

Table 23.8 *Breaking strength of wire*

Material	S_{ut} *(N/mm²)*
Improved plow steel	1380
Plow steel	1210
Mild plow steel	1100
Traction steel	900
Iron	450

Example 23.1 *A temporary elevator is assembled at the construction site to raise building materials, such as cement, to a height of 20 m. It is estimated that the maximum weight of the material to be raised is 5 kN. It is observed that the acceleration in such applications is 1 m/s². 10 mm diameter, 6 × 19 construction wire ropes with fibre core are used for this application. The tensile designation of the wire is 1570 and the factor of safety should be 10 for preliminary calculations. Determine the number of wire ropes required for this application. Neglect bending stresses.*

Solution

Given $W = 5$ kN $\quad h = 20$ m $\quad \alpha = 1$ m/s²

For wire rope, construction = 6 × 19

$d_r = 10$ mm tensile designation = 1570 (fs) = 10

Let us assume that the number of wire ropes is z. The force acting on each wire rope comprises the following factors:

(i) the weight of the material to be raised;

(ii) the weight of the wire rope; and

(iii) the force due to acceleration of the material and the wire rope.

Step I *Weight of the material*

The weight of the material raised by each wire rope is given by,

$$\left(\frac{5000}{z}\right)\text{N} \tag{i}$$

Step II *Weight of the wire rope*

From Table 23.4, the mass of 100 m long wire rope is 34.6 kg. Since the height is 20 m, the weight of the wire is given by,

$$34.6\left(\frac{20}{100}\right)(9.81) \text{ or } 67.89 \text{ N} \tag{ii}$$

Step III *Force due to acceleration*

The mass of the material raised by each wire rope is $\left[\left(\frac{5000}{9.81}\right)\left(\frac{1}{z}\right)\right]$ and that of each wire rope is $\left[34.6\left(\frac{20}{100}\right)\right]$. The force due to acceleration (i.e., mass × acceleration) is given by,

$$\left[\left(\frac{5000}{9.81}\right)\left(\frac{1}{z}\right) + 34.6\left(\frac{20}{100}\right)\right](1)$$

or $$\left[\frac{509.68}{z} + 6.92\right] \text{N}$$

Step IV *Number of wire ropes*

From Table 23.4, the breaking strength of the wire rope is 48 kN. Assuming the factor of safety to be 10,

$$\frac{48\,000}{10} = \frac{5000}{z} + 67.89 + \left[\frac{509.68}{z} + 6.92\right]$$

or $$z = 1.166 \text{ or } 2 \text{ wire ropes}$$

Example 23.2 *Assume the data of Example 23.1 and determine the true factor of safety taking into account the bending stresses. The sheave diameter can be taken as ($45d_r$).*

Solution

Given $D = 45d_r$

Step I *Bending load*

From Eq. 23.3 and Table 23.7,

$$P_b = \frac{A\,E_r d_w}{D}$$

$$= \frac{(0.40d_r^2)(83\,000)(0.063d_r)}{(45d_r)}$$

$$= 46.48d_r^2 = 46.48(10)^2 = 4648 \text{ N}$$

Step II *Total load on wire rope*

The total force acting on the wire rope consists of three factors discussed in the previous example, plus the bending load. The total force is given by,

$$\left[\frac{5000}{2} + 67.89 + \frac{509.68}{2} + 6.92\right] + 4648 \text{ or } 7477.65 \text{ N}$$

Step III *Factor of safety*

$$(fs) = \frac{48\,000}{7477.65} = 6.42$$

Example 23.3 *A 6 × 19 wire rope with fibre core and tensile designation of 1570 is used to raise the load of 20 kN as shown in Fig. 23.7. The nominal diameter of the wire rope is 12 mm and the sheave has 500 mm pitch diameter. Determine the expected life of the rope assuming 500 bends per week.*

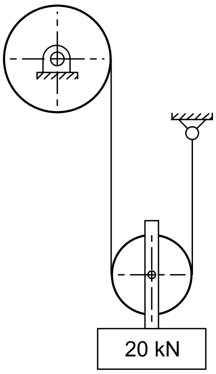


Fig. 23.7

Solution

Given $W = 20$ kN

number of bends = 500 per week

For wire rope, construction = 6 × 19

$d_r = 12$ mm tensile designation = 1570

$D = 500$ mm

Step I *(p/S_{ut}) factor*

It is observed from Fig. 23.7 that each side of the rope shares a load of (20/2) or 10 kN. Therefore, the wire rope is subjected to a maximum force of 10 kN. From Eq. 23.4,

$$p = \frac{2P}{d_r D} = \frac{2(10\times10^3)}{(12)(500)} = \left(\frac{10}{3}\right)\text{N/mm}^2$$

The ultimate tensile strength of the wires is 1570 N/mm^2. Therefore,

$$\frac{p}{S_{ut}} = \frac{10}{3(1570)} = 0.0021$$

Step II *Life of wire rope*

From Fig. 23.6, the life of the wire rope is 3.30×10^5 bends before failure,

$$\therefore \text{ life} = \frac{3.30\times10^5}{500} \text{ weeks} = \frac{3.30\times10^5}{500(52)} \text{ years}$$

$$= 12.69 \text{ years}$$

Example 23.4 *It is required to select a 6 × 19 wire rope with 1570 as tensile designation for a hoist on the basis of long life. The weight of the hoist along with the material is 5 kN. It is to be raised from a depth of 100 m. The maximum speed of 5 m/s is attained in 5 s. Determine the size of the wire rope and the sheave diameter for long life on the basis of the fatigue as failure criterion. What is the factor of safety of this wire rope under static conditions?*

Solution

Given $W = 5$ kN $h = 100$ m $v_2 = 5$ m/s

$v_1 = 0$ $t = 5$ s For wire rope,

construction = 6 × 19 tensile designation = 1570

Step I *Total load on wire rope*

The total force (P) acting on the wire rope consists of three factors—(i) the weight of the hoist; (ii) the weight of the wire rope; and (iii) the force due to acceleration. The weight of the hoist is given as

5 kN or 5000 N (i)

Referring to Table 23.4, the mass of the wire rope depends upon the nominal diameter (d_r), which is unknown at this stage. As a trial value, the mass is assumed to be 40 kg per 100 m length. Since the material is to be raised from a depth of 100 m, the length of the wire rope is assumed as 100 m. Therefore, the weight of the wire rope will be,

$$40 \times 9.81 \text{ or } 392.4 \text{ N} \qquad \text{(ii)}$$

Total mass of hoist and wire rope

$$= \left[\frac{5000}{9.81} + 40\right] \text{kg}$$

$$\text{Acceleration} = \frac{v_2 - v_1}{t} = \frac{5-0}{5} = 1 \text{ m/s}^2$$

$$\text{Acceleration force} = \left[\frac{5000}{9.81} + 40\right](1)$$

$$= 549.68 \text{ N} \qquad \text{(iii)}$$

$$\therefore \quad P = 5000 + 392.4 + 549.68 = 5942.08 \text{ N}$$

Step II *Value of (p) for long fatigue life*

For 1570 tensile designation,

$$S_{ut} = 1570 \text{ N/mm}^2$$

The wire rope has a long fatigue life if the ratio (p/S_{ut}) is equal to or less than 0.0015.

$$\therefore \quad \frac{p}{S_{ut}} = 0.0015$$

$$p = 0.0015\, S_{ut} = 0.0015(1570)$$
$$= 2.355 \text{ N/mm}^2 \qquad \text{(a)}$$

Step II *Nominal diameter of wire rope*

From Table 23.7, the recommended sheave diameter for 6 × 19 wire rope is ($45d_r$).

$$\therefore \quad D = 45d_r$$

From Eq. 23.4,

$$p = \frac{2P}{d_r D} \quad \text{or} \quad 2.355 = \frac{2P}{d_r(45d_r)}$$

$$\therefore \quad d_r^2 = \frac{P}{52.99} \text{ mm}^2 \qquad \text{(b)}$$

Substituting the value of P in the Eq. (b),

$$d_r^2 = \frac{P}{52.99} = \frac{5942.08}{52.99} \text{ or } d_r = 10.59 \text{ mm}$$

The standard nominal diameter of the wire rope is 12 mm.

$$d_r = 12 \text{ mm}$$

$$D = 45d_r = 45(12) = 540 \text{ mm}$$

Step IV *Check for design*

From Table 23.4, the mass of 100 m length wire rope of 12 mm diameter and with a fibre core is 49.8 kg.

Weight of wire = (49.8)(9.81) = 488.54 N

$$\text{Acceleration force} = \left[\frac{5000}{9.81} + 49.8\right](1)$$

$$= 559.48 \text{ N}$$

$$P = 5000 + 488.54 + 559.48 = 6048.02 \text{ N}$$

$$p = \frac{2P}{d_r D} = \frac{2(6048.02)}{12(540)} = 1.867 \text{ N/mm}^2$$

$$\frac{p}{S_{ut}} = \frac{1.867}{1570} = 0.00119$$

$$\therefore \quad p/S_{ut} < 0.0015$$

Step V *Static design*

From Table 23.4, the breaking load of a 12-mm diameter wire rope is 69 kN.

$$(fs) = \frac{69 \times 10^3}{P} = \frac{69 \times 10^3}{6048.02} = 11.4$$

23.4 ROPE SHEAVES AND DRUMS

There are three requirements for the design of rope sheaves and rope drums. They are as follows:

(i) They should have a proper size.
(ii) They should run freely on the axle.
(iii) They should have grooves with proper dimensions.

Rope sheaves and rope drums should be as large as possible to obtain maximum rope life. However, their cost increases with the size. The centrifugal force also increases with the size, particularly in high speed applications. These limitations call for relatively small sheaves and drums with consequent sacrifice in the rope life. However, there is a minimum ratio of drum or sheave diameter to rope diameter, which should be considered in the design of rope drives. The values of this ratio for different applications and wire rope constructions are given in Table 23.9.

Table 23.9 *Ratio of drum and sheave diameter to rope diameter*

Application		*Minimum ratio*		
Mining installation Winder		100		
	Haulage			
	Up to 50 kW	50		
	100 kW and above	60		
Cranes and hoists	Construction	Class-1	Class-2 and 3	Class-4
	6×37	15	17	22
	8×19			
	8×19 seale	17	18	24
	34×7 (non rotating)			
	6×19 filler wire	18	20	23
	6×19	19	23	27
	17×7 and 18×7 (non rotating)			
	6×19 seale	24	28	35

Wire rope sheaves are made of either cast iron or mild steel. Rope life depends upon contacting material of the groove on the sheave. It has been observed that ropes operating over steel sheaves wear faster than those used in conjunction with cast iron sheaves. If wear on a rope operating over a cast iron sheave is taken as unity then the wear with a steel sheave is approximately 1.1. Therefore, sheaves made of grey cast iron FG 200, are preferred over welded steel sheaves. Small sheaves are usually cast as one piece construction without ribs. Large sheaves are provided with ribs. The sheaves are always freely mounted on fixed axles. They are mounted either on rolling contact bearings or bronze bushings.

The profile of the groove in the sheave is shown in Fig. 23.8. The diameter of the groove should be always more than the rope diameter. If the groove radius (R) is too large, the rope will tend to flatten under load. On the other hand, too small a groove radius will cause rope distortion. As the rope is bent around the sheave, the strands and wires lie upon each other to adjust themselves to this curvature. When the rope is forced in an undersize groove, the strands and the wires bind against each other resulting in increased internal friction. It also hinders the readjustment of the rope under load and forces some of the wires and strands to carry more load than their share. Therefore, there are deteriorating effects of oversized and undersized grooves. The correct groove radius is given by,

Groove radius = 1.05 (nominal rope radius)

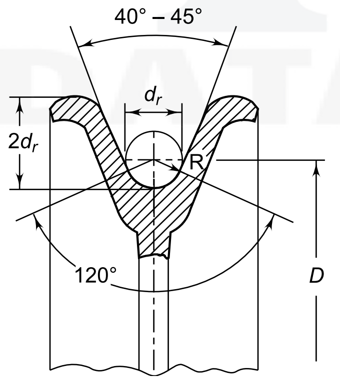


Fig. 23.8 *Sheave Groove*

The depth and the flare of the groove should be such that the rope does not rub against the flange of the sheave while entering or leaving the groove. The bottom of the groove should be a circular arc over an angle of 120°. The sides of the groove should be flared with an included angle of 40° to 45°. The depth of the groove is usually taken as twice the nominal rope diameter.

There are two types of constructions for rope drums, namely, drums with helical grooves and plain cylindrical drums without grooves. In most hoisting installations, preference is given to grooved drums instead of plain drums. The machined grooves increase the bearing surface of the drum and prevent friction between adjacent turns of the rope. Consequently, this reduces wear of rope and increases life. The drums are usually made of grey cast iron of Grade FG 200. On rare occasions, welded steel drums are used.

A grooved drum and the profile of the groove are illustrated in Fig. 23.9(a) and (b) respectively. The drum is provided with helical grooves so that the rope winds up uniformly on the drum. The radius of the helical groove should be selected so as to prevent jamming of the rope. Drums with one coiling rope have only one helical groove, i.e., right-hand helical groove. Drums designed for two rope members are provided with two helical grooves—right-hand and left-hand helical grooves. The pitch of the groove is given by,

$$t = d_r + (2 \text{ to } 3 \text{ mm})$$

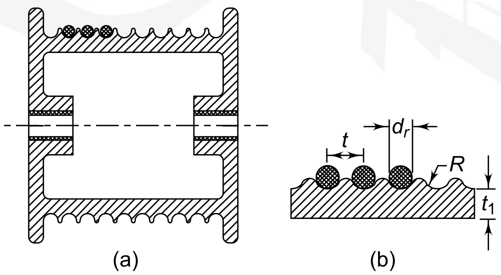


Fig. 23.9 *Rope Drum*

The shell thickness of cast iron drum is given by,

$$t_1 = 0.02\,D + (6 \text{ to } 10 \text{ mm})$$

where,

d_r = nominal diameter of rope (mm)

D = drum diameter (mm)

The groove diameter is 1.5 to 3 mm more than the rope diameter.

23.5 BUCKLING OF COLUMNS

A column or strut is a slender machine component, which has considerable length in proportion to its width, depth or diameter. A column is also called a *strut*, *pillar* or *stanchion*. Piston rods in hydraulic or pneumatic cylinders, push rods of valve mechanisms, power screws in jacks, and connecting rods are examples of columns.

When a short member is subjected to axial compressive force, as shown in Fig 23.10(a), it shortens according to Hooke's law. As the load is gradually increased, the compression of the member increases. When the compressive stress reaches the elastic limit of the material, the failure occurs in the form of bulging. However, when the length of the component is large compared with the cross-sectional dimensions, as shown in Fig. 23.10(b), the

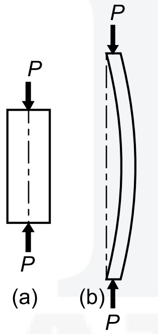


Fig. 23.10

component may fail by lateral buckling. Buckling indicates elastic instability. The load at which the buckling starts is called *critical load*, which is denoted by P_{cr}. When the axial load on the column reaches P_{cr}, there is sudden buckling and a relatively large lateral deflection occurs. Some of the rules of thumb for buckling of columns are as follows:

(i) A column made of a ductile material like steel and whose length is more than eight times of its least lateral dimension is likely to buckle and should be treated as a column.

(ii) A column made of a brittle material like cast iron and whose length is more than six times of its least lateral dimension is likely to buckle and should be treated as a column.

There is a basic difference between the lateral deflection of a beam and the buckling of columns. The lateral deflection of the beam is gradually increased as the lateral load is increased. In case of buckling, there is no such lateral deflection till the load reaches the critical load. At this point, there is sudden lateral deflection, which results in collapse of the column. The failure due to buckling is, therefore, sudden and total without any warning.

An important parameter affecting the critical load is the slenderness ratio. It is defined as,

$$\text{slenderness ratio} = \frac{l}{k} \tag{23.5}$$

where,

l = length of the column (mm)

k = least radius of gyration of the cross-section about its axis (mm)

The radius of gyration is given by,

$$k = \sqrt{\frac{I}{A}} \tag{23.6}$$

where,

I = least moment of inertia of the cross-section (mm^4)

A = area of the cross-section (mm^2)

When the slenderness ratio is less than 30, there is no effect of buckling and such components are designed on the basis of compressive stresses. Columns, with slenderness ratio greater than 30, are designed on the basis of critical load. There are two terms, namely, *short* and *long columns*, that are frequently used in buckling analysis. The rules of thumb for deciding long and short columns are as follows:

(i) Cast iron columns with a slenderness ratio not greater than 80 and steel columns with a slenderness ratio not greater than 100, are considered short columns.

(ii) Long columns are those with slenderness ratio greater than 100 for ductile materials and greater than 80 for cast iron.

There are two methods to calculate the critical load—*Euler's equation* and *Johnson's equation*.

According to Euler's equation,

$$P_{cr} = \frac{n\pi^2 EA}{(l/k)^2} \tag{23.7}$$

where,

P_{cr} = critical load (N)

n = end fixity coefficient

E = modulus of elasticity (N/mm^2)

A = area of the cross-section (mm^2)

The load carrying capacity of the column depends upon the condition of restraints at the two ends of the column. It is accounted by means of a dimensionless quantity called *end fixity coefficient* (n). The values of n are as follows:

End conditions	n
(i) Both ends hinged	1
(ii) Both ends fixed	4
(iii) One end fixed and other end hinged	2
(iv) One end fixed and other end free	0.25

According to Johnson's equation,

$$P_{cr} = S_{yt} A \left[1 - \frac{S_{yt}}{4n\pi^2 E}\left(\frac{l}{k}\right)^2\right] \tag{23.8}$$

where S_{yt} is the yield strength of the material.

In order to study the above two equations, we will consider a numerical example. Let us consider a column with both ends hinged and made of steel 45C8 (S_{yt} = 380 N/mm^2 and E = 207 000 N/mm^2). According to Euler's equation,

$$\left(\frac{P_{cr}}{A}\right)_1 = \frac{n\pi^2 E}{(l/k)^2} = \frac{(1)\pi^2(207\,000)}{(l/k)^2}$$

$$= \frac{2\,043\,008}{(l/k)^2}\ \text{N/mm}^2 \tag{a}$$

According to Johnson's equation,

$$\left(\frac{P_{cr}}{A}\right)_2 = S_{yt}\left[1 - \frac{S_{yt}}{4n\pi^2 E}\left(\frac{l}{k}\right)^2\right]$$

$$= 380\left[1 - \frac{380}{4(1)\pi^2(207\,000)} \times \left(\frac{l}{k}\right)^2\right]$$

$$= 380\left[1 - \frac{(l/k)^2}{21505}\right] \tag{b}$$

The values of (P_{cr}/A) for different values of slenderness ratio (l/k) are tabulated as follows.

(l/k)	60	80	100	103.7	120	140	160	180
$(P_{cr}/A)_1$	568	319	204	190	142	104	80	63
$(P_{cr}/A)_2$	316	267	203	190	126	34	–	–

The graph of unit load (P_{cr}/A) against slenderness ratio (l/k) is shown in Fig. 23.11. The following observations are made from Fig. 23.11.

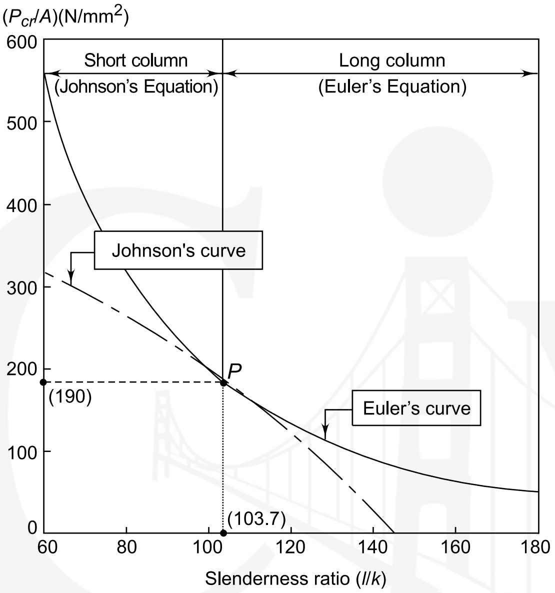


Fig. 23.11 *Variation of Unit Load Against Slenderness Ratio-Euler's & Johnson's Criteria*

(i) When the slenderness ratio is 60, the unit load according to Euler's equation is 568 N/mm^2, while the yield strength of the material is 380 N/mm^2. Therefore, Euler's equation is illogical in this range.

(ii) When the slenderness ratio is more than 150, Johnson's equation gives negative values of unit load, which is illogical.

(iii) The curves given by Euler's and Johnson's equations are tangential at the point P, where the unit load (P_{cr}/A) is equal to $(S_{yt}/2)$, i.e., 190 N/mm^2. The slenderness ratio at this point can be considered as the boundary line between short and long columns.

In design analysis, the question always arises as to whether one should use Euler's equation or Johnson's equation. From the above observations, it is concluded that Euler's equation is suitable for long columns, while Johnson's equation, for short columns. The boundary line between the two is defined by equating the unit load (P_{cr}/A) to half the yield strength.

or,

$$\frac{P_{cr}}{A} = \frac{S_{yt}}{2} \tag{c}$$

From Eq. (23.7),

$$\frac{P_{cr}}{A} = \frac{n\pi^2 E}{(l/k)^2} \tag{d}$$

From Eqs (c) and (d),

$$\frac{S_{yt}}{2} = \frac{n\pi^2 E}{(l/k)^2} \tag{23.9}$$

The ratio (l/k) obtained by Eq. (23.9) is the critical slenderness ratio between long and short columns. When the actual slenderness ratio is less than the critical slenderness ratio, Johnson's equation is used. When the actual slenderness ratio is more than the critical slenderness ratio, Euler's equation should be used.

Example 23.5 *A 25 × 50 mm bar of rectangular cross-section is made of plain carbon steel 40C8 (S_{yt} = 380 N/mm^2 and E = 207 000 N/mm^2). The length of the bar is 500 mm. The two ends of the bar are hinged and the factor of safety is 2.5. The bar is subjected to axial compressive force.*

(i) Determine the slenderness ratio;

(ii) Which of the two equations—Euler's or Johnson's—will you apply to the bar?

(iii) What is the safe compressive force for the bar?

Solution

Given For bar, cross-section = 25 × 50 mm

l = 500 mm S_{yt} = 380 N/mm^2

E = 207 000 N/mm^2 (fs) = 2.5

Step I *Slenderness ratio*

$$I = \frac{(50)(25)^3}{12}\ \text{mm}^4$$

$$k = \sqrt{\frac{I}{A}} = \sqrt{\left[\frac{(50)(25)^3}{12(50 \times 25)}\right]} = 7.22 \text{ mm}$$

$$\left(\frac{l}{k}\right) = \frac{500}{7.22} = 69.25 \qquad \text{(i)}$$

Step II *Selection of equation*

The boundary line between Johnson's and Euler's equations is given by

$$\frac{S_{yt}}{2} = \frac{n\pi^2 E}{(l/k)^2} \quad \text{or} \quad \frac{380}{2} = \frac{(1)\pi^2 (207\,000)}{(l/k)^2}$$

$$\therefore \qquad \left(\frac{l}{k}\right) = 103.7 \qquad \text{(ii)}$$

Since the slenderness ratio of the bar (69.25) is less than 103.7, the bar is treated as a short column and Johnson's equation is applicable.

Step III *Safe compressive force on bar*

$$P_{cr} = S_{yt} A \left[1 - \frac{S_{yt}}{4n\pi^2 E}\left(\frac{l}{k}\right)^2\right]$$

$$= (380)(25 \times 50)\left[1 - \frac{380}{4(1)\pi^2 (207\,000)(69.25)^2}\right]$$

or $\qquad P_{cr} = 369\,077.88$ N

The safe compressive force is given by

$$P = \frac{P_{cr}}{(fs)} = \frac{369\,077.88}{2.5} = 147\,631.15 \text{ N} \qquad \text{(iii)}$$

Example 23.6 *A trunnion mounted hydraulic cylinder is shown in Fig. 23.12. The internal diameter of the cylinder is 75 mm and the maximum operating pressure in the cylinder is 25 N/mm². The piston rod is made of steel 40Cr1 (S_{yt} = 530 N/mm² and E = 207 000 N/mm²). For buckling considerations, the effective length of the piston rod is considered as the distance between the trunnion and the clevis-mount, when the piston rod is extended to its full working stroke and this distance is 1000 mm. Determine the diameter of the piston rod, if the factor of safety is 2.5.*

Solution

Given For piston rod, $l = 1000$ mm $(fs) = 2.5$
$S_{yt} = 530$ N/mm² $E = 207\,000$ N/mm²
For cylinder, $D = 75$ mm $p = 25$ N/mm²

Step I *Estimation of critical load*

Although one end of the piston rod is fixed in the piston, considering the complete assembly between trunnion and clevis-mount, the end fixity coefficient is taken as one (both ends hinged). The maximum force on the piston rod is given by,

$$P = \frac{\pi}{4} D^2 p = \frac{\pi}{4}(75)^2 (25) = 110\,446.6 \text{ N}$$

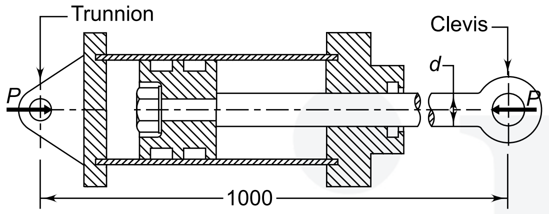


Fig. 23.12 *Hydraulic Cylinder*

Using a factor of safety of 2.5,

$P_{cr} = 2.5P = 2.5(110\,446.6) = 276\,116.5$ N

Step II *Diameter of piston rod*

For circular cross-section,

$$k = \sqrt{\frac{I}{A}} = \sqrt{\left[\frac{(\pi d^4)/64}{(\pi d^2/4)}\right]} = \left(\frac{d}{4}\right) \text{mm}$$

At this stage, it is not clear whether one should use Euler's or Johnson's equation. Using Euler's equation as a first trial,

$$P_{cr} = \frac{n\pi^2 EA}{(l/k)^2}$$

$$(276\,116.5) \text{ or } = \frac{(1)\pi^2 (207\,000)(\pi d^2/4)}{\left(\frac{1000}{d/4}\right)^2}$$

$$\therefore \qquad d = 40.73 \text{ mm}$$

Step III *Check for design*

$$\left(\frac{l}{k}\right) = \frac{1000}{(40.73/4)} = 98.21 \qquad \text{(i)}$$

The boundary line between Euler's and Johnson's equations is given by

$$\frac{S_{yt}}{2} = \frac{n\pi^2 E}{(l/k)^2} \quad \text{or} \quad \frac{530}{2} = \frac{(1)\pi^2 (207\,000)}{(l/k)^2}$$

$$\left(\frac{l}{k}\right) = 87.8 \quad \text{(ii)}$$

In this example, the slenderness ratio (98.21) is greater than the boundary value of (87.8). Therefore the assembly is treated as a long column and Euler's equation used in the first trial is justified.

$$d = 40.73 \text{ mm or } 42 \text{ mm}$$

Example 23.7 *A column of hollow rectangular cross-section and made of steel plates is shown in Fig. 23.13. The thickness of the section is 2.5 mm throughout. The plate material is steel 30C8 (S_{yt} = 400 N/mm² and E = 207 000 N/mm²). The end fixity coefficients can be taken as 1.5 and 1 for bending about long and short axes respectively. The effective length of the column is 1 m. Determine the load capacity of the column from buckling consideration.*

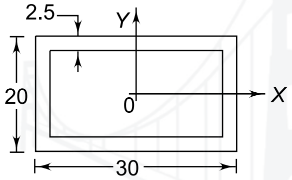


Fig. 23.13 *Cross–section of Hollow Column*

Solution

Given S_{yt} = 400 N/mm² E = 207000 N/mm²
l = 1 m n = 1.5 (for bending about long axis)
n = 1 (for bending about short axis)

Step I *Slenderness ratio about XX and YY axis*

$$A = 30 \times 20 - 25 \times 15 = 225 \text{ mm}^2$$

$$I_{xx} = \frac{1}{12}\left[30\,(20)^3 - 25\,(15)^3\right] = 12\,968.75 \text{ mm}^4$$

$$I_{yy} = \frac{1}{12}\left[20\,(30)^3 - 15\,(25)^3\right] = 25\,468.75 \text{ mm}^4$$

$$k_{xx} = \sqrt{\left(\frac{I_{xx}}{A}\right)} = \sqrt{\frac{12\,968.75}{225}} = 7.59 \text{ mm}$$

$$k_{yy} = \sqrt{\left(\frac{I_{yy}}{A}\right)} = \sqrt{\frac{25\,468.75}{225}} = 10.64 \text{ mm}$$

$$\left(\frac{l}{k_{xx}}\right) = \frac{1000}{7.59} = 131.75 \quad \text{(i)}$$

$$\left(\frac{l}{k_{yy}}\right) = \frac{1000}{10.64} = 93.98 \quad \text{(ii)}$$

Step II *Critical load along XX-axis*

$$\frac{S_{yt}}{2} = \frac{n\pi^2 E}{(l/k)^2} \quad \text{or} \quad \frac{400}{2} = \frac{(1.5)\,\pi^2\,(207\,000)}{\left(\dfrac{l}{k_{xx}}\right)^2}$$

$$\therefore \quad \left(\frac{l}{k_{xx}}\right) = 123.78 \quad \text{(iii)}$$

From (i) and (iii), the column is treated as a long column. Using Euler's equation,

$$P_{cr} = \frac{n\pi^2 EA}{(l/k_{xx})^2} = \frac{(1.5)\,\pi^2\,(207\,000)(225)}{(131.75)^2}$$

$$= 39\,723.05 \text{ N} \quad \text{(a)}$$

Step III *Critical load along YY-axis*

$$\frac{S_{yt}}{2} = \frac{n\pi^2 E}{(l/k_{yy})^2} \quad \text{or} \quad \frac{400}{2} = \frac{(1)\,\pi^2\,(207\,000)}{\left(\dfrac{l}{k_{yy}}\right)^2}$$

$$\therefore \quad \left(\frac{l}{k_{yy}}\right) = 101.07 \quad \text{(iv)}$$

From (ii) and (iv), the column is treated as a short column. Using Johnson's equation,

$$P_{cr} = S_{yt}\,A\left[1 - \frac{S_{yt}}{4n\pi^2 E}\left(\frac{l}{k_{yy}}\right)^2\right]$$

$$= (400)(225)\left[1 - \frac{400}{4(1)\pi^2\,(207\,000)}(93.98)^2\right]$$

$$\text{or} \quad P_{cr} = 51\,091.61 \text{ N} \quad \text{(b)}$$

Step IV *Load carrying capacity*

From (a) and (b), the load carrying capacity of the column is 39 723.05 N.

Example 23.8 *It is required to design the screw of a screw-jack by buckling consideration. One end* *of the screw is fixed in the nut and the other end supports a load of 20 kN. The length of the screw between the fixed and free ends is 500 mm, when the load is completely raised. The screw is made of steel 40C8 (S_{yt} = 380 N/mm^2 and E = 207 000 N/mm^2). Assuming a factor of safety of 2.5, determine the core diameter of the screw.*

Solution

Given $S_{yt} = 380$ N/mm^2 $\quad E = 207\,000$ N/mm^2
$l = 500$ mm $\quad P = 20$ kN $\quad (fs) = 2.5$

Step I *Diameter of screw using Euler's equation*

The end fixity coefficient is 0.25 when one end is fixed and the other free. As a first trial, using Euler's equation,

$$P_{cr} = \frac{n\pi^2 EA}{(l/k)^2}$$

$$(20\,000)(2.5) = \frac{(0.25)\pi^2(207\,000)\quad(\pi d^2/4)}{\left[\dfrac{500}{(d/4)}\right]^2}$$

or $\quad d = 26.57$ mm

Step II *Check for design*

$$k = d/4 = 6.64 \text{ mm}$$

$$\left(\frac{l}{k}\right) = \frac{500}{6.64} = 75.3 \qquad \text{(i)}$$

The border line between Euler's and Johnson's equations is given by

$$\frac{S_{yt}}{2} = \frac{n\pi^2 E}{(l/k)^2} \quad \text{or} \quad \frac{380}{2} = \frac{(0.25)\pi^2(207\,000)}{(l/k)^2}$$

$$\therefore \quad \left(\frac{l}{k}\right) = 51.85 \qquad \text{(ii)}$$

From (i) and (ii), it is concluded that the screw is a long column and Euler's equation considered in the first trial is justified. Therefore,

$$d = 26.57 \text{ mm or } 28 \text{ mm}$$

Example 23.9 *A piston rod of rectangular cross-section, with both ends hinged, is shown in Fig. 23.14. It is made of steel 40C8 (S_{yt} = 380 N/mm^2 and E = 207 000 N/mm^2) and subjected to an axial compressive force of 15 kN. Determine the ratio of (b/d) for equal buckling strength in either plane. Also determine the dimensions of cross-section, if the factor of safety is 4.*

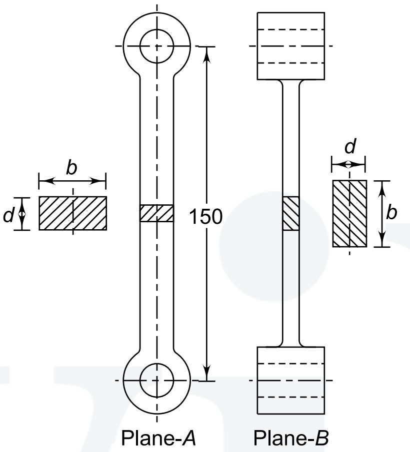


Fig. 23.14 *Piston Rod*

Solution

Given $S_{yt} = 380$ N/mm^2 $\quad E = 207\,000$ N/mm^2
$P = 15$ kN $\quad (fs) = 4 \quad l = 150$ mm

Step I *Ratio (b/d) by using Euler's equation*

For the purpose of convenience, the planes are called *A* and *B* as shown in the figure.

In the plane-*A*, the ends are hinged.

$$n_A = 1 \quad \text{and} \quad I_A = \left(\frac{db^3}{12}\right) \qquad \text{(i)}$$

In the plane-*B*, the ends are fixed.

$$n_B = 4 \quad \text{and} \quad I_B = \left(\frac{bd^3}{12}\right) \qquad \text{(ii)}$$

Using Euler's equation and equating the buckling load in two planes,

$$\frac{n_A\pi^2 EA}{\left(\dfrac{l}{k_A}\right)^2} = \frac{n_B\pi^2 EA}{\left(\dfrac{l}{k_B}\right)^2}$$

or $\quad n_A k_A^2 = n_B k_B^2$

Substituting $k^2 = I/A$,

$$n_A I_A = n_B I_B$$

$$(1)\left(\frac{db^3}{12}\right) = (4)\left(\frac{bd^3}{12}\right)$$

or
$$\left(\frac{b}{d}\right) = 2$$

Step II *Dimensions of cross-section*

As a first trial, using Johnson's equation in the plane *A*,

$$k_A^2 = \frac{I_A}{A} = \left(\frac{db^3}{12}\right)\left(\frac{1}{bd}\right) = \left(\frac{b^2}{12}\right)$$

$$\left(\frac{l}{k_A}\right)^2 = \frac{(150)^2}{\left(\frac{b^2}{12}\right)} = \frac{270\,000}{b^2}$$

$$P_{cr} = S_{yt}\, A\left[1 - \frac{S_{yt}}{4n\pi^2 E}\left(\frac{l}{k}\right)^2\right]$$

$$(15 \times 10^3 \times 4)$$

$$= (380)\,(0.5\,b^2)\left[1 - \frac{380}{4\,(1)\,\pi^2\,(207\,000)}\left(\frac{270\,000}{b^2}\right)\right]$$

$$b = 18.12 \text{ mm}$$

$$\left(\frac{l}{k_A}\right)^2 = \frac{270\,000}{b^2} = \frac{270\,000}{(18.12)^2} = 28.68$$

Since the slenderness ratio is small, Johnson's equation is justified.

$\therefore$ $b = 18.12$ mm or 20 mm

$a = b/2 = 20/2 = 10$ mm

Short-Answer Questions

23.1 What are the functions of oil seal?
23.2 What is garter spring in oil seal? What is its function?
23.3 What are the advantages of commercial oil seal unit?
23.4 What are the functions of wire rope?
23.5 Give practical applications of wire rope.
23.6 What are the advantages of wire rope?
23.7 What does the two numbers in wire rope specification indicate?
23.8 Where do you use fiber-core wire rope?
23.9 Where do you use steel-core wire rope?
23.10 What is the lay of rope?
23.11 What is Lang's-lay wire rope?
23.12 What is regular-lay wire rope?
23.13 What are the advantages of regular-lay wire rope over Lang's-lay wire rope?
23.14 What does the tensile designation of wire rope indicate?
23.15 Which is flexible wire rope—6 × 19 or 6 × 37?
23.16 Give practical examples of column.
23.17 What is slenderness ratio?
23.18 What do you understand by long and short columns?
23.19 When do you use Euler's equation for buckling of columns?
23.20 When do you use Johnson's equation for buckling of columns?

Problems for Practice

23.1 6 × 19 wire ropes with fibre core and nominal diameter of 10 mm are used for a hoist. The tensile designation of wires is 1770. The mass of the wire rope is 34.6 kg per 100 m length, while the breaking load is 54 kN. The weight of the hoist along with the material is 10 kN, which is raised through a distance of 3 m. The maximum acceleration during the operation is limited to 1 m/s^2. Neglecting bending stresses and assuming a preliminary factor of safety of 10, determine the required number of wire ropes.

[*3*]

23.2 Assume the data of Example 23.1 and determine the true factor of safety taking into account the bending stresses. The sheave diameter is 450 mm and there are three wire ropes.

[*6.48*]

23.3 A 6 × 19 wire rope with fibre core, is used to raise a load. The tensile designation of wires is 1770. The nominal diameter of the wire rope and the sheave diameter are 10 and 450 mm respectively. Assuming long life (p/S_{ut} = 0.0015) on the basis of fatigue consideration, determine the maximum load that the wire rope can carry.

[*5973.75 N*]

23.4 It is required to design the piston rod of a steam engine on the basis of buckling strength. The internal diameter of the cylinder is 200 mm, while the operating steam pressure is limited to 1 N/mm^2. The length of the piston rod is 1 m. One end of the piston rod is fixed in the piston, while the other can be considered as hinged. The piston rod is made of steel 40C8 (S_{yt} = 380 N/mm^2 and E = 207 000 N/mm^2). The factor of safety is 5. Neglecting inertia forces, determine the diameter of the piston rod.

[*29.97 mm*]

23.5 In a screw jack, one end of the screw is fixed in the nut and the other end supports a load of 10 kN. The length of the screw between the nut and the free end is 500 mm, when the load is completely raised. The screw is made of steel 40C8 (S_{yt} = 380 N/mm^2 and E = 207 000 N/mm^2). The nominal diameter and the pitch of the screw are 30 mm and 6 mm respectively. The screw has square threads. Determine the factor of safety from buckling considerations.

[*3.33*]

23.6 The link of a mechanism is subjected to an axial compressive force. It has solid circular cross-section with diameter of 6 mm and length of 300 mm. The two ends of the link are hinged. It is made of steel 30C8 (S_{yt} = 400 N/mm^2 and E = 207 000 N/mm^2). Assuming a factor of safety of 3.5, determine the safe axial force that the link can carry without buckling.

[*412.6 N*]

Statistical Considerations in Design

Chapter 24

24.1 FREQUENCY DISTRIBUTION

Statistics deals with drawing conclusions from a given or observed data. Statistical techniques are used for collection, processing, analysis and interpretation of numerical data. On the basis of statistical analysis, valid conclusions are drawn and reasonable decisions are taken. Statistics enables the engineers to understand the phenomena of variations and how to effectively predict and control them. Statistics has made valuable contributions in the areas of product design and manufacture and effective use of material and labour.

The basic data consists of observations, such as the diameters of shafts manufactured in one shift. In this case, the group of all shafts is called '*population*'. When the group is large, it is not possible to take observations of the entire population. In such cases, only a small portion of the population is examined and this portion is called a '*sample*'. *Population is defined as a collection of all elements we are studying and about which we are trying to draw conclusions. On the other hand, a sample is defined as a collection of some, but not all, of the elements of the population.* A sample is a part of the population but the converse is not true. It is easier to study a sample than studying the whole population. It costs less and takes less time. A representative sample has the characteristics of the population in the same proportions, as they are included in the entire population. Therefore, many times a sample is analysed instead of the entire population.

Let us consider an example of 100 shafts of hydrodynamic bearing, with recommended tolerance of 40e7. The shafts are manufactured on lathe and finished on grinding machine. Their diameters are measured by micrometer and the readings are tabulated in Table 24.1. The readings in Table 24.1 are called '*raw data*'. *A data is defined as the collection of numbers belonging to observations of one or more variables.* In this case, the diameter of shaft is a variable and one hundred numerical readings taken by micrometer are a data. Raw data is a data before it is arranged or analysed by any statistical method. Raw data does not show any pattern or trend of population and does not lead to any conclusions. In this chapter, the objective of statistical techniques is to rearrange and present the data in a useful way so that the decisions can be taken.

Let us rearrange the data given in Table 24.1 on the basis of diameter of shaft against the number of shafts. As shown in Table 24.2, a particular diameter such as 39.926 mm is written in the first column and each shaft with this diameter is shown by a mark X against it. Finally, the number of marks are counted and written in the last column. For example, the total number of shafts with a diameter of 39.940 mm is 8. The data in Table 24.2 is further rearranged in the form of frequency distribution table and presented in Table 24.3. *Frequency distribution is defined as an organised display of data that shows the number of observations that fall into different classes.*

Table 24.1 *Readings of diameter of 100 shafts (in mm)*

39.944	39.939	39.940	39.938	39.937
39.932	39.941	39.936	39.932	39.941
39.938	39.929	39.939	39.941	39.939
39.934	39.939	39.943	39.936	39.926
39.939	39.943	39.932	39.943	39.944
39.935	39.938	39.938	39.935	39.939
39.937	39.937	39.937	39.940	39.936
39.940	39.950	39.938	39.945	39.938
39.941	39.944	39.942	39.948	39.931
39.938	39.927	39.937	39.939	39.935
39.946	39.935	39.938	39.928	39.947
39.941	39.933	39.949	39.933	39.939
39.932	39.937	39.940	39.938	39.946
39.936	39.939	39.931	39.941	39.933
39.937	39.936	39.932	39.936	39.937
39.946	39.942	39.936	39.942	39.939
39.940	39.943	39.945	39.937	39.931
39.943	39.931	39.930	39.940	39.936
39.939	39.942	39.940	39.942	39.940
39.933	39.937	39.936	39.938	39.933

Table 24.2 *Diameter against number of shafts*

Diameter (mm)	*Number of shafts*	*Total*
39.926	X	1
39.927	X	1
39.928	X	1
39.929	X	1
39.930	X	1
39.931	XXXX	4
39.932	XXXXX	5
39.933	XXXXX	5
39.934	X	1
39.935	XXXX	4
39.936	XXXXX XXXX	9
39.937	XXXXX XXXXX	10
39.938	XXXXX XXXXX	10
39.939	XXXXX XXXXX X	11
39.940	XXXXX XXX	8
39.941	XXXXX X	6
39.942	XXXXX	5
39.943	XXXXX	5
39.944	XXX	3
39.945	XX	2
39.946	XXX	3
39.947	X	1
39.948	X	1
39.949	X	1
39.950	X	1

(Contd)

Table 24.2 *(Contd)*

Table 24.3 *Frequency distribution table*

Shaft diameter (mm)	*No. of shafts*
39.926 – 39.930	5
39.931 – 39.935	19
39.936 – 39.940	48
39.941 – 39.945	21
39.946 – 39.950	7
Total	100

It is observed from Table 24.3 that the first 'class' consists of shafts with diameters ranging from 39.926 mm to 39.930 mm. The number of observations belonging to each class is called '*class frequency*'. Thus, the class frequency of the first class is 5. The range 39.926 to 39.930 mm, which defines the class, is called the '*class interval*' and the limits 39.926 mm and 39.930 mm are called lower and upper class limits. The difference between the limits, i.e., (39.930 – 39.925) = 0.005 mm, is called '*class width*'. In Table 24.3, the classes are equal, that is, the width of the interval from beginning of one class to the beginning of the next class is same for every class. Equal classes are always preferred in statistical analysis. When classes are unequal, the distribution is more difficult to interpret.

There are two methods of representing the frequency distribution namely, *histogram* and *frequency polygon* as shown in Fig. 24.1. The histogram consists of a set of rectangles. The base of the rectangle on the *x*-axis is equal to the class width, and the area of the rectangle is proportional to the class frequency. When the class widths are of equal size, the heights of rectangles are proportional to the class frequencies. *The frequency polygon is a line graph of class frequency plotted against class marks or midpoints of class intervals*. When a large number of observations are taken and very small class widths are selected, the frequency polygon becomes an approximate curve. One such frequency curve, which is widely used in the statistical analysis, is the *normal curve* as shown in Fig. 24.2.

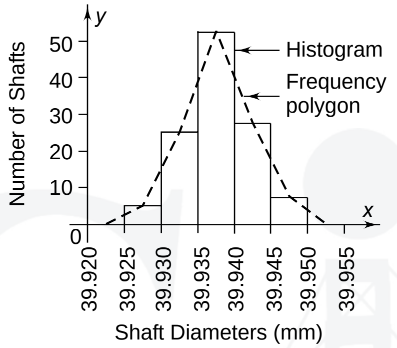


Fig. 24.1 *Frequency Distribution*

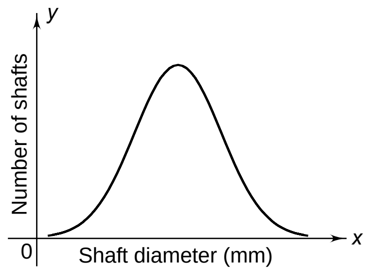


Fig. 24.2 *Frequency Curve*

24.2 CHARACTERISTICS OF FREQUENCY CURVES

It is observed from the frequency polygon and normal curve illustrated in Fig. 24.1 and Fig. 24.2, that most of the observations in engineering applications present a well behaved picture, which rises and falls smoothly. There is always a central tendency, where most of the observations cluster. *Central tendency* is the middle point of distribution. It is sometimes, referred as the '*measure of location*'. In Fig. 24.3, the central location of curve-2 lies to the right of those of curve-1 and curve-3. It is also observed that the central location of curve-1 is equal to that of curve-3. There are certain observations that tend to spread about an average value called '*variation*' or '*dispersion*' of a population. *Dispersion is defined as the spread of the data in a distribution, that is, the extent to which the observations are scattered*. In Fig. 24.4, curve-1 has a wider spread or dispersion than curve-2. The central tendency and dispersion are the two important characteristics of frequency distribution.

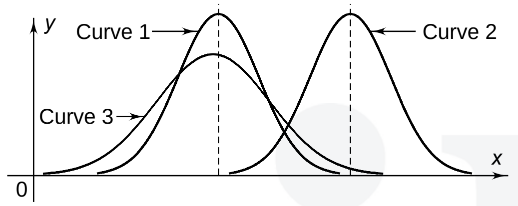


Fig. 24.3 *Comparison of Central Tendencies*

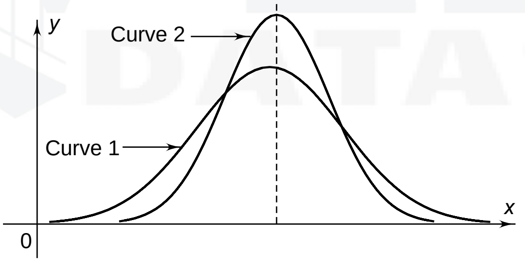


Fig. 24.4 *Comparison of Dispersions*

There are two more characteristics of population, namely, *skewness* and *kurtosis* as shown in Fig. 24.5 and Fig. 24.6 respectively. Curves representing data in statistical analysis are of two types—symmetrical and skewed. The normal curve illustrated in Fig. 24.2 is an example of a symmetrical curve. It rises and falls smoothly with a bell shape. A symmetrical curve is widely used in statistical analysis of tolerances. Curves 1 and 2 in

Fig. 24.5 are skewed. In *skewed curves*, the values in frequency distribution are concentrated at either the low end or the high end of the measuring scale on the horizontal axis. The values are not equally distributed. Curve-1 is skewed to the right because it tails off toward the high end of the scale. It is also called a *positively skewed curve*. Curve-2 is skewed

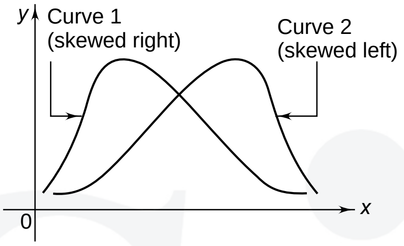


Fig. 24.5 *Comparison of Two Skewed Curves*

to the left because it tails off toward the low end of the scale. It is called a *negatively skewed curve*. The Weibull distribution used in reliability analysis of rolling contact bearing is an example of a skewed curve. *Kurtosis* is the measure of sharp peaks. In Fig. 24.6, curves 1 and 2 differ in the shape of their peaks. Curve-2 has a sharper peak than curve-1. They have the same central tendency and both are symmetrical. However, they have different degrees of kurtosis. In this chapter, we will restrict the analysis to normal curves.

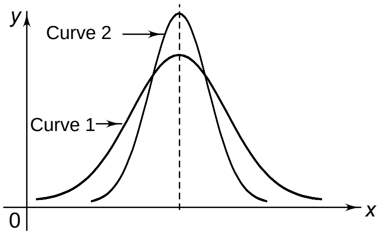


Fig. 24.6 *Comparison of Kurtosis*

24.3 MEASURES OF CENTRAL TENDENCY AND DISPERSION

There are different measures of central tendency, such as the *mean*, the *median* or the *mode*. The most popular unit to measure the central tendency is the arithmetic mean denoted by the letter μ. Suppose the population consists of N observations $X_1, X_2, \ldots, X_N$. The mean is given by

$$\mu = \frac{X_1 + X_2 + \ldots + X_N}{N}$$

or
$$\mu = \frac{\sum X_i}{N} \tag{24.1}$$

If observations X_1, X_2, …, X_k occur f_1, f_2, …, f_k times respectively (i.e., occur with frequencies f_1, f_2, …, f_k), the arithmetic mean is given by

$$\mu = \frac{f_1 X_1 + f_2 X_2 + \ldots + f_k X_k}{f_1 + f_2 + \ldots + f_k}$$

or
$$\mu = \frac{\sum f_i X_i}{\sum f_i}$$

Since, $\Sigma f_i = N$ = total number of observations, the mean is given by

$$\mu = \frac{\sum f_i X_i}{N} \tag{24.2}$$

The dispersion is measured in number of units like the *range*, the *mean deviation* or the *standard deviation*. The most popular unit for dispersion is the standard deviation denoted by the letter $\hat{\sigma}$. *The standard deviation is defined as the root mean square deviation from the mean.*

Therefore,

$$\hat{\sigma} = \sqrt{\frac{(X_1 - \mu)^2 + (X_2 - \mu)^2 + \ldots + (X_N - \mu)^2}{N}}$$

or,
$$\hat{\sigma} = \sqrt{\frac{\Sigma (X_i - \mu)^2}{N}} \tag{24.3}$$

When observations X_1, X_2, …, X_k occur at frequencies f_1, f_2, …, f_k the standard deviation is given by

$$\hat{\sigma} = \sqrt{\frac{f_1(X_1 - \mu)^2 + f_2(X_2 - \mu)^2 + \ldots + f_k(X_k - \mu)^2}{f_1 + f_2 + \ldots + f_k}}$$

$$= \sqrt{\frac{\sum f_i (X_i - \mu)^2}{\sum f_i}}$$

Therefore,

$$\hat{\sigma} = \sqrt{\frac{\sum f_i (X_i - \mu)^2}{N}} \tag{24.4}$$

Squaring both sides,

$$(\hat{\sigma})^2 = \frac{1}{N} \sum f_i (X_i - \mu)^2$$

$$= \frac{1}{N} \sum f_i (X_i^2 - 2X_i\mu + \mu^2)$$

Using the relationship,

$$\Sigma(aX_i + bY_i - cZ_i) = a\Sigma X_i + b\Sigma Y_i - c\Sigma Z_i$$

the expression for $(\hat{\sigma})^2$ is written as

$$(\hat{\sigma})^2 = \frac{\sum f_i X_i^2}{N} - \frac{2\mu \sum f_i X_i}{N} + \frac{\mu^2 \sum f_i}{N} \quad \text{(a)}$$

Since $\mu = \dfrac{\sum f_i X_i}{N}$

hence, $\Sigma f_i X_i = \mu N$ (b)

and $\Sigma f_i = N$ (c)

From (a), (b) and (c),

$$(\hat{\sigma})^2 = \frac{\sum f_i X_i^2}{N} - \mu^2$$

$$= \frac{\sum f_i X_i^2}{N} - \frac{(\sum f_i X_i)^2}{N^2}$$

$$\therefore \quad (\hat{\sigma})^2 = \frac{\sum f_i X_i^2 - \frac{(\sum f_i X_i)^2}{N}}{N} \quad (24.5)$$

When observations belong to a sample of a population, it has been observed that by replacing N by $(N - 1)$ in the denominator of Eq. 24.5, a better estimate of the standard deviation is obtained. For large values of N ($N > 30$), there is practically no difference between these two expressions. Therefore, the standard deviation in such cases is given by

$$S^2 = \frac{\sum f_i X_i^2 - \frac{(\sum f_i X_i)^2}{N}}{(N-1)} \quad (24.6)$$

where S is the standard deviation of observations belonging to the sample of the population.

Variance is defined as the square of the standard deviation.

A standard variable Z is defined as

$$Z = \frac{X - \mu}{\hat{\sigma}} \quad (24.7)$$

The standard variable measures the deviation from the mean in the units of the standard deviation.

Example 24.1 *One hundred test specimens made of grey cast iron FG 300 are tested on a universal testing machine to determine the ultimate tensile strength (S_{ut}) of the material. The results are tabulated as follows:*

Class interval (N/mm²)	*Frequency*
261–280	*2*
281–300	*12*
301–320	*50*
321–340	*32*
341–360	*4*

Calculate: (i) the mean; (ii) the variance; and (iii) the standard deviation for this sample.

Solution

Step I *Mean*

Class mark (X_i)	*Frequency (f_i)*	*f_iX_i*	*$f_iX_i^2$*
270	2	540	145 800
290	12	3 480	1 009 200
310	50	15 500	4 805 000
330	32	10 560	3 484 800
350	4	1 400	490 000
Total	100	31 480	9 934 800

From Eq. 24.2,

$$\mu = \frac{\sum f_i X_i}{\sum f_i} = \frac{31\,480}{100} = 314.8 \text{ N/mm}^2 \quad \text{(i)}$$

Step II *Variance*

From Eq. 24.6, $$S^2 = \frac{\sum f_i X_i^2 - \frac{(\sum f_i X_i)^2}{N}}{(N-1)}$$

$$= \frac{9\,934\,800 - \frac{(31\,480)^2}{100}}{(100-1)}$$

$$= 251.47 \text{ (N/mm}^2)^2 \quad \text{(ii)}$$

Step III *Standard deviation*

$$S = \sqrt{251.47} = 15.86 \text{ N/mm}^2 \quad \text{(iii)}$$

24.4 PROBABILITY

Probability is defined as the chance or likelihood that a particular event will occur. In mathematical terms, it is a number that varies from 0 to 1. The probability is 0 when the event is impossible to occur. The probability is 1 when the event is certain to occur. If there are equal chances that the event will occur or will not occur, the probability is 0.5. The concept of probability is a part of our general knowledge. If asked what is the probability that a tossed coin will come up with a head, most of the people will answer 'one half'. Or, take a six-sided cube with numbers 1 to 6 on its faces; the probability that it will come up with the number 5 is one-sixth.

In more precise terms, the probability is defined as a number p, which varies from 0 to 1 and which indicates the chance that a particular event E will occur, given that it can happen f ways out of n equally likely ways. Therefore,

$$p = P(E) = \frac{f}{n} \tag{24.8}$$

Suppose, we toss a coin twenty times (n = 20), and that we get twelve heads (f = 12). Then the probability of getting heads is given by,

$$p = \frac{f}{n} = \frac{12}{20} = 0.6 \text{ or } 60\%$$

Although the answer is 0.6, if the coin is tossed again and again for a large number of times, the probability that it will come up with heads is 0.5.

If the event does not occur, it is called 'not E' and written as $\tilde{E}$. The probability that the event will not occur is written as $P(\tilde{E})$. Therefore,

$$q = P(\tilde{E}) = 1 - P(E) \tag{24.9}$$

where q is the probability of non occurrence.

Also, $$p + q = 1 \tag{24.10}$$

Example 24.2 *Five bolts with internal cracks are accidentally mixed with 95 bolts without any defects. What is the probability that the assembly shop will use a defective bolt? Also, find out the possibility of not using the defective bolts.*

Solution

Given Number of bolts without defect = 95
Number of bolts with defect = 5

Step I *Probability of using defective bolts*

In this example, the event (E) is to use a defective bolt. Out of 100 bolts, five are defective. Therefore, the event can happen in five (f = 5) ways out of one hundred (n = 100) equally likely ways.

$$p = P(E) = \frac{f}{n} = \frac{5}{100} = 0.05$$

Step II *Probability of not using defective bolts*

The probability of not event ($\tilde{E}$), namely, not using the defective bolt, is given by,

$$q = P(\tilde{E}) = 1 - p = 1 - 0.05 = 0.95$$

Also, $p + q = 0.05 + 0.95 = 1$

The physical significance of the number 1 is that there is a certainty of using either a defective or non-defective bolt in the assembly shop.

24.5 PROBABILITY DISTRIBUTION

There is an important concept of random variables in statistical considerations. In Example 24.1, we have considered one hundred test specimens made of grey cast iron FG 300 which are tested on a universal testing machine to find out the ultimate tensile strength of the material. We get different values of UTS in each test. There are a number of factors that vary from test to test. There are variations in size of the test specimen because of tolerances. The chemical composition of each specimen also varies, although within limits. The testing on a universal testing machine is called 'random experiment' because the specimens are selected at random. The values of UTS obtained in such testing are called 'random variables'. *A random variable is defined as a variable that takes different values in random experiments.* Dimensions of component, weight, strength, forces acting on component, stress or properties of materials are all random variables. In other words, a random variable is a variable whose values depend upon the outcome of a random experiment.

We will conduct a random experiment to explain the above concept. There are two six-sided cubes with numbers 1 to 6 marked on their surfaces. Every time, the two cubes are tossed and the sum of numbers that will appear is denoted as x. Each cube can display any number from 1 to 6. It depends upon the outcome of tossing. Therefore, tossing the two cubes is a 'random experiment'. The variable x, which is the addition of two numbers, is a 'random variable'. Figure 24.7 shows the all-possible outcomes of these experiments. There are a total of 36 possible outcomes. The random variable x has a specific value for each possible outcome. For example, in the first event shown at the first row and first column, the variable x = 1 + 1 = 2. It is seen from Fig. 24.7, that x is 2 in only one out of 36 likely outcomes. Therefore, the probability of getting x = 2 is 1 out of 36 or 1/36. The values of x for each of the remaining outcomes and their probabilities are calculated in a similar manner. The results are given in Table 24.4 and shown schematically in Fig. 24.8.

1 1	1 2	1 3	1 4	1 5	1 6
2 1	2 2	2 3	2 4	2 5	2 6
3 1	3 2	3 3	3 4	3 5	3 6
4 1	4 2	4 3	4 4	4 5	4 6
5 1	5 2	5 3	5 4	5 5	5 6
6 1	6 2	6 3	6 4	6 5	6 6

Fig. 24.7 *Outcome of Toss of Two Discs*

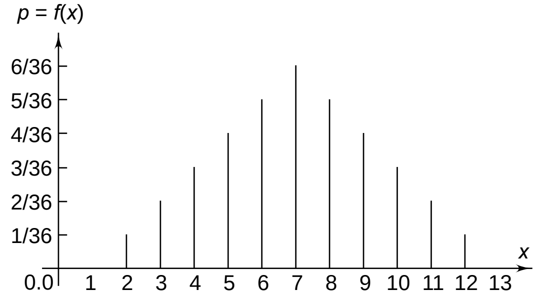


Fig. 24.8 *Probability Distribution*

Table 24.4 *Results of random experiment*

x	*Number of events*	*Probability*
1	0	0
2	1	1/36
3	2	2/36
4	3	3/36
5	4	4/36
6	5	5/36
7	6	6/36
8	5	5/36
9	4	4/36
10	3	3/36
11	2	2/36
12	1	1/36
13	0	0

Table 24.4 that gives a list of all possible values of a random variable and the corresponding probabilities is called a *probability distribution table*. Sometimes, the designer is interested in finding out the probability that x is less than a particular value. For example, in Example 24.1, the designer may be interested in finding out the probability that the ultimate tensile strength is less than 300 N/mm^2. This is because he has assumed 300 N/mm^2 as UTS for calculating the dimensions of the cast iron component. Referring to Table 24.4, the probability that x is less than x_i is obtained by adding the probabilities of all values of x up to and including x_i. For example, the probability that x is less than or equal to 4 is given by,

$$0 + \frac{1}{36} + \frac{2}{36} + \frac{3}{36} = \frac{6}{36}$$

Table 24.5 shows the values calculated by the above method. The probability that x is less than x_i, is called cumulative probability and is denoted by the cumulative probability function $F(x)$. Table 24.5 is called the cumulative probability distribution table. This distribution is also shown in Fig. 24.9.

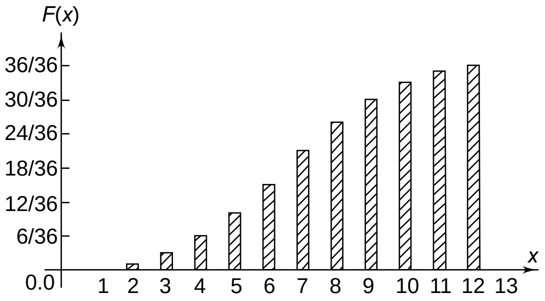


Fig. 24.9 *Cumulative Probability Distribution*

Table 24.5 *Cumulative probability distribution*

x	*Number of events*	*Probability*	*Cumulative probability*
2	1	1/36	1/36
3	2	2/36	3/36
4	3	3/36	6/36
5	4	4/36	10/36
6	5	5/36	15/36
7	6	6/36	21/36
8	5	5/36	26/36
9	4	4/36	30/36
10	3	3/36	33/36
11	2	2/36	35/36
12	1	1/36	36/36

24.6 NORMAL CURVE

In statistical analysis, the most popular probability distribution curve is the normal curve as shown in Fig. 24.10. The distribution is called normal or *Gaussian*. The equation of the normal curve in terms of the standard variable Z is given by

$$f(Z) = \frac{1}{\sqrt{2\pi}} e^{-Z^2/2} \qquad (24.11)$$

An important characteristic of the normal curve is that the total area below the curve from $Z = -\infty$ to $Z = +\infty$ is one or unity. The areas included between different values of Z are as follows:

	Percentage of total area
$Z = -1$ to $Z = +1$	68.27%
$Z = -2$ to $Z = +2$	95.45%
$Z = -3$ to $Z = +3$	99.73%

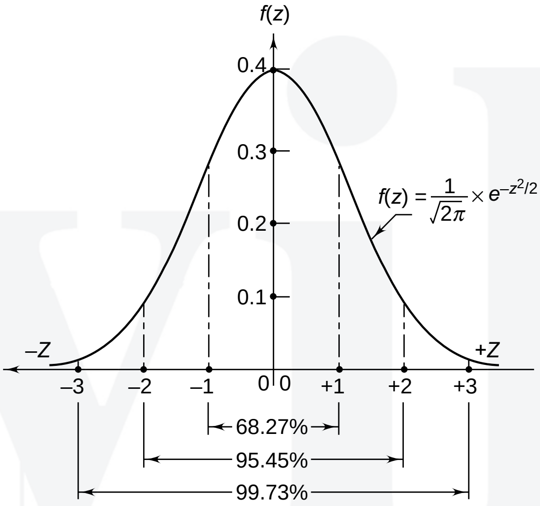


Fig. 24.10 *Normal Distribution*

In many design problems, it is required to find out the area below the normal curve from $Z = 0$ to a particular value of Z as shown by the shaded area in Fig. 24.11. The values of this area are given in Table 24.6.

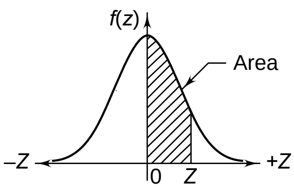


Fig. 24.11 *Area Below Normal Curve*

Table 24.6 *Areas under normal curve from 0 to Z*

Z	0	1	2	3	4	5	6	7	8	9
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.8	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000

24.7 POPULATION COMBINATIONS

There are many problems in machine design where it is required to combine two or more populations in a specific manner to obtain the resultant population. As an example, there are two populations in journal bearing—a population consisting of inner diameter of bearings and a population consisting of outer diameter of shafts. Statistically, both populations are random variables. The system is interchangeable and a shaft should match with any bearing selected at random. Further, they are fitted in such a way that there is a proper clearance between the bearing and the shaft. In this case, subtracting the shaft-population from the bearing-population can form a third population consisting of clearances. The clearance population is a random variable.

Consider a simple case of three bearings with diameters D_1, D_2 and D_3 and two shafts of diameters d_1 and d_2. The means of populations for bearing and shaft are given by,

$$\mu_D = \frac{D_1 + D_2 + D_3}{3} \tag{a}$$

$$\mu_d = \frac{d_1 + d_2}{2} \tag{b}$$

Since there are six possible combinations of the shaft and bearing, the population of clearance consists of the following six elements:

$$\begin{aligned} C_1 &= D_1 - d_1 \\ C_2 &= D_1 - d_2 \\ C_3 &= D_2 - d_1 \\ C_4 &= D_2 - d_2 \\ C_5 &= D_3 - d_1 \\ C_6 &= D_3 - d_2 \end{aligned}$$

The mean of clearance population μ_C is given by

$$\mu_C = \frac{(D_1 - d_1) + (D_1 - d_2) + (D_2 - d_1) + (D_2 - d_2) + (D_3 - d_1) + (D_3 - d_2)}{6}$$

$$= \frac{2(D_1 + D_2 + D_3) - 3(d_1 + d_2)}{6}$$

$$= \frac{(D_1 + D_2 + D_3)}{3} - \frac{(d_1 + d_2)}{2} \tag{c}$$

From (a), (b) and (c),

$$\mu_C = \mu_D - \mu_d$$

Therefore, when two populations are subtracted, the mean of the resultant population is obtained by a subtraction of their individual means. In general, when a population Y is subtracted from a population X, the mean of the resultant population is given by

$$\mu = \mu_X - \mu_Y \tag{24.12}$$

Similarly, it can be proved that when two populations are added, the mean of the resultant population is obtained by an addition of their individual means. Or,

$$\mu = \mu_X + \mu_Y \tag{24.13}$$

The standard deviations for populations of bearing and shaft are given by

$$(\hat{\sigma}_D)^2 = \frac{(D_1 - \mu_D)^2 + (D_2 - \mu_D)^2 + (D_3 - \mu_D)^2}{3} \tag{d}$$

$$(\hat{\sigma}_d)^2 = \frac{(d_1 - \mu_d)^2 + (d_2 - \mu_d)^2}{2} \tag{e}$$

Substituting,

$A_1 = (D_1 - \mu_D)$ $A_2 = (D_2 - \mu_D)$ $A_3 = (D_3 - \mu_D)$

and $B_1 = (d_1 - \mu_d)$ $B_2 = (d_2 - \mu_d)$

the above equations become

$$(\hat{\sigma}_D)^2 = \frac{A_1^2 + A_2^2 + A_3^2}{3} \tag{f}$$

$$(\hat{\sigma}_d)^2 = \frac{B_1^2 + B_2^2}{2} \tag{g}$$

The standard deviation of the clearance population is given by,

$$(\hat{\sigma}_C)^2 = \frac{(D_1 - d_1 - \mu_C)^2 + (D_1 - d_2 - \mu_C)^2 + (D_2 - d_1 - \mu_C)^2 + (D_2 - d_2 - \mu_C)^2 + \ldots}{6}$$

Since $\mu_C = \mu_D - \mu_d$

the above expression is written as

$$(\hat{\sigma}_C)^2 = \frac{(A_1 - B_1)^2 + (A_1 - B_2)^2 + (A_2 - B_1)^2 + (A_2 - B_2)^2 + (A_3 - B_1)^2 + (A_3 - B_2)^2}{6}$$

$$= \frac{2(A_1^2 + A_2^2 + A_3^2) + 3(B_1^2 + B_2^2) - 2(A_1 + A_2 + A_3)(B_1 + B_2)}{6} \tag{h}$$

Since $A_1 + A_2 + A_3 = D_1 + D_2 + D_3 - 3\mu_D = 0$

$B_1 + B_2 = d_1 + d_2 - 2\mu_d = 0$

The Eq. (h) is written as,

$$(\hat{\sigma}_C)^2 = \frac{A_1^2 + A_2^2 + A_3^2}{3} + \frac{B_1^2 + B_2^2}{2}$$

$$= (\hat{\sigma}_D)^2 + (\hat{\sigma}_d)^2$$

or

$$\hat{\sigma}_C = \sqrt{(\hat{\sigma}_D)^2 + (\hat{\sigma}_d)^2}$$

Therefore, the standard deviation follows the Pythagorean rule. It can be proved that the above result is also valid for an addition of two populations. In general, when there are two populations X and Y, the standard deviation of the resultant population, both for addition and subtraction, is given by

$$\hat{\sigma} = \sqrt{(\hat{\sigma}_X)^2 + (\hat{\sigma}_Y)^2} \qquad (24.14)$$

In this article, we have considered only addition and subtraction of two populations. Sometimes, it is required to divide or multiply one population by another. In some cases, it is required to square a population or to take an inverse of the population. Table 24.7 shows all such combinations and how to obtain mean and standard deviation of combinations. The values given in this table are approximate and standard textbooks in statistics can be referred for more exhaustive treatment. In Table 24.7,

Z = function of two independent random variables X and Y

a = constant

Table 24.7 *Mean and standard deviations of the function Z*

Function	*Mean* μ_Z	*Standard deviation* $\hat{\sigma}_Z$
$Z = a$	a	0
$Z = aX$	$a\mu_X$	$a\ \hat{\sigma}_X$
$Z = X \pm a$	$\mu_X \pm a$	$\hat{\sigma}_X$
$Z = X \pm Y$	$\mu_X \pm \mu_Y$	$\sqrt{(\hat{\sigma}_X)^2 + (\hat{\sigma}_Y)^2}$
$Z = XY$	$\mu_X \mu_Y$	$\sqrt{\mu_X^2(\hat{\sigma}_Y)^2 + \mu_Y^2(\hat{\sigma}_X)^2 + (\hat{\sigma}_X)^2(\hat{\sigma}_Y)^2}$
$Z = X/Y$	μ_X/μ_Y	$\frac{1}{\mu_Y}\left[\frac{\mu_X^2(\hat{\sigma}_Y)^2 + \mu_Y^2(\hat{\sigma}_X)^2}{\mu_Y^2 + (\hat{\sigma}_Y)^2}\right]^{1/2}$
$Z = X^2$	$\mu_x^2 + \hat{\sigma}_x^2$	$\frac{1}{2}\left(\frac{\hat{\sigma}_X}{\mu_X}\right)\left[4\mu_X^2 + (\hat{\sigma}_X)^2\right]$
$Z = X^3$	$\mu_X^3 + 3\mu_X(\hat{\sigma}_X)^2$	$3\mu_X^2(\hat{\sigma}_X) + 3(\hat{\sigma}_X)^3$
$Z = 1/X$	$\frac{1}{\mu_X}\left[1 + \left(\frac{\hat{\sigma}_X}{\mu_X}\right)^2\right]$	$\frac{\hat{\sigma}_X}{\mu_X^2}\left[1 + \left(\frac{\hat{\sigma}_X}{\mu_X}\right)^2\right]$

In statistical analysis, many times it is important to know the distribution of the resultant population obtained by combination of two or more populations. A designer cannot assume every resulting population to be a normally distributed random variable. For example, the population of stress is created by dividing the population of force by the population of area. The population of force as well as the population of area is normally distributed random variable. In order to find out the probability of a particular limiting value for stress, the designer must be sure that the resulting population of stress is a normally distributed random variable. Merely creating the resultant population and calculating values of mean and standard variable from Table 24.7 will not predict correct results. The guidelines for such problems are as follows:

(i) When two normally distributed random variables are added, the resulting population is also normally distributed.

(ii) When two normally distributed random variables are subtracted, the resulting population is also normally distributed.

(iii) When two normally distributed random variables are multiplied, the resulting population has an approximately normal distribution.

(iv) When two normally distributed random variables are divided, the resulting population does not have a strictly normal distribution. However, it may be assumed as approximately normal distribution.

Books in statistical analysis give various conditions as to where one can assume normal distribution in case of multiplication and division of two variables. However, such exhaustive treatment is avoided in this chapter.

24.8 DESIGN AND NATURAL TOLERANCES

The variations in the dimensions of a component occur due to two reasons—first, because of a large number of chance causes and, second, due to assignable causes. The variations due to chance causes occur at random. They are the characteristics of the manufacturing method and measurement technique. The variations due to assignable causes can be located and corrected. When they are corrected, the system is said to be under '*statistical control*'.

In a statistically controlled system, the dimensions of the component are normally distributed with a particular value of standard deviation. *The natural tolerance is defined as the actual capabilities of the process, and can be considered as limits within which all but a given allowable fraction of items will fall. In general, the natural tolerance of a process is the spread of the normal curve that includes 99.73% of the total population.*

Referring to the normal curve, shown in Fig. 24.10, the values of the standard variables Z_1 and Z_2 corresponding to this population are –3 and +3.

From Eq. 24.7,

$$X = \mu + Z\hat{\sigma}$$

Therefore,

$$X_1 = \mu + 3(\hat{\sigma}) \quad \text{and} \quad X_2 = \mu - 3(\hat{\sigma})$$

Therefore, the natural tolerances are $\pm 3\ (\hat{\sigma})$.

On the contrary, the design tolerances are specification limits, set somewhat arbitrarily by the designer from considerations of the proper matching of the two components and functioning of the assembly. The design tolerances can be achieved only when the manufacturing process is so selected that the natural tolerances are within the design tolerances. The percentage of rejected components depends upon the relationship between these two tolerances. Based on this relationship, the following observations are made:

(i) When the design tolerance is less than ($\pm\ 3\hat{\sigma}$), the percentage of rejected components is inevitable.

(ii) When the design tolerance is equal to ($\pm\ 3\hat{\sigma}$), there is virtually no rejection provided that the manufacturing process is centred. For an off-centre process, some components are rejected.

(iii) When the design tolerance is slightly greater than ($\pm\ 3\hat{\sigma}$), there is no rejection even if the manufacturing process is slightly off-centre.

It is necessary for the designer to select a manufacturing process for a component in such a way that the natural tolerance of the process is slightly less than design tolerance. The design tolerance should be about ($\pm 4\hat{\sigma}$).

Example 24.3 *It has been observed from a sample of 200 bearing bushes that the internal diameters are normally distributed with a mean of 30.010 mm and a standard deviation of 0.008 mm. The upper and lower limits for the internal diameter, as specified by the designer, are 30.02 and 30.00 mm respectively. Calculate the percentage of rejected bushes.*

Solution

Given $\mu = 30.010$ mm $\hat{\sigma} = 0.008$ mm
$X_1 = 30.02$ mm $X_2 = 30.00$ mm

Step I *Standard variables Z_1 and Z_2*

From Eq. 24.7,

$$Z_1 = \frac{X_1 - \mu}{\hat{\sigma}} = \frac{30.02 - 30.01}{0.008} = +1.25$$

$$Z_2 = \frac{X_2 - \mu}{\hat{\sigma}} = \frac{30.00 - 30.01}{0.008} = -1.25$$

Step II *Area below normal curve from Z_1 to Z_2*

As shown in Fig. 24.12, the shaded area below the curve represents the percentage of acceptable bushes. From Table 24.6,

Shaded area = 2 (area between $Z = 0$ and $Z = 1.25$)
= 2 (0.3944) = 0.7888

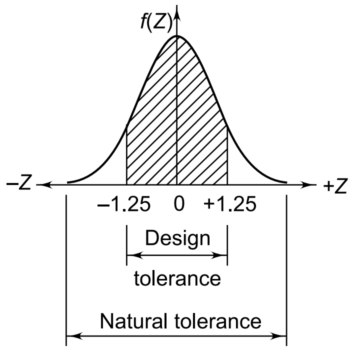


Fig. 24.12

Step III *Percentage of rejected bushes*

% of rejected bushes = (1 – 0.7888) × 100 = 21.12%

Example 24.4 *The tolerance specified by the designer for the diameters of a transmission shaft is 25.000 ± 0.025 mm. The shafts are machined on three different machines. It was observed from the sample of shafts that the diameters are normally distributed with a standard deviation of 0.015 mm for each of the three machines. However, the mean diameter of shafts fabricated on the three machines is found to be 24.99, 25.00 and 25.01 mm respectively. Determine the percentage of rejected shafts in each case and comment on the result.*

Solution

Given $\hat{\sigma} = 0.015$ mm $X_1 = 25.000 - 0.025$ mm
$X_2 = 25.000 + 0.025$ mm $\mu_1 = 24.99$ mm
$\mu_2 = 25$ mm $\mu_3 = 25.01$ mm

Step I *Percentage of rejected shafts in each machine*

The lower and upper limits specified by the designer are,

$$X_1 = 25.000 - 0.025 = 24.975 \text{ mm}$$
$$X_2 = 25.000 + 0.025 = 25.025 \text{ mm}$$

The results are tabulated in the following way:

	Machine A	*Machine B*	*Machine C*
mean μ (mm)	24.99	25.00	25.01
$Z_1 = \dfrac{X_1 - \mu}{\hat{\sigma}}$	–1	–1.67	–2.33
$Z_2 = \dfrac{X_2 - \mu}{\hat{\sigma}}$	+2.33	+1.67	+1
Area A_1 ($Z = 0$ to Z_1)	0.3413	0.4525	0.4901
Area A_2 ($Z = 0$ to Z_2)	0.4901	0.4525	0.3413
$(A_1 + A_2)$	0.8314	0.9050	0.8314
Percentage rejection	16.86	9.5	16.86

Step II *Comments on result*

The above data is illustrated in Fig. 24.13. It is observed from the figure that in case of machines *A* or *C*, the process is not centred, which results in a large percentage of rejection. With the same magnitude of tolerance, the percentage of rejection is small in case of the machine *B*, because the process is centred.

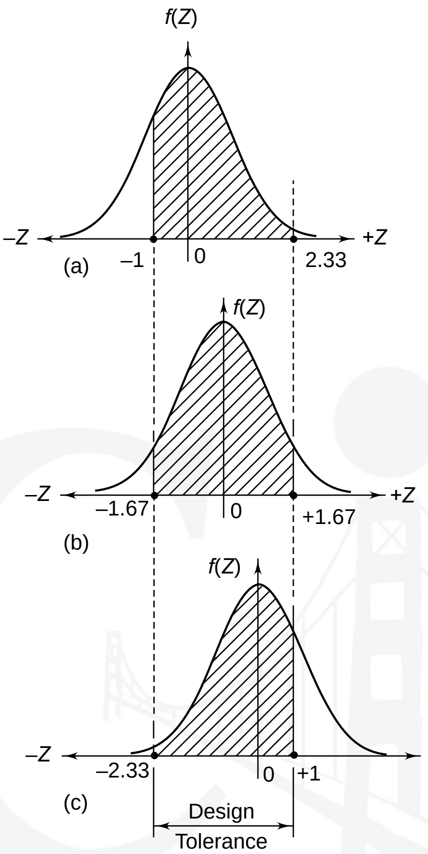


Fig. 24.13

Example 24.5 *The recommended class of transition fit between the recess and the spigot of a rigid coupling is 60H6-j5. Assuming that the dimensions of the two components are normally distributed and that the specified tolerance is equal to the natural tolerance, determine the probability of interference fit between the two components.*

Solution

Given Class of fit = 60H6-j5

There are two populations—population of recess dimension denoted by the letter R and that of spigot dimension denoted by the letter S.

Step I *Population of recess (R)*

The limiting dimensions for recess 60H6 are (Table 3.2)

$$\frac{60.019}{60.000}\text{ mm} \quad \text{or} \quad 60.0095 \pm 0.0095 \text{ mm}$$

The design tolerance and natural tolerance are equal. Therefore,

$$\mu_R = 60.0095 \text{ mm}$$

and $$\hat{\sigma}_R = \frac{0.0095}{3} = 0.00317 \text{ mm}$$

Step II *Population of spigot (S)*

The limiting dimensions for spigot 60j5 are [Table 3.3(b)]

$$\frac{60.006}{59.993}\text{ mm} \quad \text{or} \quad 59.9995 \pm 0.0065 \text{ mm}$$

Therefore,

$$\mu_S = 59.9995 \text{ and } \hat{\sigma}_S = \frac{0.0065}{3} = 0.00217 \text{ mm}$$

Step III *Population of interference (I)*

The population for interference is denoted by the letter I. It is obtained by subtracting the population of recess from the population of the spigot. From Eq. 24.12,

$$\mu_I = \mu_S - \mu_R = 59.9995 - 60.0095 = -0.01 \text{ mm}$$

From Eq. 24.14,

$$\hat{\sigma}_I = \sqrt{(\hat{\sigma}_R)^2 + (\hat{\sigma}_S)^2}$$

$$= \sqrt{(0.00317)^2 + (0.00217)^2}$$

$$= 0.00384 \text{ mm}$$

Step IV *Probability of interference fit*

When the interference is zero,

$$I = 0 \quad \text{and} \quad Z = \frac{I - \mu_I}{\hat{\sigma}_I} = \frac{0 - (-0.01)}{0.00384} = +2.6$$

As shown in Fig. 24.14, the interference will occur only when $Z > 2.60$. When Z is less than 2.60,

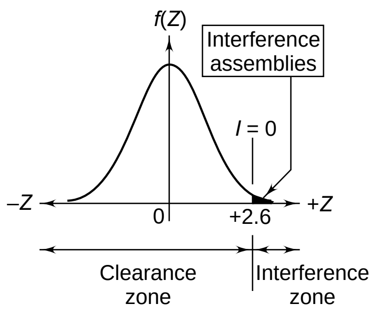


Fig. 24.14

the value of I will be negative, giving a clearance fit. The probability of interference fit is therefore given by the area below the normal curve from Z = 2.60 to Z = $+\infty$. From Table 24.6, the area below the normal curve from Z = 0 to Z = 2.60 is 0.4953. Therefore, the probability of interference fit is $(0.5 - 0.4953) \times 100$ or 0.47%.

Example 24.6 *The recommended class of fit for the journal and the bush of a hydrodynamic bearing is 40H6-e7. The dimensions of the journal and the bush are normally distributed and the natural tolerance is equal to the design tolerance. From the considerations of hydrodynamic action and bearing stability, the maximum and minimum clearances are limited to 0.08 and 0.06 mm respectively. Determine the percentage of rejected assemblies.*

Solution

Given Class of fit = 40H6-e7

$X_1 = 0.08$ mm $\quad X_2 = 0.06$ mm

There are two populations—population of the bushes denoted by the letter B and that of the journals denoted by the letter J.

Step I *Population of bushes (B)*

The limiting dimensions for the bush 40H6 are (from Table 3.2).

$$\frac{40.016}{40.000}\ \text{mm} \quad \text{or} \quad 40.008 \pm 0.008\ \text{mm}$$

Since design tolerance and natural tolerance are equal,

$$\mu_B = 40.008\ \text{mm} \qquad \hat{\sigma}_B = \frac{0.008}{3} = 0.002667\ \text{mm} \quad \text{(a)}$$

Step II *Population of journal*

The limiting dimensions for the journal 40e7 are [Table 3.3(a)]

$$\frac{39.950}{39.925}\ \text{mm} \quad \text{or} \quad 39.9375 \pm 0.0125\ \text{mm}$$

Therefore,

$$\mu_J = 39.9375\ \text{mm}$$

$$\hat{\sigma}_J = \frac{0.0125}{3} = 0.004167\ \text{mm} \qquad \text{(b)}$$

Step III *Population of clearance (C)*

The letter C denotes the population for clearance. It is obtained by subtracting the population of the journal from the population of the bushes. Therefore,

$$\mu_C = \mu_B - \mu_J = 40.008 - 39.9375 = 0.0705\ \text{mm}$$

$$\hat{\sigma}_C = \sqrt{(\hat{\sigma}_B)^2 + (\hat{\sigma}_J)^2}$$

$$= \sqrt{(0.002667)^2 + (0.004167)^2}$$

$$= 0.004947\ \text{mm}$$

Step IV *Percentage of rejected assemblies*

For maximum clearance,

$$X_1 = 0.08\ \text{mm}$$

and $$Z_1 = \frac{X_1 - \mu_C}{\hat{\sigma}_C} = \frac{0.08 - 0.0705}{0.004947} = +1.92$$

For minimum clearance,

$$X_2 = 0.06\ \text{mm}$$

and $$Z_2 = \frac{X_2 - \mu_C}{\hat{\sigma}_C} = \frac{0.06 - 0.0705}{0.004947} = -2.12$$

From Table 24.6, the areas below the normal curve from Z = 0 to 1.92 and from Z = 0 to 2.12 are 0.4726 and 0.4830 respectively. Therefore, the percentage of rejected assemblies is $[1 - (0.4726 + 0.4830)] \times 100$ or 4.44%.

Example 24.7 *An assembly of three components A, B and C is shown in Fig. 24.15. The dimensions of the three components are normally distributed and natural tolerance is equal to design tolerance as shown in the figure. Determine the percentage of assemblies where interference is likely to occur.*

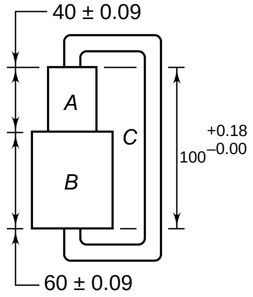


Fig. 24.15

Solution

Step I *Population of component-A (A)*

For the population of component A,

$$\mu_A = 40.00 \text{ mm} \quad \hat{\sigma}_A = \frac{0.09}{3} = 0.03 \text{ mm} \qquad \text{(a)}$$

Step II *Population of component–B (B)*

For the population of the component B,

$$\mu_B = 60.00 \text{ mm} \quad \hat{\sigma}_B = \frac{0.09}{3} = 0.03 \text{ mm} \qquad \text{(b)}$$

Step III *Population consisting of addition of populations A and B (X)*

A third population consisting of the addition of the populations A and B is formed and denoted by the letter X. Therefore,

$$\mu_X = \mu_A + \mu_B = 40 + 60 = 100 \text{ mm}$$

$$\hat{\sigma}_X = \sqrt{(\hat{\sigma}_A)^2 + (\hat{\sigma}_B)^2} = \sqrt{(0.03)^2 + (0.03)^2}$$

$$= 0.0424 \text{ mm} \qquad \text{(c)}$$

Step IV *Population of component-C (C)*

For the population of the component C,

$$\mu_C = 100.09 \text{ mm} \quad \hat{\sigma}_c = \frac{0.09}{3} = 0.03 \text{ mm} \qquad \text{(d)}$$

Step V *Population of interference (I)*

The population for interference is denoted by the letter I. It is obtained by subtracting the population C from the population X.

$$\mu_I = \mu_X - \mu_C = 100.00 - 100.09 = -0.09 \text{ mm}$$

$$\hat{\sigma}_I = \sqrt{(\hat{\sigma}_X)^2 + (\hat{\sigma}_C)^2} = \sqrt{(0.0424)^2 + (0.03)^2}$$

$$= 0.052 \text{ mm}$$

Step VI *Percentage of assemblies with interference*

When interference is zero,

$$I = 0 \text{ and } Z = \frac{I - \mu_I}{\hat{\sigma}_I} = \frac{0 - (-0.09)}{0.052} = +1.73$$

When Z is less than 1.73, I is negative, giving a clearance fit. The interference fit is given by the area below the normal curve from $Z = +1.73$ to $Z = \infty$. From Table 24.6, the area below the normal curve from $Z = 0$ to $Z = +1.73$ is 0.4582. Therefore, the percentage of interference assemblies is $(0.5 - 0.4582) \times 100$ or 4.18%.

24.9 RELIABILITY

A product is said to be reliable when it performs its intended function satisfactorily throughout its life. A domestic refrigerator, which works continuously year after year without any breakdown, is an example of a reliable product. In the past, engineers recognised the need of designing products for long life. In those days, reliability was mainly achieved through over design. Reliable products were dependable. Steam locomotives, turbines and power presses were all designed for trouble free long life. However, such products were few and comparatively simple. Modern products are complex and many factors affect their performance. The emphasis is on lightweight construction. These products work under severe working conditions than in the past. The customers expect much more reliability. All these factors have increased the importance of reliability in engineering design.

According to the International Standards Organization, reliability is defined as the ability of an item to perform a required function under stated conditions for a stated period of time. In engineering design, reliability is expressed quantitatively such as 0.9 or 90%. *In such cases, reliability is defined, as the probability that a product will perform the required function under stated conditions for a stated period of time.* This definition of reliability contains four basic elements. They are as follows:

(i) The reliability of the product is expressed as a probability.

(ii) The product is required to perform its intended function.

(iii) The period during which the product has to perform the function is specified.

(iv) The operating conditions under which the product has to function are specified.

For example, all the manufacturers of rolling contact bearing prescribe a reliability of 90% for their bearings. Let us consider a particular application, where a ball bearing is subjected to a radical load of 5 kN, while the expected life is 8000 hours. The shaft rotates at a speed of 1450 rpm. The designer in this case has calculated the required dynamic load carrying capacity and selected the bearing from the manufacturers' catalogue on the basis of this capacity. Let us consider the four basic elements in the definition of reliability with respect to this application.

(i) The reliability of the ball bearing is 90%. It indicates that if a large number of bearings are purchased and mounted in this type of application, the manufacturer gives a guarantee that 90% of the bearings will reach or exceed the life of 8000 h at 1450 rpm before fatigue failure. The remaining 10% of the bearings may or may not reach the target life. Also, the manufacturer does not give a guarantee that a particular bearing will work satisfactorily for 8000 h at 1450 rpm. In other words, the chances are that 9 out of 10 bearings will reach or exceed the expected life.

(ii) The performance of the product is the second element in the definition of the reliability. A ball bearing is intended to support the shaft, to ensure its free rotation and to transmit the load from the shaft to the housing. In the above application, the bearing is required to transmit a load of 5 kN.

(iii) The period during which the product has to perform its function is the third element in the definition of reliability. Reliability must be identified for a stated period of time. In the above example, the ball bearing is required to support the rotating shaft and transmit a force of 5 kN for 8000 h of operation of 1450 rpm.

(iv) The fourth significant element in the definition of reliability is operating conditions. The ball bearings operate under the following conditions:

(a) The ball bearings are lubricated and the lubricant is periodically replaced.

(b) The ball bearings are protected from dust particles or moisture by means of oil seals.

(c) For every ball bearing; the manufacturer prescribes a limiting speed for the shaft, which is not exceeded.

(d) The journal and housing diameters are provided tolerances as per manufacturer's specifications.

The operating conditions establish the factors that affect the life and reliability of the product.

Product reliability and product quality are closely related to each other. In fact, many times reliability is described as quality maintained during the useful life of the product. Some authors define reliability as a quality over a period of time. However, a product of high quality may not be a product with high reliability. A product may pass through all tests in the quality control department and yet it may fail when put into service due to adverse interaction between components. Even a high quality ball bearing will fail prematurely when mounted in an improper way. Quality is a special case of reliability. It is the state of reliability at the beginning of operation or at zero time.

24.10 PROBABILISTIC APPROACH TO DESIGN

Traditionally, design engineers have used factor of safety to ensure against uncertainty in magnitude of external forces acting on the component, variations in properties of materials like ultimate tensile strength or yield strength and variations in dimensions of the component due to imperfect workmanship. However, it is not possible to determine reliability using the concept of factor of safety. Since reliability is a design parameter, it should be incorporated in the product at the design stage. The factor of safety does not address reliability. Probabilistic approach is a technique to design the component for a given magnitude of reliability. In this approach, the following assumptions are made:

(i) The ultimate tensile strength or yield strength is not constant but subjected to statistical variation. The population of strength; denoted by S is under statistical control. It is normally distributed with a mean of μ_S and standard deviation $\hat{\sigma}_S$.

(ii) The stress induced in the component is not constant but subjected to statistical variation.

The population of stress, denoted by σ, is normally distributed with a mean of μ_σ and standard deviation of $\hat{\sigma}_\sigma$.

Figure 24.16 shows the normal distribution of these two populations. It is clear that the mean of strength population is more than the mean of the stress population. However, there is an overlapping area between these two curves. The forward tail of stress distribution is overlapping the rear tail of the strength population. This overlapping area represents the region of unreliability. Some failures may occur in this region.

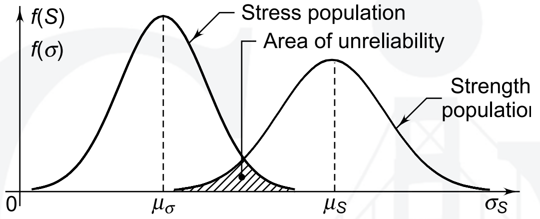


Fig. 24.16 *Normal Distribution of Strength and Stress Populations*

A third population of the margin of safety is formed by subtracting the population of stress from the population of the strength. The population of margin of safety is denoted by m. From Eq. 24.12 and 24.14,

$$\mu_m = \mu_S - \mu_\sigma \tag{24.15}$$

$$\hat{\sigma}_m = \sqrt{(\hat{\sigma}_S)^2 + (\hat{\sigma}_\sigma)^2} \tag{24.16}$$

Figure 24.17 shows the normal curve for population of margin of safety in terms of standard variable Z where,

$$Z = \frac{m - \mu_m}{\hat{\sigma}_m} \tag{24.17}$$

When the stress is equal to strength, the margin of safety is zero and failure may occur. Therefore, condition of failure is given by,

$$m = 0$$

and the corresponding standard variable Z_0 is given by,

$$Z_0 = \frac{0 - \mu_m}{\hat{\sigma}_m} = -\frac{\mu_m}{\hat{\sigma}_m} \tag{24.18}$$

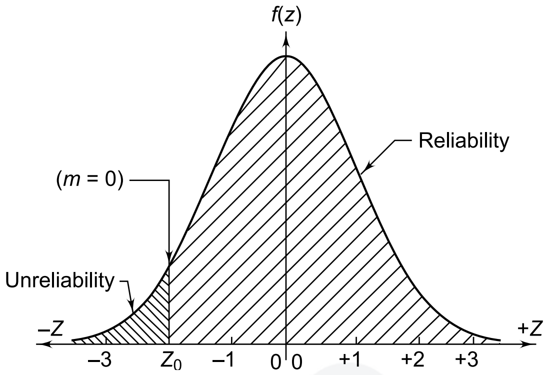


Fig. 24.17 *Normal Curve for Population of Margin of Safety*

Refer back to Fig. 24.16, where overlapping area between the strength and stress curves indicates the region of unreliability. With reference to this figure, the following four basic ways to increase reliability can be suggested:

(i) Increase the mean of strength population (μ_s) by using a better quality material for the component:

(ii) Decrease the mean of stress population (μ_σ) by increasing the size of the component.

(iii) Decrease the standard deviation ($\hat{\sigma}_\sigma$) of stress population by controlling manufacturing method, limiting tolerances and putting limitations on end use conditions.

(iv) Decrease the standard deviation ($\hat{\sigma}_S$) of strength population by controlling the quality of incoming materials.

All methods increase the cost of the component. Reliability is achieved by increasing the cost of the component. Therefore, high reliability cannot be criterion in all applications.

Example 24.8 *A tension rod is subjected to axial stress within elastic limit. According to Hooke's law,*

$$\sigma = E\varepsilon$$

It has been observed that the strain (ε) in the tension rod is a normally distributed variable with a mean of 0.001 mm/mm and a standard deviation of 0.000 07 mm/mm. The modulus of elasticity (E) *is also a normally distributed random variable with a mean of 207 000 N/mm² and a standard deviation of 6000 N/mm².*

Determine the mean and standard deviation of the corresponding stress variable (σ). Comment on the analysis.

Solution

Given $\mu_\varepsilon = 0.001$ mm/mm
$\hat{\sigma}_\varepsilon = 0.000\,07$ mm/mm $\quad \mu_E = 207\,000$ N/mm²
$\hat{\sigma}_E = 6000$ N/mm²

Step I *Population of strain (ε)*

ε denotes the population of strain. For this population,

$$\mu_\varepsilon = 0.001 \text{ mm/mm}$$

and $$\hat{\sigma}_\varepsilon = 0.000\,07 \text{ mm/mm}$$

Step II *Population of modulus of elasticity (E)*

E denotes the population of modulus of elasticity. For this population,

$$\mu_E = 207\,000 \text{ N/mm}^2$$

and $$\hat{\sigma}_E = 6000 \text{ N/mm}^2$$

Step III *Population of stress (σ)*

The third population of stress is denoted by σ. It is obtained by multiplication of the strain population by the population of the modulus of elasticity. From Table 24.7,

$$\mu_\sigma = \mu_\varepsilon \mu_E = 0.001(207\,000) = 207 \text{ N/mm}^2$$

$$\hat{\sigma}_\sigma = \sqrt{\mu_\varepsilon^2 (\hat{\sigma}_E)^2 + \mu_E^2 (\hat{\sigma}_\varepsilon)^2 + (\hat{\sigma}_\varepsilon)^2 (\hat{\sigma}_E)^2}$$

$$= \sqrt{(0.001)^2 (6000)^2 + (207\,000)^2 (0.000\,07)^2 + (0.000\,07)^2 (6000)^2}$$

$$= 15.69 \text{ N/mm}^2$$

Step IV *Comments on analysis*

The stress population has a mean of 207 N/mm² and a standard deviation of 15.69 N/mm². We can predict the mean and the standard deviation of stress population. However, stress population is not exactly a normally distributed random variable.

Example 24.9 *A rod is subjected to pure uni-axial strain, which is given by*

$$\varepsilon = \frac{\delta}{l}$$

It has been observed that the length (l) of the rod is a normally distributed random variable with a mean of 100 mm and a standard deviation of 0.5 mm. The deflection of the rod (δ) is also a normally distributed random variable with a mean of 0.075 mm and a standard deviation of 0.005 mm.

Determine the mean and standard deviation of the corresponding strain variable ε. Comment on the analysis.

Solution

Given $\mu_\delta = 0.075$ mm $\quad \hat{\sigma}_\delta = 0.005$ mm
$\mu_l = 100$ mm $\quad \hat{\sigma}_l = 0.5$ mm

Step I *Population of deflection (δ)*

δ denotes the population of deflection. For this population,

$$\mu_\delta = 0.075 \text{ mm} \quad \text{and} \quad \hat{\sigma}_\delta = 0.005 \text{ mm}$$

Step II *Population of length (l)*

l denotes the population of length of the rod. For this population,

$$\mu_l = 100 \text{ mm} \quad \text{and} \quad \hat{\sigma}_l = 0.5 \text{ mm}$$

Step III *Population of strain (ε)*

The third population of strain is denoted by ε. It is obtained by dividing the deflection population by the population of the length of the rod. From Table 24.7,

$$\mu_\varepsilon = \frac{\mu_\delta}{\mu_l} = \frac{0.075}{100} = 0.00075 \text{ mm/mm}$$

$$\hat{\sigma}_\varepsilon = \frac{1}{\mu_l}\left[\frac{\mu_\delta^2 \hat{\sigma}_l^2 + \mu_l^2 \hat{\sigma}_\delta^2}{\mu_l^2 + \hat{\sigma}_l^2}\right]^{1/2}$$

$$= \frac{1}{100}\left[\frac{0.075^2 \times 0.5^2 + 100^2 \times 0.005^2}{100^2 + 0.5^2}\right]^{1/2}$$

$$\hat{\sigma}_\varepsilon = 0.000\,05 \text{ mm/mm}$$

Step IV *Comments on analysis*

The strain population has a mean of 0.00075 mm/mm and a standard deviation of 0.000 05 mm/mm. We can predict the mean and the standard deviation of the strain population. However, strain population is not a normally distributed random variable.

Example 24.10 *A beam of circular cross-section is subjected to pure bending moment M and the bending stresses are given by the following equation:*

$$\sigma = \frac{32M_b}{\pi d^3}$$

where d is the diameter of the beam. It has been observed that the diameter (d) of the beam is a normally distributed random variable with a mean of 50 mm and a standard deviation of 0.125 mm. The bending moment (M_b) is also a normally distributed random variable with a mean of 1750 N-m and a standard deviation of 150 N-m.

Determine the mean and standard deviation of the corresponding bending stress variable (σ). Comment on the analysis.

Solution

Given $\mu_d = 50$ mm $\quad \hat{\sigma}_d = 0.125$ mm

$\mu_M = 1750 \times 10^3$ N-mm $\quad \hat{\sigma}_M = 150 \times 10^3$ N-mm

Step I *Population of diameter (d)*

d denotes the population of diameters. For this population,

$$\mu_d = 50 \text{ mm} \quad \text{and} \quad \hat{\sigma}_d = 0.125 \text{ mm}$$

Step II *Population of bending moment (M)*

M denotes the population of values of bending moment. For this population,

$$\mu_M = 1750 \times 10^3 \text{ N-mm}$$

and $\quad \hat{\sigma}_M = 150 \times 10^3$ N-mm

Step III *Population of ($\pi d^3/32$) (Z)*

Z denotes a third population. It is obtained by using the expression,

$$Z = \frac{\pi}{32} d^3$$

In the above expression ($\pi/32$) is constant and using the equations in Table 24.7

$$\mu_Z = \frac{\pi}{32}[\mu_d^3 + 3\mu_d(\hat{\sigma}_d)^2]$$

$$= \frac{\pi}{32}[50^3 + 3(50)(0.125)^2]$$

$$= 12\,272.08 \text{ mm}^3$$

$$\hat{\sigma}_Z = \frac{\pi}{32}[3\mu_d^2\hat{\sigma}_d + 3(\hat{\sigma}_d)^3]$$

$$= \frac{\pi}{32}[3(50)^2(0.125) + 3(0.125)^3]$$

$$= 92.04 \text{ mm}^3$$

Step IV *Population of bending stress (σ)*

A fourth population of bending stress is denoted by σ. It is obtained by dividing the population of values of bending moment M by the population Z. From Table 24.7,

$$\mu_\sigma = \frac{\mu_M}{\mu_Z} = \frac{1750 \times 10^3}{12\,272.08} = 142.6 \text{ N/mm}^2$$

$$\hat{\sigma}_\sigma = \frac{1}{\mu_Z}\left[\frac{\mu_M^2\hat{\sigma}_Z^2 + \mu_Z^2\hat{\sigma}_M^2}{\mu_Z^2 + \hat{\sigma}_Z^2}\right]^{1/2}$$

$$= \frac{1}{12\,272.08}\left[\frac{(1750 \times 10^3)^2 \times (92.04)^2 + (12\,272.08)^2 \times (150 \times 10^3)^2}{(12\,272.08)^2 + (92.04)^2}\right]^{1/2}$$

$$\hat{\sigma}_\sigma = 12.27 \text{ N/mm}^3$$

Step V *Comments on analysis*

The bending stress population has a mean of 142.6 N/mm^2 and standard deviation of 12.27 N/mm^2. We can predict the mean and standard deviation of this population. However, the population is not a normally distributed random variable.

Example 24.11 *The diametral tolerance for patented and cold drawn steel wire (Grade-2) used to make cold formed springs, as recommended in IS 4454-Part 1 is as follows:*

Nominal Diameter (mm)	*Tolerance (mm)*
6 – 7.5	*± 0.045*
8 –10	*± 0.050*

A 6.5 mm diameter wire is used to make a helical spring, in which the wire is subjected to torsional moment (M_t) of 10 N-m. The natural tolerance is equal to diametral tolerances. Neglect the Wahl factor and assume all other factors including torsional moment as constant. Determine the mean and standard deviation of the population of torsional shear stress.

Solution

Given $d = 6.5 \pm 0.045$ mm $M_t = 10$ N-m

The torsional shear stress in the wire is given by,

$$\tau = \frac{16M_t}{\pi d^3}$$

Step I *Population of wire diameters*

d denotes the population of wire diameters. For this population,

$$\mu_d = 6.5 \text{ mm}$$

The natural tolerance on wire diameter is $(\pm 3\hat{\sigma}_d)$. Therefore,

$$\hat{\sigma}_d = \frac{0.045}{3} = 0.015 \text{ mm}$$

Step II *Population of ($\pi d^3/16\,M_t$) (Z)*

Z denotes a second population. It is obtained by using the expression,

$$Z = \left(\frac{\pi}{16M_t}\right)d^3$$

In the above expression ($\pi/16M_t$) is constant and using the equations in Table 24.7,

$$\mu_Z = \frac{\pi}{16M_t}[\mu_d^3 + 3\mu_d(\hat{\sigma}_d)^2]$$

$$= \frac{\pi}{16(10\times 10^3)}[6.5^3 + 3(6.5)(0.015)^2]$$

$$= 5.39 \times 10^{-3} \text{ mm}^3$$

$$\hat{\sigma}_Z = \frac{\pi}{16M_t}[3\mu_d^2(\hat{\sigma}_d) + 3(\hat{\sigma}_d)^3]$$

$$= \frac{\pi}{16(10\times 10^3)}[3(6.5)^2(0.015) + 3(0.015)^3]$$

$$= 3.73 \times 10^{-5} \text{ mm}^3$$

Step III *Population of torsional shear stress (τ)*

A third population of torsional shear stress is denoted by τ. It is obtained by taking a reciprocal of population Z. From Table 24.7,

$$\mu_\tau = \frac{1}{\mu_Z}\left[1 + \left(\frac{\hat{\sigma}_Z}{\mu_Z}\right)^2\right]$$

$$= \frac{1}{(5.39\times 10^{-3})}\left[1 + \left(\frac{3.73\times 10^{-5}}{5.39\times 10^{-3}}\right)^2\right]$$

$$= 185.53 \text{ N/mm}^2$$

$$\hat{\sigma}_\tau = \frac{\hat{\sigma}_Z}{\mu_Z^2}\left[1 + \left(\frac{\hat{\sigma}_Z}{\mu_Z}\right)^2\right]$$

$$= \frac{(3.73\times 10^{-5})}{(5.39\times 10^{-3})^2}\left[1 + \left(\frac{3.73\times 10^{-5}}{5.39\times 10^{-3}}\right)^2\right]$$

$$= 1.28 \text{ N/mm}^2$$

Step IV *Comments on analysis*

The torsional shear stress population has a mean of 185.53 N/mm^2 and a standard deviation of 1.28 N/mm^2. We can predict the mean and standard deviation of this population. However, the population is not normally distributed.

Example 24.12 *A particular type of rollingcontact bearing has a normally distributed time to failure, with a mean of 10 000 hours and a standard deviation of 750 h. If there are 100 such bearings fitted at a time, how many may be expected to fail within the first 11 000 h?*

Solution

Given $\mu = 10\,000$ h $\hat{\sigma} = 750$ h $X = 11\,000$ h

Step I *Population of bearing life*

$\mu = 10\,000$ hr and $\hat{\sigma} = 750$ h

Step II *Standard variable Z for (11 000 h)*

From Eq. 24.7,

$$Z = \frac{X - \mu}{\hat{\sigma}} = \frac{11\,000 - 10\,000}{750} = 1.333$$

Step III *Number of bearings likely to fail within first 11 000 h*

From Table 24.6, area below normal curve from $Z = 0$ to $Z = 1.333$ is given by,

$$\text{Area} = 0.4082 + \frac{(0.4099 - 0.4082)}{(1.34 - 1.33)} \times (1.333 - 1.33)$$

$$= 0.40871$$

As shown in Fig. 24.18, the area below the normal curve from $Z = -\infty$ to $Z = 1.333$ represents the probability of bearings that may fail within the first 11 000 h. This area is equal to (0.5 + 0.40871) or 0.90871.

Therefore, 90.87% bearings may fail within the first 11 000 h.

Number of bearings likely to fail within the first 11 000 h = 0.90871(100) = 90.87 or 91

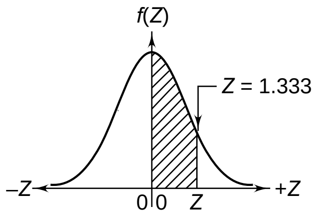


Fig. 24.18

Example 24.13 *The life of a ball bearing is a normally distributed random variable, with a mean of 10 000 h and a standard deviation of 500 h. The manufacturer of the bearings wants to give a guarantee that 90% of the bearings will reach or exceed the rated life published in his catalogue. What should be the rated life?*

Solution

Given $\mu = 10\,000$ h $\quad \hat{\sigma} = 500$ h $\quad R = 90\%$

Step I *Standard variable Z_0 for 90 % reliability*

The probability of survival of the bearings or reliability is 90% or 0.9. As shown in Fig. 24.19, the shaded area below the normal curve from $Z = Z_0$ to $Z = +\infty$ should be 0.9. The area below the normal curve from $Z = 0$ to $Z = +\infty$ is 0.5. Therefore, the area below the normal curve from $Z = 0$ to $Z = -Z_0$ should be 0.4. From Table 24.6, the corresponding value of Z_0 is approximately 1.28. Therefore,

$$Z_0 = -1.28$$

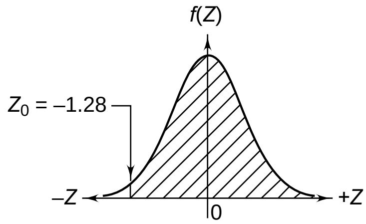


Fig. 24.19

Step II *Rated life of bearings*

$$Z_0 = \frac{X_0 - \mu}{\hat{\sigma}} \quad \text{or} \quad -1.28 = \frac{X_0 - 10\,000}{500}$$

or $X_0 = 9360$ h

Therefore 90% of the bearings will complete or exceed a life of 9360 h before fatigue failure.

Example 24.14 *Tension test is carried out on 120 specimens made of grey cast iron of grade FG300. It is observed that the ultimate tensile strength (UTS) is normally distributed with a mean of 300 N/mm² and a standard deviation of 25 N/mm².*

(i) How many specimens have UTS less than 275 N/mm²?

(ii) How many specimens have UTS between 275 and 350 N/mm²?

Solution

Given $\mu = 300$ N/mm² $\quad \hat{\sigma} = 25$ N/mm²

Step I *Number of specimens having UTS less than 275 N/mm²*

$$X_1 = 275 \text{ N/mm}^2$$

$$Z_1 = \frac{X_1 - \mu}{\hat{\sigma}} = \frac{275 - 300}{25} = -1$$

As shown in Fig. 24.20(a), the area below the normal curve from $Z_1 = -1$ to $Z = -\infty$ indicates the probability of specimens having UTS less than 275 N/mm². The normal curve is symmetrical about Y-axis. Therefore, the area below the normal curve from $Z = 0$ to $Z = -1$ is equal to the area below the curve from $Z = 0$ to $Z = +1$. From Table 24.6, the area below normal curve from $Z = 0$ to $Z = 1$ is 0.3413. Also, the area below normal curve from $Z = -\infty$ to $Z = 0$ is 0.5. Therefore,

Shaded area in Fig. 24.20(a) = 0.5 – 0.3413
= 0.1587

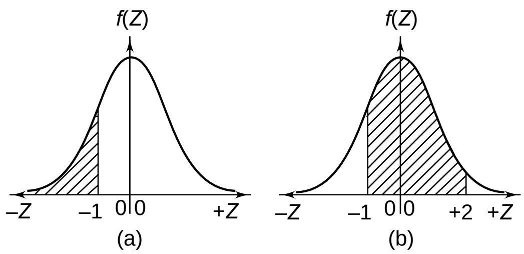


Fig. 24.20

Therefore, 15.87% of specimens will have UTS less than 275 N/mm².

No. of specimens = 0.1587 × 120 = 19.04 or 19 (i)

Step II *Number of specimens having UTS between 275 and 350 N/mm²*

$$X_2 = 350 \text{ N/mm}^2$$

$$Z_2 = \frac{X_2 - \mu}{\hat{\sigma}} = \frac{350 - 300}{25} = +2$$

As shown in Fig. 24.20(b), the area below the normal curve from $Z_1 = -1$ to $Z_2 = +2$ indicates the probability of specimens having UTS between 275 and 350 N/mm². From Table 24.6, the area below normal curve from $Z = 0$ to $Z = +2$ is 0.4772.

Shaded area in Fig. 24.20(b) = 0.3413 + 0.4772 = 0.8185

Therefore, 81.85% of specimens will have UTS between 275 to 350 N/mm².

No. of specimens = 0.8185 × 120 = 98.22 or 98 (ii)

Example 24.15 *A study of past record indicates that the load acting on a screw jack is a normally distributed random variable with a mean of 50 kN and a standard deviation of 10 kN.*

(i) What is the probability that the load selected at random will be more than 50 kN?

(ii) What is the probability that the load selected at random will be between 50 and 65 kN?

Solution

Given $\mu = 50$ kN $\quad \hat{\sigma} = 10$ kN

Step I *Probability of load more than 50 kN*

As shown in Fig. 24.21(a), half the area of the curve is located on either side of the mean load of 50 kN. The area below the normal curve from $Z = 0$ to $Z = +\infty$ indicates the probability that the load will be more than 50 kN. This area is 0.5. Therefore, we can conclude that the probability of a randomly selected load being more than 50 kN is 0.5 or 50%. (i)

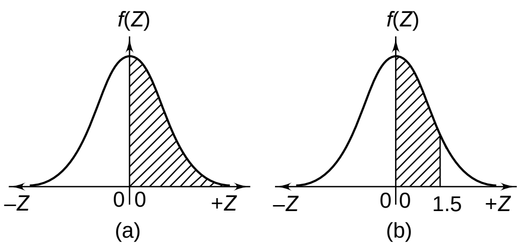


Fig. 24.21

Step II *Probability of load between 50 and 65 kN*

When the randomly selected load is 65 kN, $X = 65$ kN

$$Z = \frac{X - \mu}{\hat{\sigma}} = \frac{65 - 50}{10} = 1.5$$

As shown in Fig. 24.21(b), the area below the normal curve from $Z = 0$ to $Z = 1.5$ indicates the probability that the load selected at random will be between 50 to 65 kN. From Table 24.6, the area below the normal curve from $Z = 0$ to $Z = 1.5$ is 0.4332. Therefore, the probability that a randomly selected load will be between 50 to 65 kN is 43.32%. (ii)

Example 24.16 *It has been observed that the yield strength of the material of a component is normally distributed with a mean of 230 N/mm² and a standard deviation of 30 N/mm². The stress induced in the component is also normally distributed with a mean of 150 N/mm² and a standard deviation of 15 N/mm². Determine the reliability used in designing the component.*

Solution

Given $\mu_S = 230$ N/mm² $\quad \hat{\sigma}_S = 30$ N/mm²
$\mu_\sigma = 150$ N/mm² $\quad \hat{\sigma}_\sigma = 15$ N/mm²

Step I *Population of strength (S)*

For population of strength,

$$\mu_S = 230 \text{ N/mm}^2 \quad \text{and} \quad \hat{\sigma}_S = 30 \text{ N/mm}^2$$

Step II *Population of stress* (σ)

For population of stress,

$$\mu_\sigma = 150 \text{ N/mm}^2 \quad \text{and} \quad \hat{\sigma}_\sigma = 15 \text{ N/mm}^2$$

Step III *Population of margin of safety (m)*

The population of margin of safety is formed by subtracting the stress population from the strength population. It is denoted by *m*. Therefore,

$$\mu_m = \mu_S - \mu_\sigma = 230 - 150 = 80 \text{ N/mm}^2$$

$$\hat{\sigma}_m = \sqrt{(\hat{\sigma}_S)^2 + (\hat{\sigma}_\sigma)^2} = \sqrt{(30)^2 + (15)^2}$$

$$= 33.54 \text{ N/mm}^2$$

Step IV *Reliability*

When $m = 0$, the standard variable Z_0 is given by

$$Z_0 = \frac{m - \mu_m}{\hat{\sigma}_m} = \frac{0 - 80}{33.54} = -2.39$$

As shown in Fig. 24.22, the component will be unreliable for $Z = -\infty$ to $Z = Z_0$ or -2.39. The reliability of the component is the area below the normal curve from $Z = -2.39$ to $Z = +\infty$. From Table 24.6, area below normal curve from $Z = 0$ to $Z = 2.39$ is 0.4916. Total area below the normal curve from $Z = -2.39$ to $Z = +\infty$ is (0.4916 + 0.5) or 0.9916. Therefore, reliability used in designing the component is 99.16%.

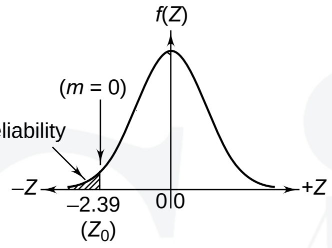


Fig. 24.22

Example 24.17 *A mechanical component is subjected to a mean stress of 100 N/mm² and a standard deviation of 10 N/mm². The material of the component has a mean strength of 130 N/mm² and a standard deviation of 15 N/mm².*

(i) Find the probability of failure for the component.

(ii) If better manufacturing control reduces the standard deviation of material strength to 10 N/mm², find the probability of failure.

(iii) If we consider only mean values of the data in design, find out the factor of safety.

Solution

Given $\mu_S = 130$ N/mm² $\quad \hat{\sigma}_S = 15$ N/mm²

$\mu_\sigma = 100$ N/mm² $\quad \hat{\sigma}_\sigma = 10$ N/mm²

Step I *Population of strength (S)*

S denotes the population of strength. For this population,

$$\mu_S = 130 \text{ N/mm}^2 \quad \text{and} \quad \hat{\sigma}_S = 15 \text{ N/mm}^2$$

Step II *Population of stress (σ)*

The population of stress is denoted by σ.

$$\mu_\sigma = 100 \text{ N/mm}^2 \quad \text{and} \quad \hat{\sigma}_\sigma = 10 \text{ N/mm}^2$$

Step III *Population of margin of safety (m)*

The population of margin of safety is denoted by m. It is obtained by subtracting the stress population from the population of strength. Therefore,

$$\mu_m = \mu_S - \mu_\sigma = 130 - 100 = 30 \text{ N/mm}^2$$

$$\hat{\sigma}_m = \sqrt{(\hat{\sigma}_S)^2 + (\hat{\sigma}_\sigma)^2} = \sqrt{(15)^2 + (10)^2}$$

$$= 18.03 \text{ N/mm}^2$$

Step IV *Probability of failure for component*

When $m = 0$, the standard variable Z_0 is given by

$$Z_0 = \frac{m - \mu_m}{\hat{\sigma}_m} = \frac{0 - 30}{18.03} = -1.664$$

As shown in Fig. 24.23(a), the area under the normal curve from $Z = -\infty$ to $Z = Z_0$ or -1.664 indicates the probability of failure of the component. From Table 24.6, the area below the normal curve from $Z = 0$ to $Z = 1.664$ is given by,

$$\text{Area} = 0.4515 + \frac{(0.4525 - 0.4515)}{(1.67 - 1.66)}(1.664 - 1.66)$$

$$= 0.4519$$

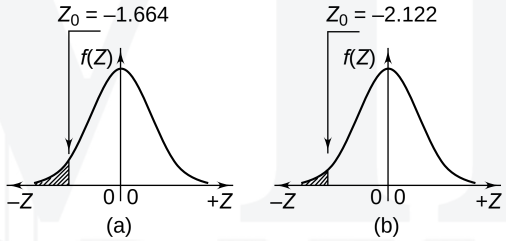


Fig. 24.23

The shaded area below the normal curve from $Z = -1.664$ to $Z = -\infty$ is (0.5 – 0.4519) or 0.0481. Therefore, the probability of failure of the components is 0.0481 or 4.81%. (i)

Step V *Probability of failure for component*

$$(\hat{\sigma}_S = 10 \text{ N/mm}^2)$$

When the manufacturing process is kept under better control,

$$\hat{\sigma}_S = 10 \text{ N/mm}^2$$

$$\hat{\sigma}_m = \sqrt{(\hat{\sigma}_S)^2 + (\hat{\sigma}_\sigma)^2} = \sqrt{(10)^2 + (10)^2}$$

$$= 14.14 \text{ N/mm}^2$$

When $m = 0$, the standard variable Z_0 is given by

$$Z_0 = \frac{m - \mu_m}{\hat{\sigma}_m} = \frac{0 - 30}{14.14} = -2.122$$

As shown in Fig. 24.23(b), the area under the normal curve from $Z = -\infty$ to $Z = Z_0$ or -2.122 indicates the probability of failure of the component. From Table 24.6, the area below the normal curve from $Z = 0$ to $Z = 2.122$ is given by,

$$\text{Area} = 0.4830 + \frac{(0.4834 - 0.4830)}{(2.13 - 2.12)}(2.122 - 2.12)$$

$$= 0.4831$$

The shaded area below the normal curve from $Z = -2.122$ to $Z = -\infty$ is $(0.5 - 0.4831)$ or 0.0169. Therefore, the probability of failure of the components is 0.0169 or 1.69%. (ii)

Step VI *Factor of safety*
When only mean values are considered.

$$\text{Factor of safety} = \frac{\mu_S}{\mu_\sigma} = \frac{130}{100} = 1.3 \qquad \text{(iii)}$$

Example 24.18 *A cantilever beam is made of plain carbon steel 25C8 having mean yield strength of 280 N/mm² and a standard deviation of 40 N/mm². It is subjected to a bending stress with a mean of 180 N/mm² and a standard deviation of 20 N/mm². Determine:*

(i) the reliability of the beam;
(ii) the average factor of safety; and
(iii) the minimum available factor of safety.

Solution

Given $\mu_S = 280$ N/mm² $\hat{\sigma}_S = 40$ N/mm²
$\mu_\sigma = 180$ N/mm² $\hat{\sigma}_\sigma = 20$ N/mm²

Step I *Population of yield strength (S)*
S denotes the population of yield strength. For this population,

$$\mu_S = 280 \text{ N/mm}^2 \quad \text{and} \quad \hat{\sigma}_S = 40 \text{ N/mm}^2$$

Step II *Population of bending stress (σ)*
The population of bending stress is denoted by σ.

$$\mu_\sigma = 180 \text{ N/mm}^2 \quad \text{and} \quad \hat{\sigma}_\sigma = 20 \text{ N/mm}^2$$

Step III *Population of margin of safety (m)*
The population of margin of safety is denoted by m. It is obtained by subtracting bending stress population from the population of yield strength. Therefore,

$$\mu_m = \mu_S - \mu_\sigma = 280 - 180 = 100 \text{ N/mm}^2$$

$$\hat{\sigma}_m = \sqrt{(\hat{\sigma}_S)^2 + (\hat{\sigma}_\sigma)^2} = \sqrt{(40)^2 + (20)^2}$$

$$= 44.72 \text{ N/mm}^2$$

Step IV *Reliability of the beam*
When $m = 0$, the standard variable Z_0 is given by

$$Z_0 = \frac{m - \mu_m}{\hat{\sigma}_m} = \frac{0 - 100}{44.72} = -2.236$$

The following observations are made with reference to Fig. 24.24,

(i) When $Z = -2.236$, the value of m is zero or there is no margin of safety.
(ii) The area below the normal curve from $Z = -\infty$ to $Z = -2.236$ indicates the region of unreliability. In this region, m is negative or the bending stress is more than the yield strength.
(iii) The area below the normal curve from $Z = -2.236$ to $Z = +\infty$ indicates the region of reliability. In this region, m is positive or the bending stress is less than the yield strength.

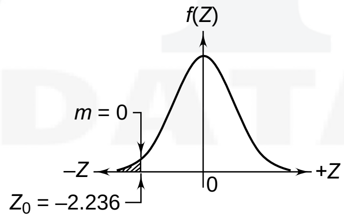


Fig. 24.24

From Table 24.6, the area below the normal curve from $Z = 0$ to $Z = +2.236$ is given by,

$$\text{Area} = 0.4871 + \frac{(0.4875 - 0.4871)}{(2.24 - 2.23)} \times (2.236 - 2.23)$$

$$= 0.4873$$

The total area below the normal curve from $Z = -2.236$ to $Z = +\infty$ consists of two parts namely, the area from $Z = -2.236$ to $Z = 0$ and area from $Z = 0$ to $Z = +\infty$. Therefore, total area is equal to $(0.4873 + 0.5)$ or 0.9873.

$\therefore$ Reliability of beam = 0.9873 or 98.73% (i)

Step V *Average factor of safety*

$$\text{Average factor of safety} = \frac{\mu_S}{\mu_\sigma} = \frac{280}{180} = 1.556 \quad \text{(ii)}$$

Step VI *Minimum available factor of safety*

Minimum yield strength

$$= \mu_S - 3\hat{\sigma}_S = 280 - 3(40) = 160 \text{ N/mm}^2$$

Maximum bending stress

$$= \mu_\sigma + 3\hat{\sigma}_\sigma = 180 + 3(20) = 240 \text{ N/mm}^2$$

$$\text{Minimum factor of safety} = \frac{160}{240} = 0.667 \quad \text{(iii)}$$

Example 24.19 *A mechanical component is subjected to a normally distributed force with a mean of 1000 N and a standard deviation of 200 N. The designer has used a factor of safety of 1.5 based on mean values. However due to variations in dimensions, the strength of the component is normally distributed with a mean of 1500 N and a standard deviation of 150 N.*

(i) What percentage of failure would be expected?

(ii) It is required to reduce the standard deviation of part strength by better quality control in order to achieve a failure rate of only 1%. What should be the standard deviation of the strength assuming other factors without any change?

(iii) It is required to improve the mean strength of the component by using better material in order to achieve a failure rate of 1%. What should be the mean strength assuming other factors unchanged?

Solution

Given $\mu_S = 1500$ N $\quad \hat{\sigma}_S = 150$ N

$(fs) = 1.5 \quad \mu_F = 1000$ N $\quad \hat{\sigma}_F = 200$ N

Step I *Population of strength (S)*

S denotes the population of strength. For this population,

$$\mu_S = 1500 \text{ N} \quad \text{and} \quad \hat{\sigma}_S = 150 \text{ N}$$

Step II *Population of force (F)*

F denotes the population of force.

$$\mu_F = 1000 \text{ N} \quad \text{and} \quad \hat{\sigma}_F = 200 \text{ N}$$

Step III *Population of margin of safety (m)*

A third population of margin of safety is denoted by m. It is obtained by subtracting the force population from the population of strength. Therefore,

$$\mu_m = \mu_S - \mu_F = 1500 - 1000 = 500 \text{ N}$$

$$\hat{\sigma}_m = \sqrt{(\hat{\sigma}_S)^2 + (\hat{\sigma}_F)^2} = \sqrt{(150)^2 + (200)^2}$$

$$= 250 \text{ N/mm}^2$$

Step IV *Percentage of failure*

When $m = 0$, the standard variable Z_0 is given by

$$Z_0 = \frac{m - \mu_m}{\hat{\sigma}_m} = \frac{0 - 500}{250} = -2$$

As shown in Fig. 24.25(a), the area under the normal curve from $Z = -\infty$ to $Z = Z_0$ or -2 indicates the probability of failure of the component. From Table 24.6, the area below the normal curve from $Z = 0$ to $Z = 2$ is 0.4772. The shaded area below the normal curve from $Z = -2$ to $Z = -\infty$ is (0.5 – 0.4772) or 0.0228. Therefore, the probability of failure of the components is 0.0228 or 2.28%. (i)

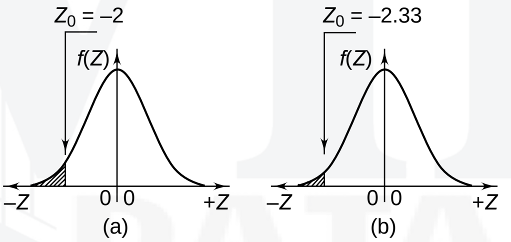


Fig. 24.25

Step V *Standard deviation to achieve 1% failure rate*

When the manufacturing process is kept under better quality control, the failure is reduced to 1%. As shown in Fig. 24.25(b), the area below the normal curve from $Z = Z_0$ to $Z = +\infty$ should be 0.99 for a failure of 1%. The area below the normal curve from $Z = 0$ to $Z = +\infty$ is 0.5. Therefore, the area below the normal curve from $Z = 0$ to $Z = Z_0$ should be (0.99 – 0.5) or 0.49. From Table 24.6, the corresponding value of Z_0 is approximately 2.33.

$$Z_0 = \frac{m - \mu_m}{\hat{\sigma}_m} \quad \text{or} \quad -2.33 = \frac{0 - 500}{\hat{\sigma}_m}$$

$$\therefore \hat{\sigma}_m = 214.59 \text{ N}$$

$$(\hat{\sigma}_m)^2 = (\hat{\sigma}_S)^2 + (\hat{\sigma}_F)^2$$

or $(214.59)^2 = (\hat{\sigma}_S)^2 + (200)^2 \quad \therefore \quad \hat{\sigma}_S = 77.77$ N

Therefore, it is necessary to reduce the standard deviation of strength from 150 to 77.77 N to reduce the failure of the components to 1%. (ii)

Step VI *Mean strength to achieve 1% failure rate*

When the mean strength of the component is improved by using a better material, the failure is reduced to 1%. As discussed in part (ii), the corresponding value of Z_0 is approximately –2.33.

$$Z_0 = \frac{m - \mu_m}{\hat{\sigma}_m} \quad \text{or} \quad -2.33 = \frac{0 - \mu_m}{250}$$

$$\therefore \quad \mu_m = 2.33(250) = 582.5 \text{ N}$$

$$\mu_m = \mu_S - \mu_F \quad \text{or} \quad 582.5 = \mu_S - 1000$$

$$\therefore \quad \mu_S = 1582.5 \text{ N}$$

Therefore, it is necessary to improve the mean strength of the material from 1500 to 1582.5 N in order to reduce the failure of the components to 1%. (iii)

Example 24.20 *Transmission shafts are machined at a machining centre. The designer has specified 25±0.05 mm as the limits for the diameter of the shafts. The natural tolerance is found to be a normally distributed random variable with a mean of 25 mm. What should be the standard deviation of the manufacturing process in order to assure that 98% of the shafts are within acceptable limits?*

Solution

Given $\mu = 25$ mm $\quad X_o = 25.05$ mm $\quad R = 98\%$

Step I *Standard variable* Z_o

The probability of acceptance is 98% or 0.98. As shown in Fig. 24.26, the shaded area below normal curve from $Z = -Z_0$ to $Z = +Z_0$ should be 0.98.

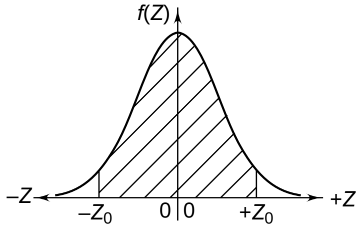


Fig. 24.26

The normal curve is symmetrical about the Y-axis. Therefore, the area below the normal curve from $Z = 0$ to $Z = +Z_0$ is (0.98/2) or 0.49. From Table 24.6, the corresponding value of Z_0 is approximately 2.33.

Step II *Standard variable*

When $Z_0 = 2.33$, the corresponding variable X_0 is (25 + 0.05) or 25.05 mm.

$$Z_o = \frac{X_0 - \mu}{\hat{\sigma}} \quad \text{or} \quad 2.33 = \frac{25.05 - 25}{\hat{\sigma}}$$

$$\therefore \quad \hat{\sigma} = 0.0215 \text{ mm}$$

Short-Answer Questions

24.1 What is frequency polygon?

24.2 What is normal curve?

24.3 What is the central tendency of population?

24.4 What is dispersion of population?

24.5 What is skewness of population?

24.6 What is kurtosis of population?

24.7 What is the measure of central tendency of population?

24.8 What is the measure of dispersion of population?

24.9 What is the mean of population?

24.10 What is standard deviation?

24.11 What is standard variable?

24.12 What is variance?

24.13 What is probability?

24.14 What is the area below normal curve from $Z = -\infty$ to $Z = +\infty$?

24.15 What is the area below normal curve from $Z = -3$ to $Z = +3$?

24.16 What is the area below normal curve from $Z = -2$ to $Z = +2$?

24.17 What is the area below normal curve from $Z = -1$ to $Z = +1$?

24.18 When population X is added to population Y, what is the mean of the resultant population?

24.19 When population Y is subtracted from population X, what is the mean of the resultant population?

24.20 When two populations X and Y are added or subtracted, what is the standard deviation of the resultant population?

24.21 What are the causes of variations in dimensions of component?

24.22 What is design tolerance?

24.23 What is natural tolerance?

24.24 What do you understand by reliable product? Give examples.

24.25 Define reliability.

Problems for Practice

24.1 The diameters of a bolt are normally distributed with a mean of 10.02 mm and a standard deviation of 0.01 mm. The design specifications for the diameter are 10 ± 0.025 mm. Calculate the percentage of bolts likely to be rejected.

[*30.85%*]

24.2 The width of a slot on an aluminium forging is normally distributed with a mean of 25 mm and a standard deviation of 0.1 mm. The design limits for the slot are 25 ± 0.15 mm. Determine the percentage of defective forgings.

[*13.36%*]

24.3 It is observed from a sample of 100 bolts produced on an automatic machine that their diameters are normally distributed with a mean of 10.5 mm and a standard deviation of 0.02 mm. Determine the tolerances specified by the designer if five bolts are rejected.

[*10.5 ± 0.0392 mm*]

24.4 The recommended class of fit between the journal and bearing is 20H7-e8. The maximum and minimum clearances are limited to 0.08 and 0.05 mm respectively. Assuming natural tolerances equal to design tolerances, determine the percentage of rejected assemblies.

[*2.78%*]

24.5 An assembly of two components *A* and *B* with an overall dimension of 40 ± 0.9 mm is shown in Fig. 24.27. The overall dimension as well as the dimensions of individual components are normally distributed, and natural tolerances are equal to design tolerances. Specify the dimensions for the component *B*.

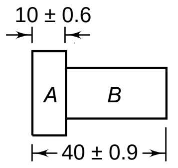


Fig. 24.27

[*30 ± 0.670 mm*]

24.6 An assembly of three components with overall dimension of 45 ± 0.9 mm is shown in Fig. 24.28. The overall dimension and the dimensions of individual components are normally distributed. The mean dimensions are shown in Fig. 24.28. The individual components have same standard deviation and their natural and design tolerances are equal. Specify the tolerances for individual components.

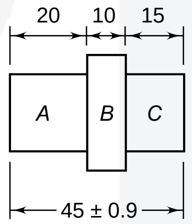


Fig. 24.28

[*± 0.52 mm*]

24.7 An assembly of two components *A* and *B* is shown in Fig. 24.29. Their dimensions are normally distributed with the following values:

$\mu_A = 75$ mm $\hat{\sigma}_A = 0.025$ mm

$\mu_B = 75.125$ mm $\hat{\sigma}_B = 0.0375$ mm

Determine the probability of interference fit between the two components.

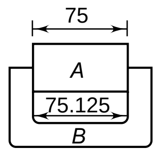


Fig. 24.29

[*0.27%*]

24.8 A rod, subjected to axial stress, has rectangular cross-section with width *w* and thickness *t*. It has been observed that the width (*w*) is a normally distributed random variable with a mean of 50 mm and a standard deviation of 0.5 mm. The thickness (*t*) is also a normally distributed random variable with a mean of 25 mm and a standard deviation of 0.3 mm.

Determine the mean and standard deviation of the cross-sectional area (*A*). Comment on the analysis.

[*1250 and 19.53 mm²*]

24.9 A mechanical component is subjected to a uni-axial stress and the factor of safety is given by

$$(fs) = \frac{\text{strength}}{\text{stress}}$$

It has been observed that the strength (*S*) of the component is a normally distributed random variable with a mean of 300 N/mm^2 and a standard deviation of 10 N/mm^2. The stress in the component (σ) is also a normally distributed random variable with a mean of 150 N/mm^2 and a standard deviation of 20 N/mm^2.

Determine the mean and standard deviation of the factor of safety (*fs*). Comment on the analysis.

[*2 and 0.27*]

24.10 A ball bearing has normally distributed time to failure, with a mean of 15 000 h and standard deviation of 1000 h. If there are 100 such bearings fitted at a time, how many may be expected to fail within the first 16 500 h?

[*93.32%*]

24.11 It has been observed that the load acting on a crane hook is a normally distributed random variable with a mean of 5 kN and a standard deviation of 0.5 kN.

(i) What is the probability that the load selected at random will be more than 5 kN?

(ii) What is the probability that the load selected at random will be between 5 and 6 kN?

[*50 and 47.72%*]

24.12 A cantilever beam is made of plain carbon steel 45C8 having mean yield strength of 380 N/mm^2 and a standard deviation of 50 N/mm^2. It is subjected to a bending stress with a mean of 250 N/mm^2 and a standard deviation of 25 N/mm^2. Determine:

(i) the reliability of the beam;

(ii) the average factor of safety; and

(iii) the minimum available factor of safety.

[*(i) 99.01% (ii) 1.52 (iii) 0.71*]

Design of IC Engine Components

Chapter 25

25.1 INTERNAL COMBUSTION ENGINE

An Internal Combustion engine (IC engine) is an engine in which the combustion of fuel, such as petrol or diesel, takes place inside the engine cylinder. In petrol engine, air and petrol is mixed in correct proportion in the carburetter and then passed into the cylinder. This mixture is ignited by means of a spark produced by the spark plug. *Since the ignition is done by spark, the petrol engine is called Spark Ignition engine (SI Engine)*. In the diesel engine, the air entrapped in the cylinder during the suction stroke is highly compressed during compression stroke. This compression increases the air temperature beyond the self-ignition temperature of diesel. The desired quantity of diesel in the form of fine spray is then admitted into the cylinder near the end of the compression stroke. The turbulent hot air ignites the diesel. *Since the ignition is done by compression of air, the diesel engine is called Compression Ignition engine (CI engine)*.

Compared with petrol engines, the diesel engines are more economical due to high thermal efficiency. They have more uniform torque over a wide range of speeds due to better volumetric efficiency. The diesel engines run at low speeds, resulting in low maintenance costs. They are more reliable and safe due to robust construction. On the other hand, petrol engines have low initial cost and higher power to weight ratio compared with diesel engine.

Internal combustion engines are also classified as 'two-stroke cycle' and 'four-stroke cycle' engines. *The two-stroke cycle engine is an engine which requires two strokes of the piston or one revolution of the crankshaft to complete one cycle. A four-stroke cycle engine is an engine which requires four strokes of the piston or two revolutions of the crankshaft to complete one cycle*. Four-stroke cycle engines have lower fuel consumption and higher efficiency. Two-stroke cycle engines are light in weight and have compact construction.

Two-stroke cycle petrol engines are mainly used in scooters, motorcycles and three-wheelers. Four-stroke cycle petrol engines are used in cars. Four-stroke cycle diesel engines are used in heavy-duty applications such as buses, trucks, locomotives and power generating sets.

Although internal combustion engine consists of a large number of parts, in this chapter we will discuss the design principles of the following main components:

(i) Cylinder and cylinder liner
(ii) Piston, piston rings and gudgeon pin
(iii) Connecting rod with big and small ends
(iv) Crankshaft, crank and crank pin
(v) Valve gear mechanism

Engine design is a specialized subject and it differs from machine design. The discussion in this chapter is restricted to basic principles applied to design of engine components.

25.2 CYLINDER AND CYLINDER LINER

There are two basic functions of an IC engine cylinder. The primary function is to retain the working fluid such as the mixture of air and petrol or air and diesel, while the secondary function is to guide the piston. The combustion of fuel takes place inside the cylinder and very high temperatures are encountered. Therefore, it is necessary to provide some arrangement for cooling the cylinder. There are two types of cooling systems—air-cooling and water-cooling. Small, single-cylinder engines are usually air-cooled. Such cylinders are provided with fins over the outer surface of the cylinder. Excess heat of combustion is transmitted by the cylinder wall to the surroundings through the fins. The fins increase the surface area of the cylinder wall and improve the overall heat transfer coefficient. Air-cooled engines are mainly used on scooters and motorcycles.

In small engines, the cylinder and frame is made of one-piece casting. In large engines, a separate cylinder liner is used. The cylinder liner, water jacket and frame are manufactured separately and then assembled. The construction of cylinder liner is illustrated in Fig. 25.1. The use of separate cylinder liner has the following advantages:

(i) Cylinder liners are more economical because they can be easily replaced after being worn out. It is not necessary to replace the complete assembly of cylinder, jacket and frame.

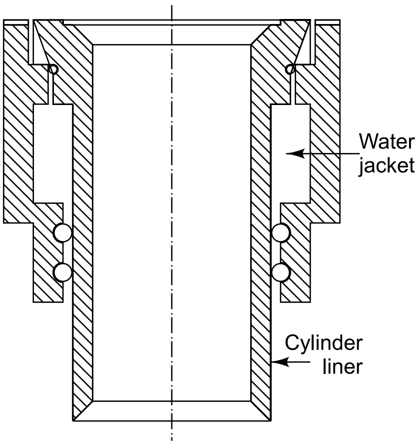


Fig. 25.1 *Cylinder liner*

(ii) Instead of using better-grade material for all parts of the cylinder assembly, only the cylinder liner is made of better-grade wear resistant cast iron. The frame and jacket can be made of ordinary cast iron.

(iii) Use of cylinder liner allows for longitudinal expansion.

There are two types of cylinder liners—dry liner and wet liner as shown in Fig. 25.2. *A dry liner is a cylinder liner which does not have any direct contact with cooling water in the jacket. A wet liner is a cylinder liner which has outer surface in direct contact with cooling water in the jacket.*

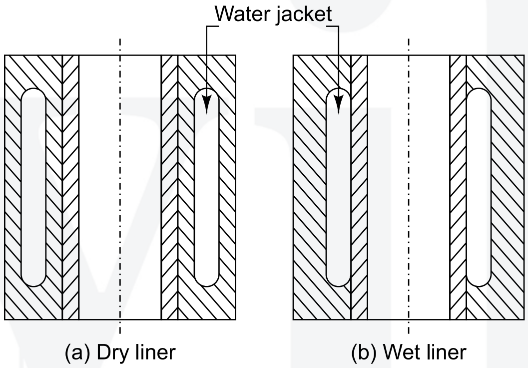


Fig. 25.2 *Dry and wet liners*

The desirable properties of materials for cylinders and cylinder liners are as follows:

(i) It should be strong enough to withstand high gas pressure during the combustion of fuel.

(ii) It should be strong enough to withstand thermal stresses due to heat transfer through the cylinder wall.

(iii) It should be hard enough to resist wear due to piston movement. It should have good surface finish to reduce friction and wear during the piston movement.

(iv) It should be corrosion resistant.

Cylinders and cylinder liners are usually made of grey cast iron with homogeneous and close grained structure. They are centrifugally cast. For heavy-duty cylinders, nickel cast iron and nickel chromium cast iron are used. In some cases, cast steel and aluminium alloys are used for cylinders.

25.3 BORE AND LENGTH OF CYLINDER

In engine terminology, 'bore' means the inner diameter of the cylinder. The main dimensions of the cylinder are calculated by using the following equations from Applied Thermodynamics.

$$\text{IP} = \frac{\text{BP}}{\eta} \tag{25.1}$$

where,

IP = indicated power or power produced inside the cylinder (W)

BP = brake power or power developed at the crankshaft (W)

η = mechanical efficiency (in fraction)

In examples where mechanical efficiency is not specified, it is assumed as 80% or 0.8.

$n = N$ (for two-stroke engines)

$n = N/2$ (for four-stroke engines) (25.2)

where,

n = number of working strokes per minute

N = engine speed (rpm)

$$\text{IP} = \frac{p_m lAn}{60} \tag{25.3}$$

where,

p_m = indicated mean effective pressure (N/mm^2 or MPa)

l = length of stroke (**m**)

A = cross-sectional area of cylinder (mm^2)

$$A = \left(\frac{\pi D^2}{4}\right)$$

n = number of working strokes/min

D = diameter of cylinder or bore (mm)

The (l/D) ratio for the cylinder is usually assumed from 1.25 to 2. In examples where (l/D) ratio is not specified, it is assumed as 1.5.

$$l = 1.5\ D \text{ (in mm)} = \left(\frac{1.5\ D}{1000}\right) \text{ (in m)}$$

The length of the cylinder is more than the length of the stroke. There is clearance on both sides of the stroke. The total clearance on two sides is taken as 15% of the stroke length.

$$L = 1.15\ l$$

where,

L = length of cylinder (mm)

25.4 THICKNESS OF CYLINDER WALL

The engine cylinder or cylinder liner is treated as a thin cylinder. Equation (22.3) of Chapter 22 derived for the thickness of thin cylinder is modified and used for engine cylinder.

$$t = \frac{p_{\text{max.}} D}{2\sigma_c} + C \tag{25.4}$$

where,

t = thickness of cylinder wall (mm)

$p_{\text{max.}}$ = maximum gas pressure inside the cylinder (N/mm^2 or MPa)

D = inner diameter of cylinder or cylinder bore (mm)

σ_c = permissible circumferential (hoop) stress for cylinder material (N/mm^2)

C = reboring allowance (mm)

Note

(i) In examples where maximum gas pressure inside the cylinder is not specified, it is assumed as 10 times of the indicated mean effective pressure,

$$p_{\text{max.}} = 10\ (p_m)$$

(ii) The circumferential hoop stress (σ_c) in Eq. (25.4) is the allowable tensile stress (σ_t). Since the cylinder material is brittle,

$$\sigma_c = \sigma_t = \frac{S_{ut}}{(fs)}$$

(iii) In examples where ultimate tensile strength of cylinder material and factor of safety are not specified, the allowable circumferential stress (σ_c) is taken as 35 to 100 N/mm^2.

(iv) The reboring allowance is taken from Table 25.1. Reboring is required to compensate uneven wear on the inner wall of the cylinder. *Reboring allowance is additional metal thickness over and above that required to withstand maximum gas pressure inside the cylinder*. It is provided to compensate for reboring at intervals during the lifetime of the cylinder.

Table 25.1 *Reboring allowance for IC engine cylinders*

D	75	100	150	200	250	300	350	400	450	500
C	1.5	2.4	4.0	6.3	8.0	9.5	11.0	12.5	12.5	12.5

(*Note: D* and *C* are in mm)

Empirical Relationships There are some empirical relationships used in cylinder design. They are as follows:

(i) The thickness of cylinder wall varies from 5 to 25 mm depending upon the cylinder bore. It can be calculated by using the following empirical equation:

$$t = 0.045\,D + 1.6 \text{ (mm)}$$

(ii) Thickness of dry liner = 0.03D to 0.035D (mm)

(iii) Thickness of water jacket wall = (1/3) t to (3/4) t,

(iv) Thickness of water jacket wall = 0.032D + 1.6 (mm)

(v) Water space between outer cylinder wall and inner jacket wall = 9 mm for 75 mm cylinder bore to 75 mm for 750 mm cylinder bore,

(vi) Water space between outer cylinder wall and inner jacket wall = 0.08D + 6.5 mm

(vii) Thickness of cylinder flange = 1.2 t to 1.4 t Also,

(viii) Thickness of cylinder flange = 1.25 d to 1.5 d (d = nominal diameter of bolt or stud)

(ix) Radial distance between outer diameter of flange and pitch circle diameter of studs = (d + 6) to (1.5d) mm

25.5 STRESSES IN CYLINDER WALL

Apparent Stresses As shown in Fig. 25.3, there are two principal stresses in engine cylinder—the circumferential hoop stress (σ_c) and longitudinal stress (σ_1). It is assumed that the stresses are uniformly distributed over the wall thickness. Considering equilibrium of forces acting on the half portion of the cylinder of unit length [Fig. 25.3(a)],

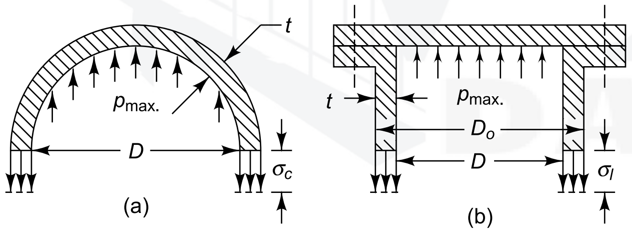


Fig. 25.3 *Stresses in thin cylinder*

$$Dp_{max.} = 2\sigma_c t$$

or
$$\sigma_c = \frac{p_{max.} D}{2t} \tag{25.5}$$

Considering equilibrium of forces in the longitudinal direction [Fig. 25.3(b)],

$$p_{max.}\left(\frac{\pi}{4}D^2\right) = \sigma_l\left[\frac{\pi}{4}(D_o^2 - D^2)\right]$$

or
$$\sigma_l = \frac{p_{max.} D^2}{(D_o^2 - D^2)} \tag{25.6}$$

Net Stresses Two principal stresses—the circumferential hoop stress (σ_c) and longitudinal stress (σ_l) are tensile stresses and they act at right angles to each other. Therefore, net stresses in these directions are reduced. The net stresses are given by,

$$(\sigma_c)_{net} = \sigma_c - \mu\sigma_l \tag{25.7}$$

$$(\sigma_l)_{net} = \sigma_l - \mu\sigma_c \tag{25.8}$$

where,

σ_c = apparent circumferential stress (N/mm^2) (obtained by Eq. 25.5)

σ_l = apparent longitudinal stress (N/mm^2) (obtained by Eq. 25.6)

$(\sigma_c)_{net}$ = net circumferential stress (N/mm^2)
$(\sigma_l)_{net}$ = net longitudinal stress (N/mm^2)
μ = Poisson's ratio

The cylinder material is usually brittle such as cast iron. The value of Poisson's ratio is taken as 0.25.

$$\mu = 0.25$$

25.6 CYLINDER HEAD

In most of the IC engines, a separate cylinder head or cylinder cover is provided. The cylinder cover accommodates the following parts:

(i) Inlet and exhaust valves
(ii) Air and gas ports
(iii) Spark plug in case of petrol engine and atomizer in case of diesel engine

The shape of the cylinder head becomes complicated due to accommodation of the above units. In general, a box type section with considerable thickness is used for the cylinder head. Calculating the various dimensions of the actual cylinder head is a difficult exercise. However, in the preliminary stages of design, the cylinder head is assumed as a flat circular plate and its thickness is calculated by the following equation:

$$t_h = D\sqrt{\frac{Kp_{max.}}{\sigma_c}} \tag{25.9}$$

where,

t_h = thickness of cylinder head (mm)
K = constant (K = 0.162)
σ_c = allowable circumferential stress (N/mm^2)

Note

(i) The circumferential hoop stress (σ_c) in Eq. (25.9) is the allowable tensile stress (σ_t). Since the material of the cylinder head is brittle,

$$\sigma_c = \sigma_t = \frac{S_{ut}}{(fs)}$$

(ii) In examples where ultimate tensile strength of cylinder head material and factor of safety are not specified, the allowable circumferential stress (σ_c) is taken as 30 to 50 N/mm^2.

25.7 DESIGN OF STUDS FOR CYLINDER HEAD

Studs are used to make the assembly of cylinder, cylinder head and gasket; and provide a leakproof joint. Initially, the studs are tightened by means of spanner to induce a preload and in working conditions they are further subjected to tensile stresses due to internal gas pressure acting on the cylinder head. There are three important parameters in design of studs—number of studs, nominal diameter and pitch of studs.

(i) Number of Studs The number of studs (z) should be between the following limits,

Minimum number of studs = $0.01D + 4$
Maximum number of studs = $0.02D + 4$ (25.10)

(ii) Diameter of Studs The core diameter of studs is obtained by equating the maximum gas force acting on the cylinder cover to the resisting force offered by all studs.

$$\text{Gas force acting on cylinder cover} = \left(\frac{\pi D^2}{4}\right)p_{max.} \tag{a}$$

$$\text{Resisting force offered by all studs} = z\left(\frac{\pi d_c^2}{4}\right)\sigma_t \tag{b}$$

Equating (a) and (b),

$$\left(\frac{\pi D^2}{4}\right)p_{max.} = z\left(\frac{\pi d_c^2}{4}\right)\sigma_t \tag{25.11}$$

where,

d_c = core or minor diameter of studs (mm)
z = number of studs
σ_t = allowable tensile stress for stud material (N/mm^2)

Note

(i) The studs are made of steel and since the material is ductile,

$$\sigma_t = \frac{S_{yt}}{(fs)}$$

(ii) In examples where the yield strength of stud material and factor of safety are not specified, the allowable tensile stress (σ_t) is taken as 35 to 70 N/mm^2.

The nominal diameter of studs is obtained by the following relationship:

$$d = \frac{d_c}{0.8} \tag{25.12}$$

(iii) Pitch of Studs The pitch circle diameter of the studs is obtained by the following empirical relationship:

$$D_p = D + 3d \tag{25.13}$$

$$\text{Pitch of studs} = \frac{\pi D_P}{z} \tag{25.14}$$

In order to obtain a leakproof joint, the pitch of studs should be between the following two limits:

$$\text{Minimum pitch} = 19\sqrt{d}$$

$$\text{Maximum pitch} = 28.5\sqrt{d} \tag{25.15}$$

It should be noted that the above analysis is elementary. In practice, various parameters are taken into consideration for calculating the size of studs. They include stiffness of gasket, stiffness of flanges of cylinder and cover and initial preload on the stud. The reader should refer to Example 22.9 of Chapter 22 for this type of treatment. Further, the studs are subjected to fluctuating load due to variation of gas pressure inside the cylinder from zero to (p_{max}). The reader should refer to Section 7.18 of Chapter 7 on 'Bolted joint under fluctuating load' for this type of treatment.

Example 25.1 *The cylinder of a four-stroke diesel engine has the following specifications:*

Brake power = 3.75 kW

Speed = 1000 rpm

Indicated mean effective pressure = 0.35 MPa

Mechanical efficiency = 80%

Determine the bore and length of the cylinder liner.

Solution

Given BP = 3.75 kW = 3750 W N = 1000 rpm
p_m = 0.35 MPa = 0.35 N/mm^2 η = 80% = 0.8

Step I *Cylinder bore*

Assumption The ratio of stroke length to cylinder diameter (l/D) is 1.5.

D = cylinder bore (mm)

A = cross-sectional area of cylinder

$$= \left(\frac{\pi D^2}{4}\right) \text{ (mm}^2\text{)}$$

$$l = \text{length of stroke in m} = \left(\frac{1.5D}{1000}\right) (m)$$

For a four-stroke engine,

$$n = \frac{N}{2} = \frac{1000}{2} = 500 \text{ strokes/min}$$

$$\text{IP} = \frac{\text{BP}}{\eta} = \frac{3750}{0.8} = 4687.5 \text{ W}$$

$$\text{IP} = \frac{p_m\, lA\, n}{60} = \frac{p_m}{60}\left(\frac{1.5D}{1000}\right)\left(\frac{\pi D^2}{4}\right)(n)$$

$$4687.5 = \frac{0.35}{60}\left(\frac{1.5D}{1000}\right)\left(\frac{\pi D^2}{4}\right)(500)$$

$$D^3 = 1364.19 \times 10^3$$

$$D = 110.9 \text{ mm or } 112 \text{ mm} \tag{i}$$

$$l = 1.5\,D = 1.5\,(112) = 168 \text{ mm}$$

The length of the stroke is 168 mm. There is clearance on both sides of the stroke. Assuming the clearance as 15% of the stroke length, the length of the cylinder (L) is given by,

$$L = 1.15\, l = 1.15\,(168) = 193.2 \text{ mm or } 195 \text{ mm} \tag{ii}$$

Example 25.2 *The cylinder of a four-stroke diesel engine has the following specifications:*

Cylinder bore = 150 mm

Maximum gas pressure = 3.5 MPa

Cylinder material = Grey cast iron FG 200
(S_{ut} = 200 N/mm^2)

Factor of safety = 5

Poisson's ratio = 0.25

Determine the thickness of the cylinder wall. Also, calculate the apparent and net circumferential and longitudinal stresses in the cylinder wall.

Solution

Given $p_{max.}$ = 3.5 MPa = 3.5 N/mm^2
D = 150 mm S_{ut} = 200 N/mm^2 (fs) = 5
μ = 0.25

Step I *Thickness of cylinder wall*

The permissible tensile stress is given by,

$$\sigma_t = \frac{S_{ut}}{(fs)} = \frac{200}{5} = 40 \text{ N/mm}^2$$

From Table 25.1, the allowance C for reboring (D = 150 mm) is 4 mm.

$$C = 4 \text{ mm}$$

The thickness of the cylinder is given by,

$$t = \frac{p_{max.} D}{2\sigma_c} + C \qquad (\sigma_c = \sigma_t = 40 \text{ N/mm}^2)$$

$$= \frac{3.5(150)}{2(40)} + 4 = 10.56 \text{ or } 12 \text{ mm} \qquad \text{(i)}$$

Step II *Apparent stresses*

Circumferential stress

$$\sigma_c = \frac{p_{max.} D}{2t} = \frac{3.5(150)}{2(12)} = 21.88 \text{ N/mm}^2$$

$$[\sigma_c < 40 \text{ N/mm}^2] \qquad \text{(ii)}$$

Longitudinal stress

$$D_o = D + 2t = 150 + 2(12) = 174 \text{ mm}$$

$$\sigma_l = \frac{p_{max.} D^2}{(D_o^2 - D^2)} = \frac{3.5(150)^2}{[(174)^2 - (150)^2]}$$

$$= 10.13 \text{ N/mm}^2 \qquad \text{(iii)}$$

Step III *Net stresses*

Circumferential stress

$$(\sigma_c)_{net} = \sigma_c - \mu\sigma_l = 21.88 - 0.25\,(10.13)$$

$$= 19.35 \text{ N/mm}^2 \qquad \text{(iv)}$$

Longitudinal stress:

$$(\sigma_l)_{net} = \sigma_l - \mu\sigma_c = 10.13 - 0.25(21.88)$$

$$= 4.66 \text{ N/mm}^2 \qquad \text{(v)}$$

Example 25.3 *The bore of a cylinder of the four-stroke diesel engine is 150 mm. The maximum gas pressure inside the cylinder is limited to 3.5 MPa. The cylinder head is made of grey cast iron FG 200 (S_{ut} = 200 N/mm^2) and the factor of safety is 5. Determine the thickness of the cylinder head.*

Studs are used to fix the cylinder head to the cylinder and obtain a leakproof joint. They are made of steel FeE 250 (S_{yt} = 250 N/mm^2) and the factor of safety is 5. Calculate.

(i) number of studs
(ii) nominal diameter of studs
(iii) pitch of studs

Solution

Given $p_{max.}$ = 3.5 MPa = 3.5 N/mm^2

D = 150 mm For cylinder head, S_{ut} = 200 N/mm^2 (fs) = 5 For studs, S_{yt} = 250 N/mm^2 (fs) = 5

Step I *Thickness of cylinder head*

The permissible stress for the cylinder head is given by,

$$\sigma_t = \frac{S_{ut}}{(fs)} = \frac{200}{5} = 40 \text{ N/mm}^2 \qquad K = 0.162$$

$$\sigma_c = \sigma_t = 40 \text{ N/mm}^2$$

$$t_h = D\sqrt{\frac{Kp_{max.}}{\sigma_c}} = (150)\sqrt{\frac{0.162(3.5)}{40}}$$

$$= 17.86 \text{ or } 18 \text{ mm}$$

Step II *Number of studs*

Limits

Minimum number of studs = $0.01D + 4$
= 0.01(150) + 4 = 5.5

Maximum number of studs = $0.02D + 4$
= 0.02 (150) + 4 = 7

The number of studs should be from 5.5 to 7. It is assumed that there are six studs.

$$z = 6$$

Step III *Nominal diameter of studs*

The permissible stress for studs is given by,

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{250}{5} = 50 \text{ N/mm}^2$$

$$\text{Force acting on cylinder head} = \left(\frac{\pi}{4}\right) D^2 p_{max.}$$

$$= \left(\frac{\pi}{4}\right)(150)^2(3.5) = 61850.11 \text{ N} \qquad \text{(a)}$$

Resisting force offered by all studs

$$= \left(\frac{\pi}{4}\right) d_c^2 z \sigma_t$$

$$= \left(\frac{\pi}{4}\right) d_c^2 (6)(50) = 235.62\, d_c^2 \qquad \text{(b)}$$

Equating (a) and (b),

$$235.62\, d_c^2 = 61\,850.11 \quad \text{or} \quad d_c = 16.2 \text{ mm}$$

$$d = \frac{d_c}{0.8} = \frac{16.2}{0.8} = 20.25 \text{ or } 20 \text{ mm}$$

Step IV *Pitch of studs*

Pitch circle diameter of studs (D_p)

$$= D + 3d = 150 + 3(20) = 210 \text{ mm}$$

$$\text{Pitch of studs} = \frac{\pi D_p}{z} = \frac{\pi(210)}{6} = 109.96 \text{ mm}$$

Limits

$$\text{Minimum pitch} = 19\sqrt{d} = 19\sqrt{20} = 84.97 \text{ mm}$$

$$\text{Maximum pitch} = 28.5\sqrt{d} = 28.5\sqrt{20} = 127.46 \text{ mm}$$

The pitch of the studs is 109.96 mm. It is within the limits of 84.97 and 127.46 mm. Therefore, the pitch of the studs is satisfactory.

Example 25.4 *The cylinder of a four-stroke diesel engine has the following specifications:*

Brake power = 7.5 kW
Speed = 1400 rpm
Indicated mean effective pressure = 0.35 MPa
Mechanical efficiency = 80%
Maximum gas pressure = 3.5 MPa

The cylinder liner and head are made of grey cast iron FG 260 (S_{ut} = 260 N/mm^2 and μ = 0.25). The studs are made of plain-carbon steel 40C8 (S_{yt} = 380 N/mm^2). The factor of safety for all parts is 6.

Calculate:

(i) bore and length of the cylinder liner
(ii) thickness of the cylinder liner
(iii) thickness of the cylinder head
(iv) size, number and pitch of studs

Solution

Given BP = 7.5 kW = 7500 W N = 1400 rpm p_m = 0.35 MPa = 0.35 N/mm^2 $p_{max.}$ = 3.5 MPa = 3.5 N/mm^2 $(fs) = 6$ η = 80% = 0.8 For studs, S_{yt} = 380 N/mm^2 For cylinder liner and head, S_{ut} = 260 N/mm^2 $\mu = 0.25$

Step I *Bore and length of cylinder liner*

Assumption The ratio of stroke length to cylinder diameter (l/D) is 1.5

D = cylinder bore (mm)

A = cross-sectional area of cylinder

$$= \left(\frac{\pi D^2}{4}\right) (\text{mm}^2)$$

$$l = \text{length of stroke in m} = \left(\frac{1.5D}{1000}\right) (m)$$

For a four-stroke engine,

$$n = \frac{N}{2} = \frac{1400}{2} = 700 \text{ strokes/min}$$

$$\text{IP} = \frac{\text{BP}}{\eta} = \frac{7500}{0.8} = 9375 \text{ W}$$

$$\text{IP} = \frac{p_m lAn}{60} = \frac{p_m}{60}\left(\frac{1.5D}{1000}\right)\left(\frac{\pi D^2}{4}\right)(n)$$

$$9375 = \frac{0.35}{60}\left(\frac{1.5D}{1000}\right)\left(\frac{\pi D^2}{4}\right)(700)$$

$$D^3 = 1948.84 \times 10^3$$

$$D = 124.9 \text{ mm or } 125 \text{ mm} \quad \text{(i)}$$

$$l = 1.5D = 1.5\,(125) = 187.5 \text{ mm}$$

The length of the stroke is 187.5 mm. There is clearance on both sides of the stroke. Assuming the clearance as 15% of stroke length, the length of the cylinder (L) is given by,

$$L = 1.15\, l = 1.15\,(187.5) = 215.63 \text{ mm}$$

or

$$L = 216 \text{ mm} \quad \text{(ii)}$$

Step II *Thickness of cylinder liner*

The permissible tensile stress is given by,

$$\sigma_t = \frac{S_{ut}}{(fs)} = \frac{260}{6} = 43.33 \text{ N/mm}^2$$

Refer to Table 25.1. By linear interpolation, the reboring allowance C for (D = 125 mm) is given by

$$C = 2.4 + \frac{(4-2.4)}{(150-100)}(125-100) = 3.2 \text{ mm}$$

The thickness of the cylinder is given by,

$$t = \frac{p_{max.} D}{2\sigma_c} + C \qquad (\sigma_c = \sigma_t = 43.33 \text{ N/mm}^2)$$

$$= \frac{3.5(125)}{2(43.33)} + 3.2 = 8.25 \text{ or } 10 \text{ mm} \quad \text{(iii)}$$

Apparent Stresses

Circumferential Stress

$$\sigma_c = \frac{p_{max.} D}{2t} = \frac{3.5(125)}{2(10)} = 21.88 \text{ N/mm}^2$$

$$[\sigma_c < 43.33 \text{ N/mm}^2]$$

Longitudinal stress

$D_o = D + 2t = 125 + 2(10) = 145$ mm

$$\sigma_l = \frac{p_{max.} D^2}{(D_o^2 - D^2)} = \frac{3.5(125)^2}{[(145)^2 - (125)^2]}$$

$$= 10.13 \text{ N/mm}^2$$

Net stresses

Circumferential stress

$(\sigma_c)_{net} = \sigma_c - \mu\sigma_1 = 21.88 - 0.25(10.13)$
$= 19.35$ N/mm^2

Longitudinal stress

$(\sigma_l)_{net} = \sigma_1 - \mu\sigma_c = 10.13 - 0.25(21.88)$
$= 4.66$ N/mm^2

Step III *Thickness of cylinder head*

The permissible stress for the cylinder head is given by,

$$\sigma_t = \frac{S_{ut}}{(fs)} = \frac{260}{6} = 43.33 \text{ N/mm}^2 \qquad K = 0.162$$

$$\sigma_c = \sigma_t = 43.33 \text{ N/mm}^2$$

$$t_h = D\sqrt{\frac{Kp_{max.}}{\sigma_c}} = (125)\sqrt{\frac{0.162(3.5)}{43.33}}$$

$$= 14.3 \text{ or } 15 \text{ mm} \qquad \text{(iv)}$$

Step IV *Number of studs*

Limits

Minimum number of studs $= 0.01D + 4$
$= 0.01\,(125) + 4 = 5.25$

Maximum number of studs $= 0.02D + 4$
$= 0.02(125) + 4 = 6.5$

The number of studs should be from 5.25 to 6.5. It is assumed that there are six studs.

$$z = 6 \qquad \text{(v)}$$

Step V *Nominal diameter of studs*

The permissible stress for studs is given by,

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{380}{6} = 63.33 \text{ N/mm}^2$$

$$\text{Force acting on cylinder head} = \left(\frac{\pi}{4}\right) D^2 p_{max.}$$

$$= \left(\frac{\pi}{4}\right)(125)^2 (3.5) = 42\,951.46 \text{ N} \qquad \text{(a)}$$

$$\text{Resisting force offered by all studs} = \left(\frac{\pi}{4}\right) d_c^2\, z\, \sigma_t$$

$$= \left(\frac{\pi}{4}\right) d_c^2 (6)(63.33) = 298.44\, d_c^2 \qquad \text{(b)}$$

Equating (a) and (b),

$298.44\, d_c^2 = 42\,951.46$ or $d_c = 12$ mm

$$d = \frac{d_c}{0.8} = \frac{12}{0.8} = 15 \text{ mm} \qquad \text{(vi)}$$

Step VI *Pitch of studs*

Pitch circle diameter of studs $(D_p) = D + 3d$
$= 125 + 3(15) = 170$ mm

$$\text{Pitch of studs} = \frac{\pi D_p}{z} = \frac{\pi(170)}{6} = 89.01 \text{ mm} \qquad \text{(vii)}$$

Limits

Minimum pitch $= 19\sqrt{d} = 19\sqrt{15} = 73.59$ mm

Maximum pitch $= 28.5\sqrt{d} = 28.5\sqrt{15}$
$= 110.38$ mm

The pitch of the studs is 89.01 mm. It is within the limits of 73.59 and 110.38 mm. Therefore, the pitch of the studs is satisfactory.

Example 25.5 *The cylinder of a four-stroke diesel engine has the following specifications:*

Brake power = 5 kW

Speed = 600 rpm

Indicated mean effective pressure = 0.5 MPa

Make suitable assumptions and calculate:

(i) bore and length of the cylinder liner

(ii) thickness of the cylinder liner

(iii) thickness of the cylinder head

(iv) size, number and pitch of studs

Solution

Given BP = 5 kW = 5000 W $N = 600$ rpm

$p_m = 0.5$ MPa $= 0.5$ N/mm^2

Step I *Bore and length of cylinder liner*

Assumption No. 1 The mechanical efficiency is 80% ($\eta = 0.8$).

$$\text{IP} = \frac{\text{BP}}{\eta} = \frac{5000}{0.8} = 6250 \text{ W}$$

Assumption No. 2 The ratio of stroke length to cylinder diameter (l/D) is 1.5

D = cylinder bore (mm)

A = cross-sectional area of cylinder

$$= \left(\frac{\pi D^2}{4}\right) \text{(mm}^2\text{)}$$

$$l = \text{length of stroke in m} = \left(\frac{1.5D}{1000}\right)(m)$$

For a four-stroke engine,

$$n = \frac{N}{2} = \frac{600}{2} = 300 \text{ strokes/min}$$

$$\text{IP} = \frac{p_m lAn}{60} = \frac{p_m}{60}\left(\frac{1.5D}{1000}\right)\left(\frac{\pi D^2}{4}\right)(n)$$

$$6250 = \frac{0.5}{60}\left(\frac{1.5D}{1000}\right)\left(\frac{\pi D^2}{4}\right)(300)$$

$$D^3 = 2122.07 \times 10^3$$

$$D = 128.5 \text{ mm or } 130 \text{ mm} \quad \text{(i)}$$

$$l = 1.5\,D = 1.5\,(130) = 195 \text{ mm}$$

The length of the stroke is 195 mm. There is clearance on both sides of the stroke. Assuming the clearance as 15% of the stroke length, the length of the cylinder (L) is given by,

$$L = 1.15\,l = 1.15\,(195) = 224.25 \text{ mm or } 225 \text{ mm} \quad \text{(ii)}$$

Step II *Thickness of cylinder liner*

Assumption No. 3 The cylinder liner is made of cast iron. The allowable tensile stress is from 35 MPa to 100 MPa. It is assumed that allowable tensile stress is 50 MPa.

$$\sigma_t = 50 \text{ N/mm}^2$$

Assumption No. 4 The maximum gas pressure ($p_{max.}$) is 10 times the mean effective pressure (p_m).

$$p_{max.} = 10\,p_m = 10\,(0.5) = 5 \text{ MPa} = 5 \text{ N/mm}^2$$

Refer to Table 25.1. By linear interpolation, the reboring allowance C for (D = 130 mm) is given by,

$$C = 2.4 + \frac{(4-2.4)}{(150-100)}(130-100) = 3.36 \text{ mm}$$

The thickness of the cylinder is given by,

$$t = \frac{p_{max.}D}{2\sigma_c} + C \quad (\sigma_c = \sigma_t = 50 \text{ N/mm}^2)$$

$$= \frac{5(130)}{2(50)} + 3.36 = 9.86 \text{ or } 10 \text{ mm} \quad \text{(iii)}$$

Apparent stresses

Circumferential stress

$$\sigma_c = \frac{p_{max.}D}{2t} = \frac{5(130)}{2(10)} = 32.5 \text{ N/mm}^2$$

$$[\sigma_c < 50 \text{ N/mm}^2]$$

Longitudinal stress

$$D_o = D + 2t = 130 + 2(10) = 150 \text{ mm}$$

$$\sigma_l = \frac{p_{max.}D^2}{(D_o^2 - D^2)} = \frac{5(130)^2}{[(150)^2 - (130)^2]}$$

$$= 15.09 \text{ N/mm}^2$$

Assumption No. 5 The Poisson's ratio for cylinder material is 0.25.

Net stresses

Circumferential stress

$$(\sigma_c)_{net} = \sigma_c - \mu\sigma_l = 32.5 - 0.25\,(15.09)$$

$$= 28.73 \text{ N/mm}^2$$

Longitudinal stress

$$(\sigma_l)_{net} = \sigma_l - \mu\sigma_c = 15.09 - 0.25(32.5)$$

$$= 6.97 \text{ N/mm}^2$$

Step III *Thickness of cylinder head*

Assumption No. 6 The cylinder head is made of cast iron. The allowable tensile stress is from 30 MPa to 50 MPa. It is assumed that allowable tensile stress is 40 MPa.

$$\sigma_t = 40 \text{ N/mm}^2$$

$$K = 0.162$$

$$\sigma_c = \sigma_t = 40 \text{ N/mm}^2$$

$$t_h = D\sqrt{\frac{Kp_{max.}}{\sigma_c}} = (130)\sqrt{\frac{0.162(5)}{40}}$$

$$= 18.5 \text{ or } 20 \text{ mm} \quad \text{(iv)}$$

Step IV *Number of studs*

Limits

Minimum number of studs $= 0.01D + 4$

$= 0.01(130) + 4 = 5.3$

Maximum number of studs $= 0.02D + 4$

$= 0.02(130) + 4 = 6.6$

The number of studs should be from 5.3 to 6.6. It is assumed that there are six studs.

$$z = 6 \quad \text{(v)}$$

Step V *Nominal diameter of studs*

Assumption No. 7 The studs are made of steel. The allowable tensile stress for studs varies from

35 MPa to 70 MPa. It is assumed that allowable tensile stress is 40 MPa.

$$\sigma_t = 40 \text{ N/mm}^2$$

Force acting on a cylinder head $= \left(\dfrac{\pi}{4}\right) D^2 p_{\max.}$

$$= \left(\frac{\pi}{4}\right)(130)^2(5) = 66\,366.15 \text{ N} \quad \text{(a)}$$

Resisting force offered by all studs $= \left(\dfrac{\pi}{4}\right) d_c^2\, z\, \sigma_t$

$$= \left(\frac{\pi}{4}\right) d_c^2 (6)(40) = 188.5\, d_c^2 \quad \text{(b)}$$

Equating (a) and (b),

$188.5 d_c^2 = 66\,366.15$ or $d_c = 18.76$ mm

$$d = \frac{d_c}{0.8} = \frac{18.76}{0.8} = 23.45 \text{ mm or } 24 \text{ mm} \quad \text{(vi)}$$

Step VI *Pitch of studs*

Pitch circle diameter of studs $(D_p) = D + 3\,d$

$= 130 + 3(24) = 202$ mm

$$\text{Pitch of studs} = \frac{\pi D_p}{z} = \frac{\pi(202)}{6} = 105.77 \text{ mm} \quad \text{(vii)}$$

Limits

Minimum pitch $= 19\sqrt{d} = 19\sqrt{24} = 93.08$ mm

Maximum pitch $= 28.5\sqrt{d} = 28.5\sqrt{24}$

$= 139.62$ mm

The pitch of the studs is 105.77 mm. It is within the limits of 93.08 and 139.62 mm. Therefore, the pitch of the studs is satisfactory.

25.8 PISTON

The piston is a reciprocating part of IC engine that performs a number of functions. The main functions of the piston are as follows:

(i) It transmits the force due to gas pressure inside the cylinder to the crankshaft through the connecting rod.

(ii) It compresses the gas during the compression stroke.

(iii) It seals the inside portion of the cylinder from the crankcase by means of piston rings.

(iv) It takes the side thrust resulting from obliquity of the connecting rod.

(v) It dissipates large amount of heat from the combustion chamber to the cylinder wall.

Trunk type piston, as shown in Fig. 25.4, is used in IC engines. It consists of the following parts:

(i) Piston Head or Crown It is the top portion of the piston which withstands the gas pressure inside the cylinder. It has flat, concave or convex shape depending upon the construction of combustion chamber.

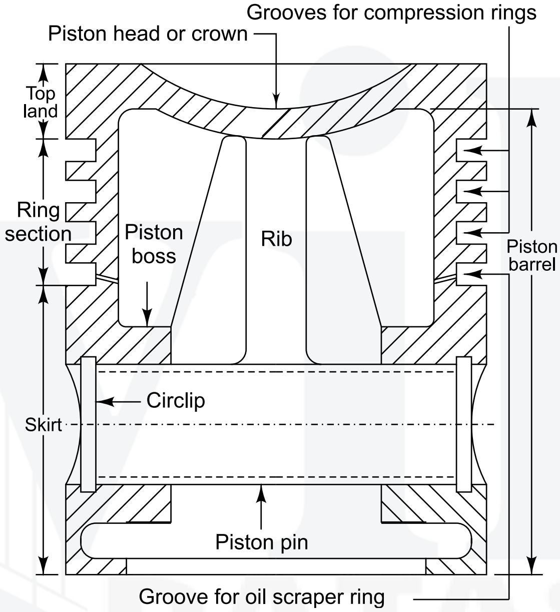


Fig. 25.4 *Piston*

(ii) Piston Rings They act as seal and prevent the leakage of gas past the piston. Piston rings are also called '*compression*' rings.

(iii) Oil Scraper Ring It prevents the leakage of lubricating oil past the piston into the combustion chamber.

(iv) Piston Skirt It is the lower part of the piston below the piston rings which acts as bearing surface for the side thrust exerted by the connecting rod.

(v) Piston Pin It connects the piston to the connecting rod. It is also called '*gudgeon*' pin or '*wrist*' pin.

The design requirements for the piston are as follows:

(i) It should have sufficient strength to withstand the force due to combustion of fuel and also the inertia forces due to reciprocating parts.
(ii) It should have sufficient rigidity to withstand thermal and mechanical distortions.
(iii) It should have adequate capacity to dissipate the heat from the crown to the cylinder wall through the piston rings and the skirt.
(iv) It should have minimum weight to reduce the inertia force due to reciprocating motion.
(v) It should form an efficient seal to prevent leakage of flue gases from combustion chamber to the crankcase past the piston. It should also prevent leakage of lubricating oil into the combustion chamber past the piston.
(vi) It should have sufficient bearing area to take the side thrust and prevent undue wear.
(vii) It should result in noiseless operation.
(viii) It should provide adequate support for the piston pin, which connects the small end of the connecting rod.

25.9 PISTON MATERIALS

Commonly used materials for IC engine pistons are cast iron, cast steel, forged steel, cast aluminium alloys and forged aluminium alloy. Compared with cast iron, aluminium alloy pistons have the following advantages:

(i) The thermal conductivity of aluminium alloys is approximately three times that of cast iron. Therefore, an aluminium alloy piston has less variation in temperature from the crown to the piston rings.
(ii) The density of aluminium alloy is about one third that of cast iron. This results in light weight construction and reduces inertia forces.

Cast iron pistons offer the following advantages:

(i) Cast iron pistons have higher strength compared with aluminium alloy pistons. As the temperature increases, the strength of aluminium alloy piston decreases rapidly compared with cast iron piston. Due to higher strength, it is possible to provide thin sections for the parts of cast iron piston.
(ii) The wear strength of a cast iron piston is more than corresponding aluminium alloy piston.
(iii) The coefficient of thermal expansion of aluminium alloy is approximately twice that of cast iron. Therefore, aluminium alloy pistons need more clearance between the cylinder wall and piston rings.

Cast iron pistons are used for moderately rated engines with piston speed below 6 m/s. Aluminium alloy pistons are used for highly rated engines with piston speeds above 6 m/s.

25.10 THICKNESS OF PISTON HEAD

There are two types of piston heads—flat plate type and cup type as shown in Fig. 25.5. The selection of the type depends upon the required volume for combustion chamber and the arrangement of valves.

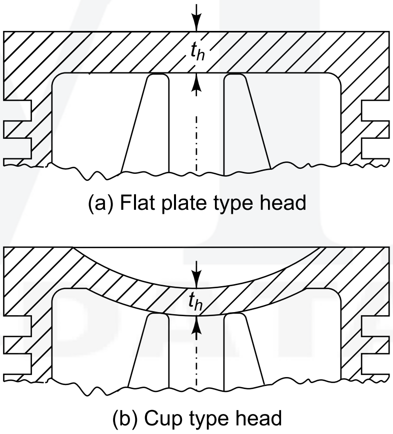


Fig. 25.5 *Types of piston head*

There are two criteria for calculating the thickness of piston head—strength and heat dissipation. On the basis of strength criterion, the piston head is treated as a flat circular plate of uniform thickness fixed at the outer edge and subjected to uniformly distributed gas pressure ($p_{max.}$) over the entire surface area. According to *Grashoff's formula*, the thickness of the piston head is given by,

$$t_h = D\sqrt{\frac{3}{16}\frac{p_{max.}}{\sigma_b}} \tag{25.16}$$

where,

t_h = thickness of piston head (mm)

D = cylinder bore (mm)

$p_{max.}$ = maximum gas pressure or explosion pressure (MPa or N/mm^2)

σ_b = permissible bending stress (N/mm^2)

Note

(i) The bending stress (σ_b) in Eq. (25.16) is the allowable tensile stress (σ_t). Since the material of the cylinder head is brittle,

$$\sigma_b = \sigma_t = \frac{S_{ut}}{(fs)}$$

(ii) In examples where ultimate tensile strength of the piston material and factor of safety are not specified, the allowable bending stress (σ_b) for grey cast iron is taken from 35 to 40 N/mm^2. For aluminium alloy, it can be assumed from 50 to 90 N/mm^2.

(iii) The maximum gas pressure ($p_{max.}$) may rise up to 8 MPa. The average value of maximum gas pressure is taken as 4 to 5 MPa or N/mm^2.

There is an empirical formula recommended by Held and Favary for the thickness of the piston head. According to this formula,

$$t_h = 0.032\,D + 1.5 \text{ mm} \qquad (25.17)$$

The piston head absorbs the heat during combustion of fuel and transmits it to the cylinder wall. It should have sufficient thickness to quickly transfer the heat to the cylinder wall. On the basis of heat dissipation, the thickness of the piston head is given by,

$$t_h = \left[\frac{H}{12.56k(T_c - T_e)}\right] \times 10^3 \qquad (25.18)$$

where,

t_h = thickness of piston head (mm)

H = amount of heat conducted through piston head (W)

k = thermal conductivity factor (W/m/°C)

T_c = temperature at the center of piston head (°C)

T_e = temperature at the edge of piston head (°C)

Note

(i) The values of thermal conductivity factor (k) are as follows:

For grey cast iron, k = 46.6 W/m/°C

For aluminium alloy, k = 175 W/m/°C

(ii) The values of permissible temperature difference ($T_c - T_e$) are as follows:

For grey cast iron, ($T_c - T_e$) = 220°C

For aluminium alloy, ($T_c - T_e$) = 75°C

The amount of heat conducted through piston head (H) is given by,

$$H = [C \times \text{HCV} \times m \times \text{BP}] \times 10^3 \qquad (25.19)$$

where,

HCV = Higher calorific value of fuel (kJ/kg)

m = mass of fuel used per brake power per second(kg/kW/s)

BP = brake power of the engine per cylinder (kW)

C is the ratio of heat absorbed by the piston to the total heat developed in the cylinder (C = 5% or C = 0.05)

Note

(i) The higher calorific values of fuels are as follows:

For diesel, HCV = 44 × 10^3 kJ/kg

For petrol, HCV = 47 × 10^3 kJ/kg

(ii) The average consumption of fuel in diesel engine is 0.24 to 0.30 kg/kW/h.

$$m = \left[\frac{0.24 \text{ to } 0.3}{60 \times 60}\right] \text{kg/kW/s}$$

25.11 PISTON RIBS AND CUP

The piston head is provided with a number of ribs for the following reasons:

(i) Ribs strengthen the piston head against the gas pressure. They increase the rigidity and prevent distortion of piston head.

(ii) Ribs transmit a large portion of combustion heat from the piston head to the piston rings. This reduces the temperature difference between the centre and edge of piston head.

(iii) The side thrust created by obliquity of connecting rod is transmitted to the piston at the piston pin. It is then transmitted to the cylinder wall through the skirt. The stiffening rib provided at the centre of boss and extending around the skirt, distributes the side thrust more uniformly and prevents distortion of the skirt.

Guidelines for ribs

(i) When the thickness of the piston head is 6 mm or less, no ribs are required. When the thickness of the piston head is more than 6 mm, a suitable number of ribs are required.

$t_h \le 6$ mm (no ribs)

$t_h > 6$ mm (provide ribs) (25.20)

(ii) The number of ribs is given by,

Number of ribs = 4 to 6 (25.21)

(iii) The thickness of ribs is given by,

$$t_R = \left(\frac{t_h}{3}\right) \text{to} \left(\frac{t_h}{2}\right) \tag{25.22}$$

where,

t_R = thickness of ribs (mm)

t_h = thickness of piston head (mm)

A cup provides additional space for combustion of fuel. Provision of cup at the top of the piston head depends upon the volume of combustion chamber. It also depends upon the arrangement of valves. If inlet and exhaust valves open and close at angles near the top dead centre, then there is possibility that either inlet or exhaust valve may strike the piston top due to overtaking. A spherical cavity in the form of cup is provided for this purpose.

Guidelines for piston cup

(i) When the ratio of stroke length to bore (l/D) is up to 1.5, a cup is required on the top of the piston.

(l/D) ≤ 1.5 (cup required)

(l/D) > 1.5 (no cup required) (25.23)

(ii) The radius of cup is given by,

radius of cup = 0.7D (25.24)

25.12 PISTON RINGS

In IC engines, two types of piston rings are used, viz., *compression rings* and *oil scraper rings*. The main function of compression rings is to maintain a seal between the cylinder wall and piston and prevent leakage of gas past the piston. They also transfer heat from the piston head to the cylinder wall. Piston rings also absorb fluctuations in side thrust. Oil scraper rings or oil control rings are provided below the compression rings. They provide proper lubrication of the cylinder liner and reduce frictional losses. Oil scraper rings allow sufficient quantity of lubricating oil to move up during the upward stroke and at the same time, scrap the excess oil from the inner surface of the liner and send it back to the crankcase. This prevents the leakage of oil into the combustion chamber.

Guidelines for design of piston rings

(i) Materials of Piston Rings Piston rings are usually made of grey cast iron and in some cases, alloy cast iron. Grey cast iron has excellent wear resistance. It also retains the spring characteristic at high temperatures. In some cases, piston rings are chromium plated to reduce wear.

(ii) Number of Piston Rings There are no strict rules for deciding the number of compression rings. The number of compression rings in automobile and aircraft engines is usually between 3 to 4. In stationary diesel engines, 5 to 7 compression rings are used. The number of oil scraper rings is usually between 1 to 3.

(iii) Dimensions of Cross-section The compression rings have rectangular cross-section as shown in Fig. 25.6(a). The radial width of the ring is given by,

$$b = D\sqrt{\frac{3p_w}{\sigma_t}} \tag{25.25}$$

where,

b = radial width of ring (mm)

p_w = allowable radial pressure on cylinder wall (N/mm^2)

σ_t = permissible tensile stress for ring material (N/mm^2)

Note

(i) The radial wall pressure is usually taken from 0.025 to 0.042 MPa.

(ii) The permissible tensile stress for cast iron rings is taken from 85 to 110 N/mm^2.

The axial thickness of piston ring is given by,

$$h = (0.7b) \text{ to } b \tag{25.26}$$

where h is the axial thickness of the piston ring in mm.

There is a limit on the minimum axial thickness. It is given by,

$$h_{\text{min.}} = \left(\frac{D}{10z}\right)$$

z = number of rings

It is preferred to provide more number of thin piston rings than a small number of thick rings. It has the following advantages:

(a) Thin rings reduce frictional loss and wear of the surface.
(b) More number of thin rings have better sealing action than a few thick rings.
(c) Thin rings occupy less piston length.
(d) More number of thin rings provide better heat transfer from the piston top to the cylinder.

(iv) Gap between Free Ends The diameter of a piston ring is slightly more than the cylinder bore (D). A part of the ring is slightly cut diagonally as shown in Fig. 25.6(b). During the assembly, the ring is compressed diagonally and passed into the liner. The gap G between the free ends of the ring is as follows:

$G = 3.5\ b$ to $4\ b$ (before assembly)

$G = 0.002\ D$ to $0.004\ D$

(after assembly in cylinder) (25.27)

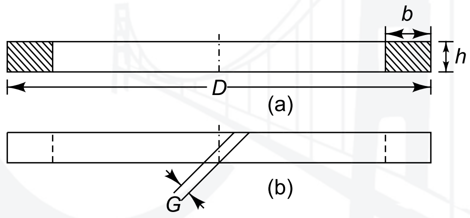


Fig. 25.6 *Piston rings*

(v) Width of Top Land and Ring Lands Refer to Fig. 25.7 for top land and ring lands.

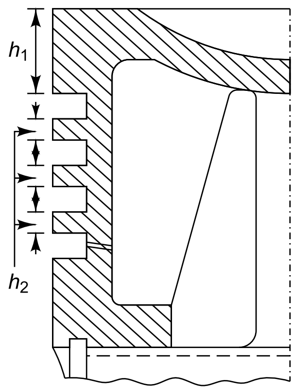


Fig. 25.7 *Grooves for piston rings*

The distance from the top of the piston to the first ring groove (h_1) is called top land. It is given by,

$$h_1 = (t_h) \text{ to } (1.2t_h) \qquad (25.28)$$

The distance between two consecutive ring grooves (h_2) is called the width of the ring groove and is given by,

$$h_2 = 0.75\ h \text{ to } h \qquad (25.29)$$

25.13 PISTON BARREL

The piston barrel is shown in Fig. 25.8. It is the cylindrical portion of the piston below the piston head.

The thickness of the piston barrel at the top end is given by,

$$t_3 = (0.03D + b + 4.9) \qquad (25.30)$$

where,

t_3 = thickness of piston barrel at the top end (mm)
b = radial width of ring (mm)

The thickness of piston barrel at the lower or open end is given by,

$$t_4 = (0.25\ t_3) \text{ to } (0.35\ t_3) \qquad (25.31)$$

t_4 = thickness piston barrel at open end (mm)

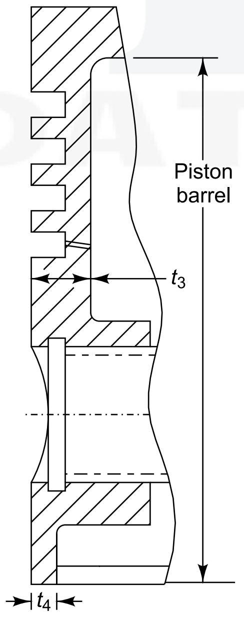


Fig. 25.8 *Piston barrel*

25.14 PISTON SKIRT

As shown in Fig. 25.4, the cylindrical portion of the piston between the last scrapper ring and the open end is called the piston skirt. The piston skirt acts as a bearing surface for the side thrust. The length of the skirt should be such that the bearing pressure due to side thrust is restricted to 0.25 MPa on the projected area. In high speed engines, the bearing pressure up to 0.5 MPa is allowed to reduce the weight of the reciprocating piston. The maximum side thrust will occur during expansion stroke.

$$\text{Maximum gas force on piston head} = \left(\frac{\pi D^2}{4}\right) p_{\max.}$$

$$\text{Side thrust} = \mu\left(\frac{\pi D^2}{4}\right) p_{\max.} \qquad \text{(a)}$$

where μ is the coefficient of friction ($\mu = 0.1$)

The side thrust taken by the skirt is also given by,

$$\text{Side thrust} = p_b\, D l_s \qquad \text{(b)}$$

where,

p_b = allowable bearing pressure (MPa or N/mm^2)

l_s = length of skirt (mm)

Equating (a) and (b),

$$\mu\left(\frac{\pi D^2}{4}\right) p_{\max.} = p_b D l_s$$

$$0.1\left(\frac{\pi D^2}{4}\right) p_{\max.} = p_b D l_s \qquad (25.32)$$

The length of the skirt is calculated by the above relationship.

The empirical relationship for the length of skirt is as follow:

$$l_s = (0.65D) \text{ to } (0.8D) \qquad (25.33)$$

The total length of the piston is given by,

$$L = \text{top land} + \text{length of ring section} + \text{length of skirt} \qquad (25.34)$$

The empirical relationship for the length of the piston is as follows.

$$L = D \text{ to } 1.5D \qquad (25.35)$$

25.15 PISTON PIN

The function of the piston pin is to connect the piston to the connecting rod. It is also called '*gudgeon*' pin or '*wrist*' *pin*. It is made of hollow circular cross-section to reduce its weight. It is often tapered on the inside and the smallest diameter is at the centre of the pin. The piston pin passes through the bosses provided on the inner side of the piston skirt and a bearing bush inside the small end of the connecting rod as shown in Fig. 25.9(a). The end movement of the piston pin is restricted by means of circlips.

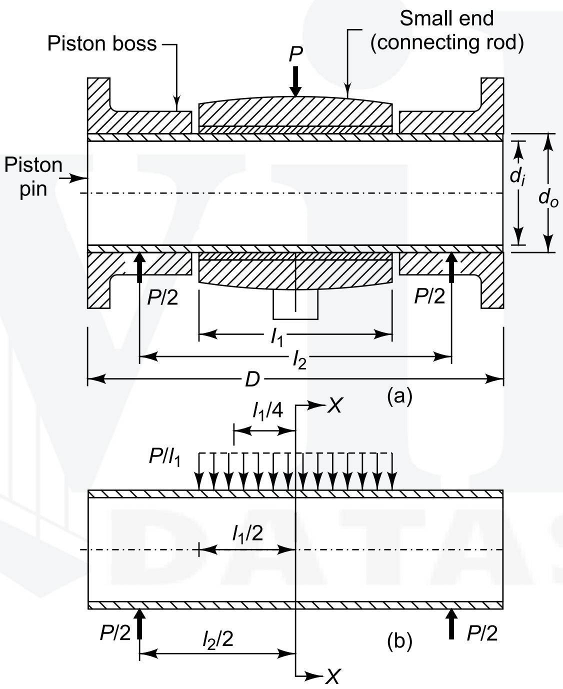


Fig. 25.9 *Piston pin as beam*

There are two types of connections between the piston pin and the small end of the connecting rod, viz., '*full-floating*' type and '*semi-floating*' type. A full-floating piston pin is free to turn both in the piston bosses as well as the bush in the small end of the connecting rod. The end movement of the pin is restricted by circlips. The semi-floating piston pin is either free to turn in the piston bosses and rigidly fixed to the small end of the connecting rod, or free to turn in the bush of the small end and rigidly fixed to the piston bosses by means of screws.

The piston pin is made of carbon steel or alloy steel. It is hardened and ground to reduce wear while turning inside the phosphor bronze bush. There are two criteria for design of the piston pin—bearing consideration and bending failure.

(i) Bearing Consideration The piston pin is partly in contact with piston bosses and partly with the bush of the connecting rod as shown in Fig. 25.9(a). The bearing area of the piston pin is approximately divided between the piston bosses and the connecting rod bush. It is assumed that the length of the pin in the connecting rod bush is 45% of the piston diameter (D) or cylinder bore. Therefore,

$$l_1 = 0.45D \tag{25.36}$$

where,

l_1 = length of piston pin in the bush of the small end of the connecting rod (mm)

The outer diameter of the piston pin (d_o) is determined by equating the force acting on the piston and the resisting bearing force offered by the piston pin:

$$\text{Force on piston} = \left(\frac{\pi D^2}{4}\right) p_{\text{max.}} \qquad \text{(a)}$$

$$\text{Resisting force} = (p_b)_1 \times d_o \times l_1 \qquad \text{(b)}$$

Equating (a) and (b),

$$\left(\frac{\pi D^2}{4}\right) p_{\text{max.}} = (p_b)_1 \times d_o \times l_1 \tag{25.37}$$

where,

$(p_b)_1$ = bearing pressure at the bushing of small end of connecting rod (MPa or N/mm^2)

d_o = outer diameter of piston pin (mm)

Note

(i) The bearing pressure at the bushing of the small end of the connecting rod $(p_b)_1$ is taken as 25 MPa.

(ii) The inner diameter of the piston pin is taken as 0.6 times of the outer diameter.

$$d_i = 0.6d_o \tag{25.38}$$

(iii) The mean diameter of the piston bosses is given by,

Mean diameter of piston bosses = $1.4d_o$ (for grey cast iron piston)

Mean diameter of piston bosses = $1.5\ d_o$ (for aluminium alloy piston)

(ii) Bending Consideration Refer to Fig. 25.9(b). The bending moment acting on the pin at the central section XX is given by,

$$M_b = \left(\frac{P}{2}\right)\times\left(\frac{l_2}{2}\right) - \left(\frac{P}{l_1}\times\frac{l_1}{2}\right)\times\left(\frac{l_1}{4}\right)$$

$$= \left(\frac{P}{2}\right)\times\left(\frac{l_2}{2}\right) - \left(\frac{P}{2}\right)\times\left(\frac{l_1}{4}\right) \qquad \text{(a)}$$

Also,

$$l_2 = \left(\frac{D + l_1}{2}\right) \qquad \text{(b)}$$

Substituting (b) in (a),

$$M_b = \left(\frac{P}{2}\right)\times\left(\frac{1}{2}\right)\left(\frac{D + l_1}{2}\right) - \left(\frac{P}{2}\right)\times\left(\frac{l_1}{4}\right)$$

$$= \left(\frac{PD}{8}\right) + \left(\frac{Pl_1}{8}\right) - \left(\frac{Pl_1}{8}\right)$$

$$M_b = \left(\frac{PD}{8}\right) \tag{25.39}$$

Also,

$$I = \frac{\pi(d_o^4 - d_i^2)}{64} \tag{25.40}$$

$$y = \left(\frac{d_o}{2}\right)$$

and $$\sigma_b = \frac{M_b y}{I}$$

Using the above relationships, the maximum bending stress in the piston pin at the central plane is calculated.

Note The allowable bending stress for the piston pin should not exceed the following values:

σ_b = 84 N/mm^2 (for case hardened carbon steel)

σ_b = 140 N/mm^2 (for heat treated alloy steels)

(iii) Piston Clearances The clearance between the cylinder liner and piston is provided to take care of thermal expansion and distortion under load. When the clearance is insufficient, '*piston seizure*' occurs. On the other hand, when clearance is excessive, '*piston slap*' occurs resulting in piston running with excessive noise. The magnitude of piston clearance varies from 0.0375 to 0.1875 mm depending upon the piston diameter and the type of engine. In aluminium alloy piston, the clearance is twice that of cast iron piston. The clearance is less when proper cooling system is provided resulting in smaller thermal expansion.

Example 25.6 *The following data is given for a four-stroke diesel engine:*

Cylinder bore = 250 mm
Length of stroke = 300 mm
Speed = 600 rpm
Indicated mean effective pressure = 0.6 MPa
Mechanical efficiency = 80%
Maximum gas pressure = 4 MPa
Fuel consumption = 0.25 kg per BP per h
Higher calorific value of fuel = 44 000 kJ/kg

Assume that 5% of the total heat developed in the cylinder is transmitted by the piston. The piston is made of grey cast iron FG 200 (S_{ut} = 200 N/mm^2 and k = 46.6 W/m/°C) and the factor of safety is 5. The temperature difference between the centre and the edge of the piston head is 220°C.

(i) Calculate the thickness of piston head by strength consideration.
(ii) Calculate the thickness of piston head by thermal consideration.
(iii) Which criterion decides the thickness of piston head?
(iv) State whether the ribs are required.
(v) If so, calculate the number and thickness of piston ribs.
(vi) State whether a cup is required in the top of the piston head.
(vii) If so, calculate the radius of the cup.

Solution

Given $D = 250$ mm $\quad l = 300$ mm $= 0.3$ m
$N = 600$ rpm $\quad p_m = 0.6$ MPa $= 0.6$ N/mm^2
$p_{\max.} = 4$ MPa $= 4$ N/mm^2 $\quad \eta = 80\% = 0.8$
$S_{ut} = 200$ N/mm^2 $\quad (fs) = 5 \quad C = 0.05$
$m = 0.25$ kg per BP per h $\quad$ HCV = 44 000 kJ/kg
$k = 46.6$ W/m/°C $\quad (T_c - T_e) = 220$°C

Step I *Thickness of piston head by strength consideration*

The permissible tensile stress is given by,

$$\sigma_t = \frac{S_{ut}}{(fs)} = \frac{200}{5} = 40 \text{ N/mm}^2$$

From Eq. 25.16,

$$t_h = D\sqrt{\frac{3}{16}\frac{p_{\max.}}{\sigma_b}} = 250\sqrt{\frac{3}{16}\frac{(4)}{(40)}}$$
$$= 34.23 \text{ or } 35 \text{ mm} \qquad \text{(i)}$$

Step II *Thickness of piston head by thermal consideration*

For a four-stroke engine,

$$n = \frac{N}{2} = \frac{600}{2} = 300 \text{ strokes/min}$$

$$\text{IP} = \frac{p_m lAn}{60} = \frac{(0.6)(0.3)}{(60)}\left(\frac{\pi(250)^2}{4}\right)(300)$$
$$= 44178.65 \text{ W or } 44.18 \text{ kW}$$

$$\text{BP} = \eta\ \text{IP} = 0.8(44.18) = 35.34 \text{ kW}$$

$$m = 0.25 \text{ kg per BP per h} = \left(\frac{0.25}{60 \times 60}\right)$$

kg per BP per second

$$= (69.44 \times 10^{-6}) \text{ kg per BP per second}$$

From Eq. 25.19,

$$H = [C \times \text{HCV} \times \text{m} \times \text{BP}] \times 10^3$$
$$= [0.05 \times 44000 \times (69.44 \times 10^{-6}) \times 35.34] \times 10^3$$
$$= 5398.82 \text{ W}$$

From Eq. 25.18,

$$t_h = \left[\frac{H}{12.56k(T_c - T_e)}\right] \times 10^3$$
$$= \left[\frac{5398.82}{12.56(46.6)(220)}\right] \times 10^3$$
$$t_h = 41.93 \text{ or } 42 \text{ mm} \qquad \text{(ii)}$$

Step III *Criterion to decide the thickness of piston head*

From (i) and (ii), thermal consideration is the criterion for piston thickness.

$$t_h = 42 \text{ mm} \qquad \text{(iii)}$$

Step IV *Requirement of piston ribs*

Since $t_h = 42$ mm $\quad t_h > 6$ mm

Therefore, ribs are required. (iv)

Step V *Number and thickness of piston ribs*
It is assumed that the number of ribs is 4.

From Eq. 25.22,

$$t_R = \left(\frac{t_h}{3}\right) \text{to} \left(\frac{t_h}{2}\right) = \left(\frac{42}{3}\right) \text{to} \left(\frac{42}{2}\right) = 14 \text{ to } 21$$

$$t_R = 18 \text{ mm} \qquad \text{(v)}$$

Step VI *Requirement of cup*

$$\left(\frac{l}{D}\right) = \left(\frac{300}{250}\right) = 1.2 \qquad \therefore \left(\frac{1}{D}\right) < 1.5$$

Therefore, a cup is required. (vi)

Step VII *Radius of cup*
From Eq. 25.24,

Radius of cup = $0.7D = 0.7(250) = 175$ mm (vii)

Example 25.7 *The following data is given for the piston of a four-stroke diesel engine:*

Cylinder bore = 250 mm
Material of piston rings = Grey cast iron
Allowable tensile stress = 100 N/mm²
Allowable radial pressure on cylinder wall = 0.03 MPa
Thickness of piston head = 42 mm
Number of piston rings = 4
Calculate:

(i) radial width of the piston rings;
(ii) axial thickness of the piston rings;
(iii) gap between the free ends of the piston ring before assembly;
(iv) gap between the free ends of the piston ring after assembly;
(v) width of the top land;
(vi) width of the ring grooves;
(vii) thickness of the piston barrel; and
(viii) thickness of the barrel at open end.

Solution

Given $D = 250$ mm $\sigma_t = 100$ N/mm^2
$p_w = 0.03$ MPa or 0.03 N/mm^2 $t_h = 42$ mm $z = 4$

Step I *Radial width of piston rings*
From Eq. 25.25,

$$b = D\sqrt{\frac{3p_w}{\sigma_t}} = 250\sqrt{\frac{3(0.03)}{100}} = 7.5 \text{ mm} \qquad \text{(i)}$$

Step II *Axial thickness of piston rings*
From Eq. 25.26,

$h = (0.7\,b)$ to $b = (0.7 \times 7.5)$ to $7.5 = 5.25$ to 7.5 mm
$h = 7$ mm (ii)
Also,

$$h_{\text{min.}} = \left(\frac{D}{10z}\right) = \left(\frac{250}{10(4)}\right) = 6.25 \text{ mm} \qquad h > h_{\text{min.}}$$

Step III *Gap between free ends of piston ring before assembly*

$G_1 = 3.5\,b$ to $4\,b$ (before assembly)
$= 3.5(7.5)$ to $4(7.5) = 26.25$ to 30 mm
$G_1 = 28$ mm (iii)

Step IV *Gap between free ends of piston ring after assembly*

$G_2 = 0.002D$ to $0.004D$ (after assembly)
$= 0.002(250)$ to $0.004(250) = 0.5$ to 1 mm
$G_2 = 0.75$ mm (iv)

Step V *Width of top land*
From Eq. 25.28,

$h_1 = (t_h)$ to $(1.2\,t_h) = (42)$ to (1.2×42)
$= 42$ to 50.4 mm
$h_1 = 45$ mm (v)

Step VI *Width of ring grooves*
From Eq. 25.29,

$h_2 = 0.75\,h$ to $h = 0.75\,(7)$ to $7 = 5.25$ to 7
$= 6$ mm (vi)

Step VII *Thickness of piston barrel*
From Eq. 25.30,

$t_3 = (0.03\,D + b + 4.9) = 0.03(250) + 7.5 + 4.9$
$= 19.9$ mm or 20 mm (vii)

Step VIII *Thickness of barrel at open end*
From Eq. 25.31,

$t_4 = (0.25\,t_3)$ to $(0.35\,t_3) =$
(0.25×20) to $(0.35 \times 20) = 5$ to 7 mm
$t_4 = 6$ mm (viii)

Example 25.8 *The following data is given for the piston of a four-stroke diesel engine:*

Cylinder bore = 250 mm
Maximum gas pressure = 4 MPa
Allowable bearing pressure for skirt = 0.4 MPa
Ratio of side thrust on liner to maximum gas load on piston = 0.1

Width of top land = 45 mm
Width of ring grooves = 6 mm
Total number of piston rings = 4
Axial thickness of piston rings = 7 mm
Calculate:
(i) length of the skirt; and
(ii) length of the piston.

Solution

Given $D = 250$ mm $p_{max.} = 4$ MPa $= 4$ N/mm^2
$\mu = 0.1$ $p_b = 0.4$ MPa $= 0.4$ N/mm^2 $h_1 = 45$ mm
$h_2 = 6$ mm $h = 7$ mm $z = 4$

Step I *Length of skirt*
From Eq. 25.32,

$$\mu\left(\frac{\pi D^2}{4}\right)p_{max.} = p_b D l_s$$

or
$$0.1\left(\frac{\pi(250)^2}{4}\right)(4) = (0.4)(250)\ l_s$$

$$l_s = 196.35 \text{ mm} \qquad \text{(i)}$$

Step II *Length of piston*
Refer to Fig. 25.10.

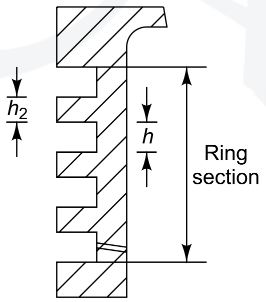


Fig. 25.10

Length of ring section $= 4h + 3h_2 = 4(7) + 3(6)$
$= 46$ mm [Refer Fig. 25.4]
Length of piston $= h_1 +$ length of ring section $+ l_s$
$= 45 + 46 + 196.35 = 287.35$ mm

$$L = 288 \text{ mm} \qquad \text{(ii)}$$

According to empirical relationship,
$L = D$ to $1.5D = 250$ to 375 mm
$\therefore$ $D < L > 1.5\ D$

Example 25.9 *The following data is given for the piston of a four-stroke diesel engine:*
Cylinder bore = 250 mm
Maximum gas pressure = 4 MPa
Bearing pressure at small end of connecting rod = 15 MPa
Length of piston pin in bush of small end = 0.45D
Ratio of inner to outer diameter of piston pin = 0.6
Mean diameter of piston boss = 1.4 × outer diameter of piston pin
Allowable bending stress for piston pin = 84 N/mm²
Calculate:
(i) outer diameter of the piston pin;
(ii) inner diameter of the piston pin;
(iii) mean diameter of the piston boss; and
(iv) check the design for bending stresses.

Solution

Given $D = 250$ mm $p_{max.} = 4$ MPa $= 4$ N/mm^2
$(p_b)_1 = 15$ MPa $= 15$ N/mm^2 $l_1 = 0.45D$
$d_i = 0.6\ d_o$ $\sigma_b = 84$ N/mm^2

Step I *Outer diameter of piston pin*

$$\text{Force on piston} = \left(\frac{\pi D^2}{4}\right)p_{max.} = \left(\frac{\pi(250)^2}{4}\right)(4)$$

$$= 196\ 349.54 \text{ N}$$

From Eq. 25.37,

$$\left(\frac{\pi D^2}{4}\right)p_{max.} = (p_b)_1 \times d_o \times l_1 \qquad [l_1 = 0.45\ \text{D}]$$

$$196\ 349.54 = 15 d_o\ (0.45 \times 250)$$

$$d_o = 116.36 \text{ or } 118 \text{ mm} \qquad \text{(i)}$$

Step II *Inner diameter of piston pin*

$$d_i = 0.6\ d_o = 0.6(116.36) = 69.82 \text{ or } 70 \text{ mm} \qquad \text{(ii)}$$

Step III *Mean diameter of piston boss*
Mean diameter of piston boss $= 1.4\ d_o$
$= 1.4(116.36) = 162.91$ mm
Mean diameter of piston boss $= 165$ mm (iii)

Step IV *Check for bending*
From Eq. 25.39,

$$M_b = \left(\frac{PD}{8}\right) = \frac{(196\,349.54)(250)}{8}$$

$$= 6135.92 \times 10^3 \text{ N-mm}$$

$$I = \frac{\pi(d_o^4 - d_i^4)}{64} = \frac{\pi\left[(118)^4 - (70)^4\right]}{64}$$

$$= (8338.36 \times 10^3)\text{ mm}^4$$

$$y = \left(\frac{d_o}{2}\right) = \left(\frac{118}{2}\right) = 59 \text{ mm}$$

$$\sigma_b = \frac{M_b y}{I} = \frac{(6135.92 \times 10^3)(59)}{8338.36 \times 10^3}$$

$$= 43.42 \text{ N/mm}^2$$

$$\therefore \qquad \sigma_b < 84 \text{ N/mm}^2 \qquad \text{(iv)}$$

Example 25.10 *Design a cast iron piston for a single acting four-stroke diesel engine with the following data:*

Cylinder bore = 300 mm
Length of stroke = 450 mm
Speed = 300 rpm
Indicated mean effective pressure = 0.85 MPa
Maximum gas pressure = 5 MPa
Fuel consumption = 0.30 kg per BP per h
Higher calorific value of fuel = 44 000 kJ/kg

Assume suitable data if required and state the assumptions you make.

Solution

Given $D = 300$ mm $l = 450$ mm $= 0.45$ m
$N = 300$ rpm $p_m = 0.85$ MPa $= 0.85$ N/mm^2
$p_{max.} = 5$ MPa $= 5$ N/mm^2 $m = 0.3$ kg per BP per h
HCV = 44 000 kJ/kg

Step I *Piston head or crown*

Assumption No. 1 The permissible tensile stress for the cast iron piston is 40 N/mm^2.

From Eq. 25.16, the thickness of the piston head by strength consideration is given by,

$$t_h = D\sqrt{\frac{3}{16}\frac{p_{max.}}{\sigma_b}} = 300\sqrt{\frac{3}{16}\frac{(5)}{(40)}}$$

$$= 45.93 \text{ mm} \qquad \text{(a)}$$

For four-stroke engine,

$$n = \frac{N}{2} = \frac{300}{2} = 150 \text{ strokes/min}$$

$$\text{IP} = \frac{p_m lAn}{60} = \frac{(0.85)(0.45)}{(60)}\left(\frac{\pi(300)^2}{4}\right)(150)$$

$$= 67\,593.33 \text{ W or } 67.59 \text{ kW}$$

Assumption No. 2 The mechanical efficiency (η) is 80% or 0.8.

$$\text{BP} = \eta\, IP = 0.8(67.59) = 54.07 \text{ kW}$$

$$m = 0.3 \text{ kg per BP per h}$$

$$= \left(\frac{0.3}{60 \times 60}\right) \text{kg per BP per second}$$

$$= (83.33 \times 10^{-6}) \text{ kg per BP per second}$$

Assumption No. 3 The ratio of heat absorbed by the piston to the total heat developed in the cylinder is 0.05 or 5% ($C = 0.05$).

From Eq. 25.19,

$$H = [C \times \text{HCV} \times \text{m} \times \text{BP}] \times 10^3$$

$$= [0.05 \times 44000 \times (83.33 \times 10^{-6}) \times 54.07] \times 10^3$$

$$= 9912.44 \text{ W}$$

Assumption No. 4 The thermal conductivity factor (k) for cast iron is 46.6 W/m/°C.

Assumption No. 5 The temperature difference ($T_c - T_e$) between the centre and the edge of the piston head is 220°C.

From Eq. 25.18, the thickness of the piston head by thermal consideration is given by,

$$t_h = \left[\frac{H}{12.56\,k(T_c - T_e)}\right] \times 10^3$$

$$= \left[\frac{9912.44}{12.56\,(46.6)\,(220)}\right] \times 10^3$$

$$t_h = 76.98 \text{ mm} \qquad \text{(b)}$$

From (a) and (b), thermal consideration is the criterion for piston thickness.

$$t_h = 76.98 \text{ or } 77 \text{ mm}$$

Step II *Radial ribs*

Since $t_h = 77$ mm $t_h > 6$ mm

Therefore, ribs are required.

Assumption No. 6 The number of radial ribs is 4.

From Eq. 25.22,

$$t_R = \left(\frac{t_h}{3}\right) \text{to} \left(\frac{t_h}{2}\right) = \left(\frac{77}{3}\right) \text{to} \left(\frac{77}{2}\right)$$
$$= 25.67 \text{ to } 38.5$$

$$t_R = 30 \text{ mm}$$

Step III *Requirement of cup*

$$\left(\frac{l}{D}\right) = \left(\frac{450}{300}\right) = 1.5$$

Therefore, a cup is required.

Radius of cup = 0.7 D = 0.7(300) = 210 mm

Step IV *Piston rings*

Assumption No. 7 The allowable radial pressure (p_w) on the cylinder wall is 0.035 MPa.

Assumption No. 8 The piston rings are made of cast iron and the permissible tensile stress is 90 N/mm^2.

Assumption No. 9 The number of compression rings is 3 and there is one oil ring.

From Eq. 25.25,

$$b = D\sqrt{\frac{3p_w}{\sigma_t}} = 300\sqrt{\frac{3(0.035)}{90}} \quad 10.25 \text{ or } 10.5 \text{ mm}$$

From Eq. 25.26,

$$h = (0.7\ b) \text{ to } b = (0.7 \times 10.5) \text{ to } 10.5$$
$$= 7.35 \text{ to } 10.5 \text{ mm}$$
$$h = 8 \text{ mm}$$
$$z = 3 + 1 = 4$$

Also,

$$h_{\text{min.}} = \left(\frac{D}{10z}\right) = \left(\frac{300}{10(4)}\right) = 7.5 \text{ mm} \quad h > h_{\text{min.}}$$

The gap between free ends of the piston ring before assembly is given by,

$$G_1 = 3.5b \text{ to } 4b$$
$$= 3.5(10.5) \text{ to } 4(10.5) = 36.75 \text{ to } 42 \text{ mm}$$
$$G_1 = 40 \text{ mm}$$

The gap between free ends of the piston ring after assembly is given by,

$$G_2 = 0.002D \text{ to } 0.004D$$
$$= 0.002(300) \text{ to } 0.004(300) = 0.6 \text{ to } 1.2 \text{ mm}$$
$$G_2 = 0.8 \text{ mm}$$

From Eq. 25.28, the width of the top land is given by,

$$h_1 = (t_h) \text{ to } (1.2\ t_h) = (77) \text{ to } (1.2 \times 77)$$
$$= 77 \text{ to } 92.4 \text{ mm}$$
$$h_1 = 85 \text{ mm}$$

From Eq. 25.29, the width of ring grooves is given by,

$$h_2 = 0.75\ h \text{ to } h = 0.75(8) \text{ to } 8 = 6 \text{ to } 8$$
$$= 7 \text{ mm}$$

Step V *Piston barrel*

From Eq. 25.30, the thickness of barrel at the top end is given by

$$t_3 = (0.03D + b + 4.9) = 0.03(300) + 10.5 + 4.9$$
$$= 24.4 \text{ mm} \quad \text{or} \quad 25 \text{ mm}$$

From Eq. 25.31, the thickness of the barrel at the open end is given by,

$$t_4 = (0.25\ t_3) \text{ to } (0.35\ t_3) = (0.25 \times 25) \text{ to } (0.35 \times 25)$$
$$= 6.25 \text{ to } 8.75 \text{ mm}$$
$$t_4 = 7 \text{ mm}$$

Step VI *Piston skirt*

Assumption No. 10 The allowable bearing pressure (p_b) for the skirt portion of the piston is 0.45 MPa.

Assumption No. 11 The ratio of side thrust on liner to maximum gas load on piston is 0.1 (μ = 0.1).

From Eq. 25.32, the length of the skirt is given by,

$$\mu\left(\frac{\pi D^2}{4}\right)p_{\text{max.}} = p_b D l_s \text{ or } 0.1\left(\frac{\pi(300)^2}{4}\right)(5) = (0.45)(300)l_s$$

$$l_s = 261.8 \text{ or } 262 \text{ mm}$$

Step VII *Piston length*

Refer to Fig. 25.10.

Length of ring section = $4h + 3h_2$ = 4(8) + 3(7)
= 53 mm [Refer Fig. 25.4]

Length of piston = h_1 + length of ring section + l_s
= 85 + 53 + 262 = 400 mm

$$L = 400 \text{ mm}$$

According to empirical relationship,

$$L = D \text{ to } 1.5D = 300 \text{ to } 450 \text{ mm}$$

$$\therefore \quad D < L > 1.5\ D$$

Step VIII *Piston pin*

Assumption No. 12 The bearing pressure $(p_b)_1$ at the bush of the small end of connecting rod is 30 MPa.

Assumption No. 13 The length of the piston pin in the bush of the small end of the connecting rod is (0.45 D).

Assumption No. 14 The piston pin is made of case hardened alloy steel and the permissible tensile stress is 140 N/mm^2.

$$P = \text{Force on piston} = \left(\frac{\pi D^2}{4}\right) p_{\max.}$$

$$= \left(\frac{\pi (300)^2}{4}\right)(5) = 353\,429.17 \text{ N}$$

From Eq. 25.37,

$$\left(\frac{\pi D^2}{4}\right) p_{\max.} = (p_b)_1 \times d_o \times l_1 \quad [l_1 = 0.45D]$$

$$353\,429.17 = 30\, d_o\,(0.45 \times 300)$$

$$d_o = 87.27 \text{ or } 90 \text{ mm}$$

In order to make the shaft hollow, we will increase the diameter to 120 mm.

$$d_o = 120 \text{ mm}$$

From Eq. 25.39,

$$M_b = \left(\frac{PD}{8}\right) = \frac{(353\,429.17)(300)}{8}$$

$$= 13\,253.59 \times 10^3 \text{ N-mm}$$

$$I = \frac{\pi (d_o^4 - d_i^4)}{64} = \frac{\pi [(120)^4 - d_i^4]}{64} = \text{mm}^4$$

$$y = \left(\frac{d_o}{2}\right) = \left(\frac{120}{2}\right) = 60 \text{ mm}$$

$$\sigma_b = \frac{M_b y}{I} \qquad 140 = \frac{(13\,253.59 \times 10^3)(60)}{\frac{\pi}{64}[(120)^4 - d_i^4]}$$

$$(120)^4 - d_i^4 = 115.71 \times 10^6$$

$$d_i = 97.84 \text{ or } 95 \text{ mm}$$

The mean diameter of the piston boss is given by,

$$\text{mean diameter of the piston boss} = 1.4 d_o$$

$$= 1.4(120) = 168 \text{ mm}$$

Example 25.11 *Design a cast iron piston for a single acting four-stroke diesel engine with the following data:*

Cylinder bore = 200 mm
Length of stroke = 250 mm
Speed = 600 rpm
Brake mean effective pressure = 0.60 MPa
Maximum gas pressure = 4 MPa
Fuel consumption = 0.25 kg per BP per h
(l/d) ratio for bush in small end of connecting rod = 1.5

Assume suitable data if required and state the assumptions you make.

Solution

Given $D = 200$ mm $\quad l = 250$ mm $= 0.25$ m
$N = 600$ rpm $\quad (p_m)_b = 0.6$ MPa $= 0.6$ N/mm^2
$p_{\max.} = 4$ MPa $= 4$ N/mm^2
$m = 0.25$ kg per BP per h $\quad (l/d)$ for bush $= 1.5$

Step I *Piston head or crown*

Assumption No. 1 The permissible tensile stress for cast iron piston is 40 N/mm^2.

From Eq. 25.16, the thickness of the piston head by strength consideration is given by,

$$t_h = D\sqrt{\frac{3}{16}\frac{p_{\max.}}{\sigma_b}} = 200\sqrt{\frac{3}{16}\frac{(4)}{(40)}} = 27.39 \text{ mm} \quad \text{(a)}$$

For a four-stroke engine,

$$n = \frac{N}{2} = \frac{600}{2} = 300 \text{ strokes/min}$$

$$\text{BP} = \frac{(p_m)_b lAn}{60} = \frac{(0.6)(0.25)}{(60)}\left(\frac{\pi (200)^2}{4}\right)(300)$$

$$= 23\,561.94 \text{ W or } 23.56 \text{ kW}$$

$$m = 0.25 \text{ kg per BP per h}$$

$$= \left(\frac{0.25}{60 \times 60}\right) \text{kg per BP per second}$$

$$= (69.44 \times 10^{-6}) \text{ kg per BP per second}$$

Assumption No. 2 The ratio of heat absorbed by the piston to the total heat developed in the cylinder is 0.05 or 5%. ($C = 0.05$).

Assumption No. 3 The higher calorific value of fuel is 44 000 kJ/kg.

From Eq. 25.19,

$$H = [C \times \text{HCV} \times m \times \text{BP}] \times 10^3$$

$$= [0.05 \times 44000 \times (69.44 \times 10^{-6}) \times 23.56] \times 10^3$$

$$= 3599.21 \text{ W}$$

Assumption No. 4 The thermal conductivity factor (k) for cast iron is 46.6 W/m/°C.

Assumption No. 5 The temperature difference $(T_c - T_e)$ between the centre and the edge of the piston head is 220°C.

From Eq. 25.18, the thickness of the piston head by thermal consideration is given by,

$$t_h = \left[\frac{H}{12.56k\,(T_c - T_e)}\right]\times 10^3$$

$$= \left[\frac{3599.21}{12.56\,(46.6)\,(220)}\right]\times 10^3$$

$$t_h = 27.95 \text{ mm} \qquad \text{(b)}$$

From (a) and (b), thermal consideration is the criterion for piston thickness.

$$t_h = 27.95 \text{ or } 28 \text{ mm}$$

Step II *Radial ribs*

Since $t_h = 28$ mm $\quad t_h > 6$ mm

Therefore, ribs are required.

Assumption No. 6 The number of radial ribs is 4.

From Eq. 25.22,

$$t_R = \left(\frac{t_h}{3}\right) \text{to} \left(\frac{t_h}{2}\right) = \left(\frac{28}{3}\right) \text{to} \left(\frac{28}{2}\right)$$

$$= 9.33 \text{ to } 14 \text{ mm}$$

$$t_R = 12 \text{ mm}$$

Step III *Requirement of cup*

$$\left(\frac{l}{D}\right) = \left(\frac{250}{200}\right) = 1.25 \quad \therefore \left(\frac{l}{D}\right) < 1.5$$

Therefore, a cup is required.

Radius of cup = $0.7D = 0.7(200) = 140$ mm

Step IV *Piston rings*

Assumption No. 7 The allowable radial pressure (p_w) on the cylinder wall is 0.04 MPa.

Assumption No. 8 The piston rings are made of cast iron and permissible tensile stress is 100 N/mm^2.

Assumption No. 9 The number of compression rings is 3 and there are two oil rings.

From Eq. 25.25,

$$b = D\sqrt{\frac{3p_w}{\sigma_t}} = 200\sqrt{\frac{3(0.04)}{100}}$$

$$= 6.93 \text{ or } 7 \text{ mm}$$

From Eq. 25.26,

$$h = (0.7\,b) \text{ to } b = (0.7 \times 7) \text{ to } 7 = 4.9 \text{ to } 7 \text{ mm}$$

$$h = 6 \text{ mm}$$

$$z = 3 + 2 = 5$$

Also,

$$h_{\min} = \left(\frac{D}{10z}\right) = \left(\frac{200}{10(5)}\right) = 4 \text{ mm} \qquad h > h_{\min.}$$

The gap between the free ends of piston ring before assembly is given by,

$$G_1 = 3.5\,b \text{ to } 4\,b$$

$$= 3.5(7) \text{ to } 4(7) = 24.5 \text{ to } 28 \text{ mm}$$

$$G_1 = 26 \text{ mm}$$

The gap between the free ends of the piston ring after assembly is given by,

$$G_2 = 0.002D \text{ to } 0.004D$$

$$= 0.002(200) \text{ to } 0.004(200)$$

$$= 0.4 \text{ to } 0.8 \text{ mm}$$

$$G_2 = 0.6 \text{ mm}$$

From Eq. 25.28, the width of the top land is given by,

$$h_1 = (t_h) \text{ to } (1.2t_h) = (28) \text{ to } (1.2 \times 28)$$

$$= 28 \text{ to } 33.6 \text{ mm}$$

$$h_1 = 30 \text{ mm}$$

From Eq. 25.29, the width of the ring grooves is given by,

$$h_2 = 0.75h \text{ to } h = 0.75(6) \text{ to } 6 = 4.5 \text{ to } 6$$

$$= 5 \text{ mm}$$

Step V *Piston barrel*

From Eq. 25.30, the thickness of barrel at the top end is given by,

$$t_3 = (0.03D + b + 4.9) = 0.03(200) + 7 + 4.9$$

$$= 17.9 \text{ mm} \quad \text{or } 18 \text{ mm}$$

From Eq. 25.31, the thickness of the barrel at the open end is given by,

$$t_4 = (0.25t_3) \text{ to } (0.35t_3)$$

$$= (0.25 \times 18) \text{ to } (0.35 \times 18) = 4.5 \text{ to } 6.3 \text{ mm}$$

$$t_4 = 6 \text{ mm}$$

Step VI *Piston skirt*

Assumption No. 10 The allowable bearing pressure (p_b) for the skirt portion of the piston is 0.4 MPa.

Assumption No. 11 The ratio of the side thrust on the liner to the maximum gas load on the piston is 0.1. $(\mu = 0.1)$

From Eq. 25.32, the length of the skirt is given by,

$$\mu\left(\frac{\pi D^2}{4}\right)p_{\max.} = p_b D l_s \text{ or}$$

$$0.1\left(\frac{\pi(200)^2}{4}\right)(4) = (0.4)\,(200)\,l_s$$

$$l_s = 157.08 \text{ or } 160 \text{ mm}$$

Step VII *Piston length*

Refer to Fig. 25.11.

Length of ring section = $5h + 4h_2$

$= 5(6) + 4(5) = 50$ mm [Refer Fig. 25.4]

Length of piston = h_1+ length of ring section + l_s

$= 30 + 50 + 160 = 240$ mm

$L = 240$ mm

According to empirical relationship,

$L = D$ to $1.5D = 200$ to 300 mm

$\therefore \quad D < L > 1.5D$

Step VIII *Piston pin*

Assumption No. 12 The bearing pressure $(p_b)_1$ at the bush of the small end of the connecting rod is 30 MPa.

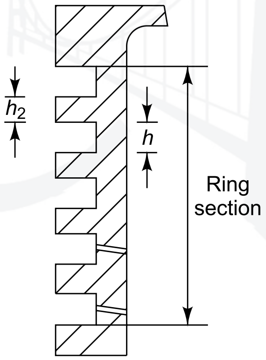


Fig. 25.11

Assumption No. 13 The piston pin is made of heat treated alloy steel and the permissible tensile stress is 140 N/mm^2.

P = Force on piston

$$= \left(\frac{\pi D^2}{4}\right)p_{\max.} = \left(\frac{\pi(200)^2}{4}\right)(4)$$

$= 125\,663.71$ N

From Eq. 25.37,

$$\left(\frac{\pi D^2}{4}\right)p_{\max.} = (p_b)_1 \times d_o \times l_1 \quad [\text{Given } l_1/d_o = 1.5]$$

$$125\,663.71 = 30d_o\,(1.5d_o)$$

$$d_o^2 = 2792.53$$

$$d_o = 52.84 \text{ mm}$$

In order to make the pin hollow, we will increase the diameter to 70 mm.

$d_o = 70$ mm

From Eq. 25.39,

$$M_b = \left(\frac{PD}{8}\right) = \frac{(125\,663.71)(200)}{8}$$

$$= 3141.59 \times 10^3 \text{ N-mm}$$

$$I = \frac{\pi(d_o^4 - d_i^4)}{64} = \frac{\pi[(70)^4 - d_i^4]}{64} \text{ mm}^4$$

$$y = \left(\frac{d_o}{2}\right) = \left(\frac{70}{2}\right) = 35 \text{ mm}$$

$$\sigma_b = \frac{M_b y}{I} \qquad 140 = \frac{(3141.59 \times 10^3)(35)}{\frac{\pi}{64}[(70)^4 - d_i^4]}$$

$$(70)^4 - d_i^4 = 16 \times 10^6$$

$$d_i = 53.2 \text{ or } 50 \text{ mm}$$

The mean diameter of piston boss is given by,

mean diameter of piston boss = $1.4d_o$

$= 1.4\,(70) = 98$ or 100 mm

25.16 CONNECTING ROD

The connecting rod consists of an eye at the small end to accommodate the piston pin, a long shank and a big end opening split into two parts to accommodate the crank pin. The construction of connecting rod is illustrated in Fig. 25.12. The basic function of the connecting rod is to transmit the push and pull forces from the piston pin to the crank pin. The connecting rod transmits the reciprocating motion of the piston to the rotary motion of the crankshaft. It also transfers lubricating oil from the crank pin to the piston pin and provides a splash or jet of oil to the piston assembly.

The connecting rod of an IC engine is made by the drop forging process and the outer surfaces are left unfinished. Most internal combustion engines have a conventional two-piece connecting rod. The whole rod is forged in one piece; the bearing cap is cut off, faced and bolted in place for final machining of the big end. The small end of the rod is generally made as a solid eye and then machined.

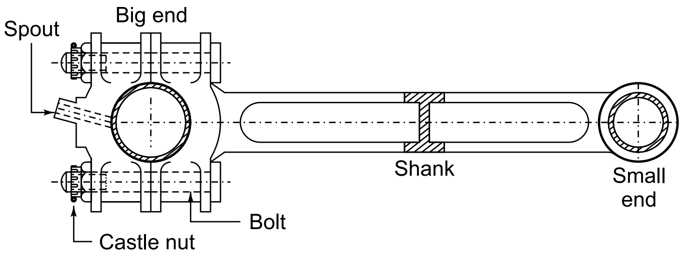


Fig. 25.12 *Connecting Rod*

The connecting rod is subjected to the force of gas pressure and the inertia force of the reciprocating part. It is one of the most heavily stressed parts of the IC engine. The materials used for the connecting rod are either medium carbon steels or alloy steels. The medium carbon steels contain 0.35 to 0.45 per cent carbon. The alloy steels include nicke chromium or chromium molybdenum steels. Medium carbon steels are used for the connecting rods of industrial engines. Alloy steels are used for connecting rods of automobile and aero engines.

There are two methods of lubrication of bearings at the two ends—splash lubrication and pressure feed lubrication. In *splash lubrication*, a spout is attached to the big end of the connecting rod and set at an angle to the axis of the rod. The spout dips into the sump of lubricating oil during the downward motion of the connecting rod and splashes the oil as the connecting rod moves up. The splashed up oil finds its way into the small end bearing. In the *pressure feed system*, oil is fed under pressure to the crank pin bearing through the holes drilled in the crankshaft. From the crank pin bearing, the oil is fed to the small end bearing through the hole drilled in the shank of the connecting rod.

The length of the connecting rod is an important consideration. When the connecting rod is short as compared to the crank radius, it has greater angular swing, resulting in greater side thrust on the piston. In high-speed engines, the ratio of the length of the connecting rod to the crank radius (L/r) is generally 4 or less. In low-speed engines, the (L/r) ratio varies from 4 to 5.

Most of the connecting rods in high-speed engines have an I-section. It reduces the weight and inertia forces. It is also easy for forging. Most rods have a rifle-drilled hole throughout the length from the small end to the big end to carry the lubricating oil to the piston pin bearing. In low-speed engines, circular cross-section is used.

25.17 BUCKLING OF CONNECTING ROD

The connecting rod is a slender engine component that has considerable length in proportion to its width and breadth. It is subjected to axial compressive force equal to maximum gas load on the piston. The compressive stress is of significant magnitude. Therefore, the connecting rod is designed as a column or a strut. The buckling of the connecting rod in two different planes—plane of motion and a plane perpendicular to the plane of motion is illustrated in Fig. 25.13. The following observations are made with reference to this figure:

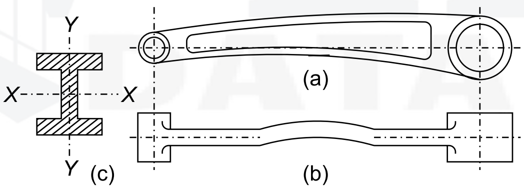


Fig. 25.13 *Buckling of Connecting Rod*

(i) The buckling of the connecting rod in the plane of motion is shown in Fig. 25.13(a). In this plane, the ends of connecting rod are hinged in the crank pin and piston pin. Therefore, for buckling about the XX-axis, the end fixity coefficient (n) is one.

(ii) The buckling of the connecting rod in a plane perpendicular to the plane of motion is shown in Fig. 25.13(b). In this plane, the ends of the connecting rod are fixed due to constraining effect of bearings at the crank pin and piston pin. Therefore, for buckling about the YY-axis, the end fixity coefficient (n) is four.

(iii) Therefore, the connecting rod is four times stronger for buckling about the YY-axis as compared to buckling about the XX-axis.

(iv) If a connecting rod is designed in such a way that it is equally resistant to buckling in either plane then

$$4I_{yy} = I_{xx} \tag{25.41}$$

where,

I = moment of inertia of cross-section (mm^4)

substituting ($I = Ak^2$),

$$4k_{yy}^2 = k_{xx}^2$$

$$k_{yy}^2 = \frac{1}{4}k_{xx}^2 \tag{25.42}$$

where,

k = radius of gyration of cross-section (mm)

(v) The above relationship proves that I-section is ideally suitable for the connecting rod. On the other hand, a circular cross-section is unnecessarily strong for buckling about the YY-axis.

Figure. 25.14 shows the typical proportions for the cross-section of the connecting rod for IC engine. For this cross-section,

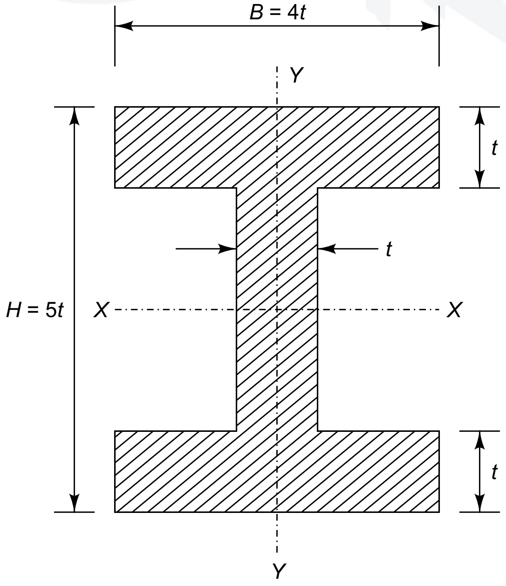


Fig. 25.14

$$A = 2(4t \times t) + (5t - 2t) \times t = 8t^2 + 3t^2 = 11t^2$$

$$A = 11t^2 \tag{25.43}$$

where,

A = area of cross-section (mm^2)

$$I_{xx} = \frac{1}{12}(4t)(5t)^3 - \frac{1}{12}(4t - t)(5t - 2t)^3$$

$$= \frac{1}{12}(500t^4 - 81t^4)$$

$$I_{xx} = \left(\frac{419}{12}\right)t^4 \tag{25.44}$$

$$k_{xx}^2 = \frac{I_{XX}}{A} = \left(\frac{419t^4}{12}\right)\left(\frac{1}{11t^2}\right) = 3.17t^2$$

$$k_{xx}^2 = 3.17t^2$$

$$k_{xx} = 1.78t \tag{25.45}$$

Also,

$$I_{yy} = 2\left[\frac{1}{12}(t)(4t)^3\right] + \frac{1}{12}(5t - 2t)(t)^3$$

$$= \frac{1}{12}[128t^4 + 3t^4]$$

$$I_{yy} = \left(\frac{131}{12}\right)t^4$$

$$k_{yy}^2 = \frac{I_{yy}}{A} = \left(\frac{131t^4}{12}\right)\left(\frac{1}{11t^2}\right) = 0.992t^2$$

$$k_{yy}^2 = 0.992t^2$$

Therefore,

$$\frac{I_{xx}}{I_{yy}} = \frac{\left(\frac{419}{12}\right)t^4}{\left(\frac{131}{12}\right)t^4} = \frac{419}{131} = 3.2$$

$$3.2I_{yy} = I_{xx} \tag{25.46}$$

It is observed from expressions (25.41) and (25.46) that the proportions of I-sections of the connecting rod are satisfactory.

25.18 CROSS-SECTION FOR CONNECTING ROD

A schematic diagram of the crank and connecting rod mechanism is shown in Fig. 25.15(a). The following notations are used:

P = force acting on the piston due to gas pressure (N)

P_s = side thrust on the cylinder wall (N)
P_c = force acting on the connecting rod (N)
φ = angle of inclination of connecting rod with line of stroke
θ = angle of inclination of crank from top dead centre position

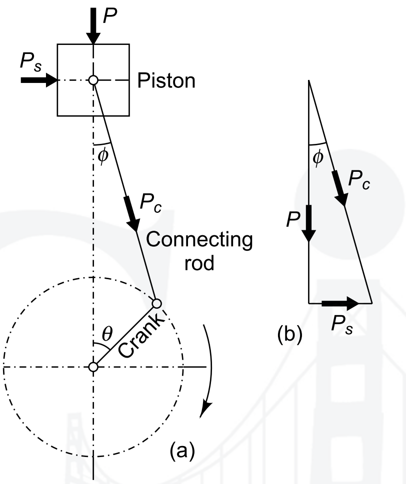


Fig. 25.15

From Fig. 25.15(b),

$$P = P_c \cos\varphi$$

$$P_c = \frac{P}{\cos\varphi} \tag{a}$$

The maximum gas load occurs shortly after the dead centre position and at this instant ($\varphi = 3.3°$).

$$\cos\varphi = \cos(3.3°) = 0.9983 \cong 1 \tag{b}$$

From (a) and (b), it is concluded that the force acting on the connecting rod is equal to the maximum force acting on the piston due to gas pressure.

$$P_c = \left(\frac{\pi D^2}{4}\right) p_{\max.} \tag{25.47}$$

The I-section illustrated in Fig. 25.14 with width as ($4t$), height at ($5t$) and thickness of web and two flanges as (t) is used for the connecting rod of IC engines. As explained in Section 25.17,

$$A = 11t^2$$

$$k_{xx} = 1.78t$$

The dimensions of cross-section are calculated by applying Rankine's formula for buckling of the connecting rod in the plane of rotation or about the XX-axis. According to this formula,

$$P_{cr} = \frac{\sigma_c A}{1 + a\left(\dfrac{L}{k_{xx}}\right)^2} \tag{25.48}$$

where,

P_{cr} = critical buckling load (N)
σ_c = compressive yield stress (N/mm^2)
A = cross-sectional area of connecting rod (mm^2)
a = constant depending upon material and end fixity coefficient
L = length of connecting rod (mm)
k_{xx} = radius of gyration (mm)

For a connecting rod made of mild steel and plain carbon steel,

$$\sigma_c = 330 \text{ N/mm}^2$$

In the plane of rotation, both ends are hinged and the equivalent length is equal to the actual length of connecting rod. The constant a for steel material is given by,

$$a = \frac{1}{7500} \tag{25.49}$$

The critical buckling load is given by,

$$P_{cr} = P_c\,(fs) \tag{25.50}$$

where,

(fs) = factor of safety (usually from 5 to 6)

The step by step procedure for finding out the dimensions of the I-section of the connecting rod is as follows:

(i) Calculate the force acting on the connecting rod.

$$P_c = \left(\frac{\pi D^2}{4}\right) p_{\max.}$$

(ii) Calculate the critical buckling load by,

$$P_{cr} = P_c\,(fs) \qquad [(fs) = 5 \text{ or } 6]$$

(iii) By Rankine's formula,

$$P_{cr} = \frac{\sigma_c A}{1 + a\left(\dfrac{L}{k_{xx}}\right)^2}$$

Substitute,

$$A = 11t^2 \quad k_{xx} = 1.78t \quad a = \frac{1}{7500} \quad \sigma_c = 330 \text{ N/mm}^2$$

Calculate the value of t.

(iv) Find out the dimensions of the cross-section of the connecting rod by using the proportions given in Fig. 25.14. The dimensions B and H are at the middle of the connecting rod.

(v) The width B is kept constant throughout the length of the connecting rod.

(vi) The height H varies from the big end to the small end in the following way:

at the middle section, $H = 5\,t$

at the small end, $H_1 = 0.75\,H$ to $0.9\,H$

at the big end, $H_2 = 1.1\,H$ to $1.25\,H$

Example 25.12 *Determine the dimensions of cross-section of the connecting rod for a diesel engine with the following data:*

Cylinder bore = 100 mm

Length of connecting rod = 350 mm

Maximum gas pressure = 4 MPa

Factor of safety = 6

Solution

Given $D = 100$ mm $p_{max.} = 4$ MPa $= 4$ N/mm^2
$L = 350$ mm $(fs) = 6$

Step I *Force acting on the connecting rod*

$$P_c = \left(\frac{\pi D^2}{4}\right) p_{max.} = \left(\frac{\pi (100)^2}{4}\right)(4) = 31\,415.93 \text{ N}$$

Step II *Critical buckling load*

$$P_{cr} = P_c\,(fs) = 31\,415.93\,(6) = 188\,495.58 \text{ N}$$

Step III *Calculation of t*

Substituting,

$$A = 11t^2 \quad k_{xx} = 1.78t \quad a = \frac{1}{7500}$$

$$\sigma_c = 330 \text{ N/mm}^2$$

$$P_{cr} = \frac{\sigma_c A}{1 + a\left(\frac{L}{k_{xx}}\right)^2}$$

or
$$188\,495.58 = \frac{(330)(11t^2)}{1 + \frac{1}{7500}\left(\frac{350}{1.78t}\right)^2}$$

$$\frac{188\,495.58}{(330)(11)} = \frac{t^2}{1 + \frac{5.16}{t^2}} \quad \text{or} \quad 51.93 = \frac{t^4}{t^2 + 5.16}$$

$$t^4 - 51.93t^2 - 267.96 = 0$$

The above expression is a quadratic equation in (t^2).

$$t^2 = \frac{51.93 \pm \sqrt{(51.93)^2 + 4(267.96)}}{2}$$

$$= \frac{51.93 \pm 61.39}{2}$$

$$t^2 = 56.66$$

$$t = 7.53 \text{ or } 8 \text{ mm}$$

Step IV *Dimensions of cross-section*

$B = 4t = 4(8) = 32$ mm

$H = 5t = 5(8) = 40$ mm

Thickness of web $= t = 8$ mm

Thickness of flanges $= t = 8$ mm

The width ($B = 32$ mm) is kept constant throughout the length of connecting rod.

Step V *Variation of height*

at the middle section, $H = 5t = 40$ mm

at the small end, $H_1 = 0.85\,H = 0.85\,(40) = 34$ mm

at the big end, $H_2 = 1.2\,H = 1.2\,(40) = 48$ mm

dimensions (B/H) of section at big end $= 32 \text{ mm} \times 48$ mm

dimensions (B/H) of section at middle $= 32 \text{ mm} \times 40$ mm

dimensions (B/H) of section at small end $= 32 \text{ mm} \times 34$ mm

25.19 BIG AND SMALL END BEARINGS

The construction of small end of the connecting rod in the piston pin is shown in Fig. 25.16(a). The piston pin bearing as shown in Fig. 25.16(b), is usually a phosphor bronze bush of 3 mm thickness. It is a solid one-piece bushing made by casting and then finished by grinding and reaming operations. It is designed by bearing considerations. The force acting on the piston pin bearing is given by,

$$P_c = \left(\frac{\pi D^2}{4}\right) p_{max.}$$

Also,

$$P_c = d_p l_p (p_b)_p \qquad (25.51)$$

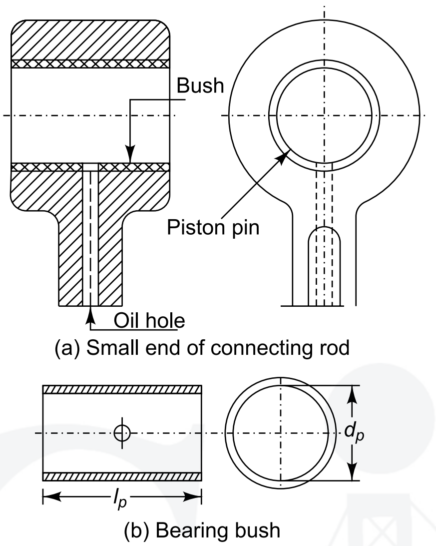


Fig. 25.16 *Small End of Connecting Rod*

where,

d_p = diameter of the piston pin or inner diameter of the bush on the piston pin (mm)

l_p = length of the piston pin or length of the bush on the piston pin (mm)

$(p_b)_p$ = allowable bearing pressure for the piston pin bush (N/mm^2)

The allowable bearing pressure for the piston pin bush is usually taken from 10 to 12.5 MPa. The (l/d) ratio for the piston pin bush is taken from 1.5 to 2.

$$\therefore \quad \left(\frac{l_p}{d_p}\right) = 1.5 \text{ to } 2 \tag{25.52}$$

The construction of the big end of the connecting rod in the crank pin is shown in Fig. 25.17(a). The crank pin bearing as shown in Fig. 25.17(b), is a lined bearing split into two halves. The lined bushing consists of steel backing with a thin lining of bearing material like babbitt. It is also designed by bearing considerations.

$$P_c = d_c l_c (p_b)_c \tag{25.53}$$

where,

d_c = diameter of the crank pin or inner diameter of the bush on the crank pin (mm)

l_c = length of the crank pin or length of the bush on the crank pin (mm)

$(p_b)_c$ = allowable bearing pressure for the crank pin bush (N/mm^2)

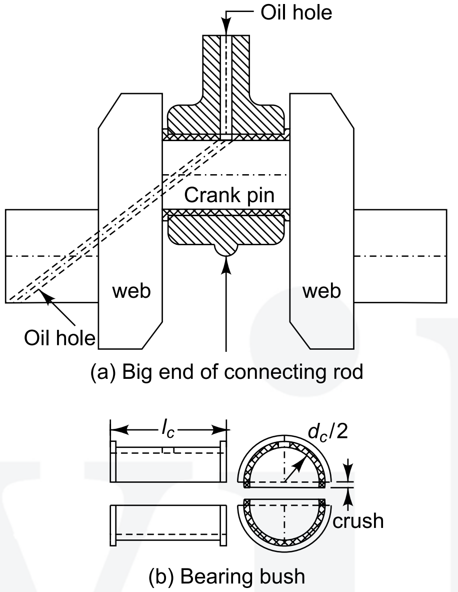


Fig. 25.17 *Big End of Connecting Rod*

The allowable bearing pressure for the crank pin bush is usually taken from 5 to 10 N/mm^2. The (l/d) ratio for the crank pin bush is taken from 1.25 to 1.5.

$$\therefore \quad \left(\frac{l_c}{d_c}\right) = 1.25 \text{ to } 1.5 \tag{25.54}$$

There is a peculiar term, '*crush*', related to big end bearings. In order to make good seating of bearing bushes in the cap and connecting rod, the sleeve height is slightly more (approximately 0.05 mm) than the half bore of the housing in which it fits. This is called bearing 'crush' as shown in Fig. 25.17(b). When the cap is tightened by bolts, the projecting bearing faces are squeezed in (or crushed) to form a press fit between the split bushes and cap and the big end of the connecting rod.

There is one more term called '*shim*'. The wear of the big end bearing is compensated by means of thin metallic strips between the cap and the fixed half. As wear takes place, one or more strips are removed and the cap is tightened. These strips are called shims.

Example 25.13 *Determine the dimensions of small and big end bearings of the connecting rod for a diesel engine with the following data:*

Cylinder bore = 100 mm
Maximum gas pressure = 4 MPa
(l/d) ratio for piston pin bearing = 2
(l/d) ratio for crank pin bearing = 1.3
Allowable bearing pressure for piston pin bearing = 12 MPa
Allowable bearing pressure for crank pin bearing = 7.5 MPa

Solution

Given $D = 100$ mm $p_{max.} = 4$ MPa $= 4$ N/mm^2
$(l_p/d_p) = 2$ $(l_c/d_c) = 1.3$
$(p_b)_p = 12$ MPa $= 12$ N/mm^2
$(p_b)_c = 7.5$ MPa $= 7.5$ N/mm^2

Step I *Maximum bearing load*

$$P_c = \left(\frac{\pi D^2}{4}\right) p_{max.} = \left(\frac{\pi (100)^2}{4}\right)(4) = 31415.93 \text{ N}$$

Step II *Piston pin bearing*

From Eq. 25.51,

$$P_c = d_p l_p (p_b)_p \quad \text{or} \quad 31\,415.93 = d_p (2d_p)(12)$$
$$d_p^2 = 1309$$
$$d_p = 36.18 \text{ or } 38 \text{ mm}$$
$$l_p = 2\,(38) = 76 \text{ mm}$$

Step III *Crank-pin bearing*

From Eq. 25.53,

$$P_c = d_c\, l_c (p_b)_c \quad \text{or} \quad 31\,415.93 = d_c\,(1.3 d_c)(7.5)$$
$$d_c^2 = 3222.15$$
$$d_c = 56.76 \text{ or } 58 \text{ mm}$$
$$l_c = 1.3\,(58) = 75.4 \text{ or } 76 \text{ mm}$$

25.20 BIG END CAP AND BOLTS

The maximum force acting on the cap and two bolts consists only of inertia force at the top dead centre on the exhaust stroke. The inertia force acting on the bolts or cap is given by,

$$P_i = m_r \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n_1}\right] \tag{25.55}$$

where,

P_i = inertia force on the cap or bolts (N)
m_r = mass of reciprocating parts (kg)
ω = angular velocity of crank or angular speed of the engine (rad/s)
r = crank radius (m)
n_1 = ratio of length of the connecting rod to the crank radius $= \left(\frac{L}{r}\right)$
L = length of connecting rod (m)
θ = angle of inclination of crank from top dead centre position

The mass of reciprocating parts is given by,

$$m_r = [\text{mass of piston assembly} + \left(\frac{1}{3}\right)\text{rd mass of connecting rod}] \tag{25.56}$$

The angular velocity of the crank is given by,

$$\omega = \left(\frac{2\pi N}{60}\right) \tag{25.57}$$

where,

N = crank speed (rpm)

The crank radius is given by,

$$r = \left(\frac{l}{2}\right) \tag{25.58}$$

where,

l = length of stroke (m)

The inertia force will be maximum at the top dead centre position where $(\theta = 0)$.

When $(\theta = 0)$,

$$\cos\theta = 1 \quad \text{and} \quad \cos 2\theta = 1$$

Substituting the above values in Eq. (25.55),

$$(P_i)_{max.} = m_r \omega^2 r \left[1 + \frac{1}{n_1}\right] \tag{25.59}$$

The forces acting on the cap and bolts are shown in Fig. 25.18. The bolts are subjected to tensile force as illustrated in Fig. 25.18(a). Since there are two bolts, each share the inertia force equally. Therefore,

$$(P_i)_{max.} = 2\left(\frac{\pi d_c^2}{4}\right)\sigma_t \tag{25.60}$$

where,

d_c = core diameter of bolts (mm)
σ_t = permissible tensile stress for bolt material (N/mm^2)

The nominal diameter (d) of the bolt is calculated by,

$$d = \left(\frac{d_c}{0.8}\right) \tag{25.61}$$

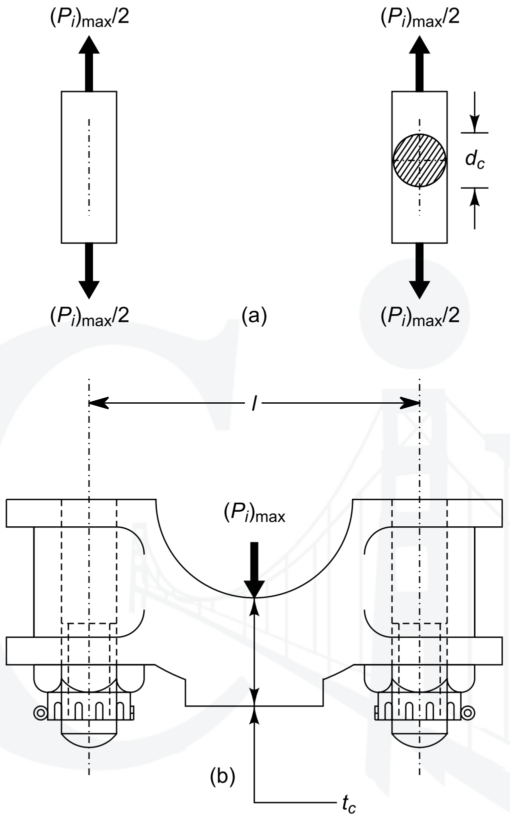


Fig. 25.18 *Forces on cap and bolts*

The cap, also called *keep plate*, is subjected to inertia force $(P_i)_{max.}$ as shown in Fig. 25.18(b). It is treated as a beam freely supported at the bolt centres and loaded in a manner intermediate between uniformly distributed and centrally concentrated load in which case the bending moment is $(Wl/6)$. Therefore,

$$M_b = \frac{(P_i)_{max.}\, l}{6} \tag{25.62}$$

where,

M_b = bending moment acting on cap (N-mm)

l = span length or distance between the bolt centers (mm)

The distance between the centres of bolts is given by,

l = diameter of crank pin + 2 [thickness of bush (3 mm)] + nominal diameter of bolt (d) + clearance (3 mm) (25.63)

The thickness of the cap (t_c) is obtained by,

$$\sigma_b = \frac{M_b y}{I}$$

where,

$$I = \left[\frac{(b_c)(t_c)^3}{12}\right] \text{ and } y = \left(\frac{t_c}{2}\right) \tag{25.64}$$

b_c = width of cap (mm); equal to length of crank pin or big end bearing (l_c).

t_c = thickness of big end cap (mm)

The thickness (t_c) as obtained by the above expressions is exclusive of the thickness of the babbitt bush on the crank pin.

Example 25.14 *The following data is given for the cap and bolts of the big end of connecting rod:*

Engine speed = 1800 rpm
Length of connecting rod = 350 mm
Length of stroke = 175 mm
Mass of reciprocating parts = 2.5 kg
Length of crank pin = 76 mm
Diameter of crank pin = 58 mm
Thickness of bearing bush = 3 mm
Permissible tensile stress for bolts = 60 N/mm²
Permissible bending stress for cap = 80 N/mm²

Calculate the nominal diameter of bolts and thickness of cap for the big end.

Solution

Given $N = 1800$ rpm $L = 350$ mm $l = 175$ mm
$m_r = 2.5$ kg $l_c = 76$ mm $d_c = 58$ mm
$\sigma_t = 60$ N/mm² $\sigma_b = 80$ N/mm²

Step I *Inertia force*

$$r = \left(\frac{l}{2}\right) = \left(\frac{175}{2}\right) = 87.5 \text{ mm} = 0.0875 \text{ m}$$

$$n_1 = \left(\frac{L}{r}\right) = \frac{350}{87.5} = 4$$

$$\omega = \left(\frac{2\pi N}{60}\right) = \left(\frac{2\pi(1800)}{60}\right) = 188.5 \text{ rad/s}$$

From Eq. 25.59,

$$(P_i)_{max.} = m_r \omega^2 r \left[1 + \frac{1}{n_1}\right]$$

$$= (2.5)(188.5)^2\ (0.0875)\left[1 + \frac{1}{4}\right]$$

$$= 9715.85 \text{ N}$$

Step II *Diameter of bolts*

From Eq. 25.60,

$$(P_i)_{max.} = 2\left(\frac{\pi d_c^2}{4}\right)\sigma_t \quad \text{or} \quad 9715.85 = 2\left(\frac{\pi d_c^2}{4}\right)(60)$$

$$d_c^2 = 103.09 \text{ mm}$$

$$d_c = 10.15 \text{ mm}$$

$$d = \left(\frac{d_c}{0.8}\right) = \left(\frac{10.15}{0.8}\right) = 12.69 \text{ or } 16 \text{ mm}$$

Step III *Thickness of cap*

$$b_c = l_c = 76 \text{ mm}$$

From Eq. 25.63,

l = diameter of crank pin + 2 (thickness of bush) + nominal diameter of bolt (d) + clearance (3 mm)

$= 58 + 2(3) + 16 + 3$

$= 83$ mm

$$M_b = \frac{(P_i)_{max.}\, l}{6} = \frac{(9715.85)(83)}{6}$$

$$= 134\,402.59 \text{ N-mm}$$

$$I = \left[\frac{(b_c)(t_c)^3}{12}\right] = \left[\frac{(76)(t_c)^3}{12}\right] = 6.33\, t_c^3$$

$$y = \left(\frac{t_c}{2}\right)$$

Substituting the above expressions,

$$\sigma_b = \frac{M_b y}{I} \quad \text{or} \quad 80 = \frac{(134\,402.59)}{(6.33\, t_c^3)}\left(\frac{t_c}{2}\right)$$

$$t_c^2 = 132.7$$

$$t_c = 11.52 \text{ mm or } 12 \text{ mm}$$

25.21 WHIPPING STRESS

The small end of the connecting rod is subjected to pure translation motion while the big end is subjected to pure rotary motion. The intermediate points on the connecting rod move in elliptical orbits. The lateral oscillations of the connecting rod induce inertia forces that act all along the length of the connecting rod causing bending. This type of action is called '*whipping*'. The bending stress due to inertia force is called '*whipping stress*'.

The mass of the connecting rod per metre length is given by,

m_1 = volume × density

= area × length × density

$m_1 = A\,(1)(\rho)$

$$m_1 = A\rho \tag{25.65}$$

where,

A = area of cross-section (m^2)

ρ = mass density of the connecting rod (kg/m^3)

For steels,

$\rho = 7800\ kg/m^3$

Refer to the cross-section of the connecting rod illustrated in Fig. 25.14. For this cross-section,

$A = 11t^2$

t = thickness of web or flanges (m)

The density of steel is taken as 7800 kg/m^3. Therefore,

$$m_1 = (11t^2)\,\rho \text{ kg/m} \tag{25.66}$$

It can be shown that the maximum bending moment occurs at a distance of $\left(\frac{L}{\sqrt{3}}\right)$ from the piston pin and its magnitude is given by,

$$(M_b)_{max.} = m\omega^2 r \frac{L}{9\sqrt{3}} \tag{a}$$

where,

m = mass of connecting rod (kg)

r = crank radius (m)

L = length of connecting rod (m)

Substituting ($m = m_1 L$) in the expression (a),

$$(M_b)_{max.} = m_l \omega^2 r \frac{L^2}{9\sqrt{3}} \tag{25.67}$$

For the cross-section of the connecting rod illustrated in Fig. 25.14,

$$I_{xx} = \left(\frac{419}{12}\right)t^4 \text{ and } y = \left(\frac{5t}{2}\right)$$

Substituting the above expressions, the whipping stress is calculated by the following equation:

$$\sigma_b = \frac{M_b y}{I}$$

Example 25.15 The following data is given for a connecting rod:

Engine speed = 1800 rpm
Length of connecting rod = 350 mm
Length of stroke = 175 mm
Density of material = 7800 kg/m^3
Thickness of web or flanges = 8 mm

Assume the cross-section illustrated in Fig. 25.14. For this cross-section,

$$A = 11t^2 \qquad I_{xx} = \left(\frac{419}{12}\right)t^4 \text{ and } y = \left(\frac{5t}{2}\right)$$

Calculate whipping stress in the connecting rod.

Solution

Given $N = 1800$ rpm $L = 350$ mm $= 0.35$ m
$l = 175$ mm $= 0.175$ m $t = 8$ mm $= 0.008$ m
$\rho = 7800$ kg/m^3

$$r = \left(\frac{l}{2}\right) = \left(\frac{0.175}{2}\right) = 0.0875 \text{ m}$$

$$\omega = \left(\frac{2\pi N}{60}\right) = \left(\frac{2\pi\,(1800)}{60}\right) = 188.5 \text{ rad/s}$$

From Eq. 25.65,

$$m_1 = (11t^2)\,\rho = 11(0.008)^2\,(7800) = 5.49 \text{ kg/m}$$

From Eq. 25.67,

$$(M_b)_{\text{max.}} = m_1\omega^2 r\,\frac{L^2}{9\sqrt{3}}$$

$$= (5.49)(188.5)^2(0.0875)\,\frac{(0.35)^2}{9\sqrt{3}}$$

$$= 134.13 \text{ N-m}$$

$$(M_b)_{\text{max.}} = (134.13 \times 10^3) \text{ N-mm}$$

$$\sigma_b = \frac{M_b y}{I} = \frac{(134.13\times10^3)\left(\frac{5\times 8}{2}\right)}{\left(\frac{419}{12}\right)(8)^4} = 18.76 \text{ N/mm}^2$$

Example 25.16 *The following data is given for the connecting rod of a diesel engine:*

Cylinder bore = 85 mm
Length of connecting rod = 350 mm
Maximum gas pressure = 3 MPa
Factor of safety against buckling failure = 5
(l/d) ratio for piston pin bearing = 1.5
(l/d) ratio for crank pin bearing = 1.25
Allowable bearing pressure for piston pin bearing = 13 MPa
Allowable bearing pressure for crank pin bearing = 11 MPa
Length of stroke = 140 mm
Mass of reciprocating parts = 1.5 kg
Engine speed = 2000 rpm
Thickness of bearing bush = 3 mm
Material of cap = steel 40C8
Yield strength of cap material = 380 N/mm^2
Factor of safety for cap = 4
Material of bolts = chromium molybdenum steel
Yield strength of bolt material = 450 N/mm^2
Factor of safety for bolts = 5
Density of connecting rod = 7800 kg/m^3
Calculate:

(i) *dimensions of the cross-section of connecting rod;*
(ii) *dimensions of small and big end bearings;*
(iii) *nominal diameter of bolts for the cap;*
(iv) *thickness of cap; and*
(v) *magnitude of whipping stress*

Solution

Given

Connecting rod shank
$D = 85$ mm $p_{\text{max.}} = 3$ MPa $= 3$ N/mm^2
$L = 350$ mm $= 0.35$ m $(fs) = 5$
Big and small end bearing
$(l_p/d_p) = 1.5$ $(l_c/d_c) = 1.25$
$(p_b)_p = 13$ MPa $= 13$ N/mm^2
$(p_b)_c = 11$ MPa $= 11$ N/mm^2
Bolts for cap
$l = 140$ mm $= 0.14$ m $N = 2000$ rpm $m_r = 1.5$ kg
$S_{yt} = 450$ N/mm^2 $(fs) = 5$
Cap $S_{yt} = 380$ N/mm^2 $(fs) = 4$
Whipping stress $\rho = 7800$ kg/m^3

Step I *Cross-section of connecting rod*

$$P_c = \left(\frac{\pi D^2}{4}\right)p_{\text{max.}} = \left(\frac{\pi(85)^2}{4}\right)(3) = 17023.51 \text{ N}$$

$$P_{cr} = P_c\,(fs) = 17\,023.51\,(5) = 85\,117.55 \text{ N}$$

Substituting,

$A = 11t^2 \quad k_{xx} = 1.78t \quad a = \dfrac{1}{7500} \quad \sigma_c = 330 \text{ N/mm}^2$

in Eq. (25.48),

$$P_{cr} = \frac{\sigma_c A}{1 + a\left(\dfrac{L}{k_{xx}}\right)^2}$$

or
$$85117.55 = \frac{(330)(11t^2)}{1 + \dfrac{1}{7500}\left(\dfrac{350}{1.78t}\right)^2}$$

$$\frac{85117.55}{(330)(11)} = \frac{t^2}{1 + \dfrac{5.16}{t^2}} \quad \text{or} \quad 23.45 = \frac{t^4}{t^2 + 5.16}$$

$t^4 - 23.45t^2 - 121 = 0$

The above expression is a quadratic equation in (t^2).

$$t^2 = \frac{23.45 \pm \sqrt{(23.45)^2 + 4(121)}}{2} = \frac{23.45 \pm 32.15}{2}$$

$t^2 = 27.8$

$t = 5.27$ or 5.5 mm

Dimensions of cross-section (Fig. 25.14)

$B = 4t = 4(5.5) = 22$ mm

$H = 5t = 5(5.5) = 27.5$ mm

Thickness of web $= t = 5.5$ mm

Thickness of flanges $= t = 5.5$ mm

The width $(B = 22$ mm) is kept constant throughout the length of the connecting rod.

Variation of height

at the middle section, $H = 5t = 27.5$ mm

at the small end, $H_1 = 0.85\,H = 0.85\,(27.5) = 23.38$ or 24 mm

at the big end, $H_2 = 1.2\,H = 1.2\,(27.5) = 33$ mm

dimensions (B/H) of section at big end $= 22 \text{ mm} \times 33$ mm

dimensions (B/H) of section at middle $= 22 \text{ mm} \times 27.5$ mm

dimensions (B/H) of section at small end $= 22 \text{ mm} \times 24$ mm

Step II *Small and big end bearings*

Piston pin bearing

From Eq. 25.51,

$$P_c = d_p\, l_p\, (p_b)_p$$

or
$$17\,023.51 = d_p\,(1.5d_p)(13)$$

$$d_p^2 = 873$$

$$d_p = 29.55 \text{ or } 30 \text{ mm}$$

$$l_p = 1.5\,(30) = 45 \text{ mm}$$

Crank pin bearing

From Eq. 25.53,

$P_c = d_c\, l_c\, (p_b)_c$ or $17\,023.51 = d_c\,(1.25\, d_c)(11)$

$d_c^2 = 1238.07$

$d_c = 35.19$ or 36 mm

$l_c = 1.25\,(36) = 45$ mm

Step III *Nominal diameter of bolts for the cap*

$$r = \left(\frac{l}{2}\right) = \left(\frac{140}{2}\right) = 70 \text{ mm} = 0.07 \text{ m}$$

$$n_1 = \left(\frac{L}{r}\right) = \frac{350}{70} = 5$$

$$\omega = \left(\frac{2\pi N}{60}\right) = \left(\frac{2\pi(2000)}{60}\right) = 209.44 \text{ rad/s}$$

From Eq. 25.59,

$$(P_i)_{\max.} = m_r \omega^2 r \left[1 + \frac{1}{n_1}\right]$$

$$= (1.5)(209.44)^2(0.07)\left[1 + \frac{1}{5}\right] = 5527 \text{ N}$$

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{450}{5} = 90 \text{ N/mm}^2$$

From Eq. 25.60,

$$(P_i)_{\max.} = 2\left(\frac{\pi d_c^2}{4}\right)\sigma_t$$

or
$$5527 = 2\left(\frac{\pi d_c^2}{4}\right)(90)$$

$$d_c^2 = 39.1$$

$$d_c = 6.25 \text{ mm}$$

$$d = \left(\frac{d_c}{0.8}\right) = \left(\frac{6.25}{0.8}\right) = 7.81 \text{ or } 8 \text{ mm}$$

Step IV *Thickness of cap*

$$b_c = l_c = 45 \text{ mm}$$

From Eq. 25.63,

l = diameter of crank pin + 2 (thickness of bush) + nominal diameter of bolt (d) + clearance (3 mm)

= 36 + 2(3) + 8 + 3

= 53 mm

$$M_b = \frac{(P_i)_{\text{max.}}\, l}{6} = \frac{(5527)(53)}{6} = 48\,821.83 \text{ N-mm}$$

$$I = \left[\frac{(b_c)(t_c)^3}{12}\right] = \left[\frac{(45)(t_c)^3}{12}\right] = 3.75 t_c^3$$

$$y = \left(\frac{t_c}{2}\right)$$

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{380}{4} = 95 \text{ N/mm}^2$$

Substituting above expressions,

$$\sigma_b = \frac{M_b y}{I} \quad \text{or} \quad 95 = \frac{(48\,821.83)}{(3.75 t_c^3)}\left(\frac{t_c}{2}\right)$$

$$t_c^2 = 68.52$$

$$t_c = 8.28 \text{ mm or } 10 \text{ mm}$$

Step V *Whipping stress*

$t = 5.5$ mm $= (5.5 \times 10^{-3})$ m

From Eq. 25.65,

$m_1 = (11t^2)\,\rho = 11(5.5 \times 10^{-3})^2\,(7800) = 2.6$ kg/m

From Eq. 25.67,

$$(M_b)_{\text{max.}} = m_1 \omega^2 r \frac{L^2}{9\sqrt{3}}$$

$$= (2.6)(209.44)^2 (0.07) \frac{(0.35)^2}{9\sqrt{3}}$$

$$= 62.74 \text{ N-m}$$

$$(M_b)_{\text{max.}} = (62.74 \times 10^3) \text{ N-mm}$$

$$I_{xx} = \left(\frac{419}{12}\right) t^4 \text{ and } y = \left(\frac{5t}{2}\right)$$

$$\sigma_b = \frac{M_b y}{I} = \frac{(62.74 \times 10^3)\left(\dfrac{5 \times 5.5}{2}\right)}{\left(\dfrac{419}{12}\right)(5.5)^4} = 27 \text{ N/mm}^2$$

Example 25.17 *Design a connecting rod for a high-speed IC engine using the following data:*

Cylinder bore = 125 mm

Length of connecting rod = 300 mm

Maximum gas pressure = 3.5 MPa

Length of stroke = 125 mm

Mass of reciprocating parts = 1.6 kg

Engine speed = 2200 rpm

Assume suitable data and state the assumptions you make.

Solution

Given $D = 125$ mm

$p_{\text{max.}} = 3.5$ MPa $= 3.5$ N/mm^2 $L = 300$ mm $= 0.3$ m

$l = 125$ mm $= 0.125$ m $N = 2200$ rpm $m_r = 1.6$ kg

Step I *Cross-section of connecting rod*

$$P_c = \left(\frac{\pi D^2}{4}\right) p_{\text{max.}} = \left(\frac{\pi (125)^2}{4}\right)(3.5) = 42\,951.46 \text{ N}$$

Assumption 1 The factor of safety against buckling failure is 5.

$$P_{cr} = P_c\,(fs) = 42\,951.46\,(5) = 214\,757.3 \text{ N}$$

Substituting,

$$A = 11t^2 \qquad k_{xx} = 1.78t \qquad a = \frac{1}{7500}$$

$$\sigma_c = 330 \text{ N/mm}^2$$

in Eq. (25.48),

$$P_{cr} = \frac{\sigma_c A}{1 + a\left(\dfrac{L}{k_{xx}}\right)^2}$$

$$\text{or} \qquad 214\,757.3 = \frac{(330)(11t^2)}{1 + \dfrac{1}{7500}\left(\dfrac{300}{1.78t}\right)^2}$$

$$\frac{214\,757.3}{(330)(11)} = \frac{t^2}{1 + \dfrac{3.79}{t^2}} \quad \text{or} \quad 59.16 = \frac{t^4}{t^2 + 3.79}$$

$$t^4 - 59.16t^2 - 224.22 = 0$$

The above expression is a quadratic equation in (t^2).

$$t^2 = \frac{59.16 \pm \sqrt{(59.16)^2 + 4(224.22)}}{2}$$

$$= \frac{59.16 \pm 66.31}{2}$$

$t^2 = 62.74$

$t = 7.92$ or 8 mm

Dimensions of cross-section (Fig. 25.14)

$B = 4t = 4(8) = 32$ mm

$H = 5t = 5(8) = 40$ mm

Thickness of web $= t = 8$ mm

Thickness of flanges $= t = 8$ mm

The width ($B = 32$ mm) is kept constant throughout the length of the connecting rod.

Variation of height

at the middle section, $H = 5t = 40$ mm

at the small end, $H_1 = 0.85\,H = 0.85\,(40) = 34$ mm

at the big end, $H_2 = 1.2\,H = 1.2\,(40) = 48$ mm

dimensions (B/H) of section at big end $= 32 \text{ mm} \times 48 \text{ mm}$

dimensions (B/H) of section at middle $= 32 \text{ mm} \times 40 \text{ mm}$

dimensions (B/H) of section at small end $= 32 \text{ mm} \times 34 \text{ mm}$

Step II *Small and big end bearings*

Assumption 2 The (l/d) ratio for piston pin bearing is 1.8.

Assumption 3 The (l/d) ratio for crank pin bearing is 1.1.

Assumption 4 The allowable bearing pressure for piston pin bearing is 14 MPa.

Assumption 5 The allowable bearing pressure for crank pin bearing is 8.5 MPa.

Piston-pin bearing

From Eq. 25.51,

$$P_c = d_p l_p (p_b)_p \quad \text{or} \quad 42\,951.46 = d_p(1.8 d_p)(14)$$

$$d_p^2 = 1704.42$$

$$d_p = 41.28 \text{ or } 42 \text{ mm}$$

$$l_p = 1.8\,(42) = 75.6 \text{ or } 76 \text{ mm}$$

Crank pin bearing

From Eq. 25.53,

$$P_c = d_c l_c (p_b)_c \quad \text{or} \quad 42\,951.46 = d_c(1.1 d_c)(8.5)$$

$$d_c^2 = 4593.74$$

$$d_c = 67.78 \text{ or } 68 \text{ mm}$$

$$l_c = 1.1\,(68) = 74.8 \text{ or } 75 \text{ mm}$$

Step III *Nominal diameter of bolts for the cap*

Assumption 6 The permissible tensile stress for the bolt material is 100 N/mm^2.

$$r = \left(\frac{l}{2}\right) = \left(\frac{125}{2}\right) = 62.5 \text{ mm} = 0.0625 \text{ m}$$

$$n_1 = \left(\frac{L}{r}\right) = \frac{300}{62.5} = 4.8$$

$$\omega = \left(\frac{2\pi N}{60}\right) = \left(\frac{2\pi\,(2200)}{60}\right) = 230.38 \text{ rad/s}$$

From Eq. 25.59,

$$(P_i)_{\max.} = m_r \omega^2 r \left[1 + \frac{1}{n_1}\right]$$

$$= (1.6)(230.38)^2(0.0625)\left[1 + \frac{1}{4.8}\right]$$

$$= 6413.22 \text{ N}$$

From Eq. 25.60,

$$(P_i)_{\max.} = 2\left(\frac{\pi d_c^2}{4}\right)\sigma_t \text{ or}$$

$$6413.22 = 2\left(\frac{\pi d_c^2}{4}\right)(100)$$

$$d_c^2 = 40.83$$

$$d_c = 6.39 \text{ mm}$$

$$d = \left(\frac{d_c}{0.8}\right) = \left(\frac{6.39}{0.8}\right) = 7.99 \text{ or } 8 \text{ mm}$$

Step IV *Thickness of cap*

Assumption 7 The permissible tensile stress for cap material is 100 N/mm^2.

$$b_c = l_c = 75 \text{ mm}$$

From Eq. 25.63,

l = diameter of crank pin + 2 (thickness of bush) + nominal diameter of bolt (d) + clearance (3 mm)

$= 68 + 2(3) + 8 + 3$

$= 85$ mm

$$M_b = \frac{(P_i)_{\max.}\,l}{6} = \frac{(6413.22)(85)}{6} = 90\,853.95 \text{ N-mm}$$

$$I = \left[\frac{(b_c)(t_c)^3}{12}\right] = \left[\frac{(75)(t_c)^3}{12}\right] = 6.25\,t_c^3$$

$$y = \left(\frac{t_c}{2}\right)$$

Substituting the above expressions,

$$\sigma_b = \frac{M_b y}{I} \quad \text{or} \quad 100 = \frac{(90853.95)}{(6.25t_c^3)}\left(\frac{t_c}{2}\right)$$

$$t_c^2 = 72.68$$

$$t_c = 8.53 \text{ mm or } 10 \text{ mm}$$

Step V *Whipping stress*

Assumption 8 The mass density of the connecting rod material is 7800 kg/m^3.

$t = 8$ mm $= (8 \times 10^{-3})$ m

From Eq. 25.65,

$m_l = (11t^2)\,\rho = 11(8 \times 10^{-3})^2\,(7800) = 5.49$ kg/m

From Eq. 25.67,

$$(M_b)_{\text{max.}} = m_l \omega^2 r \frac{L^2}{9\sqrt{3}}$$

$$= (5.49)(230.38)^2(0.0625)\frac{(0.3)^2}{9\sqrt{3}}$$

$$= 105.14 \text{ N-m}$$

$$(M_b)_{\text{max.}} = (105.14 \times 10^3) \text{ N-mm}$$

$$I_{xx} = \left(\frac{419}{12}\right)t^4 \quad \text{and} \quad y = \left(\frac{5t}{2}\right)$$

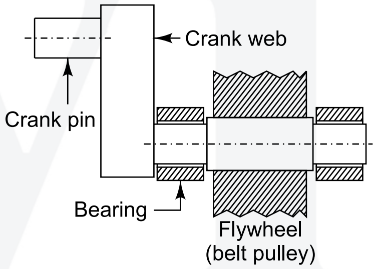
$$\sigma_b = \frac{M_b y}{I} = \frac{(105.14 \times 10^3)\left(\dfrac{5 \times 8}{2}\right)}{\left(\dfrac{419}{12}\right)(8)^4} = 14.7 \text{ N/mm}^2$$

25.22 CRANKSHAFT

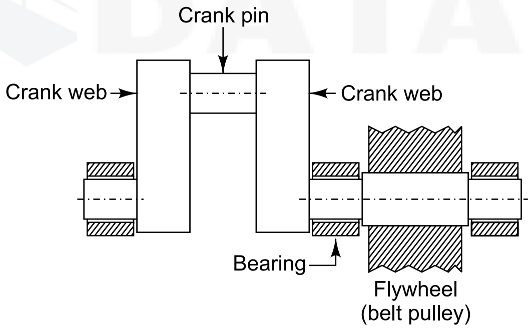
The crankshaft is an important part of IC engine that converts the reciprocating motion of the piston into rotary motion through the connecting rod. The crankshaft consists of three portions—crank pin, crank web and shaft. The big end of the connecting rod is attached to the *crank pin*. The *crank web* connects the crank pin to the shaft portion. The shaft portion rotates in the main bearings and transmits power to the outside source through the belt drive, gear drive or chain drive.

There are two types of crankshafts—*side crankshaft* and *centre crankshaft* as shown in Fig. 25.19. The side crankshaft is also called the '*overhung*' crankshaft. It has only one crank web and requires only two bearings for support. It is used in medium-size engines and large-size horizontal engines. The centre crankshaft has two webs and three bearings for support. It is used in radial aircraft engines, stationary engines and marine engines. It is more popular in automotive engines. Crankshafts are also classified as single-throw and multi-throw crankshafts depending upon the number of crank pins used in the assembly. The crankshafts illustrated in Fig. 25.19 have one crank pin and are called single-throw crankshafts. Crankshafts used in multi-cylinder engines have more than one crank pin. They are called multi-throw crankshafts.

A crankshaft should have sufficient strength to withstand the bending and twisting moments to which it is subjected. In addition, it should have sufficient rigidity to keep the lateral and angular deflections within permissible limits. The crankshaft is subjected to fluctuating stresses and, as such, it should have sufficient endurance limit stress. Crankshafts are made by the drop forging process.



(a) Side crankshaft



(b) Centre crankshaft

Fig. 25.19 *Types of Crankshafts*

The popular materials used for crankshafts are plain-carbon steels and alloy steels. The plain carbon steels include 40C8, 45C8 and 50C4. The alloy steels used for making crankshafts are nickel–chromium steels such as 16Ni3Cr2, 35Ni5Cr2 and 40Ni10Cr3Mo6.

25.23 DESIGN OF CENTRE CRANKSHAFT

A crankshaft is subjected to bending and torsional moments due to the following three forces:

(i) Force exerted by the connecting rod on the crank pin

(ii) Weight of flywheel (W) acting downward in the vertical direction

(iii) Resultant belt tensions acting in the horizontal direction ($P_1 + P_2$)

In the design of the centre crankshaft, two cases of crank positions are considered. They are as follows:

Case I The crank is at the top dead centre position and subjected to maximum bending moment and no torsional moment.

Case II The crank is at an angle with the line of dead centre positions and subjected to maximum torsional moment.

We will consider these cases separately to determine the dimensions of the crankshaft.

25.24 CENTRE CRANKSHAFT AT TOP DEAD CENTRE POSITION

The forces acting on the centre crankshaft at the top dead centre position are shown in Fig. 25.20. The crankshaft is supported on three bearings 1, 2 and 3.

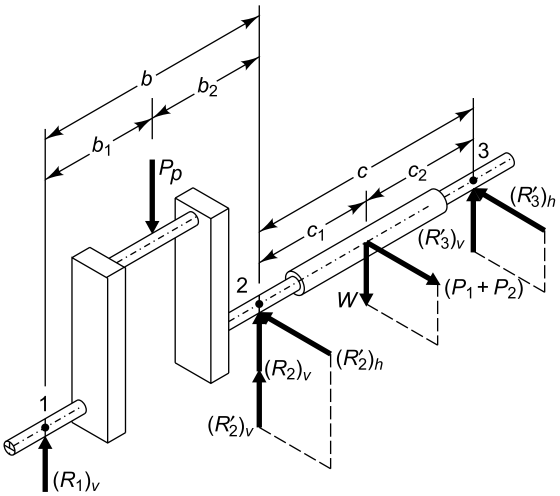


Fig. 25.20 *Centre Crankshaft at Dead Centre*

Assumptions

(i) The engine is vertical and the crank is at the top dead centre position.

(ii) The belt drive is horizontal.

(iii) The crankshaft is simply supported on bearings.

(i) Bearing Reactions

(a) The reactions at the bearings 1 and 2 due to force on the crank pin (P_p) are denoted by R_1 and R_2 followed by suffix letters v and h. The vertical component of reaction is denoted by the suffix letter v such as $(R_1)_v$. The horizontal component of reaction is denoted by the suffix letter h such as $(R_1)_h$.

(b) The reactions at the bearings 2 and 3 due to weight of the flywheel (W) and sum of the belt tensions ($P_1 + P_2$) are denoted by R_2' and R_3' followed by suffix letters v and h such as $(R_2')_v$ or $(R_3')_h$.

Suppose,

P_p = force acting on crank pin (N)
D = diameter of piston (mm)
$p_{max.}$ = maximum gas pressure inside the cylinder (MPa or N/mm^2)
W = weight of flywheel (N)
P_1 = tension in tight side of belt (N)
P_2 = tension in slack side of belt (N)
b = distance between main bearings 1 and 2
c = distance between bearings 2 and 3

At the top dead centre position, the thrust in the connecting rod will be equal to the force acting on piston.

$$P_p = \left(\frac{\pi D^2}{4}\right) p_{max.}$$

It is assumed that the portion of the crankshaft between bearings 1 and 2 is simply supported on bearings and subjected to force P_p. Taking moment of forces,

$$P_p \times b_1 = (R_2)_v \times b \quad \text{or} \quad (R_2)_v = \frac{P_p \times b_1}{b} \tag{a}$$

Similarly,

$$P_p \times b_2 = (R_1)_v \times b \quad \text{or} \quad (R_1)_v = \frac{P_p \times b_2}{b} \tag{b}$$

It is also assumed that the portion of the crankshaft between bearings 2 and 3 is simply supported on bearings and subjected to a vertical force W and horizontal force $(P_1 + P_2)$. Taking moment of forces,

$$W \times c_1 = (R_3')_v \times c \quad \text{or} \quad (R_3')_v = \frac{W \times c_1}{c} \tag{c}$$

$$W \times c_2 = (R_2')_v \times c \quad \text{or} \quad (R_2')_v = \frac{W \times c_2}{c} \tag{d}$$

$$(P_1 + P_2) \times c_1 = (R_3')_h \times c$$

or
$$(R_3')_h = \frac{(P_1 + P_2) \times c_1}{c} \tag{e}$$

$$(P_1 + P_2) \times c_2 = (R_2')_h \times c$$

or
$$(R_2')_h = \frac{(P_1 + P_2) \times c_2}{c} \tag{f}$$

The resultant reactions at the bearings are as follows:

$$R_1 = (R_1)_v$$

$$R_2 = \sqrt{[(R_2)_v + (R_2')_v]^2 + [(R_2')_h]^2}$$

$$R_3 = \sqrt{[(R_3')_v]^2 + [(R_3')_h]^2}$$

Note When the distance b between bearings 1 and 2 is not specified, it is assumed by the following empirical relationship:

$b = 2 \times$ piston diameter or $b = 2D$

(ii) Design of Crank Pin As shown in Fig. 25.21, the central plane of the crank pin is subjected to maximum bending moment. Suppose,

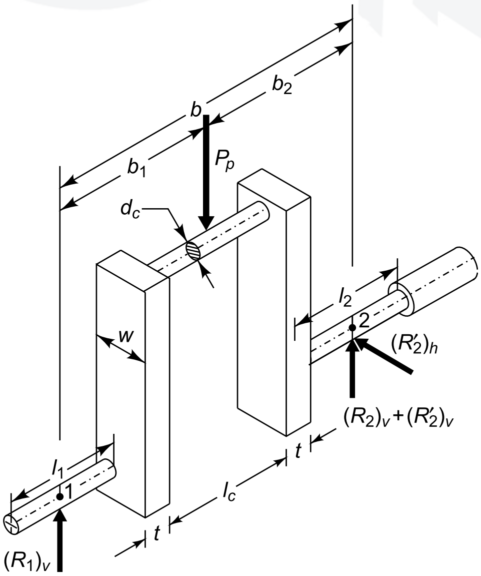


Fig. 25.21 *Crank Pin and Web*

d_c = diameter of crank pin (mm)
l_c = length of crank pin (mm)
σ_b = allowable bending stress for crank pin (N/mm^2)

The bending moment at the central plane is given by,

$$(M_b)_c = (R_1)_v\, b_1 \tag{25.68a}$$

$$I = \left(\frac{\pi d_c^4}{64}\right) \quad y = \left(\frac{d_c}{2}\right) \text{ and } \sigma_b = \frac{(M_b)_c\, y}{I}$$

Substituting,

$$(M_b)_c = \left(\frac{\pi d_c^3}{32}\right)\sigma_b \tag{25.68b}$$

The diameter of the crank pin can be determined from Eqs. (25.68a) and (25.68b).

Note In absence of data, the allowable bending stress for the crank pin can be assumed as 75 N/mm^2.

The length of the crank pin is determined by bearing consideration. Suppose,

p_b = allowable bearing pressure at the crank pin bush (MPa or N/mm^2)

$$p_b = \frac{P_p}{d_c l_c} \quad \text{or} \quad l_c \frac{P_p}{d_c p_b} \tag{25.68c}$$

(iii) Design of Left-hand Crank Web

Suppose,

w = width of crank web (mm)
t = thickness of crank web (mm)

The dimensions of crank web are calculated by empirical relationships and checked for the stresses. The empirical relationships are as follows:

$$t = 0.7\, d_c \tag{25.68 d}$$

$$w = 1.14\, d_c \tag{25.68 e}$$

where,

d_c = diameter of crank pin (mm)

As shown in Fig. 25.21, the left-hand crank web is subjected to eccentric load $(R_1)_v$. There are two types of stresses in the central plane of the crank web, viz., direct compressive stress and bending stress due to eccentricity of reaction $(R_1)_v$.

The direct compressive stress is given by,

$$\sigma_c = \frac{(R_1)_v}{wt} \tag{25.68f}$$

The bending moment at the central plane is given by,

$$M_b = (R_1)_v \left[b_1 - \frac{l_c}{2} - \frac{t}{2} \right]$$

$$I = \left(\frac{wt^3}{12} \right) \quad y = \left(\frac{t}{2} \right) \quad \text{and} \quad \sigma_b = \frac{M_b y}{I}$$

Substituting,

$$\sigma_b = \frac{(R_1)_v \left[b_1 - \frac{l_c}{2} - \frac{t}{2} \right] \left(\frac{t}{2} \right)}{\left(\frac{wt^3}{12} \right)}$$

$$= \frac{6(R_1)_v \left[b_1 - \frac{l_c}{2} - \frac{t}{2} \right]}{wt^2} \qquad (25.68g)$$

The total compressive stress is given by,

$$(\sigma_c)_t = \sigma_c + \sigma_b \qquad (25.68h)$$

The total compressive stress should be less than the allowable bending stress.

(iv) Design of Right-hand Crank Web The right-hand and left-hand webs should be identical from balancing considerations. Therefore, the thickness and width of the right-hand crank web are made equal to that of the left-hand crank web.

(v) Design of Shaft Under Flywheel The forces acting on the shaft under the flywheel are shown in Fig. 25.22. The central plane of the shaft is subjected to maximum bending moment. Suppose,

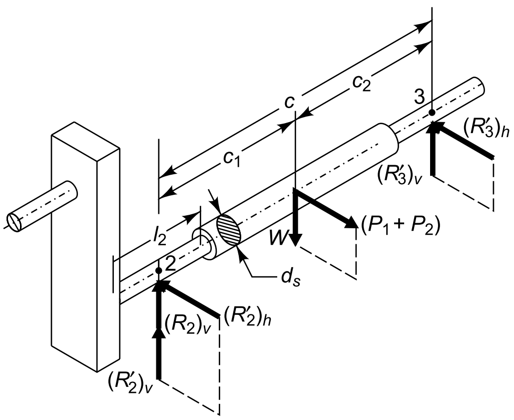


Fig. 25.22 *Shaft Under Flywheel*

d_s = diameter of shaft under flywheel (mm)

The bending moment in the vertical plane due to weight of flywheel is given by,

$$(M_b)_v = (R'_3)_v \, c_2$$

The bending moment in the horizontal plane due to resultant belt tension is given by,

$$(M_b)_h = (R'_3)_h \, c_2$$

The resultant bending moment is given by,

$$M_b = \sqrt{(M_b)_v^2 + (M_b)_h^2}$$

$$= \sqrt{[(R'_3)_v c_2]^2 + [(R'_3)_h c_2]^2} \qquad (25.68i)$$

Using similarity of Eq. (25.68b),

$$M_b = \left(\frac{\pi d_s^3}{32} \right) \sigma_b \qquad (25.68j)$$

From Eqs. (25.68i) and (25.68j), the diameter of shaft under flywheel (d_s) can be calculated.

25.25 CENTRE CRANKSHAFT AT ANGLE OF MAXIMUM TORQUE

(i) Components of Force on Crank Pin The position of the crank when it makes an angle (θ) with the line of dead centres is shown in Fig. 25.23. The torque is maximum when the tangential component of force on the crank pin is maximum. For this condition, the crank angle from the top dead centre position (θ) is usually 25° to 35° for petrol engines and 30° to 40° for diesel engines. In Fig. 25.23, the following notations are used:

P_p = force acting on piston top due to gas pressure (N)

P_q = thrust on connecting rod (N)

P_t = tangential component of force on crank pin (N)

P_r = radial component of force on crank pin (N)

φ = angle of inclination of connecting rod with the line of dead centres (deg)

θ = angle of inclination of crank with line of dead centres (deg)

Suppose p' is the gas pressure on the piston top for maximum torque condition.

$$P_p = \left(\frac{\pi D^2}{4} \right) p' \qquad (25.69a)$$

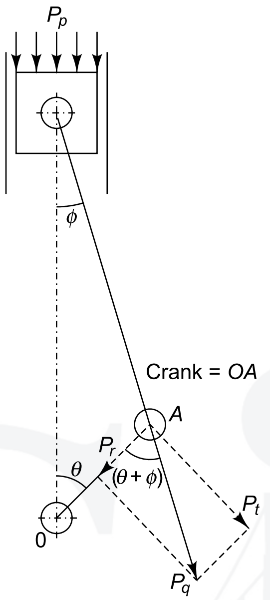


Fig. 25.23 *Force Acting on Crank*

The relationship between φ and θ is given by,

$$\sin\varphi = \frac{\sin\theta}{(L/r)} \tag{25.69b}$$

where (L/r) is the ratio of length of the connecting rod to the radius of the crank.

The thrust on the connecting rod (P_q) is given by,

$$P_q = \frac{P_p}{\cos\varphi} \tag{25.69c}$$

P_t and P_r are tangential and radial components of P_q at the crank pin. Therefore,

$$P_t = P_q \sin(\theta + \varphi) \tag{25.69d}$$

$$P_r = P_q \cos(\theta + \varphi) \tag{25.69e}$$

(ii) Bearing Reactions The forces acting on the centre crankshaft at an angle of maximum torque are shown in Fig. 25.24. The crankshaft is supported on three bearings 1, 2 and 3.

It is assumed that the portion of the crankshaft between bearings 1 and 2 is simply supported on bearings and subjected to tangential force P_t and radial force P_r at the crank pin as shown in Fig. 25.25. Due to the tangential component P_t, there are reactions $(R_1)_h$ and $(R_2)_h$ at bearings 1 and 2 respectively. Similarly, due to the radial component P_r, there are reactions $(R_1)_v$ and $(R_2)_v$ at bearings 1 and 2 respectively.

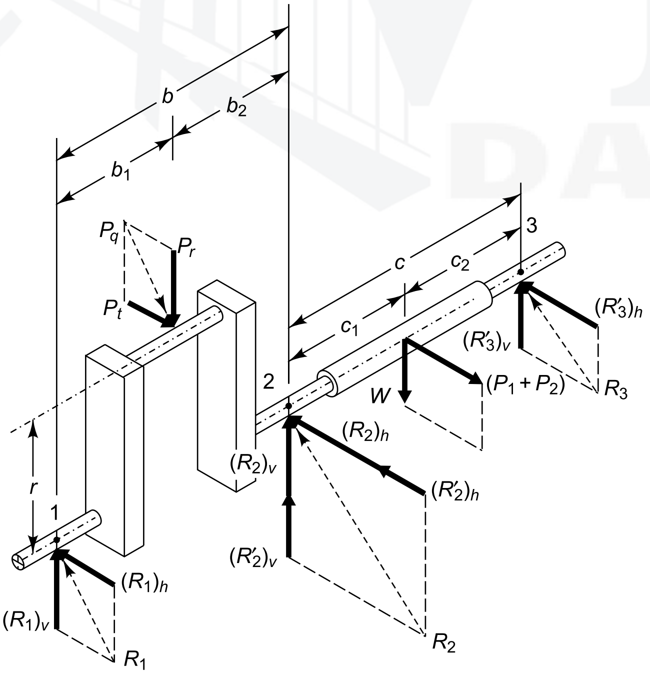


Fig. 25.24 *Centre Crankshaft at Angle of Maximum Torque*

Taking moment of horizontal forces about bearing 1,

$$P_t \times b_1 = (R_2)_h \times b \quad \text{or} \quad (R_2)_h = \frac{P_t \times b_1}{b} \qquad \text{(a)}$$

Taking moment of horizontal forces about the bearing 2,

$$P_t \times b_2 = (R_1)_h \times b \quad \text{or} \quad (R_1)_h = \frac{P_t \times b_2}{b} \qquad \text{(b)}$$

Similarly, it can be proved that

$$(R_2)_v = \frac{P_r \times b_1}{b} \qquad \text{(c)}$$

$$(R_1)_v = \frac{P_r \times b_2}{b} \qquad \text{(d)}$$

It is also assumed that the portion of the crankshaft between bearings 2 and 3 is simply supported on bearings and subjected to vertical force W and horizontal force $(P_1 + P_2)$. The reactions at bearings 2 and 3 due to the weight W and belt tensions $(P_1 + P_2)$ will be same as discussed in the previous article [Eqs. (c) to (f)]. They are rewritten here:

$$(R_3')_v = \frac{W \times c_1}{c} \qquad \text{(e)}$$

$$(R_2')_v = \frac{W \times c_2}{c} \qquad \text{(f)}$$

$$(R_3')_h = \frac{(P_1 + P_2) \times c_1}{c} \qquad \text{(g)}$$

$$(R_2')_h = \frac{(P_1 + P_2) \times c_2}{c} \qquad \text{(h)}$$

Referring to Fig. 25.24, the resultant reactions at the bearings are as follows:

$$R_1 = \sqrt{[(R_1)_v]^2 + [(R_1)_h]^2}$$

$$R_2 = \sqrt{[(R_2)_v + (R_2')_v]^2 + [(R_2)_h + (R_2')_h]^2} \qquad \text{(25.69f)}$$

$$R_3 = \sqrt{[(R_3')_v]^2 + [(R_3')_h]^2}$$

(iii) Design of Crank Pin As shown in Fig. 25.25, the central plane of the crank pin is subjected to bending moment M_b due to $(R_1)_v$ and torsional moment M_t due to $(R_1)_h$.

$$M_b = (R_1)_v \times b_1$$
$$M_t = (R_1)_h \times r$$

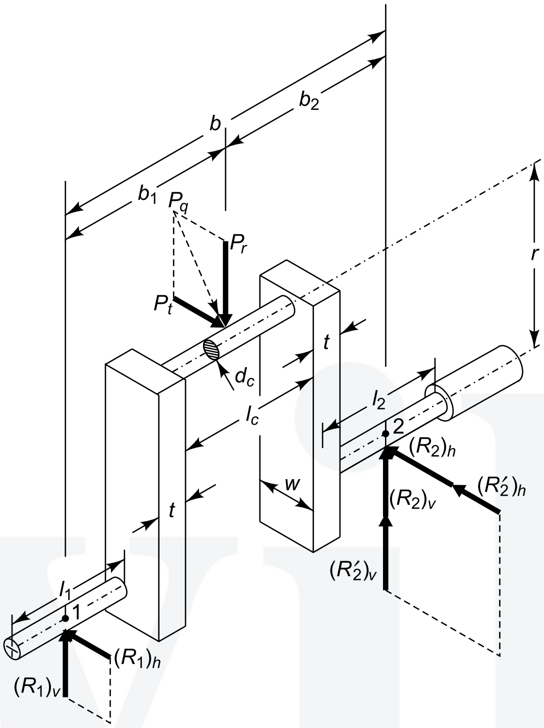


Fig. 25.25 *Crank Pin and Web*

The diameter of the crank pin (d_c) is calculated by following equation:

$$d_c^3 = \frac{16}{\pi \tau} \sqrt{(M_b)^2 + (M_t)^2}$$

$$d_c^3 = \frac{16}{\pi \tau} \sqrt{[(R_1)_v \times b_1]^2 + [(R_1)_h \times r]^2} \qquad \text{(25.69g)}$$

where (τ) is the allowable shear stress.

Note In absence of data, the allowable shear stress can be taken as 40 N/mm^2.

The length of the crank pin (l_c) is determined by bearing consideration. Suppose,

p_b = allowable bearing pressure at the crank pin bush (MPa or N/mm^2)

$$p_b = \frac{P_q}{d_c l_c} \quad \text{or} \quad l_c = \frac{P_q}{d_c p_b} \qquad \text{(25.69h)}$$

(iv) Design of Shaft Under Flywheel The forces acting on the shaft under the flywheel are shown in Fig. 25.24. Suppose,

d_s = diameter of shaft under flywheel (mm)

The central plane of the shaft is subjected to maximum bending moment due to the reaction R_3.

$$M_b = (R_3) \times c_2$$

It is also subjected to torsional moment M_t due to the tangential component P_t.

$$M_t = P_t \times r$$

The diameter of the shaft (d_s) is calculated by the following equation:

$$d_s^3 = \frac{16}{\pi \tau}\sqrt{(M_b)^2 + (M_t)^2}$$

$$d_s^3 = \frac{16}{\pi \tau}\sqrt{[R_3 \times c_2]^2 + [P_t \times r]^2} \qquad (25.69i)$$

where (τ) is the allowable shear stress.

(v) Design of Shaft at the Juncture of Right-Hand Crank Web Suppose,

d_{s1} = diameter of shaft at the juncture of right-hand crank web (mm)

The cross-section of shaft under flywheel at the juncture of the right-hand crank web is subjected to the following moments: [Fig. 25.25]

(i) Bending moment in vertical plane $(M_b)_v$ due to forces in vertical plane, viz., $(R_1)_v$ and P_r

(ii) Bending moment in horizontal plane $(M_b)_h$ due to forces in the horizontal plane, viz., $(R_1)_h$ and P_t

(iii) Torsional moment M_t due to tangential component P_t

$$(M_b)_v = (R_1)_v\left[b_1 + \frac{l_c}{2} + \frac{t}{2}\right] - P_r\left[\frac{l_c}{2} + \frac{t}{2}\right] \qquad (25.69j)$$

$$(M_b)_h = (R_1)_h\left[b_1 + \frac{l_c}{2} + \frac{t}{2}\right] - P_t\left[\frac{l_c}{2} + \frac{t}{2}\right] \qquad (25.69k)$$

$$M_t = P_t \times r \qquad (25.69l)$$

The resultant bending moment M_b is given by,

$$M_b = \sqrt{[(M_b)_v]^2 + [(M_b)_h]^2} \qquad (25.69m)$$

The diameter of shaft d_{s1} is calculated by the following expression:

$$d_{s1}^3 = \frac{16}{\pi \tau}\sqrt{(M_b)^2 + (M_t)^2} \qquad (25.69n)$$

where (τ) is the allowable shear stress.

The diameter of the shaft at the juncture of the right-hand crank web is calculated by using equations from (25.69j) to (25.69n).

(vi) Design of Right-Hand Crank Web As shown in Fig. 25.25, the right-hand crank web is subjected to the following stresses:

(i) Bending stresses in the vertical and horizontal planes due to radial component P_r and tangential component P_t respectively.

(ii) Direct compressive stress due to radial component P_r.

(iii) Torsional shear stresses.

The bending moment due to radial component is given by,

$$(M_b)_r = (R_2)_v\left[b_2 - \frac{l_c}{2} - \frac{t}{2}\right] \qquad (25.69o)$$

Since,

$$Z = \frac{1}{6}wt^2$$

Substituting,

$$(M_b)_r = (\sigma_b)_r \times Z = (\sigma_b)_r\left[\frac{1}{6}wt^2\right]$$

or
$$(M_b)_r = (\sigma_b)_r\left[\frac{1}{6}wt^2\right] \qquad (25.69p)$$

From Eqs. (25.69o) and (25.69p), the bending stress due to radial component is calculated.

The bending moment due to tangential component at the juncture of the crank web and shaft is given by,

$$(M_b)_t = P_t\left[r - \frac{d_{s1}}{2}\right] \qquad (25.69q)$$

where,

d_{s1} = diameter of shaft at the juncture of the right-hand crank web (mm)

Since,

$$Z = \frac{1}{6}tw^2$$

Substituting,

$$(M_b)_t = (\sigma_b)_t \times Z = (\sigma_b)_t\left[\frac{1}{6}tw^2\right]$$

or
$$(M_b)_t = (\sigma_b)_t\left[\frac{1}{6}tw^2\right] \qquad (25.69r)$$

From Eqs. (25.69q) and (25.69r), the bending stress due to tangential component is calculated.

The direct compressive stress due to radial component is given by,

$$(\sigma_c)_d = \frac{P_r}{2wt} \qquad (25.69s)$$

The maximum compressive stress (σ_c) is given by,

$$\sigma_c = (\sigma_b)_r + (\sigma_b)_t + (\sigma_c)_d \qquad (25.69t)$$

The torsional moment on the arm is given by,

$$M_t = (R_1)_h\left[b_1 + \frac{l_c}{2}\right] - P_t\left[\frac{l_c}{2}\right] = (R_2)_h\left[b_2 - \frac{l_c}{2}\right] \qquad (25.69u)$$

$$\tau = \frac{M_t}{Z_p} = \frac{4.5M_t}{wt^2} \qquad (25.69v)$$

where,

$$Z_p = \text{polar section modulus} = \frac{wt^2}{4.5}$$

The maximum compressive stress is given by,

$$(\sigma_c)_{\text{max.}} = \frac{\sigma_c}{2} + \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\tau^2} \qquad (25.69w)$$

The value of $(\sigma_c)_{\text{max.}}$ should be less than the allowable compressive stress. If it exceeds the allowable compressive stress, the width of the web w can be increased because it does nor affect the other calculations.

(vii) Design of Left-hand Crank Web The left-hand crank web is not severely stressed to the extent of the right-hand crank web. Therefore, it is not necessary to check the stresses in the left-hand crank web. The thickness and width of the left-hand crank web are made equal to that of the right-hand crank web from balancing consideration.

(viii) Design of Crankshaft Bearing Bearing 2 is subjected to maximum stress. The reaction at this bearing is given by Eq. (25.69f).

$$R_2 = \sqrt{[(R_2)_v + (R_2')_v]^2 + [(R_2)_h + (R_2')_h]^2}$$

The diameter of the journal at the bearing 2 is (d_{s1}). The length l_2 is calculated by bearing consideration. The bearing pressure is given by, [Fig. 25.25]

$$p_b = \frac{R_2}{d_{s1}\,l_2} \quad \text{or} \quad l_2 = \frac{R_2}{d_{s1}\,p_b} \qquad (25.69x)$$

Note The above-mentioned force analysis of the centre crankshaft is elementary in nature. The centre crankshaft is supported on three bearings and as such, it is a *statically 'indeterminate' structure*. Such problems are solved by using three equations, one for summation of vertical forces, one for summation of moments and the third by taking into consideration the deflection of the shaft. The geometry of the crankshaft is such that it is not possible to write down analytical equations. The problem is solved by using finite element method in practice.

Example 25.18 *Design a centre crankshaft for a single-cylinder vertical engine using the following data:*

Cylinder bore = 125 mm

(L/r) ratio = 4.5

Maximum gas pressure = 2.5 MPa

Length of stroke = 150 mm

Weight of flywheel cum belt pulley = 1 kN

Total belt pull = 2 kN

Width of hub for flywheel cum belt pulley = 200 mm

The torque on the crankshaft is maximum when the crank turns through 25° from the top dead centre and at this position the gas pressure inside the cylinder is 2 MPa. The belts are in the horizontal direction.

Assume suitable data and state the assumptions you make.

Solution

Given $D = 125$ mm $p_{\text{max.}} = 2.5$ MPa $= 2.5$ N/mm^2
$(L/r) = 4.5$ $l = 150$ mm $W = 1$ kN $= 1000$ N
$\theta = 25°$ $(P_1 + P_2) = 2$ kN $= 2000$ N
$p' = 2$ MPa $= 2$ N/mm^2

The crank radius r is given by,

$$r = \left(\frac{l}{2}\right) = \frac{150}{2} = 75 \text{ mm}$$

Case I The crank is at the top dead centre position and subjected to maximum bending moment and no torsional moment.

Step I *Bearing reactions*

At the top dead centre position, the thrust in the connecting rod will be equal to the force acting on the piston.

$$P_p = \left(\frac{\pi D^2}{4}\right) p_{\text{max.}} = \left(\frac{\pi(125)^2}{4}\right)(2.5)$$

$$= 30\,679.62 \text{ N}$$

Refer to Fig. 25.20. The distance b between bearings 1 and 2 is not specified.

Assumption 1 The centre to centre distance between the main bearings 1 and 2 is twice of the piston diameter. Therefore,

$$b = 2 \times \text{piston diameter}$$

or $$b = 2\,(125) = 250 \text{ mm}$$

It is further assumed that $b_1 = b_2$

Therefore,

$$b_1 = b_2 = \frac{b}{2} = \frac{250}{2} = 125 \text{ mm}$$

By symmetry,

$$(R_1)_v = (R_2)_v = \left(\frac{P_p}{2}\right) = \left(\frac{30\,679.62}{2}\right) = 15\,339.81 \text{ N}$$

Similarly, it is assumed that, $c_1 = c_2$

Therefore, by symmetry,

$$(R_2')_v = (R_3')_v = \left(\frac{W}{2}\right) = \left(\frac{1000}{2}\right) = 500 \text{ N}$$

$$(R_2')_h = (R_3')_h = \left(\frac{P_1 + P_2}{2}\right) = \left(\frac{2000}{2}\right) = 1000 \text{ N}$$

Step II *Design of crank pin*

Assumption 2 The allowable bending stress for the crank pin is 75 N/mm^2.

Assumption 3 The allowable bearing pressure for the crank pin bushing is 10 N/mm^2.

Suppose,

d_c = diameter of crank pin (mm)

l_c = length of crank pin (mm)

As shown in Fig. 25.21, the central plane of the crank pin is subjected to maximum bending moment.

The bending moment at the central plane is given by,

$$(M_b)_c = (R_1)_v\, b_1 = 15\,339.81(125)$$

$$= 1917.48 \times 10^3 \text{ N-mm}$$

$$(M_b)_c = \left(\frac{\pi d_c^3}{32}\right)\sigma_b$$

Substituting,

$$1917.48 \times 10^3 = \left(\frac{\pi d_c^3}{32}\right)(75)$$

or $$d_c^3 = 260.42 \times 10^3$$

$$d_c = 63.86 \text{ or } 65 \text{ mm} \qquad \text{(i)}$$

Assumption 4 The (l/d) ratio for the crank pin bearing is 1.

$$\left(\frac{l_c}{d_c}\right) = 1$$

$$l_c = d_c = 65 \text{ mm} \qquad \text{(ii)}$$

From Eq. (25.68c),

$$p_b = \frac{P_p}{d_c\, l_c} = \frac{30\,679.62}{(65)(65)} = 7.26 \text{ N/mm}^2$$

$$\therefore \quad p_b < 10 \text{ N/mm}^2$$

Step III *Design of left-hand crank web*

Suppose,

w = width of crank web (mm)

t = thickness of crank web (mm)

The dimensions of the crank web are calculated by empirical relationships and checked for the stresses. The empirical relationships are as follows:

$$t = 0.7\, d_c = 0.7(65) = 45.5 \text{ mm}$$

or $$t = 46 \text{ mm} \qquad \text{(iii)}$$

$$w = 1.14\, d_c = 1.14(65) = 74.1 \text{ mm}$$

or $$w = 75 \text{ mm} \qquad \text{(iv)}$$

As shown in Fig. 25.21, the left-hand crank web is subjected to eccentric load $(R_1)_v$. There are two types of stresses in the central plane of the crank web, viz., direct compressive stress and bending stress due to reaction $(R_1)_v$.

The direct compressive stress is given by,

$$\sigma_c = \frac{(R_1)_v}{wt} = \frac{15\,339.81}{(75)(46)} = 4.45 \text{ N/mm}^2$$

From Eq. (25.68g),

$$\sigma_b = \frac{6(R_1)_v\left[b_1 - \frac{l_c}{2} - \frac{t}{2}\right]}{wt^2}$$

$$= \frac{6(15\,339.81)\left[125 - \frac{65}{2} - \frac{46}{2}\right]}{75(46)^2}$$

$$= 40.31 \text{ N/mm}^2$$

The total compressive stress is given by,

$(\sigma_c)_t = \sigma_c + \sigma_b = 4.45 + 40.31 = 44.76$ N/mm^2

The total compressive stress is less than the allowable bending stress of 75 N/mm^2 and the design of the crank web is safe.

Step IV *Design of right-hand crank web*

The right-hand and left-hand webs are made identical from balancing considerations. Therefore, the thickness and width of the right-hand crank web are made equal to that of the left-hand crank web.

Step V *Design of shaft under flywheel*

The width of the hub for flywheel cum belt pulley is given as 200 mm. It is observed from Fig. 25.20 that the centre to centre distance between bearings 2 and 3 should be more than 200 mm to accommodate the bearings. We will assume,

$c = 200$ + margin for length of two bearings 2 and 3

$c = 200 + 100 = 300$ mm

$$c_1 = c_2 = \frac{c}{2} = \frac{300}{2} = 150 \text{ mm}$$

Suppose,

d_s = diameter of shaft under flywheel (mm)

The forces acting on the shaft under the flywheel are shown in Fig. 25.22. The central plane of the shaft is subjected to maximum bending moment. The bending moment in the vertical plane due to weight of the flywheel is given by,

$$(M_b)_v = (R'_3)_v\, c_2 = 500(150) = 75 \times 10^3 \text{ N-mm}$$

The bending moment in horizontal plane due to resultant belt tension is given by,

$$(M_b)_h = (R'_3)_h\, c_2 = 1000(150) = 150 \times 10^3 \text{ N-mm}$$

The resultant bending moment is given by,

$$M_b = \sqrt{(M_b)_v^2 + (M_b)_h^2}$$
$$= \sqrt{(75 \times 10^3)^2 + (150 \times 10^3)^2}$$
$$= 167.71 \times 10^3 \text{ N-mm}$$

The allowable bending stress is 75 N/mm^2

From Eq. (25.68j),

$$M_b = \left(\frac{\pi d_s^3}{32}\right)\sigma_b \quad \text{or} \quad 167.71 \times 10^3 = \left(\frac{\pi d_s^3}{32}\right)(75)$$

$d_s^3 = 22.78 \times 10^3$

$d_s = 28.35$ or 30 mm (v)

Case II The crank is at an angle with the line of the dead centre positions and subjected to maximum torsional moment.

Step I *Components of force on crank pin*

The crank angle (θ) for maximum torsional moment is given as 25°.

Since, p' is the gas pressure on the piston top for maximum torque condition.

$$P_p = \left(\frac{\pi D^2}{4}\right)p' = \left(\frac{\pi (125)^2}{4}\right)(2) = 24\,543.69 \text{ N}$$

$$\sin\varphi = \frac{\sin\theta}{(L/r)} = \frac{\sin(25)}{(4.5)} = 0.09392$$

$$\varphi = \sin^{-1}(0.09392) = 5.39°$$

The thrust on the connecting rod (P_q) is given by,

$$P_q = \frac{P_p}{\cos\varphi} = \frac{24\,543.69}{\cos(5.39)} = 24\,652.69 \text{ N}$$

P_t and P_r are the tangential and radial components of P_q at the crank pin. Therefore,

$P_t = P_q \sin(\theta + \varphi) = 24\ 652.69 \sin(25 + 5.39)$
$= 12\ 471.38$ N

$P_r = P_q \cos(\theta + \varphi) = 24\ 652.69 \cos(25 + 5.39)$
$= 21\ 265.46$ N

Step II *Bearing reactions*

The forces acting on the centre crankshaft at an angle of maximum torque are shown in Fig. 25.24. The crankshaft is supported on three bearings 1, 2 and 3. As decided in the previous case,

$b = 250$ mm and $c = 300$ mm

$$b_1 = b_2 = \frac{b}{2} = \frac{250}{2} = 125 \text{ mm}$$

$$c_1 = c_2 = \frac{c}{2} = \frac{300}{2} = 150 \text{ mm}$$

By symmetry,

$$(R_1)_v = (R_2)_v = \left(\frac{P_r}{2}\right) = \left(\frac{21\,265.46}{2}\right)$$
$$= 10\,632.73 \text{ N}$$

$$(R_1)_h = (R_2)_h = \left(\frac{P_t}{2}\right) = \left(\frac{12\,471.38}{2}\right)$$
$$= 6235.69 \text{ N}$$

$$(R_2')_v = (R_3')_v = \left(\frac{W}{2}\right) = \left(\frac{1000}{2}\right) = 500 \text{ N}$$

$$(R_2')_h = (R_3')_h = \left(\frac{P_1 + P_2}{2}\right) = \left(\frac{2000}{2}\right) = 1000 \text{ N}$$

Step III *Design of crank pin*

As shown in Fig. 25.25, the central plane of the crank pin is subjected to the bending moment M_b due to $(R_1)_v$ and torsional moment M_t due to $(R_1)_h$.

In absence of data, the allowable shear stress is taken as 40 N/mm^2.

The diameter of the crank pin (d_c) is calculated by (25.69g).

$$d_c^3 = \frac{16}{\pi\tau}\sqrt{(M_b)^2 + (M_t)^2}$$

$$= \frac{16}{\pi\tau}\sqrt{[(R_1)_v \times b_1]^2 + [(R_1)_h \times r]^2}$$

$$= \frac{16}{\pi(40)}\sqrt{[10\,632.73 \times 125]^2 + [6235.69 \times 75]^2}$$

$$= 179.4 \times 10^3$$

$$d_c = 56.4 \text{ mm}$$

In calculations of previous case, the diameter (d_c) is 65 mm and the length of the crank pin (l_c) is 65 mm. Since the diameter is more, the first case is the criterion of deciding the diameter of the crank pin. Therefore,

$$d_c = l_c = 65 \text{ mm}$$

Step IV *Design of shaft under flywheel*

The forces acting on the shaft under the flywheel are shown in Fig. 25.24. Suppose,

d_s = diameter of shaft under flywheel (mm)

The central plane of the shaft is subjected to maximum bending moment due to the reaction R_3.

$$M_b = (R_3) \times c_2$$

It is also subjected to torsional moment M_t due to tangential component P_t.

$$M_t = P_t \times r$$

$$R_3 = \sqrt{[(R_3')_v]^2 + [(R_3')_h]^2}$$

$$= \sqrt{(500)^2 + (1000)^2} = 1118.03 \text{ N}$$

The diameter of the shaft (d_s) is calculated by the following equation:

$$d_s^3 = \frac{16}{\pi\tau}\sqrt{(M_b)^2 + (M_t)^2}$$

$$= \frac{16}{\pi\tau}\sqrt{[R_3 \times c_2]^2 + [P_t \times r]^2}$$

$$= \frac{16}{\pi(40)}\sqrt{[(1118.03) \times (150)]^2 + [(12\,471.38) \times (75)]^2}$$

$$= 120.99 \times 10^3$$

$$d_s = 49.46 \text{ mm or } 50 \text{ mm}$$

In calculations of the previous case, the diameter (d_s) is 30 mm. Since the diameter is less, the second case is the criterion of deciding the diameter of the shaft under flywheel. Therefore,

$$d_s = 50 \text{ mm}$$

Step V *Design of shaft at the juncture of right-hand crank web*

Suppose,

d_{s1} = diameter of the shaft at the juncture of the right-hand crank web (mm)

Refer to Fig. 25.25. The cross-section of the shaft under flywheel at the juncture of the right-hand crank web is subjected to the following moments:

(i) Bending moment in vertical plane $(M_b)_v$ due to forces in vertical plane, viz., $(R_1)_v$ and P_r

(ii) Bending moment in horizontal plane $(M_b)_h$ due to forces in horizontal plane, viz., $(R_1)_h$ and P_t

(iii) Torsional moment M_t due to tangential component P_t

$$(M_b)_v = (R_1)_v\left[b_1 + \frac{l_c}{2} + \frac{t}{2}\right] - P_r\left[\frac{l_c}{2} + \frac{t}{2}\right]$$

$$= (10\,632.73)\left[125 + \frac{65}{2} + \frac{46}{2}\right]$$

$$-(21265.46)\left[\frac{65}{2} + \frac{46}{2}\right]$$

$$= 738.97 \times 10^3 \text{ N-mm}$$

$$(M_b)_h = (R_1)_h\left[b_1 + \frac{l_c}{2} + \frac{t}{2}\right] - P_t\left[\frac{l_c}{2} + \frac{t}{2}\right]$$

$$= (6235.69)\left[125 + \frac{65}{2} + \frac{46}{2}\right] - (12\,471.38)\left[\frac{65}{2} + \frac{46}{2}\right]$$

$$= 433.38 \times 10^3 \text{ N-mm}$$

The resultant bending moment M_b is given by,

$$M_b = \sqrt{[(M_b)_v]^2 + [(M_b)_h]^2}$$

$$= \sqrt{[738.97 \times 10^3]^2 + [433.38 \times 10^3]^2}$$

$$= 856.68 \times 10^3 \text{ N-mm}$$

$$M_t = P_t \times r = 12\,471.38 \times 75$$
$$= 935.36 \times 10^3 \text{ N-mm}$$

The diameter of the shaft d_{s1} is calculated by the following expression:

$$d_{s1}^3 = \frac{16}{\pi \tau}\sqrt{(M_b)^2 + (M_t)^2}$$

$$= \frac{16}{\pi(40)}\sqrt{(856.68 \times 10^3)^2 + (935.36 \times 10^3)^2}$$

$$= 161.5 \times 10^3$$

$$d_{s1} = 54.46 \text{ or } 55 \text{ mm}$$

Step VI *Design of right-hand crank web*

As shown in Fig. 25.25, the right-hand crank web is subjected to the following stresses:

(i) Bending stresses in vertical and horizontal planes due to radial component P_r and tangential component P_t respectively.

(ii) Direct compressive stress due to radial component P_r.

(iii) Torsional shear stresses.

The bending moment due to the radial component is given by,

$$(M_b)_r = (R_2)_v\left[b_2 - \frac{l_c}{2} - \frac{t}{2}\right]$$

$$= (10\,632.73)\left[125 - \frac{65}{2} - \frac{46}{2}\right]$$

$$= 738.97 \times 10^3 \text{ N-mm}$$

Also,

$$(M_b)_r = (\sigma_b)_r\left[\frac{1}{6}wt^2\right]$$

$$738.97 \times 10^3 = (\sigma_b)_r\left[\frac{1}{6}(75)(46)^2\right]$$

$$(\sigma_b)_r = 27.94 \text{ N/mm}^2 \quad \text{(a)}$$

The bending moment due to tangential component at the juncture of the crank web and shaft is given by,

$$(M_b)_t = P_t\left[r - \frac{d_{s1}}{2}\right] = (12471.38)\left[75 - \frac{55}{2}\right]$$

$$= 592.39 \times 10^3 \text{ N-mm}$$

Also,

$$(M_b)_t = (\sigma_b)_t\left[\frac{1}{6}tw^2\right]$$

or

$$592.39 \times 10^3 = (\sigma_b)_t\left[\frac{1}{6}(46)(75)^2\right]$$

$$(\sigma_b)_t = 13.74 \text{ N/mm}^2 \quad \text{(b)}$$

The direct compressive stress due to radial component is given by,

$$(\sigma_c)_d = \frac{P_r}{2wt} = \frac{21265.46}{2(75)(46)} = 3.08 \text{ N/mm}^2 \quad \text{(c)}$$

The maximum compressive stress (σ_c) is given by,

$$\sigma_c = (\sigma_b)_r + (\sigma_b)_t + (\sigma_c)_d$$
$$= 27.94 + 13.74 + 3.08 = 44.76 \text{ N/mm}^2 \quad \text{(d)}$$

The torsional moment on the arm is given by,

$$M_t = (R_2)_h\left[b_2 - \frac{l_c}{2}\right] = (6235.69)\left[125 - \frac{65}{2}\right]$$

$$= 576.80 \times 10^3 \text{ N-mm}$$

$$\tau = \frac{M_t}{Z_p} = \frac{4.5M_t}{wt^2} = \frac{4.5(576.80 \times 10^3)}{(75)(46)^2}$$

$$= 16.36 \text{ N/mm}^2$$

The maximum compressive stress is given by,

$$(\sigma_c)_{max.} = \frac{1}{2}\left[\sigma_c + \sqrt{(\sigma_c)^2 + 4\tau^2}\right]$$

$$(\sigma_c)_{max.} = \frac{1}{2}\left[44.76 + \sqrt{(44.76)^2 + 4(16.36)^2}\right]$$

$$= 50.10 \text{ N/mm}^2$$

The above value of $(\sigma_c)_{max.}$ is less than the allowable compressive stress (75 N/mm^2) and the design is safe.

Step VII *Design of left-hand crank web*

The left-hand crank web is not severely stressed to the extent of the right-hand crank web. Therefore, it is not necessary to check the stresses in the left-hand crank web. The thickness and width of the left-hand crank web are made equal to that of the right-hand crank web from balancing consideration.

Step VIII *Design of crankshaft bearing*

Bearing 2 is subjected to maximum stress. The reaction at this bearing is given by Eq. (25.69f).

$$R_2 = \sqrt{[(R_2)_v + (R_2')_v]^2 + [(R_2)_h + (R_2')_h]^2}$$

$$= \sqrt{[10\,632.73 + 500]^2 + [6235.69 + 1000]^2}$$

$$= 13\,277.53 \text{ N}$$

The diameter of the journal at the bearing 2 is (d_{s1}).

$$d_{s1} = 55 \text{ mm}$$

The (l/d) ratio for bearing is assumed as 1.

$$l_2 = d_{s1} = 55 \text{ mm}$$

The bearing pressure is given by,

$$p_b = \frac{R_2}{d_{s1}\, l_2} = \frac{13\,277.53}{(55)(55)} = 4.39 \text{ N/mm}^2$$

$$\therefore \quad p_b < 10 \text{ N/mm}^2$$

The bearing pressure is within the limits and design is safe.

25.26 SIDE CRANKSHAFT AT TOP-DEAD CENTRE POSITION

The forces acting on a side crankshaft at the top dead centre position are shown in Fig. 25.26. The crankshaft is supported on two bearings, 1 and 2.

Assumptions

(i) The engine is vertical and the crank is at the top dead centre position.

(ii) The belt drive is horizontal.

(iii) The crankshaft is simply supported on bearings 1 and 2.

(i) Bearing Reactions

(a) The reactions at the bearings 1 and 2 due to force on the crank pin (P_p) are denoted by $(R_1)_v$ and $(R_2)_v$.

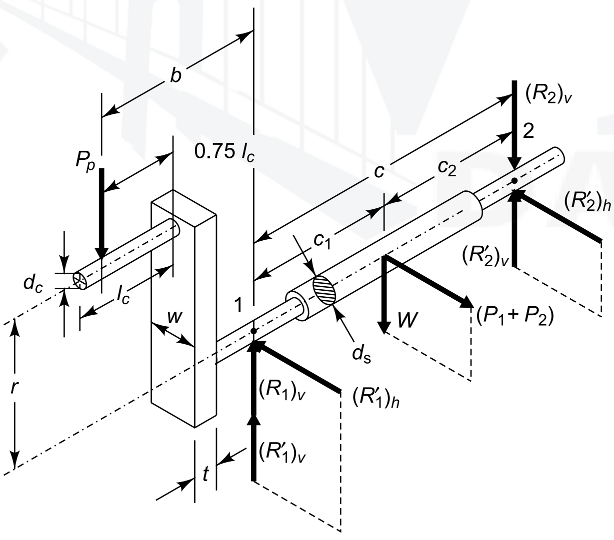


Fig. 25.26 *Side Crankshaft at Dead Centre*

(b) The reactions at the bearings 1 and 2 due to weight of the flywheel (W) and sum of belt tensions ($P_1 + P_2$) are denoted by R_1' and R_2' followed by suffix letters v and h such as $(R_1')_v$ or $(R_1')_h$. The suffix v is used for vertical component and suffix h for horizontal component of reactions.

Suppose,

P_p = force acting on crank pin (N)
D = diameter of piston (mm)
$p_{max.}$ = maximum gas pressure inside the cylinder (MPa or N/mm^2)
W = weight of flywheel cum belt pulley (N)
P_1 = tension in tight side of belt (N)
P_2 = tension in slack side of belt (N)
b = overhang distance of force P_p from bearing 1
c = centre distance between bearings 1 and 2

At the top dead centre position, the thrust in the connecting rod will be equal to the force acting on the piston.

$$P_p = \left(\frac{\pi D^2}{4}\right) p_{max.} \qquad (25.70a)$$

The forces acting on the crankshaft in the vertical and horizontal planes are shown in Fig. 25.27.

Consider the reactions at bearings due to force P_p [Fig. 25.27(a)]. Taking moment of forces about bearing 2,

$$P_p \times (b + c) = (R_1)_v \times c$$

or
$$(R_1)_v = \frac{P_p \times (b+c)}{c} \qquad (a)$$

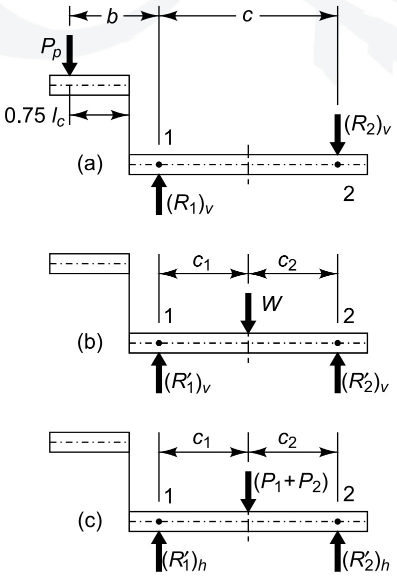


Fig. 25.27 *Side crankshaft-bearing reactions*

Similarly, by taking moment of forces about bearing 1,

$$P_p \times b = (R_2)_v \times c$$

or
$$(R_2)_v = \frac{P_p \times b}{c} \qquad (b)$$

Consider the reactions at bearings due to weight W [Fig. 25.27(b)]. Taking moment of forces,

$$W \times c_1 = (R'_2)_v \times c$$

or
$$(R'_2)_v = \frac{W \times c_1}{c} \qquad (c)$$

$$W \times c_2 = (R'_1)_v \times c$$

or
$$(R'_1)_v = \frac{W \times c_2}{c} \qquad (d)$$

Consider the reactions at bearings due to belt pull $(P_1 + P_2)$ [Fig. 25.27(c)].

$$(P_1 + P_2) \times c_1 = (R'_2)_h \times c$$

or
$$(R'_2)_h = \frac{(P_1 + P_2) \times c_1}{c} \qquad (e)$$

$$(P_1 + P_2) \times c_2 = (R'_1)_h \times c$$

or
$$(R'_1)_h = \frac{(P_1 + P_2) \times c_2}{c} \qquad (f)$$

(ii) Design of Crank Pin

Suppose,

d_c = diameter of crank pin (mm)
l_c = length of crank pin (mm)
σ_b = allowable bending stress for crank pin (N/mm^2)
p_b = allowable bearing pressure at the crank pin bush (MPa or N/mm^2)

The bearing pressure at the crank pin is given by,

$$p_b = \frac{P_p}{d_c\, l_c} \qquad (25.70b)$$

The (l/d) ratio for the crank pin is usually from 0.60 to 1.4.

$$\left(\frac{l_c}{d_c}\right) = 0.60 \text{ to } 1.4 \qquad (25.70c)$$

The allowable bearing pressure at the crank pin is generally assumed from 10 to 12 MPa.

$$p_b = 10 \text{ to } 12 \text{ N/mm}^2$$

Assuming the above value of permissible bearing pressure, the dimensions d_c and l_c are calculated by using Eq. (25.70b) and Eq. (25.70c).

After calculating the dimensions, it is necessary to check the bending stresses in the crank pin.

As shown in Fig. 25.26, the crank pin acts as a cantilever and subjected to bending stresses. If it is assumed that the load (P_p) is uniformly distributed on the length of the crank pin, the maximum bending moment is given by,

$$M_b = P_p\left(\frac{l_c}{2}\right) \qquad \text{(a)}$$

On the other hand, if we assume that the load is not uniformly distributed and located at the end of the crank pin, the maximum bending moment is given by,

$$M_b = P_p l_c \qquad \text{(b)}$$

Equations (a) and (b) indicate limiting values in two conditions. We will assume a mean value in calculations. Therefore,

$$M_b = \left(\frac{3}{4}\right) P_p l_c \qquad (25.70d)$$

In other words, we assume that the load (P_p) acts at a distance of ($0.75\ l_c$) from the crank web.

The section modulus for the crank pin is given by,

$$Z = \frac{\pi\, d_c^3}{32}$$

Therefore,

$$M_b = \sigma_b\left(\frac{\pi\, d_c^3}{32}\right) \qquad (25.70e)$$

The bending stresses in the crank pin are calculated by using Eqs. (25.70d) and (25.70e). They should be within the permissible limit.

(iii) Design of Bearings

Suppose,

d_1 = diameter of journal or shaft at bearing 1 (mm)

l_1 = length of bearing 1 (mm)

σ_b = allowable bending stress for shaft at bearing (N/mm^2)

t = thickness of web (mm)

The thickness of the web is calculated by the following empirical relationship:

$$t = 0.45\ d_c \text{ to } 0.75\ d_c \qquad (25.70f)$$

Refer to Fig. 25.26. The bending moment at the bearing 1 is given by,

$$M_b = P_p\,[0.75 l_c + t + 0.5 l_1] \qquad (25.70g)$$

The length of the bearing is calculated by the following empirical relationship:

$$l_1 = 1.5 d_c \text{ to } 2 d_c \qquad (25.70h)$$

The bending stress in the shaft of the bearing 1 is given by,

$$M_b = \sigma_b\left(\frac{\pi d_1^3}{32}\right) \qquad (25.70i)$$

The diameter of the shaft at the bearing 1 is calculated by using Eqs. (25.70g) and (25.70i).

The dimensions d_1 and l_1 are to be checked for bearing pressure. The bearing pressure is given by,

$$p_b = \frac{R_1}{d_1\, l_1} \qquad (25.70j)$$

The allowable bearing pressure at the bearing is generally assumed from 10 to 12 MPa.

$$p_b = 10 \text{ to } 12 \text{ N/mm}^2$$

The reaction R_1 is given by,

$$R_1 = \sqrt{[(R_1)_v + (R_1')_v]^2 + [(R_1')_h]^2} \qquad (25.70k)$$

Bearings 1 and 2 are made identical.

(iv) Design of Crank Web

Suppose,

w = width of crank web (mm)

As shown in Fig. 25.26, the crank web is subjected to eccentric load P_p. There are two types of stresses in the central plane of the crank web, viz., direct compressive stress and bending stress due to P_p.

The direct compressive stress is given by,

$$\sigma_c = \frac{P_P}{wt} \qquad (25.70l)$$

The bending moment at the central plane is given by,

$$M_b = P_P\left[0.75\, l_c + \frac{t}{2}\right] \qquad (25.70m)$$

$$Z = \left(\frac{wt^2}{6}\right) \quad \text{and} \quad \sigma_b = \frac{M_b}{Z}$$

The total compressive stress is given by,

$$(\sigma_c)_t = \sigma_c + \sigma_b \tag{25.70n}$$

The total compressive stress should be less than the allowable bending stress.

(v) Design of Shaft Under Flywheel The forces acting on the shaft under the flywheel are shown in Fig. 25.26. The central plane of the shaft is subjected to maximum bending moment. Suppose,

d_s = diameter of shaft under flywheel (mm)

The bending moment in vertical plane is given by,

$$(M_b)_v = P_p(b + c_1) - [(R_1)_v + (R_1')_v](c_1) \tag{25.70o}$$

The bending moment in the horizontal plane is given by,

$$(M_b)_h = (R_1')_h c_1 \tag{25.70p}$$

The resultant bending moment is given by,

$$M_b = \sqrt{(M_b)_v^2 + (M_b)_h^2} \tag{25.70q}$$

Also,

$$M_b = \left(\frac{\pi d_s^3}{32}\right)\sigma_b \tag{25.70r}$$

The diameter of the shaft under the flywheel (d_s) can be calculated by using equations from (25.70o) to (25.70r).

25.27 SIDE CRANKSHAFT AT ANGLE OF MAXIMUM TORQUE

The torque is maximum when the tangential component of the force on the crank pin is maximum. For this condition, the crank angle from the top dead centre position (θ) is usually 25° to 35° for petrol engines and 30° to 40° for diesel engines.

Assumptions

(i) The engine is vertical.

(ii) The belt drive is horizontal.

(iii) The crankshaft is simply supported on bearings 1 and 2.

Suppose,

P_p = force acting on piston top due to gas pressure (N)

P_q = thrust on connecting rod (N)

P_t = tangential component of force on crank pin (N)

P_r = radial component of force on crank pin (N)

φ = angle of inclination of connecting rod with the line of dead centres (deg)

θ = angle of inclination of crank with line of dead centres (deg)

(i) Components of Force on Crank Pin Suppose p' is the gas pressure on the piston top for maximum torque condition.

$$P_p = \left(\frac{\pi D^2}{4}\right)p' \tag{25.71a}$$

The relationship between φ and θ is given by,

$$\sin\varphi = \frac{\sin\theta}{(L/r)} \tag{25.71b}$$

where (L/r) is ratio of length of the connecting rod to the radius of the crank.

The thrust on the connecting rod (P_q) is given by,

$$P_q = \frac{P_p}{\cos\varphi} \tag{25.71c}$$

P_t and P_r are tangential and radial components of P_q at the crank pin. Therefore,

$$P_t = P_q \sin(\theta + \varphi) \tag{25.71d}$$

$$P_r = P_q \cos(\theta + \varphi) \tag{25.71e}$$

(ii) Bearing Reactions The forces acting on the side crankshaft at an angle of maximum torque are shown in Fig. 25.28. The crankshaft is supported on two bearings, 1 and 2.

The forces acting on the crankshaft in vertical and horizontal planes are shown in Fig. 25.29.

(a) Forces in vertical plane [*Fig. 25.29(a)*]

(i) Reactions of P_r

$(R_1)_v$ and $(R_2)_v$ are the reactions at bearings due to the radial component P_r. Taking moment of forces about bearing 2,

$$P_r \times (b + c) = (R_1)_v \times c$$

or
$$(R_1)_v = \frac{P_r \times (b + c)}{c} \tag{a}$$

Taking moment of forces about the bearing 1,

$$P_r \times b = (R_2)_v \times c$$

or
$$(R_2)_v = \frac{P_r \times b}{c} \tag{b}$$

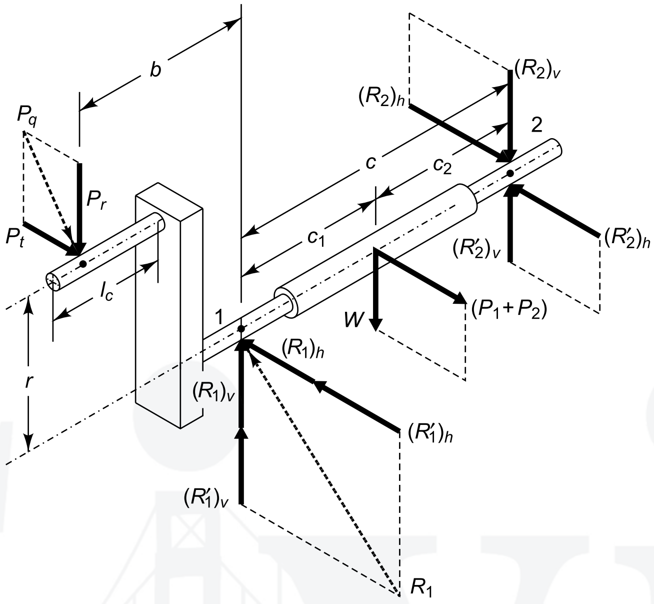


Fig. 25.28 *Side Crankshaft at Angle of Maximum Torque*

(ii) Reactions of W

$(R_1')_v$ and $(R_2')_v$ are the reactions at bearings due to the weight W. Taking moment of forces about the bearing 2,

$$W \times c_2 = (R_1')_v \times c$$

or $$(R_1')_v = \frac{W \times c_2}{c} \quad \text{(c)}$$

Taking moment of forces about the bearing 1,

$$W \times c_1 = (R_2')_v \times c$$

or $$(R_2')_v = \frac{W \times c_1}{c} \quad \text{(d)}$$

(b) Forces in horizontal plane [*Fig. 25.29(b)*]

(iii) Reactions of P_t

$(R_1)_h$ and $(R_2)_h$ are the reactions at bearings due to tangential component P_t. Taking moment of forces about the bearing 2,

$$P_t \times (b + c) = (R_1)_h \times c$$

or $$(R_1)_h = \frac{P_t \times (b + c)}{c} \quad \text{(e)}$$

Taking moment of forces about the bearing 1,

$$P_t \times b = (R_2)_h \times c$$

or $$(R_2)_h = \frac{P_t \times b}{c} \quad \text{(f)}$$

(iv) Reactions of $(P_1 + P_2)$

$(R_1')_h$ and $(R_2')_h$ are the reactions at bearings due to belt pull $(P_1 + P_2)$. Taking moment of forces about the bearing 2,

$$(P_1 + P_2) \times c_2 = (R_1')_h \times c$$

or $$(R_1')_h = \frac{(P_1 + P_2) \times c_2}{c} \quad \text{(g)}$$

Taking moment of forces about bearing 1,

$$(P_1 + P_2) \times c_1 = (R_1')_h \times c$$

or $$(R_2')_h = \frac{(P_1 + P_2) \times c_1}{c} \quad \text{(h)}$$

(iii) Design of Crank Pin

Check for bending stresses

It was discussed in the previous article that the load on the crank pin acts at a distance of $(0.75 l_c)$ from the crank web. We will assume the force components P_r and P_t act at a distance of $(0.75 l_c)$ from the crank web. The crank pin acts as a cantilever and subjected to maximum bending moment at the crank web as shown in Fig. 25.28. The bending moment in the vertical plane is given by,

$$(M_b)_v = P_r \times (0.75 l_c) \quad \text{(25.71f)}$$

The bending moment in the horizontal plane is given by,

$$(M_b)_h = P_t \times (0.75 l_c) \qquad (25.71g)$$

The resultant bending moment M_b is given by,

$$M_b = \sqrt{[(M_b)_v]^2 + [(M_b)_h]^2} \qquad (25.71h)$$

The section modulus for the crank pin is given by,

$$Z = \frac{\pi d_c^3}{32}$$

Therefore,

$$M_b = \sigma_b \left(\frac{\pi d_c^3}{32} \right) \qquad (25.71i)$$

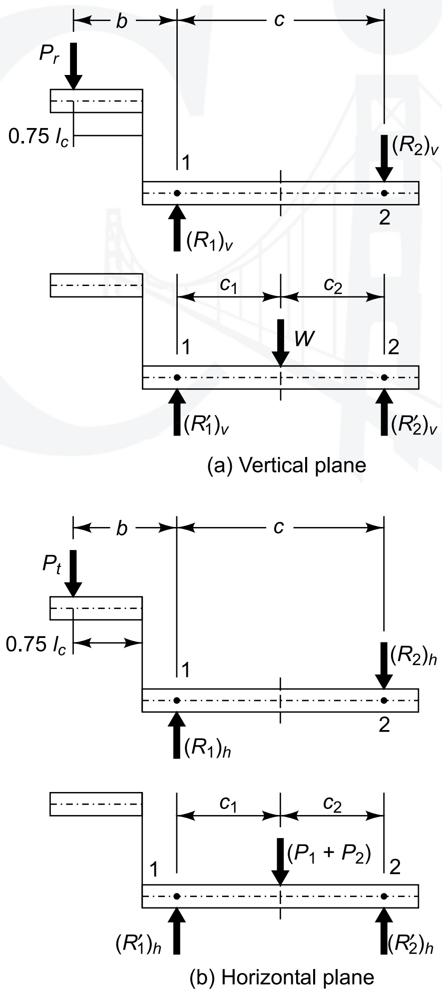


Fig. 25.29 *Side Crankshaft-bearing Reactions*

The bending stresses in the crank pin are calculated by using equations from (25.71f) and (25.71i). They should be within the permissible limit.

(iv) Design of Crank Web

Check for bending stresses

As shown in Fig. 25.28, the crank web is subjected to the following stresses:

(a) Bending stresses in vertical and horizontal planes due to radial component P_r and tangential component P_t respectively

(b) Direct compressive stress due to radial component P_r

(c) Torsional shear stresses due to tangential component P_t

The bending moment due to radial component at the central plane is given by,

$$(M_b)_r = P_r\, [0.75 l_c + 0.5t] \qquad (25.71j)$$

Since,

$$Z = \frac{1}{6} wt^2$$

Substituting,

$$(M_b)_r = (\sigma_b)_r \times Z = (\sigma_b)_r \left[\frac{1}{6} w t^2 \right]$$

or

$$(M_b)_r = (\sigma_b)_r \left[\frac{1}{6} w t^2 \right] \qquad (25.71k)$$

The bending stress due to radial component is calculated by using Eqs. (25.71j) and (25.71k). The bending moment due to tangential component at the juncture of the crank web and shaft is given by,

$$(M_b)_t = P_t \left[r - \frac{d_1}{2} \right] \qquad (25.71l)$$

where,

d_1 = diameter of shaft at the juncture of the crank web (mm)

Since,

$$Z = \frac{1}{6} tw^2$$

Substituting,

$$(M_b)_t = (\sigma_b)_t \times Z = (\sigma_b)_t \left[\frac{1}{6} t\, w^2 \right]$$

or

$$(M_b)_t = (\sigma_b)_t \left[\frac{1}{6} t\, w^2 \right] \qquad (25.71m)$$

The bending stress due to tangential component is calculated by Eqs. (25.71l) and (25.71m). The direct compressive stress due to radial component is given by,

$$(\sigma_c)_d = \frac{P_r}{wt} \quad (25.71n)$$

The maximum compressive stress (σ_c) is given by,

$$\sigma_c = (\sigma_b)_r + (\sigma_b)_t + (\sigma_c)_d \quad (25.71o)$$

The torsional moment on the arm is given by,

$$M_t = P_t\,[0.75 l_c + 0.5t] \quad (25.71p)$$

$$\tau = \frac{M_t}{Z_p} = \frac{4.5\,M_t}{wt^2} \quad (25.71q)$$

where,

Z_p = polar section modulus = $\dfrac{wt^2}{4.5}$

The maximum compressive stress is given by,

$$(\sigma_c)_{\max.} = \frac{\sigma_c}{2} + \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\tau^2} \quad (25.71r)$$

The value of $(\sigma_c)_{\max.}$ should be less than the allowable compressive stress. If it exceeds the allowable compressive stress, the width of the web w can be increased because it does not affect the other calculations.

(v) Design of Shaft at the Juncture of Crank Web

Check for torsional shear stresses

Suppose,

d_{s1} = diameter of shaft at the juncture of crank web (mm)

The cross-section of the shaft at the juncture of the crank web is subjected to the following moments:

(a) Bending moment in vertical plane $(M_b)_v$ due to P_r

(b) Bending moment in horizontal plane $(M_b)_h$ due to P_t

(c) Torsional moment M_t due to tangential component P_t

$$(M_b)_v = P_r\,[0.75 l_c + t] \quad (25.71s)$$

$$(M_b)_h = P_t\,[0.75 l_c + t] \quad (25.71t)$$

$$M_t = P_t \times r \quad (25.71u)$$

The resultant bending moment M_b is given by,

$$M_b = \sqrt{[(M_b)_v]^2 + [(M_b)_h]^2}$$

The torsional shear stresses in the shaft are calculated by the following expression:

$$\tau = \frac{16}{\pi d_{s1}^3}\sqrt{(M_b)^2 + (M_t)^2} \quad (25.71v)$$

where (τ) is the allowable shear stress.

(vi) Design of Shaft Under Flywheel

Check for torsional shear stresses

The forces acting on shaft under the flywheel are shown in Fig. 25.28. Suppose,

d_s = diameter of shaft under flywheel (mm)

The central plane of the shaft is subjected to maximum bending moment. The bending moment in vertical plane $(M_b)_v$ is given by,

$$(M_b)_v = P_r\,(b + c_1) - [(R_1)_v + (R_1')_v]c_1 \quad (25.71w)$$

The bending moment in the horizontal plane $(M_b)_h$ is given by,

$$(M_b)_h = P_t\,(b + c_1) - [(R_1)_h + (R_1')_h]c_1 \quad (25.71x)$$

The resultant bending moment M_b is given by,

$$M_b = \sqrt{[(M_b)_v]^2 + [(M_b)_h]^2}$$

The shaft is also subjected to torsional moment M_t due to the tangential component P_t.

$$M_t = P_t \times r \quad (25.71y)$$

The torsional shear stress in the shaft is calculated by the following equation:

$$\tau = \frac{16}{\pi d_s^3}\sqrt{(M_b)^2 + (M_t)^2} \quad (25.71z)$$

where (τ) is the allowable shear stress.

Example 25.19 *Design an overhung crankshaft for a 300 × 350 mm single cylinder vertical engine using the following data:*

Maximum gas pressure = 2.5 MPa

(L/r) ratio = 4.5

Weight of flywheel cum belt pulley = 10 kN

Total belt pull = 5 kN

Width of hub for flywheel cum belt pulley = 150 mm

The torque on the crankshaft is maximum when the crank turns through 35° from the top dead centre and at this position the gas pressure inside the cylinder is 1 MPa. The belts are in the horizontal direction.

Assume suitable data and state the assumptions you make.

Solution

Given $D = 300$ mm

$p_{max.} = 2.5$ MPa $= 2.5$ N/mm^2 $(L/r) = 4.5$

$l = 350$ mm $W = 10$ kN $= 10\,000$ N $\theta = 35°$

$(P_1 + P_2) = 5$ kN $= 5000$ N

$p' = 1$ MPa $= 1$ N/mm^2

The crank radius r is given by,

$$r = \left(\frac{l}{2}\right) = \frac{350}{2} = 175 \text{ mm}$$

Case I The crank is at the top dead centre position and subjected to maximum bending moment and no torsional moment.

At the top dead centre position, the thrust in the connecting rod will be equal to the force acting on piston.

$$P_p = \left(\frac{\pi D^2}{4}\right) p_{max.} = \left(\frac{\pi(300)^2}{4}\right)(2.5)$$

$$= 176\,714.59 \text{ N}$$

Step 1 *Design of crank pin*

Suppose,

d_c = diameter of crank pin (mm)

l_c = length of crank pin (mm)

σ_b = allowable bending stress for crank pin (N/mm^2)

p_b = allowable bearing pressure at the crank pin bush (MPa or N/mm^2)

Assumption 1 The allowable bearing pressure at the crank pin is 10 MPa.

Assumption 2 The allowable bending stress for crank pin is 60 N/mm^2.

Assumption 3 The (l/d) ratio for the crank pin is 0.80.

$$\left(\frac{l_c}{d_c}\right) = 0.8 \quad \text{or} \quad l_c = 0.8\, d_c$$

The bearing pressure at the crank pin is given by,

$$p_b = \frac{P_p}{d_c\, l_c} \quad \text{or} \quad 10 = \frac{176\,714.59}{d_c(0.8\, d_c)}$$

$$d_c^2 = 22089.32$$

$$d_c = 148.62 \quad \text{or} \quad 150 \text{ mm}$$

$$l_c = 0.8\, d_c = 0.8(150) = 120 \text{ mm}$$

After calculating the dimensions, it is necessary to check the bending stresses in the crank pin.

The crank pin acts as a cantilever and the maximum bending moment is given by,

$$M_b = \left(\frac{3}{4}\right) P_p l_c = \left(\frac{3}{4}\right)(176\,714.59)(120)$$

$$= 15\,904.31 \times 10^3 \text{ N-mm}$$

The section modulus for the crank pin is given by,

$$Z = \frac{\pi d_c^3}{32} = \frac{\pi\,(150)^3}{32} = 331.34 \times 10^3 \text{ mm}^3$$

$$M_b = \sigma_b \left(\frac{\pi\, d_c^3}{32}\right)$$

or $$15\,904.31 \times 10^3 = \sigma_b\,(331.34 \times 10^3)$$

$$\sigma_b = 48 \text{ N/mm}^2 \qquad \sigma_b < 60 \text{ N/mm}^2$$

The bending stress in the crank pin is within the permissible limit.

Step 2 *Design of bearings*

Refer to Fig. 25.26. Suppose,

d_1 = diameter of journal or shaft at the bearing 1 (mm)

l_1 = length of the bearing 1 (mm)

σ_b = allowable bending stress for shaft (N/mm^2)

t = thickness of web (mm)

The thickness of the web is calculated by the following empirical relationship:

$$t = 0.60 d_c = 0.60(150) = 90 \text{ mm}$$

The length of the bearing is calculated by the following empirical relationship:

$$l_1 = 1.75 d_c = 1.75(150) = 262.5 \quad \text{or} \quad 265 \text{ mm}$$

Refer to Fig. 25.26. The bending moment at the bearing 1 is given by,

$$M_b = P_p\,[0.75 l_c + t + 0.5 l_1]$$

$$= 176\,714.59\,[0.75(120) + 90 + 0.5(265)]$$

$$= 55.22 \times 10^6 \text{ N-mm}$$

The bending stress in the shaft of the bearing 1 is given by,

$$M_b = \sigma_b \left(\frac{\pi\, d_1^3}{32}\right)$$

or $$55.22 \times 10^6 = 60 \left(\frac{\pi d_1^3}{32}\right)$$

$$d_1^3 = 9374.44 \times 10^3 \text{ mm}^3$$

$$d_1 = 210.85 \quad \text{or} \quad 215 \text{ mm}$$

Step 3 *Dimensions of b and c*

Refer to Fig. 25.26. The dimension b is given by,

$$b = \left[0.75 l_c + t + \frac{l_1}{2}\right] = \left[0.75(120) + 90 + \frac{(265)}{2}\right]$$

$= 312.5$ or 320 mm

The dimension c is given by,

$$c = \left\{\frac{l_1}{2} + \text{flywheel width} + \frac{l_2}{2} + \text{margin}\right\}$$

$$c = \left\{\frac{265}{2} + 150 + \frac{265}{2} + 15\right\} = 430 \text{ mm}$$

We will assume,

$$c_1 = c_2 = \frac{c}{2} = \frac{430}{2} = 215 \text{ mm}$$

The dimensions are as follows:

$b = 320$ mm $\quad c = 430$ mm $\quad c_1 = c_2 = 215$ mm

Step 4 *Bearing reactions*

$$(R_1)_v = \frac{P_p \times (b + c)}{c}$$

$$= \frac{176\,714.59 \times (320 + 430)}{430}$$

$$= 308.22 \times 10^3 \text{ N}$$

$$(R_2)_v = \frac{P_p \times b}{c}$$

$$= \frac{176\,714.59 \times 320}{430}$$

$$= 131.51 \times 10^3 \text{ N}$$

Since, $\quad c_1 = c_2$

By symmetry,

$$(R_1')_v = (R_2')_v = \frac{W}{2} = \frac{10 \times 10^3}{2} = 5000 \text{ N}$$

$$(R_1')_h = (R_2')_h = \frac{(P_1 + P_2)}{2} = \frac{5000}{2} = 2500 \text{ N}$$

Step 5 *Check for bearing pressure at the bearing 1*

The dimensions d_1 and l_1 are to be checked for bearing pressure.

The reaction R_1 is given by,

$$R_1 = \sqrt{[(R_1)_v + (R_1')_v]^2 + [(R_1')_h]^2}$$

$$= \sqrt{[(308.22 \times 10^3) + (5 \times 10^3)]^2 + (2.5 \times 10^3)^2}$$

$$= 313.23 \times 10^3 \text{ N}$$

The bearing pressure is given by,

$$p_b = \frac{R_1}{d_1 l_1} = \frac{313.23 \times 10^3}{(215)(265)}$$

$$= 5.5 \text{ N/mm}^2 \qquad p_b < 10 \text{ N/mm}^2$$

Bearings 1 and 2 are made identical.

Step 6 *Design of crank web*

Assumption 4 The allowable compressive stress is 60 N/mm^2.

Suppose,

w = width of crank web (mm)

As shown in Fig. 25.26, the crank web is subjected to the eccentric load P_p. There are two types of stresses in the central plane of the crank web, viz., direct compressive stress and bending stress due to the reaction P_p.

The direct compressive stress is given by,

$$\sigma_c = \frac{P_P}{w t} = \frac{176\,714.59}{w(90)} = \left(\frac{1963.5}{w}\right) \text{N/mm}^2$$

The bending moment at the central plane is given by,

$$M_b = P_P\left[0.75 l_c + \frac{t}{2}\right]$$

$$= 176\,714.59\left[0.75(120) + \frac{90}{2}\right]$$

$$= 23\,856.47 \times 10^3 \text{ N-mm}$$

$$Z = \left(\frac{wt^2}{6}\right) = \left(\frac{w(90)^2}{6}\right) = (1350w) \text{ mm}^3$$

$$\sigma_b = \frac{M_b}{Z} = \frac{23\,856.47 \times 10^3}{(1350w)} = \left(\frac{17671.46}{w}\right) \text{N/mm}^2$$

The total compressive stress is given by,

$$(\sigma_c)_t = \sigma_c + \sigma_b = \left(\frac{1963.5}{w}\right) + \left(\frac{17\,671.46}{w}\right)$$

$$= \left(\frac{19\,634.96}{w}\right)$$

$$(\sigma_c)_t = \left(\frac{19\,634.96}{w}\right) \quad \text{or} \quad 60 = \left(\frac{19\,634.96}{w}\right)$$

$w = 327.25$ or 330 mm

Step 6 *Design of shaft under flywheel*

The forces acting on the shaft under the flywheel are shown in Fig. 25.26. The central plane of the shaft is subjected to maximum bending moment. Suppose,

d_s = diameter of shaft under flywheel (mm)

The bending moment in vertical plane is given by,

$$(M_b)_v = P_p(b + c_1) - [(R_1)_v + (R_1')_v](c_1)$$
$$= 176\,714.59(320 + 215)$$
$$- [308.22 \times 10^3 + 5000]\,(215)$$
$$= 27.2 \times 10^6 \text{ N-mm}$$

The bending moment in the horizontal plane is given by,

$$(M_b)_h = (R_1')_h\, c_1 = 2500(215) = 0.54 \times 10^6 \text{ N-mm}$$

The resultant bending moment is given by,

$$M_b = \sqrt{(M_b)_v^2 + (M_b)_h^2}$$
$$= \sqrt{(27.2 \times 10^6)^2 + (0.54 \times 10^6)^2}$$
$$= 27.2 \times 10^6 \text{ N-mm}$$

Also,

$$M_b = \left(\frac{\pi d_s^3}{32}\right)\sigma_b \quad \text{or} \quad 27.2 \times 10^6 = \left(\frac{\pi d_s^3}{32}\right)(60)$$

$$d_s^3 = 4617.62 \times 10^3$$
$$d_s = 166.52 \text{ or } 170 \text{ mm}$$

The diameter (d_s) should be more than the diameter (d_1).

$$d_s > d_1 \quad \text{or} \quad d_s > 215 \text{ mm}$$

Let us assume the diameter (d_s) as 250 mm.

Case II The crank is at an angle with the line of dead centre positions and subjected to maximum torsional moment.

Step 1 *Components of force on crank pin*

$$P_p = \left(\frac{\pi D^2}{4}\right)p' = \left(\frac{\pi(300)^2}{4}\right)(1.0)$$
$$= 70\,685.83 \text{ N}$$

$$\sin\varphi = \frac{\sin\theta}{(L/r)} = \frac{\sin(35)}{(4.5)} = 0.1275$$

$$\varphi = \sin{-1}\,(0.1275) = 7.32°$$

The thrust on the connecting rod (P_q) is given by,

$$P_q = \frac{P_p}{\cos\varphi} = \frac{(70\,685.83)}{\cos(7.32)} = 71\,266.65 \text{ N}$$

P_t and P_r are the tangential and radial components of P_q at the crank pin.

$$P_t = P_q \sin(\theta + \varphi) = 71266.65 \sin(35 + 7.32)$$
$$= 47\,981.74 \text{ N}$$
$$P_r = P_q \cos(\theta + \varphi) = 71\,266.65 \cos(35 + 7.32)$$
$$= 52\,694.28 \text{ N}$$

Step 2 *Bearing reactions*

The forces acting on the side crankshaft at an angle of maximum torque are shown in Fig. 25.28. The crankshaft is supported on two bearings, 1 and 2. The forces acting on crankshaft in vertical and horizontal planes are shown in Fig. 25.29.

(a) Forces in vertical plane **[*Fig. 25.29(a)*]**

(i) Reactions of P_r

$(R_1)_v$ and $(R_2)_v$ are the reactions at bearings due to radial component P_r.

$$(R_1)_v = \frac{P_r \times (b + c)}{c} = \frac{52\,694.28 \times (320 + 430)}{430}$$
$$= 91\,908.63 \text{ N}$$

$$(R_2)_v = \frac{P_r \times b}{c} = \frac{52\,694.28 \times 320}{430} = 39\,214.35 \text{ N}$$

(ii) Reactions of W

By symmetry,

$$(R_1')_v = (R_2')_v = \frac{W}{2} = \frac{10 \times 10^3}{2} = 5000 \text{ N}$$

(b) Forces in horizontal plane [Fig. 25.29(b)]

(iii) Reactions of P_t

$$(R_1)_h = \frac{P_t \times (b + c)}{c} = \frac{47\,981.74 \times (320 + 430)}{430}$$
$$= 83\,689.08 \text{ N}$$

$$(R_2)_h = \frac{P_t \times b}{c} = \frac{47\,981.74 \times 320}{430} = 35\,707.34 \text{ N}$$

(iv) Reactions of ($P_1 + P_2$)

By symmetry,

$$(R_1')_h = (R_2')_h = \frac{(P_1 + P_2)}{2} = \frac{5000}{2} = 2500 \text{ N}$$

Step 3 *Design of crank pin*

Check for bending stresses

The crank pin acts as a cantilever and subjected to maximum bending moment at the crank web as shown in Fig. 25.28. The bending moment in a vertical plane is given by,

$$(M_b)_v = P_r \times (0.75 l_c) = 52694.28\,(0.75 \times 120)$$
$$= 4742.49 \times 10^3 \text{ N-mm}$$

The bending moment in horizontal plane is given by,

$(M_b)_h = P_t \times (0.75\ l_c) = 47981.74(0.75 \times 120)$
$= 4318.36 \times 10^3$ N-mm

The resultant bending moment M_b is given by,

$$M_b = \sqrt{[(M_b)_v]^2 + [(M_b)_h]^2}$$

$$= \sqrt{[4742.49 \times 10^3]^2 + [4318.36 \times 10^3]^2}$$

$= 6414 \times 10^3$ N-mm

The section modulus for the crank pin is given by,

$$Z = \frac{\pi d_c^3}{32} = \frac{\pi(150)^3}{32} = 331.34 \times 10^3 \text{ mm}^3$$

$M_b = \sigma_b Z$ or $6414 \times 10^3 = \sigma_b (331.34 \times 10^3)$

$\sigma_b = 19.36$ N/mm² $\sigma_b < 60$ N/mm²

The bending stresses in the crank pin are within the permissible limit.

Step 4 *Design of crank web*

Check for bending stresses

The bending moment due to radial component at the central plane is given by,

$(M_b)_r = P_r\ [0.75 l_c + 0.5t]$
$= 52\,694.28\ [0.75(120) + 0.5(90)]$
$= 7113.73 \times 10^3$ N-mm

$$Z = \frac{1}{6} wt^2 = \frac{1}{6}(330)(90)^2 = 445.5 \times 10^3 \text{ mm}^3$$

$(M_b)_r = (\sigma_b)_r \times Z$

or $7113.73 \times 10^3 = (\sigma_b)_r\ (445.5 \times 10^3)$

$(\sigma_b)_r = 15.97$ N/mm² (a)

The bending moment due to tangential component at the juncture of the crank web and shaft is given by,

$$(M_b)_t = P_t\left[r - \frac{d_1}{2}\right] = 47\,981.74\left[175 - \frac{215}{2}\right]$$

$= 3238.77 \times 10^3$ N-mm

$$Z = \frac{1}{6} tw^2 = \frac{1}{6}(90)(330)^2 = 1633.5 \times 10^3 \text{ mm}^3$$

$(M_b)_t = (\sigma_b)_t Z$ or

$3238.77 \times 10^3 = (\sigma_b)_t\ 1633.5 \times 10^3$

$(\sigma_b)_t = 1.98$ N/mm² (b)

The direct compressive stress due to radial component is given by,

$$(\sigma_c)_d = \frac{P_r}{wt} = \frac{52694.28}{(330)(90)} = 1.77 \text{ N/mm}^2 \qquad \text{(c)}$$

The maximum compressive stress (σ_c) is given by,

$\sigma_c = (\sigma_b)_r + (\sigma_b)_t + (\sigma_c)_d = 15.97 + 1.98 + 1.77$
$= 19.72$ N/mm²

The torsional moment on the arm is given by,

$M_t = P_t\ [0.75 l_c + 0.5t]$
$= 47\,981.74[0.75(120) + 0.5(90)]$
$= 6477.53 \times 10^3$ N-mm

$$\tau = \frac{M_t}{Z_p} = \frac{4.5 M_t}{wt^2} = \frac{4.5\,(6477.53 \times 10^3)}{(330)(90)^2}$$

$= 10.9$ N/mm²

The maximum compressive stress is given by,

$$(\sigma_c)_{\max.} = \frac{\sigma_c}{2} + \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\tau^2}$$

$$= \frac{(19.72)}{2} + \frac{1}{2}\sqrt{(19.72)^2 + 4(10.9)^2}$$

$= 24.56$ N/mm²

$(\sigma_c)_{\max.} < 60$ N/mm²

The value of $(\sigma_c)_{\max.}$ is less than the allowable compressive stress and design is safe.

Step 5 *Design of shaft at the juncture of crank web*

Check for torsional shear stresses

Suppose,

d_{s1} = diameter of shaft at the juncture of the crank web (mm)

$d_{s1} = d_1 = 215$ mm

$(M_b)_v = P_r[0.75 l_c + t]$
$= 52\,694.28\ [0.75(120) + 90]$
$= 9484.97 \times 10^3$ N-mm

$(M_b)_h = P_t[0.75 l_c + t]$
$= 47\,981.74[0.75(120) + 90]$
$= 8636.71 \times 10^3$ N-mm

The resultant bending moment M_b is given by,

$$M_b = \sqrt{[(M_b)_v]^2 + [(M_b)_h]^2}$$

$$= \sqrt{(9484.97 \times 10^3)^2 + (8636.71 \times 10^3)^2}$$

$= 12827.99 \times 10^3$ N-mm

$M_t = P_t \times r = 47\,981.74 \times 175$
$= 8396.8 \times 10^3$ N-mm

The torsional shear stress in shaft is calculated by the following expression,

$$\tau = \frac{16}{\pi d_{s1}^3}\sqrt{(M_b)^2 + (M_t)^2}$$

$$= \frac{16}{\pi (215)^3} \sqrt{(12\,827.99 \times 10^3)^2 + (8396.8 \times 10^3)^2}$$

$= 7.86$ N/mm^2 $\quad \tau < 40$ N/mm^2

The shear stress in the shaft is within permissible limits and the design is safe.

Step 6 *Design of shaft under flywheel*

Check for torsional shear stresses

Suppose,

d_s = diameter of shaft under flywheel (mm)

The central plane of the shaft is subjected to maximum bending moment. The bending moment in the vertical plane $(M_b)_v$ is given by,

$$(M_b)_v = P_r(b + c_1) - [(R_1)_v + (R_1')_v]c_1$$
$$= 52\,694.28\,(320 + 215)$$
$$-[91908.63 + 5000](215)$$
$$= 7.36 \times 10^6 \text{ N-mm}$$

The bending moment in the horizontal plane $(M_b)_h$ is given by,

$$(M_b)_h = P_t(b + c_1) - [(R_1)_h + (R_1')_h]c_1$$
$$= 47981.74(320 + 215)$$
$$-[83\,689.08 + 2500](215)$$
$$= 7.14 \times 10^6 \text{ N-mm}$$

The resultant bending moment M_b is given by,

$$M_b = \sqrt{[(M_b)_v]^2 + [(M_b)_h]^2}$$
$$= \sqrt{[7.36 \times 10^6]^2 + [7.14 \times 10^6]^2}$$
$$= 10.25 \times 10^6 \text{ N-mm}$$

The shaft is also subjected to torsional moment M_t due to tangential component P_t.

$M_t = P_t \times r = 47\,981.74(175)$

$= 8.4 \times 10^6$ N-mm

The torsional shear stress in the shaft is calculated by the following equation:

$$\tau = \frac{16}{\pi d_s^3} \sqrt{(M_b)^2 + (M_t)^2}$$
$$= \frac{16}{\pi (250)^3} \sqrt{(10.25 \times 10^6)^2 + (8.4 \times 10^6)^2}$$

$= 4.32$ N/mm^2 $\qquad \tau < 40$ N/mm^2

The shear stress in the shaft is within permissible limit and the design is safe.

25.28 VALVE GEAR MECHANISM

The valve gear mechanism is a subassembly of the IC engine and its function is to open and close the inlet and exhaust valves at a proper time with respect to the position of the piston and crankshaft. The fuel is admitted into the cylinder when the inlet valve is open. Also, the burnt gases escape when the exhaust valve is open. The valve gear mechanisms for vertical and horizontal engines are illustrated in Figs. 25.30 and 25.31 respectively. The main parts of a valve-gear mechanism are valve, spring, tappet, rocker arm, push rod, cam and camshaft.

Figure 25.30 shows the valve gear mechanism for a vertical engine with overhead valves. The camshaft is rotated by means of a belt drive from the crankshaft. As the camshaft rotates, the cam pushes the follower and the push rod upwards. The rocker arm is pivoted at its centre by means of a fulcrum pin. When the right end of the rocker arm is pushed up by the push rod, the left end moves downward. This compresses the spring and pushes the valve rod down in the cylinder, thereby causing the valve to open. When the follower moves over the circular portion of the cam, the spring expands and closes the valve. The spring pushes the left end of the rocker arm upward. This causes the right end to move downward and keeps the follower in contact with the cam.

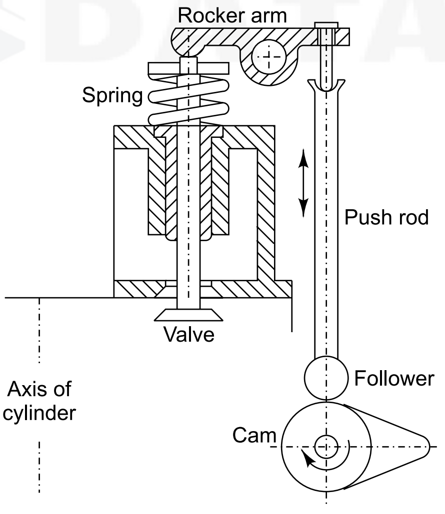


Fig. 25.30 *Valve gear mechanism (vertical engine)*

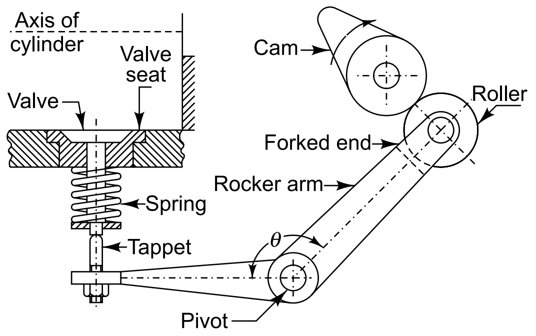


Fig. 25.31 *Valve gear mechanism (horizontal engine)*

It should be noted that the valve is closed by a helical compression spring. One end of the spring is secured to the valve stem and the other end rests on the cylinder. The spring is initially compressed. The force due to initial compression presses the valve down on its seat. However, the opening of the valve is due to oscillatory motion of the rocker arm.

25.29 DESIGN OF VALVES

Two types of valves are used in IC engines, viz., *poppet* or *mushroom* valves and *sleeve* valves. There are limited applications of sleeve valves. They are used in very few automobile engines. The poppet valves, on the other hand, are frequently used in IC engines. The construction of a poppet valve is shown in Fig. 25.32. It consists of a disk called '*head*' at the end of a rod called '*stem*'. The 'face' of the valve is a conical surface that comes in contact with a valve seat. It has a face angle varying from 30° to 45°. The conical surface of the valve results in self-centering property. There is small margin between the head and the face. There are two reasons of providing the margin—it avoids the sharp edges and a provision is made for regrinding of the valve. The lower end of valve stem has a groove. A spring retainer is fitted in this groove, which supports one end of the valve spring.

In slow-speed engines, valves have composite construction with a cast iron head and steel stem. In high-speed engines, one piece construction is used and valves are forged. The exhaust valves are subjected to high temperatures, as high as 1900°C to 2200°C, during the peak of explosion. The material used for making valves should have the following characteristics:

(i) It should be heat resistant.
(ii) It should have a good thermal conductivity.
(iii) It should be corrosion resistant.
(iv) It should have a wear resistant surface.
(v) It should have shock resistance.

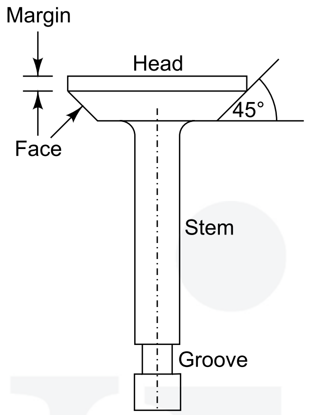


Fig. 25.32 *Poppet Valve*

The inlet valve is subjected to comparatively less temperature than the exhaust valve. Therefore, inlet valves are made of nickel–chromium steel. The exhaust valves are made of heat resistant silicon–chromium steel. For heavy duty engines, valves are made of chromium–vanadium steel. The valves are heat treated and surface hardness for inlet and exhaust valves is in the range of 250 to 300 HB. The design of valves consists of the following steps:

(i) Diameter of Valve Port Suppose,

a = area of piston
v = mean velocity of piston
a_p = area of port
v_p = mean velocity of gas flowing through the port

Then,

$$a_p \times v_p = a \times v \quad \text{or} \quad a_p = \frac{av}{v_p} \tag{25.72}$$

Also,

$$a_p = \left(\frac{\pi d_p^2}{4}\right)$$

where,

d_p = diameter of port (mm)

It should be noted that the mean velocity of the piston (v) is given by,

$$v = 2l\left(\frac{N}{60}\right)$$

where,

v = mean velocity of piston (m/s)

l = length of stroke (m)

N = engine speed (rpm)

The allowable mean velocities of the gas through the port (v_p) are given in Table 25.2.

Table 25.2 *Allowable mean velocities of the gas (v_p)*

Type of engine	*Mean velocity of gas (m/s)*	
	Inlet valve	*Exhaust valve*
Low-speed engine	33–40	40–50
Medium-speed engine	35–45	50–60
High-speed engine	80–90	90–100

The inlet ports are made 20% to 40% larger than the exhaust ports for better cylinder charging and scavenging.

(ii) Diameter of Valve Head The projected width w of the valve seat is shown in Fig. 25.33. For a seat angle of 45°, the projected width of the valve seat is given by the following empirical relationship:

$w = (0.05 \text{ to } 0.07)\, d_p$ or $w = 0.06\, d_p$

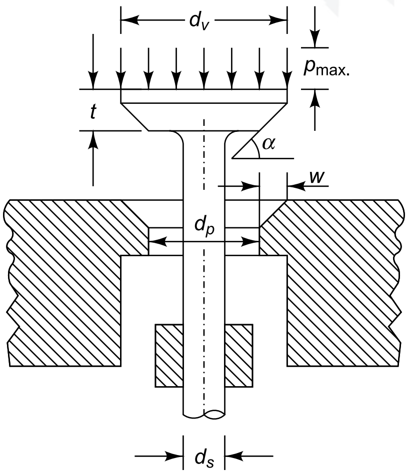


Fig. 25.33 *Force on Valve*

The diameter of the valve head (d_v) is given by,

$$d_v = (d_p + 2w)$$

(iii) Thickness of Valve Disk The valve disk is assumed as a circular disk freely supported around its periphery and subjected to a uniformly distributed load $p_{max.}$ as shown in Fig. 25.33. The thickness of the disk is calculated by the following equation:

$$t = kd_p\sqrt{\frac{p_{max.}}{\sigma_b}} \qquad (25.73)$$

where,

t = thickness of valve disk (mm)

k = constant (k = 0.42 for steel and k = 0.54 for cast iron)

d_p = diameter of port (mm)

$p_{max.}$ = maximum gas pressure (MPa or N/mm^2)

σ_b = permissible bending stress (N/mm^2)

The values of permissible bending stress (σ_b) are as follows:

For carbon steel σ_b = 50 to 60 N/mm^2

For alloy steel σ_b = 100 to 120 N/mm^2

The margin or thickness of the valve disk at the edges is given by,

Thickness of valve disk at edges = 0.75 to 0.85t

(iv) Diameter of Valve Stem The diameter of the valve stem is calculated by the following empirical relationship:

$$d_s = \left[\frac{d_p}{8} + 6.35\right] \text{ to } \left[\frac{d_p}{8} + 11\right] \qquad (25.74)$$

d_s = diameter of valve stem (mm)

The valve is subjected to a spring force when seated. The spring force is assumed as a concentrated force at the centre. In this position, the stress in the valve is given by,

$$\sigma_t = \frac{1.4P_s}{t^2}\left[1 - \frac{2d_s}{3d_p}\right] \qquad (25.75)$$

where,

P_s = spring force (N)

(v) Maximum Valve Lift Suppose,

$h_{max.}$ = maximum lift of valve (mm)

For continuous flow, the mean velocity of the gas flow through the valve is equal to that through the opening of the valve when it is lifted to a maximum. Therefore, the maximum lift of the valve is obtained by equating the area across the valve seat to the area of the port. As shown in Fig. 25.33,

$$(\pi d_p) \times (h_{max.} \cos\alpha) = \left(\frac{\pi d_p^2}{4}\right)$$

$$h_{max.} = \frac{d_p}{4\cos\alpha} \tag{25.76}$$

For flat headed valves, $(\alpha = 0)$ $(\cos\alpha = 1)$

$$h_{max.} = \frac{d_p}{4}$$

The valve seat has the same angle as the valve seating surface. In practice, the angle of the valve seat is made 0.5° to 1° more than the valve angle. This difference is sometimes called '*interference*' angle. It results in a more effective seal.

Example 25.20 *Design an exhaust valve for a horizontal diesel engine using the following data:*

Cylinder bore = 150 mm
Length of stroke = 275 mm
Engine speed = 500 rpm
Maximum gas pressure = 3.5 MPa
Seat angle = 45°
Calculate:
(i) diameter of the valve port;
(ii) diameter of the valve head;
(iii) thickness of the valve head;
(iv) diameter of the valve stem; and
(v) maximum lift of the valve.

Solution

Given $D = 150$ mm $l = 275$ mm N = 500 rpm
$p_{max.} = 3.5$ MPa = 3.5 N/mm² $\alpha = 45°$

Step I *Diameter of the valve port*

$$a = \text{area of piston} = \left(\frac{\pi(150)^2}{4}\right) \text{mm}^2$$

v = mean velocity of piston

$$= 2l\left(\frac{N}{60}\right) = 2\left(\frac{275}{1000}\right)\left(\frac{500}{60}\right) \text{ m/s}$$

Assume [from Table 25.2],

$$v_p = 50 \text{ m/s}$$

$$a_p = \left(\frac{\pi d_p^2}{4}\right) \text{mm}^2$$

Therefore,

$$a \times v = a_p \times v_p$$

$$\left(\frac{\pi(150)^2}{4}\right) \times 2\left(\frac{275}{1000}\right)\left(\frac{500}{60}\right) = \left(\frac{\pi d_p^2}{4}\right) \times (50)$$

$$d_p^2 = 2062.5 \text{ mm}^2$$

$$d_p = 45.41 \text{ mm or } 46 \text{ mm} \tag{i}$$

Step II *Diameter of valve head*

For a seat angle of 45°, the projected width of the valve seat is given by the following empirical relationship:

$$w = (0.05 \text{ to } 0.07)\, d_p$$

or $$w = 0.06 d_p = 0.06\,(46) = 2.76 \text{ or } 3 \text{ mm}$$

The diameter of the valve head (d_v) is given by,

$$d_v = (d_p + 2w) = 46 + 2 \times 3 = 52 \text{ mm} \tag{ii}$$

Step III *Thickness of the valve head*

For a steel valve,

$k = 0.42$ $\sigma_b = 50$ N/mm²

$$t = kd_p\sqrt{\frac{p_{max.}}{\sigma_b}} = (0.42)(46)\sqrt{\frac{3.5}{50}} = 5.11$$

$$= 5.5 \text{ mm} \tag{iii}$$

Step IV *Diameter of valve stem*

$$d_s = \left[\frac{d_p}{8} + 6.35\right] \text{ to } \left[\frac{d_p}{8} + 11\right]$$

$$= \left[\frac{46}{8} + 6.35\right] \text{ to } \left[\frac{46}{8} + 11\right]$$

$$= 12.1 \text{ to } 16.75 \text{ mm}$$

$$d_s = 15 \text{ mm} \tag{iv}$$

Step V *Maximum lift of valve*

$$h_{max.} = \frac{d_p}{4\cos\alpha} = \frac{46}{4\cos(45)} = 16.26 \text{ mm} \tag{v}$$

25.30 DESIGN OF A ROCKER ARM

The basic function of rocker arm is to open or close the inlet or exhaust valve with respect to the motion of the cam and follower. The rocker arm for operating the exhaust valve is shown in Fig. 25.34. It is a two arm lever with the fulcrum in the middle. One end of the rocker arm that is actuated by the cam is forked and carries a roller. The other arm of the rocker arm consists of a plain boss where a tappet is screwed. In order to reduce the weight and inertia force, the cross-section of the rocker arm is

I-section. The arms of the rocker arm are made of uniform strength by tapering the width and depth dimensions. However, the thickness of the web and flanges is kept uniform.

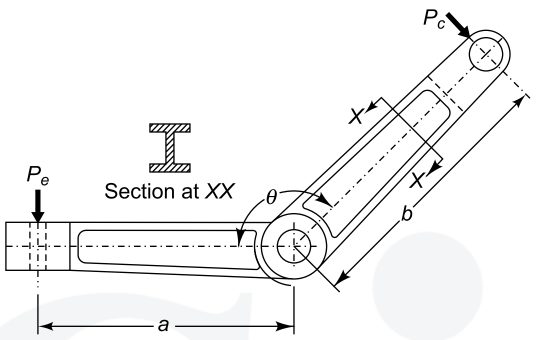


Fig. 25.34 *Rocker Arm for Exhaust Valve*

Rocker arms are made of grey cast iron, malleable cast iron or cast steel. In four-stroke cycle engines, the rocker arm of the exhaust valve is more heavily loaded. On the other hand, the force required to operate the inlet valve is comparatively less. However, it is usual practice to make rocker arms for inlet and exhaust valves identical. This results in ease of manufacturing. The main objective of the rocker arm as a 'lever' is to change the direction of force and not the multiplication of the effort. Therefore, the two arms of the rocker arm are often made equal in moderate and low speed engines. In high speed engines, the (a/b) ratio is taken as (1/1.3).

The forces acting on the rocker arm of the exhaust valve are illustrated in Fig. 25.35. The maximum force exerted by the valve rod on the tappet will determine the bending moment for the design of the cross-section of the rocker arm. This force consists of the following three factors:

(i) The gas pressure on the valve when it opens
(ii) The inertia force when the valve moves up
(iii) The initial spring force to hold the valve on its seat against suction or negative pressure inside the cylinder during the suction stoke

The gas load P_g is given by,

P_g = area of valve × gas pressure when the exhaust valve opens

$$P_g = \left(\frac{\pi d_v^2}{4}\right) p_c \qquad (25.77)$$

where,

d_v = diameter of the valve head (mm)

p_c = cylinder pressure or back pressure when the exhaust valve opens (MPa)

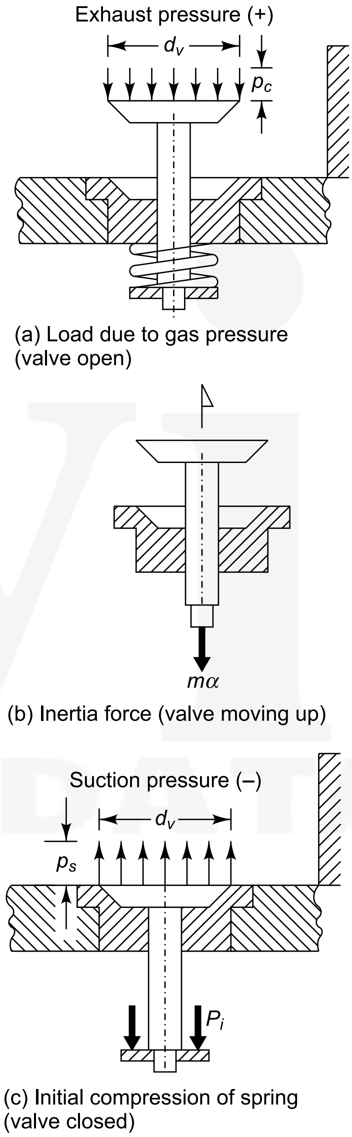


Fig. 25.35 *Forces Acting on Tappet*

As the valve moves up, the inertia force acts opposite to the direction of motion. The downward inertia force P_a is given by,

$$P_a = m\alpha \qquad (25.78)$$

where,

m = mass of valve (kg)

α = acceleration of the valve (m/s^2)

The initial spring force P_i is given by,

P_i = area of valve × maximum suction pressure

$$P_i = \left(\frac{\pi d_v^2}{4}\right) p_s \qquad (25.79)$$

where,

p_s = maximum suction pressure (MPa)

Adding (25.77), (25.78) and (25.79), the total force (P_e) on the rocker arm of the exhaust valve is given by,

$$P_e = P_g + P_a + P_i \qquad (25.80)$$

It should be noted that the total force (P_i) on the rocker arm of the inlet valve consists of only two factors and it is given by,

$$P_i = P_a + P_i \qquad (25.81)$$

After calculating the forces, various portions of the rocker arm are designed in the following way.

(i) Design of Fulcrum Pin As shown in Fig. 25.36, the reaction at the fulcrum pin is given by,

$$R_f = \sqrt{(P_e)^2 + (P_c)^2 - 2P_eP_c\cos\theta} \qquad (25.82)$$

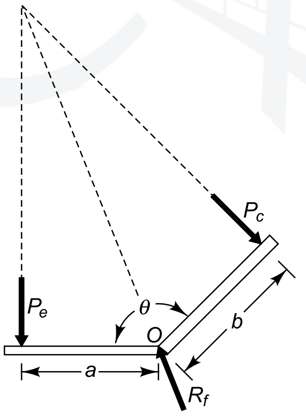


Fig. 25.36 *Fulcrum Reaction*

When arms are equal,

$a = b$ and $P_e = P_c$

Suppose,

d_1 = diameter of fulcrum pin (mm)

l_1 = length of fulcrum pin (mm)

Considering the bearing at the fulcrum pin,

$$p_b = \frac{R_f}{d_1 l_1} \qquad (25.83)$$

The (l/d) ratio for the fulcrum pin is taken as 1.25. Therefore,

$$\left(\frac{l_1}{d_1}\right) = 1.25 \qquad (25.84)$$

The bearing pressure can be taken as 5 MPa.

The dimensions of the pin can be determined by using equations from (25.82) to (25.84). The pin should be checked for double shear stress. The shear stress is given by,

$$\tau = \frac{R_f}{2\left(\frac{\pi d_1^2}{4}\right)} \qquad (25.85)$$

The permissible shear stress for the fulcrum pin is 42 N/mm^2.

A phosphor-bronze bush of 3 mm thickness is press fitted in the pinhole of the rocker arm to reduce friction. It should be noted that the rocker arm oscillates freely over the fulcrum pin. The outside diameter of the boss of the rocker arm at the fulcrum pin is taken as twice the diameter of the fulcrum pin.

(ii) Design of Forked End The construction of the forked end of the rocker arm is shown in Fig. 25.37. The forked end carries a roller that revolves on the pin. The force acting on the roller pin is P_c.

Suppose,

d_2 = diameter of roller pin (mm)

l_2 = length of roller pin (mm)

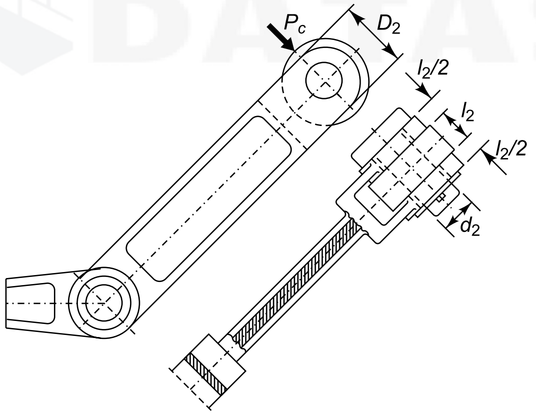


Fig. 25.37 *Forked Arm*

Considering the bearing at the roller pin,

$$p_b = \frac{P_c}{d_2 l_2} \qquad (25.86)$$

The (l/d) ratio for the roller pin is taken as 1.25. Therefore,

$$\left(\frac{l_2}{d_2}\right) = 1.25 \qquad (25.87)$$

The bearing pressure can be taken as 5 MPa.

The dimensions of the pin can be determined by using Eqs. (25.86) and (25.87). The pin should be checked for double shear stress. The shear stress is given by,

$$\tau = \frac{P_c}{2\left(\frac{\pi d_2^2}{4}\right)} \qquad (25.88)$$

The permissible shear stress for the fulcrum pin is 42 N/mm^2.

The roller pin is fixed in the eye. The dimensions of the eye are as follows:

Thickness of each eye = $l_2/2$

Outer diameter of eye = $D_2 = 2d_2$

The diameter of the roller should be more than the outer diameter of the eye by at least 5 mm.

When the roller pin is tight in the two eyes of the fork, failure occurs due to shear. On the other hand, when the pin is loose, it is subjected to bending moment. It acts as a beam as shown in Fig. 25.38. It is assumed that the load acting on the pin is uniformly distributed in the portion that is in contact with the roller. It is uniformly varying in two parts of the fork.

For triangular distribution of the load between the pin and the eyes,

$$x = \left(\frac{1}{3}\right)\left(\frac{l_2}{2}\right) = \left(\frac{l_2}{6}\right)$$

For a uniformly distributed load between the pin and the roller,

$$z = \left(\frac{1}{2}\right)\left(\frac{l_2}{2}\right) = \left(\frac{l_2}{4}\right)$$

The bending moment is maximum at the central plane of the pin. It is given by,

$$M_b = \left(\frac{P_c}{2}\right)\left(\frac{l_2}{2} + x\right) - \left(\frac{P_c}{2}\right)(z)$$

$$= \left(\frac{P_c}{2}\right)\left(\frac{l_2}{2} + \frac{l_2}{6}\right) - \left(\frac{P_c}{2}\right)\left(\frac{l_2}{4}\right)$$

$$= \left(\frac{P_c}{2}\right)\left(\frac{2l_2}{3}\right) - \left(\frac{P_c}{2}\right)\left(\frac{l_2}{4}\right) = \left(\frac{P_c}{2}\right)\left(\frac{5l_2}{12}\right)$$

$$M_b = \left(\frac{5}{24}\right)P_c l_2 \qquad (25.89)$$

Also,

$$Z = \frac{\pi d_2^3}{32} \quad \text{and} \quad \sigma_b = \frac{M_b}{Z}$$

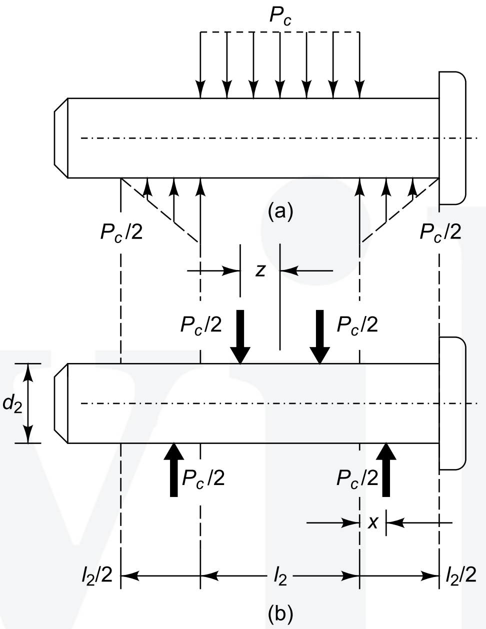


Fig. 25.38 *Roller Pin as Beam: (a) Actual Distribution of Forces (b) Simplified Diagram of Forces*

The bending stress calculated by the above equations should be within the limits.

(iii) Design of Cross-section of Rocker Arm As shown in Fig. 25.39, the arm is subjected to bending moment under the action of force P_e. The bending moment is maximum at the centre of the pivot. In general, the central section passing through the fulcrum pin is sufficiently strong to withstand bending moment. Therefore, we will consider the cross-section of the arm near the boss of the fulcrum pin. The outside diameter of the boss of the rocker arm at the fulcrum pin is twice the diameter of the fulcrum pin ($2d_1$). Therefore,

$$M_b = P_e[a - d_1] \qquad (25.90)$$

The proportions of cross-section for the rocker arm are shown in Fig. 25.39(b). The moment of inertia of the cross-section is determined by subtracting the moment of inertia of the inner rectangle from that of the outer rectangle. The size of the outer rectangle is $(2.5t \times 6t)$. The inner rectangle is formed by adding two vertical strips, each of width $[(2.5t - t)/2]$ and height of $(4t)$.

$$I = \left[\frac{1}{12}(2.5t)(6t)^3 - \frac{1}{12}(1.5t)(4t)^3\right] = (37t^4) \tag{25.91}$$

$$y = (3t)$$

$$\sigma_b = \frac{M_b y}{I}$$

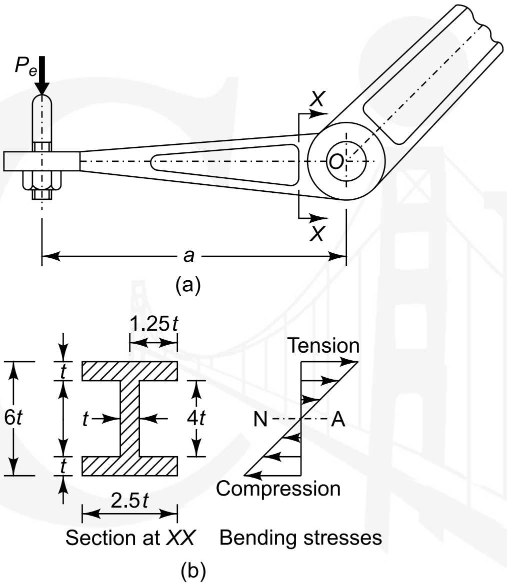


Fig. 25.39 *Bending Stresses in Rocker Arm*

The permissible values of bending stresses are as follows:

For cast steel, $\sigma_b = 50$ to 60 N/mm^2

For forged steel, $\sigma_b = 70$ to 80 N/mm^2

Assuming the above values of permissible bending stresses, the dimensions of the cross-section of the rocker arm are calculated.

(iv) Design of Tappet A tappet is a stud, which is subjected to compressive force (P_e). Suppose,

d_c = core diameter of stud (mm)

σ_c = permissible compressive stress for stud (N/mm^2)

$$\sigma_c = \frac{P_e}{\left(\dfrac{\pi d_c^2}{4}\right)} \tag{25.92}$$

The permissible compressive stress can be taken as 50 N/mm^2 for steel studs. Assuming this value, the core diameter of the stud is calculated. The nominal diameter (d) of the tappet is calculated by the following relationship:

$$d = \left(\frac{d_c}{0.8}\right)$$

The diameter of the circular end of the rocker arm (D_3) and its depth (t_3) are calculated by the following relationships:

$$D_3 = 2d \quad \text{and} \quad t_3 = 2d$$

25.31 DESIGN OF VALVE SPRING

The purpose of the valve spring is to exert a force on the rocker arm in order to maintain contact between the follower and cam at the other end. During the suction stroke, the initial spring force keeps the valve closed against negative pressure inside the cylinder. The total force acting on the spring of the exhaust valve consists of two factors, viz., the initial spring force and force required to lift the valve. The initial spring force is given by,

P_i = area of valve × maximum suction pressure

$$P_i = \left(\frac{\pi d_v^2}{4}\right) p_s \tag{a}$$

where,

p_s = maximum suction pressure (MPa)

The force required to lift the valve (P_2) is given by,

$$P_2 = k\delta \tag{b}$$

where,

k = stiffness of spring (N/mm)

δ = maximum lift of valve (mm)

Adding (a) and (b), the maximum force on spring ($P_{max.}$) is given by,

$$P_{max.} = P_i + k\delta \tag{25.93}$$

The following assumptions are made in the design of valve springs:

(i) The spring is made of oil-hardened and tempered valve spring wire of Grade-VW.
(ii) The stiffness of the spring is 10 N/mm.
(iii) The allowable torsional shear stress for spring material is 250 to 350 N/mm^2.
(iv) The spring index (D/d) is 8.
(v) The spring has square and ground ends.

The dimensions of the spring are calculated by the following steps:

(i) Wire Diameter

$$\tau = K\left(\frac{8P_{max.}C}{\pi d^2}\right)$$

where,

τ = allowable torsional shear stress (250 to 350 N/mm^2)

C = spring index (8)

d = wire diameter (mm)

K = Wahl factor

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

(ii) Mean Coil Diameter

$$D = Cd$$

where,

D = mean coil diameter (mm)

(iii) Number of Active Turns

$$N = \frac{Gd^4}{8D^3k}$$

where,

N = number of active turns

G = modulus of rigidity (84×10^3 N/mm^2)

k = stiffness of spring (10 N/mm)

(iv) Total Number of Turns For square and ground ends

$$N_t = N + 2$$

where,

N_t = total number of turns

(v) Maximum Compression of Spring

$$\delta_{max.} = \frac{8P_{max.}D^3N}{Gd^4}$$

where,

$\delta_{max.}$ = maximum compression of spring (mm)

(vi) Solid Length of Spring

$$\text{Solid length} = N_t\, d$$

(vii) Free Length of Spring There should be a gap between consecutive coils when the spring is subjected to maximum force ($P_{max.}$). The total gap is assumed as 15% of the maximum compression.

Total gap = 0.15 $\delta_{max.}$

Free length = solid length + $\delta_{max.}$ + 0.15 $\delta_{max.}$

(viii) Pitch of Coils

$$\text{Pitch of coils} = \frac{\text{free length}}{(N_t - 1)}$$

When the natural frequency of vibrations of spring coincides with the frequency of external periodic force, which acts on it, resonance occurs. In this state, the spring is subjected to a wave of successive compressions of coils that travels from one end to the other and back. This type of vibratory motion is called the '*surge*' of spring. Surge is found in valve springs, which are subjected to periodic force. The natural frequency of helical compression springs held between two parallel plates is given by,

$$\omega = \frac{1}{2}\sqrt{\frac{k}{m}}$$

where,

k = stiffness of spring (N/m)

m = mass of spring (kg)

For heavy-duty diesel engines, two concentric springs of different natural frequencies are used to eliminate the surge. In this case, the outer spring is designed to carry 60 to 70 per cent of maximum spring force on the combination and the inner spring is designed for the remaining force. One of the springs is wound in the right hand and other in the left hand to avoid interlocking.

Note The above analysis of valve spring is elementary. The valve spring is subjected to fluctuating loads. In IC engines, it is subjected to millions of stress cycles during its lifetime. They are designed as per the procedure explained in Example 10.15 in Chapter10 on 'Springs'.

25.32 DESIGN OF PUSH ROD

Push rods are used in overhead valve and side valve engines. It is a long column introduced between the cam and rocker arm so that the camshaft can be located at a lower level. The push rods are made of bright drawn steel tubes with 4% carbon or duralumin tubes. The ends of the push rod depend upon the general configuration of valve gear mechanism.

When the push rod is guided, the ends are flat plugs. When the push rod is not guided, ball and socket joints are used at the ends. The flat plugs or sockets are force fitted at the ends of the push rod.

The push rod is designed as a column on the basis of buckling criterion. The slenderness ratio of push rod is such that it falls in the category of a '*long*' column.

Suppose,

d_o = outer diameter of push rod (mm)

d_i = inner diameter of push rod (mm)

The ratio of (d_i/d_o) for push rod is 0.6 to 0.8.

$$\left(\frac{d_i}{d_o}\right) = 0.6 \text{ to } 0.8$$

The moment of inertia (I) of the cross-section is given by,

$$I = \frac{\pi(d_o^4 - d_i^4)}{64}$$

The area (A) of the cross-section is given by,

$$A = \frac{\pi(d_o^2 - d_i^2)}{4}$$

The radius of gyration (k) is given by,

$$I = Ak^2$$

or,

$$k^2 = \frac{I}{A} = \left(\frac{\pi(d_o^4 - d_i^4)}{64}\right)\left(\frac{4}{\pi(d_o^2 - d_i^2)}\right)$$

$$= \frac{1}{16}\frac{(d_o^4 - d_i^4)}{(d_o^2 - d_i^2)}$$

$$= \frac{1}{16}\frac{(d_o^2 + d_i^2)(d_o^2 - d_i^2)}{(d_o^2 - d_i^2)}$$

$$k^2 = \frac{(d_o^2 + d_i^2)}{16}$$

The dimensions of cross-section of the push rod are calculated by applying Rankine's formula. According to this formula,

$$P = \frac{\sigma_c A}{1 + a\left(\frac{l}{k}\right)^2}$$

where,

P = force acting on the push rod (N)

σ_c = permissible compressive stress (N/mm^2)

A = cross-sectional area of push rod (mm^2)

a = constant depending upon material and end fixity coefficient

l = actual length of push rod (mm)

k = radius of gyration (mm)

For push rod made of mild steel and plain carbon steel,

$$\sigma_c = 70 \text{ N/mm}^2 \quad \text{(initial value)}$$

The constant a for steel push rod is given by,

$$a = \frac{1}{7500}$$

The value of permissible compressive stress depends upon the (length/diameter) ratio of push rod. They are given in Table 25.3.

Table 25.3 *Values of permissible compressive stress (σ_c)(N/mm^2)*

(l/d_o)	10	20	30	40	50	60	70	80	90	100
σ_c	100	70	50	35	28	20	15	12	9	7.5

The step by step procedure for design of the cross-section of a push rod is as follows:

(i) Assume,

$$\sigma_c = 70 \text{ N/mm}^2 \quad a = \frac{1}{7500}$$

$$\left(\frac{d_i}{d_o}\right) = 0.8 \quad k^2 = \frac{(d_o^2 + d_i^2)}{16}$$

$$A = \frac{\pi(d_o^2 - d_i^2)}{4}$$

Calculate the diameter d_o by,

$$P = \frac{\sigma_c A}{1 + a\left(\frac{l}{k}\right)^2}$$

(ii) Calculate the ratio (l/d_o) and find out the permissible compressive stress from Table 25.3. If it is less, go back to Step (i) with this new value of permissible compressive stress and repeat the calculations.

(iii) The problem is solved by trial and error method.

Example 25.21 *Design a rocker arm for the exhaust valve of a four-stroke engine using the following data:*

Effective length of each arm = 180 mm
Angle between two arms = 135°
Diameter of valve head = 75 mm
Lift of valve = 25 mm
Mass of valve = 0.5 kg
Engine speed = 600 rpm
Back pressure when the exhaust valve opens = 0.4 MPa
Maximum suction pressure = 0.02 MPa below atmosphere

The valve opens 33° before the outer dead centre and closes 1° after the inner dead centre. The motion of the valve is SHM without dwell in the fully opened condition. Assume suitable data and state the assumptions you make.

Solution

Given $a = 180$ mm $\theta = 135°$ $d_v = 75$ mm
$m = 0.5$ kg $p_c = 0.4$ MPa $= 0.4$ N/mm^2
$p_s = 0.02$ MPa $= 0.02$ N/mm^2 $h = 25$ mm
$N = 600$ rpm

Step I *Total force on rocker arm*

The gas load P_g is given by,

$$P_g = \left(\frac{\pi d_v^2}{4}\right) p_c = \left(\frac{\pi (75)^2}{4}\right)(0.4)$$
$$= 1767.15 \text{ N} \qquad \text{(a)}$$

The initial spring force P_i is given by,

$$P_i = \left(\frac{\pi d_v^2}{4}\right) p_s = \left(\frac{\pi (75)^2}{4}\right)(0.02)$$
$$= 88.36 \text{ N} \qquad \text{(b)}$$

The acceleration of the valve is calculated by using the following steps:

(i) *Speed of camshaft*

For a four-stroke engine,

$$\text{speed of camshaft} = \frac{1}{2}\text{(speed of crankshaft)}$$
$$= \frac{N}{2} = \frac{600}{2} = 300 \text{ rpm}$$

(ii) *Angle turned by camshaft per second*

Since, 1 revolution = 360°

Angle turned by camshaft per second = (Number of revolutions/s)(360°)

$$= \left(\frac{300}{60}\right)(360) = 1800° \text{ per s}$$

(iii) Total crankshaft angle when the valve is open = 33 + 180 + 1 = 214°

(iv) Total angle of cam action

For a four-stroke engine,

$$\text{Angle of camshaft} = \frac{1}{2}\text{(angle of crankshaft)}$$
$$= \frac{1}{2}(214) = 107°$$

(v) Time taken by valve to open and close

Since, 1800° of cam rotation = 1 second

Time taken by the valve to open

$$= \frac{\text{Angle of cam action}}{\text{Angle turned by camshaft per second}}$$
$$= \frac{107}{1800} = 0.059 \text{ s}$$

(vi) $$\omega = \frac{2\pi}{t} = \left(\frac{2\pi}{0.059}\right)$$

(vii) $$r = \frac{h}{2} = \frac{25}{2} = 12.5 \text{ mm} = (12.5 \times 10^{-3}) \text{ m}$$

(vii) $$\alpha = \omega^2 r = \left(\frac{2\pi}{0.059}\right)^2 (12.5 \times 10^{-3})$$
$$= 141.76 \text{ m/s}^2$$

The inertia force P_a is given by,

$$P_a = m\alpha = 0.5\,(141.76) = 70.88 \text{ N} \qquad \text{(c)}$$

Adding (a), (b) and (c), the total force (P_e) on the rocker arm of the exhaust valve is given by,

$$P_e = P_g + P_a + P_i = 1767.15 + 70.88 + 88.36$$
$$= 1926.39 \text{ N}$$

Assumption 1 The permissible bending stress is 70 N/mm^2.

Assumption 2 The permissible shear stress is 42 N/mm^2.

Assumption 3 The permissible compressive stress is 50 N/mm^2.

Assumption 4 The permissible bearing pressure is 5 MPa.

Step II *Design of fulcrum pin*

The reaction at the fulcrum pin is given by,

$$R_f = \sqrt{(P_e)^2 + (P_c)^2 - 2P_eP_c\cos\theta}$$

When arms are equal,

$a = b$ $P_e = P_c = 1926.39$ N and $\theta = 135°$

$$R_f = \sqrt{(1926.39)^2 + (1926.39)^2 - 2(1926.39)(1926.39)\cos(135^\circ)}$$

$$= 1926.39\sqrt{1 + 1 - 2\cos(135)} = 3559.5 \text{ N}$$

Suppose,

d_1 = diameter of fulcrum pin (mm)

l_1 = length of fulcrum pin (mm)

Considering the bearing at the fulcrum pin,

$$p_b = \frac{R_f}{d_1 l_1} \qquad (p_b = 5 \text{ N/mm}^2)$$

The (l/d) ratio for fulcrum pin is assumed as 1.25. Therefore,

$$5 = \frac{3559.5}{d_1(1.25\,d_1)} \qquad d_1^2 = 569.52$$

$d_1 = 23.86$ or 25 mm

$l_1 = 1.25 d_1 = 1.25\,(25) = 31.25$ or 32 mm

The pin should be checked for double shear stress. The shear stress is given by,

$$\tau = \frac{R_f}{2\left(\dfrac{\pi d_1^2}{4}\right)} = \frac{3559.5}{2\left(\dfrac{\pi(25)^2}{4}\right)}$$

$$= 3.63 \text{ N/mm}^2 \qquad \tau < 42 \text{ N/mm}^2$$

The outside diameter of the boss of the rocker arm at the fulcrum pin is assumed as twice the diameter of the fulcrum pin.

Outside diameter of the boss at the fulcrum pin = $D_1 = 2d_1 = 2(25) = 50$ mm

A phosphor-bronze bush of 3-mm thickness is press fitted in the pinhole of the rocker arm.

Inside diameter of hole at the boss

$= d_1 + 3 \times 2 = 25 + 3 \times 2 = 31$ mm

The cross-section of the rocker arm passing through the fulcrum pin is shown in Fig. 25.40. Let us check this cross-section for bending stress.

$$M_b = P_e \times a = 1926.39 \times 180$$
$$= 346.75 \times 10^3 \text{ N-mm}$$

$$I = \frac{(32)\times(50)^3}{12} - \frac{(32)\times(31)^3}{12} = 253.89 \times 10^3 \text{ mm}^4$$

$$y = \frac{50}{2} = 25 \text{ mm}$$

$$\sigma_b = \frac{M_b y}{I} = \frac{(346.75 \times 10^3)(25)}{(253.89 \times 10^3)} = 34.14 \text{ N/mm}^2$$

$$\sigma_b < 70 \text{ N/mm}^2$$

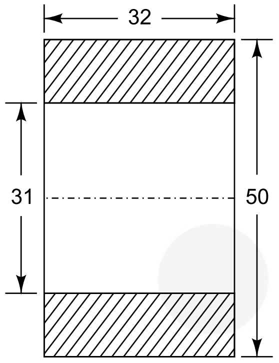


Fig. 25.40

Step III *Design of forked end*

The forked end carries a roller that revolves on the pin. The force acting on the roller pin is P_c.

$$P_c = P_e = 1926.39 \text{ N}$$

Suppose,

d_2 = diameter of roller pin (mm)

l_2 = length of roller pin (mm)

Considering the bearing at the roller pin,

$$p_b = \frac{P_c}{d_2 l_2} \qquad (p_b = 5 \text{ N/mm}^2)$$

The (l/d) ratio for the roller pin is taken as 1.25.

$$5 = \frac{1926.39}{d_2(1.25 d_2)} \qquad d_2^2 = 308.22$$

$d_2 = 17.56$ or 20 mm

$l_2 = 1.25\, d_2 = 1.25\,(20) = 25$ mm

The shear stress is given by,

$$\tau = \frac{P_c}{2\left(\dfrac{\pi d_2^2}{4}\right)} = \frac{1926.39}{2\left(\dfrac{\pi(20)^2}{4}\right)} = 3.07 \text{ N/mm}^2$$

$$\tau < 42 \text{ N/mm}^2$$

The roller pin is fixed in the eye. The dimensions of the eye are as follows:

Thickness of each eye = $l_2/2 = 25/2 = 12.5$ mm

Outer diameter of eye = $D_2 = 2d_2 = 2\,(20)$
= 40 mm

The diameter of the roller should be more than the outer diameter of the eye by at least 5 mm.

Diameter of roller = 40 + 5 = 45 or 50 mm

When the roller pin is tight in the two eyes of the fork, failure occurs due to shear. On the other hand, when the pin is loose, it is subjected to bending moment. It acts as a beam as shown in Fig. 25.38. It is assumed that the load acting on the pin is uniformly distributed in the portion that is in contact with the roller. It is uniformly varying in two parts of the fork.

The bending moment is maximum at the central plane of the pin. From Eq. 25.89,

$$M_b = \left(\frac{5}{24}\right) P_c l_2 = \left(\frac{5}{24}\right)(1926.39)(25)$$

$$= 10\,033.28 \text{ N-mm}$$

$$Z = \frac{\pi d_2^3}{32} = \frac{\pi (20)^3}{32} = 785.4 \text{ mm}^3$$

$$\sigma_b = \frac{M_b}{Z} = \frac{10\,033.28}{785.4} = 12.77 \text{ N/mm}^2$$

$$\sigma_b < 70 \text{ N/mm}^2$$

Step IV *Design of cross-section of rocker arm*

As shown in Fig. 25.39(a), the arm is subjected to bending moment under the action of the force P_e. We will consider the cross-section of arm near the boss of the fulcrum pin. The outside diameter of the boss of rocker arm at the fulcrum pin is twice the diameter of fulcrum pin ($2d_1$). Therefore,

$$M_b = P_e [a - d_1] = 1926.39 [180 - 25]$$

$$= 298.59 \times 10^3 \text{ N-mm}$$

The proportions of cross-section for the rocker arm are shown in Fig. 25.39(b).

From Eq. 25.91,

$$I = (37t^4) \quad \text{and} \quad y = (3t)$$

$$\sigma_b = \frac{M_b y}{I} \quad \text{or} \quad 70 = \frac{298.59 \times 10^3 (3t)}{(37t^4)}$$

$$t^3 = 345.86$$

$$t = 7.02 \text{ or } 8 \text{ mm}$$

The dimensions of cross-section as shown in Fig. 25.39(b) are as follows:

Width of flange = $2.5t = 2.5(8) = 20$ mm

Depth of section = $6t = 6(8) = 48$ mm

Step V *Design of tappet*

A tappet is a stud which is subjected to compressive force (P_e). Suppose,

d_c = core diameter of stud (mm)

$$\sigma_c = \frac{P_e}{\left(\frac{\pi d_c^2}{4}\right)} \quad \text{or} \quad 50 = \frac{1926.39}{\left(\frac{\pi d_c^2}{4}\right)}$$

$$d_c^2 = 49.06$$

$$d_c = 7 \text{ mm}$$

The nominal diameter (d) of the stud is given by,

$$d = \frac{d_c}{0.8} = \frac{7}{0.8} = 8.75 \text{ or } 10 \text{ mm}$$

The diameter of the circular end of the rocker arm (D_3) and its depth (t_3) are calculated by the following relationships:

$$D_3 = 2d = 2(10) = 20 \text{ mm}$$

$$t_3 = 2d = 2(10) = 20 \text{ mm}$$

Example 25.22 *Design a valve spring for the exhaust valve of a four-stroke engine using the following data:*

Diameter of valve head = 75 mm

Lift of valve = 25 mm

Maximum suction pressure = 0.02 MPa below atmosphere

Stiffness of spring = 10 N/mm

Spring index = 8

Permissible torsional shear stress for spring wire = 300 N/mm²

Modulus of rigidity = 84 × 10³ N/mm²

Total gap between consecutive coils, when the spring is subjected to maximum force, can be taken as 15% of the maximum compression.

Solution

Given $d_v = 75$ mm

$p_s = 0.02$ MPa $= 0.02$ N/mm² $\quad h = 25$ mm

$k = 10$ N/mm $\quad C = 8 \quad \tau = 300$ N/mm²

$G = 84 \times 10^3$ N/mm²

Step I *Maximum force on spring*

The initial spring force P_i is given by,

$$P_i = \left(\frac{\pi d_v^2}{4}\right) p_s = \left(\frac{\pi (75)^2}{4}\right)(0.02) = 88.36 \text{ N}$$

The maximum force on spring ($P_{max.}$) is given by,

$$P_{max.} = P_i + kd = 88.36 + 10\,(25) = 338.36 \text{ N} \quad (\delta = h)$$

Step II *Wire diameter*
The Wahl factor is given by,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4(8)-1}{4(8)-4} + \frac{0.615}{8} = 1.184$$

$$\tau = K\left(\frac{8P_{max.}C}{\pi d^2}\right) \text{ or } 300 = 1.184\left(\frac{8(338.36)(8)}{\pi d^2}\right)$$

$d^2 = 27.2$
$d = 5.22$ or 6 mm

Step III *Mean coil diameter*

$$D = Cd = 8(6) = 48 \text{ mm}$$

Step IV *Number of active turns*

$$N = \frac{Gd^4}{8D^3k} = \frac{(84\times10^3)(6)^4}{8(48)^3(10)} = 12.3 \text{ or } 13$$

Step V *Total number of turns*
For square and ground ends

$$N_t = N + 2 = 13 + 2 = 15$$

Step VI *Maximum compression of spring*

$$\delta_{max.} = \frac{8P_{max.}D^3N}{Gd^4} = \frac{8(338.36)(48)^3(13)}{(84\times10^3)(6)^4}$$

$$= 35.75 \text{ mm}$$

Step VII *Solid length of spring*

Solid length $= N_t\, d = 15(6) = 90$ mm

Step VIII *Free length of spring*

Free length = solid length + $\delta_{max.} + 0.15\delta_{max.}$
$= 90 + 35.75 + 0.15\,(35.75)$
$= 131.11$ mm

Step IX *Pitch of coils*

$$\text{Pitch of coils} = \frac{\text{free length}}{(N_t - 1)} = \frac{131.11}{(15-1)} = 9.37 \text{ mm}$$

Example 25.23 *Design the valve gear mechanism for the exhaust valve of a horizontal diesel engine using the following data:*

Cylinder bore = 250 mm
Length of stroke = 300 mm
Engine speed = 450 rpm
Maximum gas pressure = 3.5 MPa
Effective length of each arm = 150 mm
Angle between two arms = 165°
Seat angle of valve = 45°
Mass of valve = 0.5 kg
Back pressure when the exhaust valve opens = 0.4 MPa
Maximum suction pressure = 0.02 MPa (below atmosphere)

The valve opens 33° before outer dead centre and closes 1° after inner dead centre. The valve is to open and close with constant acceleration and deceleration for each half of the lift. Assume suitable data and state the assumptions you make.

Solution

Given $D = 250$ mm $l = 300$ mm $N = 450$ rpm
$p_{max.} = 3.5$ MPa $= 3.5$ N/mm^2 $a = 150$ mm
$\theta = 165°$ $m = 0.5$ kg $\alpha = 45°$
$p_c = 0.4$ MPa $= 0.4$ N/mm^2
$p_s = 0.02$ MPa $= 0.02$ N/mm^2

Part 1: Design of valve

Step I *Diameter of valve port*

$$a = \text{area of piston} = \left(\frac{\pi(250)^2}{4}\right) \text{mm}^2$$

v = mean velocity of piston

$$= 2l\left(\frac{N}{60}\right) = 2\left(\frac{300}{1000}\right)\left(\frac{450}{60}\right) \text{ m/s}$$

Assume [from Table 25.2],

$v_p = 50$ m/s

$$a_p = \left(\frac{\pi d_p^2}{4}\right) \text{mm}^2$$

Therefore,

$$a \times v = a_p \times v_p$$

$$\left(\frac{\pi(250)^2}{4}\right)\times 2\left(\frac{300}{1000}\right)\left(\frac{450}{60}\right) = \left(\frac{\pi d_p^2}{4}\right)\times(50)$$

$d_p^2 = 5625$ mm^2
$d_p = 75$ mm (i)

Step II *Diameter of valve head*

For a seat angle of 45°, the projected width of the valve seat is given by following empirical relationship:

$$w = (0.05 \text{ to } 0.07)\, d_p$$

or $\quad w = 0.06\, d_p = 0.06\,(75) = 4.5$ mm

The diameter of the valve head (d_v) is given by,

$$d_v = (d_p + 2w) = 75 + 2 \times 4.5 = 84 \text{ mm} \qquad \text{(ii)}$$

Step III *Thickness of the valve head*

For steel valve,

$k = 0.42 \qquad$ Assume $\sigma_b = 70$ N/mm^2

$$t = kd_p \sqrt{\frac{p_{\max.}}{\sigma_b}} = (0.42)(75)\sqrt{\frac{3.5}{70}} = 7.04 \text{ or } 8 \text{ mm} \qquad \text{(iii)}$$

Step IV *Diameter of valve stem*

$$d_s = \left[\frac{d_p}{8} + 6.35\right] \text{ to } \left[\frac{d_p}{8} + 11\right]$$

$$= \left[\frac{75}{8} + 6.35\right] \text{ to } \left[\frac{75}{8} + 11\right]$$

$$= 15.73 \text{ to } 20.38 \text{ mm}$$

$$d_s = 18 \text{ mm} \qquad \text{(iv)}$$

Step V *Maximum lift of valve*

$$h_{\max.} = \frac{d_p}{4\cos\alpha} = \frac{75}{4\cos(45)°} = 26.52 \text{ or } 27 \text{ mm} \qquad \text{(v)}$$

Part 2: Forces acting on rocker arm

The gas load P_g is given by,

$$P_g = \left(\frac{\pi d_v^2}{4}\right) p_c = \left(\frac{\pi (84)^2}{4}\right)(0.4)$$

$$= 2216.71 \text{ N} \qquad \text{(a)}$$

The initial spring force P_i is given by.

$$P_i = \left(\frac{\pi d_v^2}{4}\right) p_s = \left(\frac{\pi (84)^2}{4}\right)(0.02)$$

$$= 110.84 \text{ N} \qquad \text{(b)}$$

The acceleration of the valve is calculated by using the following steps:

(*i*) *Speed of camshaft*

For four-stroke engine,

$$\text{Speed of camshaft} = \frac{1}{2}(\text{speed of crankshaft})$$

$$= \frac{N}{2} = \frac{450}{2} = 225 \text{ rpm}$$

(*ii*) *Angle turned by camshaft per second*

Since, 1 revolution = 360°

Angle turned by camshaft per second = (Number of revolutions/s)(360°)

$$= \left(\frac{225}{60}\right)(360) = 1350° \text{ per s}$$

In other words, 1350° of cam rotation takes 1 second.

(*iii*) *Total crankshaft angle when the valve is open* = 33 + 180 + 1 = 214°

(*iv*) *Total angle of cam action* For four-stroke engine,

$$\text{angle of camshaft} = \frac{1}{2}(\text{angle of crankshaft})$$

$$= \frac{1}{2}(214) = 107°$$

As shown in Fig. 25.41, during half of this rotation, the valve will open and in the remaining half, the valve will close. Therefore, cam rotation for valve opening is (107/2) or 53.5°. Out of this, half the angle is occupied by constant acceleration and half the angle by constant deceleration. Therefore, cam rotation during the constant acceleration period is (53.5/2) or 26.75°. The valve lift is 27 mm. The valve will be raised by (27/2) or 13.5 mm during constant acceleration and 13.5 mm during the constant deceleration period.

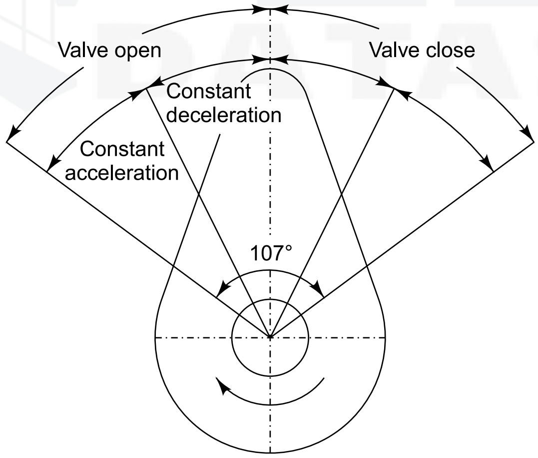


Fig. 25.41

Cam angle during constant acceleration = 26.75°

Valve lift during constant acceleration (s)
$= 13.5 \text{ mm} = 13.5 \times 10^{-3}$ m

(v) *Time taken by valve during constant acceleration period*

Since, 1350° of cam rotation = 1 second

Time taken by valve during constant acceleration period (t)

$$= \frac{\text{Angle of cam action}}{\text{Angle turned by camshaft per second}}$$

$$= \frac{26.75}{1350} = 0.0198 \text{ s}$$

$$s = ut + \frac{1}{2}\alpha t^2$$

$s = 13.5 \times 10^{-3}$ m $\quad u = 0 \quad t = 0.0198$ s

Substituting,

$$13.5 \times 10^{-3} = 0\,(0.0198) + \frac{1}{2}\alpha\,(0.0198)^2$$

$\alpha = 68.87 \text{ m/s}^2$

The inertia force P_a is given by,

$$P_a = m\alpha = 0.5(68.87) = 34.44 \text{ N} \qquad \text{(c)}$$

Adding (a), (b) and (c), the total force (P_e) on the rocker arm of the exhaust valve is given by,

$$P_e = P_g + P_a + P_i = 2216.71 + 34.44 + 110.84$$
$$= 2361.99 \quad \text{or} \quad 2362 \text{ N}$$

Assumptions

(i) The permissible bending stress is 70 N/mm^2.

(ii) The permissible shear stress is 42 N/mm^2.

(iii) The permissible compressive stress is 50 N/mm^2.

(iv) The permissible bearing pressure is 5 MPa.

(v) The (l/d) ratio for fulcrum and roller pins is 1.25.

Part 3: Design of fulcrum pin

The reaction at the fulcrum pin is given by,

$$R_f = \sqrt{(P_e)^2 + (P_c)^2 - 2P_eP_c\cos\theta}$$

When arms are equal,

$a = b \quad P_e = P_c = 2362$ N $\quad$ and $\quad \theta = 165°$

$$R_f = \sqrt{(2362)^2 + (2362)^2 - 2(2362)(2362)\cos(165)}$$

$$= 2362\sqrt{1 + 1 - 2\cos(165)} = 4683.59 \text{ N}$$

Suppose,

d_1 = diameter of fulcrum pin (mm)

l_1 = length of fulcrum pin (mm)

Considering the bearing at the fulcrum pin,

$$p_b = \frac{R_f}{d_1 l_1} \qquad (p_b = 5 \text{ N/mm}^2)\ (l_1/d_1 = 1.25)$$

Substituting,

$$5 = \frac{4683.59}{d_1(1.25d_1)} \qquad d_1^2 = 749.37$$

$d_1 = 27.37$ or 30 mm

$l_1 = 1.25d_1 = 1.25\,(30) = 37.5$ or 40 mm

The pin should be checked for double shear stress. The shear stress is given by,

$$\tau = \frac{R_f}{2\left(\frac{\pi d_1^2}{4}\right)} = \frac{4683.59}{2\left(\frac{\pi(30)^2}{4}\right)} = 3.31 \text{ N/mm}^2$$

$\tau < 42$ N/mm^2

The outside diameter of the boss of the rocker arm at the fulcrum pin is assumed as twice the diameter of the fulcrum pin.

Outside diameter of boss at fulcrum pin = D_1 = $2\,d_1 = 2(30) = 60$ mm

A phosphor bronze bush of 3 mm thickness is press fitted in the pinhole of the rocker arm.

Inside diameter of hole at the boss = $d_1 + 3 \times 2$ = $30 + 3 \times 2 = 36$ mm

The cross-section of the rocker arm passing through the fulcrum pin is shown in Fig. 25.42. Let us check this cross-section for bending stress.

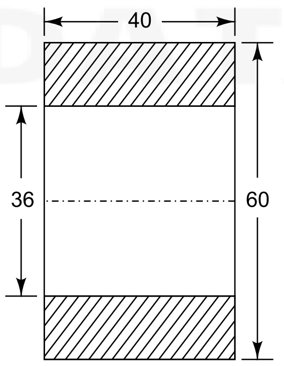


Fig. 25.42

$$M_b = P_e \times a = 2362 \times 150 = 354.3 \times 10^3 \text{ N-mm}$$

$$I = \frac{(40)\times(60)^3}{12} - \frac{(40)\times(36)^3}{12}$$

$$= \frac{1}{12}(40)[(60)^3 - (36)^3] = 564.48 \times 10^3 \text{ mm}^4$$

$$y = \frac{60}{2} = 30 \text{ mm}$$

$$\sigma_b = \frac{M_b y}{I} = \frac{(354.3\times10^3)(30)}{(564.48\times10^3)} = 18.83 \text{ N/mm}^2$$

$$\sigma_b < 70 \text{ N/mm}^2$$

Part 4: Design of forked end

The forked end carries a roller that revolves on the pin. The force acting on the roller pin is P_c.

$$P_c = P_e = 2362 \text{ N}$$

Suppose,

d_2 = diameter of roller pin (mm)

l_2 = length of roller pin (mm)

Considering the bearing at the roller pin,

$$p_b = \frac{P_c}{d_2 l_2} \qquad (p_b = 5 \text{ N/mm}^2)\ (l_2/d_2 = 1.25)$$

Substituting,

$$5 = \frac{2362}{d_2(1.25d_2)} \qquad d_2^2 = 377.92$$

$$d_2 = 19.44 \text{ or } 20 \text{ mm}$$

$$l_2 = 1.25d_2 = 1.25\ (20) = 25 \text{ mm}$$

The shear stress is given by,

$$\tau = \frac{P_c}{2\left(\frac{\pi d_2^2}{4}\right)} = \frac{2362}{2\left(\frac{\pi(20)^2}{4}\right)} = 3.76 \text{ N/mm}^2$$

$$\tau < 42 \text{ N/mm}^2$$

The roller pin is fixed in the eye. The dimensions of the eye are as follows:

Thickness of each eye = $l_2/2 = 25/2 = 12.5$ mm

Outer diameter of eye = $D_2 = 2d_2 = 2(20)$
= 40 mm

The diameter of the roller should be more than the outer diameter of the eye by at least 5 mm.

Diameter of roller = 40 + 5 = 45 or 50 mm

When the roller pin is tight in the two eyes of the fork, failure occurs due to shear. On the other hand, when the pin is loose, it is subjected to bending moment. It acts as a beam as shown in Fig. 25.38. It is assumed that the load acting on the pin is uniformly distributed in the portion that is in contact with the roller. It is uniformly varying in two parts of the fork.

The bending moment is maximum at the central plane of the pin. From Eq. 25.89,

$$M_b = \left(\frac{5}{24}\right)P_c l_2 = \left(\frac{5}{24}\right)(2362)(25)$$
$$= 12\,302.08 \text{ N-mm}$$

$$Z = \frac{\pi d_2^3}{32} = \frac{\pi(20)^3}{32} = 785.4 \text{ mm}^3$$

$$\sigma_b = \frac{M_b}{Z} = \frac{12\,302.08}{785.4} = 15.66 \text{ N/mm}^2$$

$$\sigma_b < 70 \text{ N/mm}^2$$

Part 5: Design of cross-section of rocker arm

As shown in Fig. 25.39(a), the arm is subjected to bending moment under the action of the force P_e. We will consider the cross-section of the arm near the boss of the fulcrum pin. The outside diameter of the boss of the rocker arm at the fulcrum pin is twice the diameter of the fulcrum pin ($2d_1$). Therefore,

$$M_b = P_e\,[a - d_1] = 2362\,[150 - 30]$$
$$= 283.44 \times 10^3 \text{ N-mm}$$

The proportions of cross-section for the rocker arm are shown in Fig. 25.43.

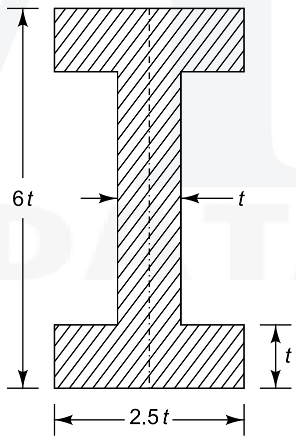


Fig. 25.43

From Eq. 25.91,

$$I = (37t^4) \quad \text{and} \quad y = (3t)$$

$$\sigma_b = \frac{M_b y}{I} \quad \text{or} \quad 70 = \frac{283.44\times10^3(3t)}{(37t^4)}$$

$$t^3 = 328.31$$

$$t = 6.9 \text{ or } 7 \text{ mm}$$

The dimensions of cross-section as shown in Fig. 25.43 are as follows:

Width of flange = $2.5t = 2.5(7) = 17.5$ mm
Depth of section = $6t = 6(7) = 42$ mm

Part 6: Design of tappet

A tappet is a stud, which is subjected to compressive force (P_e). Suppose,

d_c = core diameter of stud (mm)

$$\sigma_c = \frac{P_e}{\left(\frac{\pi d_c^2}{4}\right)} \quad \text{or} \quad 50 = \frac{2362}{\left(\frac{\pi d_c^2}{4}\right)}$$

$$d_c^2 = 60.15$$

$$d_c = 7.76 \text{ mm}$$

The nominal diameter (d) of the stud is given by,

$$d = \frac{d_c}{0.8} = \frac{7.76}{0.8} = 9.7 \text{ or } 10 \text{ mm}$$

The diameter of the circular end of the rocker arm (D_3) and its depth (t_3) are calculated by the following relationships:

$$D_3 = 2d = 2(10) = 20 \text{ mm}$$
$$t_3 = 2d = 2(10) = 20 \text{ mm}$$

Part 7: Design of valve spring

Assumptions

(i) The spring index is 8.
(ii) The stiffness of spring is 10 N/mm.
(iii) Permissible torsional shear stress for the spring wire is 300 N/mm^2
(iv) Modulus of rigidity for the spring wire is 84×10^3 N/mm^2
(v) Total gap between consecutive coils, when the spring is compressed by maximum force, is 15% of maximum compression.

Step I *Maximum force on spring*

The initial spring force P_i is calculated as,

$$P_i = 110.84 \text{ N}$$

The maximum force on spring ($P_{max.}$) is given by,

$$P_{max.} = P_i + k\delta = 110.84 + 10\,(27)$$
$$= 380.84 \text{ N} \qquad (\delta = h = 27 \text{ mm})$$

Step II *Wire diameter*

The Wahl factor is given by,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4(8)-1}{4(8)-4} + \frac{0.615}{8} = 1.184$$

$$\tau = K\left(\frac{8P_{max.}C}{\pi d^2}\right) \quad \text{or} \quad 300 = 1.184\left(\frac{8(380.84)(8)}{\pi d^2}\right)$$

$$d^2 = 30.62$$
$$d = 5.53 \text{ or } 6 \text{ mm}$$

Step III *Mean coil diameter*

$$D = Cd = 8\,(6) = 48 \text{ mm}$$

Step IV *Number of active turns*

$$N = \frac{Gd^4}{8D^3k} = \frac{(84\times10^3)(6)^4}{8(48)^3(10)} = 12.3 \text{ or } 13$$

Step V *Total number of turns*

For square and ground ends

$$N_t = N + 2 = 13 + 2 = 15$$

Step VI *Maximum compression of spring*

$$\delta_{max.} = \frac{8P_{max.}D^3N}{Gd^4} = \frac{8(380.84)(48)^3(13)}{(84\times10^3)(6)^4}$$
$$= 40.24 \text{ mm}$$

Step VII *Solid length of spring*

Solid length = $N_t\,d = 15\,(6) = 90$ mm

Step VIII *Free length of spring*

Free length = solid length + $\delta_{max.}$ + 0.15 $\delta_{max.}$
= 90 + 40.24 + 0.15 (40.24)
= 136.28 mm

Step IX *Pitch of coils*

$$\text{Pitch of coils} = \frac{\text{free length}}{(N_t - 1)} = \frac{136.28}{(15-1)}$$
$$= 9.73 \text{ mm}$$

Part 8: Design of cam

The diameter of camshaft (D') is obtained by the following empirical relationship:

$$D' = 0.16D + 12.5$$

where,

D = cylinder bore (mm)

$$D' = 0.16D + 12.5 = 0.16\,(250) + 12.5$$
$$= 52.5 \text{ or } 55 \text{ mm}$$

The base circle diameter of the cam is at least 5 mm more than the camshaft diameter.

Base circle diameter of cam = 55 + 5 = 60 mm

The diameter of the roller is calculated in the design of the forked arm.

Roller diameter = 50 mm
Roller width = l_2 = 25 mm
Cam width = roller width = 25 mm
Lift of valve = 27 mm

Displacement diagram

The displacement diagram is shown in Fig. 25.44. The step by step approach to construct this diagram is as follows:

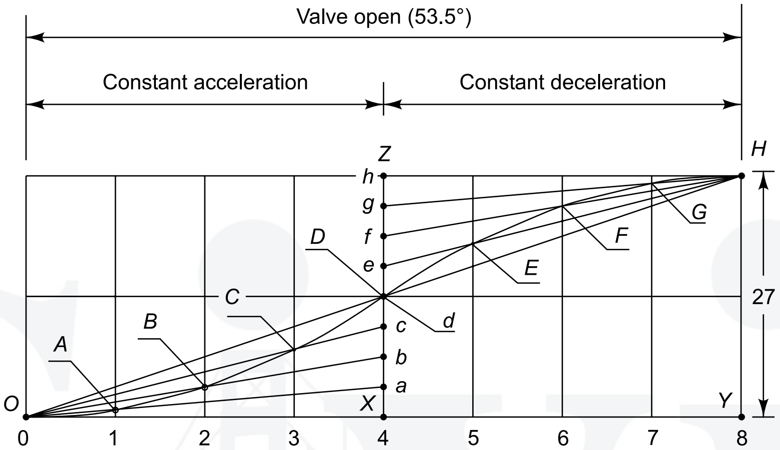


Fig. 25.44 *Displacement Diagram*

(i) Draw a horizontal line *OY*. It represents the angular rotation of the cam when the valve opens, i.e., 53.5°. It is divided into two equal parts—*OX* and *XY*. The line *OX* indicates the constant acceleration phase (26.75°) and line *XY* represents the constant deceleration phase (26.75°).

(ii) Divide lines *OX* and *XY* each into four equal parts. The total of eight parts are shown by numbers 1, 2, 3, …, 8. Each part represents (53.5º/8) or 6.6875º of cam rotation.

(iii) Draw a vertical line *YH* equal to the valve lift, i.e., 27 mm.

(iv) Draw ordinates through points 1, 2, 3, ……, 8.

(v) Draw a vertical line *XZ* equal to the valve lift (i.e., 27 mm). Divide the line *XZ* into eight equal parts. They are shown by points *a*, *b*, *c*, …, *h*. Therefore, each part represents (27/8) or 3.375 mm of the valve lift.

(vi) Join lines *Oa*, *Ob*, *Oc*, *Od* that intersects the ordinates through 1, 2, 3 and 4 at points *A*, *B*, *C*, and *D* respectively.

(vii) Also, join lines *Hd*, *He*, *Hf*, and *Hg* that intersect the ordinates through 4, 5, 6 and 7 at points *D*, *E*, *F*, and *G* respectively.

(viii) Draw a smooth curve passing through points *O*, *A*, *B*, *C* and *D*. It represents the parabolic curve for the constant acceleration phase.

(ix) Similarly, draw a smooth curve passing through points *D*, *E*, *F*, *G*, and *H*. It represents the parabolic curve for the constant deceleration phase.

Cam profile

The profile of the cam is shown in Fig. 25.45. The step by step approach to construct this profile is as follows:

(i) Draw a base circle with O_1 as centre and radius equal to 30 mm.

(ii) Draw a prime circle with O_1 as centre and radius equal to (radius of base circle + radius of roller), i.e., (30 + 25) or 55 mm.

(iii) Draw angle $\angle OO_1H$ equal to 53.5º. It represents the angular rotation of cam when the valve opens.

(iv) Divide the angle $\angle OO_1H$ (53.5º) into eight equal parts. Therefore, each part represents (53.5º/8) or 6.6875º of cam rotation. The eight parts are shown by angles $\angle O\ O_1 1$, $\angle 1 O_1 2$, $\angle 2\ O_1 3$, etc.

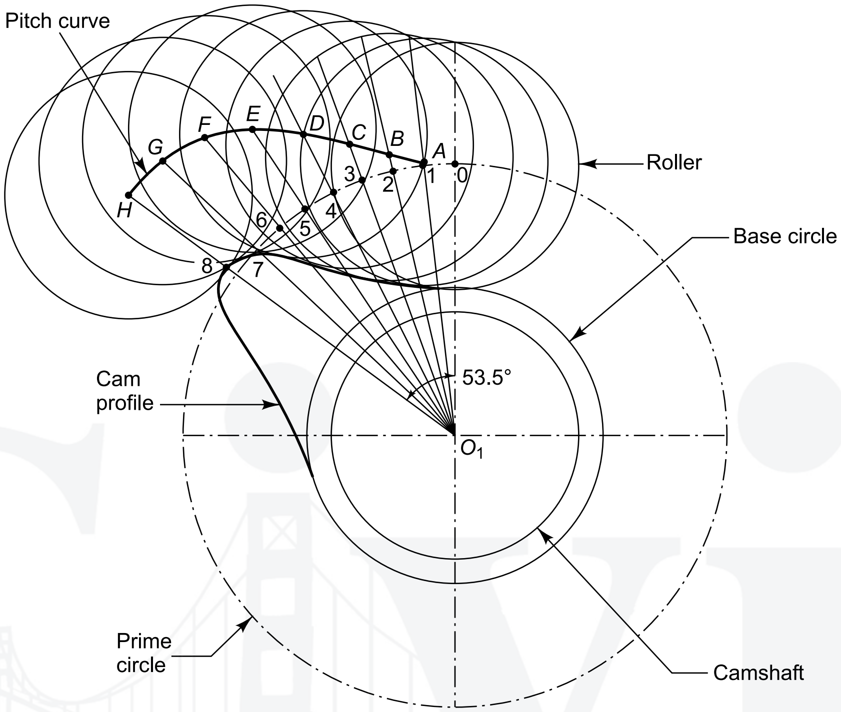


Fig. 25.45 *Cam Profile*

(v) Join points 1, 2, 3, etc., with the centre O_1 and extend the radial lines beyond the prime circle as shown in Fig. 25.45.

(vi) Mark the distances 1*A*, 2*B*, 3*C*,… etc., from the displacement diagram shown in Fig. 25.44, on the extension of radial lines beyond the prime circle.

(vii) Draw a smooth curve passing through points *O*, *A*, *B*, *C*, *D*, *E*, *F*, *G*, and *H*. This curve is called the 'pitch curve'.

(viii) Draw circles with radii equal to the radius of the roller (25 mm) and centres at points *O*, *A*, *B*, *C*, *D*, *E*, *F*, *G*, and *H*.

(ix) The profile of the cam is a curve drawn tangential to these circles at the bottom as shown in Fig. 25.45. This curve indicates the cam profile when the valve is open. Only half the profile is drawn, but it is symmetrical about the line O_1H.

Short-Answer Questions

25.1 What are the functions of engine cylinder?

25.2 What are the cooling systems for engine cylinders? Where do you use them?

25.3 What are the advantages of cylinder liner?

25.4 What are dry and wet cylinder liners?

25.5 What are the desirable properties of cylinder materials?

25.6 Name the materials used for engine cylinder.

25.7 What do you understand by 'bore' of cylinder?

25.8 What are the functions of piston?

25.9 What are the design requirements of piston?

25.10 Name the materials used for engine piston.

25.11 What are the advantages and disadvantages of aluminium piston over cast iron piston?

25.12 Why is piston made lightweight?

25.13 Name two criteria for calculating the thickness of piston head.

25.14 Why is piston clearance necessary? What is its usual value?
25.15 What are the functions of piston ribs?
25.16 What is the function of the cup on piston head?
25.17 What are the functions of compression piston rings?
25.18 What are the functions of oil scraper rings?
25.19 Name the materials used for piston rings.
25.20 Why are more number of thin piston rings preferred over small number of thick rings?
25.21 Name two design criteria for piston pin.
25.22 How is piston pin secured to piston?
25.23 Why is piston pin located at or above the middle of the skirt length?
25.24 What is the function of connecting rod?
25.25 What is the manufacturing method for connecting rod?
25.26 Name the materials used for connecting rod.
25.27 What are the lubricating methods for bearings at small and big ends of the connecting rod?
25.28 What are the forces acting on the connecting rod?
25.29 Why are connecting rods made of I sections?
25.30 What is the force on bolts of big end of connecting rod?
25.31 What is the difference between centre and overhung crankshafts?
25.32 Where do you use overhung crankshafts?
25.33 Where do you use centre crankshafts?
25.34 What is the main advantage of overhung crankshafts?
25.35 Name the materials for crankshafts.
25.36 What is the manufacturing method for crankshaft?
25.37 When do you use push rod?
25.38 Why is the design of exhaust valve more critical than that of an inlet valve?
25.39 Why is the area of inlet valve port more than that of an exhaust valve?
25.40 Why do inlet and exhaust valves have conical heads and seats?
25.41 What is the function of rocker arm?
25.42 Why is rocker arm made of I section?
25.43 Name the materials for rocker arm.
25.44 What is tappet? What is the stress in tappet?
25.45 What is the purpose of valve spring?
25.46 What is the criterion for design of push rod?

Problems for Practice

25.1 The cylinder of a four-stroke diesel engine has the following specifications:
Brake power = 3 kW
Speed = 800 rpm
Indicated mean effective pressure = 0.3 MPa
Mechanical efficiency = 80%
Determine the bore and length of the cylinder liner.
[*D* = *118 mm (116.75); L = 204 mm (203.55)*]

25.2 The cylinder of a four-stroke diesel engine has the following specifications:
Cylinder bore = 150 mm
Maximum gas pressure = 3 MPa
Allowable tensile stress = 50 N/mm^2
Determine the thickness of cylinder wall. Also, calculate the apparent and net circumferential and longitudinal stresses in cylinder wall.
[t = *10 mm (8.5);* σ_c = *22.5 N/mm*^{2}; σ_l = *10.55 N/mm*^{2}; $(\sigma_c)_{net}$ = *19.86 N/mm*^{2}; $(\sigma_l)_{net}$ = *4.93 N/mm*^{2}]

25.3 The bore of a cylinder of the four-stroke diesel engine is 120 mm. The maximum gas pressure inside the cylinder is limited to 4 MPa. The cylinder head is made of cast iron and allowable tensile stress is 40 N/mm^2. Determine the thickness of cylinder head. The studs, which are made of steel, have allowable stress as 50 N/mm^2. Calculate (i) number of studs, (ii) nominal diameter of studs, and (iii) pitch of studs.
[t_h = *16 mm (15.27); z = 6; d = 18 mm (17.33); pitch = 91.11 mm*]

25.4 The following data is given for a four-stroke diesel engine:
Cylinder bore = 100 mm
Length of stroke = 125 mm
Speed = 2000 rpm
Brake mean effective pressure = 0.65 MPa
Maximum gas pressure = 5 MPa

Fuel consumption = 0.25 kg per BP per h
Higher calorific value of fuel = 42 000 kJ/kg
Assume that 5% of the total heat developed in the cylinder is transmitted by the piston. The piston is made of grey cast iron and the permissible tensile stress is 37.5 N/mm^2 (k = 46.6 W/m/°C). The temperature difference between the centre and edge of the piston head is 220°C.

(i) Calculate the thickness of the piston head by strength consideration.
(ii) Calculate the thickness of the piston head by thermal consideration.
(iii) Which criterion decides the thickness of the piston head?
(iv) State whether the ribs are required.
(v) If so, calculate the number and thickness of piston ribs.
(vi) State whether a cup is required in the top of piston head.
(vii) If so, calculate the radius of the cup.

[*(i) 15.81 mm (ii) 12.05 mm (iii) strength (t_h = 16 mm) (iv) yes (v) 4 ribs of 7 mm thickness (vi) yes (vii) 70 mm*]

25.5 The following data is given for the piston of a four-stroke diesel engine:
Cylinder bore = 100 mm
Material of piston rings = Grey cast iron
Allowable tensile stress = 90 N/mm^2
Allowable radial pressure on cylinder wall = 0.035 MPa
Thickness of piston head = 16 mm
Number of piston rings = 4
Calculate: (i) radial width of piston rings; (ii) axial thickness of piston rings; (iii) gap between the free ends of the piston ring before assembly; (iv) gap between the free ends of the piston ring after assembly; (v) width of top land; (vi) width of ring grooves; (vii) thickness of piston barrel; and (viii) thickness of barrel at open end.

[*(i) 4(3.42) mm (ii) 3 mm (iii) 15 mm (iv) 0.3 mm (v) 18 mm (vi) 2.5 mm (vii) 12 (11.9) mm (viii) 4 mm*]

25.6 The following data is given for the piston of a four-stroke diesel engine:
Cylinder bore = 100 mm
Maximum gas pressure = 5 MPa
Allowable bearing pressure for skirt = 0.45 MPa
Ratio of side thrust on liner to maximum gas load on piston = 0.1
Width of top land = 18 mm
Width of ring grooves = 2.5 mm
Total number of piston rings = 4
Axial thickness of piston rings = 3 mm
Calculate: (i) length of skirt; and (ii) length of piston.

[*(i) 87.27 mm (ii) 125 (124.77) mm*]

25.7 The following data is given for the piston of a four-stroke diesel engine:
Cylinder bore = 100 mm
Maximum gas pressure = 5 MPa
Bearing pressure at small end of connecting rod = 25 MPa
Length of piston pin in bush of small end = 0.45 D
Mean diameter of piston boss = 1.4 × outer diameter of piston pin
Allowable bending stress for piston pin = 140 N/mm^2
Calculate: (i) outer diameter of piston pin; (ii) inner diameter of piston pin; (iii) mean diameter of piston boss; and (iv) check the design for bending stresses.

[*(i) 35 (34.91) mm (ii) 20 (21) mm (iii) 50 (49) mm (iv) 130.53 N/mm^2*]

25.8 Determine the dimensions of cross-section of the connecting rod, illustrated in Fig. 25.14, for a diesel engine with the following data:
Cylinder bore = 100 mm
Length of connecting rod = 320 mm
Maximum gas pressure = 2.45 MPa
Factor of safety against buckling failure = 5

[*t = 6 (5.5) mm*]

25.9 Determine the dimensions of small and big end bearings of the connecting rod for a diesel engine with the following data:
Cylinder bore = 100 mm
Maximum gas pressure = 2.45 MPa
(l/d) ratio for piston pin bearing = 1.5
(l/d) ratio for crank pin bearing = 1.4

Allowable bearing pressure for piston pin bearing = 15 MPa
Allowable bearing pressure for crank pin bearing = 10 MPa

[d_p = 30 (29.24) mm l_p = 45 mm
d_c = 38 (37.07) mm l_c = 54 (53.2) mm]

25.10 The following data is given for the cap and bolts of the big end of the connecting rod:
Engine speed = 1500 rpm
Length of connecting rod = 320 mm
Length of stroke = 140 mm
Mass of reciprocating parts = 1.75 kg
Length of crank pin = 54 mm
Diameter of crank pin = 38 mm
Permissible tensile stress for bolts = 120 N/mm^2
Permissible bending stress for cap = 120 N/mm^2

Calculate the nominal diameter of bolts and thickness of cap for the big end.

[d = 6 mm (d_c = 4.42 mm), t_c = 6 (5.49) mm]

25.11 The following data is given for a connecting rod:
Engine speed = 1500 rpm
Length of connecting rod = 320 mm
Length of stroke = 140 mm
Density of material = 7830 kg/m^3
Thickness of web or flanges = 6 mm

Assume the cross-section illustrated in Fig. 25.14. For this cross-section,

$$A = 11t^2 \quad I_{xx} = \left(\frac{419}{12}\right)t^4 \text{ and } y = \left(\frac{5t}{2}\right)$$

Calculate whipping stress in connecting rod.

[11.66 N/mm^2]

25.12 The following data is given for the centre crankshaft of a single-cylinder vertical engine:
Cylinder bore = 150 mm
(L/r) ratio = 4.75
Maximum gas pressure = 4 MPa
Length of stroke = 200 mm
Weight of flywheel cum belt pulley = 3.5 kN
Total belt pull = 1.8 kN
Allowable bending stress = 75 N/mm^2
Allowable compressive stress = 75 N/mm^2
Allowable shear stress = 40 N/mm^2
Allowable bearing pressure = 10 MPa

The main bearings are 350 mm apart and the third bearing is 400 mm apart from the main bearing on its side. The belts are in the horizontal direction.

Consider the case of the crank at the top dead centre position and subjected to maximum bending moment. Assume [(l/d) ratio = 1] for crank pin and calculate: (i) vertical and horizontal components of reactions at three bearings; (ii) Diameter and length of crank pin; (iii) width and thickness of crank web; (iv) total compressive stress in crank web; and (v) diameter of the shaft under flywheel-cum belt pulley.

[(i) $(R_1)_v = (R_2)_v$ = 35342.92 N; $(R'_2)_v = (R'_3)_v$ = 1750 N; $(R'_2)_h = (R'_3)_h$ = 900 N
(ii) d_c = 95(94.35) mm and l_c = 95 mm
(iii) t = 70(66.5) mm and w = 110(108.3) mm
(iv) 40.98 N/mm^2 (v) d_s = 40 (37.67) mm]

25.13 Assume the data of Example 25.12 for the centre crankshaft of a single cylinder vertical engine. The torque on the crankshaft is maximum when the crank turns through 25° from top dead centre and at this position the gas pressure inside the cylinder is 3.75 MPa. Consider this position of the crank and the calculate: (i) components of force on crank pin; (ii) vertical and horizontal components of reactions at three bearings; (iii) diameter and length of the crank pin; (iv) diameter of shaft under flywheel; (v) diameter of shaft at the juncture of the right-hand crank web; (vi) maximum compressive stress in right-hand crank web; and (vii) maximum reaction at main bearing and bearing pressure.

[(i) P_t = 33 366.19 N and P_r = 57 559.7 N
(ii) $(R_1)_v = (R_2)_v$ = 28779.85 N; $(R_1)_h = (R_2)_h$ = 16 683.10 N; $(R'_2)_v = (R'_3)_v$ = 1750 N; $(R'_2)_h = (R'_3)_h$ = 900 N
(iii) $d_c = l_c$ = 95(87.74) mm (iv) d_s = 80(75.35) mm
(v) d_{s1} = 85(83.3) mm (vi) $(\sigma_c)_{max.}$ = 52.92 N/mm^2
(vii) R_2 = 32 231.2 N and p_b = 4.46 N/mm^2]

25.14 Design an exhaust valve for a horizontal diesel engine using the following data:

Cylinder bore = 250 mm
Length of stroke = 300 mm
Engine speed = 600 rpm
Maximum gas pressure = 4 MPa
Seat angle = 45°
Mean velocity of gas through port = 50 m/s
Allowable bending stress for valve = 50 N/mm^2

Calculate:
(i) diameter of the valve port; (ii) diameter of the valve head; (iii) thickness of the valve head; (iv) diameter of the valve stem; and (v) maximum lift of the valve

[*(i) 90 (86.6) mm (ii) 101 mm (iii) 11(10.69) mm (iv) 20 mm (17.6 to 22.25) (v) 31.82 mm*]

25.15 Design a rocker arm for the exhaust valve of a four-stroke engine using the following data:

Effective length of each arm = 150 mm
Angle between two arms = 150°
Diameter of valve head = 56 mm
Lift of valve = 20 mm
Mass of valve = 0.3 kg
Engine speed = 500 rpm
Back pressure when the exhaust valve opens = 0.35 MPa
Maximum suction pressure = 0.025 MPa (below atmosphere)
(l/d) ratio for fulcrum and roller pins = 1.25
Thickness of phosphor bronze bushings = 3 mm
Permissible bending stress = 70 N/mm^2
Permissible shear stress = 42 N/mm^2
Permissible compressive stress = 50 N/mm^2
Permissible bearing pressure = 5 MPa

The valve opens 39° before the outer dead centre and closes 8° after the inner dead centre. The motion of the valve is SHM without dwell in the fully opened condition. The proportions of cross-section of the rocker arm are shown in Fig. 25.39(b).

Calculate: (i) acceleration of the valve; (ii) total force on the rocker arm; (iii) diameter and length of the fulcrum pin; (iv) bending stress in cross-section of the rocker arm at the fulcrum pin; (v) diameter and length of the roller pin; (vi) bending stress in the roller pin; (vii) dimensions of cross-section of the rocker arm; and (viii) nominal diameter of the tappet.

[*(i) 68.35 m/s^2 (ii) 944.14 N (iii) 20(17.08) and 25 mm (iv) 29.28 N/mm^2 (v) 15(12.29) and 20(18.75) mm (vi) 11.87 N/mm^2 (vii) t = 6(5.22) mm (viii) 6(5.83) mm*]

25.16 Design a valve spring for the exhaust valve of a four-stroke engine using the following data:

Diameter of valve head = 56 mm
Lift of valve = 20 mm
Maximum suction pressure = 0.025 MPa (below atmosphere)
Stiffness of spring = 10 N/mm
Spring index = 8
Permissible torsional shear stress for spring wire = 300 N/mm^2
Modulus of rigidity = 84×10^3 N/mm^2

Total gap between consecutive coils, when the spring is subjected to maximum force, can be taken as 15% of maximum compression.

Calculate: (i) maximum force on the spring; (ii) wire diameter; (iii) mean coil diameter; (iv) number of active turns; (v) total number of turns; (vi) free length of the spring; and (vii) pitch of coils.

[*(i) 261.58 N (ii) 5 (4.59) mm (iii) 40 mm (iv) 11(10.25) (v) 13 (vi) 97.27 mm (vii) 8.11 mm*]

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